

Accelerator Laboratory: Introduction to Beam Diagnostics and Instrumentation

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DESY / MDI

- Introduction
- Beam Position Monitor
- Transverse Emittance / Beam Profile

• instrumentation

- › catchword for all technologies needed to produce primary measurements of beam parameters

• diagnostics

- › making use of these instruments in order to
 - operate the accelerators
 - orbit control
 - improve the accelerator performance
 - feedback, emittance preservation
 - deduce additional beam parameters or performance indicators of the machine by further data processing
 - chromaticity measurements, betatron matching, ... (examples for circular accelerator)
 - detect equipment faults

H. Schmickler, Introduction to Beam Diagnostics, CAS 2005

• outline

 **emphasis on beam instrumentation**

Beam Instrumentation for...

- **beam position**

- orbit, lattice parameters, tune, chromaticity, feedback,...

- **beam intensity**

- dc & bunch current, coasting beam, lifetime, efficiencies,...

- **beam profile**

- longitudinal and transverse distributions, emittances,...

- **beam loss**

- identify position of losses, prevent damage of components,...

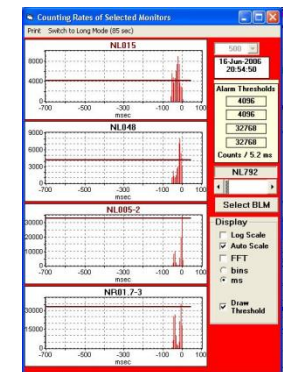
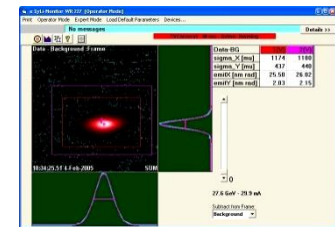
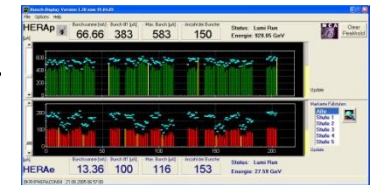
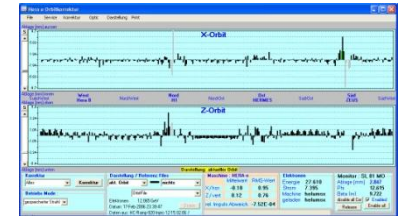
- **beam energy**

- mainly required by users,...

- **luminosity (collider)**

- key parameter, collision optimization...

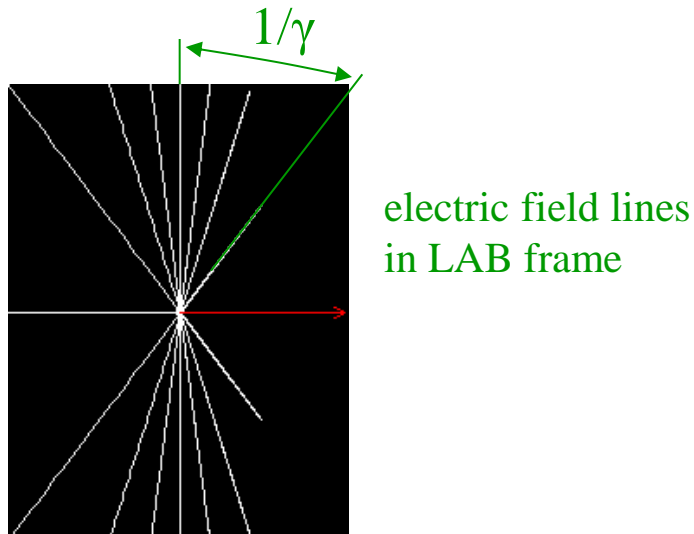
and even more: charge states, mass numbers, timing...



