Status of the CKM matrix

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Why precision CKM studies?

- The SM accomodates flavour & CP violation, but ***we have no theory of flavour***
- We expect New Physics at the EW scale, and most models predict additional flavour and CP violation.
- The CKM mechanism is very successful ➠ ***flavour and CP problem*** (NP must preserve agreement with data)
- To uncover small signals of physics beyond CKM, we need precision tests, in many ways a challenge for our QCD understanding

The CKM matrix

Weak and mass eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{\rm CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Wolfenstein parameterization $\lambda \sim 0.22$, A, ρ, η *are* $O(1)$

To improve the accuracy, define to all orders in λ

The Cabibbo angle

$$\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\lambda = \sin(\theta_{\rm Cabibbo}) = V_{us}$$

Universality of charged currents ⇔ CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{ub}|^2$ → $O(10^{-5})$

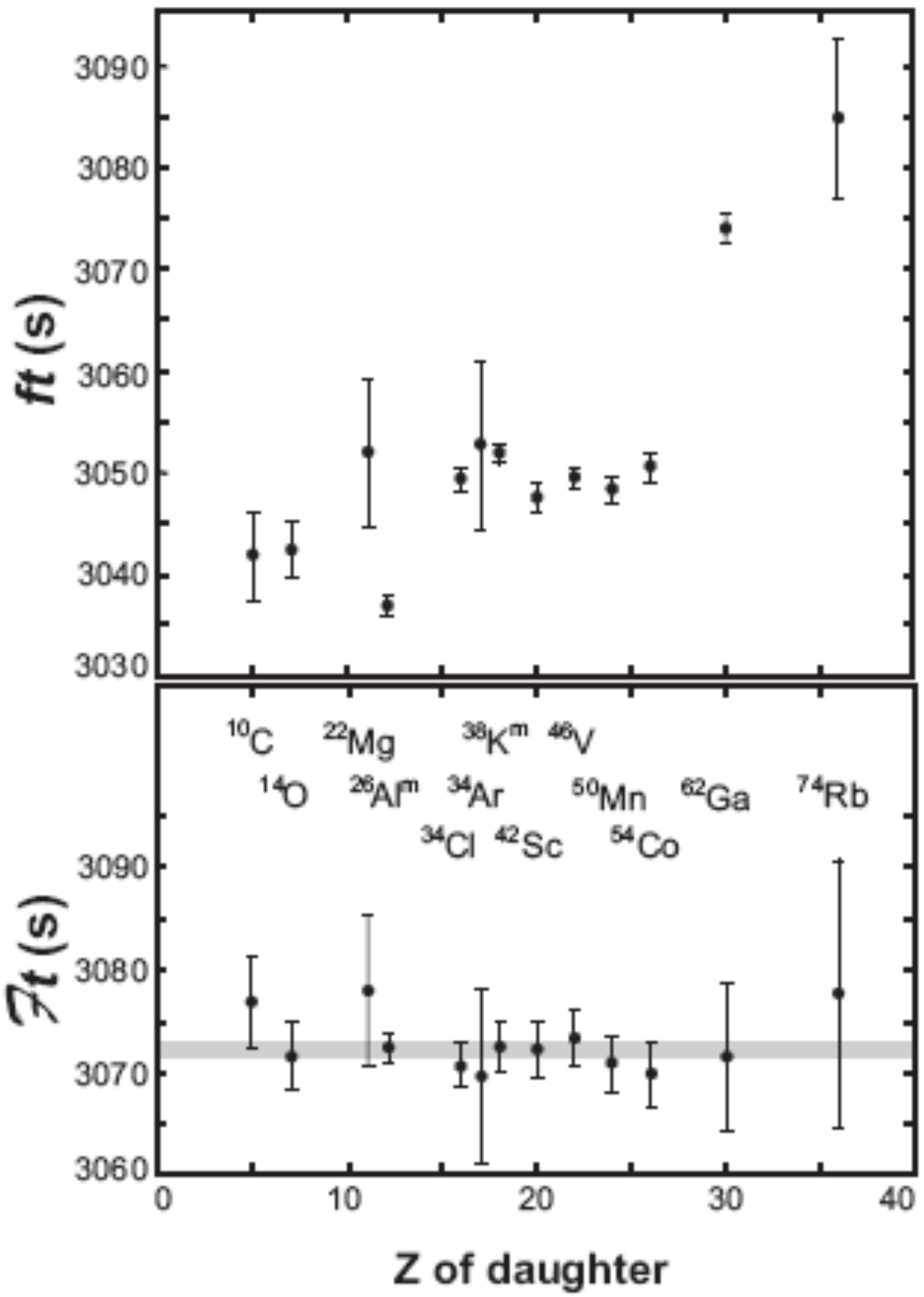
Comparison between V_{ud}, V_{us} determinations of λ tests unitarity of the first line of V_{CKM}

λ could also be measured from 2nd line, V_{cd} (DIS) at 10%,
W decays at LEP constrains $\Sigma_{ij}|V_{ij}|^2$ at 1.3% V_{cs} at 1.3%

λ from V_{ud}: Fermi transitions

Superallowed Fermi transitions *(0^+->0^+ β decay)*

$$\langle p_f; 0^+|\bar{u}\gamma_\mu d|p_i; 0^+\rangle = \sqrt{2}(p_i + p_f)_\mu$$



$$\mathcal{F}t \equiv ft(1+\delta'_R)(1+\delta_{NS}-\delta_C)$$

isospin breaking, nuclear structure

$$V_{ud}^2 = \frac{K}{2G_F^2(1+\Delta_R^V)\overline{\mathcal{F}t}}$$

$$\Delta_R^V = (2.361 \pm 0.038)\%$$

Marciano Sirlin 2006

$$|V_{ud}| = 0.97425 \pm 0.00022$$

Hardy Towner 2008

➠ *λ=0.2254(10) using unitarity*

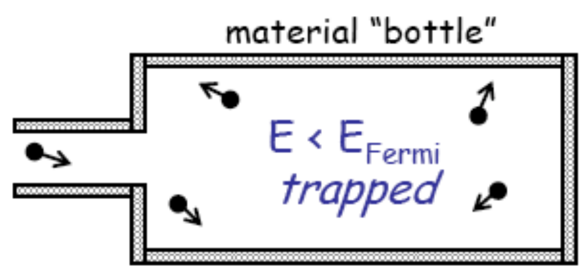
Dominant error from structure indep RC, next structure dep ones.
Great exp advances (Penning traps etc)

Other V_{ud} determinations

neutron β decay *not pure vector, needs g_A/g_V but no nuclear structure. δV_{ud}~0.002, will be improved through asymmetry measuremnts at PERKEO, Heidelberg and UCNA, LANL. 2005 measurement of n lifetime (6σ away) serious problem!*

$$V_{ud} = 0.9746(4)_{\tau_n}(18)_{g_A}(2)_{\rm RC}$$

Ultracold neutrons



Long interaction times in apparatus

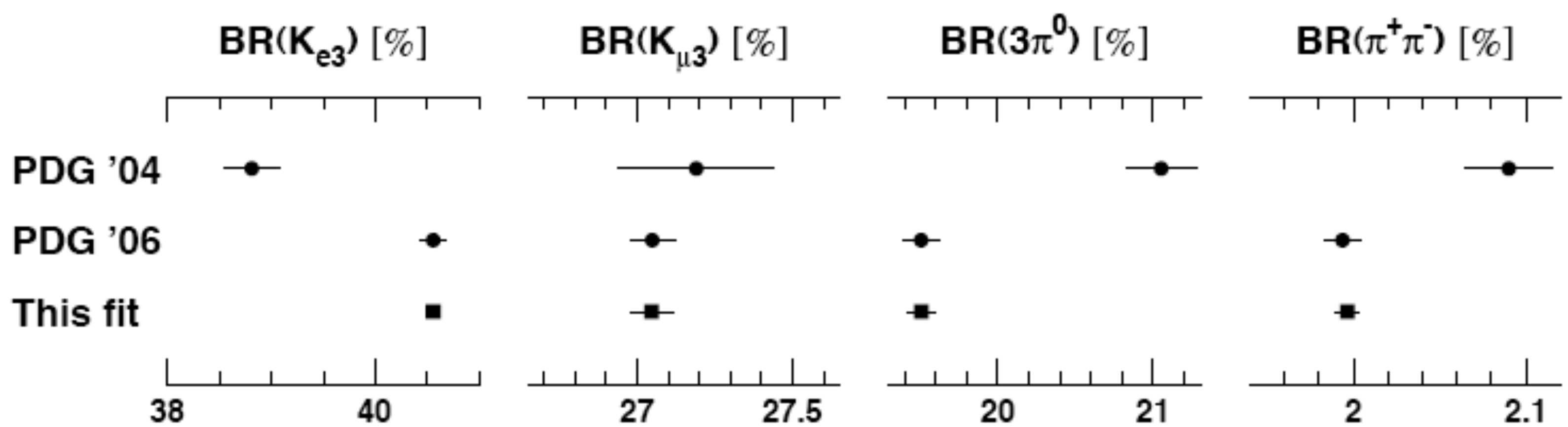
↓

Need relatively small number of neutrons

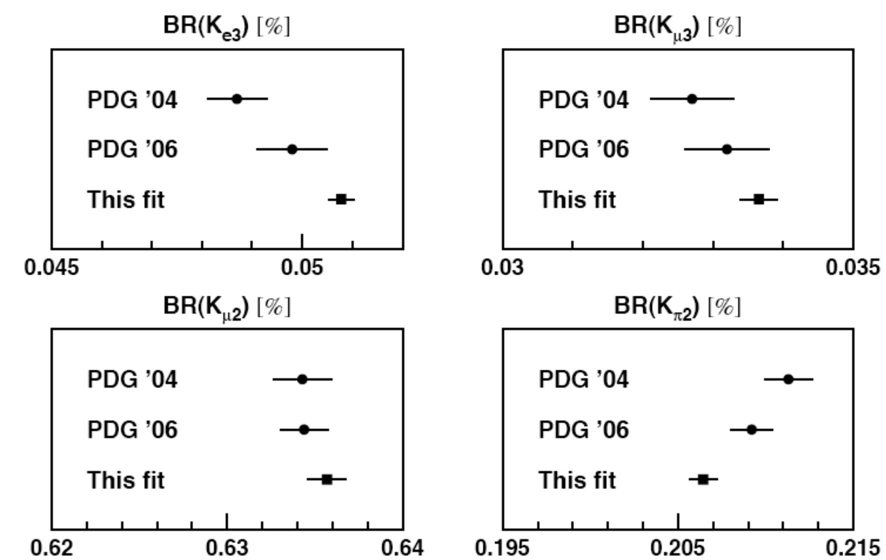
π^+ decay to $\pi^0 e\nu$ *th cleanest, promising in long term but BR~10^{-8} PIBETA at PSI has δV_{ud}~0.003*

$$V_{ud} = 0.9749(26)\left[\frac{BR(\pi^+\to e^+\nu_e(\gamma))}{1.2352\times 10^{-4}}\right]^{\frac{1}{2}}$$

