
Electromagnetic properties of neutrino and applications in astrophysics
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La Thuile,
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Moscow State University

Carlo Giunti, A.S. :

- *"Neutrino electromagnetic properties"*
  arXiv:0812.3646, to appear in Phys.Atom.Nucl. (2009)

A.S. :

- *"Neutrino magnetic moment: a window to new physics"*
  arXiv:0812.4716, to appear in Nucl.Phys.B (2009)

…Why

Electromagnetic properties of ν

provide a kind of window / bridge to

NEW Physics ?

… Up to now, in spite of reasonable efforts,

- ***NO** any unambiguous experimental confirmation in favour of nonvanishing ν **em** properties ,*

- *available experimental data in the field do not rule out possibility that ν have **"ZERO" em** properties.*

- *… However, in course of recent development of knowledge on ν mixing and oscillations,*

Recent studies (exp. & theor.) of

flavour conversion of

solar, atmospheric, reactor and accelerator

neutrinos have conclusively established that

neutrinos have non-zero mass

and they mix among themselves

that provides the first evidence of new physics

beyond the standard model

Neutrino mass

$$m_\nu \neq 0 \;!$$

$$\Rightarrow$$

Neutrino mass

$$m_\nu \neq 0 \;!$$

$\Longrightarrow$

Neutrino magnetic moment ν ⊙

$$\mu_\nu \neq 0 \;?\; (!)$$

⊛ Lee, Shrock 1977; Lee, Shrock, Fujikawa 1980

...Massive neutrino electromagnetic properties...

Theory (Standard Model with ν_R)

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3\cdot 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1\,eV}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee
Shrock, 1977; Fujikawa
Shrok, 1980

In the Standard Model : $m_\nu = 0$,
there is no ν_R $\Rightarrow$
ν magnetic moment $\mu_\nu = 0$.

Thus, $\mu_\nu \neq 0$ ← beyond the SM. !

… puzzling

ν electromagnetic properties

something that is tiny or probably even does not exist at all…

ν exhibits unexpected properties (puzzles)

W. Pauli, 1930

Pauli himself wrote to **Baade**:
"Today I did something a physicist should never do. I predicted something which will never be observed experimentally...".

- neutron — **now we know that it is** ***neutrino*** *E.Fermi, 1933*
- neutral — **now we know that** $\boxed{q_\nu \neq 0}$ **in plasma**
- and probably $\mu_\nu \neq 0$? !
- massless particle — **now we know that** $m_\nu \neq 0$
- ν **very important player** **(astrophysics, cosmology etc. . .)**

Outline

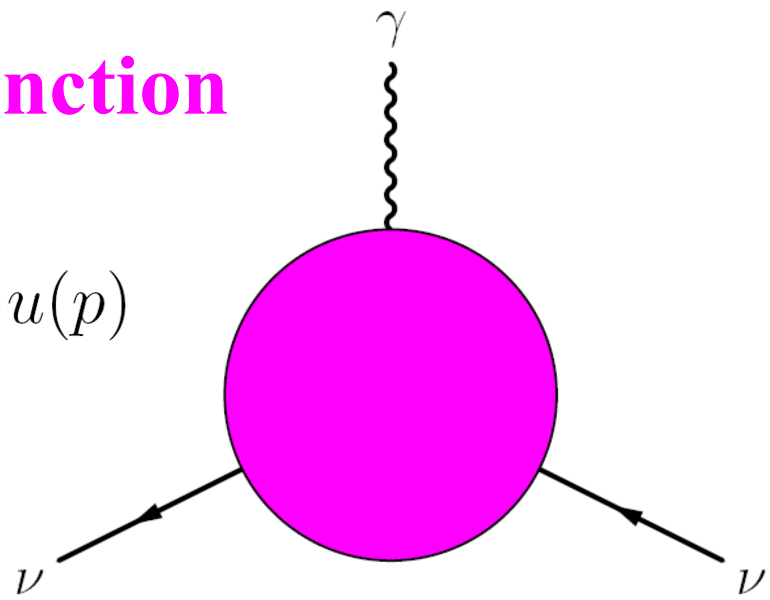
- ν electromagnetic properties - theory
- ν magnetic moment - experiment
- ν magnetic moment - astrophysical bounds

Outline

0. Introduction
1. ν magnetic moment in experiments
2. New experimental result on μ_ν
3. ν electromagnetic properties - theory
   - 3.1 ν vertex function
   - 3.2 μ_ν (arbitrary masses)
   - 3.3 relationship between m_ν and μ_ν
   - 3.4 ν vertex function in case of flavour mixing
   - 3.5 ν dipole moments in case of mixing
   - 3.6 μ_ν in left-right symmetry models
   - 3.7 astrophysical bounds on μ_ν
   - 3.8 ν millicharge (Red Gaints cooling etc)
   - 3.9 ν charge radius and anapole moment
   - 3.10 ν electromagnetic properties in matter and e.m.f.
4. Effects of ν electromagnetic properties
   - 3.11 ν radiative decay, *Ch* radiation and *Spin Light of* ν in matter
   - 3.12 ν radiative $2\times\gamma$ - decay
   - 3.13 ν spin-flavour oscillations
5. Direct-Indirect influence of e.m.f. on
6. Conclusion

electromagnetic vertex function

$$< \psi(p')|J_\mu^{EM}|\psi(p) >= \bar{u}(p')\Lambda_\mu(q,l)u(p)$$



Matrix element of electromagnetic current is a Lorentz vector

$\Lambda_\mu(q,l)$ ***should be constructed using***

matrices $\hat{\mathbf{1}}, \quad \gamma_5, \quad \gamma_\mu, \quad \gamma_5\gamma_\mu, \quad \sigma_{\mu\nu},$

tensors $g_{\mu\nu}, \quad \epsilon_{\mu\nu\sigma\gamma}$

vectors q_μ ***and*** l_μ

$$q_\mu = p'_\mu - p_\mu, \; l_\mu = p'_\mu + p_\mu$$

Vertex function $\Lambda_\mu(q,l)$ → **there are three sets of operators:**

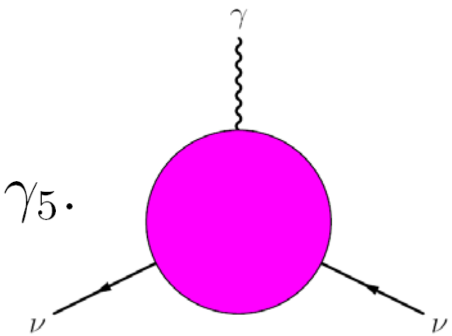
- $\hat{\mathbf{1}} q_\mu, \quad \hat{\mathbf{1}} l_\mu, \quad \gamma_5 q_\mu, \quad \gamma_5 l_\mu$

  $\not{q} q_\mu, \quad \not{l} q_\mu, \quad \gamma_5 q_\mu, \quad \gamma_5 \not{q} q_\mu, \quad \gamma_5 \not{l} q_\mu, \quad \sigma_{\alpha\beta} q^\alpha l^\beta q_\mu, \quad \{q_\mu \leftrightarrow l_\mu\}$

- $\gamma_\mu, \quad \gamma_5 \gamma_\mu, \quad \sigma_{\mu\nu} q^\nu, \quad \sigma_{\mu\nu} l^\nu.$

- $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} q^\nu, \quad \epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} l^\nu, \quad \epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} q_\beta q^\sigma l^\gamma,$

  $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} l_\beta q^\sigma l^\gamma, \quad \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \hat{\mathbf{1}}, \quad \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \gamma_5.$



vertex function (using Gordon-like identities)

$$\boxed{\begin{aligned}\Lambda_\mu(q,l) = {} & f_1(q^2) q_\mu + f_2(q^2) q_\mu \gamma_5 + f_3(q^2) \gamma_\mu + \\ & f_4(q^2) \gamma_\mu \gamma_5 + f_5(q^2) \sigma_{\mu\nu} q^\nu + f_6(q^2) \epsilon_{\mu\nu\rho\gamma} \sigma^{\rho\gamma} q^\nu,\end{aligned}}$$

the only dependence on q^2 **remains because** $p^2 = p'^2 = m^2, \; l^2 = 4m^2 - q^2$

Requirement of current conservation (electromagnetic gauge invariance)

$$\boxed{\partial_\mu j^\mu = 0}$$

$$f_1(q^2)q^2 + f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0,$$

$$f_1(q^2) = 0, \qquad f_2(q^2)q^2 + 2mf_4(q^2) = 0$$

$$\Lambda_\mu(q) \;=\; f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

charge

dipole electric and magnetic

anapole

Form Factors

Matrix element of electromagnetic current between neutrino states

$$\boxed{\langle\nu(p')|J_\mu^{EM}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p)}$$

where vertex function generally contains 4 *form factors*

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\sigma_{\mu\nu}q^\nu-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5+f_A(q^2)(q^2\gamma_\mu-q_\mu \not{q})\gamma_5$$

1. electric ***2. magnetic*** ***dipole*** ***3. electric*** ***4. anapole***

Hermiticity and **discrete symmetries** of EM current J_μ^{EM} **put constraints on form factors**

Dirac ν

1) ***CP invariance + hermiticity*** $\Longrightarrow$ $f_E = 0$,

2) ***at zero momentum transfer only electric charge*** $f_Q(0)$ ***and magnetic moment*** $f_M(0)$ ***contribute to*** $H_{int} \sim J_\mu^{EM} A^\mu$,

3) ***hermiticity* itself** $\Longrightarrow$ ***three form factors are real:*** $Im f_Q = Im f_M = Im f_A$.

Majoran ν

1) ***from CPT invariance (regardless CP or ~~CP~~) .***

$$f_Q = f_M = f_E = 0$$

EM properties $\Longrightarrow$ a way to distinguish **Dirac** and **Majorana** ν

Effective Lagrangian for the spin component of ν vertex

$$\boxed{L = \frac{1}{2}\bar{\nu}_j \sigma_{\eta\xi}(\beta_{ij} + \varepsilon_{ij}\gamma_5)\nu_i F^{\eta\xi} + \text{h.c.}\,,}$$

magnetic and **electric** moments

which couple together mass eigenstates

e.m. field tensor

$(\nu_i)_L$ and $(\nu_j)_R$ ⇒ change of the helicity states

- $\nu_i = \nu_j$ ⇒ diagonal moments
- $\nu_i \neq \nu_j$ ⇒ transitional moments

- $\varepsilon_{ii} = \beta_{ii} = 0$ for Majorana ν

E.M. properties

↳ a way to distinguish Dirac and Majorana ν

In general case matrix element of J_μ^{EM} can be considered between ν ν different initial $\psi_i(p)$ and final $\psi_j(p')$ states of different masses $p^2=m_i^2,\ p'^2=m_j^2$:

$$\boxed{<\psi_j(p')|J_\mu^{EM}|\psi_i(p)>=\bar{u}_j(p')\Lambda_\mu(q)u_i(p)}$$

and

$$\boxed{\begin{aligned}\Lambda_\mu(q)=\Big(f_Q(q^2)_{ij}+f_A(q^2)_{ij}\gamma_5\Big)(q^2\gamma_\mu-q_\mu\not{q})+\\ f_M(q^2)_{ij}i\sigma_{\mu\nu}q^\nu+f_E(q^2)_{ij}\sigma_{\mu\nu}q^\nu\gamma_5\end{aligned}}$$

! *form factors are matrices in* ν *mass eigenstates space.*

Dirac ν *(off-diagonal case* $i\neq j$*)* **Majorana** ν

1) ***hermiticity*** itself ***does not apply restrictions on form factors,***

2) ***CP invariance + hermiticity***

$\downarrow$

$f_Q(q^2),\ f_M(q^2),\ f_E(q^2),\ f_A(q^2)$

are relatively real (no relative phases) .

1) ***CP invariance + hermiticity***

$$\boxed{\mu_{ij}^M=2\mu_{ij}^D\ \ and\ \ \epsilon_{ij}^M=0}$$ ***or***

$$\boxed{\mu_{ij}^M=0\ \ and\ \ \epsilon_{ij}^M=2\epsilon_{ij}^D}$$

…two remarks ….

