Which Top Mass is Measured at Hadron Colliders ?

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Outline

- Why the question is relevant.
- Methods for top mass determinations
- 2 (yet) incomplete answers:
  - Factorization Theorem
  - Fixing up the top mass in MC's
- Outlook and Conclusions

Top Quark is Special !

- Heaviest known quark (related to SSB?)
- Important for quantum effects affecting many observables
- Very unstable, decays "before hadronization" ($\Gamma_t \approx 1.5$ GeV)

$$m_t = 172.4 \pm 1.2 \text{ GeV}$$

FERMILAB-TM-2380-E

$$\cancel{m_t = 172.6 \pm 1.4 \text{ GeV}}$$

13th March 2007

$$\cancel{M_t = 170.9 \pm 1.8 \text{ GeV}/c^2}$$

Combination of CDF and DØ Results on the Mass of the Top Quark

The Tevatron Electroweak Working Group[1]
for the CDF and DØ Collaborations

Abstract

We summarize the top-quark mass measurements from the CDF and DØ experiments at Fermilab. We combine published Run-I (1992-1996) measurements with the most recent preliminary Run-II (2001-present) measurements using up to 1 fb^{-1} of data. Taking correlated uncertainties properly into account the resulting preliminary world average mass of the top quark is $M_t = 170.9 \pm 1.1(\text{stat}) \pm 1.5(\text{syst})$ GeV/c^2, which corresponds to a total uncertainty of 1.8 GeV/c^2. The top-quark mass is now known with a precision of 1.1%.

<1% precision !

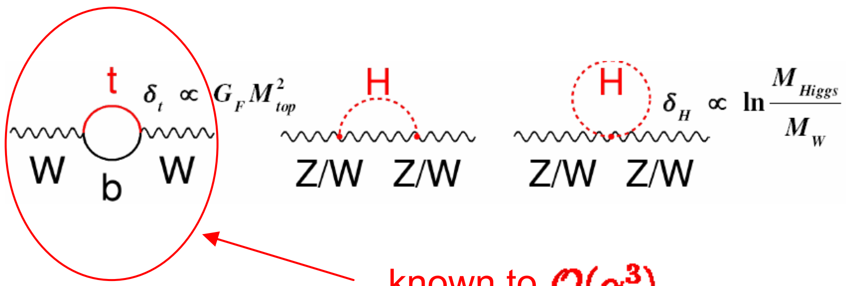
How shall we theorists judge the error ?

What is the theoretical error ?

What mass is it ?

Need for a precise Top mass

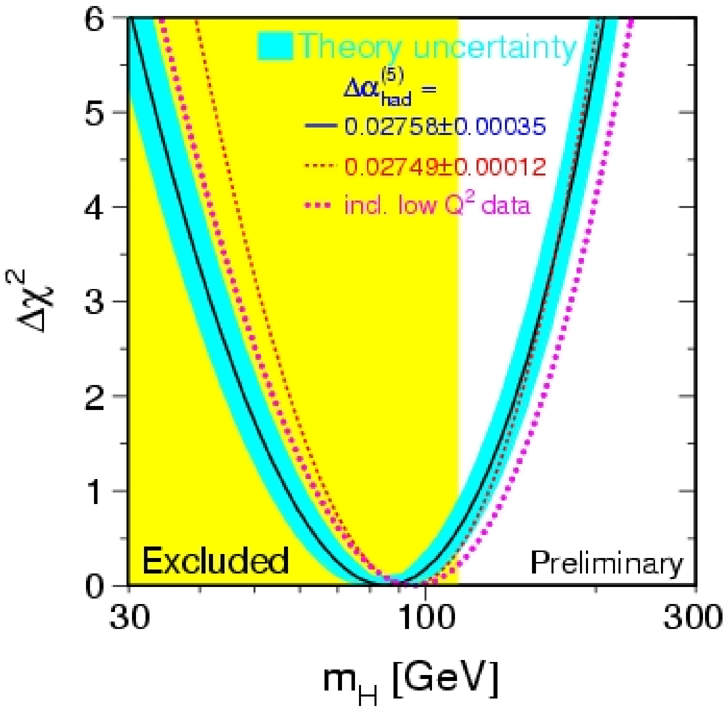
Fit to electroweak precision observables



known to $\mathcal{O}(\alpha_s^3)$

$$\sin\theta_W \times \left(1 + \delta(m_t, m_H, \ldots)\right)$$

$$= 1 - \frac{M_W^2}{M_Z^2}$$



$$m_H = 76^{+33}_{-24}\ \text{GeV}$$

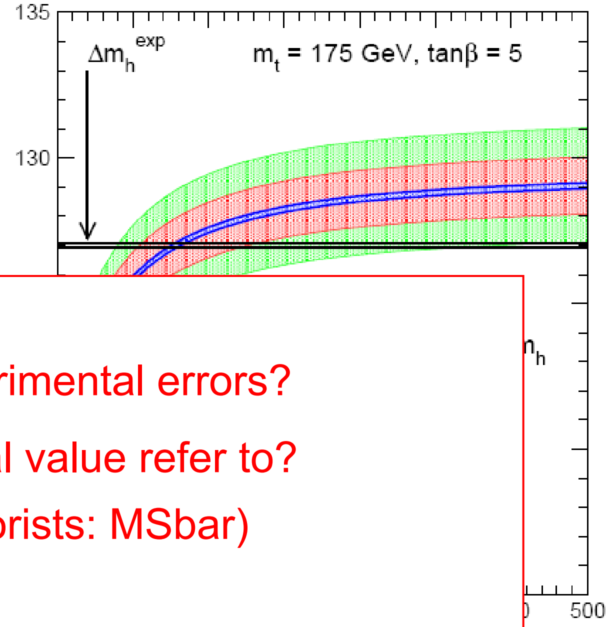
$$m_H < 182\ \text{GeV}\ \text{(95\%CL)}$$

$$m_t = 170.9 \pm 1.8\ \text{GeV}$$

2 GeV error: 15% change in m_H

Need for a precise Top mass

Mass of Lightest MSSM Higgs Boson



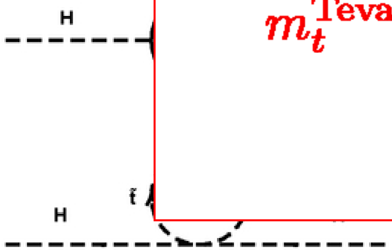
5 Higgs bosons:

m_h (scalar, neutral)

m_H (scal

m_A (spe

$m_H^{\pm}$ (cha



- How to reduce theoretical and experimental errors?
- What mass scheme does the central value refer to? (preferred mass scheme for NP-theorists: MSbar)

$$m_t^{\mathrm{Tevatron}} = \quad m_t^{\mathrm{pole}}? \qquad m_t^{\mathrm{1S}}?$$

$$\overline{m_t}(\mu)? \qquad m_t^{\mathrm{PS}}(\mu)? \qquad m_t^{\mathrm{kin}}(\mu)?$$

