Transverse Phase Space: Beam Size and Emittance

Tasks: Emittance Diagnostics

- estimate the image resolution for an optical synchrotron radiation profile monitor
  - modern 3rd generation light source: $E = 6$ GeV, $\lambda_{obs} = 500$ nm , $\sigma_y = 10$ µm

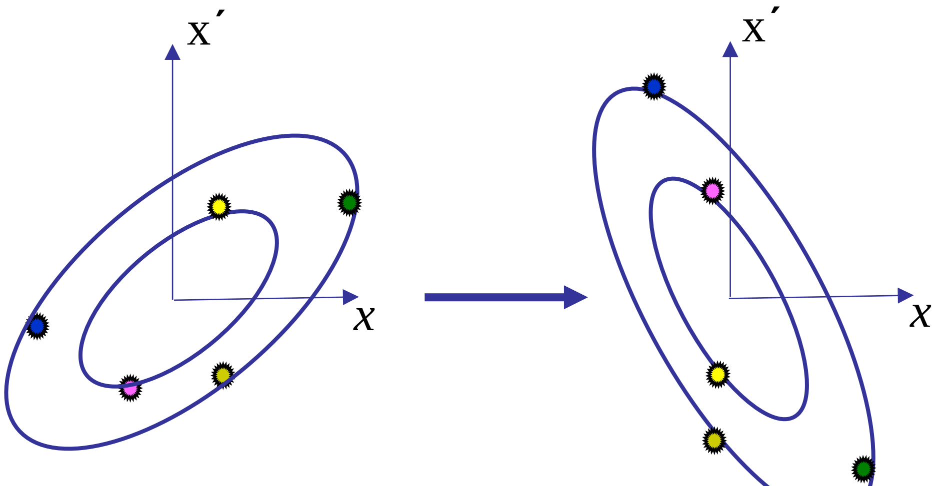
    → assume „self diffaction", i.e. aperture limitation imposed by radiation angular distribution (1/γ)
- derive the single particle transport matrix for a drift space
  - assume paraxial approximation

    → $\sin(x') \approx x'$
- calculate the evolution of the beam size after a drift space
  - use the beam matrix transformation together with the transport matrix R for a drift space
- investigate the performance of the CCD
  - spatial calibration → dot grid target (0.5 mm spacing)
  - resolution → Siemens star, USAF 1951 target
- measure the emittance of the laser beam
  - measure spot sizes for different distances of the lens
  - analyse the horizontal profiles as function of the lens position
  - calculate the laser beam emittance → use the simplest way with only 2 values
  - (repeat with a different scintillator thickness)

Transverse Emittance

- projection of phase space volume
  - separate horizontal, vertical and longitudinal plane
- accelerator key parameter
  - defines **luminosity** / **brilliance**
- linear forces
  - any particle moves on an ellipse in phase space (x,x´)
  - ellipse rotates in magnets and shears along drifts

    → but area is preserved: **emittance**



$$\varepsilon = \gamma\, x^2 + 2\alpha\, x\, x' + \beta\, x'^2$$

(α, β, γ, ε: Courant-Snyder or Twiss parameters)

- transformation along accelerator
  - knowledge of the magnet structure (beam optics) → transformation from initial (i) to final (f) location

→ single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \underbrace{\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}}_{R} \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

→ transformation of optical functions

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} R_{11}{}^2 & -2R_{11}R_{12} & R_{12}{}^2 \\ -R_{11}R_{21} & 1+R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}{}^2 & -2R_{21}R_{22} & R_{22}{}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$