λ from K_{l3} - Experimental progress



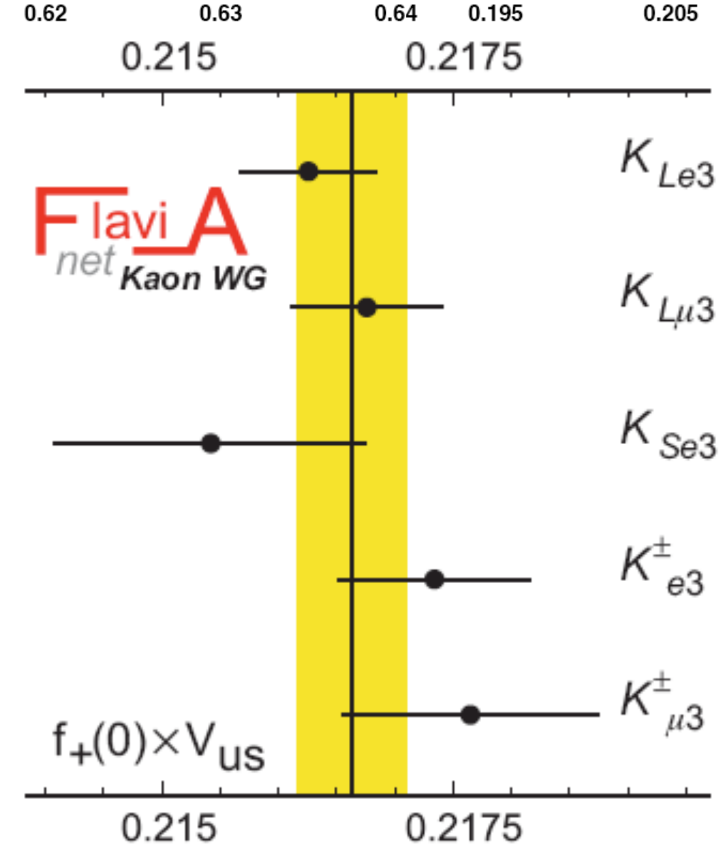
FlaviAnet Kaon WG 2008



$$\Gamma_{K\ell 3} = \frac{G_F^2 M_K^5}{192\pi^3} S_{EW} (1 + \delta_K^\ell + \delta_{SU2}) C^2 \left|V_{us}\right|^2 f_+^2(0) I_K^\ell.$$

$$\boxed{f_+(0)|V_{us}| = 0.21673 \pm 0.00046}$$

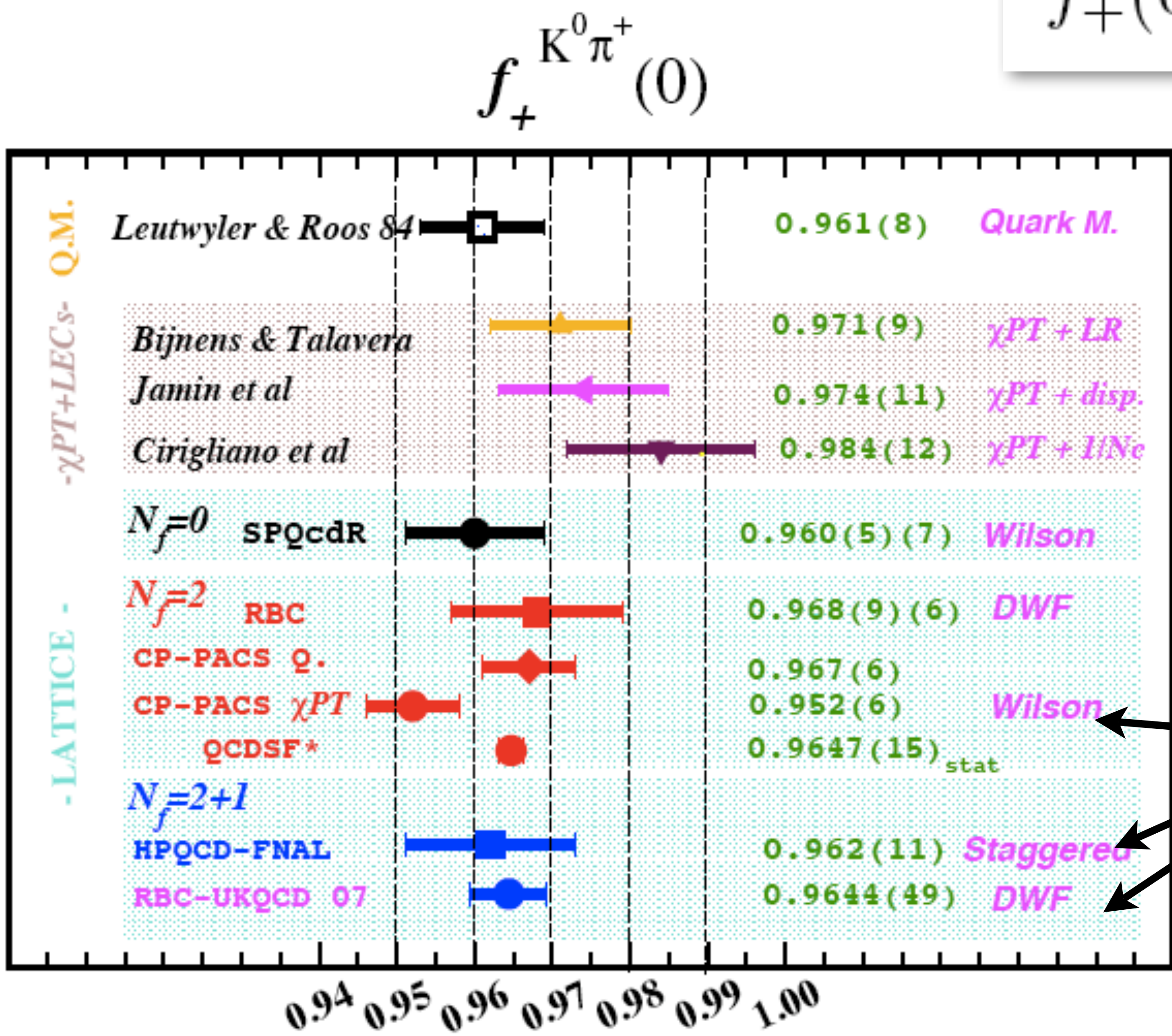
0.25% accuracy!
muon channels perfectly consistent



λ from K_{l3} - Theoretical progress

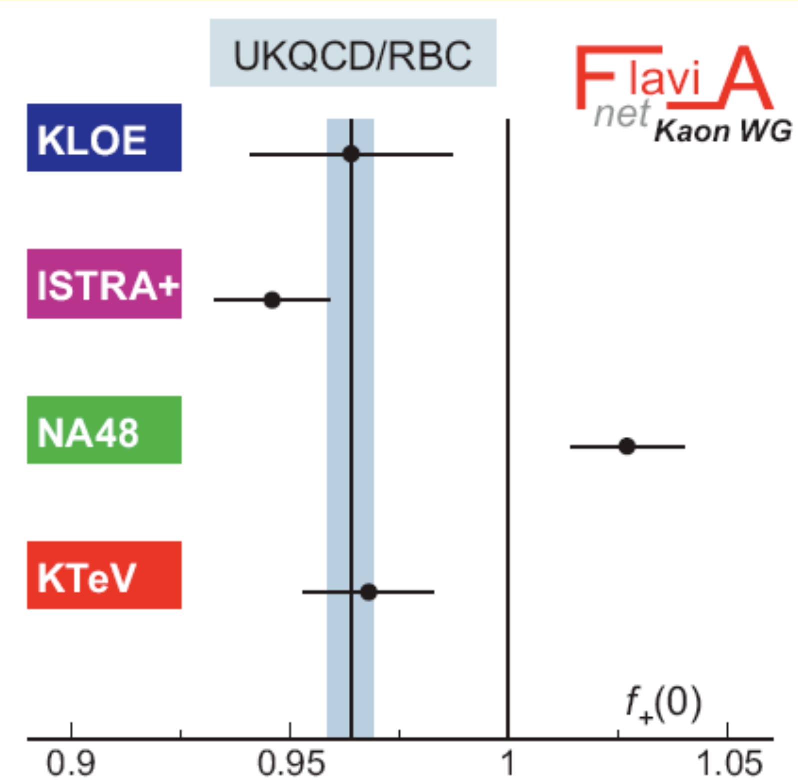
$$f_+(0) = 1 + f_2 + f_4 + \ldots$$ *SU(3) symmetry, Ademollo Gatto*

$$= 0.964(5)$$ *RBC-UKQCD 07*



Various lattice actions

Tests of lattice are now possible from measurements of the shapes, eg from Callan-Treiman and f_K/f_π



$$|V_{us}| = 0.2246 \pm 0.0012 \qquad [K_{\ell 3} \text{ only}]$$

λ from K_{l2}

$$\frac{\Gamma(K^{\pm}_{\ell 2(\gamma)})}{\Gamma(\pi^{\pm}_{\ell 2(\gamma)})} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2 m_K}{f_\pi^2 m_\pi} \left(\frac{1-m_\ell^2/m_K^2}{1-m_\ell^2/m_\pi^2}\right)^2 \times (1+\delta_{\rm em})$$

Marciano 2004

Kloe, NA48/2 find

$$R_K = (2.457 \pm 0.032) \times 10^{-5},$$

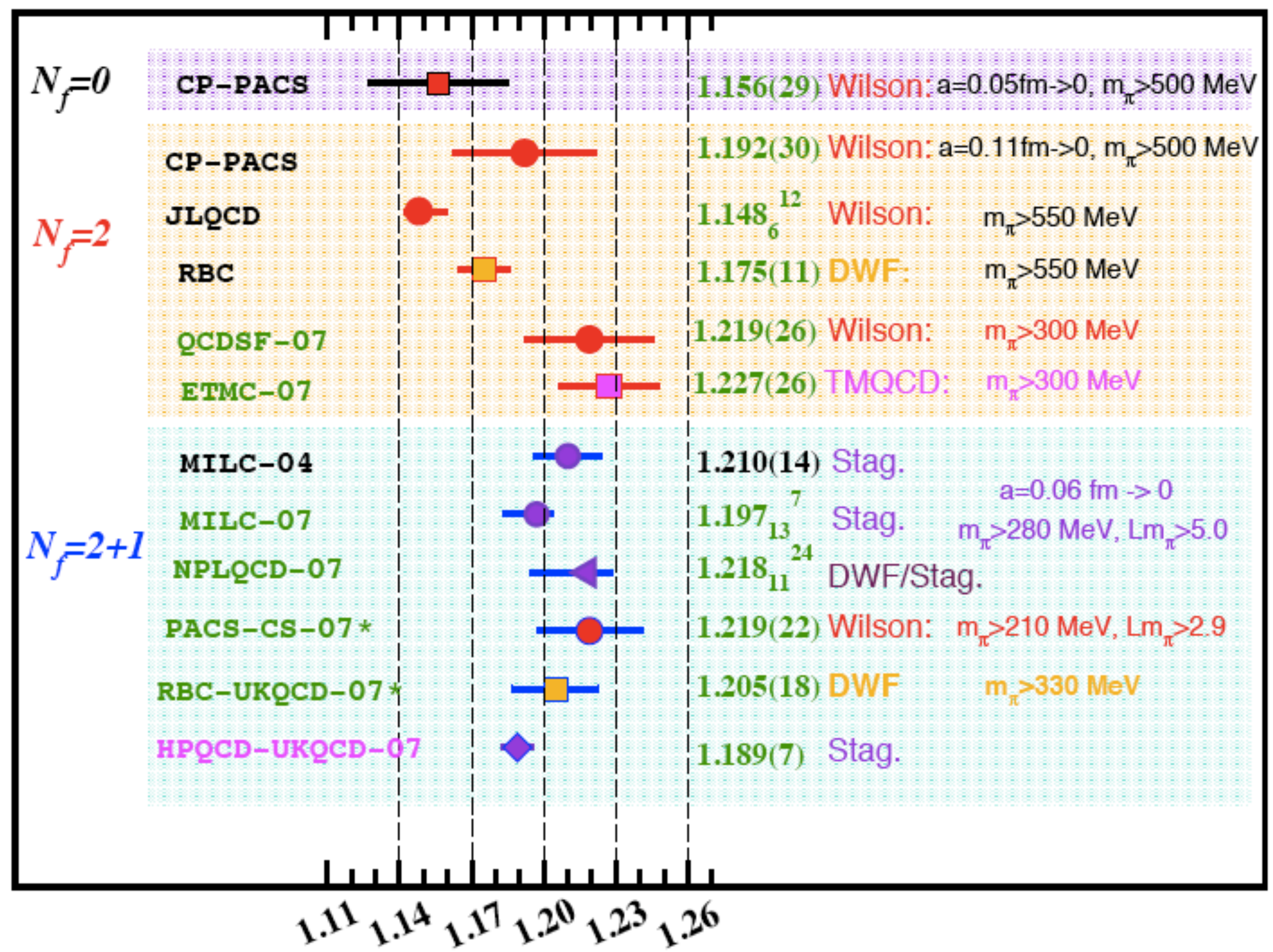
$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.2321 \pm 0.0015$$ *Only K_{l2}*

Very similar from $\Gamma(\tau^- \to K^- \nu)/\Gamma(\tau^- \to \pi^- \nu)$ *measured at Babar:* $\left|\frac{V_{us}}{V_{ud}}\right| = 0.2315 \pm 0.0024$

Cross-check of K_{l2}!