$\pi^2 \sin^2\beta \quad (\quad m_t^2 \quad)$

$\mathcal{O}(\alpha_s^2)$ corrections known

Top Quark Pole Mass

- Based on (unphysical) concept of top quark being a free parton

$$p - m_t - \Sigma(p, m_t)\big|_{p^2=m_t^2}$$

- No physical quantity (i.e. renormalization condition) exists that is tied to the pole mass scheme., also not the peak of the top invariant mass distribution.
- Pole mass renormalization condition introduces artificially large corrections.

$\overline{m}_t(\overline{m}_t)$ [GeV]	$M_t^{\rm pole}$ [GeV]		
	1-loop	2-loop	3-loop
160.00	167.44	169.05	169.56
165.00	172.64	174.28	174.80
170.00	177.84	179.52	180.05

$\alpha_s(M_z) = 0.119$

1.6 GeV

- Pole mass measurements are:
  - order-dependent
  - strongly correlated to other theory parameters

Main Methods at Tevatron

Template Method

- Principle: perform kinematic fit and reconstruct top mass event

$$\chi^2 = \sum_{i=\ell,4jets} \frac{(p_T^{i,fit} - \ldots}{\ldots} + \frac{(M_{\ell\nu} - M_W)^2}{\Gamma_W^2}$$

Usually pick

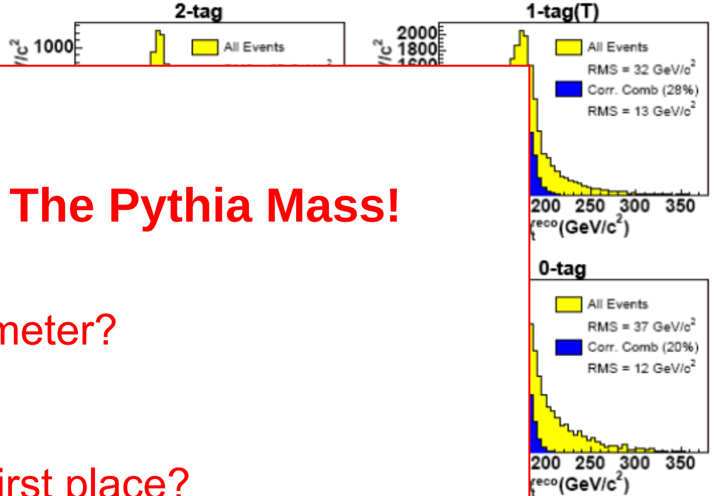
Dynamic

- Principle: a function objects in t Maximize s

What mass is measured?: The Pythia Mass!

- What is the Pythia mass parameter?
- It’s not the pole mass !
- How reliable is the MC in the first place?
- How can we approach the issue?
- Should we be worried concerning top physics at LHC ?

Lepton+jets (≥1 b-tag); Signal-only templates

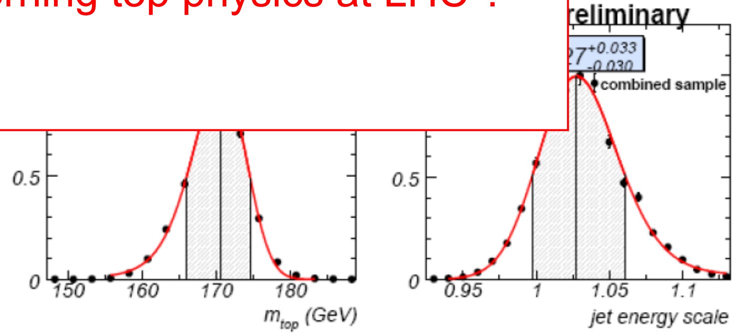


$$P(x;m_t) = \frac{1}{\sigma}\int d^n\sigma(y;m_t)\,dq_1 dq_2\, f(q_1) f(q_2)\, W(x\,|\,y)$$

parton distribution functions

differential cross section (LO matrix element)

transfer function: mapping from parton-level variables (y) to reconstructed-level variables (x)



MC and the Top Mass

- MC is an excellent tool to describe many **physical** cross sections.

- Concept of mass in the MC depends on the **structure and reliability of the perturbative part** and the **interplay of perturbative and nonperturbative part** in the MC.

Parton shower (kinematic expansion)

Perturbative corrections

top mass definition
contained here !

scheme-dependent
separation
(shower cut)

Hadronization Models

affect all top mass
dependent observables

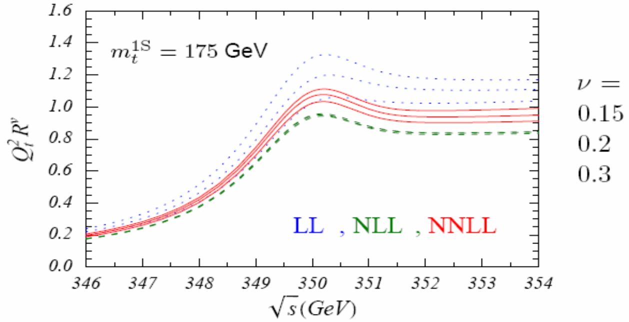
Basic Direct Methods

Threshold Scan $Q \approx 2m_t$ **ILC**

Miquel, Martinez; Boogert, Gounaris

- count number of $t\bar{t}$ events
- color singlet state
- background is non-resonant
- physics well understood (renormalons, summations)

Manohar,Stewart,Teubner,AH



$\rightarrow \delta m_t^{\rm exp} \simeq 50$ MeV

$\rightarrow \delta m_t^{\rm th} \simeq 100$ MeV

What mass?