Beam Monitors: Physical Processes

- influence of particle electromagnetic field
 - non-propagating fields, i.e. electro-magnetic influence of moving charge on environment
 - beam transformers, pick-ups, ...
 - propagating fields, i.e. emission of photons
 - synchrotron radiation monitors, (OTR), ...

particle electromagnetic field



relativistic contraction characterized by Lorentz factor

$$\gamma = E / m_0 c^2$$

E : total energy

$m_0 c^2$: rest mass energy

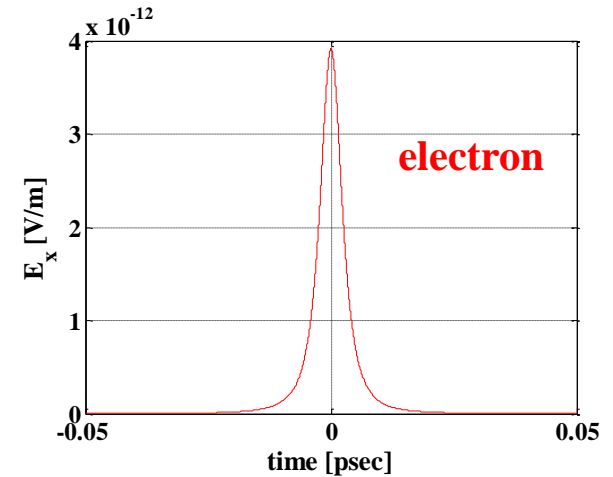
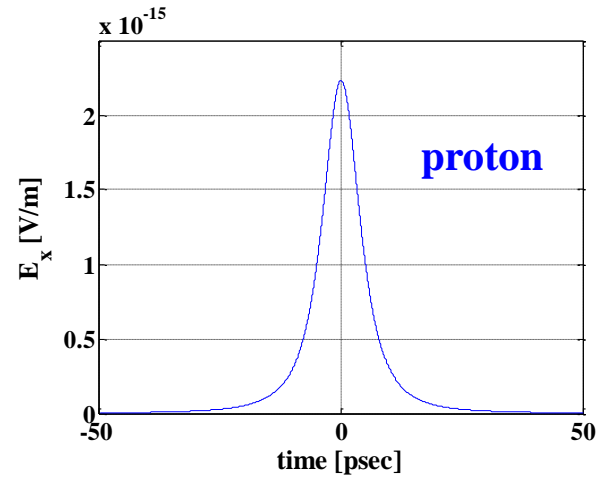
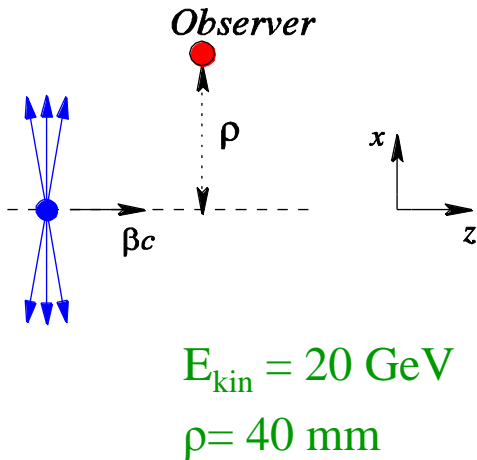
proton: $m_p c^2 = 938.272 \text{ MeV}$

electron: $m_e c^2 = 0.511 \text{ MeV}$

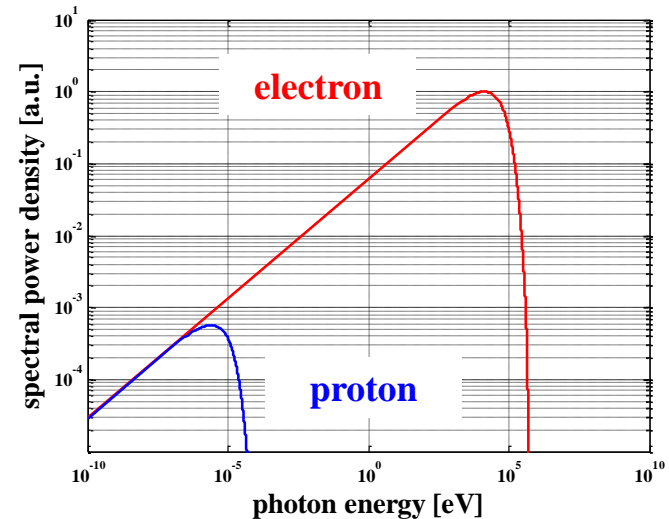
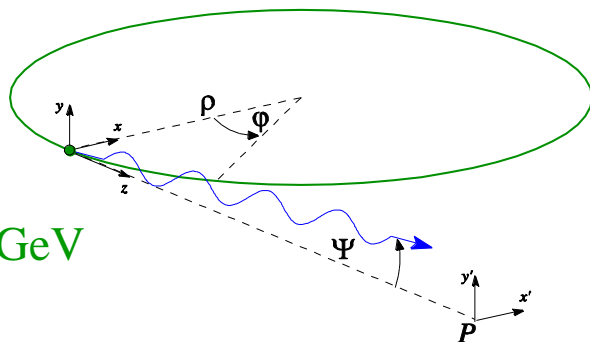
Beam Monitors: Physical Processes