f_K/f_π

Not protected by Ademollo Gatto



Unitarity of the first row

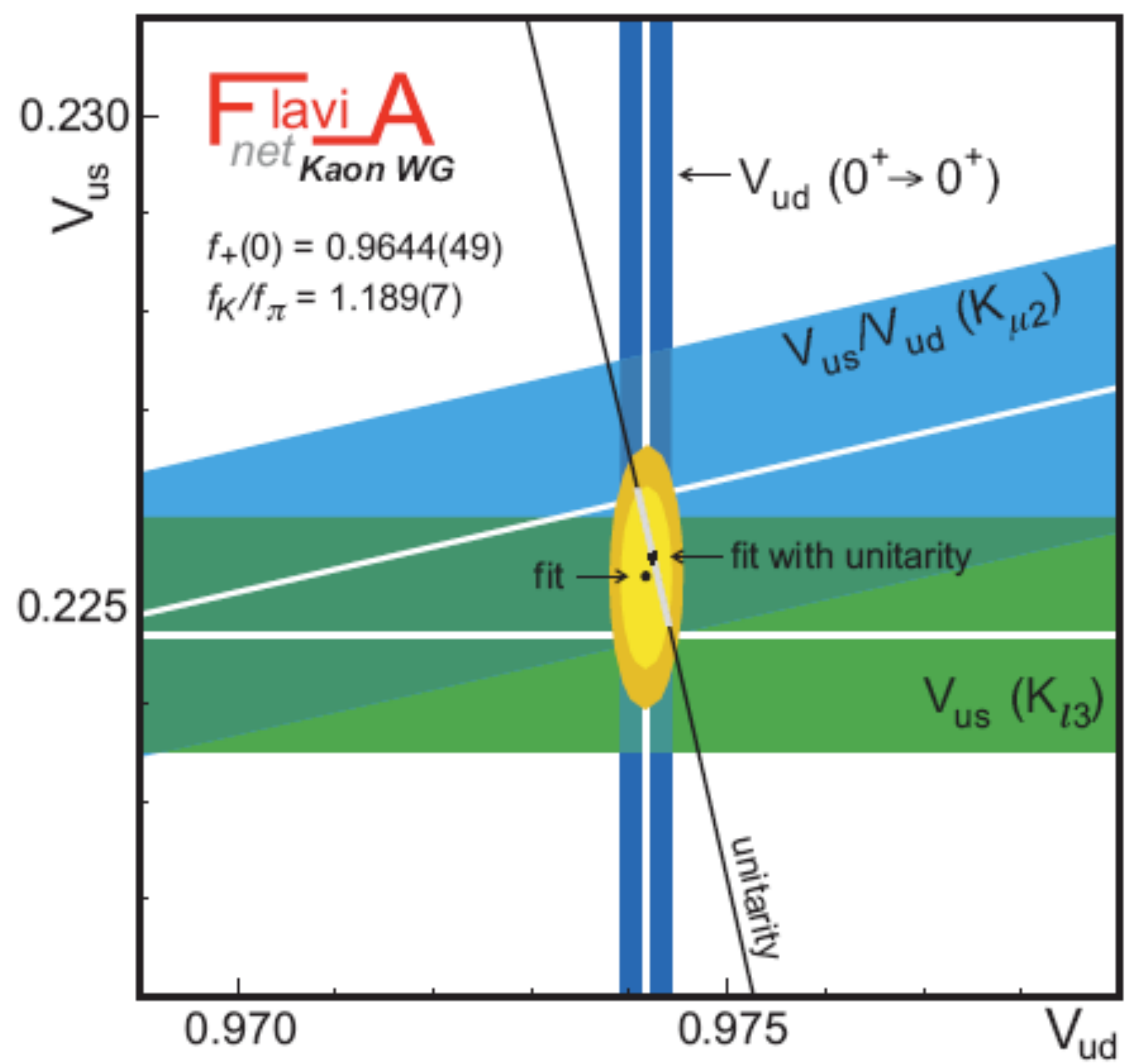
$$|V_{us}| = \sin\theta_C = \lambda = 0.2255(7) \qquad \text{[with unitarity]}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 =$$

=0.9999±0.0006

Inclusive Tau decays also give λ with ~1% error but need $m_s(m_\tau)$.
Preliminary Belle and Babar data suggest 0.2165(27) but there are some doubts on experimental analyses (missing modes)

Gamiz et al 2007, ...



Constraints on New Physics

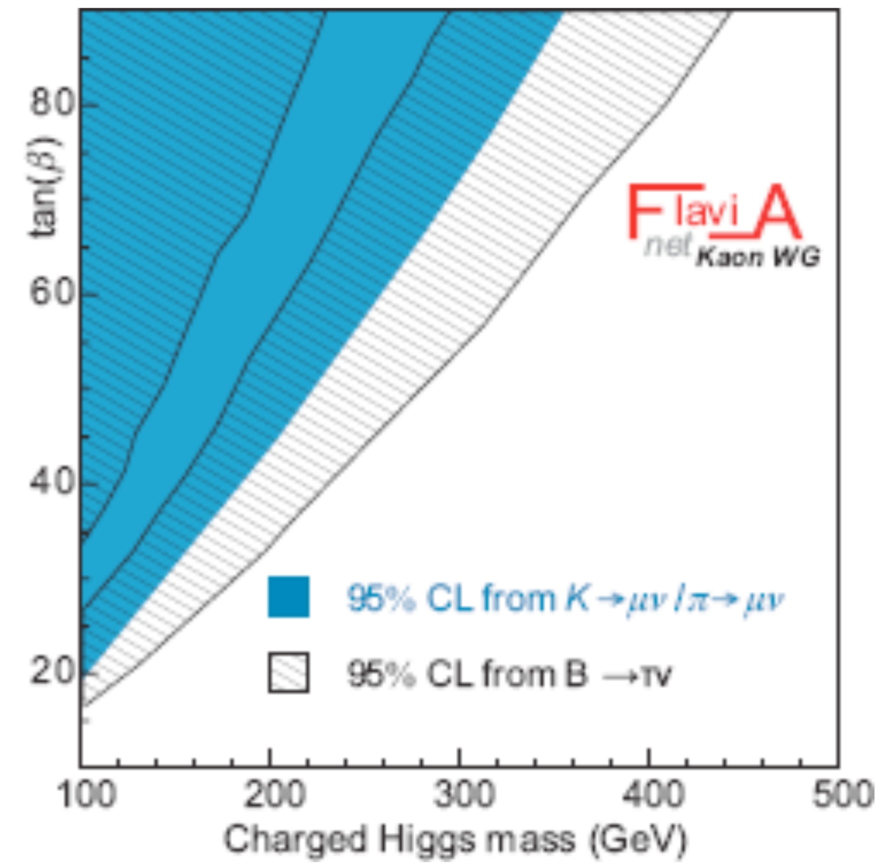
- *Bounds on **non-universality** :* $G_F^{CKM} = 1.1663(4)10^{-5}\ GeV^{-2}$

$$G_\mu = 1.166371(6)10^{-5}\ GeV^{-2}$$

$$G_F^{EW} = 1.1656(11)10^{-5}\ GeV^{-2}$$

Can be tree-level (mixing with heavy quark) or loop induced (squarks vs sleptons)

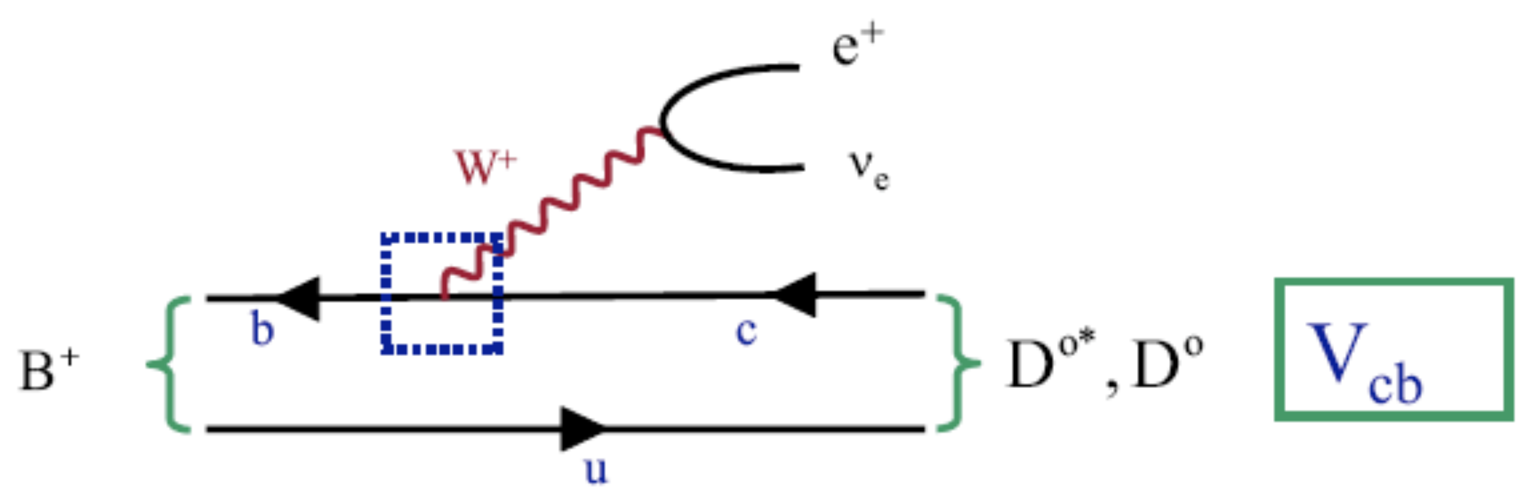
- *Bounds on scalar currents, eg charged Higgs interactions in K_{l2}*



Determination of A

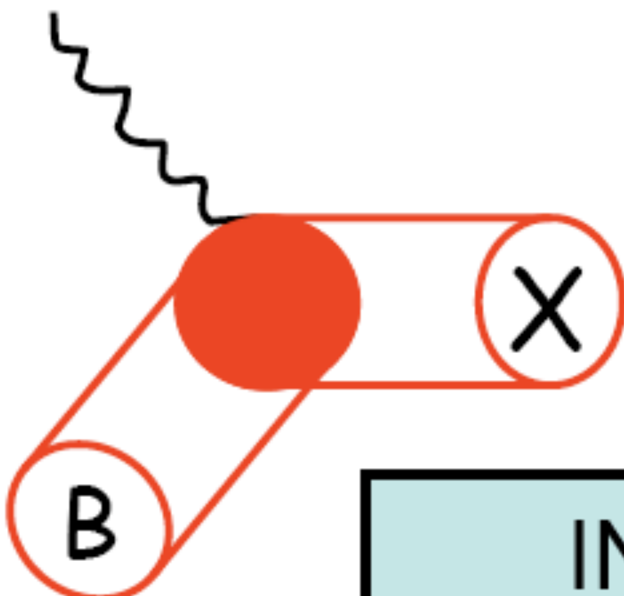
$$\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

A can be determined from either V_{cb} or V_{ts}



Two roads to V_{cb}: inclusive and exclusive

Inclusive vs exclusive B decays



Simplicity: ew or em currents probe the B dynamics

INCLUSIVE	EXCLUSIVE
OPE: non-pert physics described by B matrix elemnts of local operators can be extracted by exp suppressed by $1/m_b^2$	Form factors: in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

As precision increases, simplicity evaporates...