1 Difference between electromagnetic vertex function of massive and massless ν

Dirac Form factor

- *m=0* : $$\boxed{\bar{u}(p')\Lambda_\mu(q)u(p) = f_D(q^2)\bar{u}(p')\gamma_\mu(1+\gamma_5)u(p)}$$

electric charge $f_Q(q^2)$ and anapole moment $f_A(q^2)$ ***FF*** *are related to* ***DF*** *(and to each other):*

$$\boxed{f_Q(q^2) = f_D(q^2), \quad f_A(q^2) = f_D(q^2)/q^2}$$

- *In case $m \neq 0$ there is no such simple relation (because term $q_\mu \not{q} \gamma_5$ in anapole* ***FF*** *cannot be neglected).*

2

ν form factors in gauge models

$$\boxed{<\psi_j(p')|J_\mu^{EM}|\psi_i(p)>=\bar{u}_j(p')\Lambda_\mu(q)u_i(p)}$$

Form Factors at zero momentum transfer ($q^2 = 0$) are elements of scattering matrix

In any consistent theoretical model FF in matrix element ⇒ (*gauge independent and finite.*)

Then

FF at $q^2 = 0$ determine static properties of ν that can be probed (measured) in direct interaction with external em fields .

This is the case for
$f_Q(q^2),\ f_M(q^2),\ f_E(q^2)$
in minimally extended SM
($f_A(q^2)$ is an exceptional case)

In non-Abelian gauge models,

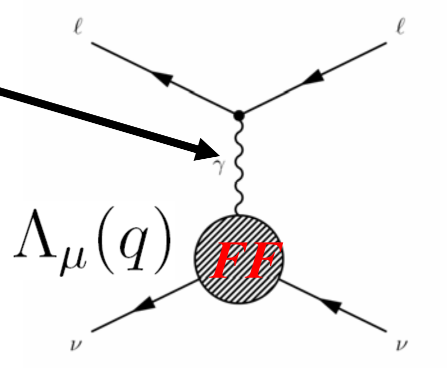
FF at $q^2 \neq 0$ can be not invariant under gauge transformation

because (in general) off-shell photon propagator *is gauge dependent !*

… One-photon approximation is not enough to get physical quantity…

… FF in matrix element cannot be directly measured in experiment with em field …

… FF can contribute to higher order processes accessible for experimental observation.



Dipole magnetic $\boxed{f_M(q^2)}$ **and electric** $\boxed{f_E(q^2)}$

are most well studied and theoretically understood among form factors

…because even in the limit $q^2 \to 0$ they may have nonvanishing values

$$\boxed{\mu_\nu = f_M(0)} \Longleftarrow \nu \text{ magnetic moment}$$

$$\boxed{\epsilon_\nu = f_E(0)} \Longleftarrow \nu \text{ electric moment ???}$$

ν **magnetic moment ?**

1

ν magnetic moment in experiments

Samuel Ting
(wrote on the wall at Department of Theoretical Physics of Moscow State University) :

"Physics is an experimental science"

Studies of ν–e scattering - most sensitive method of experimental investigation of μ_ν

Cross-section:

- $$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\rm SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu},$$

where the Standard Model contribution

- $$\left(\frac{d\sigma}{dT}\right)_{\rm SM} = \frac{G_{\rm F}^2 m_e}{2\pi}\left[(g_V + g_A)^2 + (g_V - g_A)^2\left(1 - \frac{T}{E_\nu}\right)^2 + (g_A^2 - g_V^2)\frac{m_e T}{E_\nu^2}\right],$$

T is the electron recoil energy and

$$g_V = \begin{cases} 2\sin^2\theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2\sin^2\theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \qquad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases}$$

for anti-neutrinos $g_A \to -g_A$,

to incorporate **charge radius**: $g_V \to g_V + \frac{2}{3}M_W^2\langle r^2\rangle \sin^2\theta_W$.

$$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\rm SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$

ν-γ coupling

- $$\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1 - T/E_\nu}{T}\right]\mu_\nu^2$$

with change of helicity, contrary to SM

T is the electron recoil energy: $0 \le T \le \dfrac{2E_\nu^2}{2E_\nu + m_e}$

If neutrino has electric dipole moment,
or electric or magnetic transition moments,
these quantities would also contribute to scattering cross section

$$\mu_\nu^2 = \sum_{j=\ \nu_e,\ \nu_\mu,\ \nu_\tau} |\ \mu_{ij} - \epsilon_{ij}\ |^2\ ,$$ *i* *refers to initial neutrino flavour*

Possibility of ***distractive interference*** between **magnetic** and **electric** transition moments of **Dirac** neutrino
(**Majorana** neutrino has only magnetic or electric transition moments, but not both if CP is conserved)

Effective ν_e magnetic moment measured in ν-e scattering experiments ? $\boxed{\mu_e^2}$

Two steps:

1) consider ν_e as superposition of mass eigenstates (*i=1,2,3*) at some distance L, and then sum up magnetic moment contributions to ν-e scattering amplitude of each of mass components induced by their magnetic moments

$$\boxed{A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji}}$$

2) amplitudes combine incoherently in total cross section

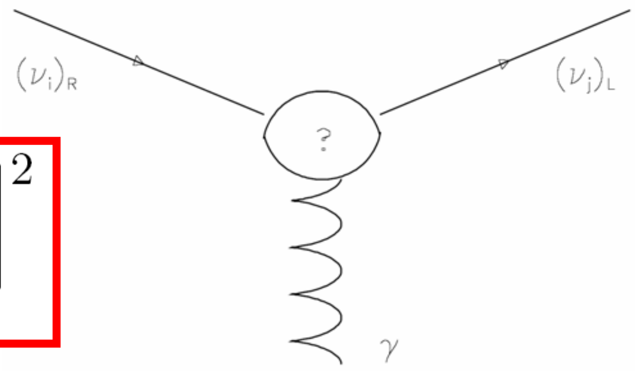
$$\boxed{\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2}$$

J.Beacom, P.Vogel, 1999

NB! Summation over *j=1,2,3* is outside the square because of **incoherence** of different final mass states contributions to cross section.

ν magnetic moment in experiments

(for neutrino produced as ν_l with energy E_ν and after traveling a distance L)



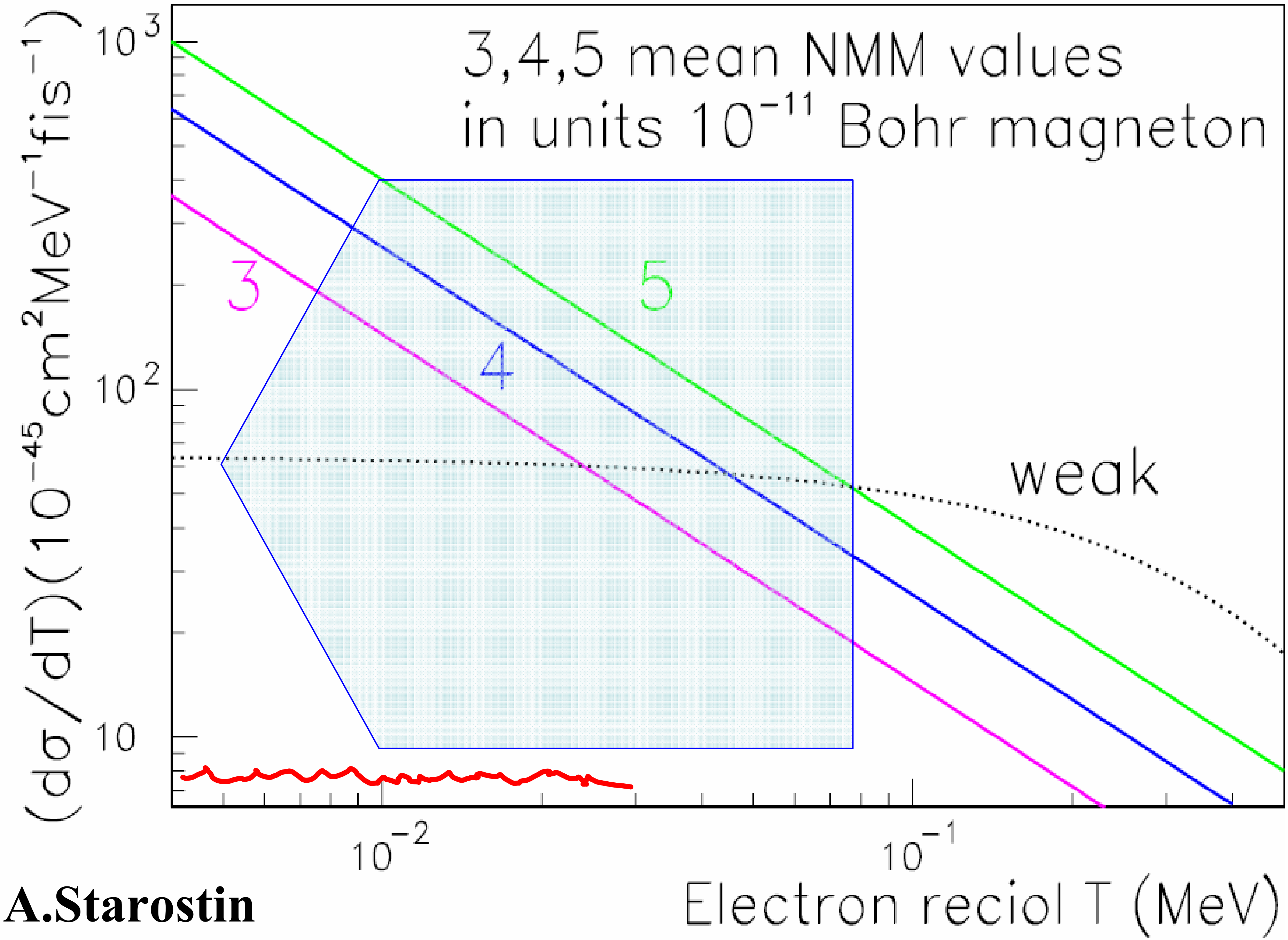
$$\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_i L} \mu_{ji} \right|^2$$

where neutrino mixing matrix $\quad \mu_{ij} \equiv |\beta_{ij} - \varepsilon_{ij}|$

Observable μ_ν is an effective parameter that depends on neutrino flavour composition at the detector.

H.Wong, H.-B.Li, 2005

Implications of μ_ν limits from different experiments (reactor , solar $^8\mathrm{B}$ and $^7\mathrm{Be}$) are different.