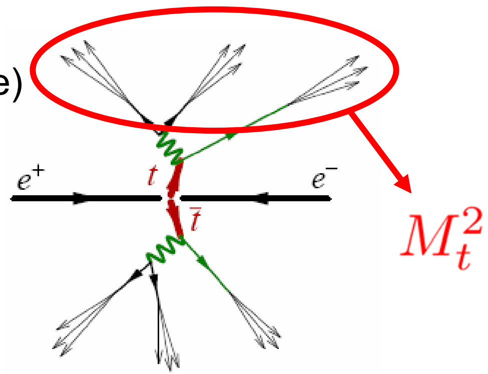
$\sqrt{s}_{\rm rise} \sim 2m_t^{\rm thr}$ + pert.series

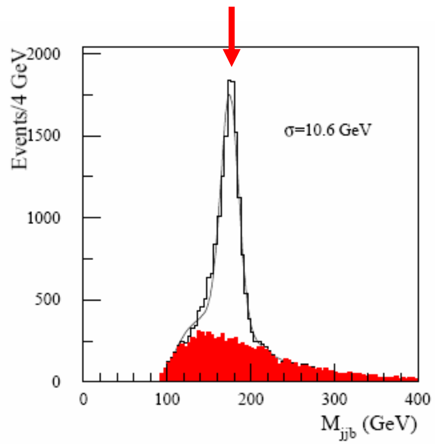
(short distance mass: 1S $\leftrightarrow \overline{\rm MS}$)

"threshold masses"

Invariant Mass Reconstruction $Q \geq 2m_t$ **Tev +LHC + ILC**

- Relation: quark masses $\longleftrightarrow$ resonance mass
- Mass-scheme dependence (best convergence)
- Radiative corrections
- Needs consistent separation
  - perturbative effects
  - non-perturbative effects





Reconstruction Methods

Lepton + Jets

Atlas study:
Borjanovic etal

Plain Method:

- W reconstruction + b tagging
- Inv. mass from M_{jjb}
- $\Delta R = 0.4$
- 100.000 events after cuts

Major error sources:

- b jet energy scale: $x \times 0.7$ GeV
- FSR jets: 1 GeV

Kinematic Fit:

- reconstruct entire event
- impose constraints (e.g. $M_{jjb} = M_{jl\nu}$)
- vary unknows freely

- b jet energy scale: $x \times 0.7$ GeV
- FSR jets: < 0.5 GeV

Continuous Jet Definition:

- plain method for varying cone size: $\Delta R = 0.3 \ldots 1.0$
- take weighted average
- FSR error reduced

Event shape-like !
Has a chance to be computed analytically.

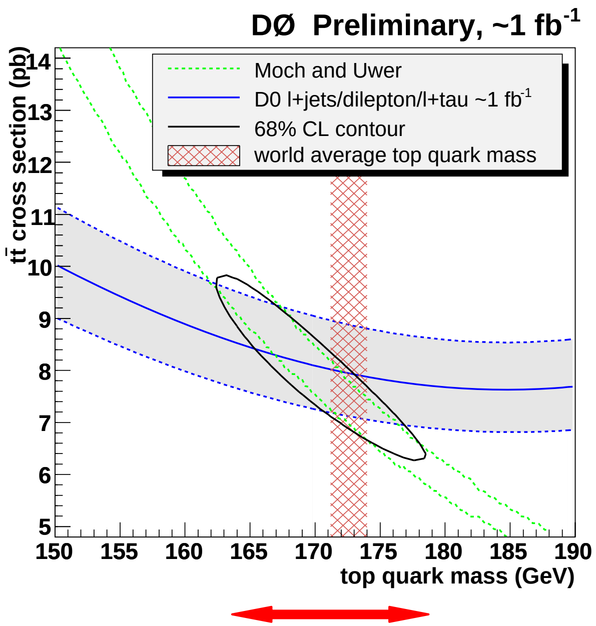
Large p_T Events:

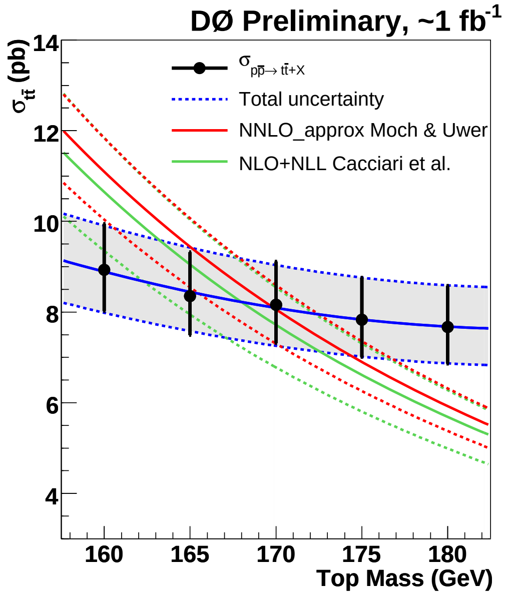
- select events with $p_T > 200$ GeV
- top pair back-to-back ⟹ decay products in different hemispheres
- large cone size around top/antitop jet axes: $\Delta R = 0.8 \ldots 1.8$
- M_t and $M_{\bar{t}}$ from in-cone momenta
- strong sensitivity to soft jets + Underlying Events
- Mass scale calibrations (W mass)

→

- UE: 1.3 GeV
- calibration 1 GeV

Top Mass from the Cross Section





Should be computed in the MSbar scheme !

- Theoretical cross section taken from theorists - computed in the pole scheme.
- More sensitivity to uncertainties that affect the normalization of the cross section.

- Experimental total cross section determined with MC, depends on MC top mass.
- MC top mass identified with the pole mass.
- Top mass dependence can be reduced by modifying the analysis.

2 ways to go…

Factorization Theorems:

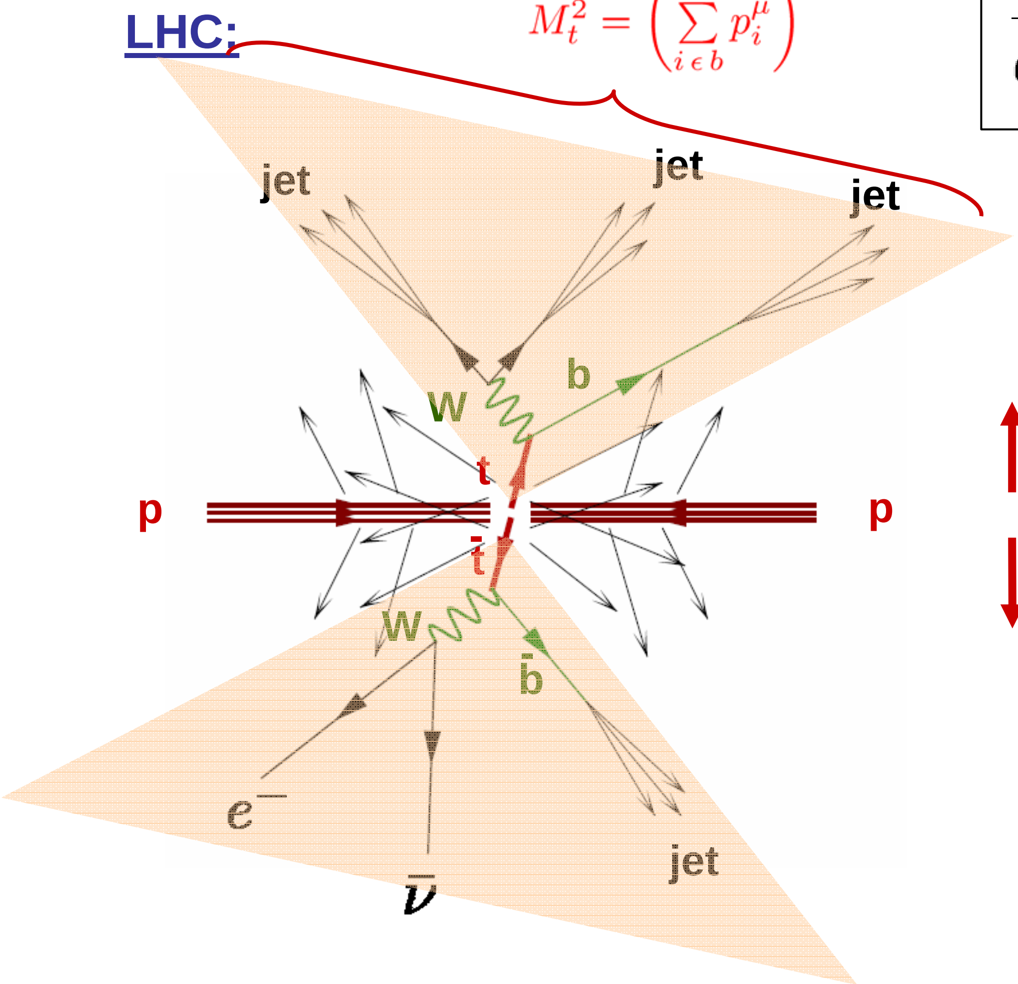
- Derive and compute factorization theorems for the invariant mass distribution
- Achieve independence of the MC reign
- needs as much as possible inclusive definition (hadron event shape)
- conceptually clear and systematic
- full analytic control over perturbative and non-perturbative contribution and the top mass scheme
- very much MC-independent
- new challenging problems to resolve (e.g. underlying events)

Define the top mass in MC's:

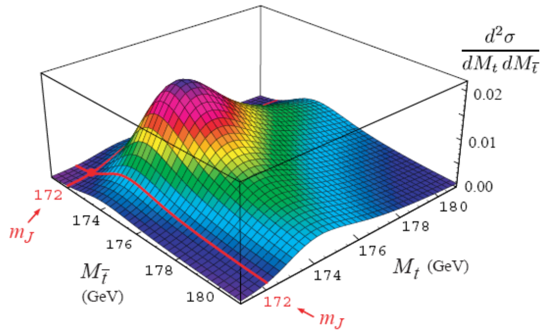
- Determine which mass definition is in the MC's.
- Seems to be the most convenient resolution.
- Relies on how much MC's are indeed systematic tools to do QCD computations.
- Could require a new generation of MC's.

Factorization Theorems

Large- p_T Method at the LHC



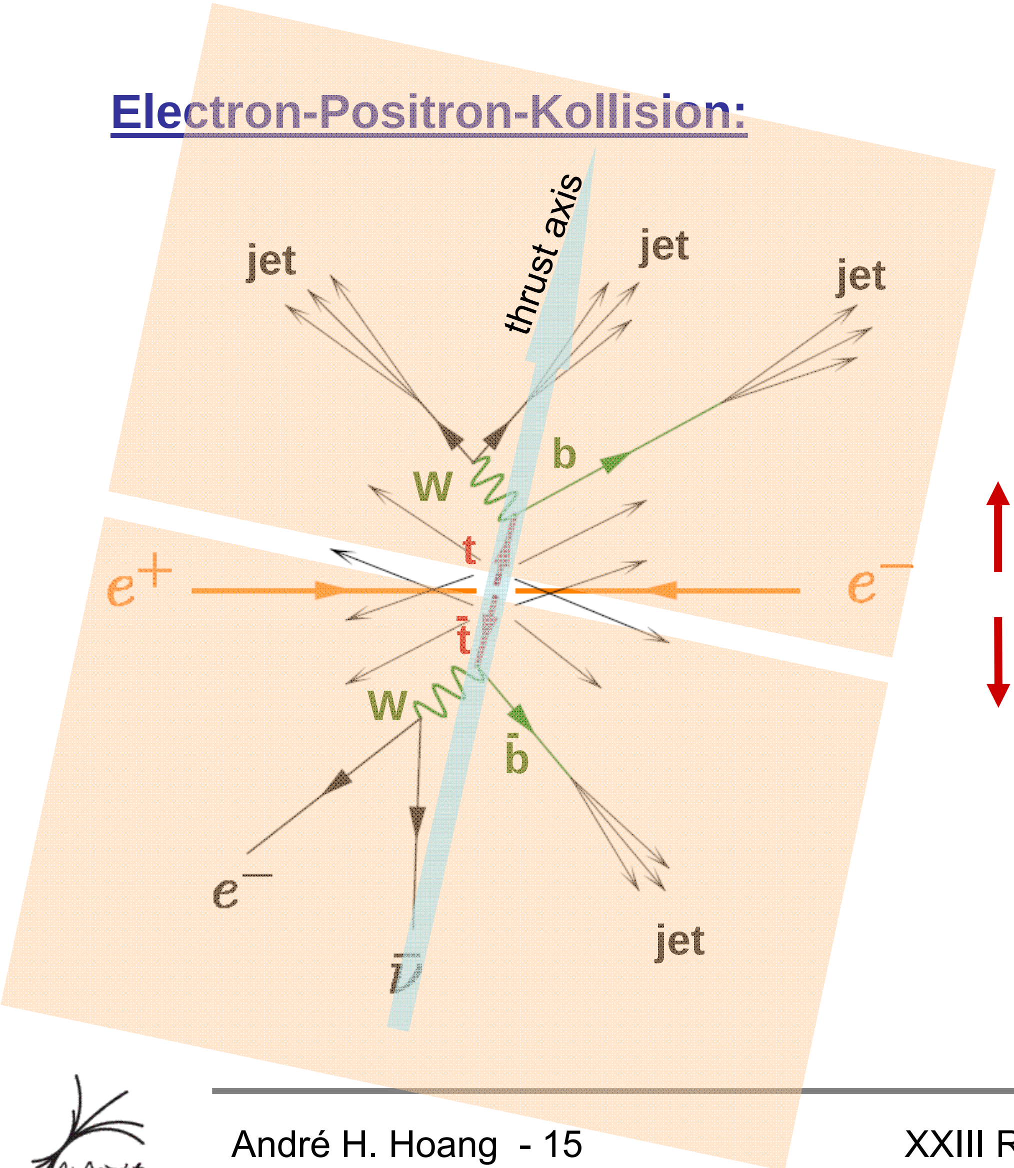
$$\frac{d^2\sigma}{dM_t^2\, dM_{\bar{t}}^2}$$



$$p_T \gg m_t$$

Tevatron: no large p_T events !