Transverse Emittance Ellipse

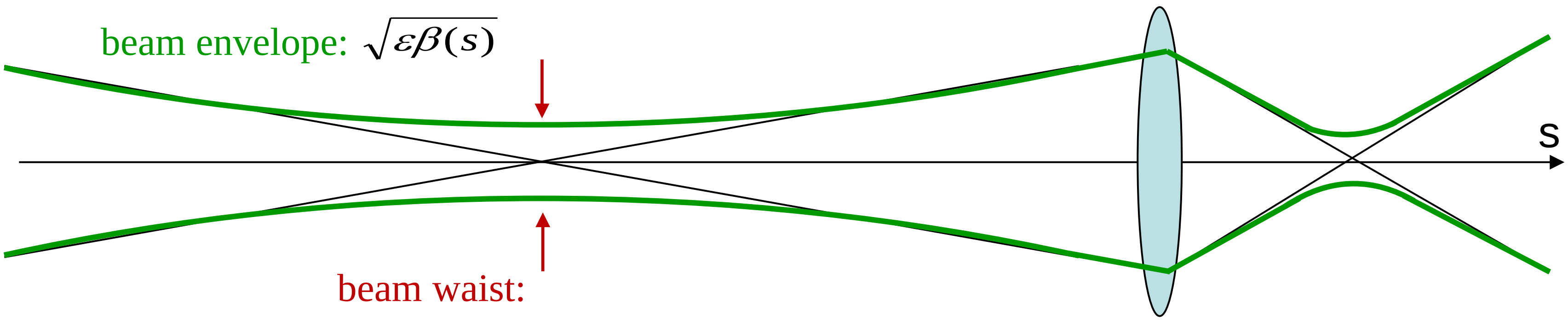
- propagation along accelerator
  - change of ellipse shape and orientation → area is preserved

$$\varepsilon = \gamma(s)\,x(s)^2 + 2\alpha(s)\,x(s)\,x'(s) + \beta(s)\,x'(s)^2$$

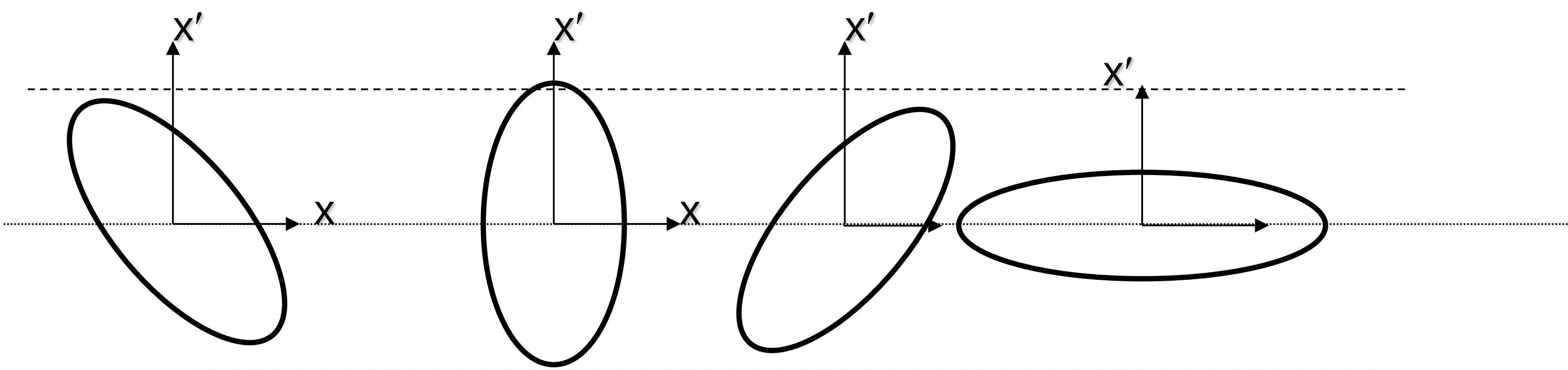
$$\alpha(s) = -\frac{\beta'(s)}{2}$$

$$\gamma(s) = \frac{1+\alpha^2(s)}{\beta(s)}$$

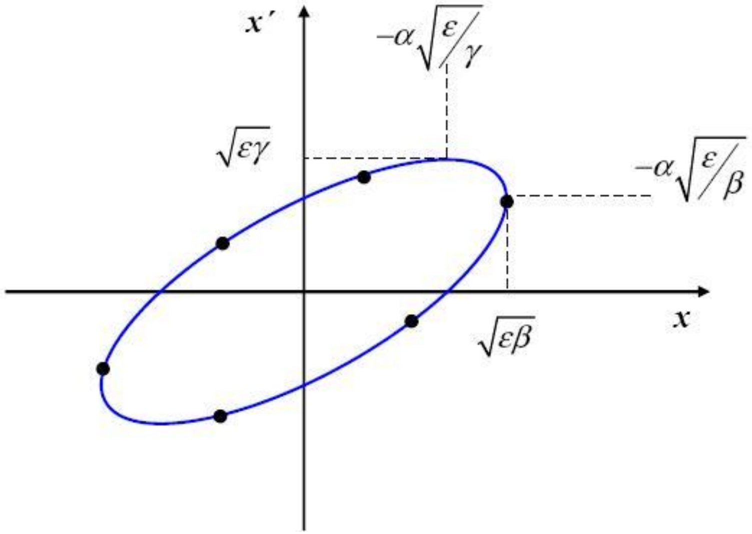
$$x(s) = \sqrt{\varepsilon\beta(s)} \cdot \cos\left[\Psi(s) + \Phi\right]$$