3,4,5 mean NMM values
in units 10^{-11} Bohr magneton
3
4
5
weak
$(d\sigma/dT)(10^{-45}cm^2MeV^{-1}fis^{-1})$
10^3
10^2
10
10^{-2}
10^{-1}
Electron reciol T (MeV)

A.Starostin

Magnetic moment contribution is dominated at low electron recoil energies

and $\boxed{\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}}$ when $\boxed{\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4} \mu_\nu^2}$

… the lower the smallest measurable electron recoil energy is,

the smaller values of μ_ν^2 can be probed in scattering experiments:

- $\boxed{\mu_\nu \leq 2 \div 4 \times 10^{-10} \mu_B}$ **Savannah River (1976),** ***first observation of ν - e***
  Vogel, Engel, 1989
  Kurchatov, Krasnoyarsk (1992), Rovno (1993) reactors
- $\boxed{\mu_\nu \leq 1.1 \times 10^{-10} \mu_B}$ **SuperKamiokande (2004)**
- $\boxed{\mu_\nu \leq 9 \times 10^{-11} \mu_B}$ **MUNU (Bugey reactor, 2005)**
- $\boxed{\mu_\nu \leq few \times 10^{-11} \mu_B}$ *…in the future…* **Beta-beams**
  McLaughlin, Volpe, 2004

MUNU experiment at Bugey reactor (2005)

$$\boxed{\mu_\nu \leq 9 \times 10^{-11} \mu_B}$$

TEXONO collaboration at Kuo-Sheng power plant (2006)

$$\boxed{\mu_\nu \leq 7 \times 10^{-11} \mu_B}$$

GEMMA (2007)

$$\boxed{\mu_\nu \leq \; 5.8 \times 10^{-11} \mu_B}$$

GEMMA I 2005 - 2007

BOREXINO (2008)

$$\boxed{\mu_\nu \leq \; 5.4 \times 10^{-11} \mu_B}$$

reported at Neutrino'08 Conference (New Zealand), and NOW'08, see also talk of Sandra Zavatarelli

$$\boxed{\mu_\nu \leq \; 8.5 \times 10^{-11} \mu_B \quad (\nu_\tau, \; \nu_\mu)}$$

Montanino, Picariello, Pulido, PRD 2008

2

New Result of Neutrino Magnetic Moment Measurement in GEMMA Experiment (2008)

A.Starostin et al, in: "Particle Physics on the Eve of LHC", ed. by A.Studenikin, World Scientific (Singapore), p.112, 2009,
www.icas.ru (13th Lomonosov Conference)

A.Beda et al, Phys.Atom.Nucl. 70 (2007) 1873

3

… a bit of ν electromagnetic properties theory

3.1 ν vertex function

The most general study of the massive neutrino vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R accounting for masses of particles in polarization loops

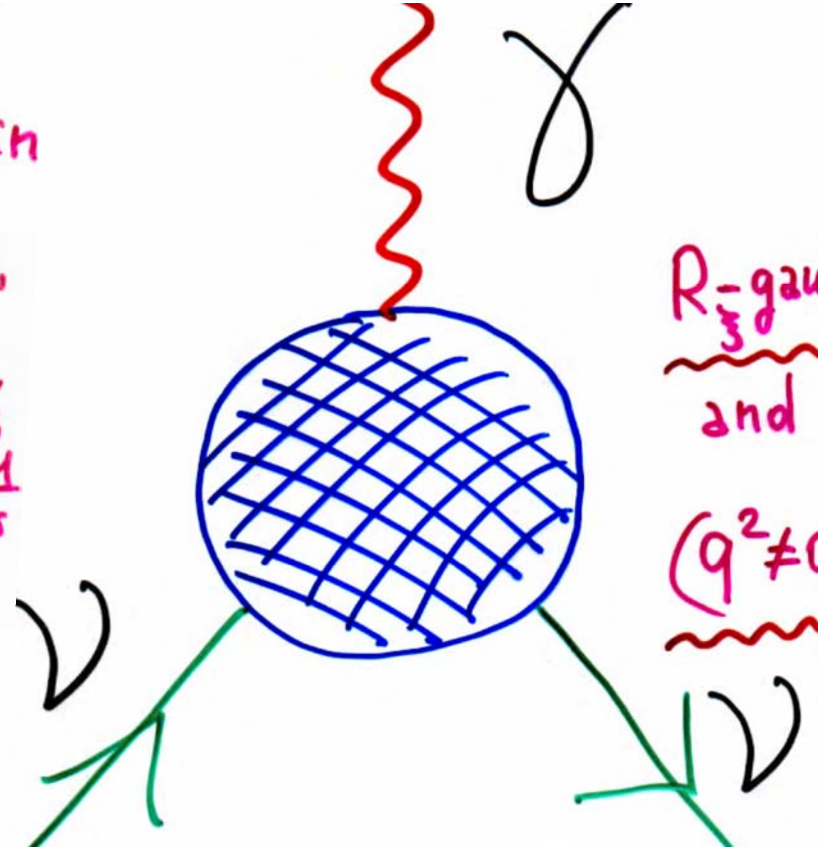
M. Dvornikov, A. Studenikin

* Phys. Rev. D 63, 073001, 2004,

"Electric charge and magnetic moment of massive neutrino";

JETP 126 (2004), N8, 1

* "Electromagnetic form factors of a massiv neutrino."



R_ξ-gauge

and

$(q^2 \neq 0)$

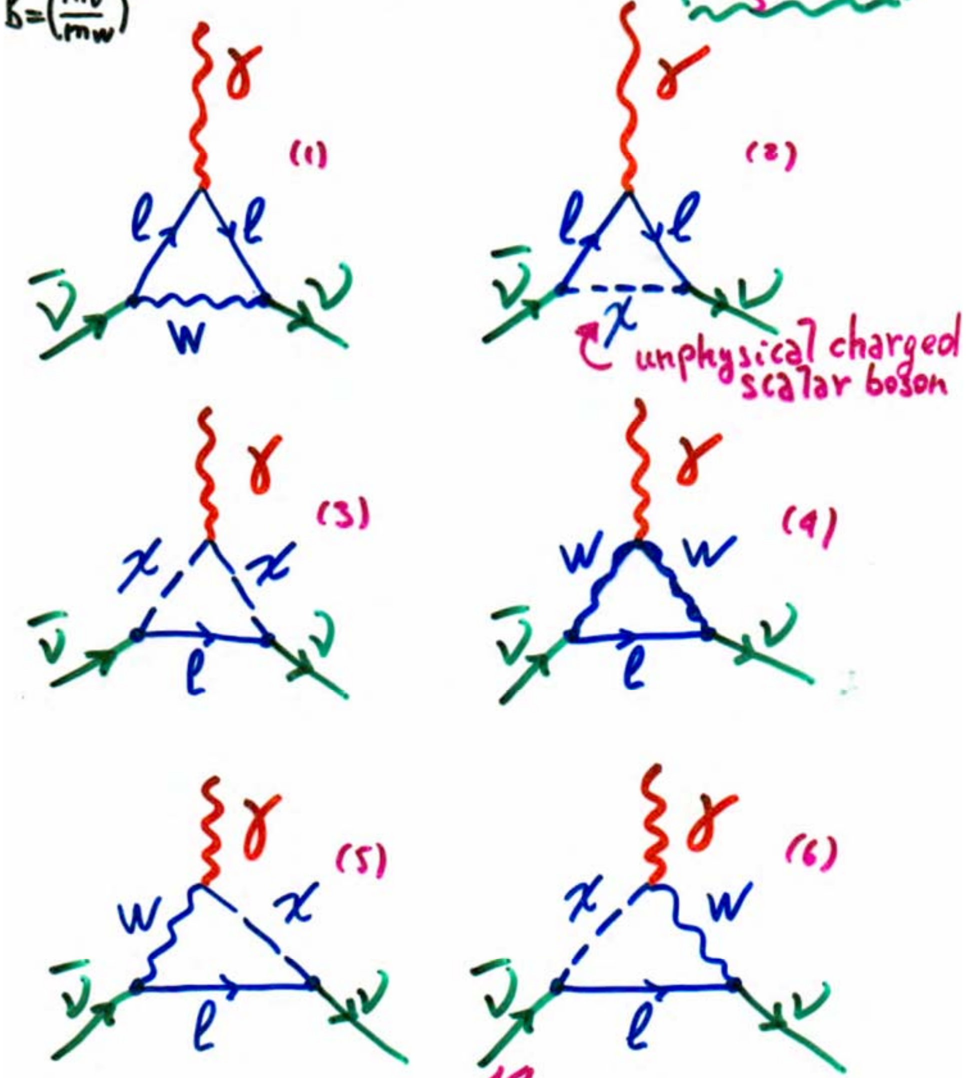
$$\Lambda_\mu(q) = \underbrace{f_Q}_{\text{charge}}(q^2)\gamma_\mu + \underbrace{f_M}_{\text{magnetic moment}}(q^2)\, i\sigma_{\mu\nu} q^\nu - \underbrace{f_E}_{\text{electric moment}}(q^2)\, i\sigma_{\mu\nu} q^\nu \gamma_5 - \underbrace{f_A}_{\text{anapole moment}}(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

$a=\left(\frac{m_\ell}{m_W}\right)^2,$

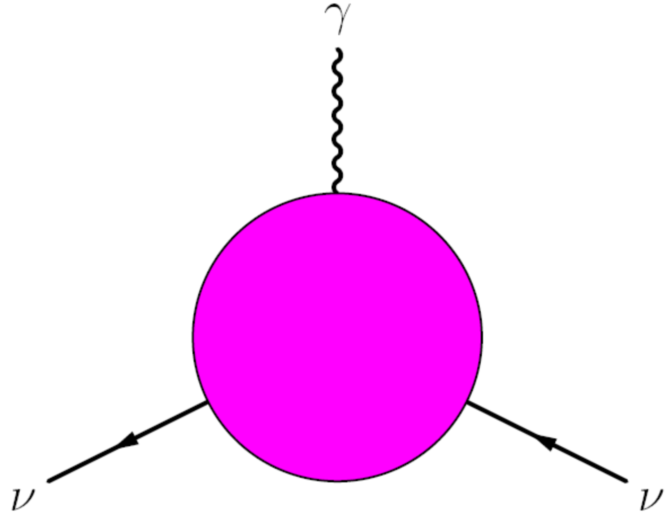
$b=\left(\frac{m_\nu}{m_W}\right)^2$

Proper vertices

R_ξ-gauge



$$\Lambda_\mu(q) = \sum_{i=1}^{14} \Lambda_\mu^i(q)$$



$$\Lambda_\mu(q)$$

Contributions of proper vertices diagrams

(dimensional-regularization scheme)

- $$\Lambda_\mu^{(1)} = i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\left[g^{\kappa\lambda}-(1-\alpha)\frac{k^\kappa k^\lambda}{k^2-\alpha M_W^2}\right]\times\frac{\gamma_\kappa^L(\not{p}'-\not{k}+m_\ell)\gamma_\mu(\not{p}-\not{k}+m_\ell)\gamma_\lambda^L}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-M_W^2]},$$

- $$\Lambda_\mu^{(2)} = i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}\frac{(m_\nu P_L-m_\ell P_R)(\not{p}'-\not{k}+m_\ell)\gamma_\mu(\not{p}-\not{k}+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-\alpha M_W^2]},$$

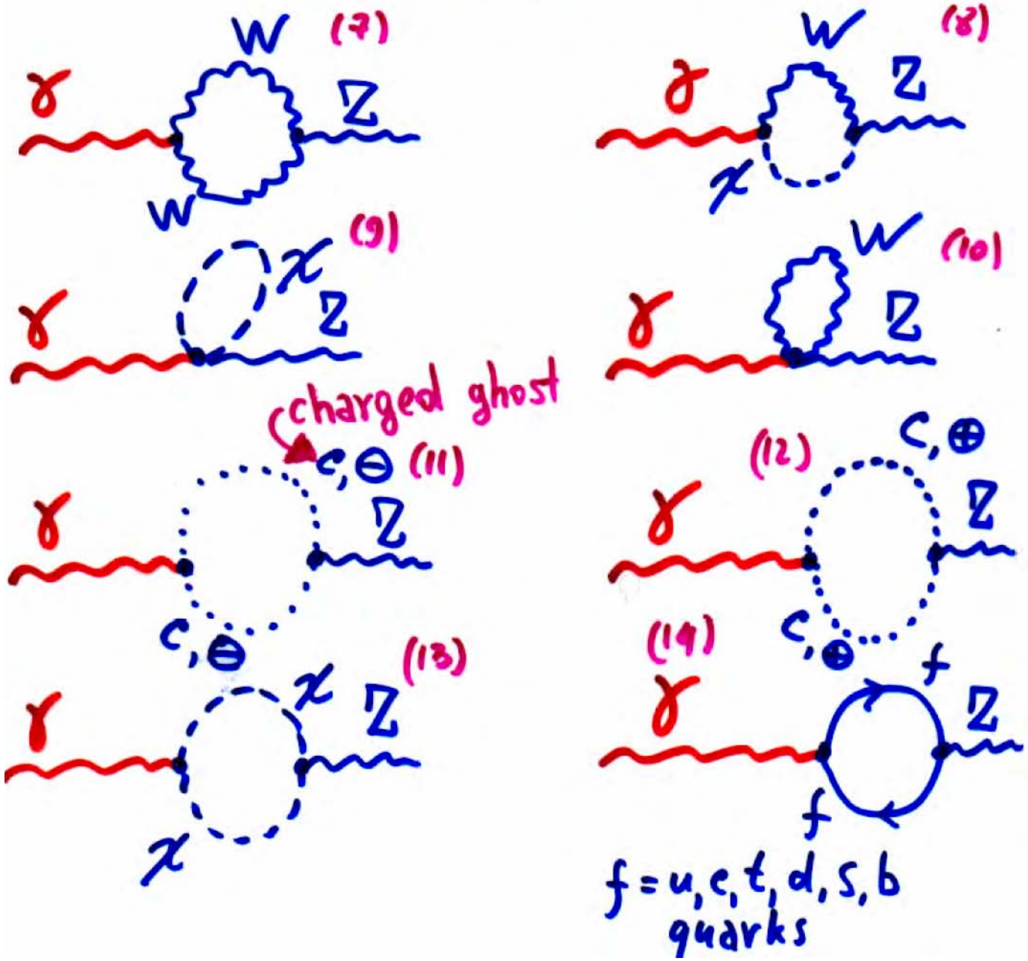
- $$\Lambda_\mu^{(3)} = i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}(2k-p-p')_\mu\frac{(m_\nu P_L-m_\ell P_R)(\not{k}+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-\alpha M_W^2][k^2-m_\ell^2]},$$

