

V_{us} and rare K decays at KLOE

Mario Antonelli for the KLOE
collaboration

LNF-INFN

Results and Perspectives in Particle Physics

La Thuile, March 4 2009

V_{us} and CKM unitarity

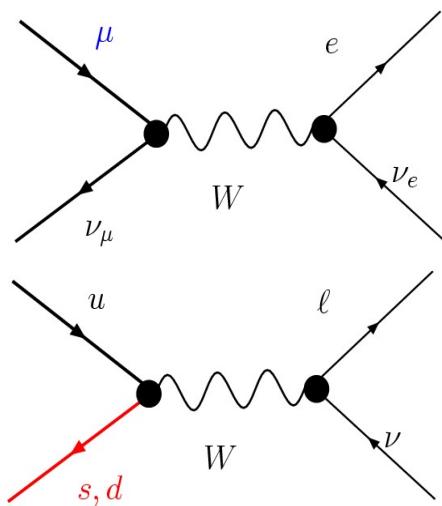
Unitarity test of CKM: G_F universality

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \equiv 1$$



Universality of Weak coupling- $G_F = (g_w/M_w)^2$

$$G_F^2 \equiv G_{CKM}^2 = (|V_{ud}|^2 + |V_{us}|^2) G_F^2$$



$$G_F = 1.166371(6) \times 10^{-5} \text{ GeV}^{-2}$$

V_{us} at 0.5%

$$G_{CKM} = 1.16xx(4) \times 10^{-5} \text{ GeV}^{-2}$$

Sensitivity to new physics :
naively

$$G_{CKM} = G_F [1 + a(M_w/M_M)^2]$$

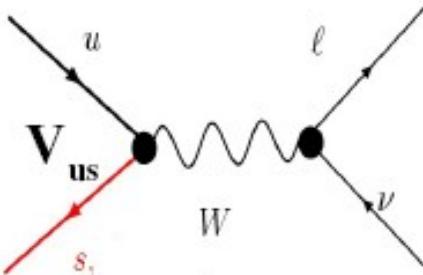
Tree level $a \sim 1$

$M_M \sim 10 \text{ TeV}$

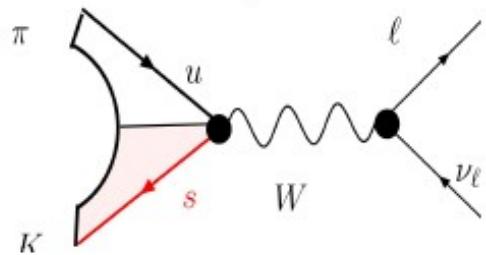
loops $a \sim g_w^2 / (16\pi^2)$

$M_M \sim 1 \text{ TeV}$

Kaon high precision observables



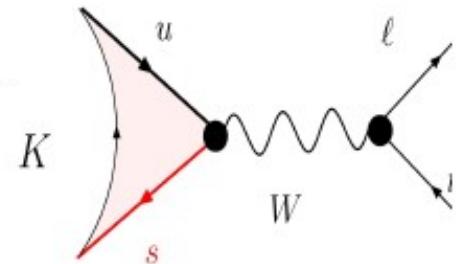
$K_{\ell 3}$: $K \rightarrow \pi \ell \nu$



Experimental processes

Short distance physics

$K_{\ell 2}$: $K \rightarrow \ell \nu$



Vector transition protected against $SU(3)$ corrections: [Ademollo Gatto]

$$\Gamma(K_{\ell 3(\gamma)}) \propto G_F^2 |V_{us}|^2 |f_+^{K^0 \pi^-}(0)|^2$$

Small uncertainties in f_K/f_π from lattice [Mariciano]

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K^2}{f_\pi^2}$$

the KLOE's role

PLB 632 (2006)

$$\text{BR}^{(0)}(K_L \rightarrow \pi e v) = 0.4049(21)$$

PLB 626 (2005)

$$\tau_L = 50.92(30) \text{ ns}$$

PLB 636 (2006)

$$\lambda'_+ \times 10^3$$

$$25.6 \pm 1.8$$

$$\lambda''_+ \times 10^3$$

$$1.5 \pm 0.8$$

$$\lambda_0 \times 10^3$$

$$15.4 \pm 2.1$$

$f_+(t)$

JHEP0712:105

PLB 636 (2006)

$$\text{BR}(K_s \rightarrow \pi e v) = 7.046(91) \times 10^{-4}$$

K_s

PLB 636 (2006)

$$\text{BR}(K^+ \rightarrow \mu^+ \nu(\gamma)) = 0.6366(17)$$

K^\pm

JHEP0802:098

$$\text{BR}(K^\pm \rightarrow \pi e v) = 0.04965(52)$$

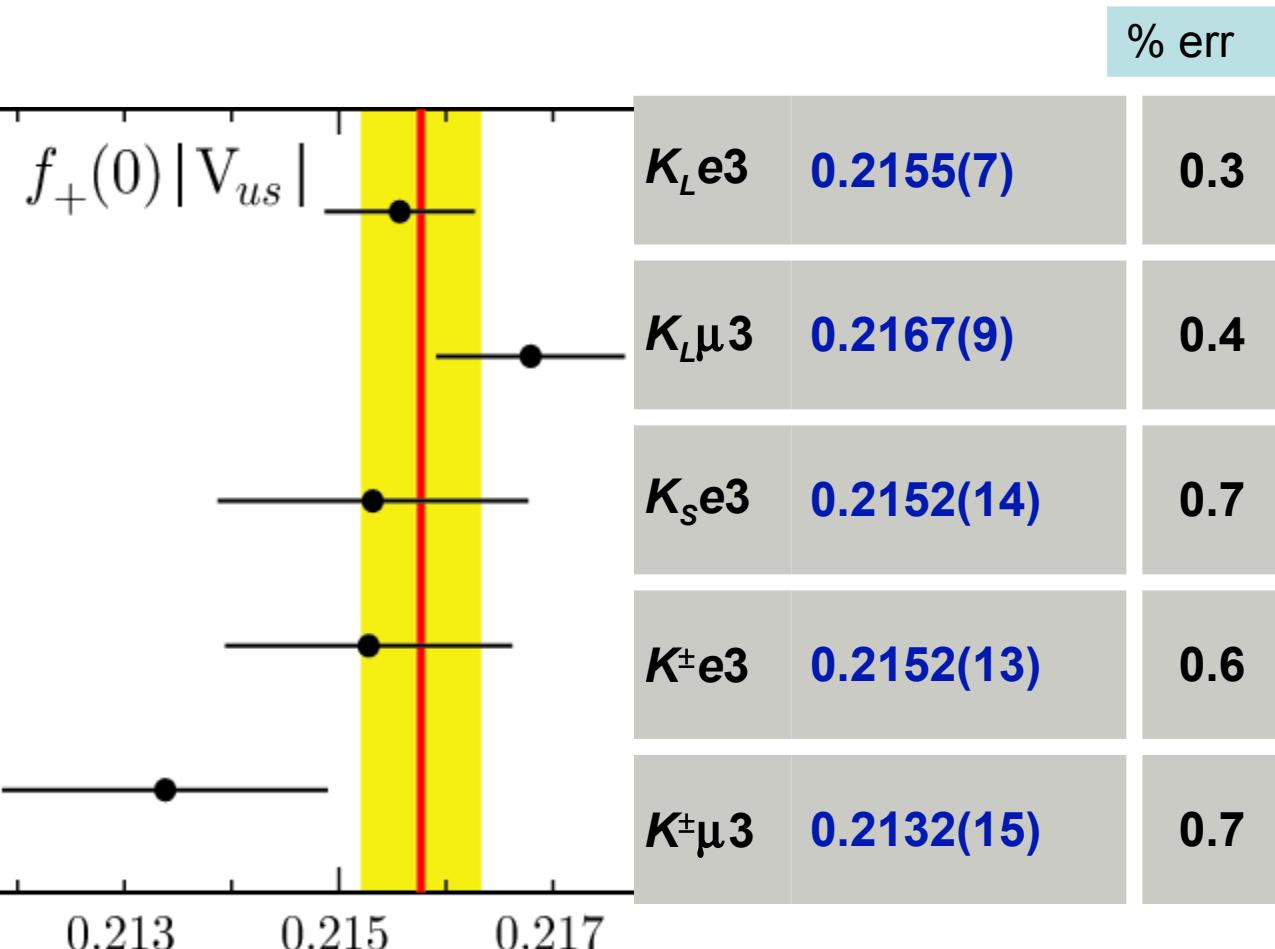
JHEP0801:073

$$\text{BR}(K^\pm \rightarrow \pi \mu v) = 0.03233(39)$$

$$\tau_\pm = 12.347(30) \text{ ns}$$

V_{us} from KLOE K_{l3} data

JHEP
0804:059



$$|V_{us}| f_+(0)$$

KLOE Avg:
 $0.2157(6)$
 $\chi^2/\text{ndf} = 7/4$ (13%)

World Avg:
 $0.2166(5)$

$$f_+(0) = 0.964(5)$$

RBC/UKQCD

$$V_{ud} = 0.97418(26)$$

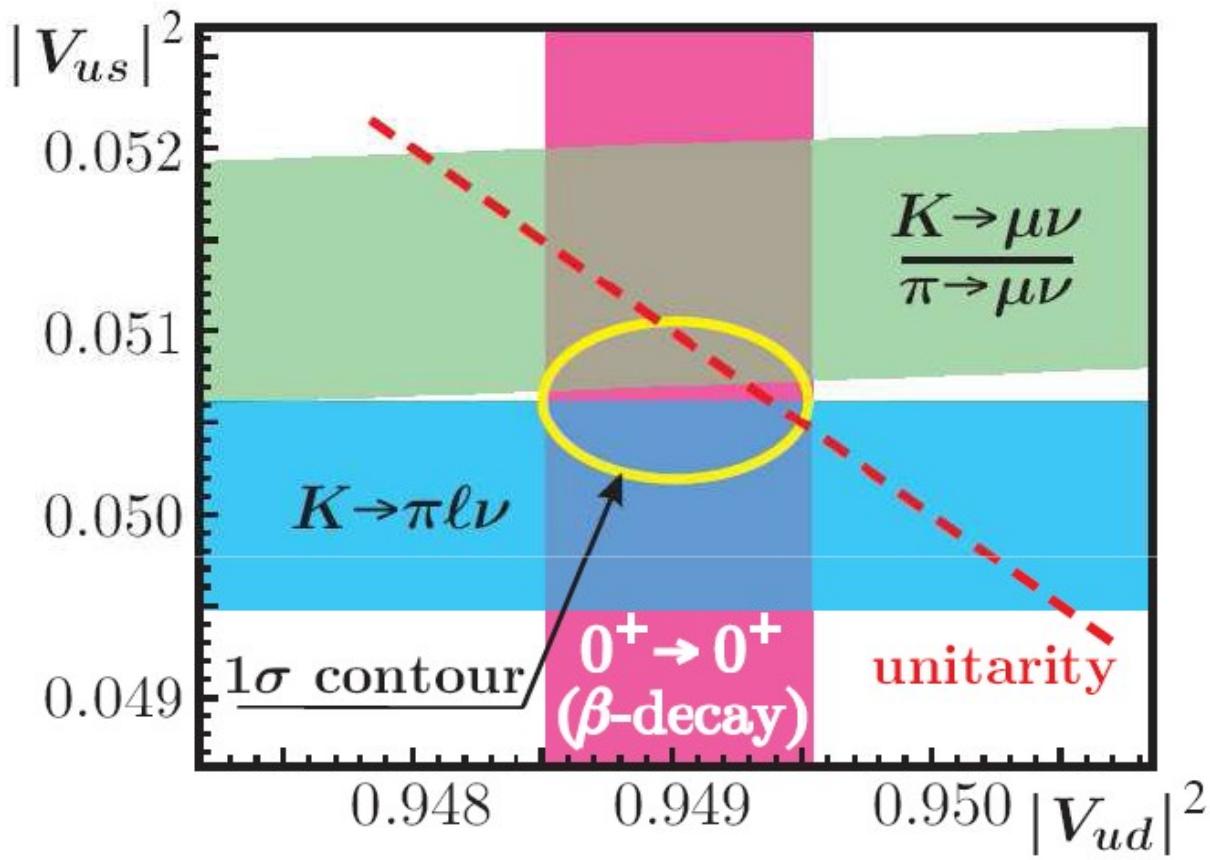
arXiv:0710.3181

$$\Rightarrow V_{us} = 0.2237(13)$$

$$\Rightarrow 1 - V_{ud}^2 - V_{us}^2 = 9(8) \times 10^{-4}$$

V_{ud} , V_{us} and V_{us}/V_{ud}

JHEP
0804:059



no constraint:

$$V_{ud}^2 = 0.9490(5)$$

$$V_{us}^2 = 0.0506(4)$$

$$\chi^2/\text{ndf} = 2.3/1 \text{ (13\%)}$$

agreement with
unitarity:

$$1 - V_{ud}^2 - V_{us}^2 = 4(7) \times 10^{-4}$$

@ 0.6σ

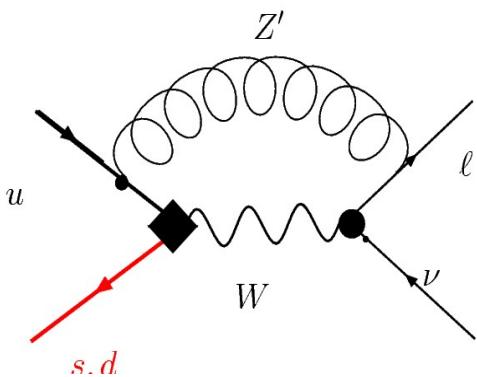
$|V_{ud}| = 0.97418(26)$ [Towner & Hardy arXiv:0710.3181]

$f_+(0) = 0.964(5)$ UKQCD/RBC NF=2+1, DWF

$f_K/f_\pi = 1.189(7)$ HPQCD-UKQCD(MILC) NF=2+1, Stag

sensitivity to NP: Z'ooogy

1)



$$G_F = G_{CKM} [1 - 0.007 Q_{eL} (Q_{\mu L} - Q_{dL}) \frac{2 \ln(m_{Z'}/m_W)}{(m_{Z'}^2/m_W^2 - 1)}]$$

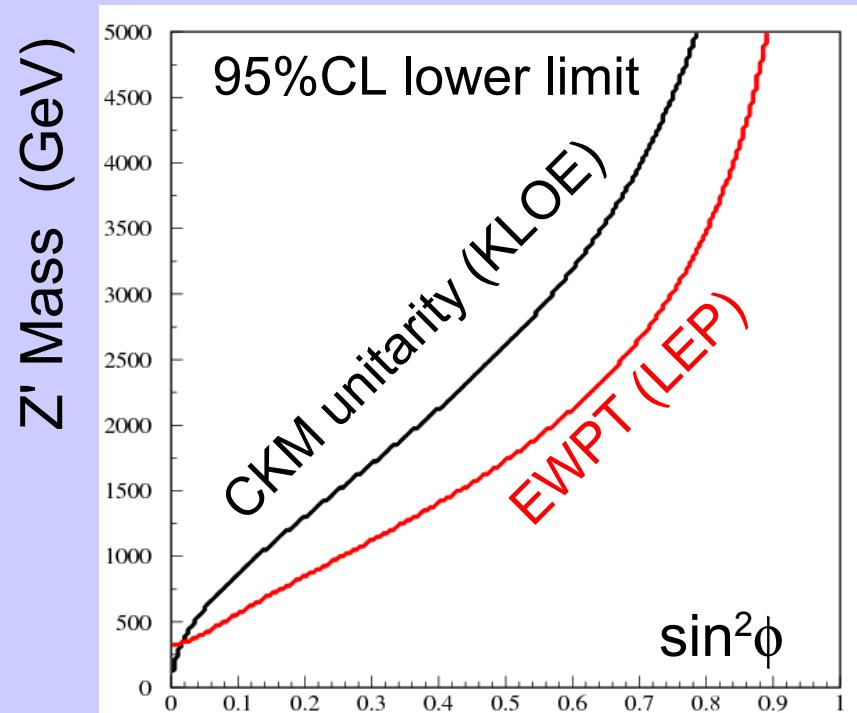
SO(10) Z_χ Boson: $Q_{eL} = Q_{\mu L} = -3Q_{dL} = 1$ [Marciano]

$m_{Z\chi} > 750 \text{ GeV } 95\% \text{ CL}$

2)

[K.Y. Lee]

Tree level breaking of unitarity in models with non-universal gauge interaction



sensitivity to NP: charged Higgs

Pseudoscalar currents, e.g. due to H^\pm , affect the K width:

JHEP
0804:059

$$\frac{\Gamma(M \rightarrow \ell\nu)}{\Gamma_{SM}(M \rightarrow \ell\nu)} = \left[1 - \tan^2 \beta \left(\frac{m_{s,d}}{m_u + m_{s,d}} \right) \frac{m_M^2}{m_H^2} \right]^2 \quad \text{for } M = K, \pi$$

Hou, Isidori-Paradisi

The observable

$$R_{\ell 23} = \left| \frac{V_{us}(K_{\mu 2})}{V_{us}(K_{\ell 3})} \times \frac{V_{ud}(0^+ \rightarrow 0^+)}{V_{ud}(\pi_{\mu 2})} \right|$$

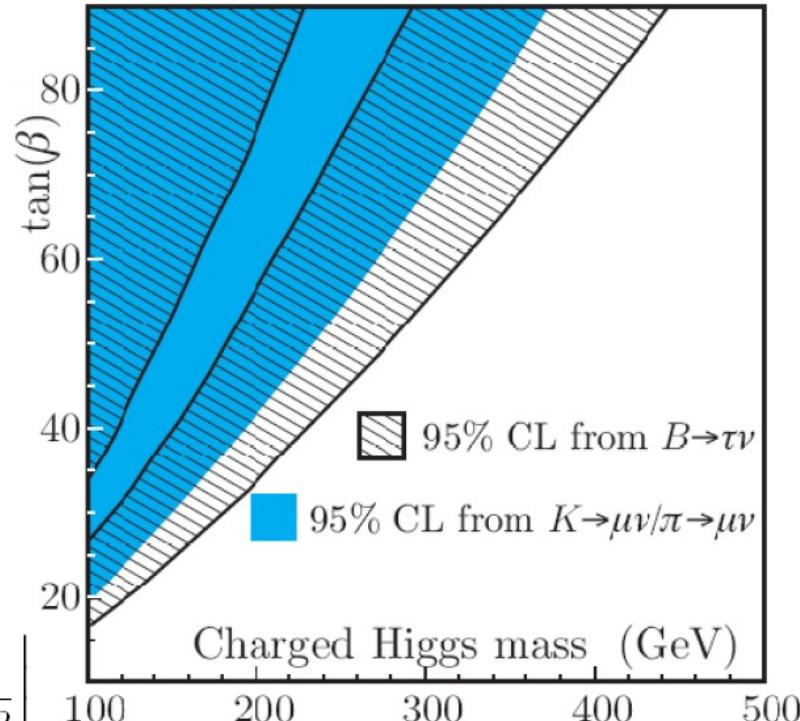
we get:

- $R_{\ell 23} = 1.008(8)$

(unitarity for K_{l3} and β -decays is used)