→ minimum in envelope → minimum in β → β′ = 0 → α = 0



Emittance and Beam Matrix



- via Twiss parameters

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

- statistical definition

P.M. Lapostolle, IEEE Trans. Nucl. Sci. NS-18, No.3 (1971) 1101

$$\varepsilon_{rms} = \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

2nd moment of beam distribution ρ(x)

$$\langle x^2\rangle = \frac{\int_{-\infty}^{\infty} dx\, x^2 \cdot \rho(x)}{\int_{-\infty}^{\infty} dx\, \rho(x)}$$

- ε_{rms} is measure of spread in phase space
- root-mean-square (rms) of distribution

$$\sigma_x = \langle x^2\rangle^{1/2}$$

- ε_{rms} useful definition for non-linear beams

→ usually restriction to certain range

(c.f. 90% of particles instead of [-∞,+∞])

- beam matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2\rangle & \langle xx'\rangle \\ \langle xx'\rangle & \langle x'^2\rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11}\cdot\Sigma_{22} - \Sigma_{12}^2}$$

- transformation of beam matrix

$$\Sigma^1 = R\Sigma^0 R^T \qquad R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

Beam Matrix based Measurements

- **starting point:** beam matrix $$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

- **emittance determination**
  - measurement of **3** matrix elements $\mathbf{\Sigma_{11}}$, $\mathbf{\Sigma_{12}}$, $\mathbf{\Sigma_{22}}$
  - **remember:** beam matrix σ depends on location, i.e. Σ(s)

    → determination of matrix elements at same location required

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11} \cdot \Sigma_{22} - \Sigma_{12}^2}$$

- **access to matrix elements**
  - profile monitor determines only $\sigma = \sqrt{\Sigma_{11}}$
  - other matrix elements can be inferred from beam profiles taken under various transport conditions

    → knowledge of transport matrix R required

$$\Sigma^b = R \cdot \Sigma^a \cdot R^T \qquad R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

- **measurement of at least 3 profiles for 3 matrix elements**

$$\Sigma_{11}^a$$

$$\Sigma_{11}^b = R_{11}^2 \cdot \Sigma_{11}^a + 2R_{11}R_{12} \cdot \Sigma_{12}^a + R_{12}^2 \cdot \Sigma_{22}^a$$

$$\Sigma_{11}^c = \overline{R}_{11}^2 \cdot \Sigma_{11}^a + 2\overline{R}_{11}\overline{R}_{12} \cdot \Sigma_{12}^a + \overline{R}_{12}^2 \cdot \Sigma_{22}^a$$

  - **measurement :** profiles $\sigma^{a,b,c} = \sqrt{\Sigma_{11}^{a,b,c}}$
  - **known:** transport optics $R, \overline{R}$
  - **deduced:** matrix elements $\Sigma_{11}^a, \Sigma_{12}^a, \Sigma_{22}^a$

→ more than 3 profile measurements favourable, data subjected to least-square analysis