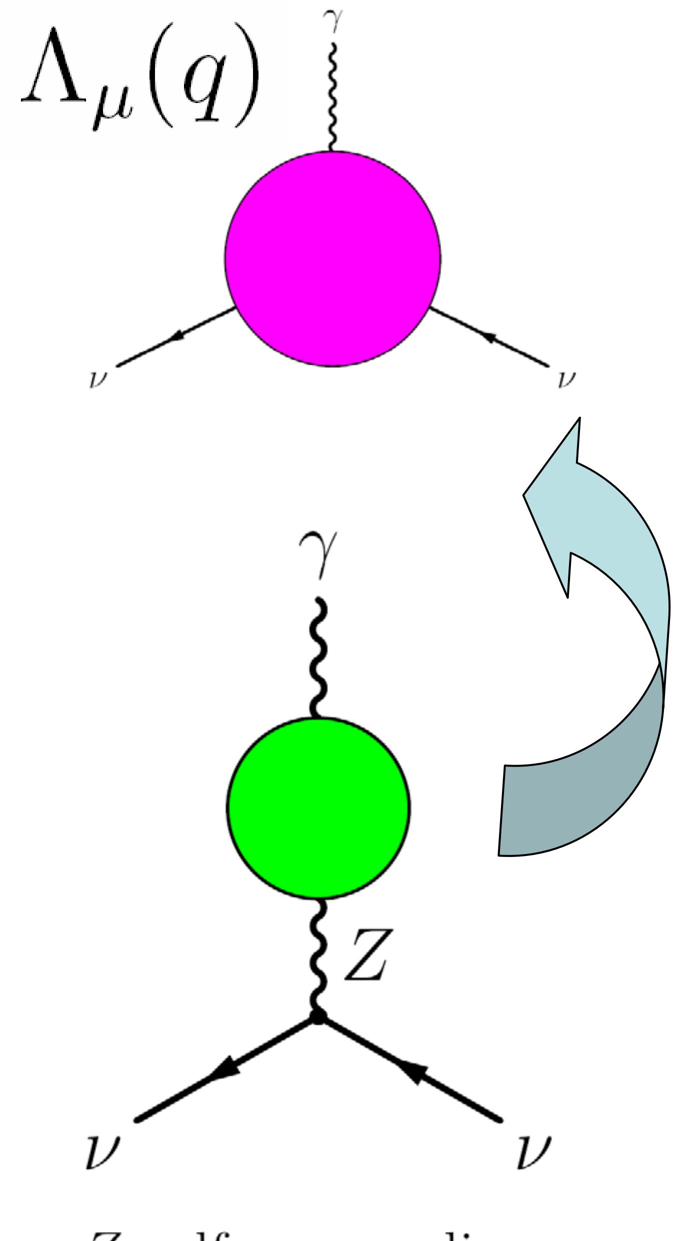
- $$\Lambda_\mu^{(4)} = i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\gamma_\kappa^L(\not{k}+m_\ell)\gamma_\lambda^L\left[\delta_\beta^\kappa-(1-\alpha)\frac{(p'-k)^\kappa(p'-k)_\beta}{(p'-k)^2-\alpha M_W^2}\right]\left[\delta_\gamma^\lambda-(1-\alpha)\frac{(p-k)^\lambda(p-k)_\gamma}{(p-k)^2-\alpha M_W^2}\right]$$
$$\times\frac{\delta_\mu^\beta(2p'-p-k)^\gamma+g^{\beta\gamma}(2k-p-p')_\mu+\delta_\mu^\gamma(2p-p'-k)^\beta}{[(p'-k)^2-M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]},$$

- $$\Lambda_\mu^{(5)+(6)} = i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}$$
$$\times\left\{\frac{\gamma_\beta^L(\not{k}-m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-M_W^2][(p-k)^2-\alpha M_W^2][k^2m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p'-k)^\beta(p'-k)_\mu}{(p'-k)^2-\alpha M_W^2}\right]\right.$$
$$\left.-\frac{(m_\nu P_L-m_\ell P_R)(\not{k}-m_\ell)\gamma_\beta^L}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p-k)^\beta(p-k)_\mu}{(p-k)^2-\alpha M_W^2}\right]\right\}$$

$$\Lambda_\mu^j(q) = \frac{g}{2\cos\theta_W}\,\Pi_{\mu\nu}^{(j)}(q)\,\frac{1}{q^2-M_Z^2}$$
$$\times\left\{g^{\nu\alpha} - (1-\alpha_Z)\frac{q^\nu q^\alpha}{q^2-\alpha_Z M_Z^2}\right\}\gamma_\alpha^4,\; j=7,\dots,14$$

γ-Z self-energy diagrams





$\gamma - Z$ self-energy diagrams

Matrix element of electromagnetic current between massive and zero-mass neutrino states differ radically

- **For massless** ν

$$f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

$$\boxed{\overline{u}(p')\Lambda_\mu(q)u(p) = f_D(q^2)\overline{u}(p')\gamma_\mu(1+\gamma_5)u(p)}$$

$\boxed{f_Q(q^2) = f_D(q^2)}$ ***electric*** **form factor** ***anapole*** $\boxed{f_A(q^2) = f_D(q^2)/q^2}$

- **For massive** ν

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$
$$\boxed{+ f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5}$$

one cannot disregard

- **Calculations of massive** ν **vertex function (calculation the complete set of Feynman diagrams)**

Dvornikov, Studenikin, 2004

additional term $\boxed{\Lambda_\mu(q) \sim f_5(q^2)\gamma_\mu\gamma_5}$

- **Direct calculation of these contributions**

$$\boxed{f_5(q^2) = f_5^{(\gamma - Z)}(q^2) + f_5^{(\text{prop. vert.})}(q^2) = 0}$$

Direct calculations of complete set of one-loop contributions to ν vertex function in minimally extended Standard Model

M.Dvornikov, A.S. 2004

Massive Dirac neutrino:

… in case CP conservation

- $\Lambda_\mu(q) \Rightarrow f_Q(q^2),\ f_M(q^2),\ \cancel{f_E(q^2)},\ f_A(q^2)$
- **Electric charge** $f_Q(0) = \mathbf{0}$ **and is gauge-independent**
- **Magnetic moment** $f_M(0)$ **is finite and gauge-independent**

ν

ν magnetic moment

(for arbitrary neutrino mass, heavy neutrino…)

- LEP data $\Rightarrow$

only 3 light νs coupled to Z°,

for any additional neutrino

$$\boxed{m_\nu \geq 45 \;\; Gev}$$

3.2

Calculation of ν magnetic moment (massive ν, arbitrary R_ξ- gauge)

Dvornikov, Studenikin, PRD 2004

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2) i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

magnetic moment

$$\mu(a,b,\alpha) = f_M(q^2=0)$$

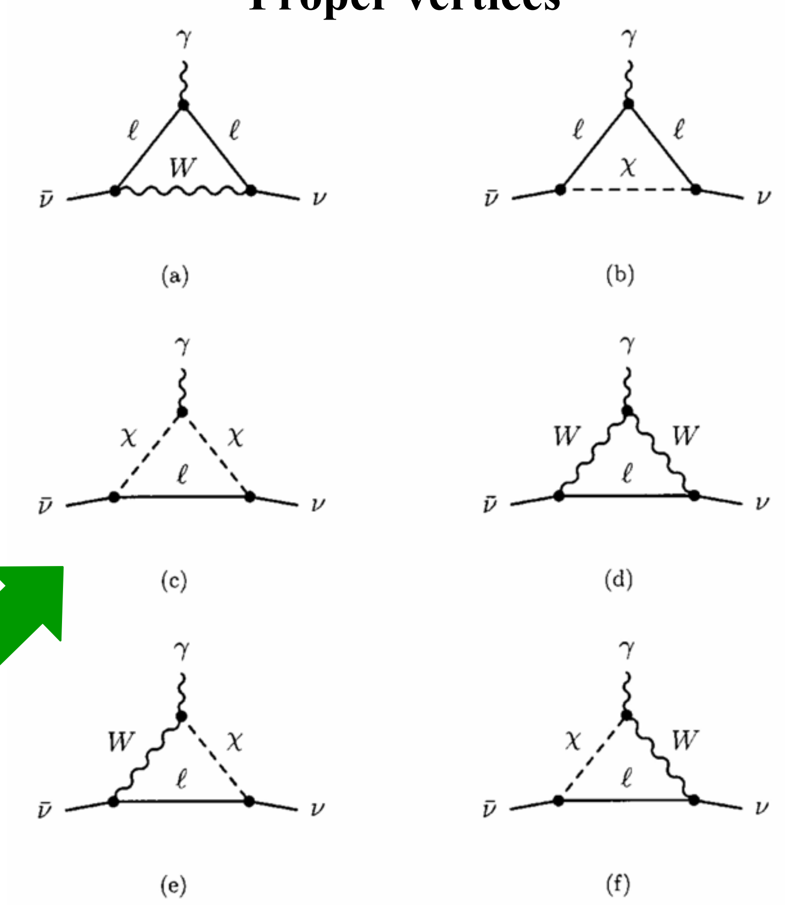
two mass parameters $a = \left(\frac{m_\ell}{M_W}\right)^2$ $b = \left(\frac{m_\nu}{M_W}\right)^2$

and gauge-fixing parameter $\alpha = \frac{1}{\xi}$

$\xi = 0$ **- unitary gauge,** $\xi = 1$ **- 't Hooft-Feynman gauge**

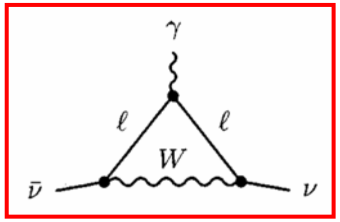
$$\mu(a,b,\alpha) = \sum_{i=1}^{6} \mu^{(i)}(a,b,\alpha)$$

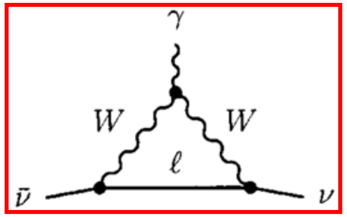
Proper vertices



… after loop integrals calculations (e.g., for diagrams (a) and (d) contributing in unitary gauge)

$$\mu^{(1)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\int_0^1 dz\, z(1-z^2)\frac{1}{D}-\frac{1}{2}\int_0^1 dz(1-z)^3(a-bz)\left[\frac{1}{D_\alpha}-\frac{1}{D}\right]\right.$$
$$\left.-\frac{1}{2}\int_0^1 dz(1-z)(1-3z)[\ln D_\alpha-\ln D]\right\},$$





$$\mu^{(4)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\frac{1}{2}\int_0^1 dz\, z^2(1+2z)\frac{1}{D}\right.$$
$$+\frac{b}{2}\int_0^1 dz\int_0^z dy(1-z)^2[z(1-z)-2y]\left[\frac{1}{D_\alpha+y(1-\alpha)}-\frac{1}{D}\right]$$
$$\left.+\frac{1}{2}\int_0^1 dz\int_0^z dz(-2+9z-4z^2-6y)\{\ln[D_\alpha+y(1-\alpha)]-\ln D\}\right\},$$

where $D_\alpha=a+(\alpha-a)z-bz(1-z)$ **and** $D=D_{\alpha=1}$

Dvornikov, Studenikin,
PRD 2004, JETP 2004

… within exact calculations it is possible to expand over mass parameter $b = \left(\frac{m_\nu}{M_W}\right)^2$

$$\mu(a,b,\alpha) = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \sum_{i=1}^{6} \{\overline{\mu}_0^{(i)}(a,\alpha) + b\,\overline{\mu}_1^{(i)}(a,\alpha) + \mathcal{O}(b^2)\}$$