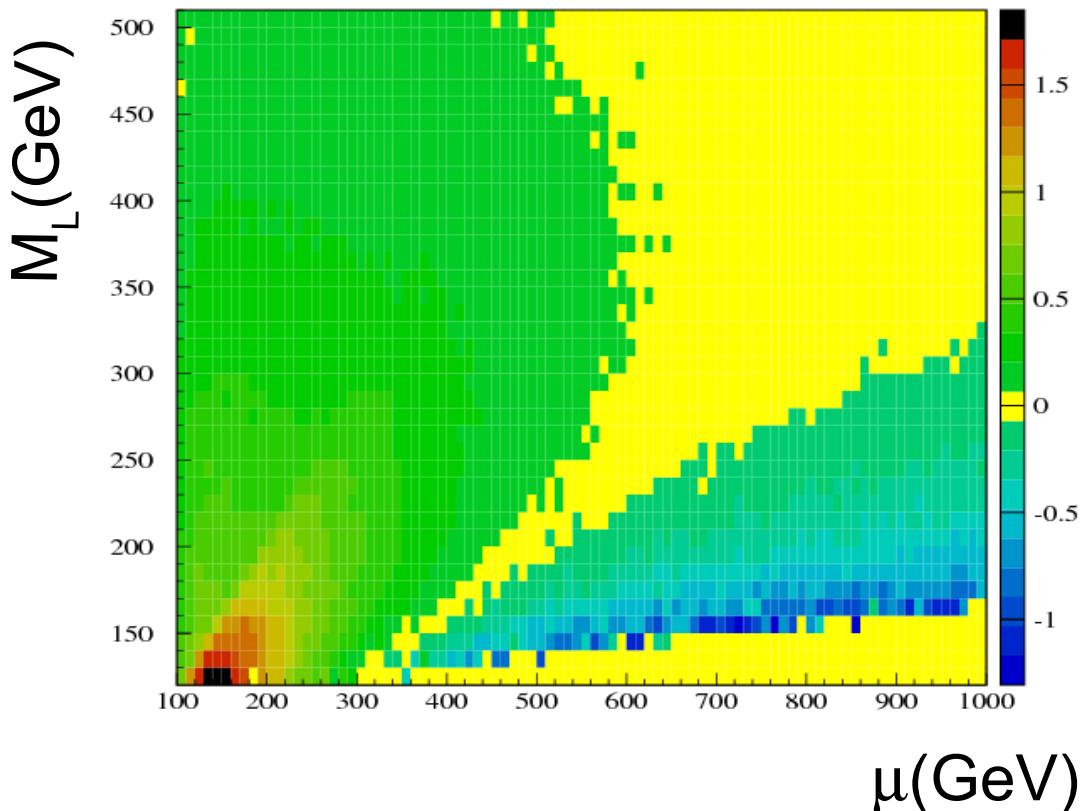
$R_{\ell 23}$ sensitivity to H^\pm exchange

$$R_{\ell 23} = \left| 1 - \frac{m_{K^+}^2}{m_{H^+}^2} \left(1 - \frac{m_{\pi^+}^2}{m_{K^+}^2} \right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right|$$

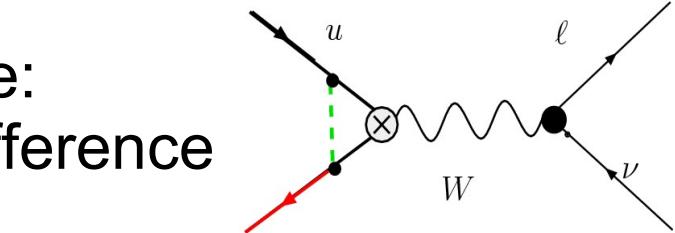


sensitivity to NP: MSSM

Scan over MSSM parameters space:
sensitive to squark slepton mass difference



direct and indirect limits included



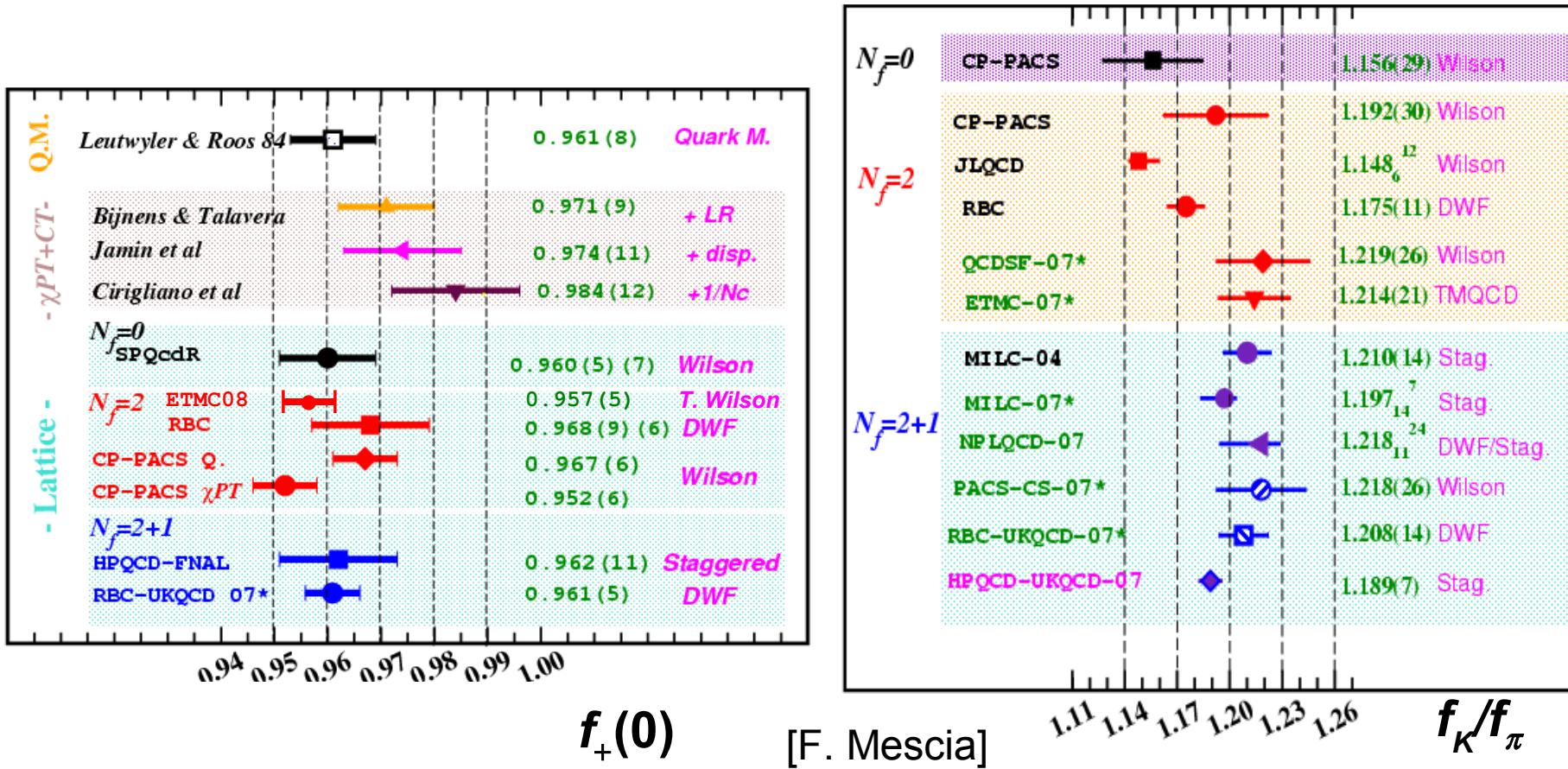
$$\frac{1 - (\sqrt{V_{ud}^2 + V_{us}^2})^{1/2}}{10^4}$$

Need a factor 2 improvement to enter the game

[R. Barbieri '85,
K.Hagiwara et al
'95, A. Kurylov
et al '00]

[code by
Mescia,
Paradisi]

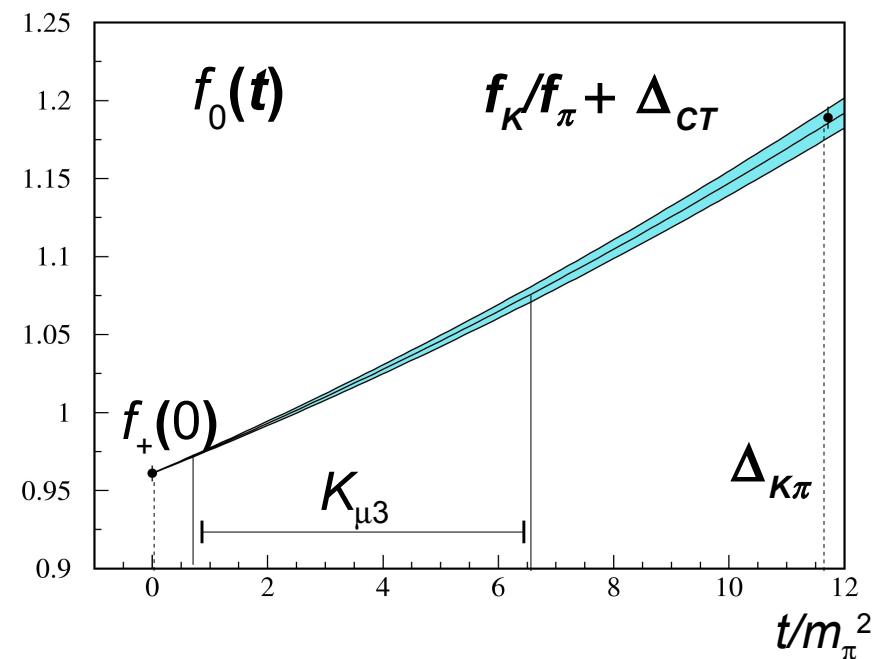
Evaluations of $f_+(0)$ and f_K/f_π



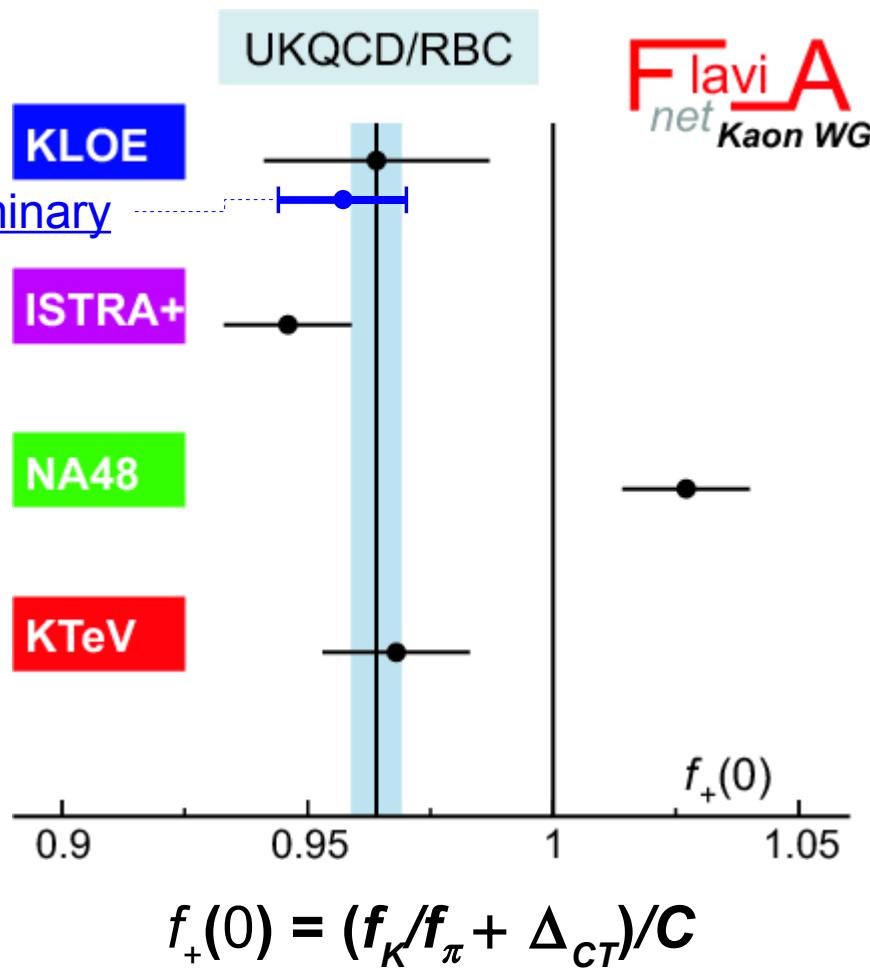
Lattice results continuously improving

Callan-Treiman relation

Check from measurement of scalar
ff slopes in $K\mu 3$ and use of
dispersive parametrization
[Stern et al] [Pich et al] (further info [preliminary](#)
from τ)



$$f_K/f_\pi = 1.189(7) \quad \text{from HPQCD-UKQCD}$$



$$R_K = \Gamma(K_{e2})/\Gamma(K_{\mu 2})$$

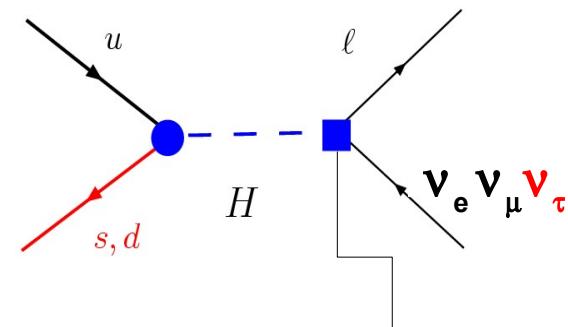
The special role of $\Gamma(K_{e2})/\Gamma(K_{\mu 2})$

SM: very well known no hadronic uncertainties (no f_K)

In MSSM, LFV can give up to % deviations: [Masiero, Paradisi,

NP dominated by contribution of $e\nu_\tau$ final state:

$$R_K \approx \frac{\Gamma(K \rightarrow e\nu_e) + \Gamma(K \rightarrow e\nu_\tau)}{\Gamma(K \rightarrow \mu\nu_\mu)}$$

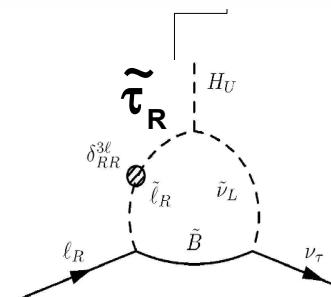


$$R_K \approx R_K^{\text{SM}} \left[1 + \frac{m_K^4}{m_H^4} \frac{m_\tau^2}{m_e^2} |\Delta_R^{31}|^2 \tan^6 \beta \right]$$

$$eH^\pm \nu_\tau \rightarrow \frac{g_2}{\sqrt{2}} \frac{m_\tau}{M_W} \Delta_R^{31} \tan^2 \beta$$

1% effect ($\Delta_R^{31} \sim 5 \times 10^{-4}$, $\tan \beta \sim 40$, $m_H \sim 500 \text{ GeV}$)
not unnatural

Present accuracy on R_K @ 6% Need for precise measurements



$K_{e2}/K_{\mu 2}$: SM prediction

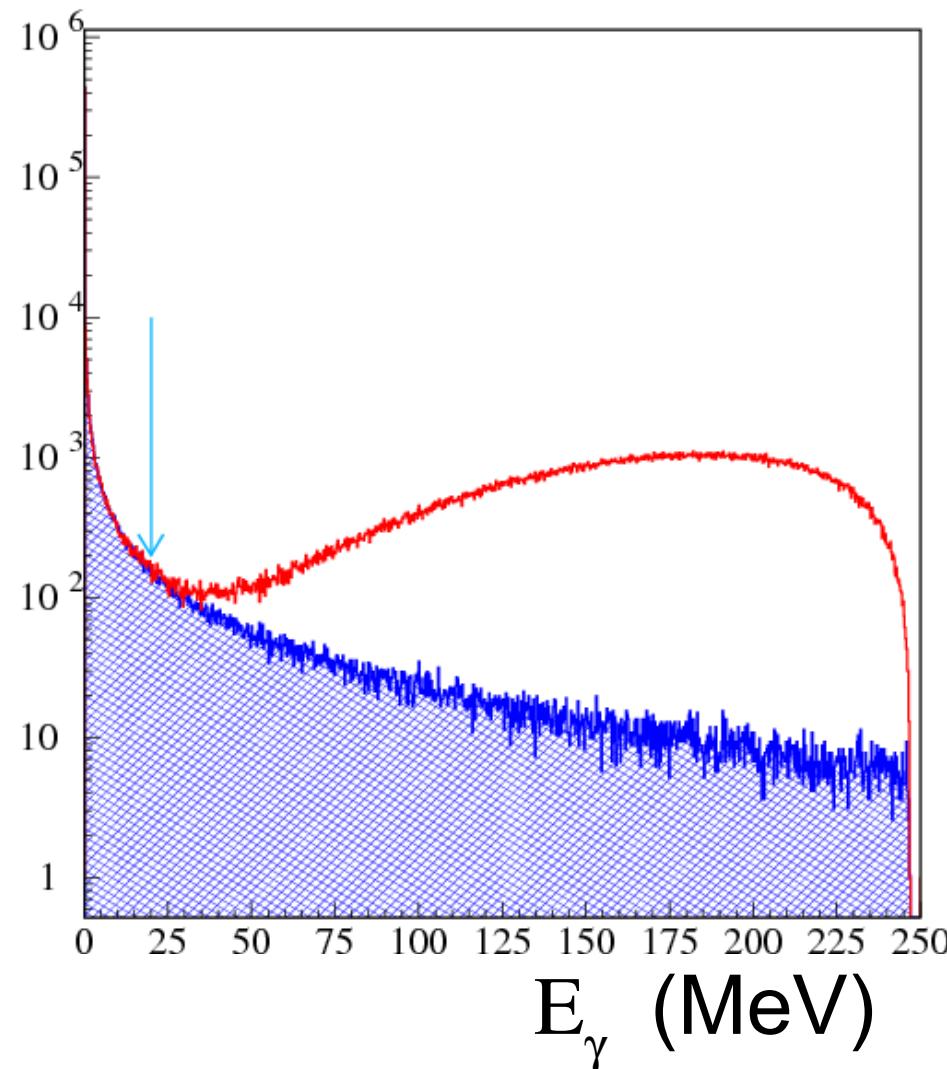
SM prediction made in terms of IB process only (unobservable)

$$R_K = 2.477(1) \times 10^{-5} \quad [\text{Cirigliano, Rosell}]$$

Radiative corrections: **IB** + **DE** amplitudes in MC generator

Signal: $K \rightarrow e\nu(\gamma)$, $E_\gamma < 20$ MeV

DE is negligible in this range



Key issues for a ~1% measurement

Perform **direct search** for K_{e2} without tag: **gain \times 4 of statistics**