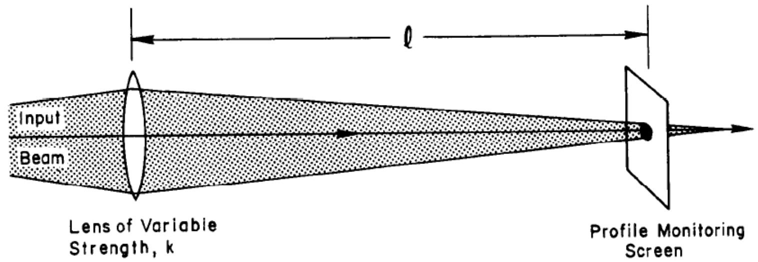
Miscellaneous

- transfer matrices
  - drift

$$R_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

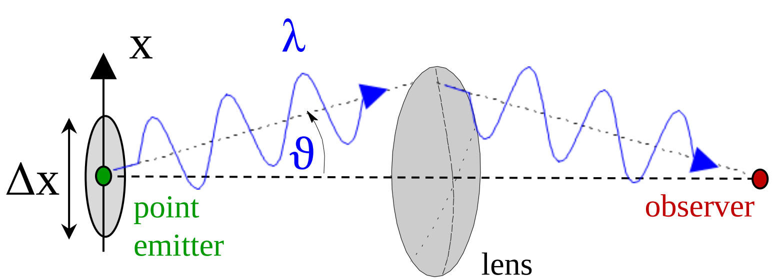
  - quadrupole

$$R_{quad} = \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$



R_{quad} (f = 1/K) R_{drift} (drift space) → $R_{total} = R_{drift}\, R_{quad}$

- fundamental resolution limit
  - point observer detecting photons from point emitter → location of emission point ?



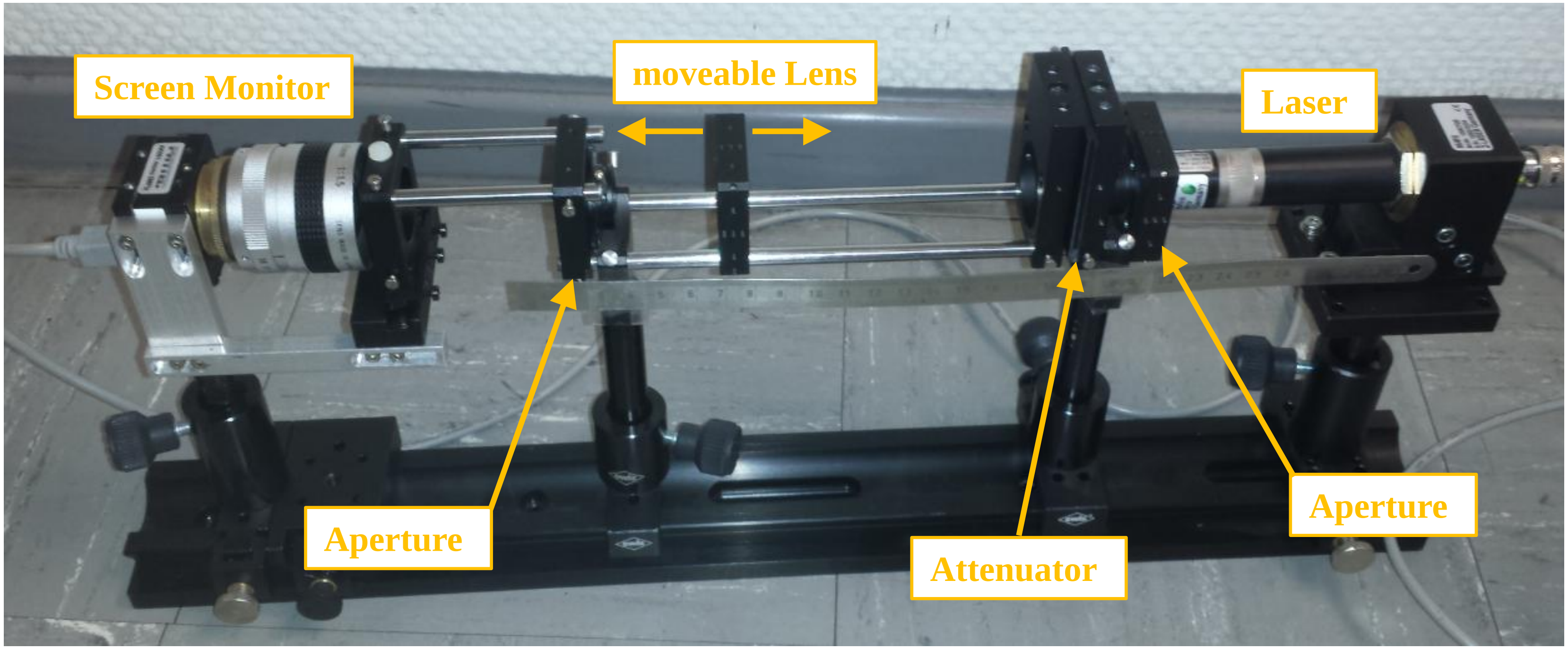
NA = sinϑ:

numerical aperture

uncertainty principle: $\Delta x \cdot \Delta p_x \approx h \Rightarrow \Delta x \approx \frac{\lambda}{2 \sin \vartheta}$