Exclusive decays: $B \to D^* l\nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A (1 + \delta_{1/m^2})$$

Recent progress in the measurement of slopes and shape parameters Despite extrapolation, exp error ~2%

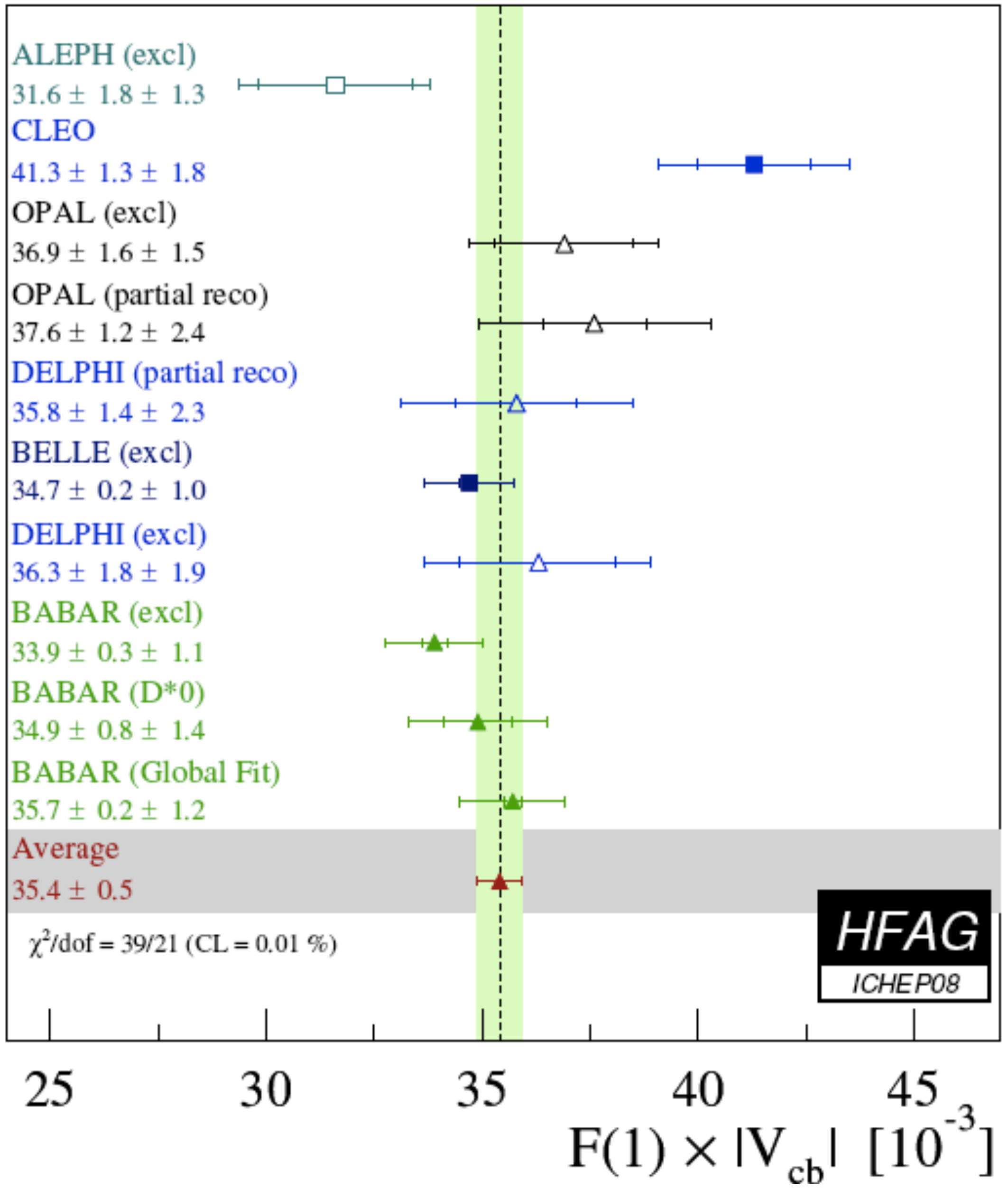
Main problem is normalization F(1): ***non-pert quantities relevant for excl decays cannot be experimentally determined***

New and **only** unquenched Lattice QCD:
F(1) =0.921(24) Laiho et al 2008, HQET, double ratio

$|V_{cb}|=38.2(0.5)(1.1)\times 10^{-3}$

~2.4σ from inclusive determination

NB Heavy Quark Sum rules give higher $|V_{cb}|$: F(1)=0.87(4) Uraltsev in agreement with inclusive *work in progress*



Lattice promising alternative: step scaling, w dependence, only quenched de Divitiis et al

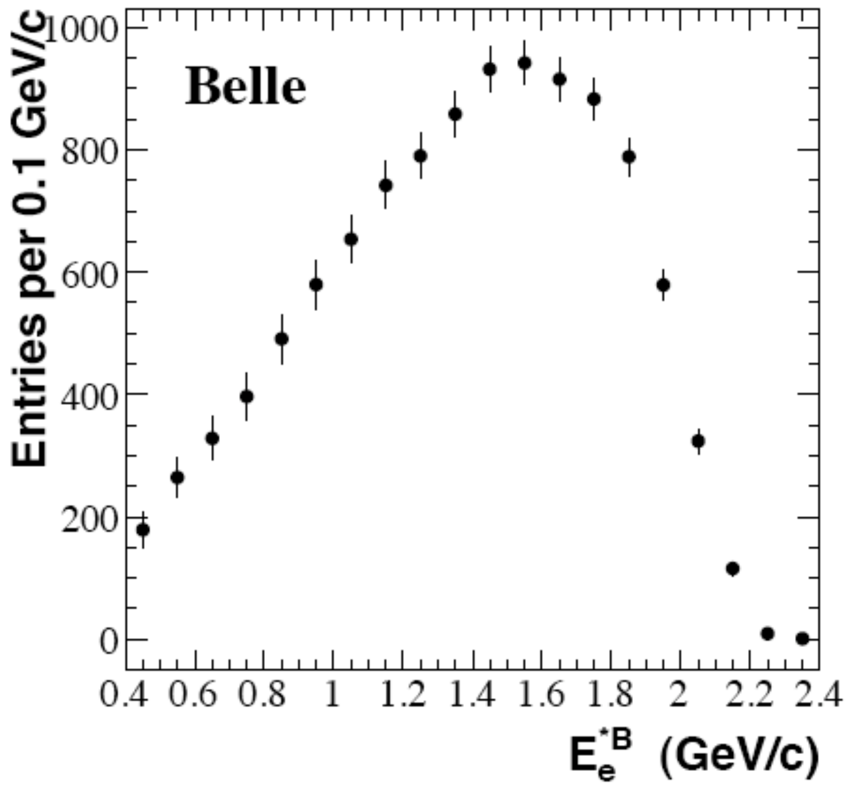
$B \to Dl\nu$ gives consistent but much less precise results

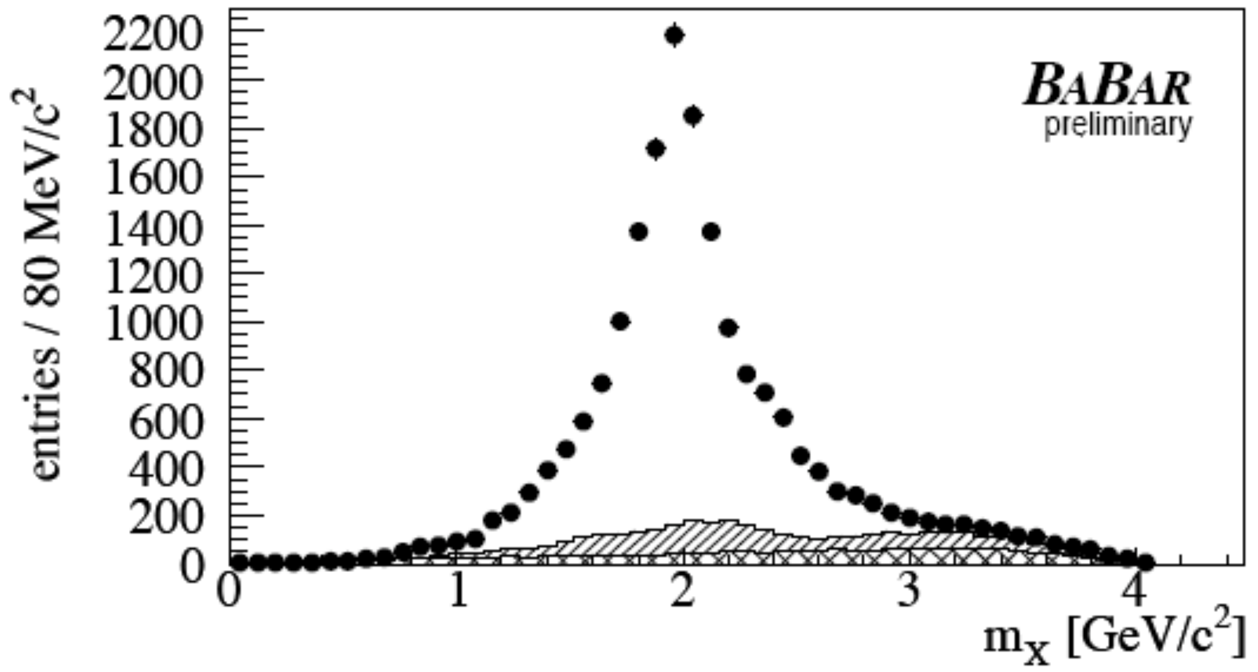
Inclusive $|V_{cb}|$: basic features

- ***Simple idea:*** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of matrix elements of local operators
- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***
- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle_\mu$$

Fitting OPE parameters to the moments





Total ***rate*** *gives $|V_{cb}|$, global* ***shape*** *parameters (moments of the distributions) tell us about B structure*

OPE parameters describe universal properties of the B meson and of the quarks

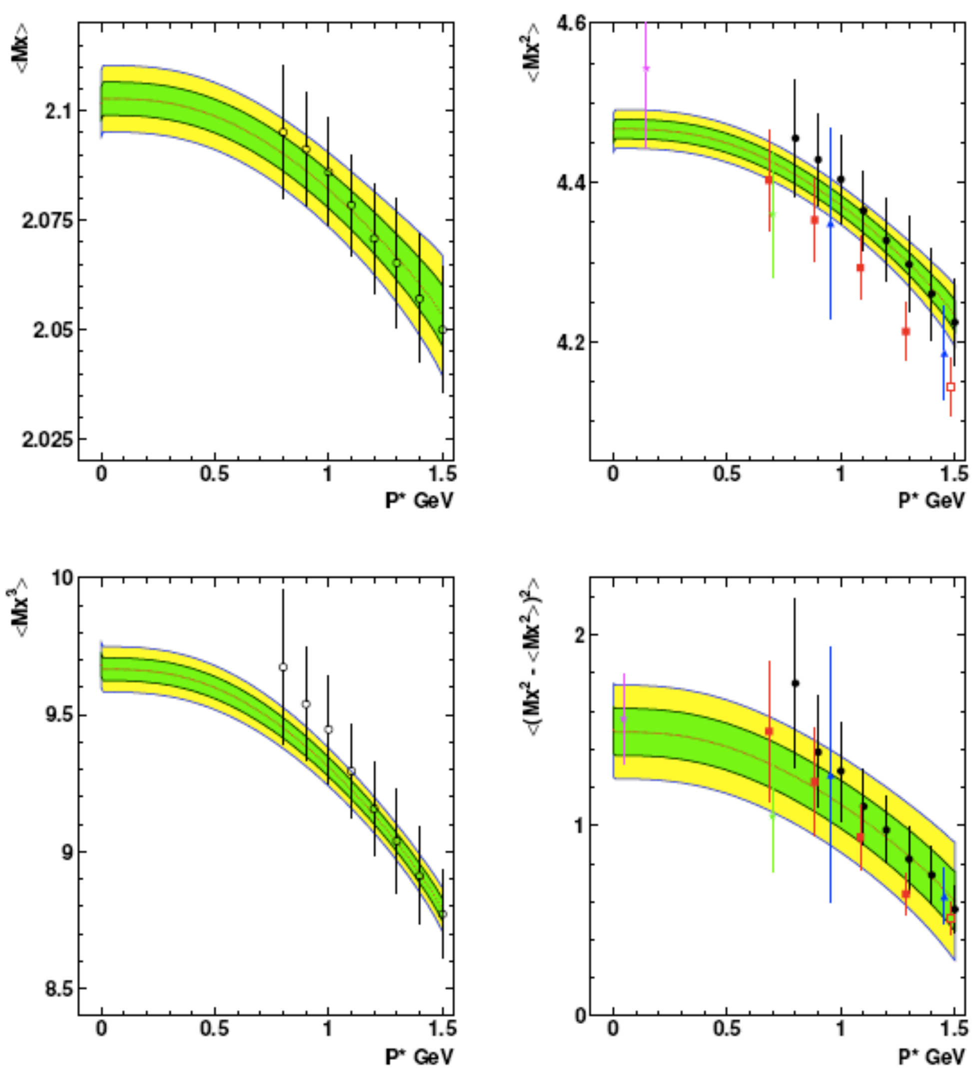
Global fit (kinetic scheme)