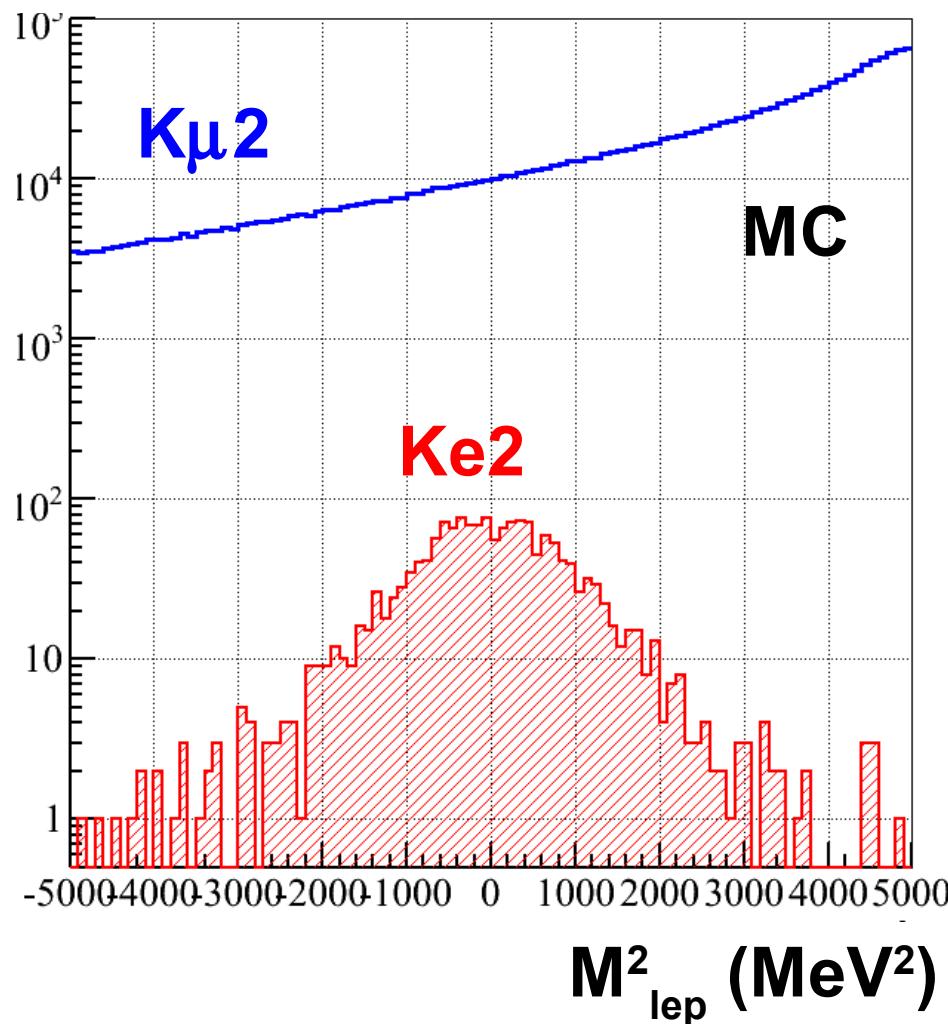
~ 25K of reconstructed $Ke2$ expected in the entire KLOE data set (2.2 fb^{-1})

K_{l2} kinematics:

$m_\nu \sim 0$ get M^2_{lep}

Rule of the game:

Reject background by 4 order of magnitudes with
~ 50% signal efficiency



Background rejection

Use ϕ decay kinematics for a redundant determination of K momentum

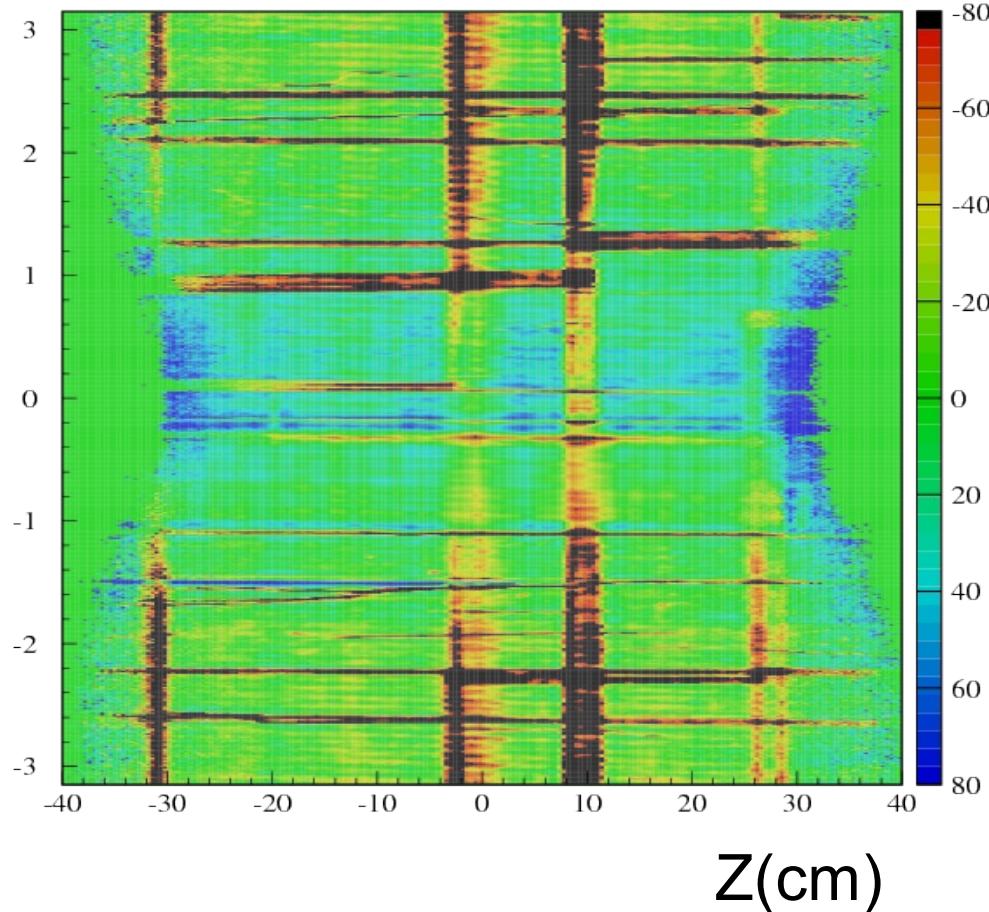
but... $\beta_K \sim 0.2$ → precise knowledge of material budget mandatory

no redundant measurement for secondary → select well reconstructed tracks

$\chi^2 +$ expected error on M_{lep}^2

Drift Chamber inner wall radiography

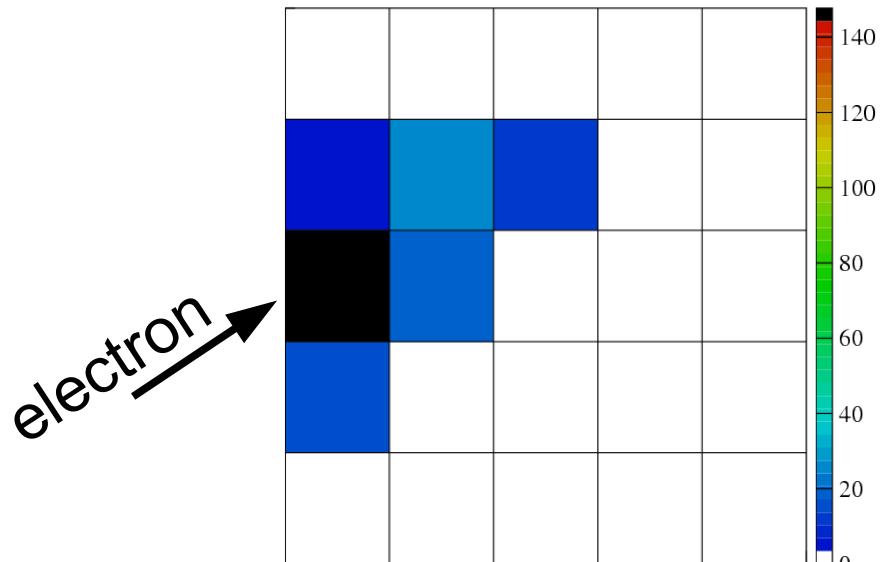
$\Delta (\mu\text{m})$



Particle Identification

E (MeV)

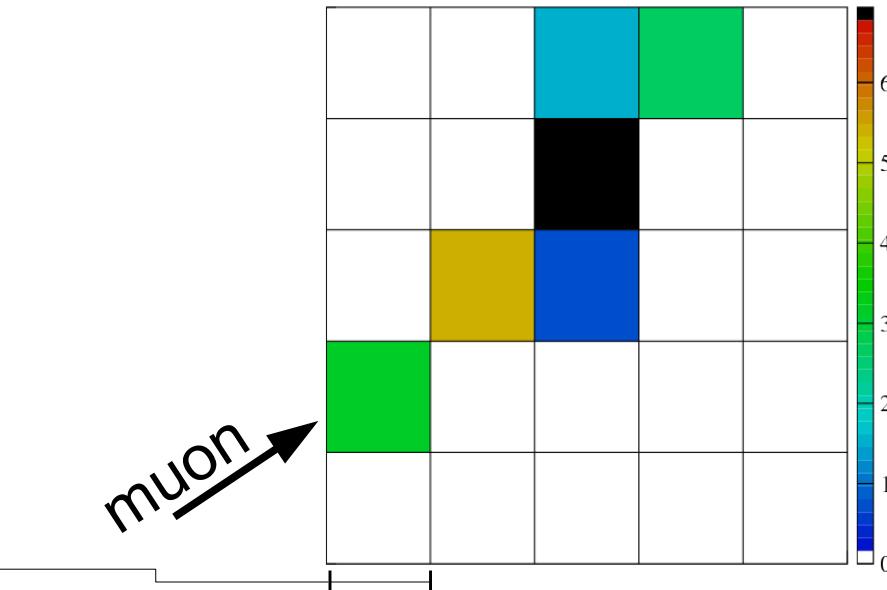
particle ID exploits EmC
granularity: energy deposits
into 5 layers in depth



Combine infos with a neural
network

use pure sample of $K_L e3$ to
correct cell response in MC
and for NN training

4.4 cm

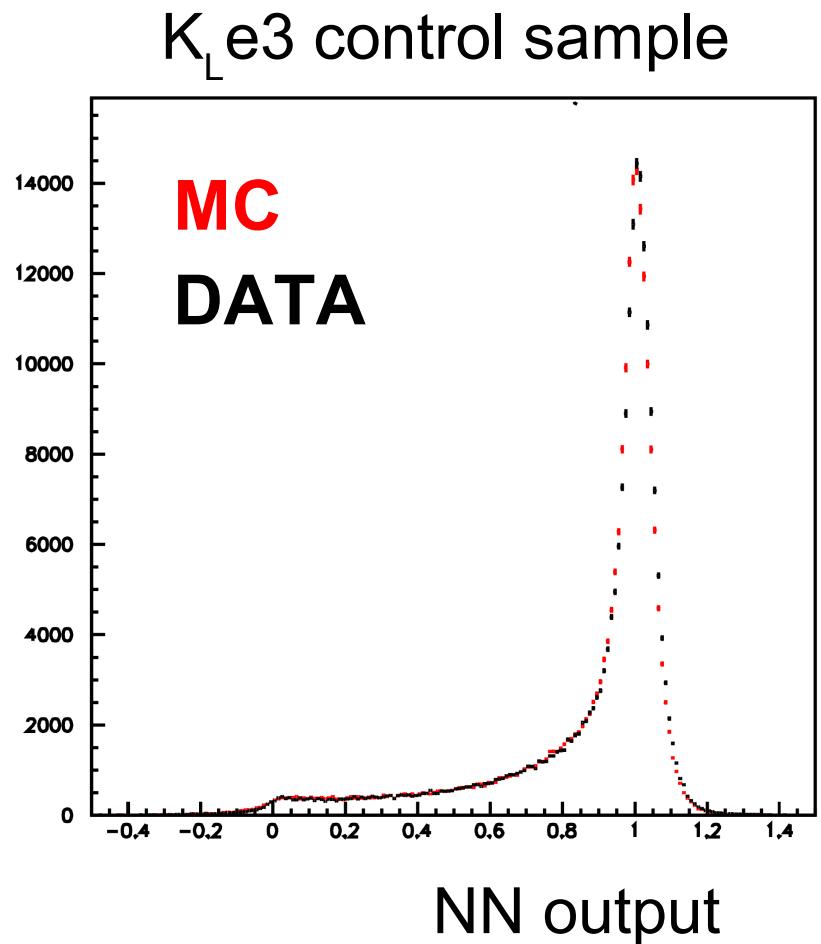


Particle Identification

particle ID exploits EmC
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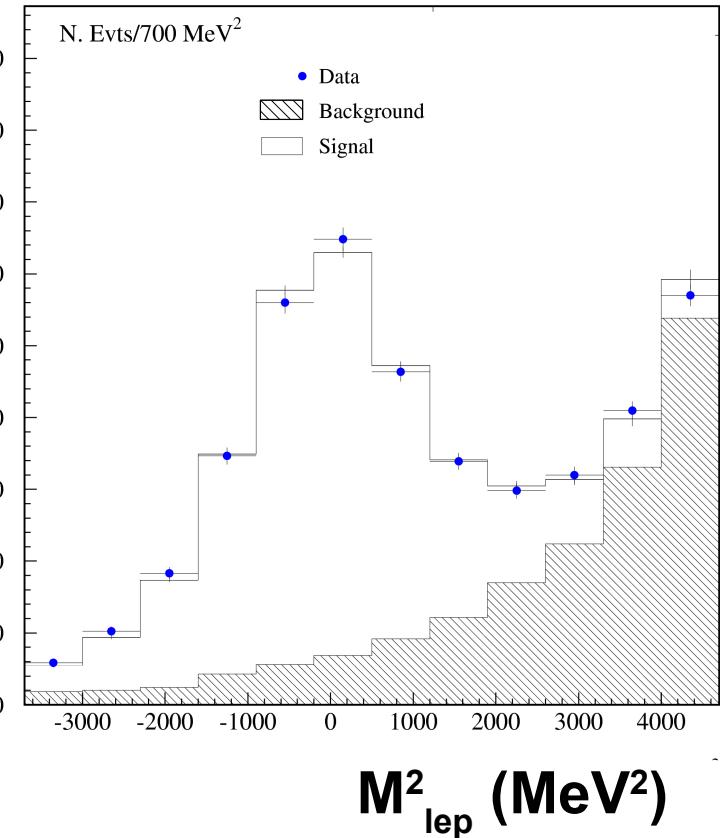
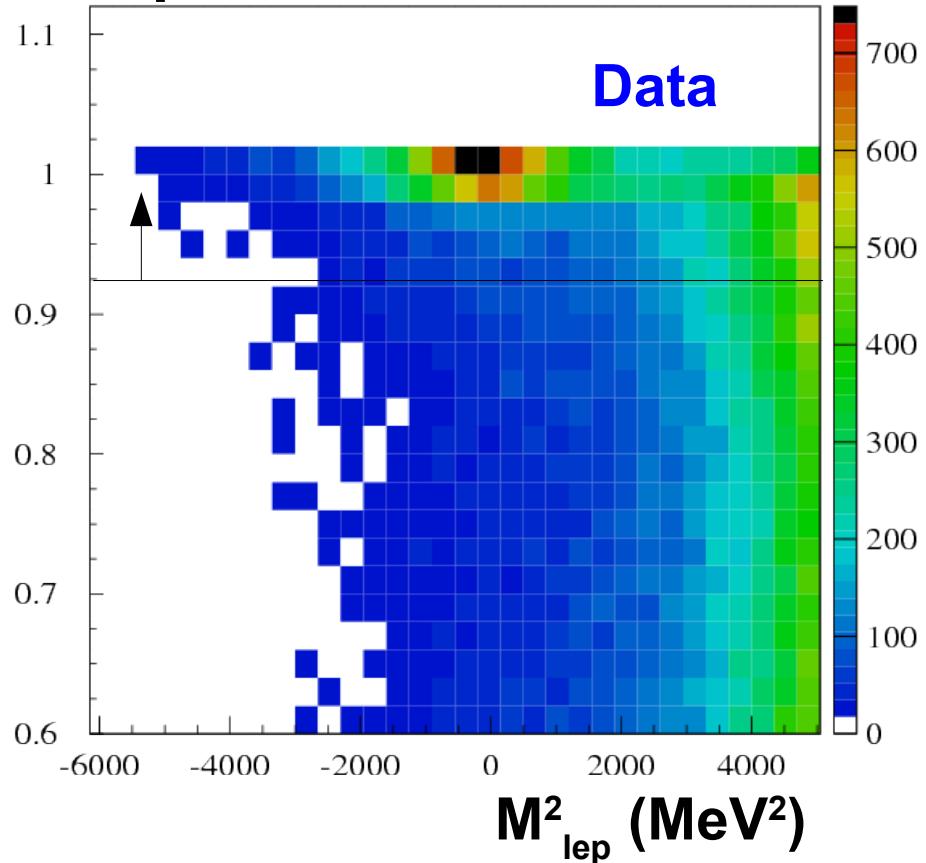
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Counting K_{e2} events