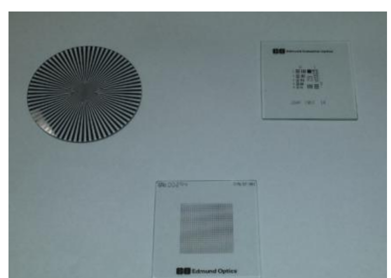
Emittance Measurement Test Setup

- test setup



  - screen : Al_2O_3:Cr (Chromox), thickness 1.0 mm / 0.5 mm / 0.3 mm

- calibration / resolution targets



  - Siemens star: focusing
  - USAF 1951-target: resolution
  - dot grid target: calibration

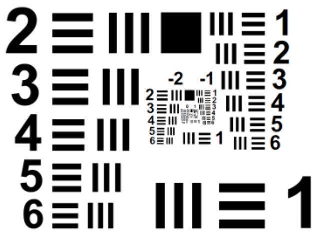
Parameters

- CCD
  - Phytec USB-CAM 051H

Resolution	**2592 x 1944 (5 MPix)**, 2048 x 1536 (3,1MPix), 1600 x 1200 (2MPix), 1280 x 960 (1,2MPix) 1024 x 768 (0,8MPix), 640 x 480 (VGA)			
Model	**USB-CAM-051H**	**USB-CAM-151H**	**USB-CAM-052H**	**USB-CAM-152H**
color / monochrom	monochrom		color	
Sensor Format	1/2,5“			
Image Sensor	Aptina MT9P031, CMOS			
Pixel Size	2,2 µm x 2,2 µm			
Color format	Y8		RGB32, RGGB (Raw)	
Lens Holder	C / CS – Mount			
fps	6 fps to 52 fps			
Dynamic Range	8 bit			
Shutter	Rolling			
Light sensitivity	1,4 V/lux-sec			
Interface	USB 2.0 High Speed			
Exposure time	1/10.000 s to 30 s			
Gain	0 dB to 18 dB			
White Balance	-		-6 dB bis +6 dB	
Power supply	4,5 V bis 5,5V DC			
Power Consumption	Circa 250 mA bei 5V			
Feature (optional)	-	ext. Trigger, Digital-Output	-	ext. Trigger, Digital-Output
Temperature range	-5°C bis +45°C			
Dimensions (B x L x H)	36 mm x 36 mm x 25 mm			
Fixing	1/4“ and M6x8 on all sides			
Weight	70 g			
Connection	USB Mini-B			
Feature- Connection	-	Hirose HR10A-7R-4P	-	Hirose HR10A-7R-4P

- screen
  - material: Al_2O_3:Cr (Chromox)
  - thickness: 1.0 mm / 0.5 mm / 0.3 mm