Top Production at the ILC



- No beam remnant : all soft radiation can be assigned to top or anti-top

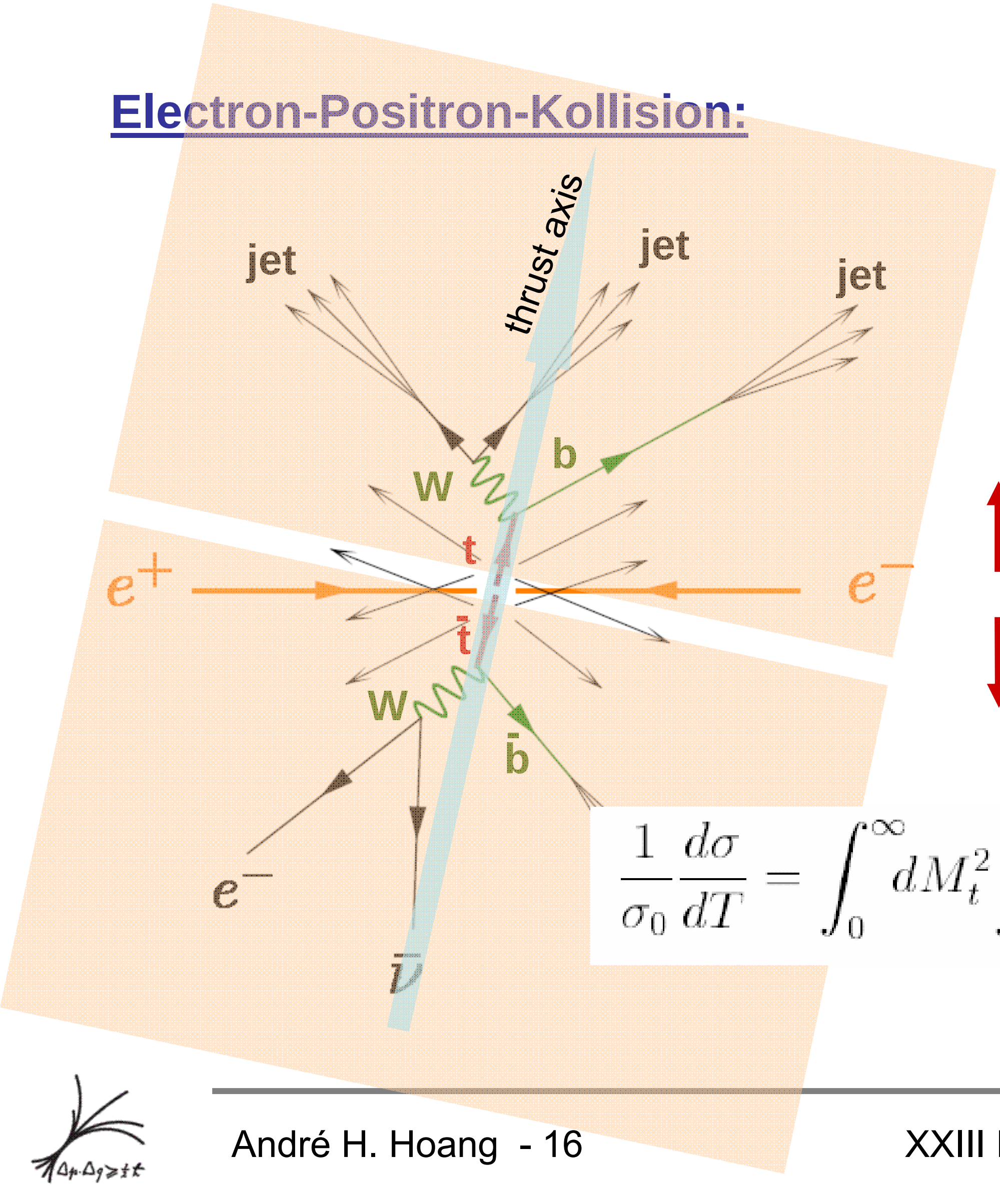
  → k_T jet algorithm

  → hemisphere prescription

$$Q \gg m_t$$

- Avoids : Underying Events & Inititial State Radiation
- Event-shape: complete event characterized/controlled by a few IR-safe variables