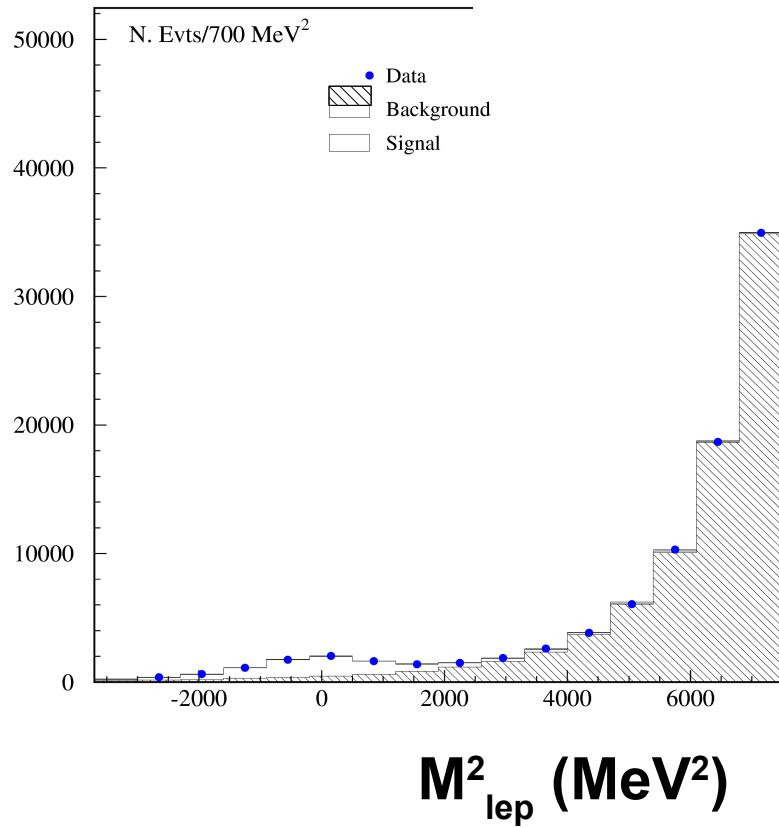
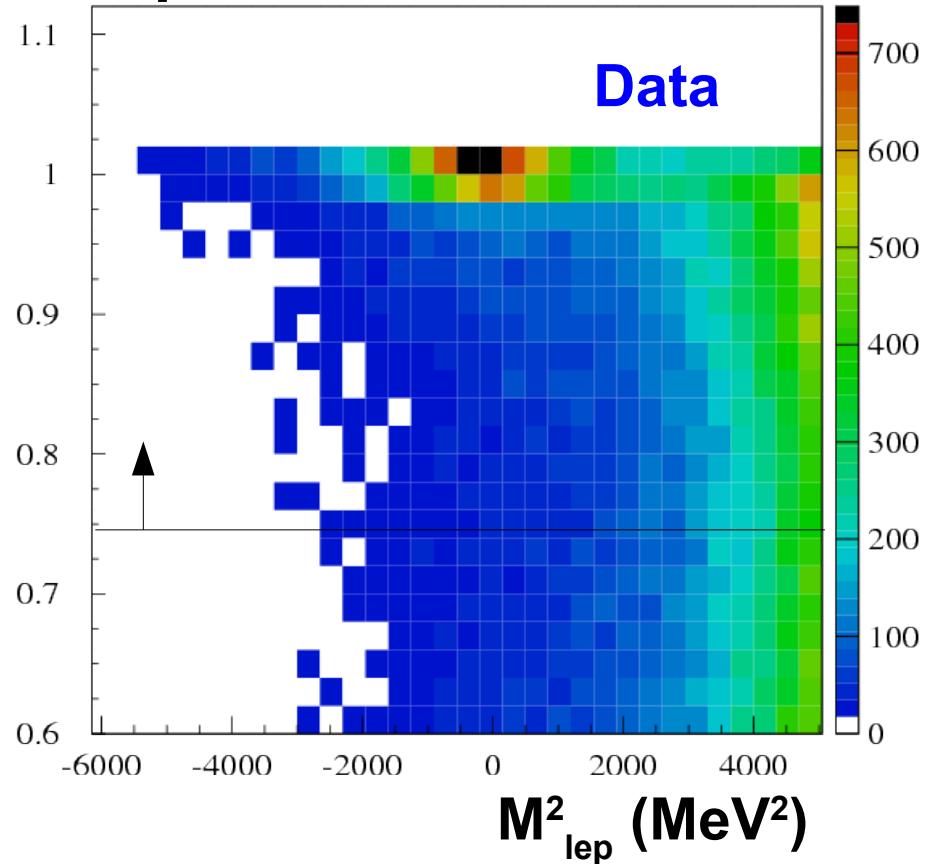
NN output



Two-dimensional binned likelihood fit in the plane
NN output - M^2_{lep} **count 7060 + 6750 Ke2 events**

Counting K_{e2} events

NN output



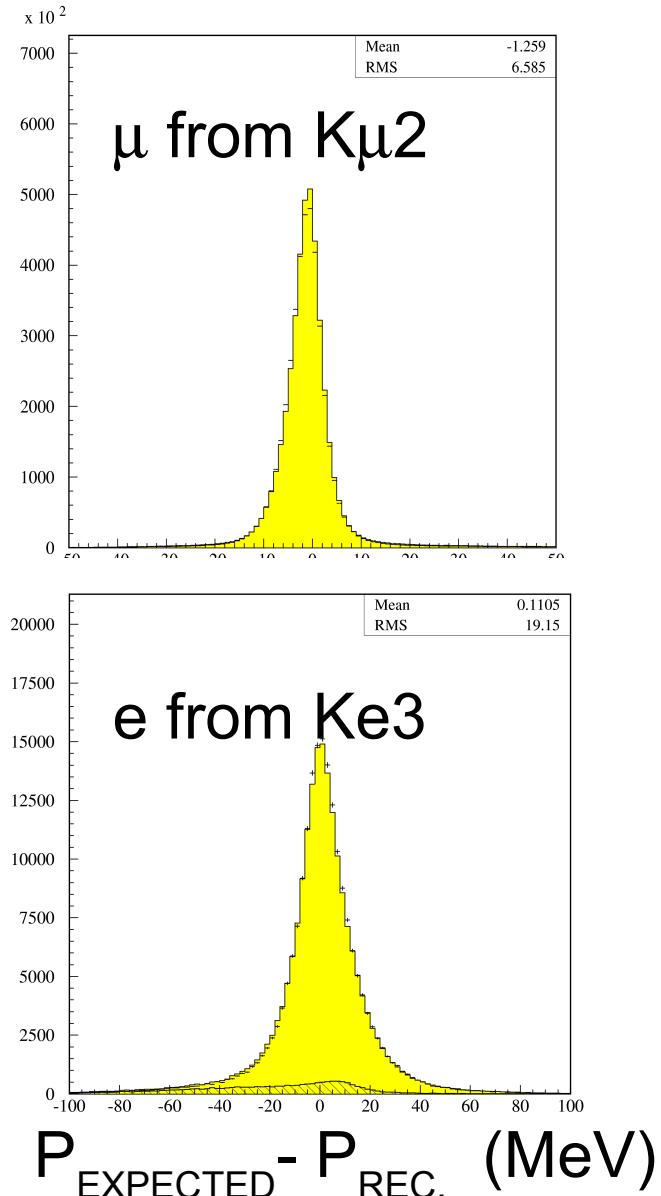
Two-dimensional binned likelihood fit in the plane
NN output - M^2_{lep} count 7060 + 6750 Ke2 events

Efficiencies

Reconstruction efficiencies
from MC with corrections from
Ke3 and K μ 2 decays control
samples

CS selected in tagged events
with additional criteria based
only on EMC infos

$$C = 0.921 \pm 0.004 \pm 0.004$$



Systematics and checks

Reconstruction	0.006	Control Sample
Trigger	0.004	downscaled evts
Fit	0.003	range variation
DE	0.001	measure on data
CLUSTERING	0.003	K _L Control samples

Check: Use same algorithms to measure
 $B(Ke3)/B(K\mu 3)$

$$1.507 \pm 0.005 \text{ for } K^+$$

$$1.510 \pm 0.006 \text{ for } K^-$$

$$1.506 \pm 0.003 \text{ SM (FlaviaNet inputs)}$$

Result

$$R_K = (2.493 \pm 0.025 \pm 0.019) \times 10^{-5}$$

SM prediction:

$$R_K = (2.477 \pm 0.001) \times 10^{-5}$$

The systematic uncertainty has significant contribution from statistics of CS (0.015×10^{-5})

results for K^+ and K^- (uncorrelated errors only)

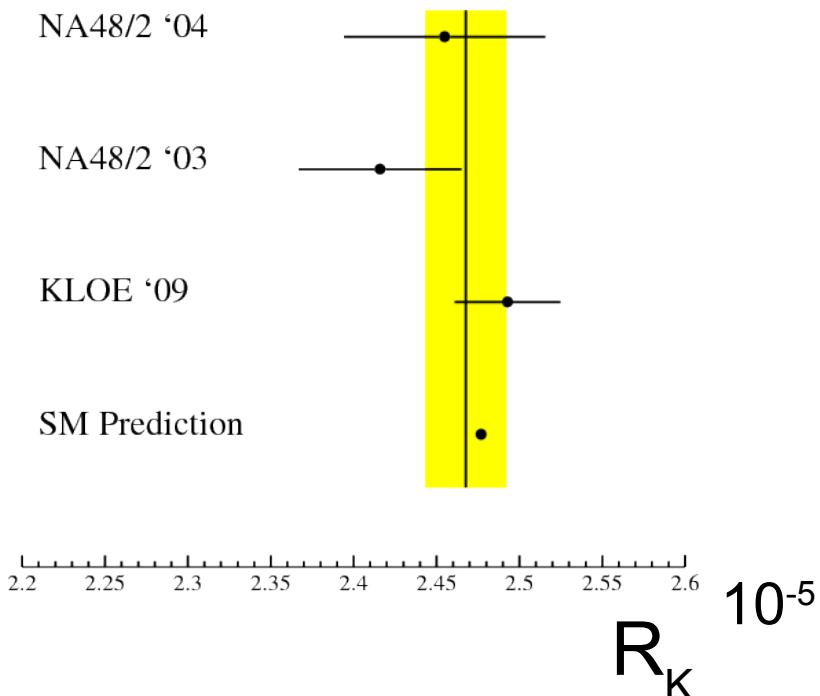
$$K^+: R_K = (2.496 \pm 0.037) \times 10^{-5}$$

$$K^-: R_K = (2.490 \pm 0.038) \times 10^{-5}$$

R_K world average

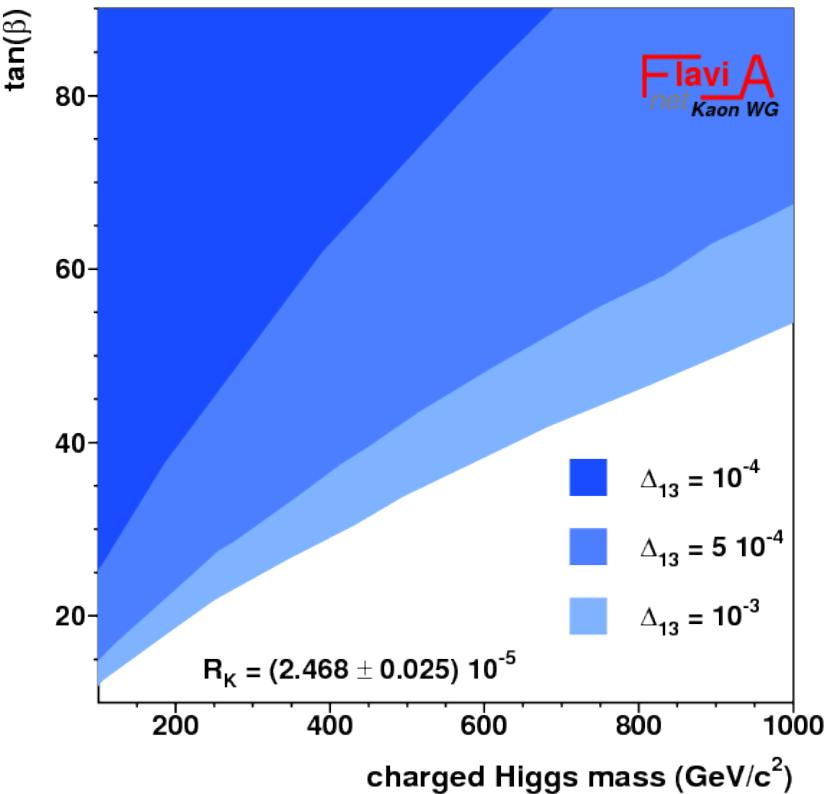
Uncertainty @ 1%

$$R_K = (2.468 \pm 0.025) \times 10^{-5}$$



95%-CL excluded regions in the
 $\tan\beta - M_H$ plane, for

$$\Delta_{13} = 10^{-3}, 0.5 \times 10^{-3}, 10^{-4}$$



CONCLUSIONS

$$V_{us} = 0.2237(13)$$

agreement with
unitarity:

$$V_{us}/V_{ud} = 0.2323(15)$$

$$1 - V_{ud}^2 - V_{us}^2 = 4(7) \times 10^{-4}$$

@ 0.6σ (0.07 %)

Important constraints for physics BSM

$$R_K = (2.493 \pm 0.025 \pm 0.019) \times 10^{-5}$$

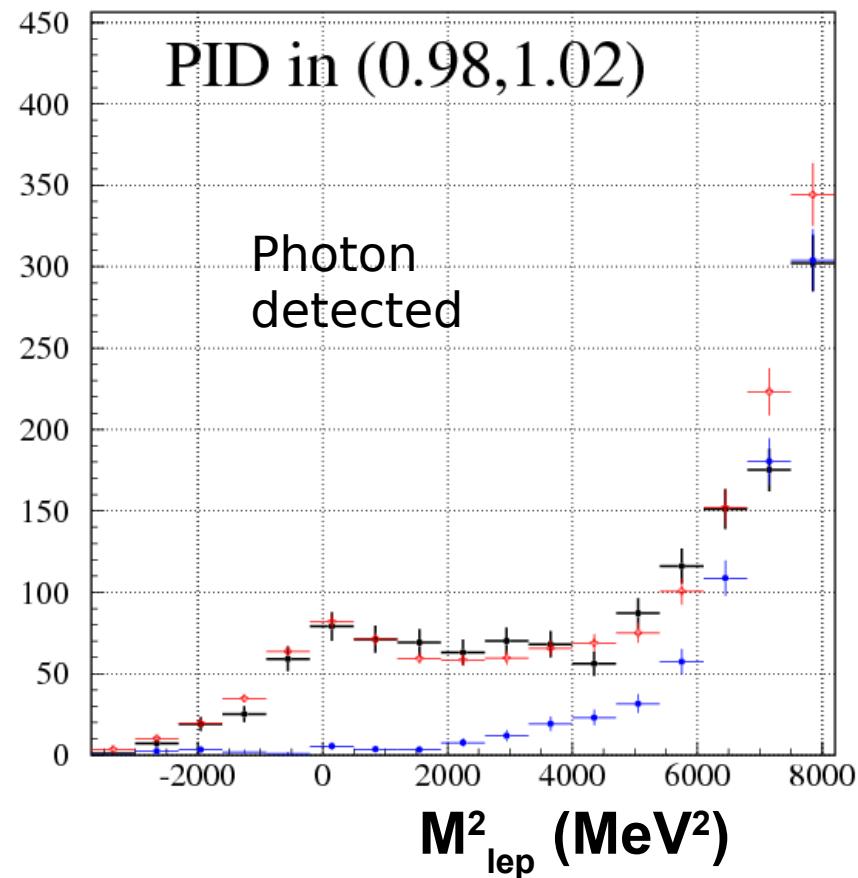
Check Ke₂ γ (DE)

Require γ cluster ($E > 20$ meV), enhance DE fraction:
from 0.1 (no γ) to ~ 0.6 (with γ)

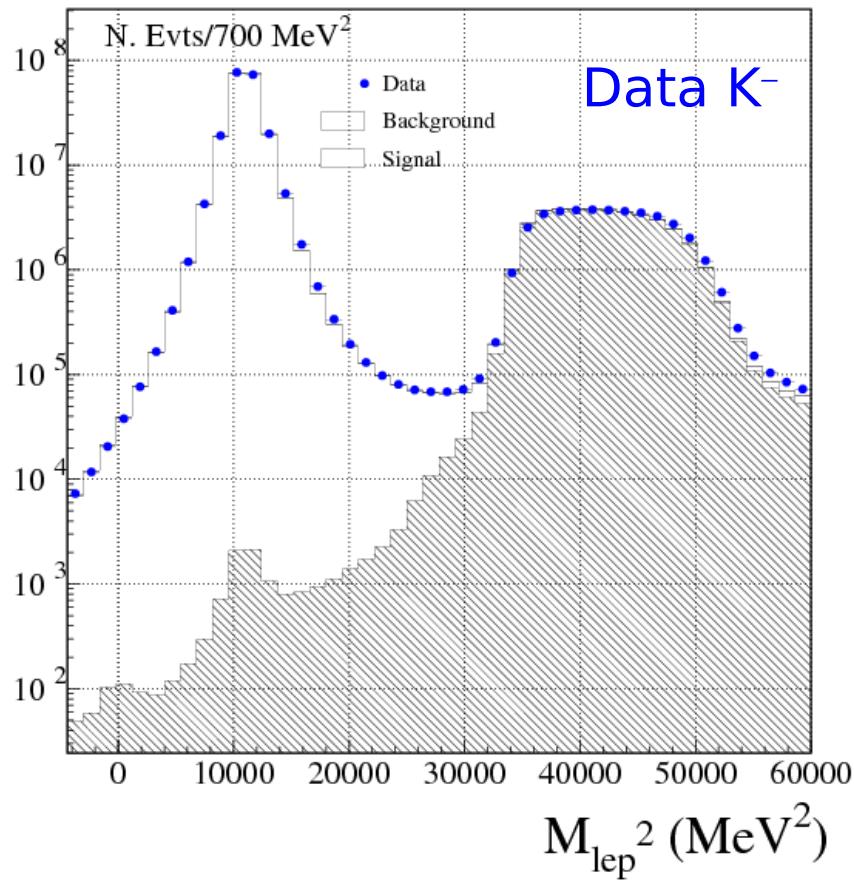
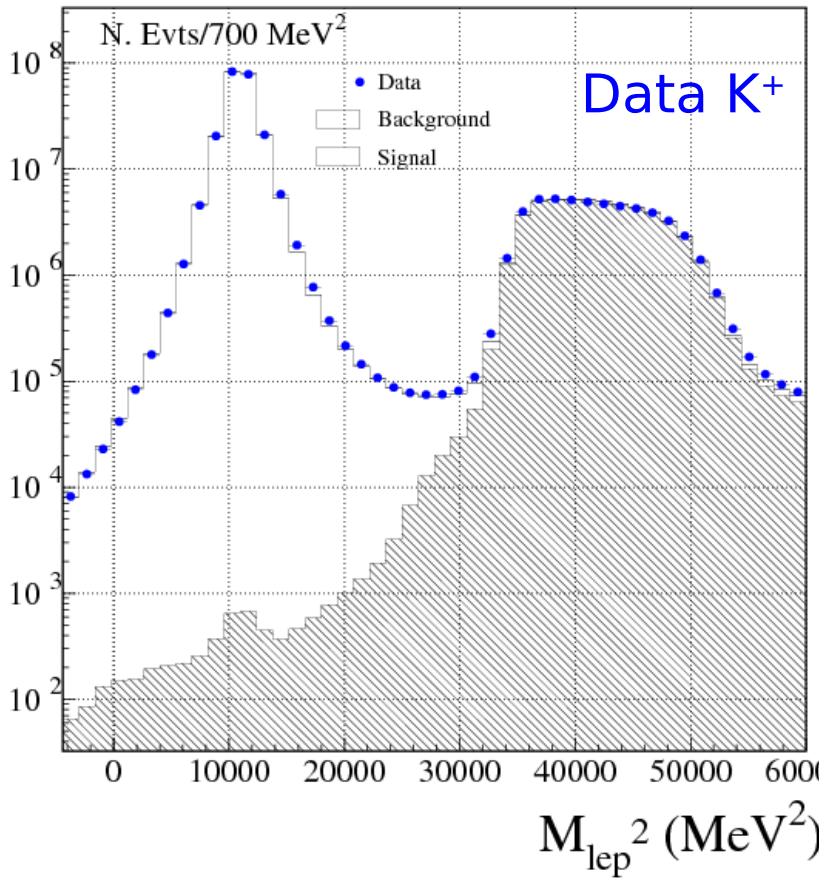
Check N(Ke₂) variation with respect to standard analysis

Expectation from theory
well matched,
difference is zero within 3%_{stat}

Vary DE contribution in the
standard fit by 3%
result variation 0.1%



counting $K\mu 2$



RESULTS FROM $0^+ \rightarrow 0^+$ DECAY IN 2008

1) G_V constant

$$\mathcal{F}t = \frac{K}{2G_V^2 (1 + \Delta_R)}$$

✓ verified to $\pm 0.013\%$

2) Scalar current zero

✓ limit, $C_s/C_V = 0.0011$ (14)