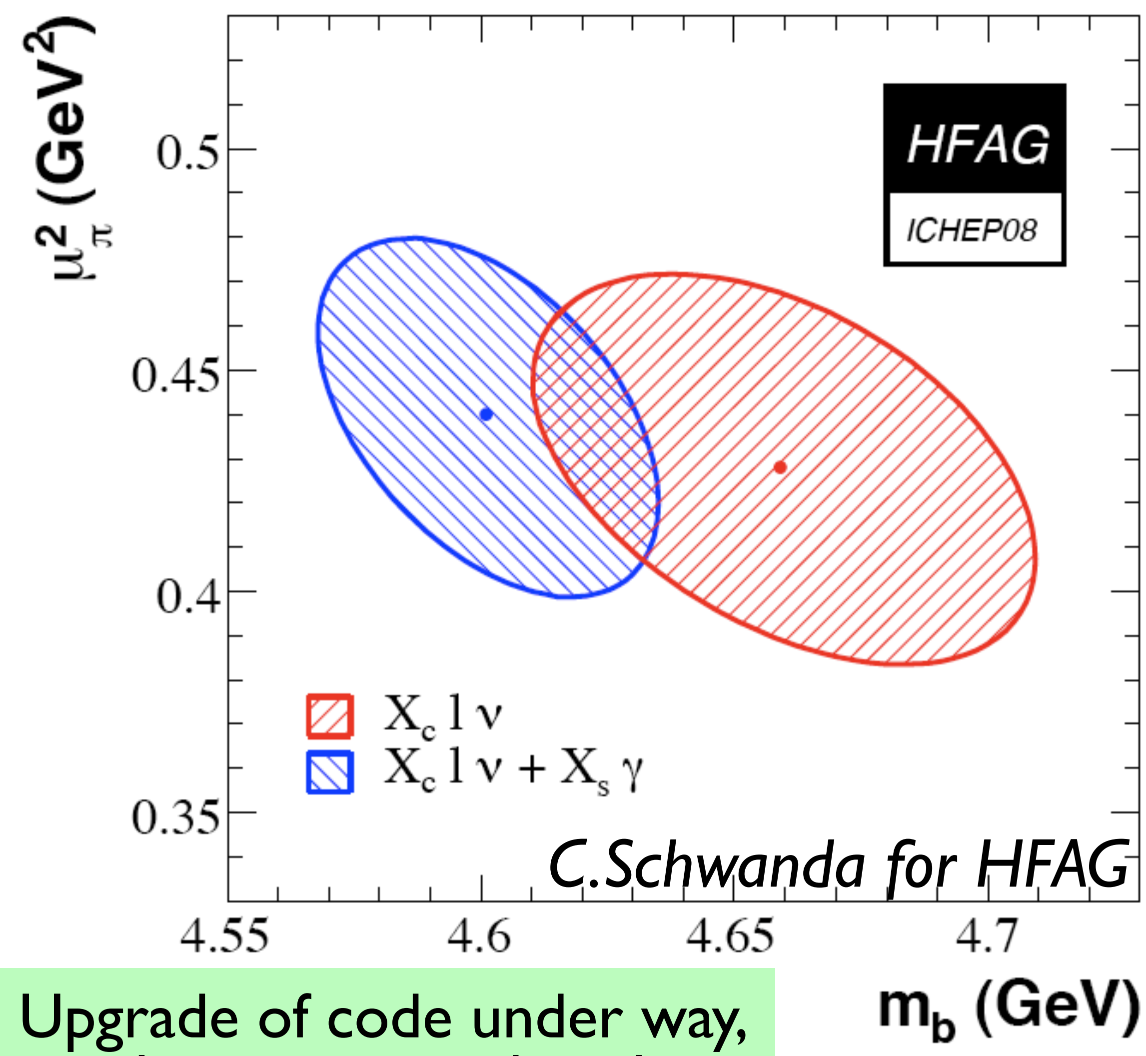
Inputs	$\lvert V_{cb}\rvert\ 10^3$	m_b^{kin}	χ^2/ndf
$b\to c$ & $b\to s\gamma$	41.67(44)(58)	4.601(34)	29.7/57
$b\to c$ only	41.48(48)(58)	4.659(49)	24.1/46

Based on PG, Uraltsev & Benson et al

***In the kinetic scheme** the contributions of gluons with energy below $\mu\approx 1$GeV are absorbed in the OPE parameters*

Here *scheme* means also a number of different assumptions and a recipe for theory errors





Upgrade of code under way, discrepancy reduced

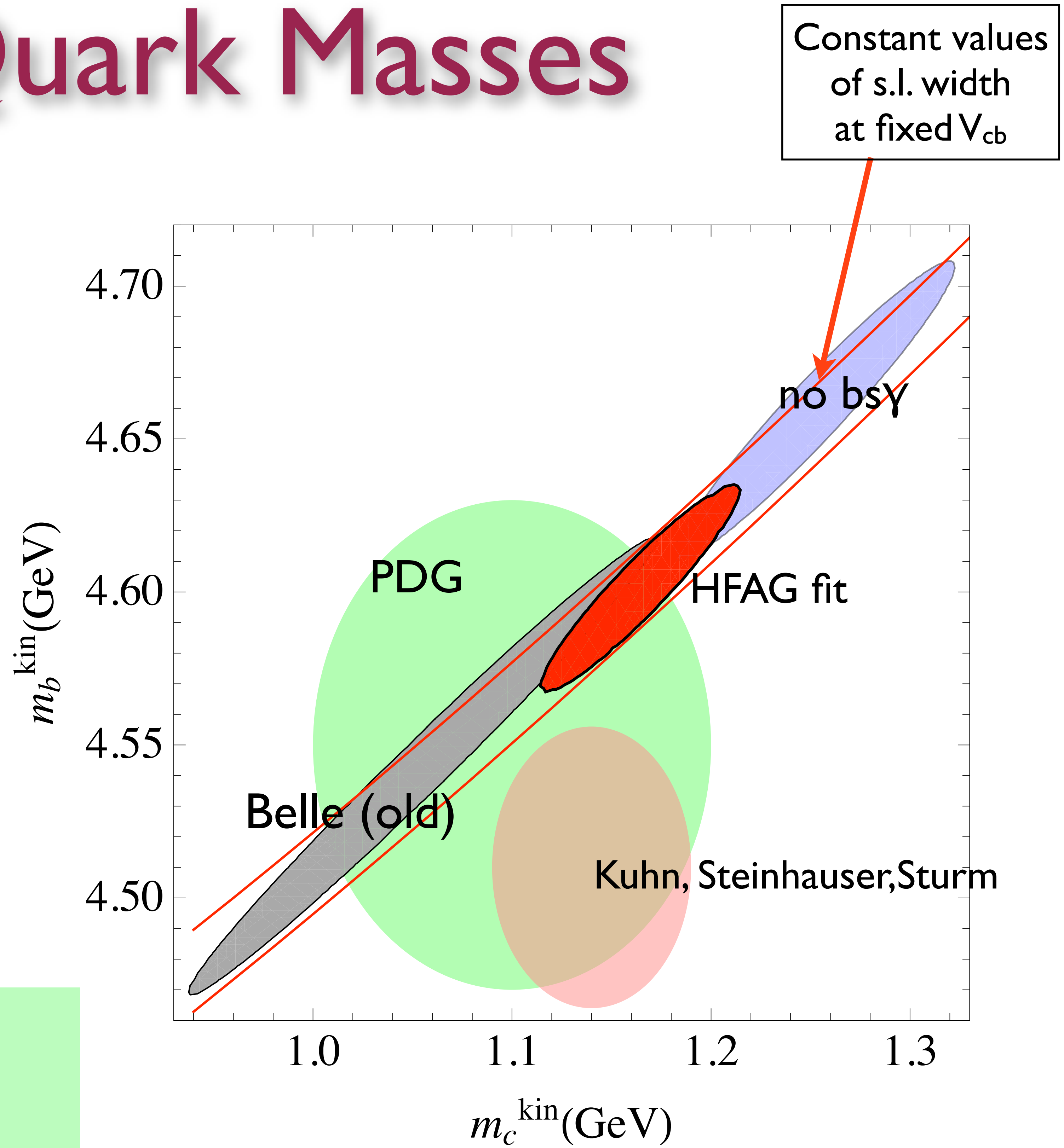
Fits & Quark Masses

Assume quark-hadron duality but self-consistently check it

*Semileptonic moments identify a strip in (m_b,m_c) plane along which the minimum is **shallow**.*

Inclusion of radiative moments controversial as OPE fails at O(α_s). At present the role of radiative moments in the fits is similar to using PDG bound on m_b.

Incl. |V_{cb}| looks OK
Heavy quark masses?