Top Production at the ILC



- No beam remnant : all soft radiation can be assigned to top or anti-top

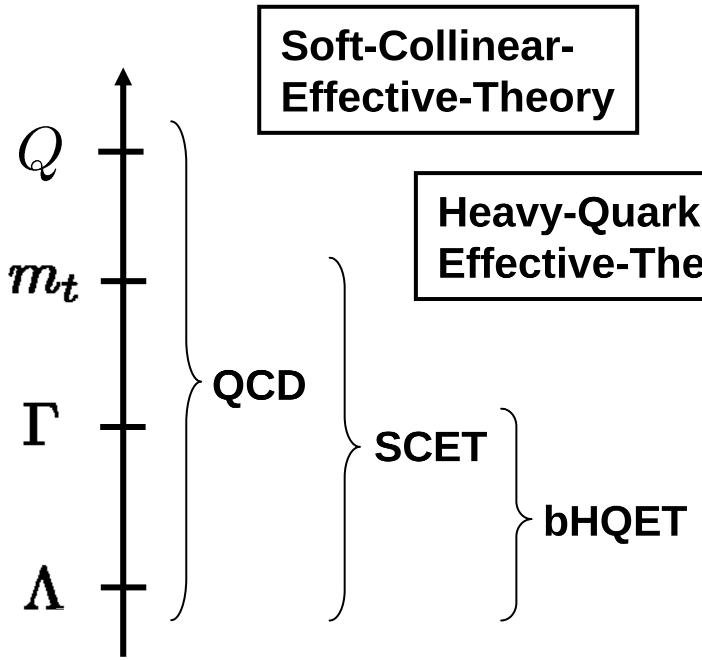
  → k_T jet algorithm

  → hemisphere prescription

$$Q \gg m_t$$

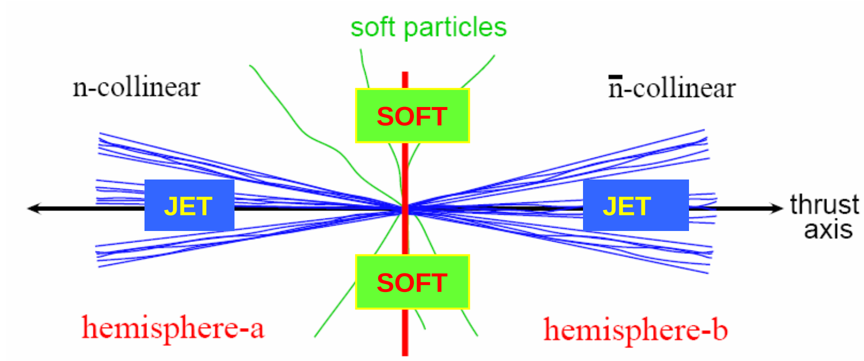
$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = \int_0^\infty dM_t^2 \int_0^\infty dM_{\bar{t}}^2 \;\; \delta\left(\tau - \frac{M_t^2 + M_{\bar{t}}^2}{Q^2}\right) \frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2}$$

Factorization Theorem



Fleming, Mantry, Stewart, AH
PRD77, 074010 (2008)
PRD77, 114003 (2008)

$$\hat{s} = \frac{M_t^2 - m_t^2}{m_t} \ll m_t$$



LO factorization theorem for the double resonance region

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar{t}}^2}\right)_{\text{hemi}} = \sigma_0\, H_Q(Q,\mu_m) H_m\left(m,\frac{Q}{m},\mu_m,\mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\left(\hat{s}_t - \frac{Q\ell^+}{m},\Gamma,\mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m},\Gamma,\mu\right) S_{\text{hemi}}(\ell^+,\ell^-,\mu)$$

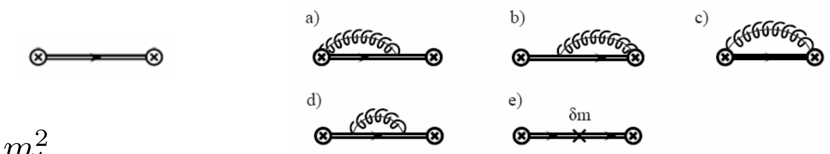
JET JET SOFT

Factorization Theorem

$$\left(\frac{d^2\sigma}{dM_t^2\,dM_{\bar t}^2}\right)_{\rm hemi} = \sigma_0\, H_Q(Q,\mu_m) H_m\Big(m,\frac{Q}{m},\mu_m,\mu\Big) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^-\, B_+\Big(\hat s_t - \frac{Q\ell^+}{m},\Gamma,\mu\Big)\, B_-\Big(\hat s_{\bar t} - \frac{Q\ell^-}{m},\Gamma,\mu\Big)\, S_{\rm hemi}(\ell^+,\ell^-,\mu)$$

Jet functions: $B_+(2v_+\cdot k) = \frac{-1}{8\pi N_c m}\,{\rm Disc}\int d^4x\, e^{ik\cdot x}\langle 0|{\rm T}\{\bar h_{v_+}(0)W_n(0)W_n^\dagger(x)h_{v_+}(x)\}|0\rangle$

- perturbative, <u>any mass scheme</u>
- depends on m_t, Γ_t
- Breit-Wigner at tree level



$$B_\pm(\hat s,\Gamma_t) = \frac{1}{\pi m_t}\frac{\Gamma_t}{\hat s^2+\Gamma_t^2} \qquad \hat s = \frac{M^2-m_t^2}{m_t}$$

Soft function: $S_{\rm hemi}(\ell^+,\ell^-,\mu) = \frac{1}{N_c}\sum_{X_s}\delta(\ell^+ - k_s^{+a})\delta(\ell^- - k_s^{-b})\langle 0|\overline{Y}_{\bar n}\, Y_n(0)|X_s\rangle\langle X_s|Y_n^\dagger\,\overline{Y}_{\bar n}^\dagger(0)|0\rangle$

- non-perturbative (fragement. fct.)
- dep. on treatement of soft radiation
- also governs **<u>massless dijet thrust and jet mass event distributions</u>**

Korshemsky, Sterman, etal.
Bauer, Manohar, Wise, Lee

➡ **Short distance top mass can (in principle) be determined to better than $\Lambda_{\rm QCD}$.**

NLL Numerical Analysis

Double differential invariant mass distribution:

$$Q = 5 \times 172\ \text{GeV}$$

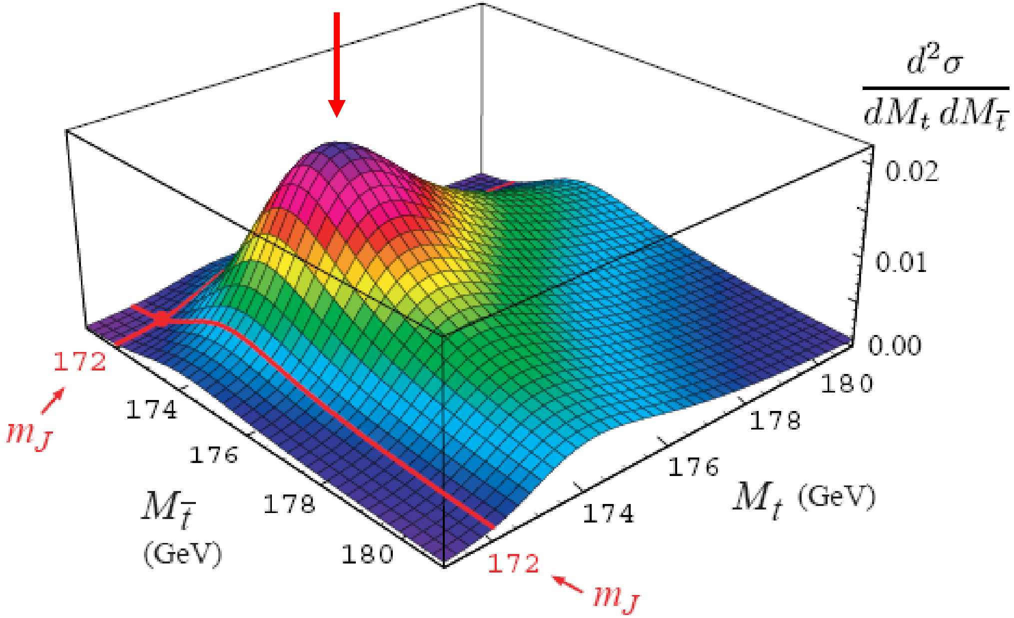
$$\Gamma = 1.43\ \text{GeV}$$

$$m_J(2\,\text{GeV}) = 172\ \text{GeV}$$

$$\mu_\Gamma = 5\ \text{GeV}$$

$$\mu_\Lambda = 1\ \text{GeV}$$

$$a = 2.5\,, \quad b = -0.4$$
$$\Lambda = 0.55\ \text{GeV}$$



Non-perturbative effects shift the peak by <u>+2.4 GeV</u> and broaden the distribution.

NLL Numerical Analysis

Peak Position in any Mass Scheme:

$$M^{\mathrm{peak}} = \underbrace{\underbrace{m}_{\text{scheme dependent}} + \Gamma_t(\alpha_s + \underbrace{\alpha_s^2}_{\text{scheme dependent}} + \ldots)}_{\text{scheme independent}} + \frac{Q\Lambda_{\mathrm{QCD}}}{m}$$

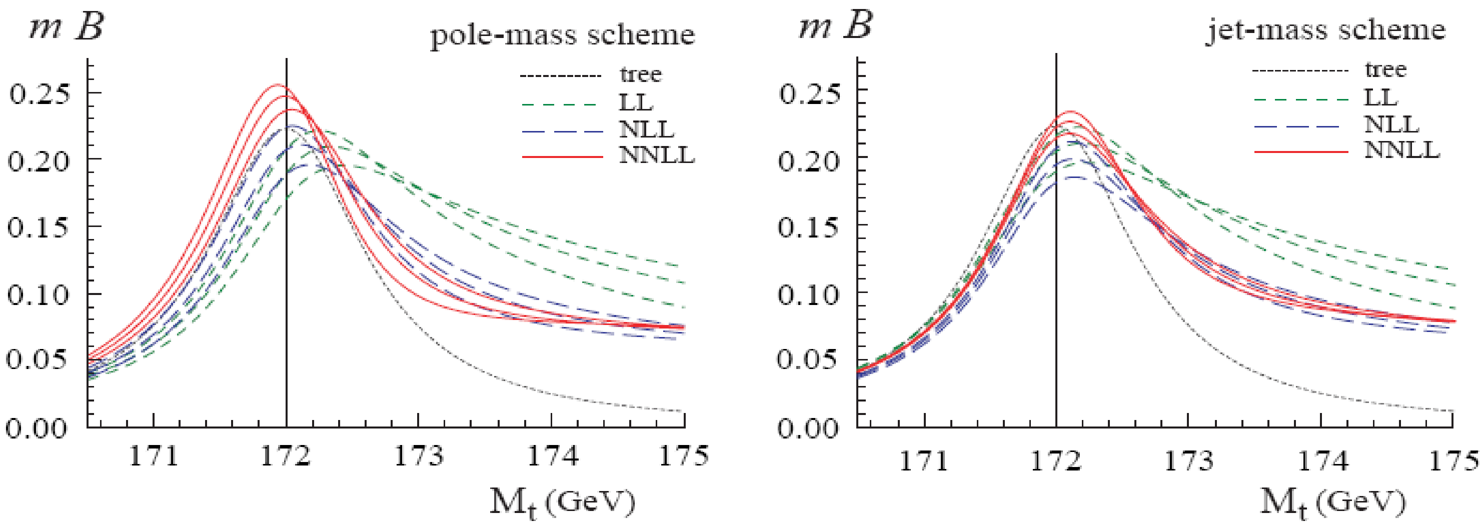
- Good mass scheme: has well behaved perturbative series.
- Jet mass scheme: small momentum contribution of the jet function are absorbed into the mass

Top Quark Mass Scheme

→ Jet function has an $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon in the pole mass scheme

$$\mathcal{B}_\pm(\hat{s},0,\mu,\delta m) = -\frac{1}{\pi m}\frac{1}{\hat{s}+i0}\left\{1+\frac{\alpha_s C_F}{4\pi}\left[4\ln^2\left(\frac{\mu}{-\hat{s}-i0}\right)+4\ln\left(\frac{\mu}{-\hat{s}-i0}\right)+4+\frac{5\pi^2}{6}\right]\right\} - \frac{1}{\pi m}\frac{2\delta m}{(\hat{s}+i0)^2}$$

$$\delta m = m_t^{\rm scheme} - m_t^{\rm pole}$$



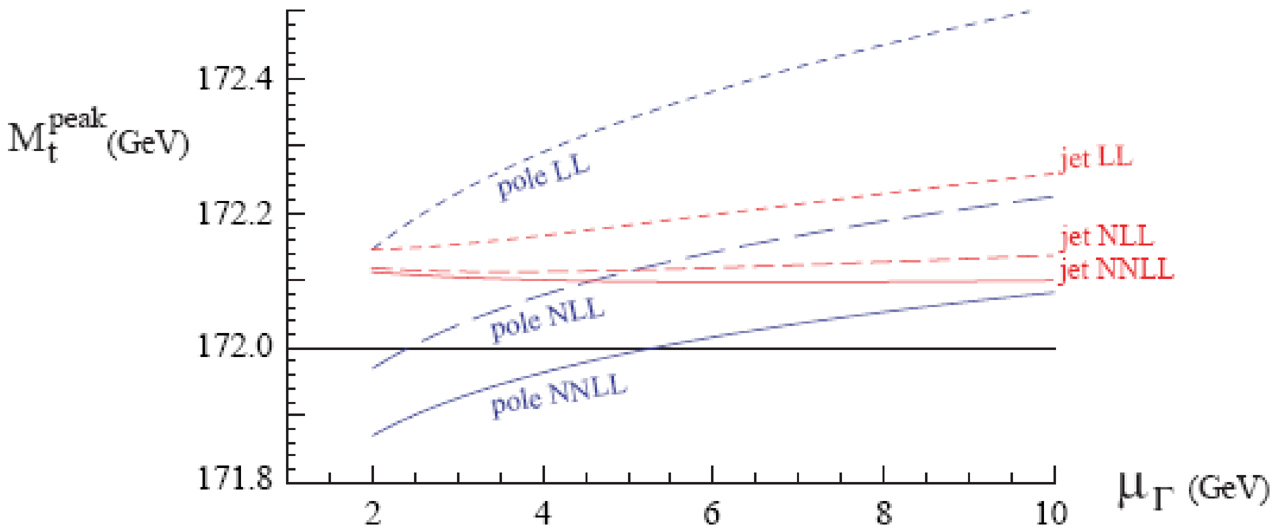
Jain, Scimemi, Stewart PRD77, 094008(2008)