Cabral-Rosetti, Bernabéu, Vidal, Zepeda, EPJ 2000

$$\mu_0(a,\alpha) = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}(2 - 7a + 6a^2 - 2a^2\ln a - a^3) + \mathcal{O}(a^2)$$

$a = \left(\frac{m_\ell}{M_W}\right)^2$

$$\overline{\mu}_1(a,\alpha) = \sum_{i=1}^{6} \overline{\mu}_1^{(i)}(a,\alpha) = \frac{1}{12(1-a)^5}(5 - 26a + 6a\ln a - 36a^2 - 60a^2\ln a + 58a^3 - 18a^3\ln a - a^4)$$

… μ_ν **gauge independent and finite value…**

$m_\nu \ll m_e \ll M_W$ **light** ν

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$

$$\boxed{\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}\left(2-7a+6a^2-2a^2\ln a - a^3\right)}, \quad a = \left(\frac{m_e}{M_W}\right)^2$$

Gabral-Rosetti, Bernabeu, Vidal, Zepeda, Eur.Phys.J C 12 (2000) 633

Dvornikov, Studenikin, Phys.Rev.D 69 (2004) 073001; JETP 99 (2004) 254

$m_e \ll m_\nu \ll M_W$ **intermediate** ν

$$\boxed{\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1 + \frac{5}{18} b\right\}}, \quad b = \left(\frac{m_\nu}{M_W}\right)^2$$

$m_e \ll M_W \ll m_\nu$ **heavy** ν

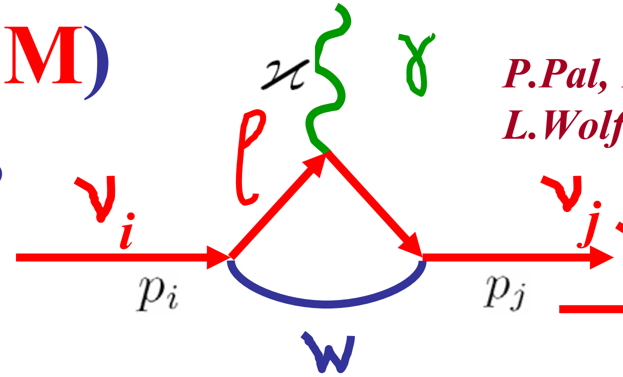
$$\boxed{\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu}$$

$$\sim 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1\,eV}\right)$$

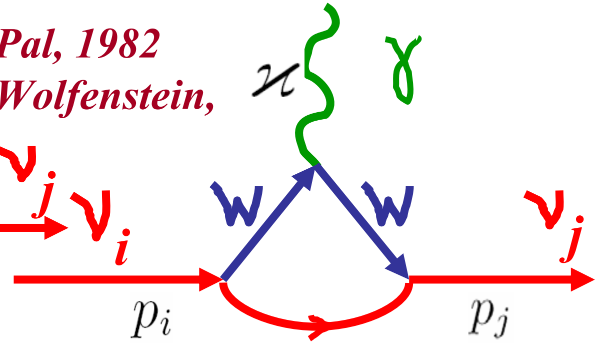
3.5

Neutrino (SM) dipole moments

(+ **transition moments**)



P.Pal, 1982
L.Wolfenstein,



- **Dirac neutrino**

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\sum_{l=e,\,\mu,\,\tau} f(r_l)U_{lj}U^*_{li}$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

ℓ $m_e = 0.5\ MeV$
$m_\mu = 105.7\ MeV$
$m_\tau = 1.78\ GeV$
$m_W = 80.2\ GeV$

- $m_i, m_j \ll m_l, m_W$

$$f(r_l) \approx \frac{3}{2}\left(1 - \frac{1}{2}r_l\right), \quad r_l \ll 1$$

transition moments vanish because unitarity of ***U*** implies that its rows or columns represent orthogonal vectors

- **transition moments** are suppressed, **Glashow-Iliopoulos-Maiani cancellation**,
- **for diagonal there is no GIM cancelation**

- **Majorana neutrino only for** $i \neq j$

$$\mu^M_{ij} = 2\mu^D_{ij} \quad and \quad \epsilon^M_{ij} = 0$$

or

$$\mu^M_{ij} = 0 \quad and \quad \epsilon^M_{ij} = 2\epsilon^D_{ij}$$

… depending on relative ***CP*** phase of ν_i and ν_j

The first nonzero contribution from

$$f_{r_l} \to \cancel{-\frac{3}{2}} + \frac{3}{4}\left(\frac{m_l}{m_W}\right)^2 \ll 1$$

GIM cancellation

neutrino transition moments

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\left(\frac{m_\tau}{m_W}\right)^2 \sum_{l=e,\ \mu,\ \tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U_{li}^*$$

$$\mu_B = \frac{e}{2m_e}$$

- $$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = 4 \times 10^{-23}\mu_B\left(\frac{m_i \pm m_j}{1\ eV}\right) \sum_{l=e,\ \mu,\ \tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U_{li}^*$$

… neutrino radiative decay is very slow

- **Dirac ν diagonal *(i=j)* magnetic moment** $\epsilon_{ii}^D = 0$ **for CP-invariant interactions**

$$\mu_{ii}^M = \epsilon_{ii}^M = 0$$

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2}\left(1 - \frac{1}{2}\sum_{l=e,\mu,\tau} r_l \mid U_{li}\mid^2\right) \approx 3.2 \times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B$$

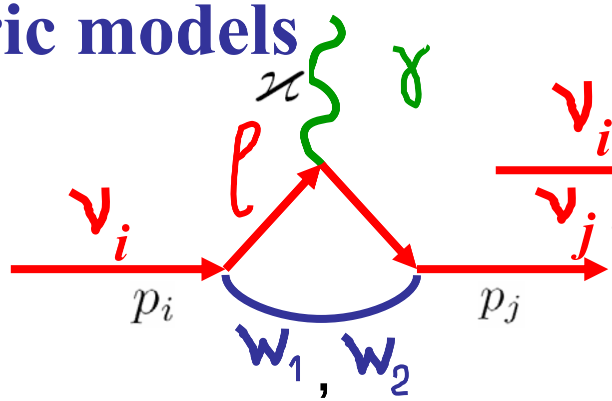
$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

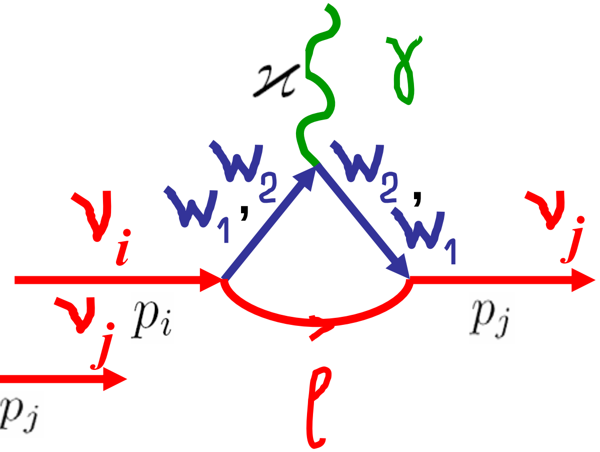
Lee, Shrock, Fujikawa, 1977

- ***no GIM cancellation***
- μ_{ii}^D **- to leading order - independent on U_{li} and $m_{l=e,\ \mu,\ \tau}$**
- $$\mu_e^2 = \sum_{i=1,2,3} \mid U_{ie}\mid^2 \mu_{ii}^2$$
- $\mu_{ii}^D = 0$ **for massless ν (in the absence of right-handed charged currents)**

3.6 Neutrino magnetic moment in left-right symmetric models

$SU_L(2) \times SU_R(2) \times U(1)$





Gauge bosons mass states $W_1 = W_L \cos\xi - W_R \sin\xi$

$W_2 = W_L \sin\xi + W_R \cos\xi$

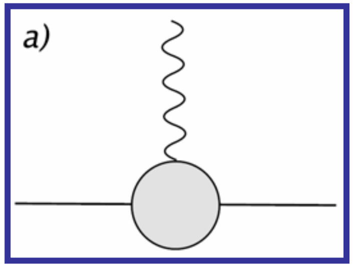
with mixing angle ξ **of gauge bosons** $W_{L,R}$ **with pure** $(V \pm A)$ **couplings**

Kim, 1976; Marciano, Sanda, 1977;
Beg, Marciano, Ruderman, 1978

$$\boxed{\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2}\left[m_l\left(1 - \frac{m^2_{W_1}}{m^2_{W_2}}\right)\sin 2\xi + \frac{3}{4}m_{\nu_l}\left(1 + \frac{m^2_{W_1}}{m^2_{W_2}}\right)\right]}$$

3.3 Naïve relationship between the size of m_ν and μ_ν

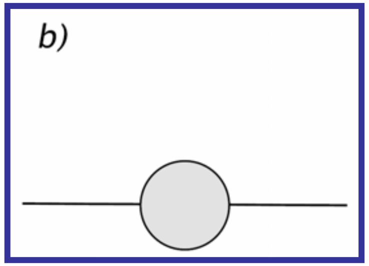
If μ_ν is generated by physics beyond the SM at energy scale Λ,



then $\boxed{\mu_\nu \sim \frac{eG}{\Lambda},}$

P.Vogel e.a., 2006

contribution to m_ν given by



then $\boxed{m_\nu \sim G\Lambda}$

$$\Rightarrow \quad m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{10^{-18}\mu_B}[\Lambda(\mathrm{TeV})]^2 \ \mathrm{eV}$$

from quadratic divergence appearing in renormalization
of dimension four neutrino mass operator

large magnetic moment

$\mu_\nu = \mu_\nu(m_\nu, m_{B^+}, m_{e^-})$

$\Uparrow$

- In the $\underline{L\text{-}R}$ symmetric models $\left(SU(2)_L \times SU(2)_R \times U(1)\right)$

Kim, 1976

Beg, Marciano, Ruderman, 1978

- *M.Voloshin, 1988*

"On compatibility of small m_ν with large μ_ν of neutrino", Sov.J.Nucl.Phys. 48 (1988) 512

… there may be $SU(2)_\nu$ symmetry that forbids m_ν but not μ_ν

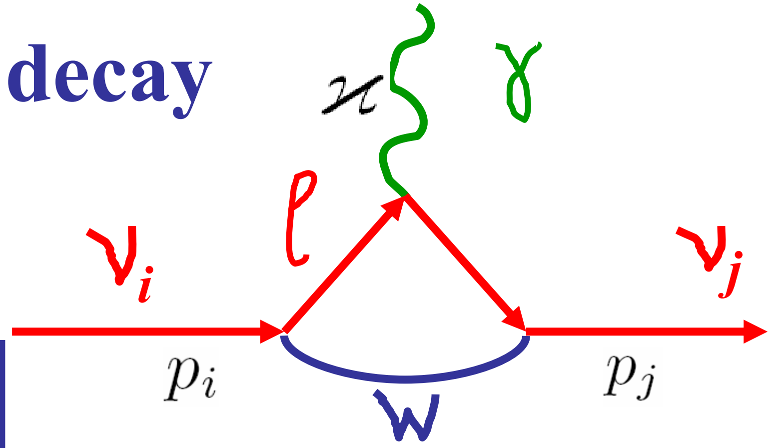
- supersymmetry
- extra dimensions

$$\boxed{\textit{considerable enhancement of } \mu_\nu \textit{ to experimentally relevant range}}$$

3.7 Neutrino radiative decay

$$\nu_i \longrightarrow \nu_j + \gamma$$

$$m_i > m_j$$



$$\boxed{L_{int} = \frac{1}{2}\bar{\psi}_i \sigma_{\alpha\beta}(\sigma_{ij} + \epsilon_{ij}\gamma_5)\psi_j F^{\alpha\beta} + h.c.}$$