3) Precise value determined for V_{ud}

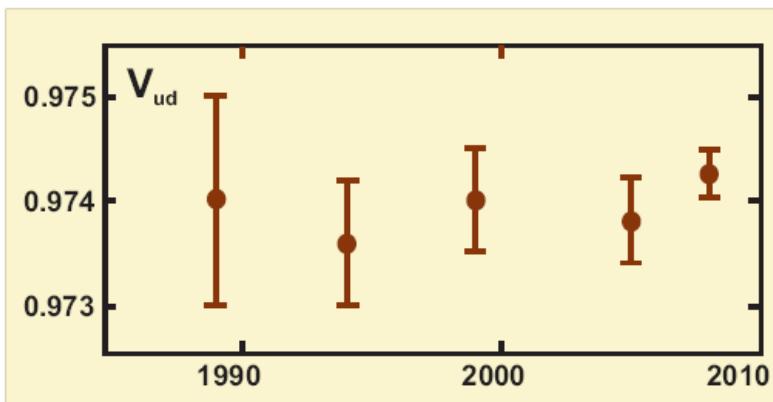
$$V_{ud} = G_V/G_\mu$$

$$V_{ud} = 0.97425 \pm 0.00023$$

Compare:

neutron $V_{ud} = 0.9746 \pm 0.0019$

pion $V_{ud} = 0.9749 \pm 0.0026$

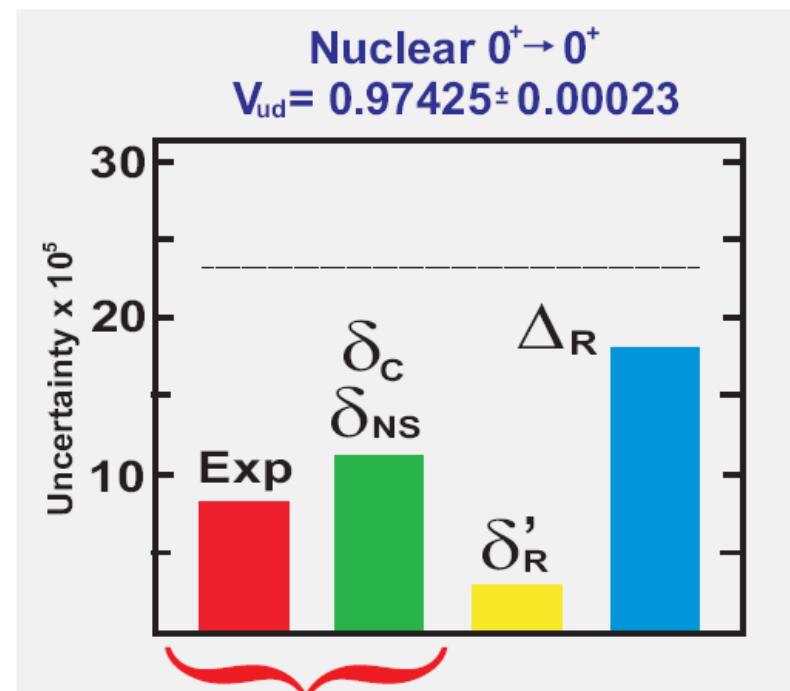


I. S. Towner
@ CKM08

OPPORTUNITIES FOR IMPROVEMENT

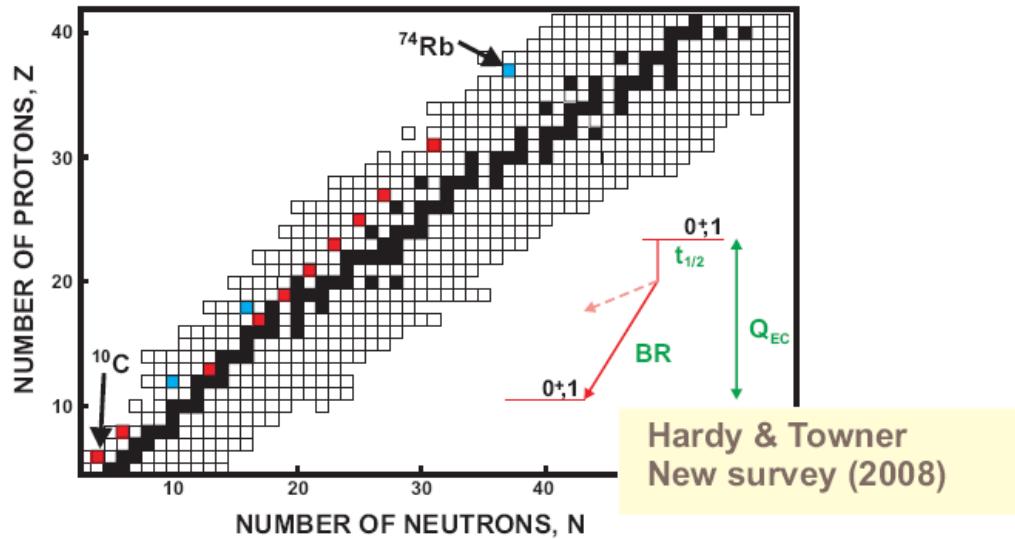
- Goal remains to tighten the window for new physics by reducing the uncertainty on V_{ud} .
- Uncertainty on calculated radiative correction Δ_R is the dominant contribution to the error budget.
- Nuclear-structure-dependent corrections, δ_c and δ_{ns} , can be tested by experiment; this has already led to improvements, but more are still possible.

Data on “well known” transitions can be made more precise, and new cases can be measured.



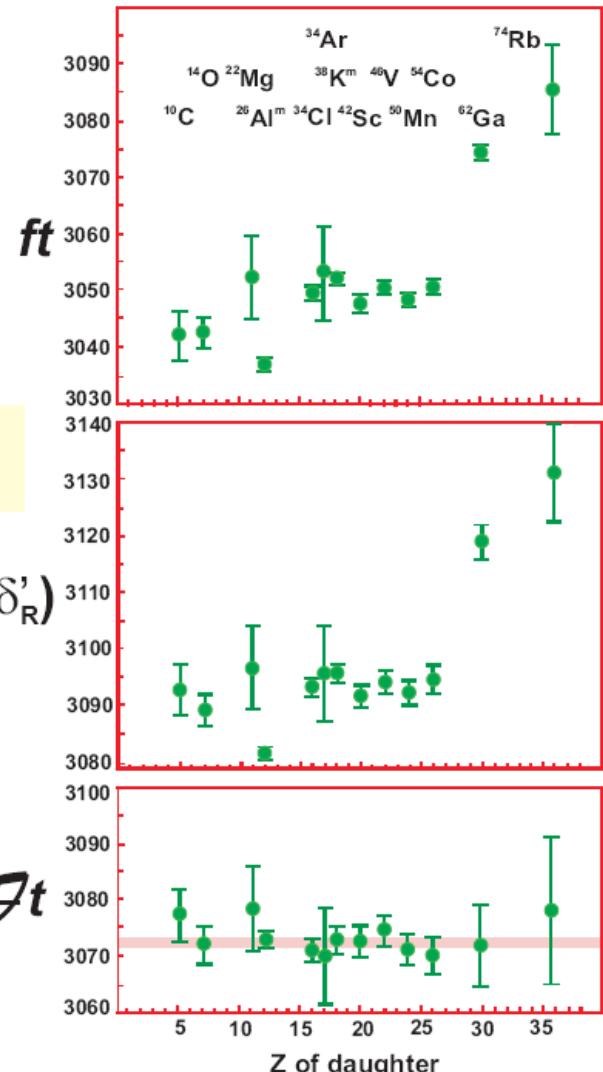
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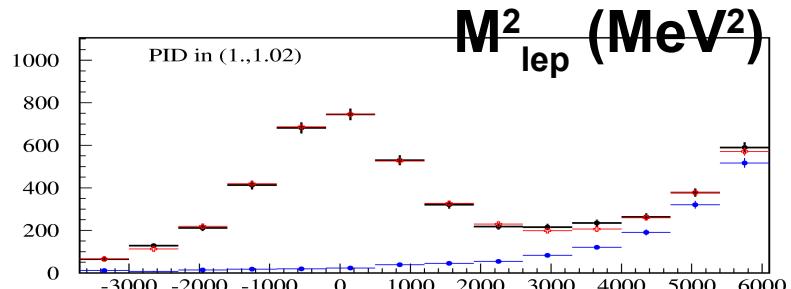
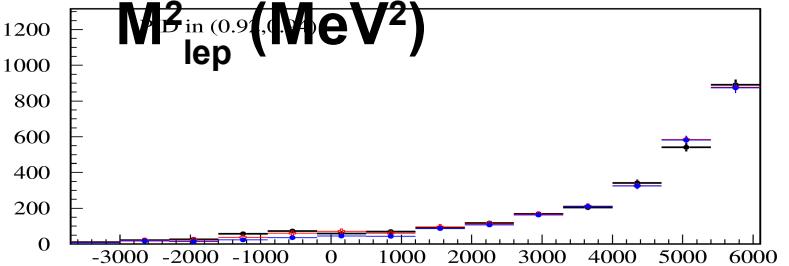
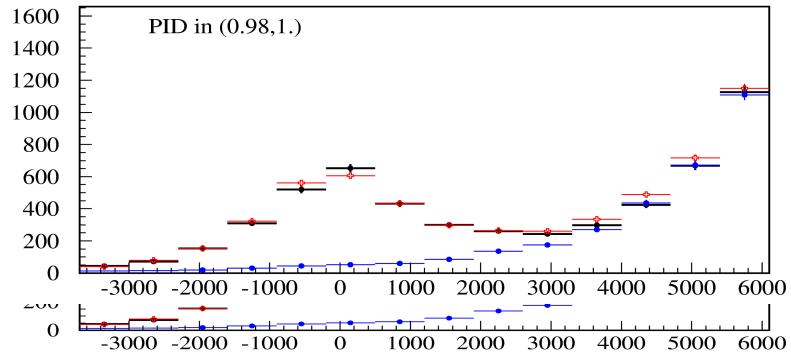
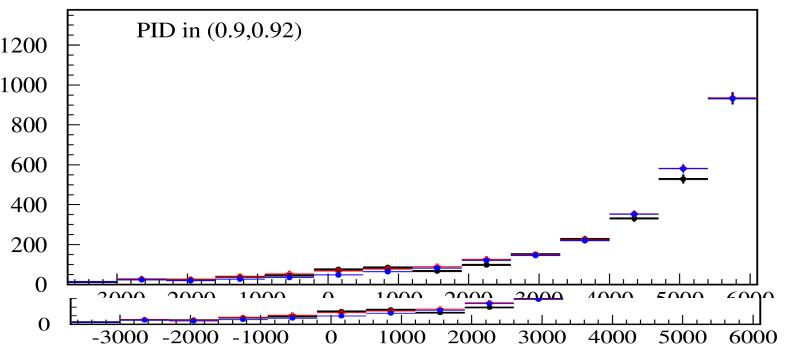
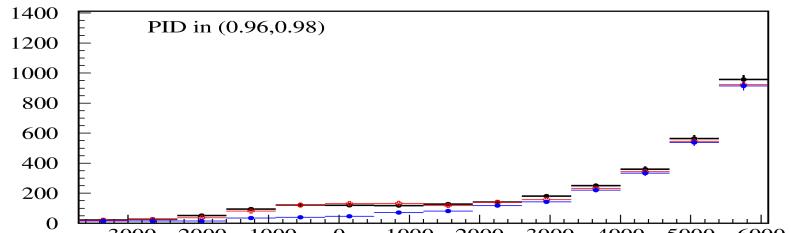
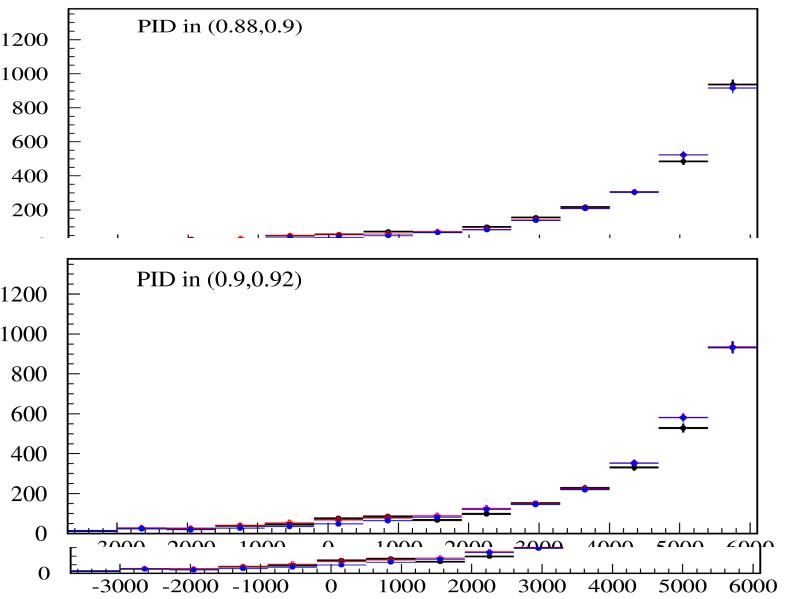
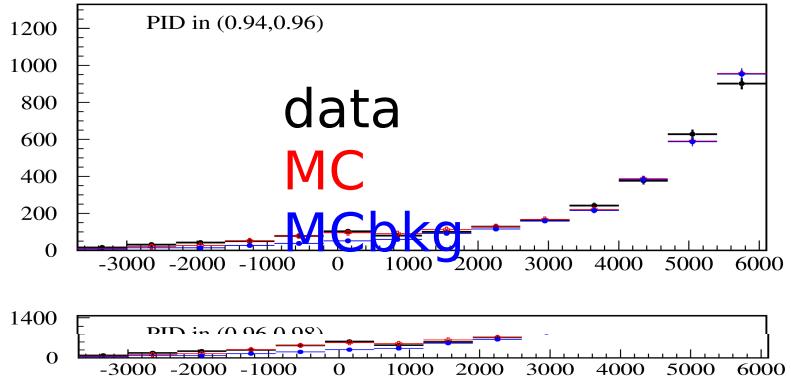
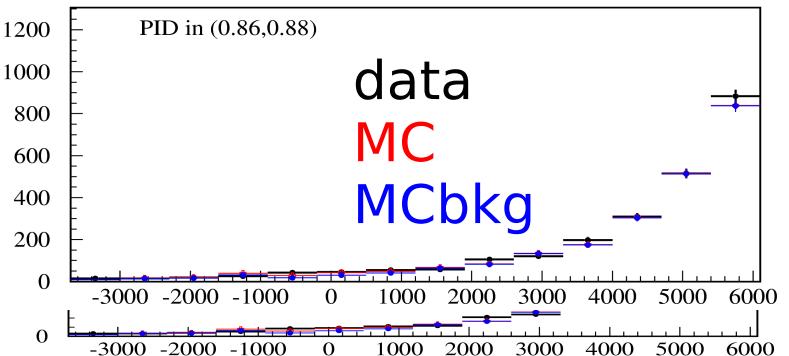
WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2008



- 10 cases with ft -values measured to $\sim 0.1\%$ precision; 3 more cases with $< 0.3\%$ precision.
- ~ 150 individual measurements with compatible precision

$$\mathcal{F}t = ft(1 + \delta'_R)[1 - (\delta_c - \delta_{NS})] = \frac{K}{2G_V^2 (1 + \Delta_R)}$$





Dominant K_L branching ratios

Absolute BR mmnts to 0.5-1% using K_L beam tagged by $K_S \rightarrow \pi^+ \pi^-$

328 pb $^{-1}$ '01 + '02 data

$13 \times 10^6 K_L$'s for counting (25%)

75% used to evaluate efficiencies

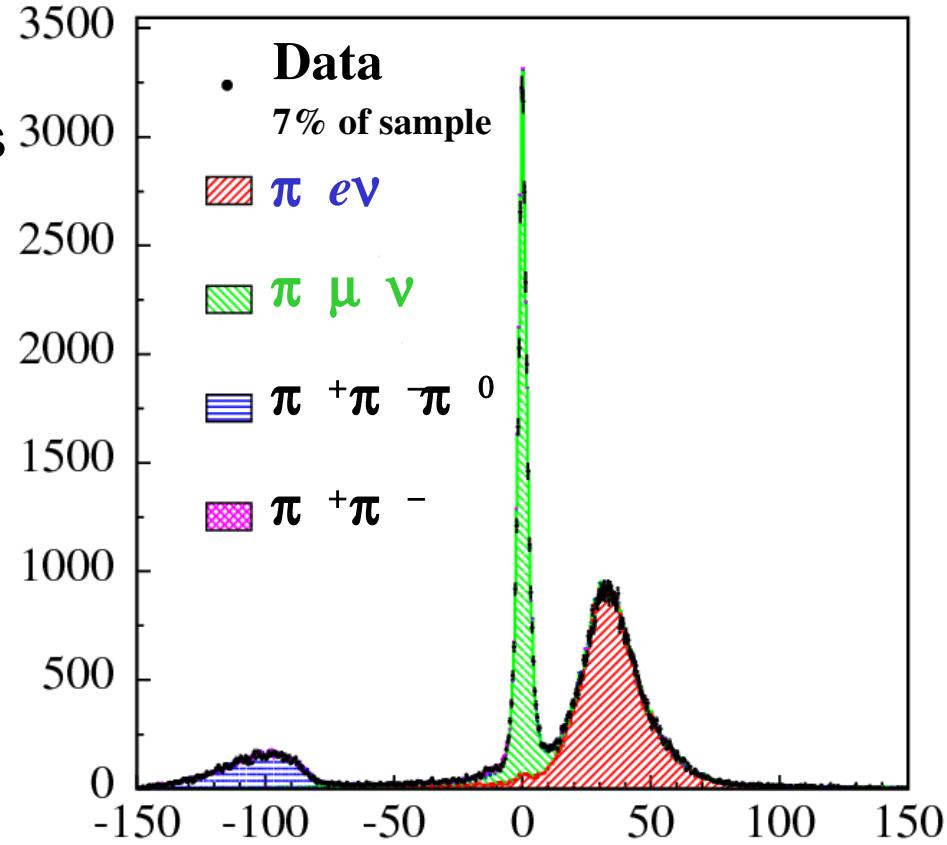
BR's to $\pi e \nu$, $\pi \mu \nu$, and

$\pi^+ \pi^- \pi^0$:

- K_L vertex reconstructed in DC
- PID using decay kinematics
- Fit with MC spectra including radiative processes and optimized EmC response to $\mu/\pi/K_L$

BR to $\pi^0 \pi^0 \pi^0$:

- vertex by EmC TOF (≥ 3 clusters)
- $\epsilon_{\text{rec}} = 99\%$, background < 1%



Lesser of $p_{\text{miss}} - E_{\text{miss}}$ in $\pi \mu \nu$ or $\mu \pi \nu$ hyp. (MeV)

$K_L e 3$ form factor slopes

K_L decays tagged by $K_s \rightarrow \pi^+ \pi^-$ and loose cuts on kinematics
background rejection and PID using TOF (from tracks associated to clusters)
form factor slopes measured using kinematical variables in Kaon rest frame
Binned log-L fit of t/m_{π}^2 distribution, accounts for statistical fluctuation of
efficiency corrections
Analysis by charge

**2x10⁶ Ke3 events
1% Kμ 3 background**

KLOE

PLB 636 (2006)