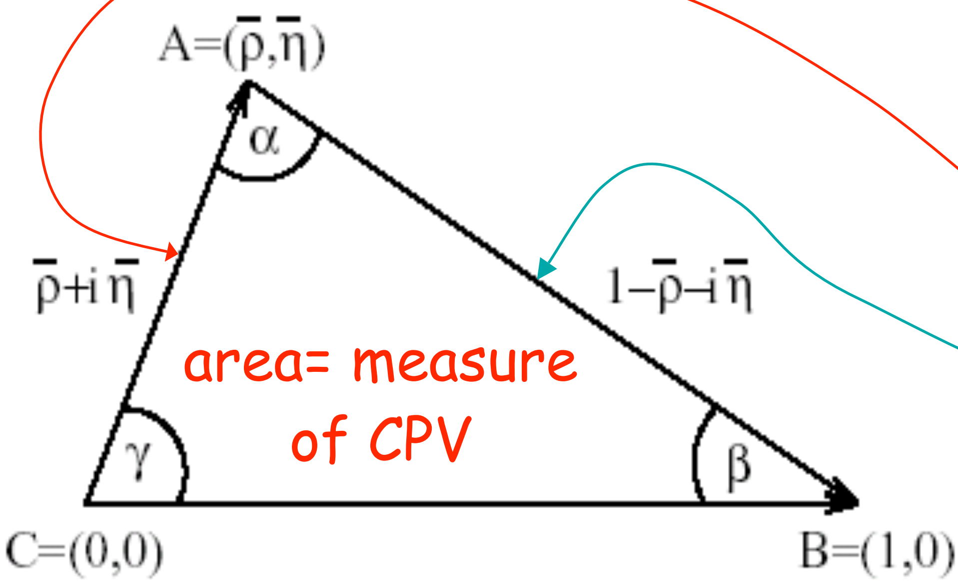
The Unitarity Triangle

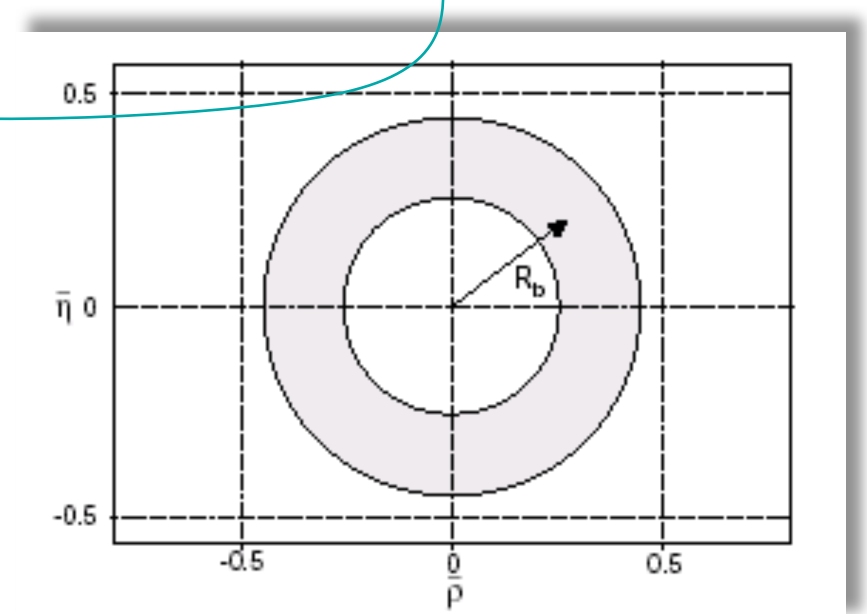
$$V_{ji}V^*_{jk} = \delta_{ik}$$

Unitarity determines several triangles in complex plane

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \quad O(\lambda^3)$$

$$1 + \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0$$

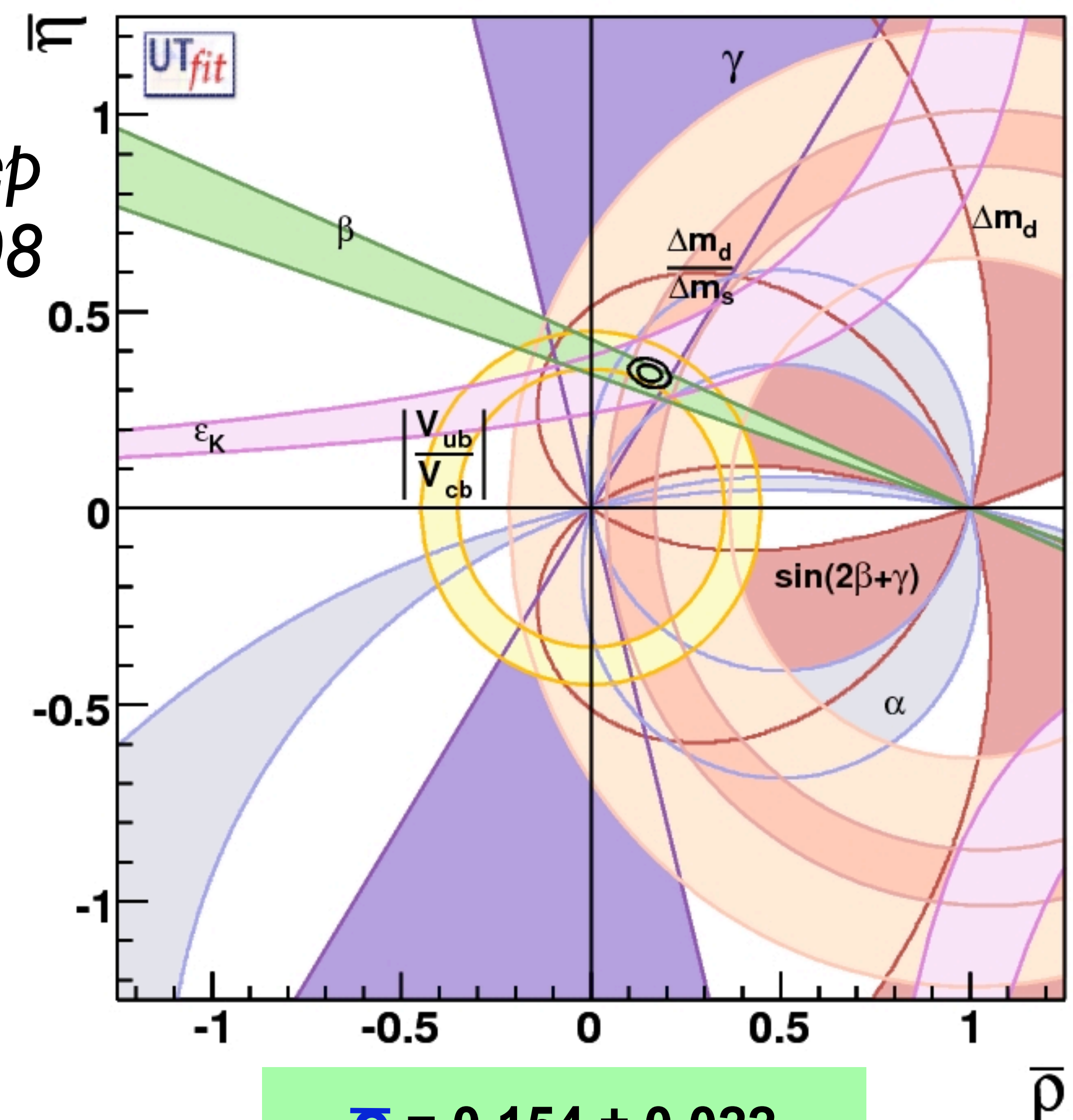


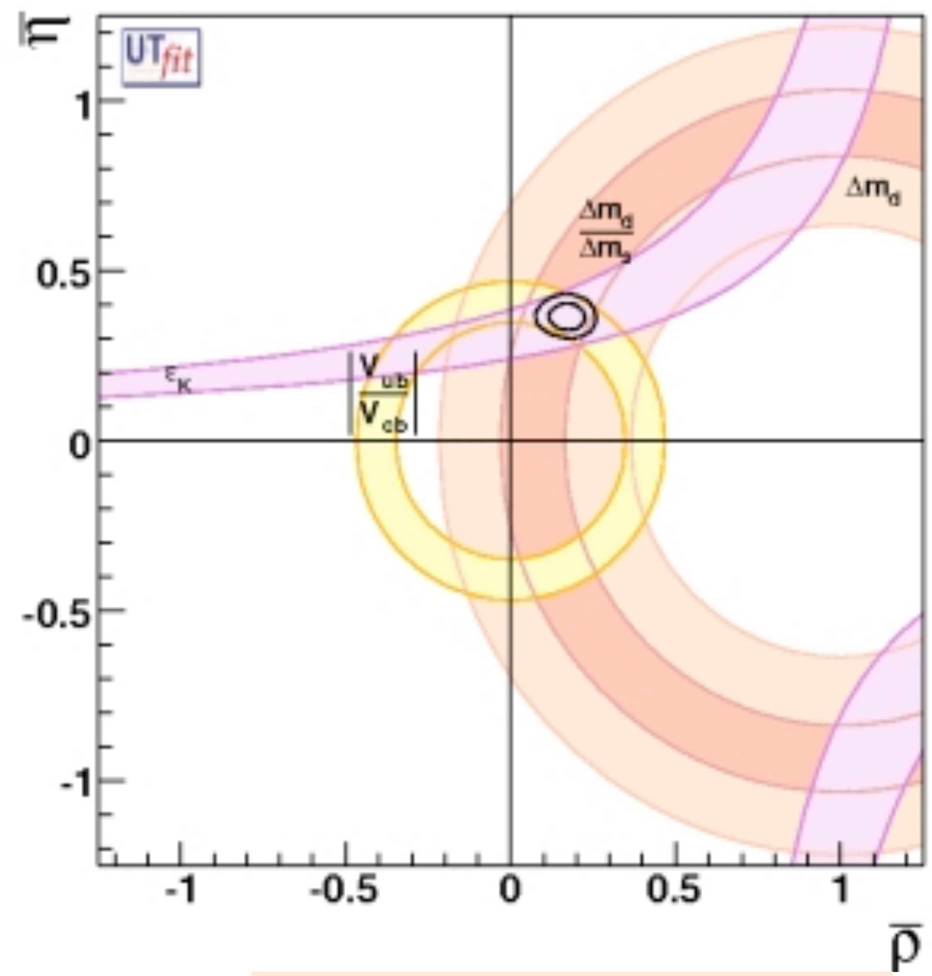


V_{td} **cannot** be accessed directly: need FCNC loops sensitive to new physics eg B_d, B_s mixing

The Unitarity Triangle

Ichep 2008





$\bar{\rho} = 0.154 \pm 0.022$

$\bar{\eta} = 0.342 \pm 0.014$

UTfit inputs:
$\xi=1.21(4)$ $B_K=0.75(7)$

Almost identical results by CKMfitter @ ICHEP 2008

$B_K=0.725(50)$ Lellouch LAT08

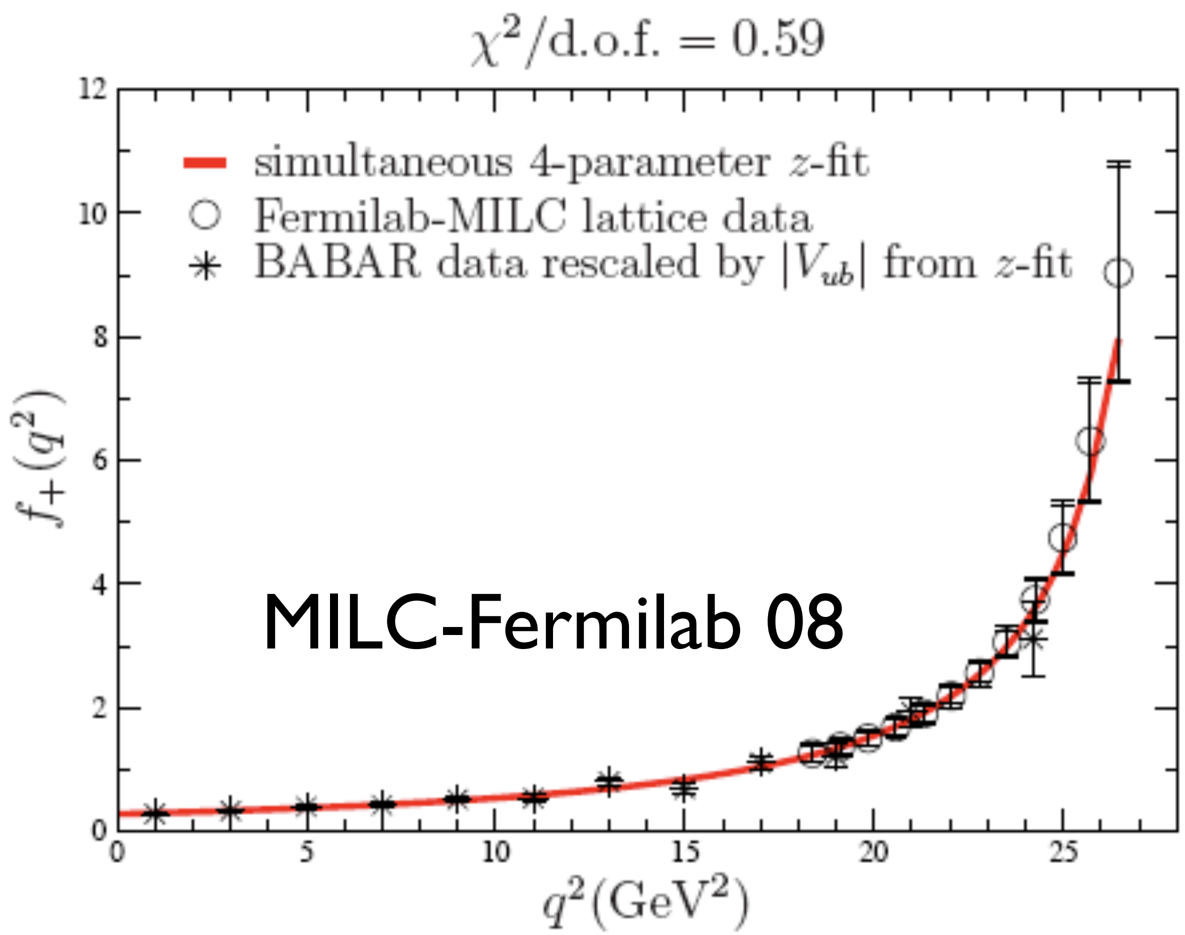
$\sin 2\beta_{\text{charmonium}} = 0.672 \pm 0.024$ HFAG

getting closer to 5% accuracy?