Matrix element squired :

$$|\, M \,|^2 = 8\mu_{eff}^2 (\varkappa \cdot p_i)(\varkappa \cdot p_j)$$

$$\boxed{\mu_{eff}^2 = |\, \mu_{ij} \,|^2 + |\, \epsilon_{ij} \,|^2}$$

Radiative decay rate

$$\Gamma_{\nu_i \to \nu_j + \gamma} = \frac{\mu_{eff}^2}{8\pi}\left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3 \approx 5\left(\frac{\mu_{eff}}{\mu_B}\right)^2 \left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3 \left(\frac{m_i}{1\ eV}\right)^3 s^{-1}$$

- Radiative decay has been constrained from absence of decay photons:

  1) reactor $\bar{\nu}_e$ and solar ν_e fluxes,

  2) SN 1987A ν burst (all flavours),

  3) spectral distortion of CMBR

Raffelt 1999

Kolb, Turner 1990;

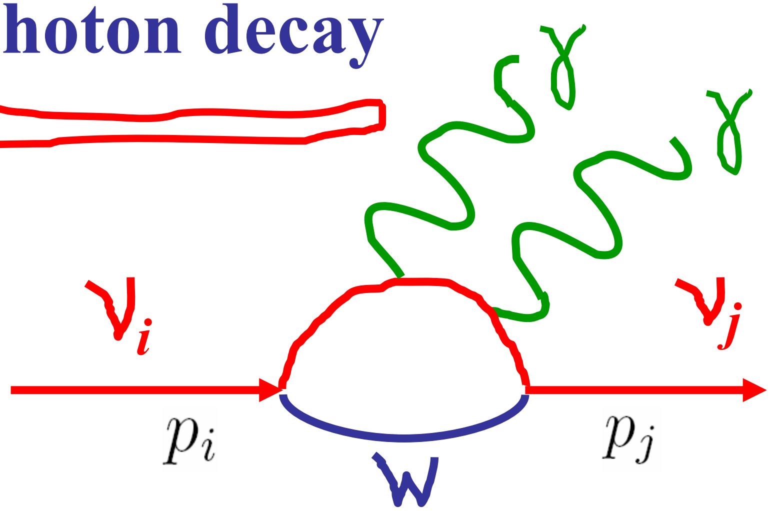
Ressell, Turner 1990

3.8

Neutrino radiative two-photon decay

$\nu_i \longrightarrow \nu_j + \gamma + \gamma$

$m_i > m_j$



fine structure constant $\ll 1$

$$\boxed{\Gamma_{\nu_i\to\nu_j+\gamma+\gamma} \sim \frac{\alpha_{QED}}{4\pi}\Gamma_{\nu_i\to\nu_j+\gamma}}$$

$$f(r_l) \approx \frac{3}{2}\left(\cancel{1} - \frac{1}{2}\left(\frac{m_l}{m_W}\right)^2\right) \to (m_i/m_l)^2$$

✓ *… there is no GIM cancellation…*

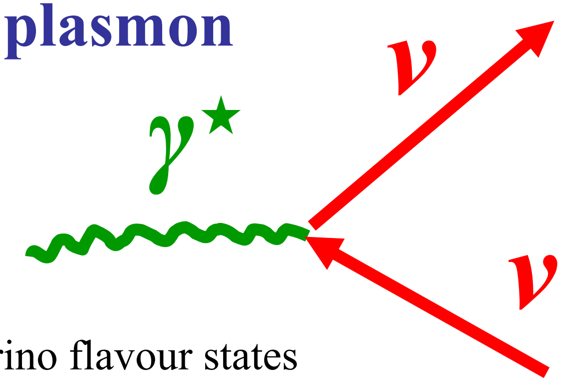
Nieves, 1983; Ghosh, 1984

… can be of interest for certain range of ν masses…

3.9 The tightest astrophysical bound on μ_ν

G.Raffelt, PRL 1990

comes from cooling of red giant stars by plasmon decay $\gamma^\star \longrightarrow \nu\nu$



$$L_{int} = \frac{1}{2}\sum_{a,b}\left(\mu_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\psi_b + \epsilon_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\gamma_5\psi_b\right)$$

neutrino flavour states

Matrix element

$$\epsilon_\alpha k^\alpha = 0$$

$$|M|^2 = M_{\alpha\beta}p^\alpha p^\beta, \quad M_{\alpha\beta} = 4\mu^2(2k_\alpha k_\beta - 2k^2\epsilon^*_\alpha\epsilon_\beta - k^2 g_{\alpha,\beta}),$$

Decay rate

$$\Gamma_{\gamma\to\nu\bar{\nu}} = \frac{\mu^2}{24\pi}\frac{(\omega^2-k^2)^2}{\omega}$$

= 0 in vacuum $\omega = k$

In the classical limit $\gamma^\star$ - like a massive particle with $\omega^2 - k^2 = \omega_{pl}^2$

Energy-loss rate per unit volume

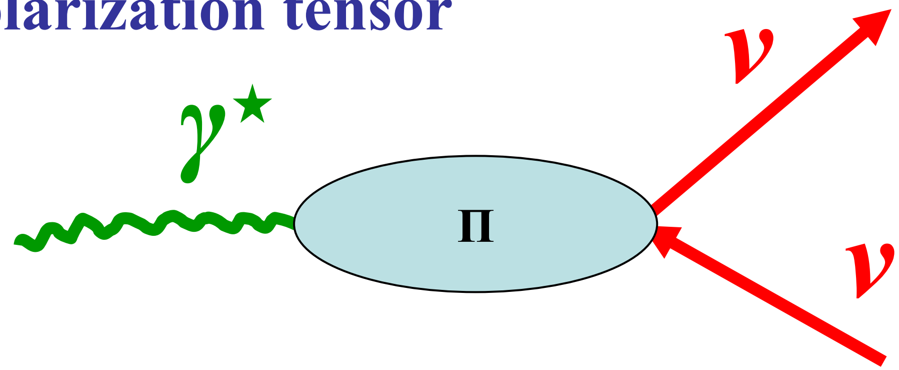
$$Q_\mu = g\int\frac{d^3k}{(2\pi)^3}\omega f_{BE}\Gamma_{\gamma\to\nu\bar{\nu}}$$

$$\mu^2 \to \sum_{a,b}\left(|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2\right)$$

distribution function of plasmons

$$\boxed{Q_\mu = g\int \frac{d^3k}{(2\pi)^3}\,\omega f_{BE}\Gamma_{\gamma\to\nu\bar{\nu}}}$$

Magnetic moment plasmon decay enhances the Standard Model photo-neutrino cooling by photon polarization tensor



more fast cooling of the star.

In order not to delay helium ignition (≤5% in Q)

$$\boxed{\mu^2 \leq 3\times 10^{-12}\mu_B}$$

$$\mu^2 \to \sum_{a,b}\left(|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2\right)$$

G.Raffelt, PRL 1990

Astrophysics bounds on μ_ν

$$\boxed{\mu_\nu(\text{astro}) < 10^{-10}\text{–}10^{-12}\ \mu_B}$$

Mostly derived from consequences of **helicity-state change** in astrophysical medium:

- available degrees of freedom in BBN,
- stellar cooling via plasmon decay,
- cooling of SN1987a.

Red Giant lumin.
! $\mu_\nu < 3 \cdot 10^{-12} \mu_B$
G. Raffelt, D. Dearborn, J. Silk, 1989.

The bounds depend on

- modeling of the astrophysical systems,
- on assumptions on the neutrino properties.

Generic assumption:

- absence of other nonstandard interactions except for μ_ν .

A global treatment would be desirable, incorporating **oscillation** and **matter effects** as well as the complications due to interference and competitions among various channels

… A remark on electric charge of ν

ν neutrality ***Q=0*** is attributed to

gauge invariance
+
anomally cancellation constraints

imposed in SM of electroweak interactions

Foot, Joshi, Lew, Volkas, 1990;
Foot, Lew, Volkas, 1993;
Babu, Mohapatra, 1989, 1990

…General proof:

- *In SM :* $\boxed{Q = \underbrace{I_3}_{SU(2)_L} + \frac{\overbrace{Y}^{U(1)_Y}}{2}}$
- *In SM (without ν_R) triangle anomalies cancellation constraints* ⇒ *certain relations among particle hypercharges Y, that is enough to fix all Y so that they, and consequently **Q**, are quantized*
- $\boxed{Q=0}$ *is proven also by direct calculation in SM within different gauges and methods*

$\boxed{Q=0}$

Bardeen, Gastmans, Lautrup, 1972;
Cabral-Rosetti, Bernabeu, Vidal, Zepeda, 2000;
Beg, Marciano, Ruderman, 1978;
Marciano, Sirlin, 1980; Sakakibara, 1981;
M.Dvornikov, A.S., 2004 (for extended SM in one-loop calculations)

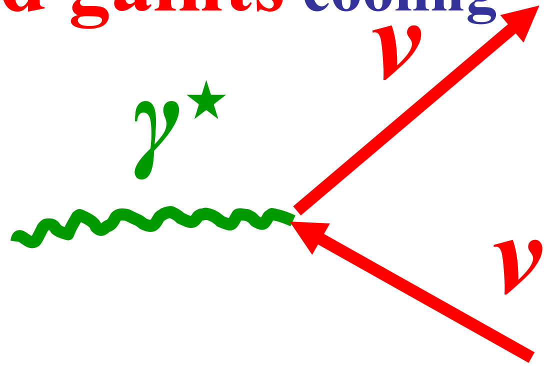
- *… However, strict requirements for **Q** quantization may disappear in extensions of standard $SU(2)_L \times U(1)_Y$ EW model if ν_R with $Y \neq 0$ are included : in the absence of **Y** quantization electric charges **Q** gets dequantized* ⇒ millicharged ν

3.10

Dobroliubov, Ignatiev (1990); Babu, Volkas (1992); Mohapatra, Nussinov (1992) …

- **Constraints on neutrino millicharge from red gaints cooling**

$\gamma^\star \longrightarrow \nu\nu$



Interaction Lagrangian $\quad \boxed{L_{int} = -iq_\nu \bar{\psi}_\nu \gamma^\mu \psi_\nu A^\mu}$

millicharge

Decay rate

$$\boxed{\Gamma_{q_\nu} = \frac{{q_\nu}^2}{12\pi}\omega_{pl}\left(\frac{\omega_{pl}}{\omega}\right)}$$

- $\boxed{q_\nu \leq 2 \times 10^{-14} e}$ …to avoid helium ignition in low-mass **red gaints**

Halt, Raffelt, Weiss, PRL1994

- $\boxed{q_\nu \leq 3 \times 10^{-17} e}$ … absence of anomalous energy-dependent dispersion of SN1987A ν signal, most model independent

- **… from "charge neutrality" of neutron…** $\boxed{q_\nu \leq 3 \times 10^{-21} e}$

3.11 ν anapole moment and charge radius

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\sigma_{\mu\nu}q^\nu-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$
$$+f_A(q^2)(q^2\gamma_\mu-q_\mu \not{q})\gamma_5$$

1. electric *dipole* *2. magnetic* *3. electric* *4. anapole*

Although it is usually assumed that ν are electrically neutral (charge quantization implies $Q\sim\frac{1}{3}e$), ν can dissociates into charged particles so that $f_Q(q^2)\neq 0\ for\ q^2\neq 0$:

$$f_Q(q^2)=f_Q(0)+q^2\frac{df_Q}{dq^2}(0)+\cdots,$$

where the massive ν charge radius $\langle r_\nu^2\rangle=-6\,\dfrac{df_Q}{dq^2}(0)$

For massless ν

anapole moment $a_\nu=f_A(q^2)=\frac{1}{6}\langle r_\nu^2\rangle$

Interpretation of charge radius as an observable is rather delicate issue: $\langle r_\nu^2\rangle$ represents a correction to tree-level electroweak scattering amplitude between ν and charged particles, which receives radiative corrections from several diagrams (including γ exchange) to be considered simultaneously ⇒ calculated CR is infinite and gauge dependent quantity. For massless ν, a_ν and $\langle r_\nu^2\rangle$ can be defined (finite and gauge independent) from scattering cross section. ⇒

Bernabeu, Papavassiliou, Vidal, 2004

For massive ν ? ? ?