non-propagating field

transverse electrical field components



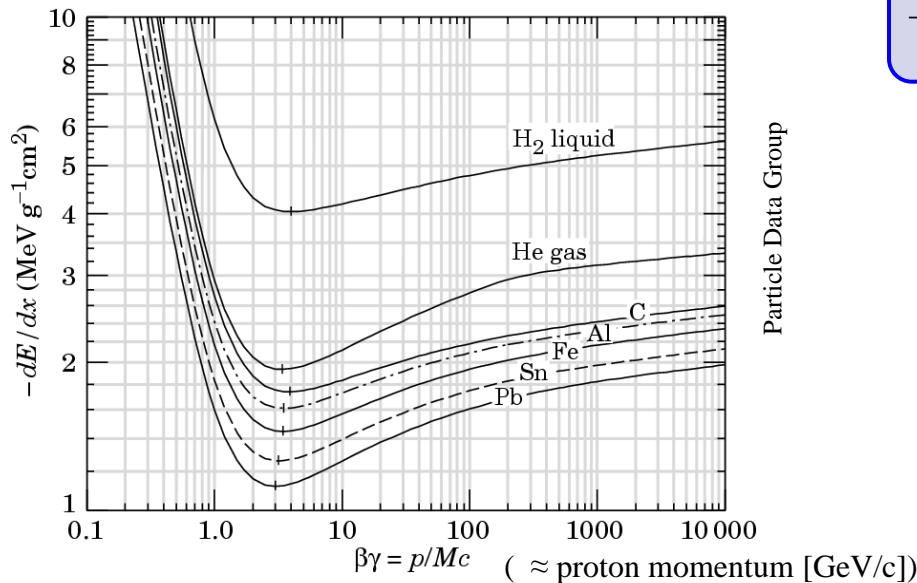
propagating field (synchrotron radiation)



Beam Monitors: Physical Processes

Coulomb interaction of charged particle penetrating matter

→ viewing screens, residual gas monitors, ...



$$-\frac{dE}{dx} = 4\pi N_A r_e^2 m_e c^2 \frac{Z_T}{A_T} \rho \frac{Z_p^2}{\beta^2} \left[\ln \frac{2m_e c^2 \gamma^2 \beta^2}{I} - \beta^2 \right]$$

Bethe Bloch Equation („low-energy approximation“)

• **constants:**

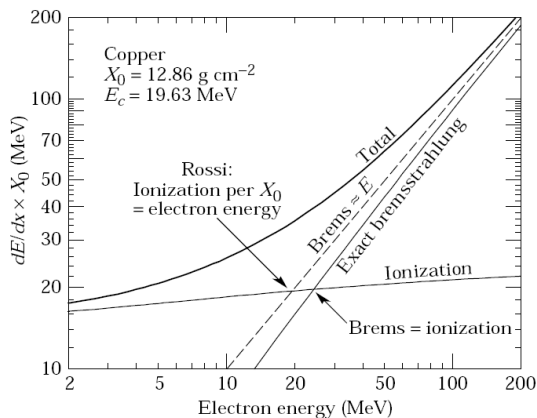
N_A : Avogadro number
 m_e, r_e : electron rest mass, classical electron radius
 c : speed of light

• **target material properties:**

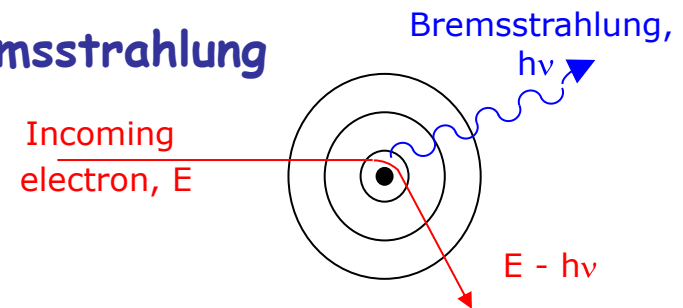
ρ : material density
 A_T, Z_T : atomic mass, nuclear charge
 I : mean excitation energy