Exclusive determination of $|V_{ub}|$

High exp accuracy. Various parameterizations based on analyticity etc + experimental data on the q^2 spectrum: model independently

$$|V_{ub}|f_+(0) = (9.1 \pm 0.6(\text{shape}) \pm 0.3(\text{BR})) \times 10^{-4}$$



ff on lattice or with LC sum rules, no symmetry helps. LCSR cannot be much improved, while lattice can

LCSR: $|V_{ub}| = (3.5 \pm 0.4 \pm 0.1) \times 10^{-4}$

Ball-Zwicky

$$|V_{ub}| = \left(3.5 \pm 0.4|_{th} \pm 0.2|_{shape} \pm 0.1|_{BR}\right) \times 10^{-3}$$

Duplancic et al

$$|V_{ub}| \times 10^3 = 3.38 \pm 0.36.$$

MILC-Fermilab 08

$|V_{ub}| \text{x} 10^3 = 3.36 \pm 0.23$ *Bourrelly et al 08*

Combines older lattice results +LCSR with larger (!) errors and a new parameterization

$|V_{ub}|$ inclusive

$|V_{ub}|$ from total BR($b\to ul\nu$) like incl $|V_{cb}|$ but we need kinematic cuts to avoid the ~100x larger $b\to cl\nu$ background:

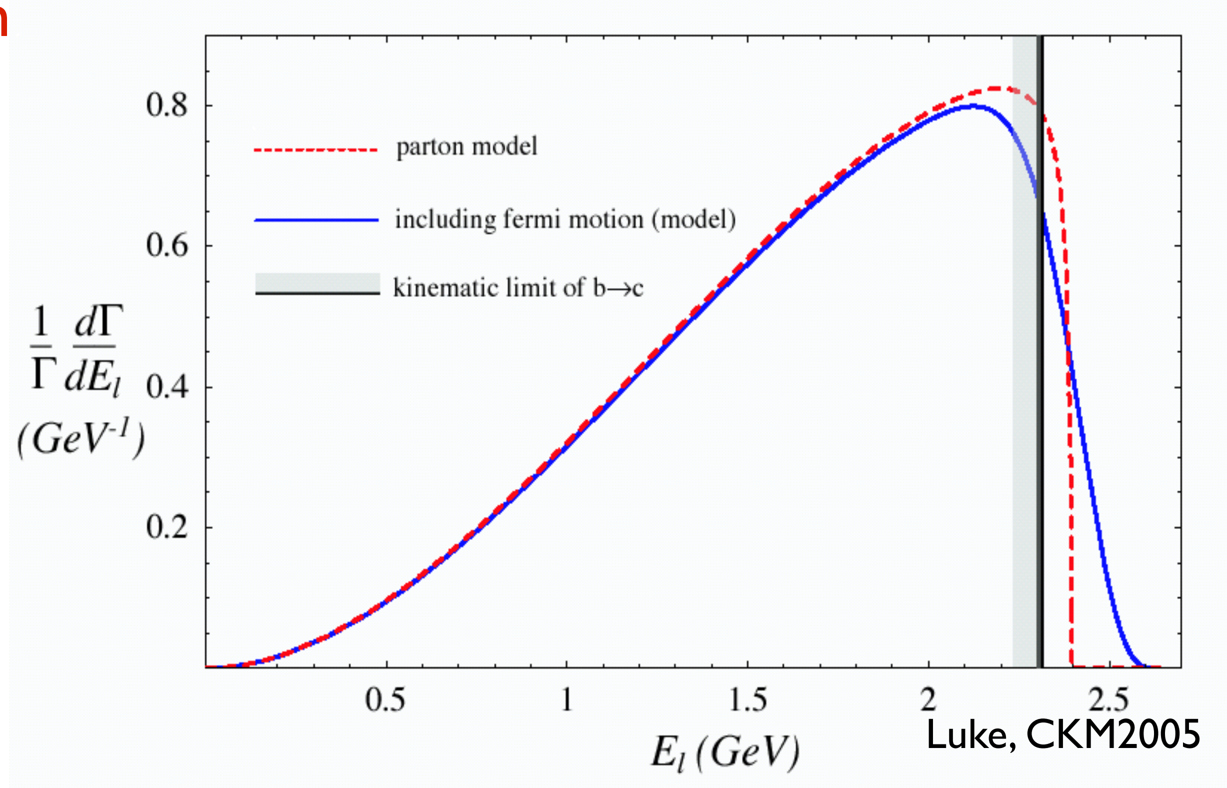
$m_X < M_D$ $\quad$ $E_l > (M_B^2-M_D^2)/2M_B$ $\quad$ $q^2 > (M_B-M_D)^2$ ...

or combined (m_X,q^2) cuts

The cuts destroy convergence of the OPE that work so well in $b\to c$.

OPE expected to work only away from pert singularities

Rate becomes sensitive to "local" b-quark wave function properties like Fermi motion   Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** f(k+)



Luke, CKM2005

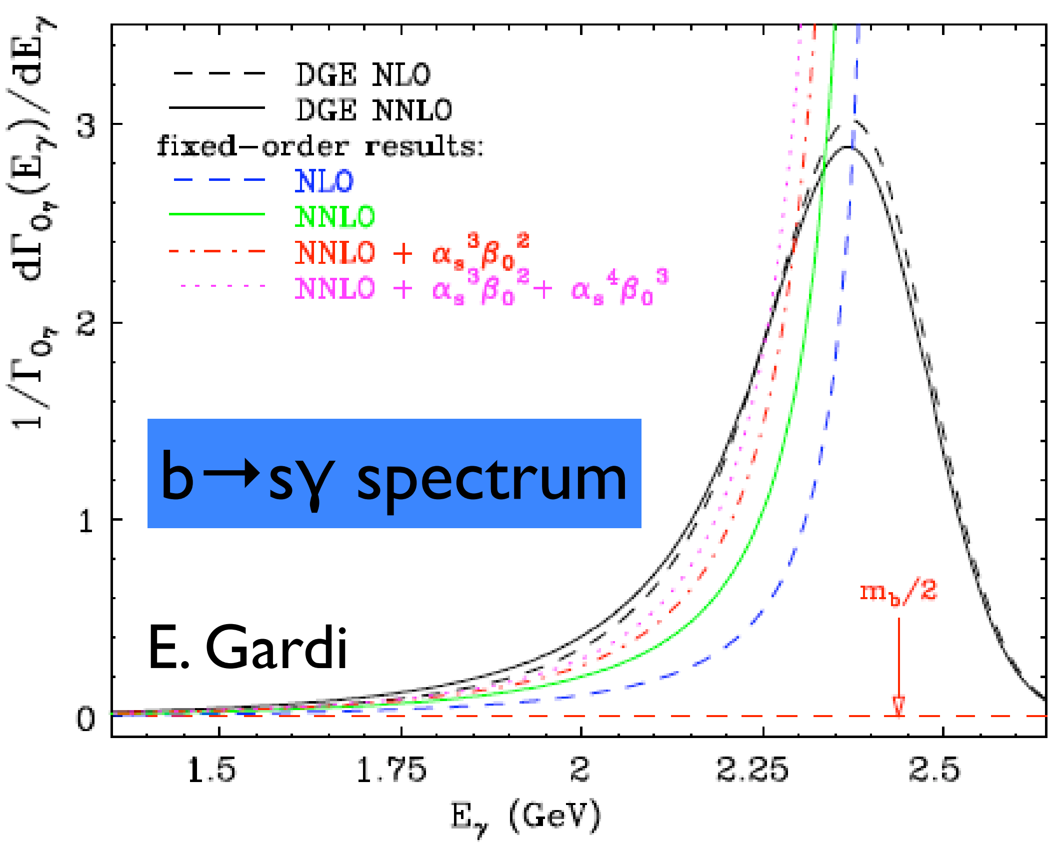
SF from perturbation theory

Resummed perturbation theory is qualitatively different: Support properties; stability! *(E. Gardi)*

b *quark* SF emerges from soft-gluon resummed pQCD but needs resummation of running coupling corrections and power corrections for b →B

Dress Gluon Exponentiation (DGE) by Gardi et al employs renormalon resummation to define Fermi motion. Power corrections can be partly accomodated.

Aglietti et al (ADFR) use Analytic Coupling in the IR



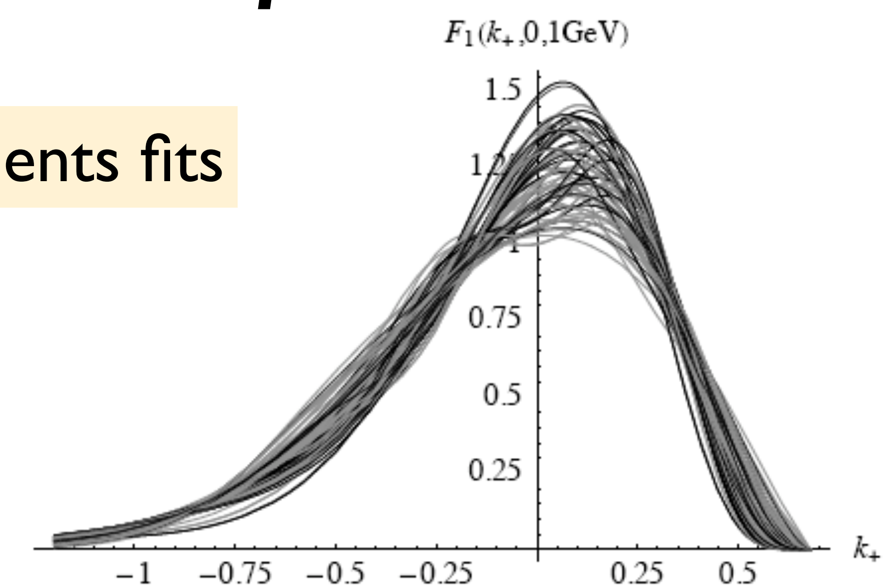
The SF in the OPE

Local OPE has also threshold singularities and SF can be equivalently introduced resumming dominant singularities *Bigi et al, Neubert*

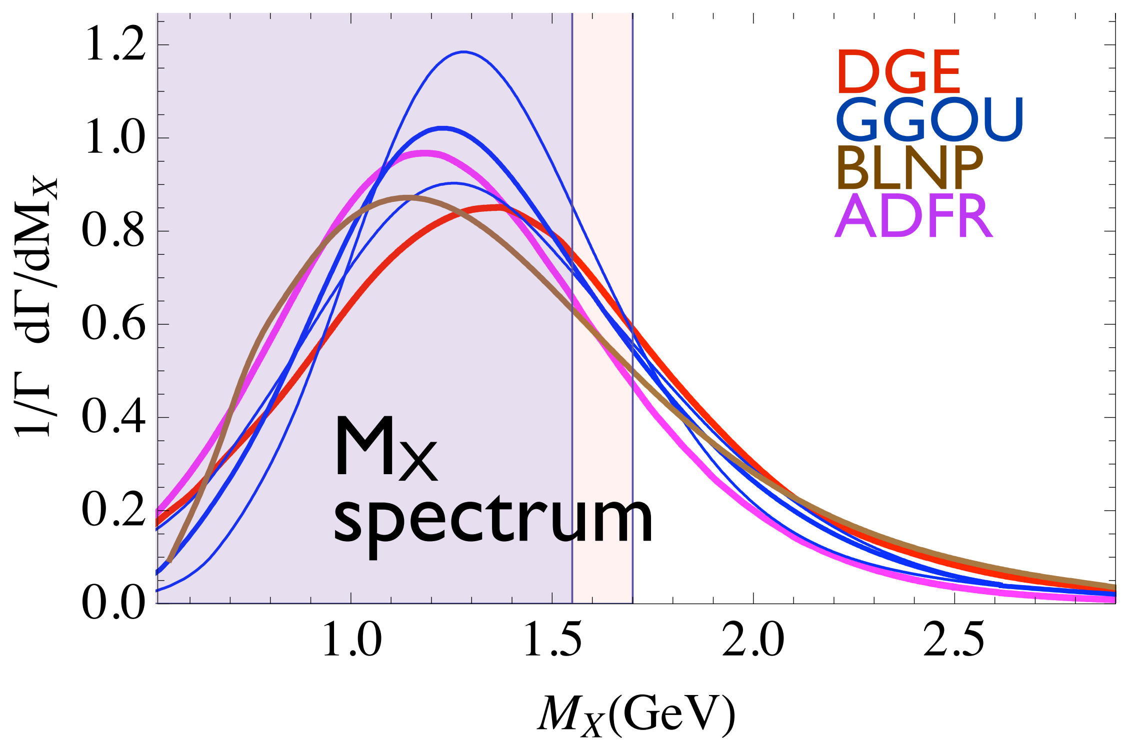
Fermi motion can be parameterized within the OPE like PDFs in DIS. At leading order in m_b only a single universal function of one parameter enters (SF).