ν charge radius

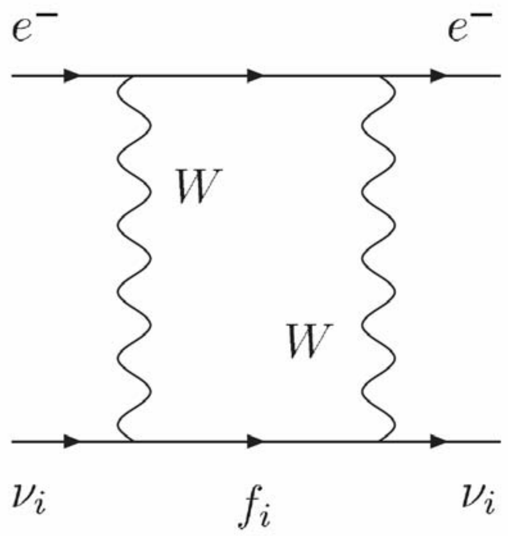
Even if the electric charge of a neutrino is vanishing, the electric form factor $f_Q(q^2)$ can still contain nontrivial information about neutrino static properties. A neutral particle can be characterized by a superposition of two charge distributions of opposite signs so that the particle's form factor $f_Q(q^2)$ can be non zero for $q^2 \neq 0$. The application of this notion to neutrinos has a long-standing history and is puzzling. In the case of a electrically neutral neutrino, one usually introduces the mean charge radius, which is determined by the second term in the expansion of the neutrino charge form factor $f_Q(q^2)$ in series of powers of q^2,

$$\boxed{f_Q(q^2) = f_Q(0) + q^2 \frac{df_Q(q^2)}{dq^2}_{|q^2=0} + \ \ldots}$$

$$\boxed{\langle r_\nu^2 \rangle = -6 \frac{df_Q(q^2)}{dq^2}|_{q^2=0}}$$

To obtain ν charge radius as physical (finite, not divergent) quantity

Bernabeu, Papavassiliou, Vidal, 2004



Contribution of box diagram to

$$\nu_l + l' \to \nu_l + l'.$$

$i = e, \mu, \tau$

$$\boxed{\langle r^2_{\nu_i} \rangle = \frac{G_F}{4\sqrt{2}\pi^2}\left[3 - 2\log\left(\frac{m_i^2}{m_W^2}\right)\right]}$$

$$\boxed{\langle r^2_{\nu_e} \rangle = 4 \times 10^{-33}\,\mathrm{cm}^2}$$

…contribution to ν - e scattering experiments through

$$\boxed{g_V \to \frac{1}{2} + 2\sin^2\theta_W + \frac{2}{3} m_W^2 \langle r^2_{\nu_e} \rangle \sin^2\theta_W}$$

!

… theoretical predictions and present experimental limits are in agreement within one order of magnitude…

ν anapole form factor

Giunti, AS, 2008
Dubovik, Kuznetsov, 1998;
Bukina, Dubovik, Kuznetsov

- **Anapole form factor is the most mysterious one!**

To understand the physical meaning of the anapole form factor, as well as the meaning of other form factors, it is instructive to couple the correspondent term of the current to an external electromagnetic field (given by a potential A_μ), to derive the corresponding Dirac equation of motion for a neutrino field ψ of mass m, and finally to obtain the interaction energy with a static electromagnetic field in the nonrelativistic limit. From

$$\Lambda_\mu(q)_A = f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

- ***In nonrelativistic limit, the correspondent interaction energy***

Zeldovich, JETP, 1957

$$\boxed{H_{int} \propto f_A(0)\big(\boldsymbol{\sigma} \cdot curl\ \mathbf{B} - \dot{\mathbf{E}}\big),}$$

which corresponds to a T-invariant toroidal (anapole) interaction of the neutrino that does not conserve the P and C parities. This interaction defines the axial-vector interaction with an external electromagnetic field. The poloidal currents on a torus can be considered as a geometrical model for the anapole [92].

Direct calculation of γ - Z and **proper-vertex** diagrams contribution

ν **anapole moment** is **infinite** and **gauge dependent**

$m=0$, Lucio, Rosado, Zepeda, 1985
$m\neq 0$, Dvornikov, Studenikin, 2004

is not a static quantity,
can't be measured with external field

Physical definition of anapole moment:

Dubovik, Kuznetsov, 1998

- **through diagrammes contributing to** $\boxed{\nu_l\ l' \to \nu_l\ l'}$
- **with inclusion of all** ν **anapole diagrammes**
- **finite and gauge independent**
- **does not depend on charged lepton** l' **.**

3.12

ν e.m. form factors are affected by matter and B

* magnetic moment $\mu_\nu = \mu_\nu(B)$

Egorov, Studenikin 1997

Borisov, Zhukovsky, Kurilin, Ternov, 1985

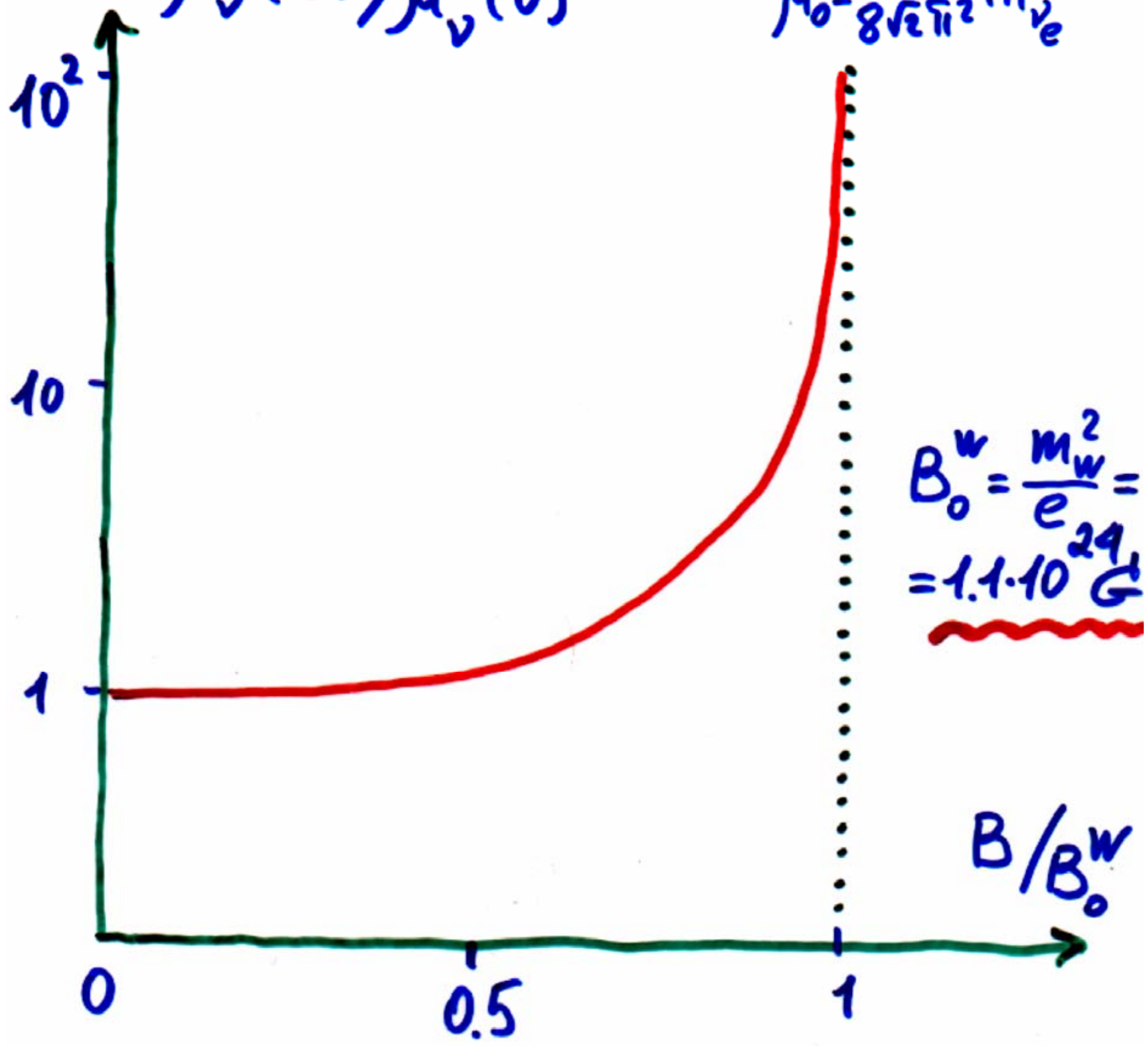
* induced electric charge of ν in magnetized matter

* Oraevsky, Semikoz, Smorodinsky, 1986

Bhattacharaya, Ganguly, Konar, 2002

Nieves, 2003

Neutrino magnetic moment



$\mu_0 = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$

Borisov, Zhukovs'ky, Kurilin, Ternov, 1985;

$B_0^W = \frac{m_W^2}{e} = 1.1 \cdot 10^{24}\,G$

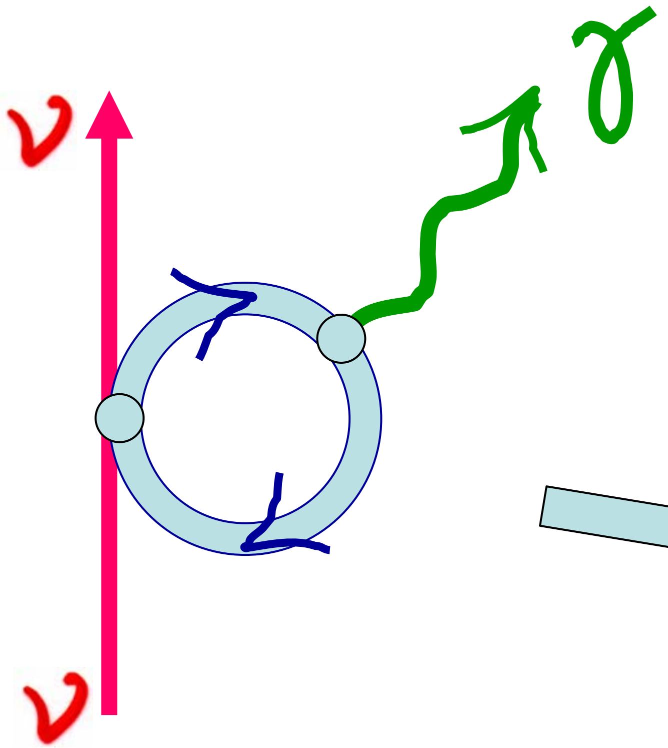
Masood,
Perez Rojas,
Gaitan,
Rodrigues-Romo,
1999

ν "effective electric charge" in magnetized plasma

- νs do not couple with γs in vacuum,

  . . . however, when

- ν in thermal medium (e^- and e^+)



V.Oraevsky, V.Semikoz, Ya.Smorodinsky, JETP Lett. 43 (1986) 709;
J.Nieves, P.Pal, Phys.Rev.D 49 (1994) 1398;
T.Altherr, P.Salati, Nucl.Phys.B421 (1994) 662;
K.Bhattacharya, A.Ganguly, 2002

…different νγ interactions in astrophysical and cosmological media

4 ν spin and spin-flavour oscillations in $B_\perp$

Consider **two different neutrinos**: $\nu_{e_L}, \quad \nu_{\mu_R}, \quad m_L \neq m_R$
with **magnetic moment interaction**

$$\boxed{L \sim \bar{\nu}\sigma_{\lambda\rho}F^{\lambda\rho}\nu\ ' = \bar{\nu}_L\sigma_{\lambda\rho}F^{\lambda\rho}\nu_R\ ' + \ \bar{\nu}_R\sigma_{\lambda\rho}F^{\lambda\rho}\nu_L\ '}.$$

Twisting magnetic field $B = |\mathbf{B}_\perp|e^{i\phi(t)}$ ⇐ **for solar** ν **etc …**