• **particle properties:**

Z_p : charge
 β : velocity, with $\gamma^2 = \frac{1}{1-\beta^2}$



Electrons : Bremsstrahlung



Beam Monitors: Physical Processes

• nuclear or elementary particle physics interactions

→ beam loss monitors, luminosity monitors...

electrons

- › simple (point) objects
- › interaction cross sections into final states can be calculated precisely

hadrons

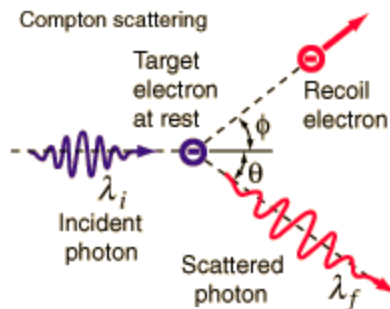
- › constituent nature (collection of quarks and gluons)
- › interaction cross sections not precisely calculable

• interaction of particles with photon beams

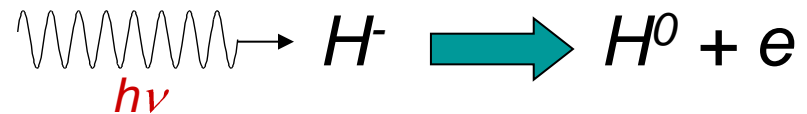
→ laser wire scanners, Compton polarimeters, ...

electrons: Compton scattering

hadrons: laser photo neutralization (H^- beam)



$$\frac{d\sigma}{d\Omega} \propto \left(\frac{e^2}{m_0 c^2} \right)^2$$



applied for high power H^- beam profile diagnostics

Beam Position Monitor (BPM)

Beam Position Monitors

short version of E-XFEL BPM specification

specified charge range: 0.1 – 1nC

	Number	Beam Pipe	Length	Type	Single Bunch Resolution (RMS)	Train Averaged Resolution (RMS)	Optimum Resolution Range	Relaxed Resolution Range	x/y Crosstalk	Bunch to Bunch Crosstalk	Trans. Alignment Tolerance (RMS)
		mm	mm		μm	μm	mm	mm	%	μm	μm
Standard BPM	219	40.5	200/ 100	Button	50	10	± 3.0	± 10	1	10	200
Cold BPM	102	78	170	Button/ Re-entrant	50	10	± 3.0	± 10	1	10	300
Cavity BPM Beam Transfer Line	12	40.5	255	Cavity	10	1	± 1.0	± 2	1	1	200
Cavity BPM Undulator	117	10	100	Cavity	1	0.1	± 0.5	± 2	1	0.1	50
IBFB	4	40.5	255	Cavity	1	0.1	± 1.0	± 2	1	0.1	200



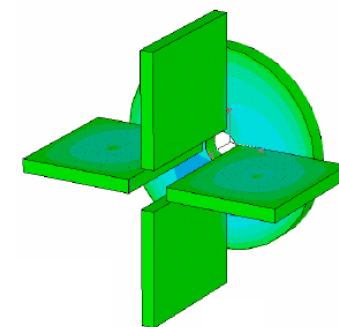
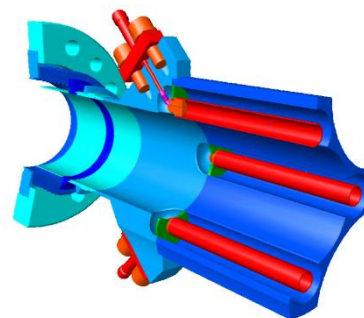
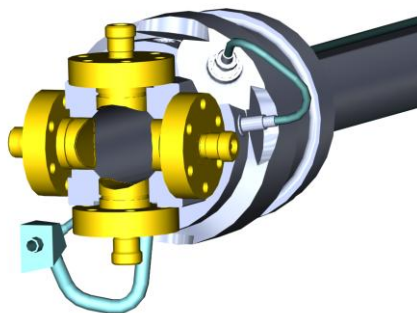
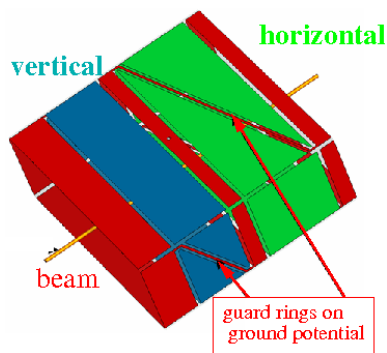
different BPM types to meet different requirements

courtesy: D.Nölle (DESY)