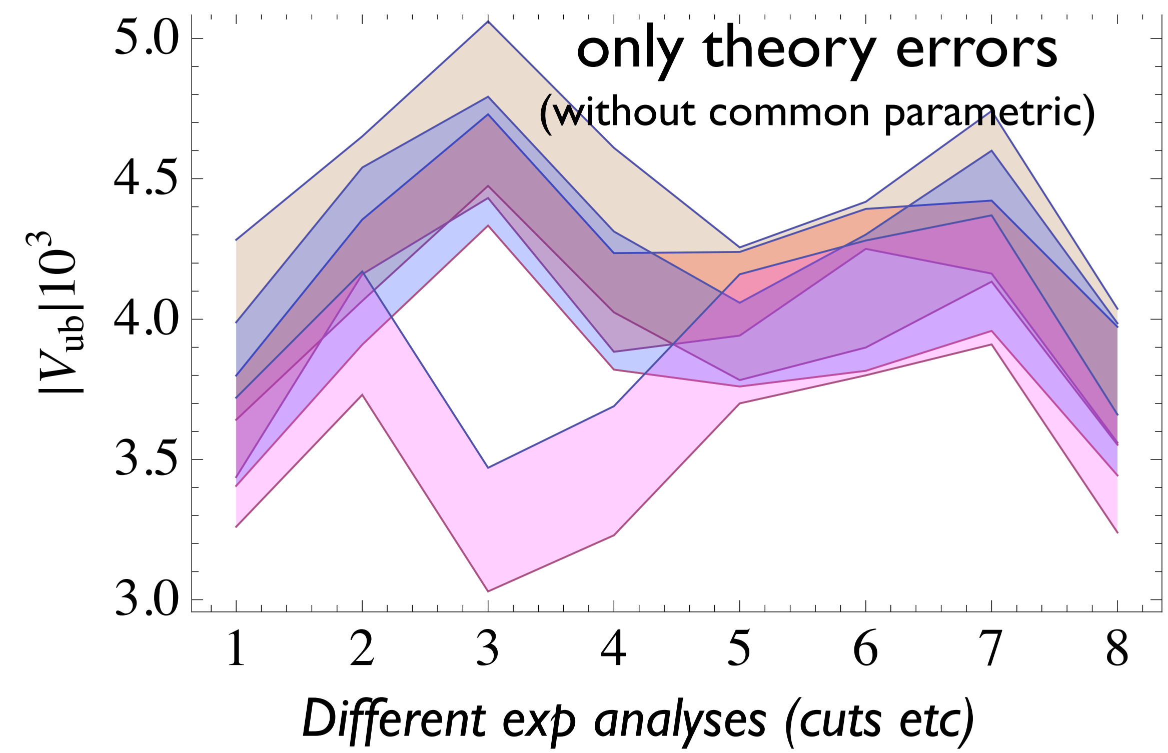
Unlike resummed pQCD, ***the OPE does not predict the SF****, only its first few moments. One then* ***needs an ansatz for its functional form****.*

$$\int dk_+ \; k_+^n \; F_i(k_+, q^2) =$$ local OPE prediction ⇐ moments fits

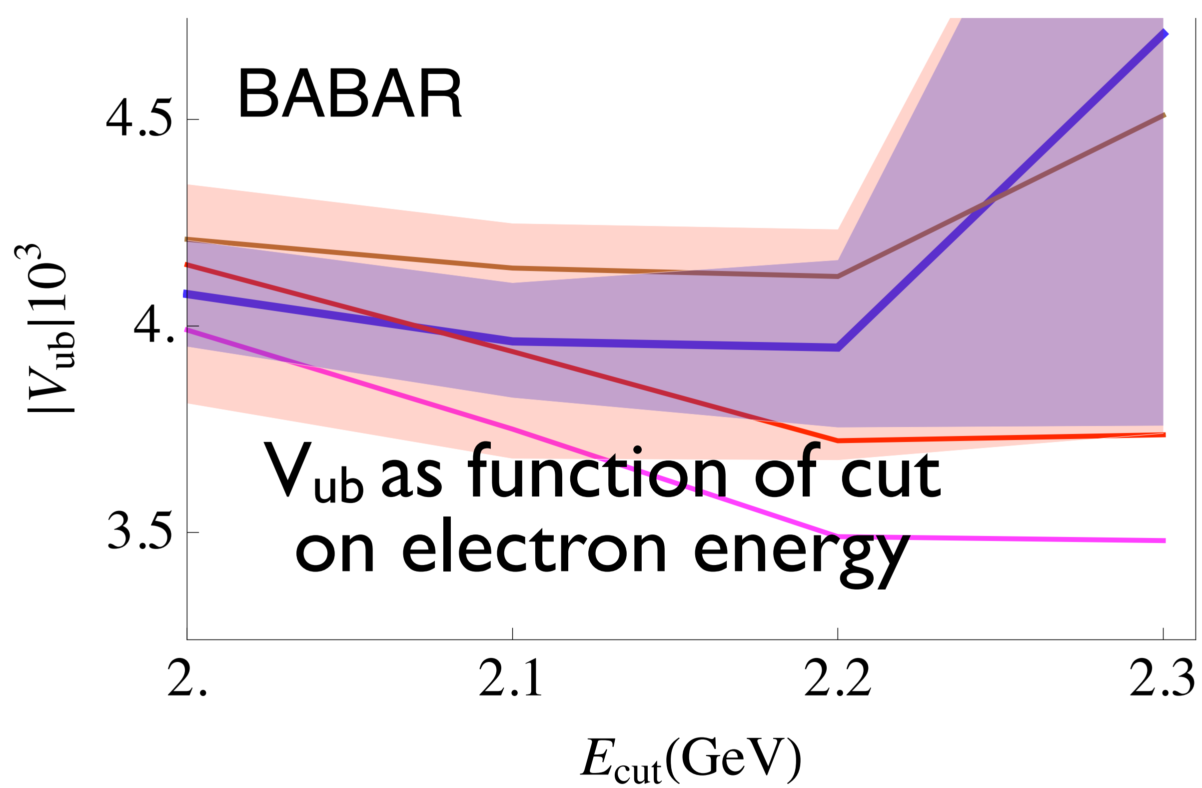


Two very different implementations:
PG,Giordano,Ossola,Uraltsev (GGOU)
Bosch,Lampe,Neubert,Paz (BLNP)





- Comparison at common inputs
- Overall good agreement with one exception **SPREAD WITHIN TH ERRORS**
- Precise measurement of spectra may allow to discriminate between them, but difficult for exp
- Strong m_b dependence $V_{ub} \propto m_b^{4.5}$



- *Not all observables are equally* clean. eg high q^2 tail is sensitive to WA
- Need spectra: only way to test frameworks (see E_l spectrum).
- More inclusive measurements, less dependence on m_b
- Theory errors are partly parametric (m_b)

	Average $\|V_{ub}\|$x10^3
DGE	$4.26(14)_{ex}{}^{+19}_{-13}$
BLNP	$4.31(16)_{ex}{}^{+32}_{-27}$
GGOU	$3.96(15)_{ex}{}^{+20}_{-23}$

2.1, 1.9, 1.3σ from $B\to\pi l\nu$ (MILC-FNAL)

3.1, 2.4, 1.5σ from UTFit (because of sin2β)

NEW preliminary Belle Multivariate analysis only E_l>1GeV

$$|V_{ub}| = (4.42 \pm 0.26^{+0.14}_{-0.22}) \times 10^{-3}$$

GGOU

2.1σ from excl, 2.5σ from UTFit

This includes about 90% of the rate really inclusive measurement, no need for SF. Only crucial input m_b needs to be confirmed!

NEW PHYSICS?

eg LR models Chen,Nam


More
tension?

CP violation in B vs K decays

Recent lattice results for B_K and previously neglected contributions lead to 15% smaller ε_K, in conflict with $\sin 2\beta$

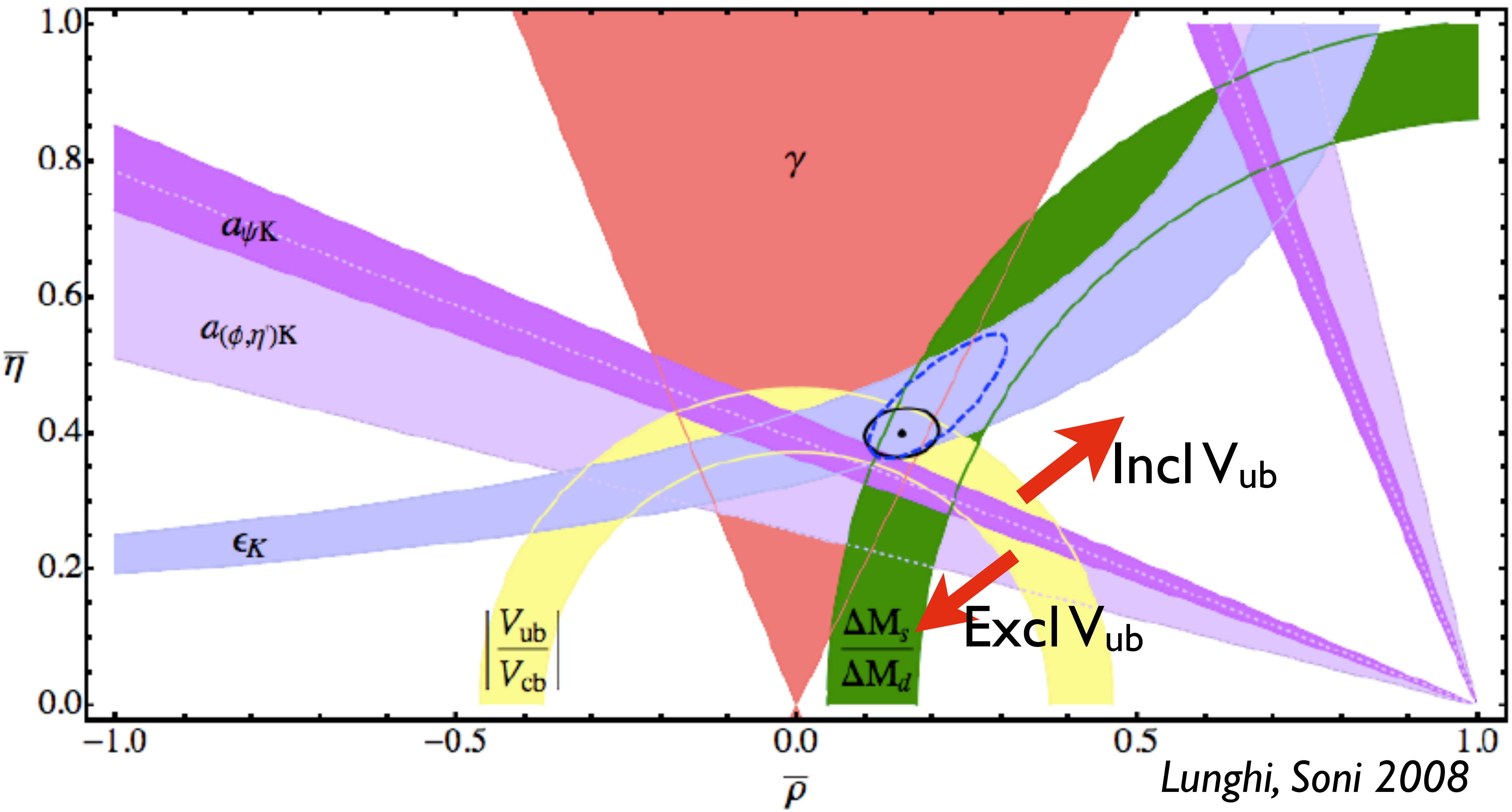
Assuming SM, use $\sin 2\beta$, $|V_{cb}|$, λ, $\Delta M_s/\Delta M_d$, ξ, B_K=0.720(39) *RBC-UKQCD*

$|\varepsilon_K|^{SM} = (1.78 \pm 0.25) 10^{-3}$ vs $|\varepsilon_K|^{exp} = (2.229 \pm 0.012) 10^{-3}$

1.8-2.1σ *depending on assigned errors* *Buras,Guadagnoli,Lunghi Soni*

Easy to find new physics explanations, even in CMFV

UTfit without $\sin 2\beta_{exp}$=0.672(24) gives $\sin 2\beta$=0.732±0.034 ➠ 1.5σ
CKMfitter finds 2.5 tension between $\sin 2\beta_{exp}$ and $B \to \tau\nu$ (depends strongly on B_B)



Lunghi, Soni 2008

Perhaps sin2β is simply too low...

... or incl V_{ub} and B_K both wrong

Conclusions

CKM is overall in a good shape

Great progress in lattice calculations

A few ~2σ problems: inclusive and exclusive $|V_{xb}|$ tend to clash, *sin*2β seems a bit low, too early to invoke new physics

We need better $m_{b,c}$ *determinations* and to exploit data to check theory calculations (shapes, distributions etc)

Important constraints on new physics but no time to discuss them