ν **evolution equation**

$$\boxed{i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = H\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}}$$

$$\boxed{H = \begin{pmatrix} E_L & \mu_{e\mu}Be^{-i\phi} \\ \mu_{e\mu}Be^{+i\phi} & E_R \end{pmatrix} = \ldots \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tilde{H}}$$

!

$$\boxed{\tilde{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{V_{\nu_e}}{2} & \mu_{e\mu}Be^{-i\phi} \\ \mu_{e\mu}Be^{+i\phi} & \frac{\Delta m^2}{4E} - \frac{V_{\nu_e}}{2} \end{pmatrix}}$$

Probability of $\nu_{e_L} \Longleftrightarrow \nu_{\mu_R}$ oscillations in $\boxed{B = |\mathrm{B}_\perp| e^{i\phi(t)}}$ and matter

- $$\boxed{P_{\nu_L \nu_R} = \sin^2\beta \, \sin^2 \Omega z, \quad \sin^2\beta = \frac{(\mu_{e\mu} B)^2}{(\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}}$$

$$\boxed{\Delta_{LR} = \frac{\Delta m^2}{2}(\cos 2\theta + 1) - 2EV_{\nu_e} + 2E\dot{\phi}}$$

$$\boxed{\Omega^2 = (\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}$$

- **Resonance amplification of oscillations in matter:**

$$\boxed{\Delta_{LR} \to 0} \Rightarrow \boxed{\sin^2\beta \to 1}$$

Akhmedov, 1988
Lim, Marciano

In magnetic field

$\nu_{e_L} \ \nu_{\mu_R}$

$$i \frac{d}{dz} \nu_{e_L} = -\frac{\Delta_{LR}}{4E} \nu_{e_L} + \mu_{e\mu} B \nu_{\mu_R}$$

$$i \frac{d}{dz} \nu_{\mu_L} = \frac{\Delta_{LR}}{4E} \nu_{\mu_L} + \mu_{e\mu} B \nu_{e_R}$$

Neutrino conversions and oscillations in magnetic field

- ⊛ $\nu_\odot$ problem $\nu_L \overset{B}{\leftrightarrow} \nu_R$ …for recent analysis see *J.Pulido, 2006* *A.Balantekin, C.Volpe, 2005*

  Cisneros, 1971

  * Voloshin, Vysotsky, Okun, 1986; Barbieri, Fiorentini, 1988

  ⊙ twisting B: Smirnov, 1991; Akhmedov, Petcov, Smirnov, 1993

- ⊛ Supernova $\nu_L \overset{B}{\leftrightarrow} \nu_R$

  Dar, 1987

  Fujikawa, Shrock, 1988

  Voloshin, 1988

Spin-flavour oscillations in early universe – strong $B_\perp$ ⇒ *population of* ν *wrong-helicity states (r.h.) would accelerate expansion of universe (???)*

Periodicity of the active solar neutrino flux is probably the most important issue to be investigated after LMA has been ascertained as the dominant solution to the $\odot$ ν problem. If confirmed it will imply the existence of a sizable neutrino magnetic moment μ_ν and hence a wealth of new physics.

- **Idea was introduced in 1986 by**

  Voloshin, Vysotsky and Okun

$$Strong\ B_\odot \rightarrow large\ \mu_\nu B_\odot \rightarrow large\ conversion$$

- **For recent analysis see**

  J.Pulido, 2006 …see also A.Balantekin and C.Volpe, 2005

- **… Spin-flavour precession resonance and MSW resonance take place very close to each other inside sun…**

SPIN FLAVOUR PRECESSION AND LMA

João M. Pulido

CFTP - Instituto Superior Técnico, Lisbon

12th Lomonosov Conference on Elementary Particle Physics, Moscow, August 2005

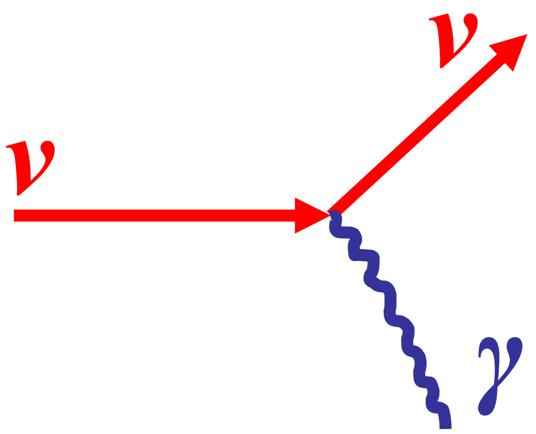
Long term periodicity may have been observed by the Gallium experiments. In fact

Period	1991-97	1998-03
SAGE+Ga/GNO	77.8 ± 5.0	63.3 ± 3.6
Ga/GNO only	77.5 ± 7.7	62.9 ± 6.0
no. of suspots	52	100

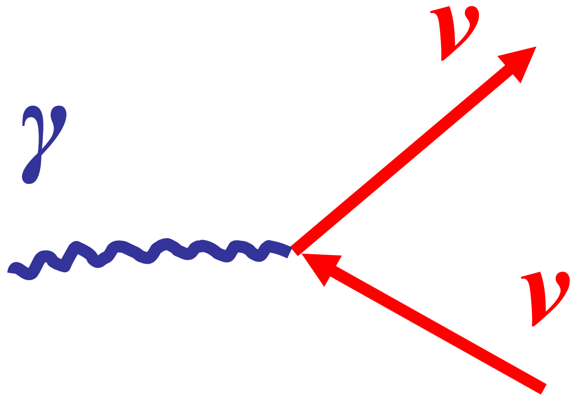
Notice a 2.4σ discrepancy in the combined results over the two periods. This is suggestive of an anticorrelation of Ga event rate with the 11-year solar sunspot cycle.

Conclusion

Neutrino – photon couplings (I)

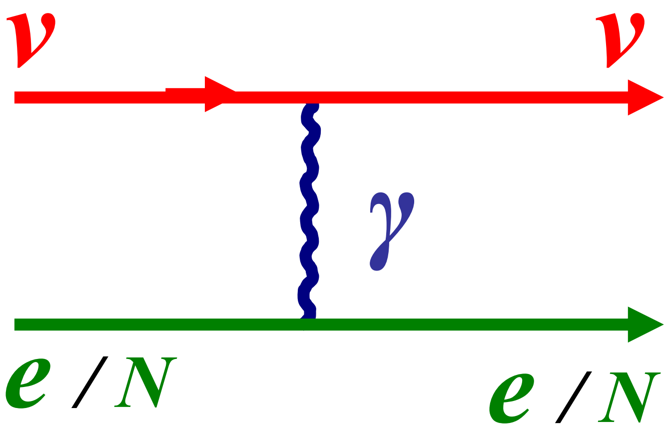


ν **decay, Cherenkov radiation**

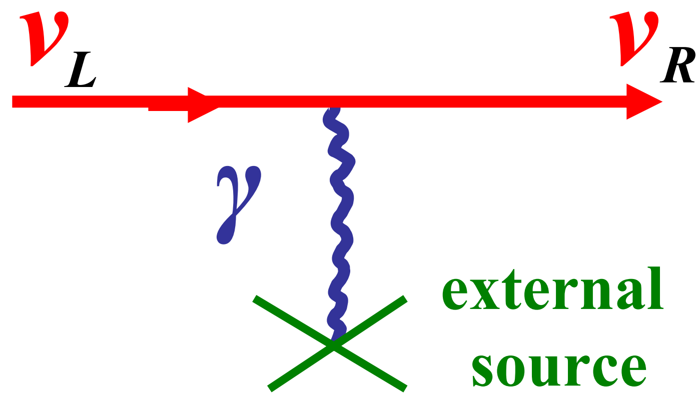


γ **decay in plasma**

!!!



Scattering



Spin precession

ν spin evolution in presence of general external fields

M.Dvornikov, A.Studenikin,
JHEP 09 (2002) 016

General types non-derivative interaction with external fields

$$\boxed{\begin{aligned}-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \\ + \frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g'_t}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu,\end{aligned}}$$

scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor fields:

$$s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}),$$
$$T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$$

Relativistic equation (quasiclassical) for ν *spin vector:*

$$\begin{aligned}\dot{\vec{\zeta}}_\nu = 2g_a\left\{A^0[\vec{\zeta}_\nu\times\vec{\beta}] - \frac{m_\nu}{E_\nu}[\vec{\zeta}_\nu\times\vec{A}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{A}\vec{\beta})[\vec{\zeta}_\nu\times\vec{\beta}]\right\} \\ +2g_t\left\{[\vec{\zeta}_\nu\times\vec{b}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{b})[\vec{\zeta}_\nu\times\vec{\beta}] + [\vec{\zeta}_\nu\times[\vec{a}\times\vec{\beta}]]\right\} + \\ +2ig'_t\left\{[\vec{\zeta}_\nu\times\vec{c}] - \frac{E_\nu}{E_\nu+m_\nu}(\vec{\beta}\vec{c})[\vec{\zeta}_\nu\times\vec{\beta}] - [\vec{\zeta}_\nu\times[\vec{d}\times\vec{\beta}]]\right\}.\end{aligned}$$

- *Neither* ***s*** *nor* $\boldsymbol{\pi}$ *nor* $\boldsymbol{V}$ *contributes to spin evolution*

- **Electromagnetic interaction**

$$T_{\mu\nu} = F_{\mu\nu} = (\vec{E}, \vec{B})$$

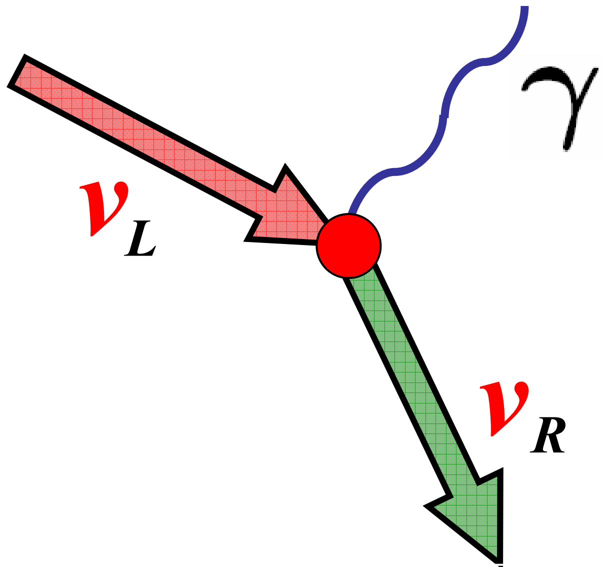
- **SM weak interaction**

$$G_{\mu\nu} = (-\vec{P}, \vec{M}) \qquad \begin{aligned}\vec{M} &= \gamma(A^0\vec{\beta} - \vec{A}) \\ \vec{P} &= -\gamma[\vec{\beta}\times\vec{A}],\end{aligned}$$

New mechanism of electromagnetic radiation

$SL\nu$

Spin light of neutrino ***in matter***



- We predict the existence of a **new mechanism** of the electromagnetic process stimulated by the presence of matter, in which a neutrino with **non-zero magnetic moment** emits light.

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A.Grigoriev, A.Lobanov, A.Studenikin, **Phys.Lett.B 535** (2002) 187

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μ_ν is presently known to be in the range

$$\boxed{10^{-20}\mu_B \le \mu_\nu \le 10^{-10}\mu_B}$$

μ_ν provides a tool for exploration possible physics beyond the Standard Model

Due to smallness of neutrino-mass-induced magnetic moments,

$$\boxed{\mu_{ii} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1\ eV}\right) \mu_B}$$

any indication for non-trivial electromagnetic properties of ν , that could be obtained within reasonable time in the future, would give evidence for interactions beyond extended Standard Model

… situation with

ν electromagnetic properties

is better then it was in the case of W. Pauli, 1930

… once they will be observed experimentally

… are important in astrophysics

… there is a need for theoretical studies

Experimental and theoretical studies of

ν electromagnetic properties

is a tedious task

$\Downarrow$

important impact on understanding of

fundamentals of particle physics

(Dirac $\leftrightarrow$ Majorana etc) and

applications in astrophysics