Comparison of BPM Types

BPM Type	Application	Precaution	Advantage	Disadvantage
Shoe-Box	p-synchrotrons heavy-ion accelerators	long bunches $f_{RF} < 10$ MHz	very linear no x-y coupling sensitive	complex mechanics capacitive coupling between plates
Button	p-linacs all e-accelerators	$f_{RF} > 10$ MHz	simple mechanics	non-linear x-y coupling possible signal deformation
Stripline	colliders p-linacs all e-accelerators	best for $\beta \approx 1$ short bunches	directivity large signal	complex 50 Ω matching complex mechanics
Cavity	e-linacs (e.g. FELs)	short bunches, special applic.	very sensitive	very complex high frequency

P. Forck, "Lecture Notes on Beam Instrumentation and Diagnostics", JUAS 2011



Beam Position Monitor

- most common: capacitive pickups

- signal generation via beam electric field
- popular design: **button-type pickup**
 - simple, cheap, ...
 - moderate resolution

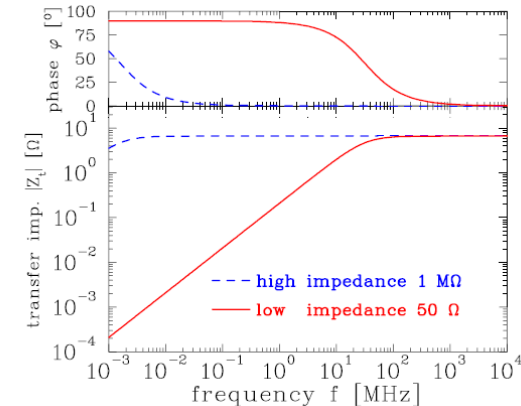
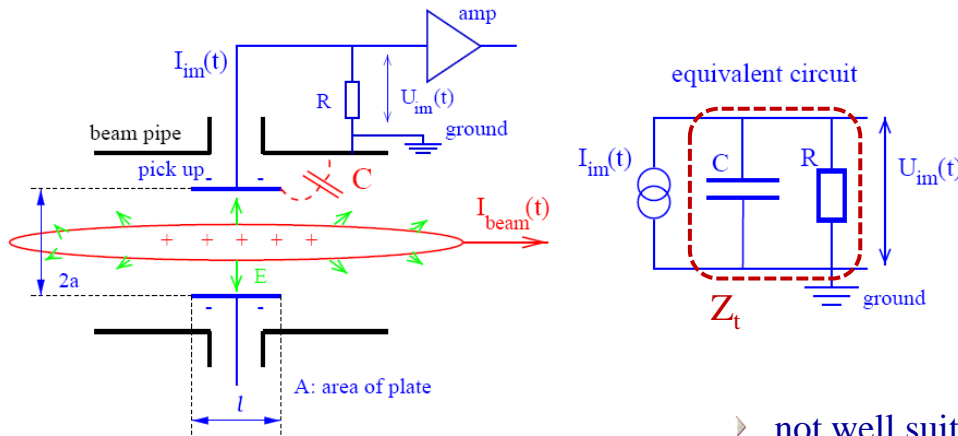
LHC button pickup
courtesy: R.Jones (CERN)



- operation principle

- electric field induces image charge on pick-up
 - pick-up mounted isolated inside vacuum chamber
 - amount of induced charge depends on distance between beam and pick-up