- dot grid target
  - spacing: 0.5 mm



- USAF 1951 target
  - Wikipedia says…

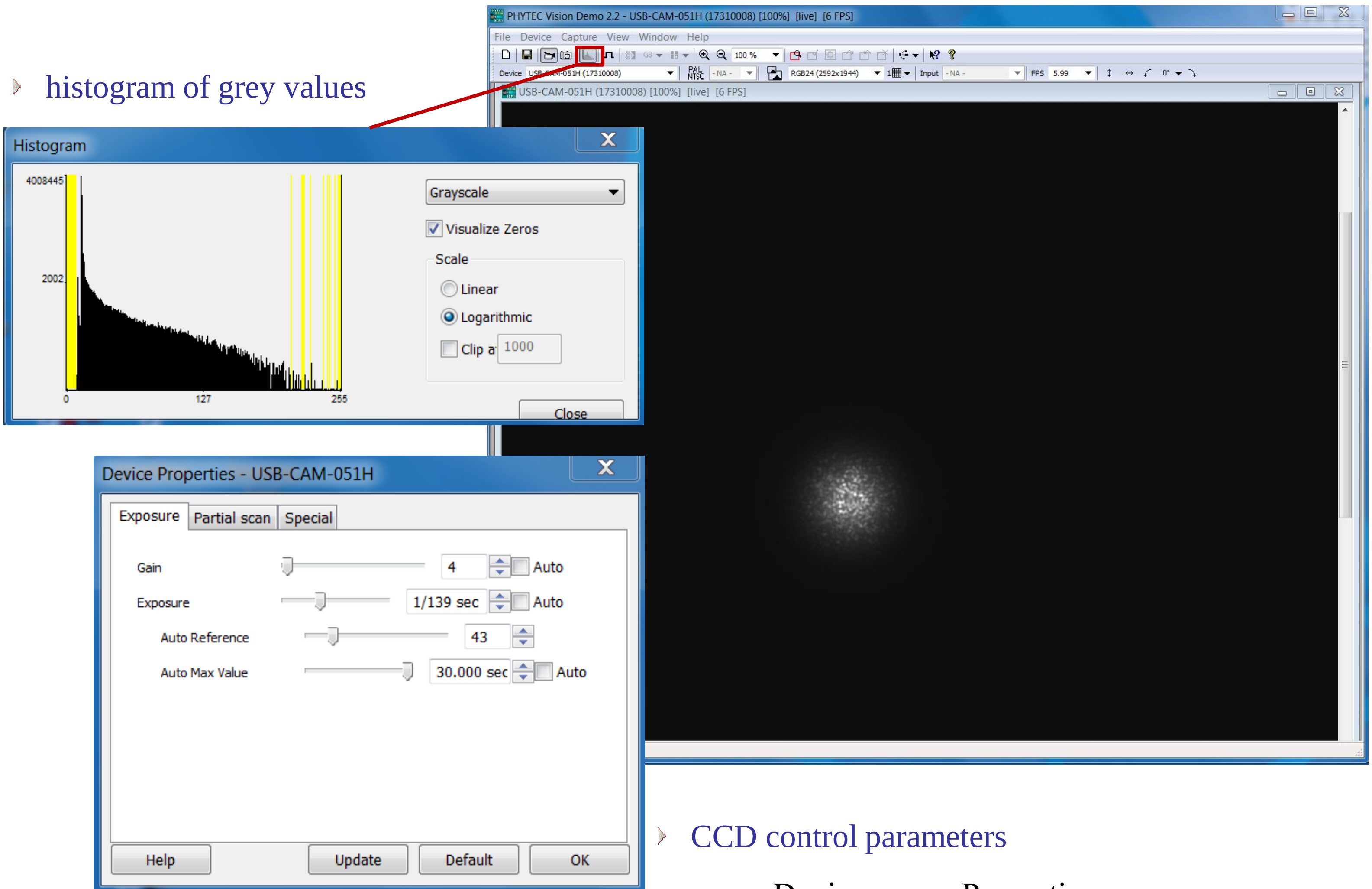
The **1951 USAF resolution test chart** is a resolution test pattern conforming to MIL-STD-150A standard, set by US Air Force in 1951. It is still widely accepted to test the resolution of optical imaging systems such as microscopes, cameras and image scanners, although MIL-STD-150A was cancelled on October 16, 2006. The pattern consists of groups of three bars with dimensions from big to small. The largest bar the imager cannot discern is the limitation of its resolving power...

Number of Line Pairs / mm in USAF Resolving Power Test Target 1951					
	Group Number				
Element	1	2	3	4	5
1	2.00	4.00	8.00	16.00	32.0
2	2.24	4.49	8.98	17.96	35.9
3	2.52	5.04	10.08	20.16	40.3
4	2.83	5.66	11.31	22.63	45.3
5	3.17	6.35	12.70	25.40	50.8
6	3.56	7.13	14.25	28.51	57.0