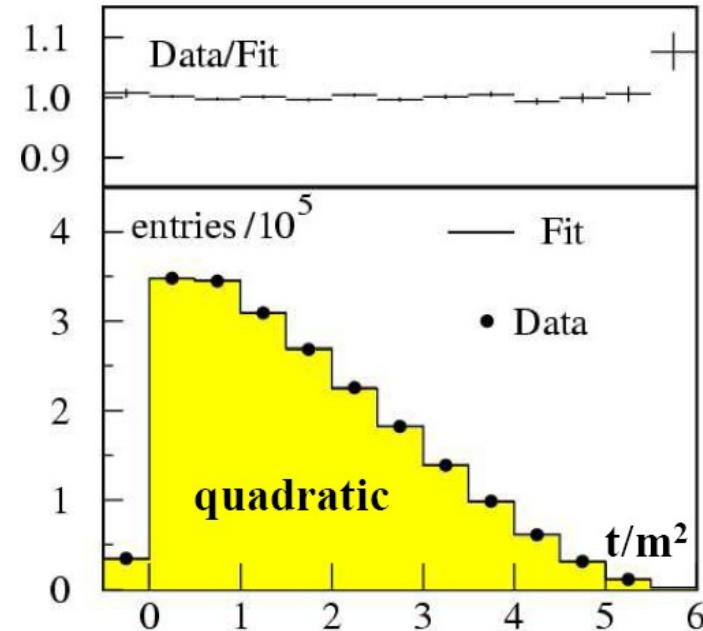
Correlation
-0.95

$$\lambda'_{+} \times 10^3$$

$$25.5 \pm 1.8$$

$$\lambda''_{+} \times 10^3$$

$$1.4 \pm 0.8$$



*Agreement between results from quadratic and pole parametrization
and between different experiments*

Unique to KLOE: BR($K_s \rightarrow \pi^- e^+ \nu_e$)

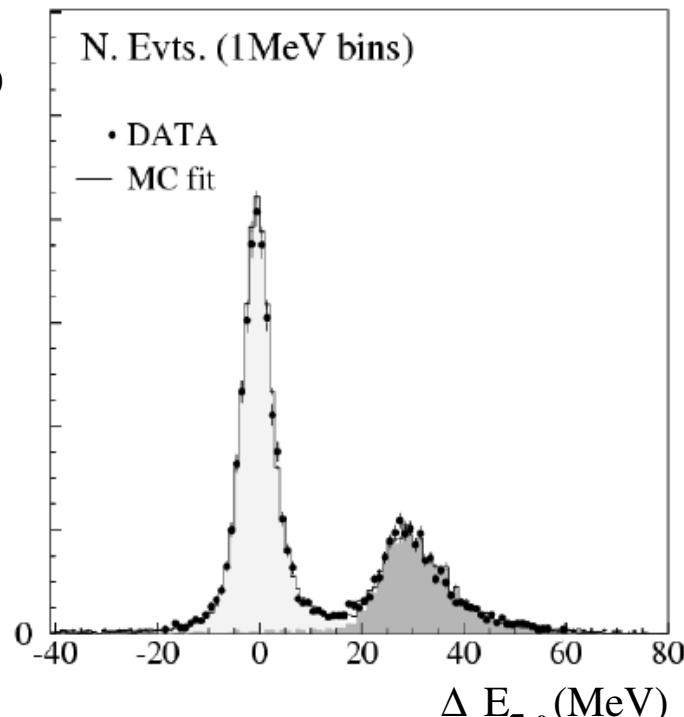
K_s tagged by KL interacting in EMC

- $K_s \rightarrow \pi^- e^+ \nu_e, \pi^- e^+ \nu_e$:
extrapolate tracks from IP to EMC
p/e discrimination from TOF
signal count from fit of distribution
of multiple kinetic variables

- $K_s \rightarrow \pi^+ \pi^-$: extrapolate tracks to the IP
accept events with two tracks

Normalize $K_s \rightarrow \pi^- e^+ \nu_e, \pi^- e^+ \nu_e$ to $K_s \rightarrow \pi^+ \pi^-$

Correct for selection efficiencies by charge



KLOE

PLB 636 (2006)

Using tagged K_S beam

$$\text{BR}(K_S \rightarrow \pi e \nu) / \text{BR}(K_S \rightarrow \pi^+ \pi^-) = 10.19(13) \times 10^{-4}$$

KLOE

EPJC 48 (2006)

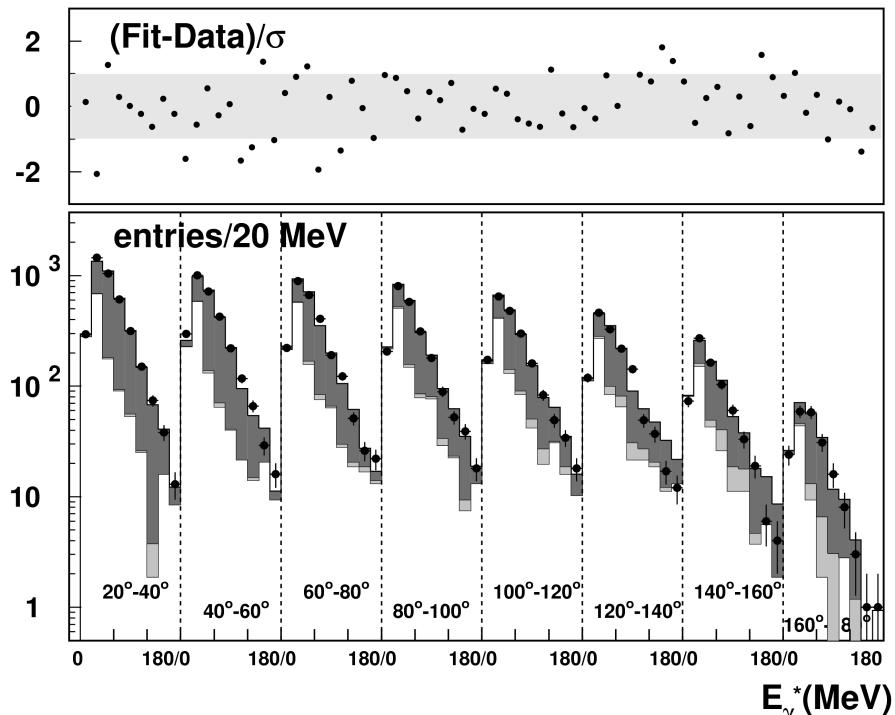
410 pb⁻¹, averaged with KLOE '02 result (17 pb⁻¹)

$$\text{BR}(K_S \rightarrow \pi^+ \pi^-) / \text{BR}(K_S \rightarrow \pi^0 \pi^0) = 2.2549(54)$$

These two measurements completely determine main K_S BRs

$$\text{BR}(K_S \rightarrow \pi e \nu) = 7.046(91) \times 10^{-4} \quad (\text{total error dominated by statistics})$$

$BR(K_L \rightarrow \pi e \nu \gamma)$

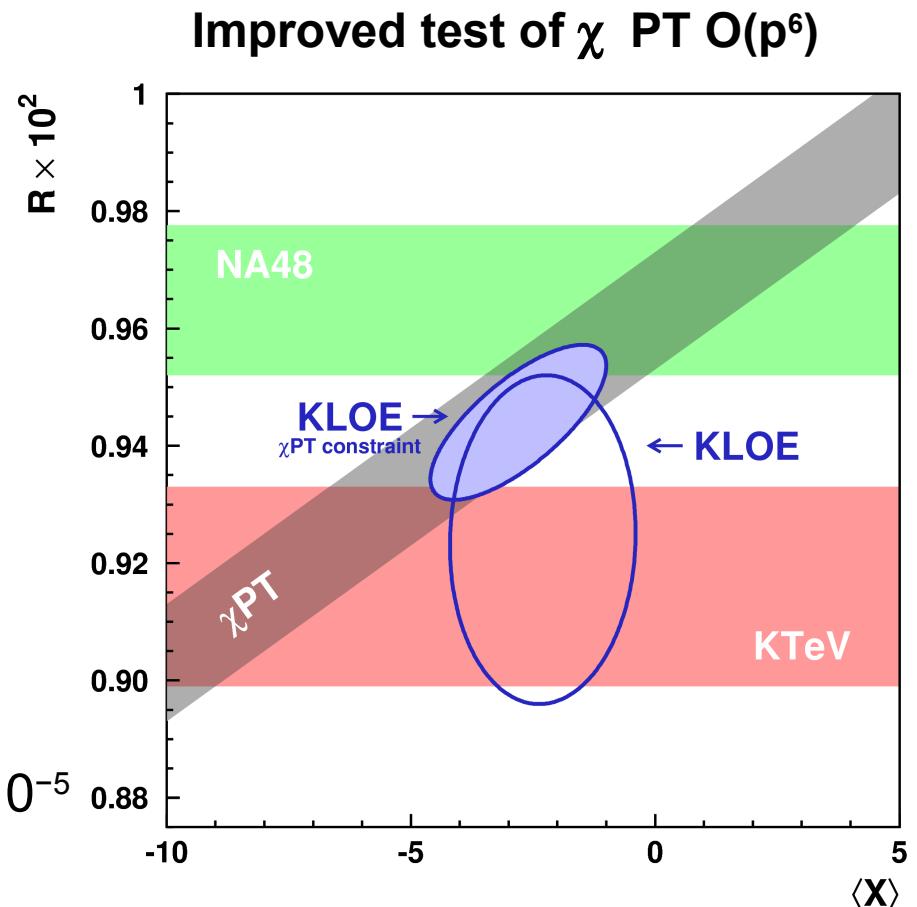


$$R = \frac{BR(K_L \rightarrow \pi e \nu \gamma)}{BR(K_L \rightarrow \pi e \nu)} = (924 \pm 23_{\text{stat}} \pm 16_{\text{syst}}) \times 10^{-5}$$

$$\langle X \rangle = -2.3 \pm 1.3_{\text{stat}} \pm 1.4_{\text{syst}}$$

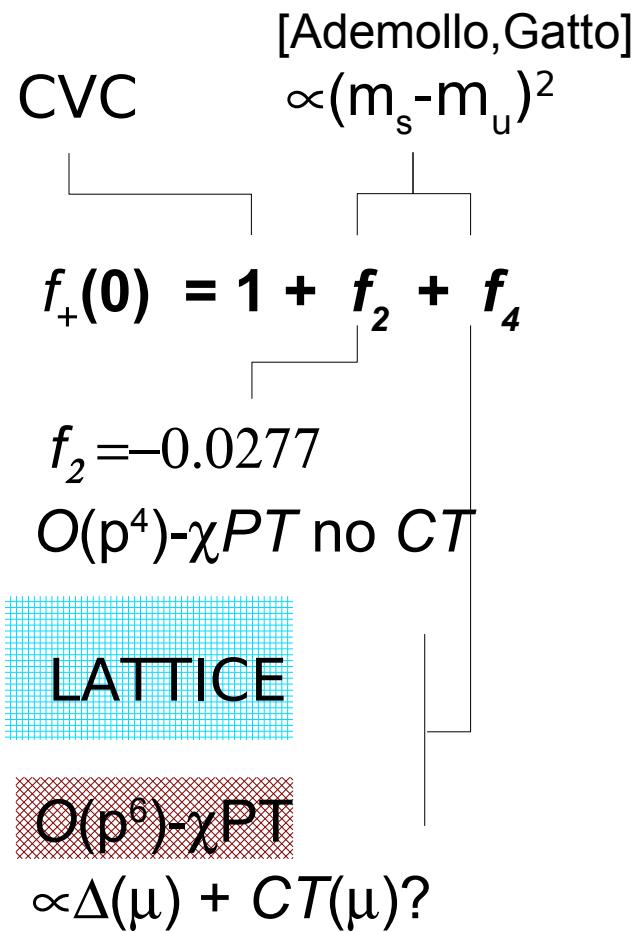
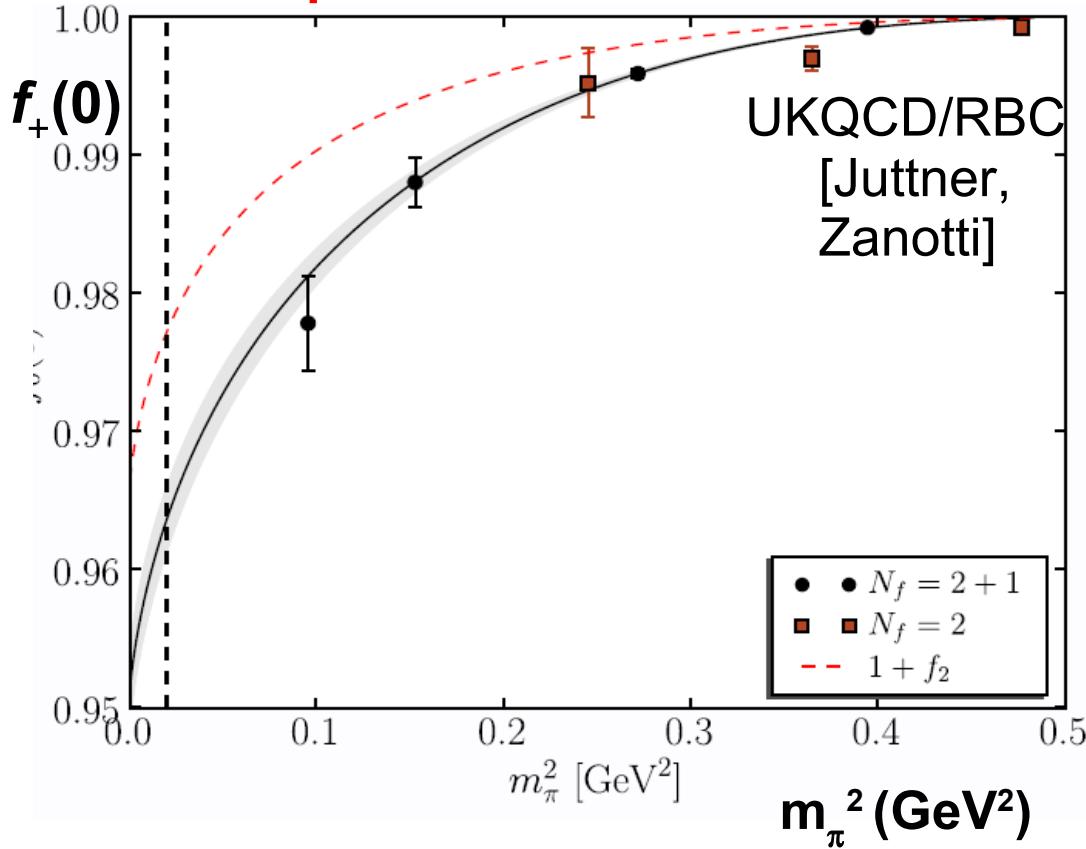
Effective strength parameter describing direct emission

$E_\gamma^* > 30 \text{ MeV}$ $\theta_{e\gamma} > 20^\circ$



Evaluations of $f_+(0)$

Chiral extrapolation seen for the first time



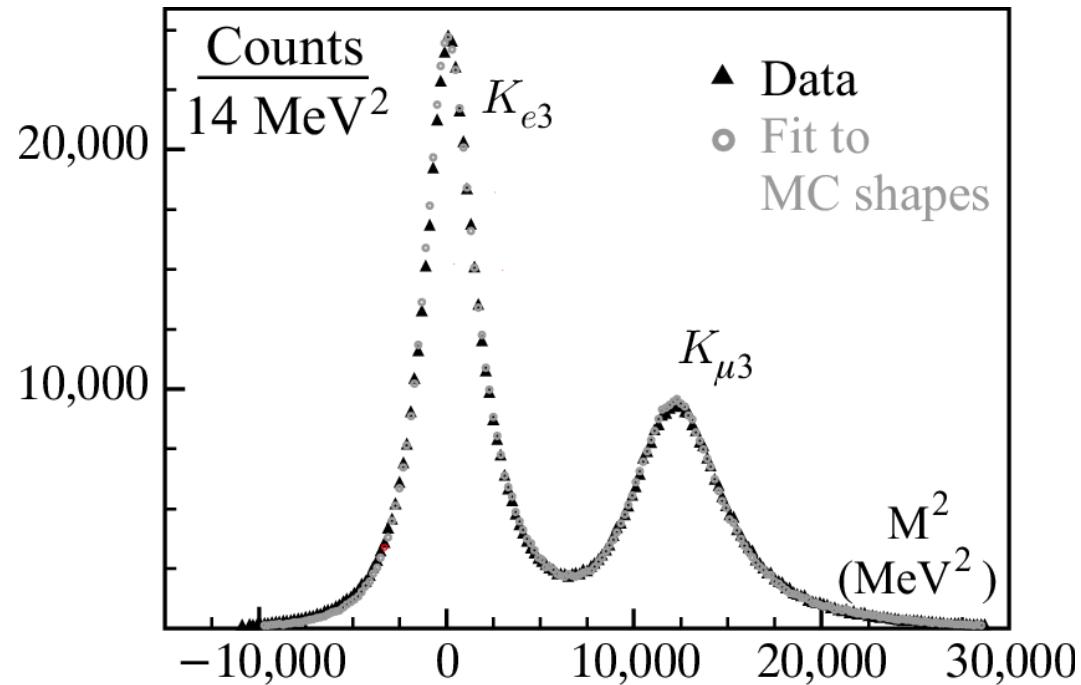
encouraging results from UKQCD/RBC NF=2+1, DWF, $m_\pi >= 300\text{MeV}$:
 37

$$f_+(0) = 0.964(5)$$

Absolute BR for $K^\pm \rightarrow l^\pm \nu_l$

- 4 independent tag samples: $K^\pm \rightarrow \mu^\pm \nu_\mu$, $K^\pm \rightarrow \pi^\pm \pi^0$
- Number of signal events from a fit of distribution of lepton mass squared (M^2) known from TOF
- Perform measurement of absolute BR on each tag sample separately, check consistency