$$m_{\rm pole} = m_J(\mu) + e^{\gamma_E} R\frac{\alpha_s(\mu)\,C_F}{\pi}\left[\ln\frac{\mu}{R}+\frac{1}{2}\right] + \mathcal{O}(\alpha_s^2)$$

$$\boxed{R \sim \Gamma_t}$$

NLL Numerical Analysis

Scale-dependence of peak position

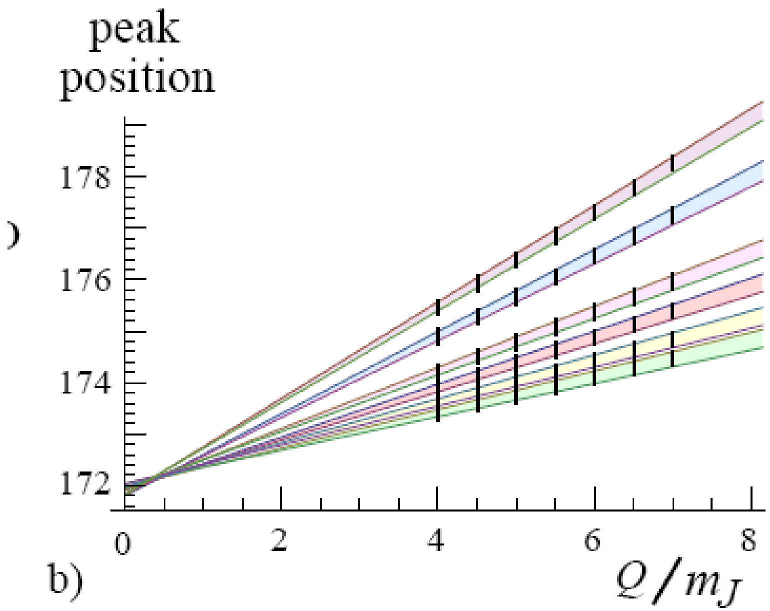


- Jet mass scheme: significanly better perturbative behavior.
- Renormalon problem of pole scheme already evident at NLL.

NLL Numerical Analysis

Q-dependence of peak position

$$M_t^{\text{peak}} \approx m_J(\mu_\Gamma) + \frac{Q}{m_J}\,\text{const}$$



different soft models

different treatment for soft particles between jets

Fairly precise determination of jet mass from determination of Q-dependence of the peak position and extrapolation Q to zero

LHC: $M_t(p_T^{\text{cut}})$

Theory Issues for $pp \to t\bar{t} + X$

- ★ definition of jet observables → Hadron event shapes $$T_{\perp,g} \equiv \max_{\vec{n}_T} \frac{\sum_i |\vec{q}_{\perp i} \cdot \vec{n}_T|}{\sum_i q_{\perp i}}$$ Banfi, Salam, Zanderighi
- ★ initial state radiation
- ★ final state radiation
- • underlying events → Soft function ?
- ★ color reconnection & soft gluon interactions
- ★ beam remnant
- ★ parton distributions
- ★ summing large logs $Q \gg m_t \gg \Gamma_t$
- ★ relation to Lagrangian short distance mass

★ **Can be addressed in the framework of a LC.**

★ **Requires extensions of LC concepts and other known concepts**

The Monte Carlo Top Mass

MC Top Mass

→ Use analogies between MC set up and factorization theorem

Final State Shower

- Start: at transverse momenta of primary partons, evolution to smaller scales.
- Shower cutoff $R_{sc} \sim 1 \text{ GeV}$
- Hadronization models fixed from reference processes

Factorization Theorem

- Renormalization group evolution from transverse momenta of primary partons to scales in matrix elements.
- Subtraction in jet function that defines the mass scheme
- Soft function model extracted from another process with the same soft function

Additional Complications:

Initial state shower, underlying events, combinatorial background, etc

Let's assume that these aspects are treated correctly in the MC

MC Top Mass

Conclusion:

constant of order unity

$$m_t^{\mathrm{MC}}(R_{sc}) \;=\; m_t^{\mathrm{pole}} \;-\; R_{sc}\, c \left[\frac{\alpha_s}{\pi}\right] \quad \pm\, \Delta m^{\mathrm{conc.}}$$

Determination of the MSbar mass:

$$m_t^{\mathrm{TeV}} \;=\; m_t^{\mathrm{MC}}(R_{sc}) \;=\; 172.6 \pm 0.8(stat) \pm 1.1(syst) \;\pm\, \Delta m^{\mathrm{conc.}}$$

$\Downarrow$ 3-loop R-evolution equation

AHH,Jain, Scimemi, Stewart
PRL 101,151602(2008)

$$\overline{m}_t(\overline{m}_t) \;=\; 163.0 \pm 1.3\,^{+0.6}_{-0.3}\ \mathrm{GeV} \qquad (c = 3^{+6}_{-2}) \;\pm\, \Delta m^{\mathrm{conc.}}$$

$$(\Delta m^{\mathrm{conc.}})^{\mathrm{TeV}} \;\neq\; (\Delta m^{\mathrm{conc.}})^{\mathrm{LHC}} \qquad \text{“top mass anomaly”}$$

Outlook & Conclusion

Plans:

$\longrightarrow$ Determine top mass in Pythia(e+e-) using the factorization theorem

$\longrightarrow$ Derivation of factorization theorem for large p_T top events at LHC

$\longrightarrow$ Hadron event shapes

$\longrightarrow$ “More systematic” MC’s, e.g. GenEva framework $\longrightarrow \Delta m^{\mathrm{conc.}}$

$\longrightarrow$ NNNLL event shape analysis of LEP data: soft function & α_s & m_b

Conclusion:

$\longrightarrow$ I find is scary that this simple question isn’t answered, while so many very mart new physics models are just waiting to be pulled out of the drawers, if the top mass anomaly is found to be large.

$\longrightarrow$ But real new physics might actually help (e.g. $Z' \longrightarrow t\bar{t}$)