CCD Readout: Introduction

- **readout program** PHYTEC Vision Demo 2.2

  - histogram of grey values



  - CCD control parameters

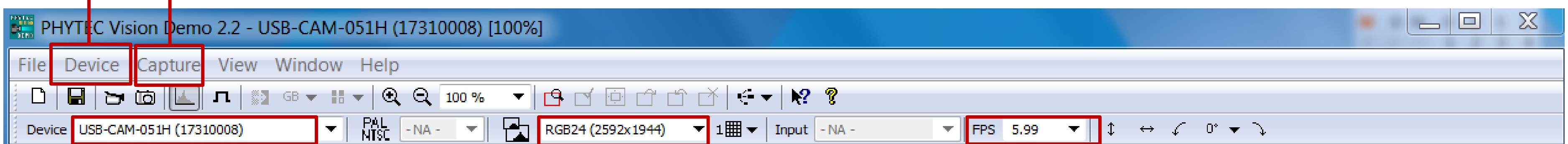
    → Device → Properties

CCD Readout: Introduction

Start/Stop acquisition → Device → Live (Shortcut: Ctrl + L)

CCD control parameters → Device → Properties

Save image → Capture → Save Image (Shortcut: Ctrl + U): save as Jpeg images



CCD type

readout format

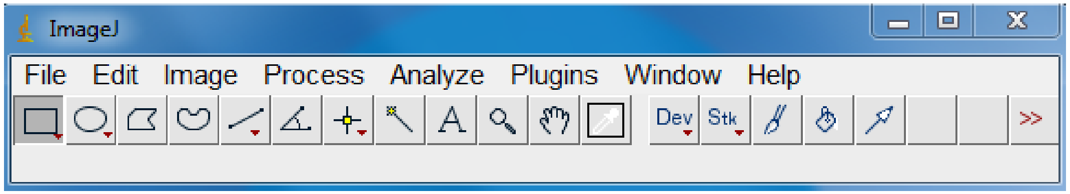
(RGB, 2592 x 1944 pixel)

readout rate

(5.99 frames per second)

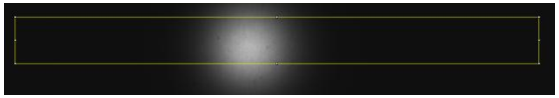
ImageJ: Introduction

- press icon   access to start panel

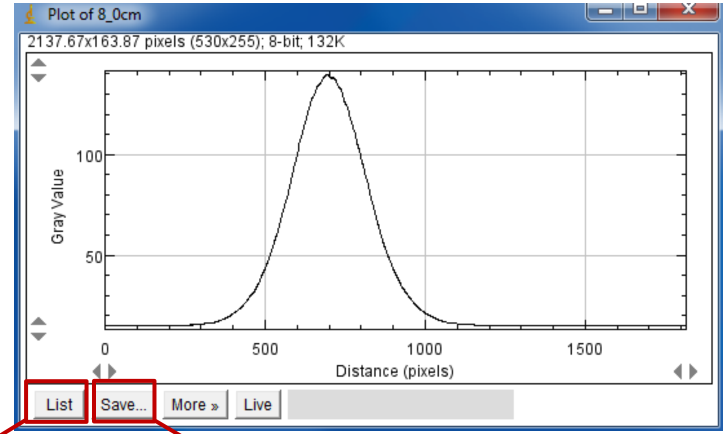


- load image file   → File → Open   (Shortcut: Ctrl + O)
- select ROI: in start panel: select left button (below "File"), usually already pre-selected

  then with left mouse button: draw rectangular ROI



- plot horizontal projection   → Analyze → Plot Profile   (Shortcut: Ctrl + k)

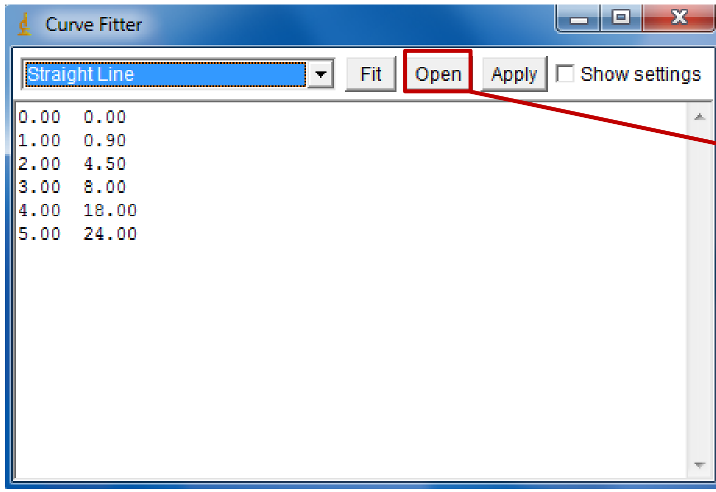


list data points

save data as Excel sheet (required for profile fitting)