JHEP0802:098



$$\text{BR}(K^\pm \rightarrow e^\pm \nu_e) = 4.965(52)\%$$

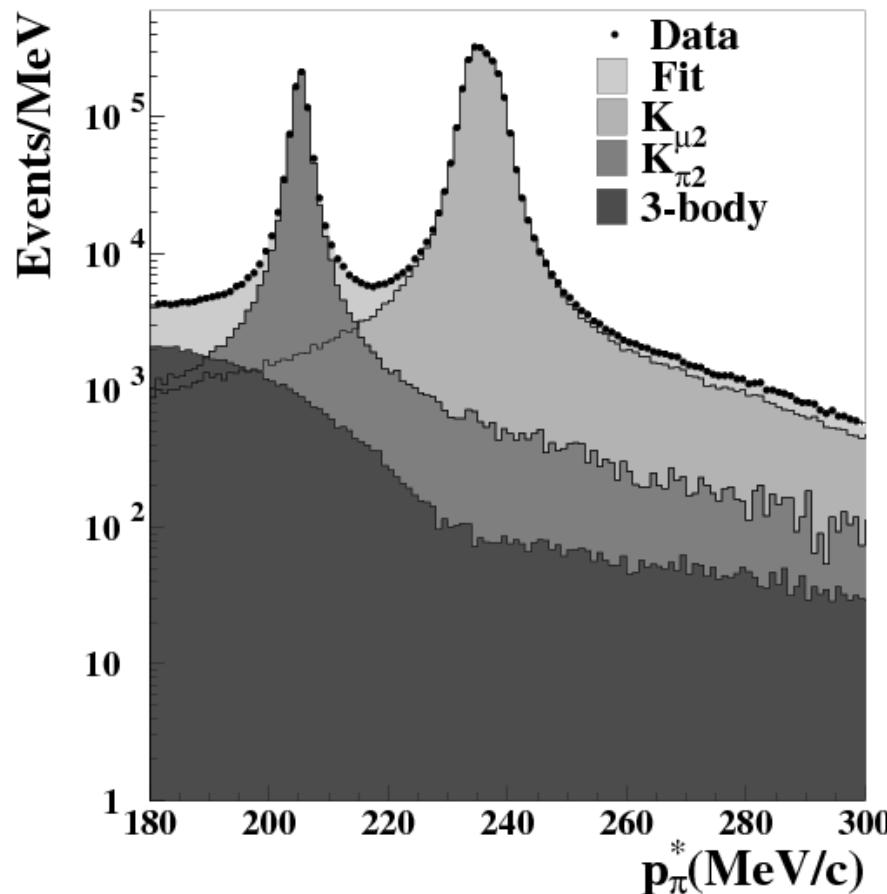
$$\text{BR}(K^\pm \rightarrow \mu^\pm \nu_\mu) = 3.233(39)\%$$

at $\tau_\pm = 12.385$ ns, with
 $d\text{BR}/\text{BR} = -0.5d\tau_\pm/\tau_\pm$

$\sigma_{\text{rel}} \sim 1\%$

Absolute BR for $K^+ \rightarrow \pi^+ \pi^0$

- Needed to perform a global fit to K^+ BRs
- K_{l3}/K_{π^2} measured by NA48 and ISTRA
- Available measurement dates back to '72 (no radiative corrections)
- Normalization given by $K^- \rightarrow \mu^- \nu$ tag
- Number of $K^+ \rightarrow \pi^+ \pi^0$ events from a fit of the distribution of the momentum of the secondary particle in K rest frame, p^*



$$\text{BR} = (20.65 \pm 0.05_{\text{stat}} \pm 0.08_{\text{syst}})\%$$

-1.3% respect to PDG'06

$\sigma_{\text{rel}} \sim 0.5\%$

ArXiv: 0707.4631

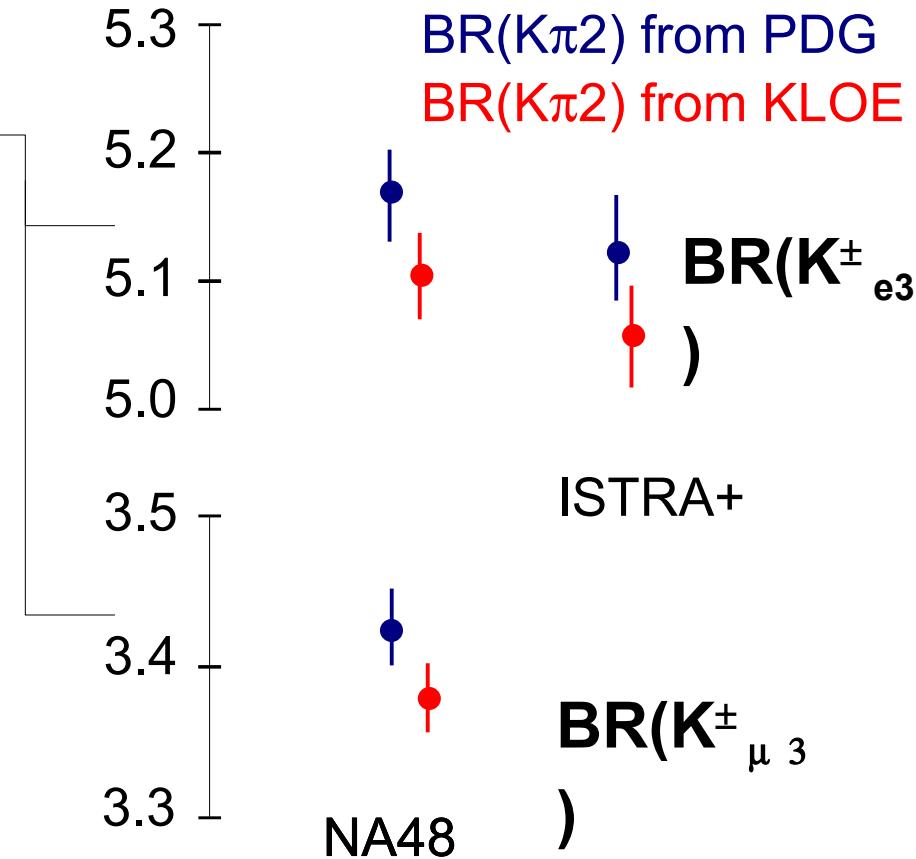
Absolute BR for $K^+ \rightarrow \pi^+ \pi^0$

- Needed to perform a global fit to K^+ BRs
- K_{l3}/K_{π_2} measured by NA48 and ISTRA
- Available measurement dates back to '72 (no radiative corrections)
- Normalization given by $K^- \rightarrow \mu^- \nu$ tag
- Number of $K^+ \rightarrow \pi^+ \pi^0$ events from a fit of the distribution of the momentum of the secondary particle in K rest frame, p^*

submitted to PLB

$$\text{BR} = (20.65 \pm 0.05_{\text{stat}} \pm 0.08_{\text{syst}})\%$$

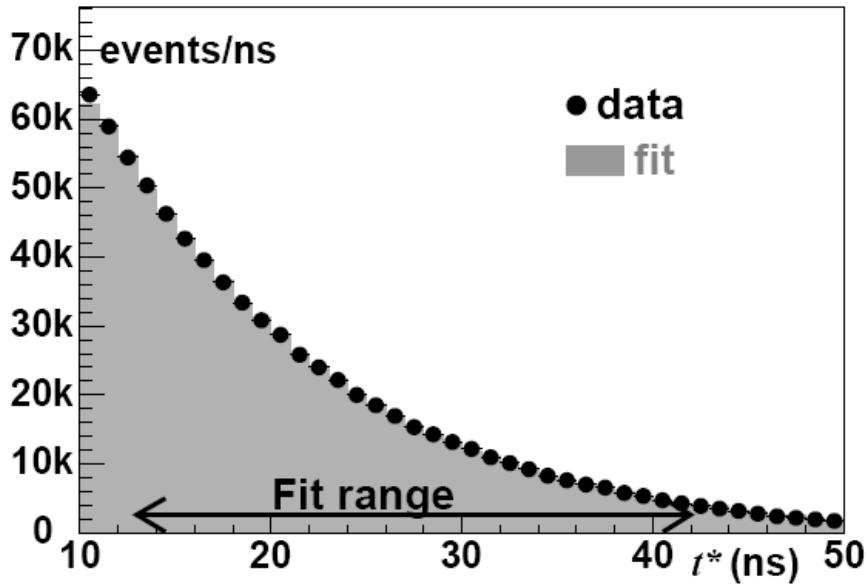
-1.3% respect to PDG'06



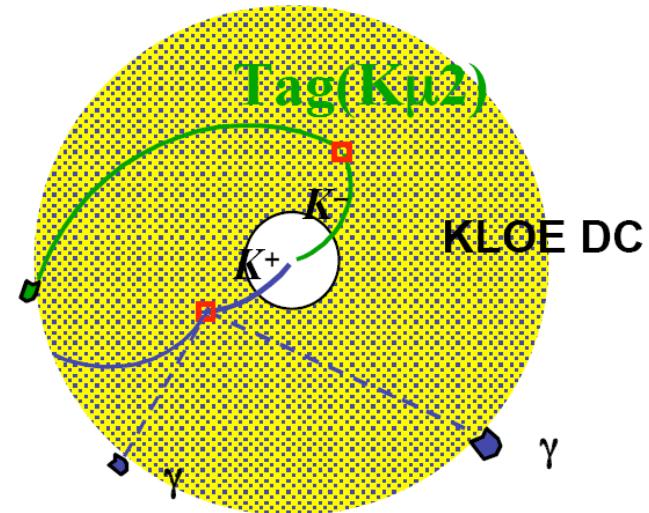
$\sigma_{\text{rel}} \sim 0.5\%$

ArXiv: 0707.4631

K^\pm lifetime



- Tag events with $K^\pm \rightarrow \mu^- \nu_\mu$ decay
- Identify a kaon decay on the opposite side



2 different methods:

from the K decay length

$$\tau_{\pm} = 12.364(31)(31) \text{ ns}$$

$$\rho = 0.34$$

JHEP0801:073

$$\tau_{\pm} = 12.347(30) \text{ ns}$$

from the K decay time

$$\tau_{\pm} = 12.337(30)(20) \text{ ns}$$

$$\sigma_{\text{rel}} \sim 0.4\%$$

K_{Le^3} form factor slopes

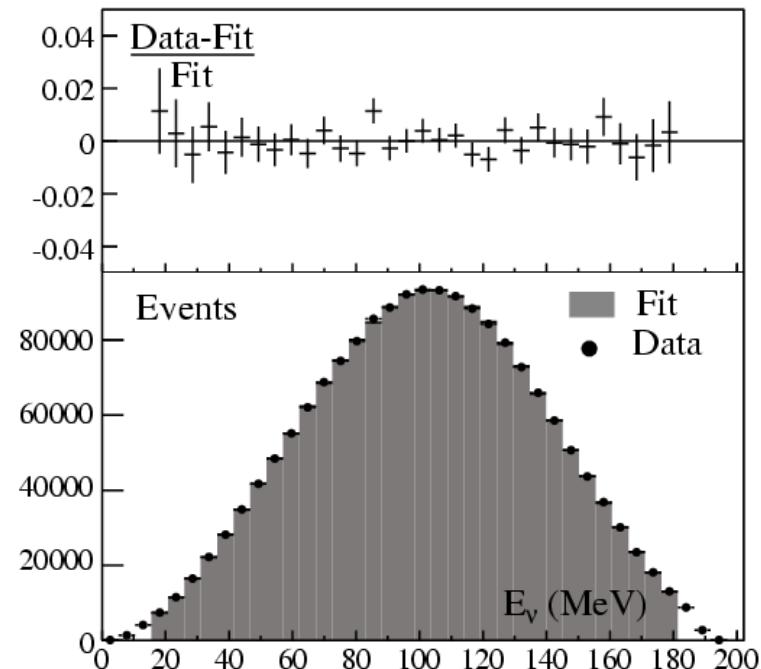
Standard method: fit t-spectrum,
 $t = (p_K - p_\pi)^2$

π/μ separation at low energies
is difficult

at the end of the spectrum, +1%
in signal counts $\rightarrow +15\%$ in λ_0

Fit E_ν spectrum, sensitivity
loss: $\times 2\text{-}3$ on $\sigma_{\text{stat}}(\lambda_0)$

$\dots \times 1.3$ with a combined fit with K_{Le^3}



$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$
25.6 ± 1.8	1.5 ± 0.8	15.4 ± 2.1

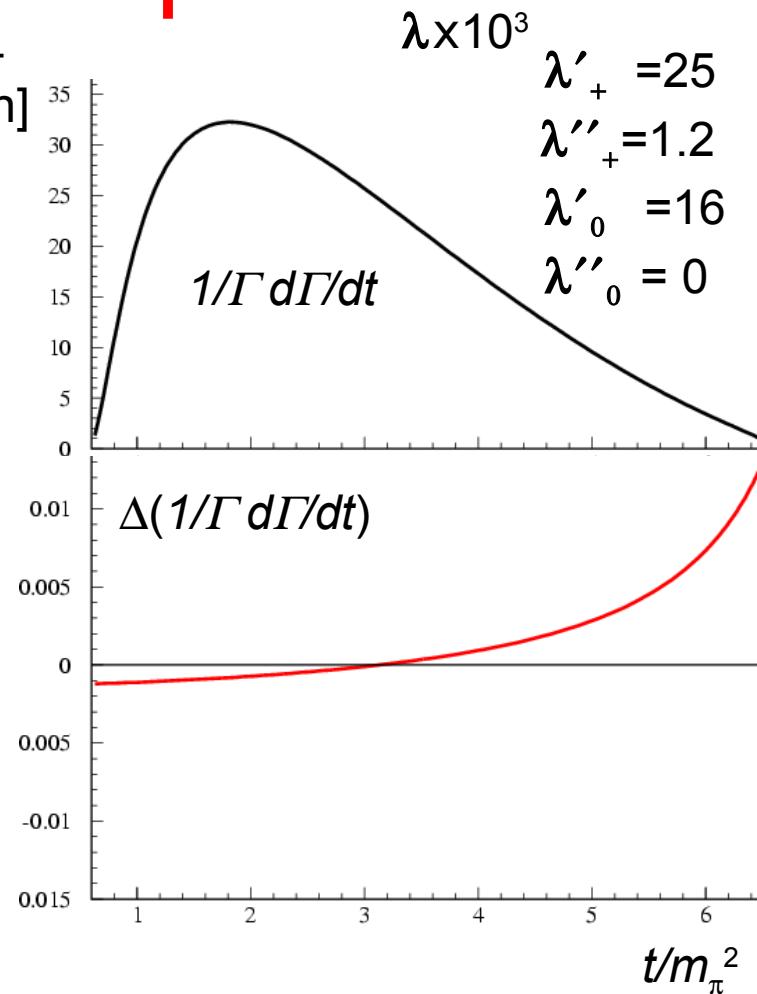
JHEP0712:105

$K_{\mu 3}$ form-factor slopes

- Knowledge of $\tilde{f}_0(t)$ important to test [Callan-Treiman] QCD parameters: $f_0(\Delta_{K\pi} = m_K^2 - m_\pi^2) = f_K / f_\pi$
- Linear parametrization not a good physics approximation: hints for λ''_0 ?
- Fractional partial width difference by varying slopes values :

$$\Delta(1/\Gamma d\Gamma/dt) \quad [\lambda''_0 = 0.4, 0]$$

$$\lambda \times 10^3$$

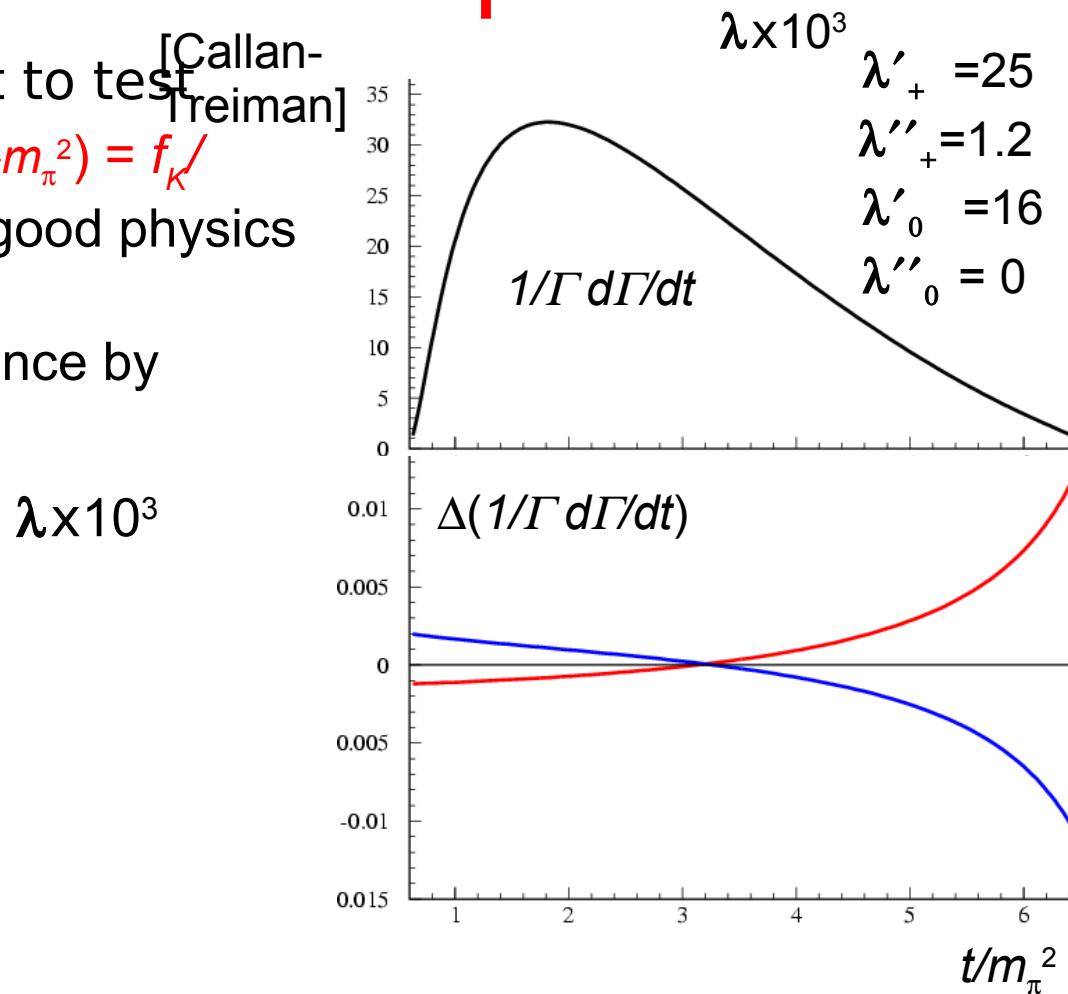


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$$\Delta(1/\Gamma d\Gamma/dt) [\lambda''_0 = 0.4, 0]$$

$$\Delta(1/\Gamma d\Gamma/dt) [\lambda'_0 = 14.7, 16]$$



$K_{\mu 3}$ form-factor slopes

- Knowledge of $\tilde{f}_0(t)$ important to test QCD parameters: $f_0(\Delta_{K\pi} = m_K^2 - m_\pi^2) = f_K / f_\pi$
- [Callan-Treiman] Linear parametrization not a good physics approximation: hints for λ''_0 ?
- Fractional partial width difference by varying slopes values :

$\Delta(1/\Gamma d\Gamma/dt)$ **[$\lambda''_0 = 0.4, 0$]**

$\Delta(1/\Gamma d\Gamma/dt)$ **[$\lambda'_0 = 14.7, 16$]**

Almost exact cancellation

$\Delta(1/\Gamma d\Gamma/dt)$ **[$\lambda'_0 = 14.7, 16; \lambda''_0 = 0.4, 0$]**

Correlation matrix from Ideal t-spectrum experiment:

λ'_0	1	-0.9996	-0.97
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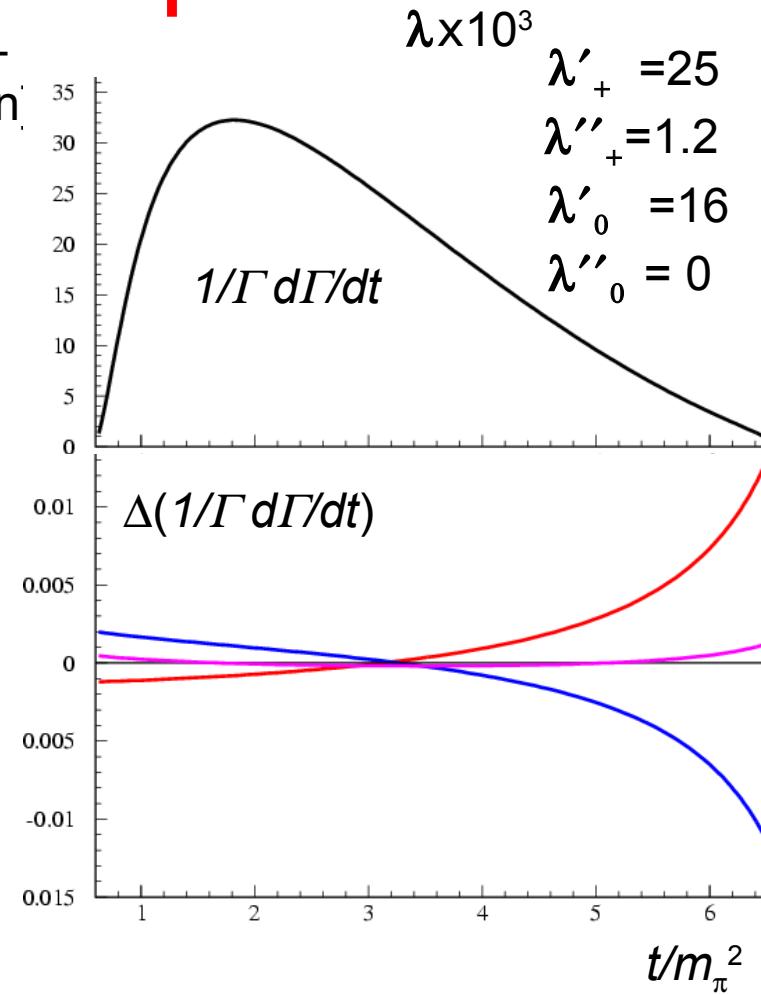
[Franzini]

λ''_0	0.91
---------------	------

λ'_+	1	0.98
--------------	---	------

λ''_+	-0.92
---------------	-------

1



Simultaneous λ'_0, λ''_0 measurement not possible

$K_{\text{L}\mu 3}$ form factor slopes

Preliminary results with $\sim 1 \text{ fb}^{-1}$

$5.8 \times 10^6 K\mu 3$ decays selected

Sensitivity to all FF's parameters

$$\lambda_+ = (25.7 \pm 5.1 \pm 2.5) \times 10^{-3}$$

$$\lambda_+'' = (2.9 \pm 2.5 \pm 1.3) \times 10^{-3}$$

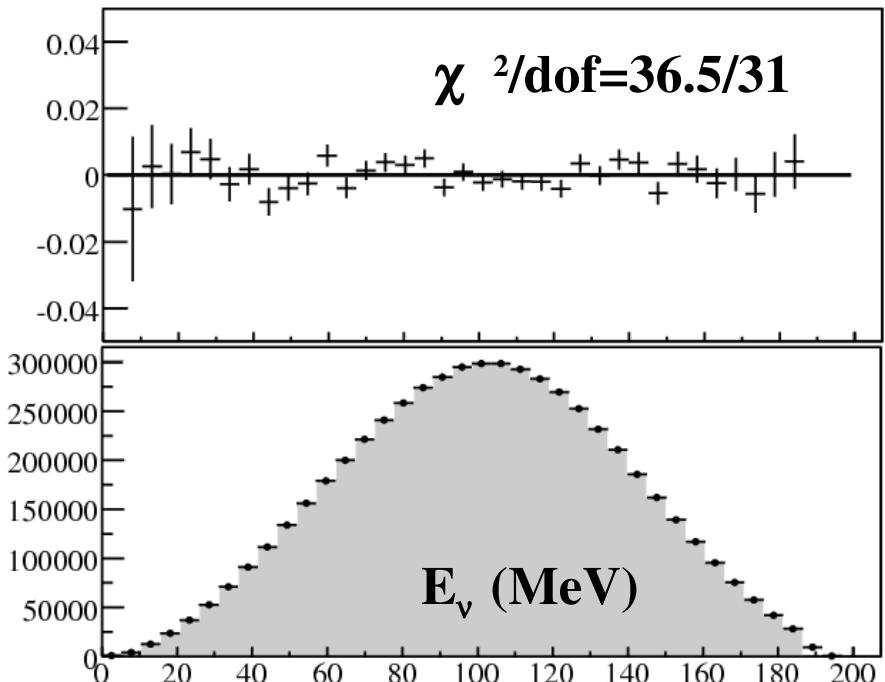
$$\lambda_0 = (14.3 \pm 2.9 \pm 2.4) \times 10^{-3}$$

$$\begin{pmatrix} -0.97 & 0.90 \\ & -0.80 \end{pmatrix}$$

Results obtained with dispersive relations for $f_{+,0}(t)$
averaged with published results

$$\lambda_+ = (26.0 \pm 0.5_{\text{STAT+SYST}}) \times 10^{-3}$$

$$\lambda_0 = (15.1 \pm 1.4_{\text{STAT+SYST}}) \times 10^{-3}$$



$$\log C = 0.217 \pm 0.016$$

Beyond quadratic parametrization

[Stern et al]

Dispersion relation for $\ln f_0(t)$ subtracted at $t = 0$ and $t = m_K^2 - m_\pi^2$, giving:

$$\tilde{f}_0(t) = \exp \left[\frac{t}{m_K^2 - m_\pi^2} (\ln C - G(t)) \right]$$

$G(t)$ evaluated using $K\pi$ scattering data

1 fit parameter:
 $\log C$

$$\log C = 0.204 \pm 0.023$$

JHEP0712:105

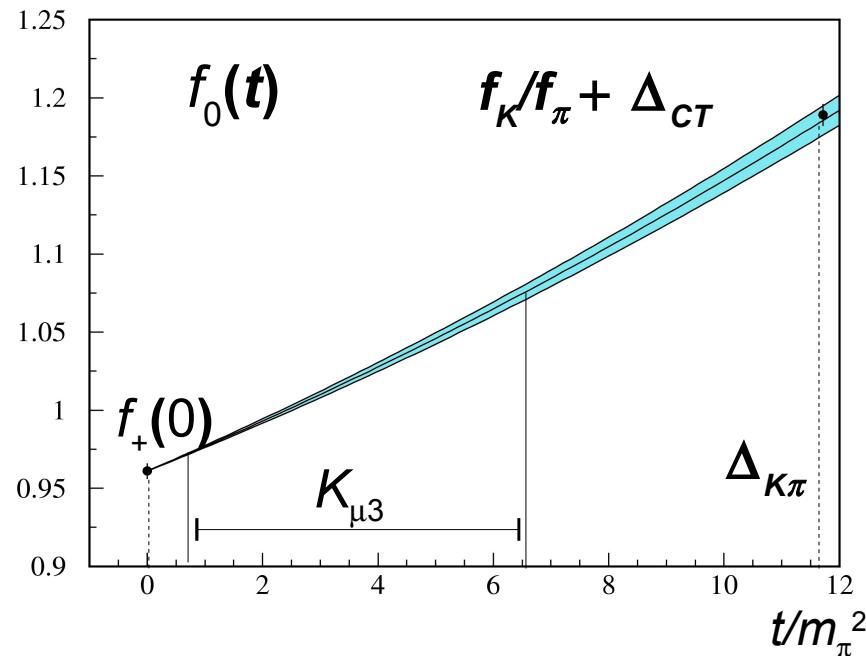
Very precise relation between $f_0(0)^*$
and f_K/f_π :

$$f_0(\Delta_{K\pi}) = f_K/f_\pi + \Delta_{CT}$$

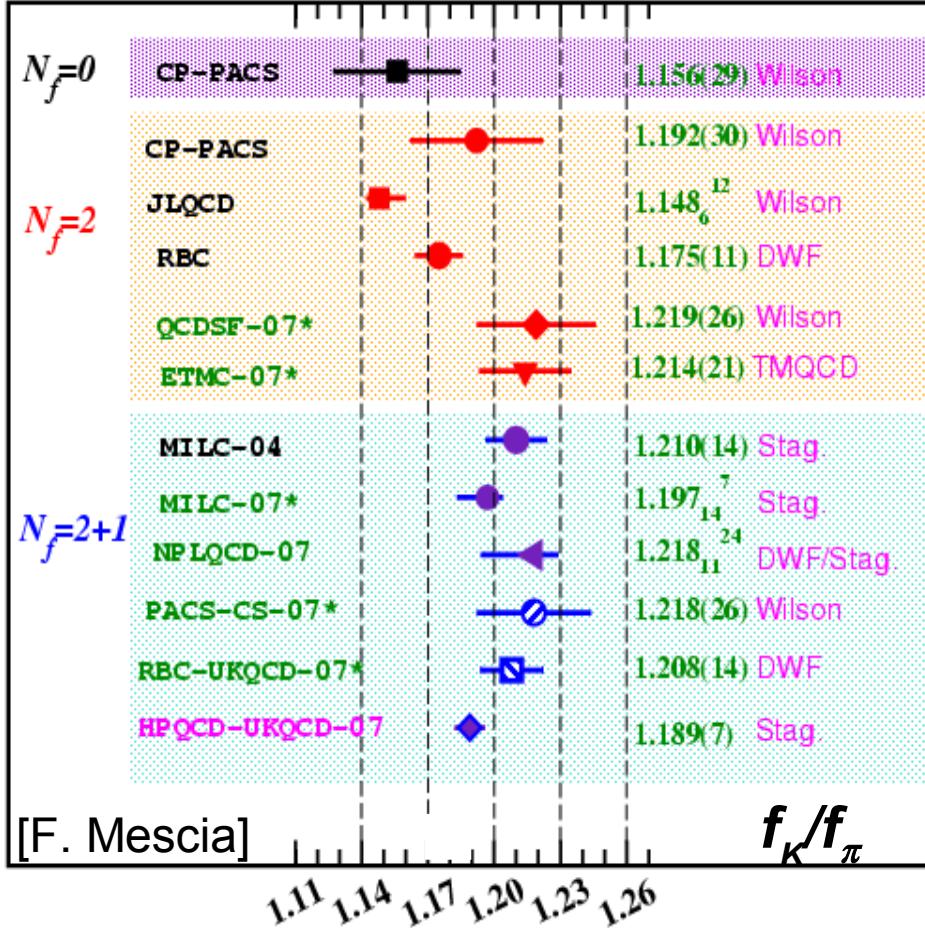
$$\sim$$

$$f_+(0) f_0(\Delta_{K\pi}) = f_K/f_\pi + \Delta_{CT}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2 ; \Delta_{CT} = 3.5 \times 10^{-3} \text{ SU}(2)$$



Evaluations of $f_+(0)$ and f_K/f_π



No symmetry protection

$$\propto m_s - m_u$$

$$f_K/f_\pi = 1 + \text{loops}(\mu) + CT(\mu)$$

χPT CT already at NLO

Only LATTICE

delicate:

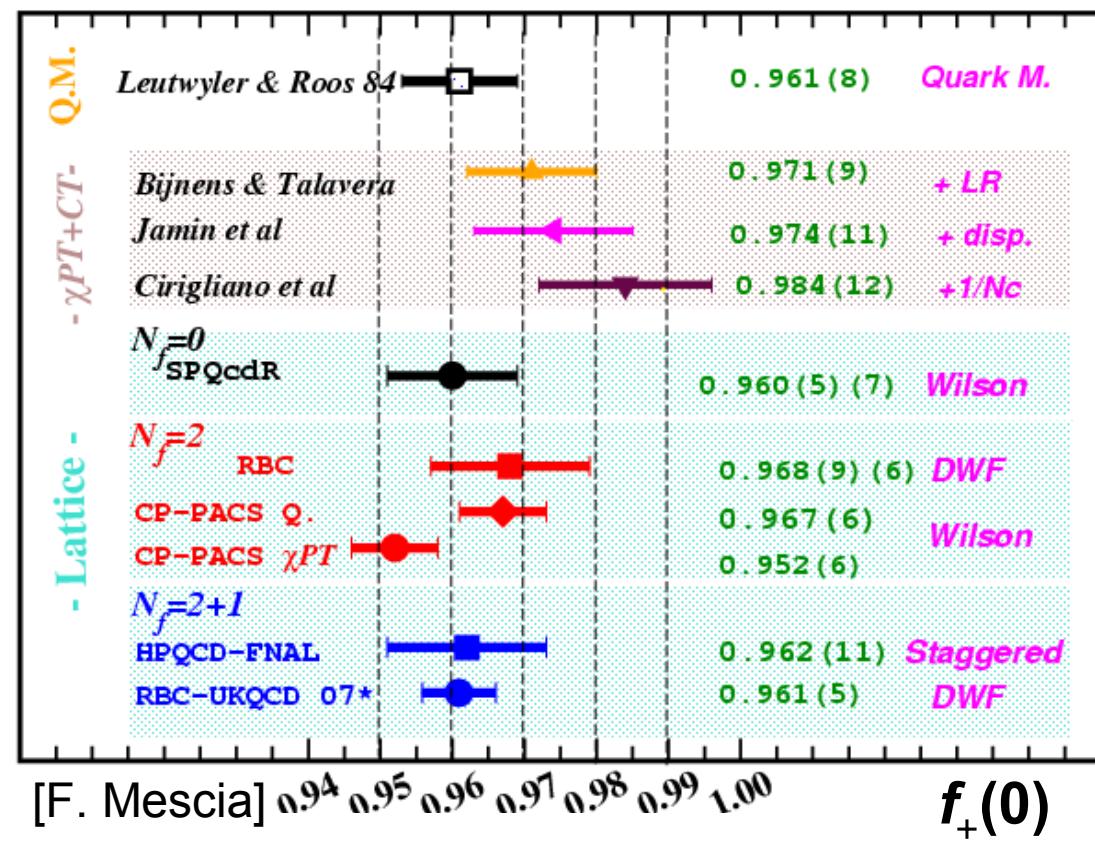
large chiral extrapolation

no need of q^2 extrapolation

updated result from HPQCD-UKQCD(MILC) $N_f=2+1$, Stag, $m_\pi \sim 240$ MeV!

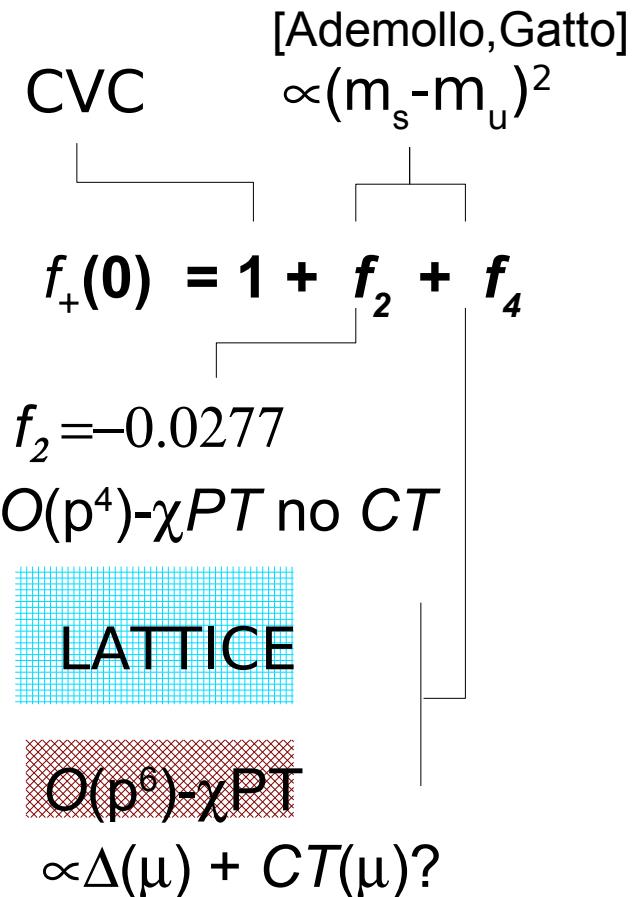
continuum limit: $f_K/f_\pi = 1.189(7)$

Evaluations of $f_+(0)$



encouraging results from UKQCD/RBC NF=2+1, DWF, $m_\pi \geq 300$ MeV:

$$f_+(0) = 0.964(5)$$



V_{us}/V_{ud} from $K_{\mu 2}$

Marciano '04

$$\frac{\Gamma(K^\pm \rightarrow \mu^\pm \nu(\gamma))}{\Gamma(\pi^\pm \rightarrow \mu^\pm \nu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_\mu^2/m_K^2)^2}{|V_{ud}|^2 f_\pi^2 m_\pi (1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

Uncertainty from SD virtual corrections

HP/UKQCD '07

arXiv:0706.1726

$$f_K/f_\pi = 1.189(7)$$

$$N_f = (2+1)_{\text{stag}}$$

Cancellation of lattice-scale uncertainties

PLB 636 (2006)

$$\text{BR}(K^+ \rightarrow \mu^+ \nu(\gamma)) = 0.6366(17)$$

Uses $K^- \rightarrow \mu^- \nu$ to tag 2-body K decays

Counts $K^+ \rightarrow \mu^+ \nu$ from decay-momentum spectrum

$$V_{us}/V_{ud} = 0.2323(15)$$

$K_{\ell 3}$ decays

Vector transition protected against $SU(3)$ corrections: [Ademollo-Gatto]

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 M_K^5}{192\pi^3} S_{EW} G_F^2 |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times$$

$$I_{K\ell}(\{\lambda\}_{K\ell}) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{EM})$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+ , 1 for K^0

S_{EW} Universal SD EW correction (1.0232)

Inputs from theory:

$$f_+^{K^0\pi^-}(0)$$

Hadronic matrix element
(form factor) at zero
momentum transfer ($t = 0$)

$$\Delta_K^{SU(2)}$$

Form-factor correction for
 $SU(2)$ breaking

$$\Delta_{K\ell}^{EM}$$

Form-factor correction for
long-distance EM effects

Inputs from experiment:

$$\Gamma(K_{\ell 3(\gamma)})$$

Rates with well-determined
treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$$I_{K\ell}(\{\lambda\}_{K\ell})$$

Integral of dalitz density
(includes ff) over phase
space:

- K_{e3} : Only λ_+ (or λ'_+ , λ''_+)
- $K_{\mu 3}$: Need λ_+ and λ_0

$K_{\ell 2}$ decays

[Mariciano]

Small uncertainties in f_K/f_π from lattice → determine V_{us}/V_{ud}
Reduced uncertainty from e.m. Structure Dependence corrections

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K^2}{f_\pi^2} \times \frac{M_K(1-m_\mu^2/M_K^2)^2}{m_\pi(1-m_\mu^2/m_\pi^2)^2} \times 1 + \alpha(C_K - C_\pi)$$

Inputs from theory:

f_K/f_π Ratio of pseudoscalar decay constants

C_K, C_π Radiative inclusive electroweak corrections

$$1 + \alpha(C_K - C_\pi) = 0.9930(35)$$

Reduced uncertainty from SD virtual corrections

Inputs from experiment:

$\Gamma(K_{\mu 2(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- lifetimes