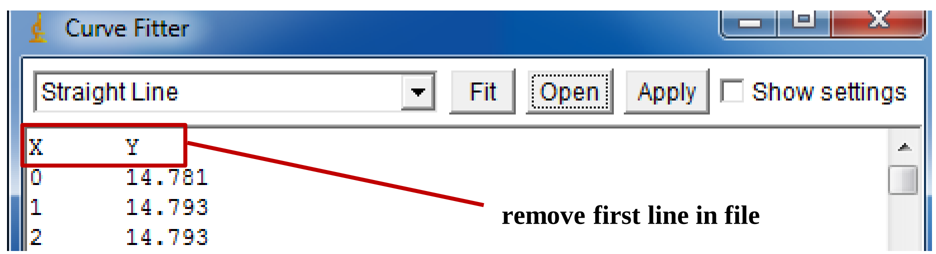
ImageJ: Introduction

- profile fitting → Analyze → Tools → Curve Fitting…
  - load profile data:



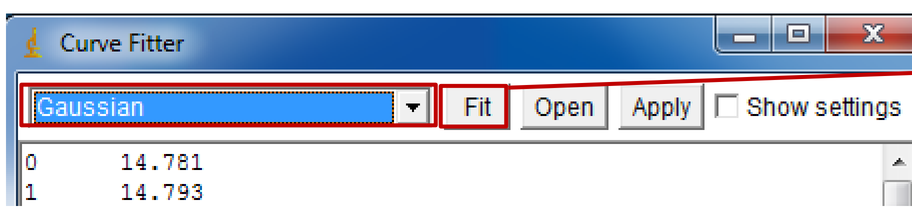
load data file

  - delete bad data:



remove first line in file

  - select fit function:

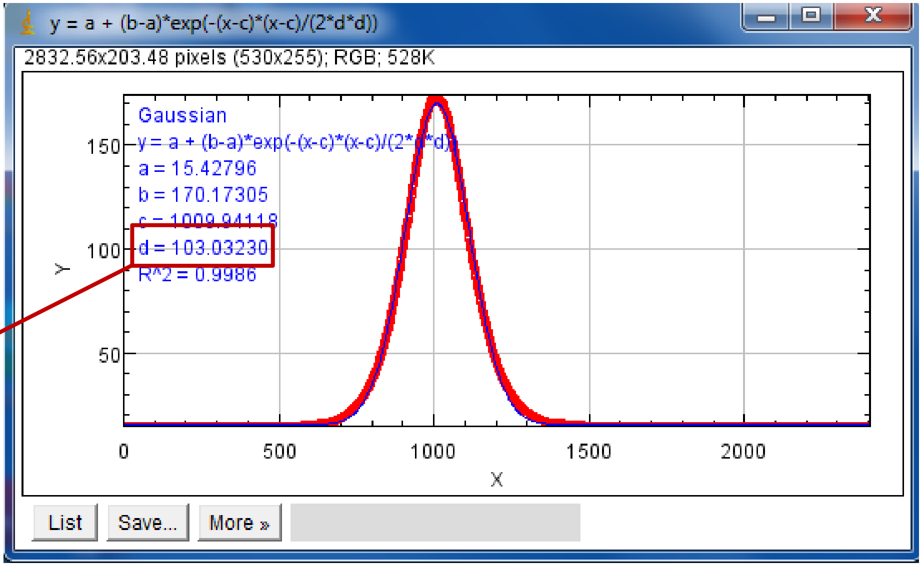


fit profile data

$$y = a + (b-a)\cdot e^{-\frac{(x-c)^2}{2d^2}}$$

ImageJ: Introduction

- fit results:



1 σ-width (in pixel)

additional data fitting

- create data file → e.g. simple ASCII text file with Notepad
- repeat fitting as described before