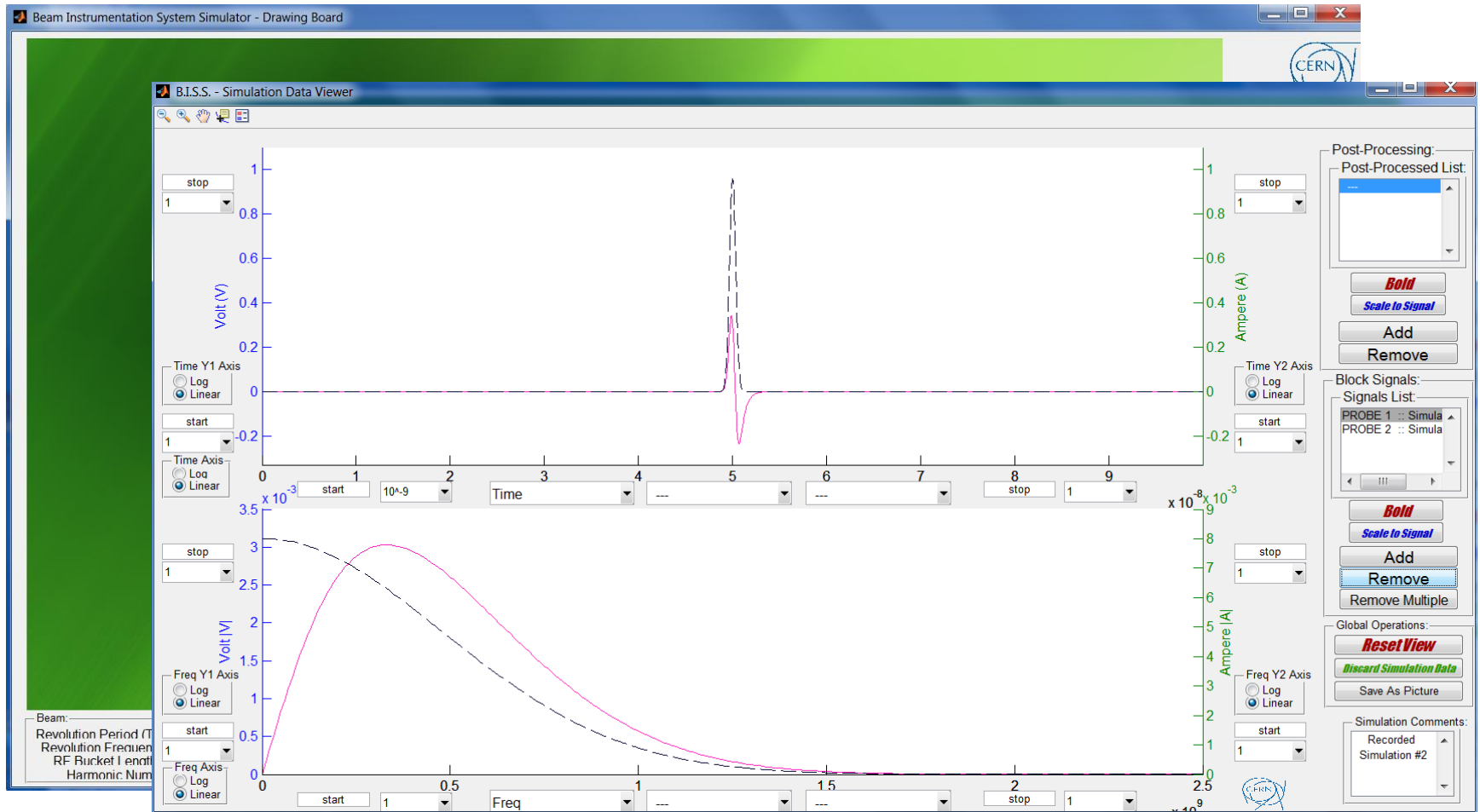
- button pickup: high pass characteristics



- not well suited for long bunches
 - especially: low energy hadron beams, i.e. heavy ion beams
 - small coupling between pickup and bunch

BPM Signal Calculation

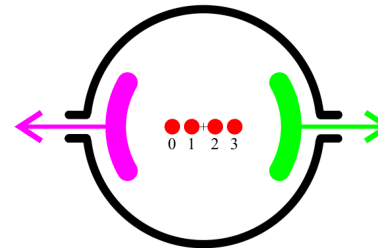
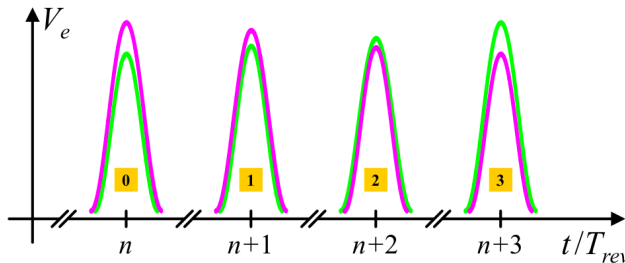
- Beam Instrumentation System Simulator (B.I.S.S.)
 - calculation from BPM signals in time- and frequency domain
 - study influence of various parameters



BPM Signals

● observation (1): signals are short with small modulation

- single bunch response → nsec or sub-nsec pulse signals
- beam position information → **amplitude modulated** on large (common mode) beam intensity signal!

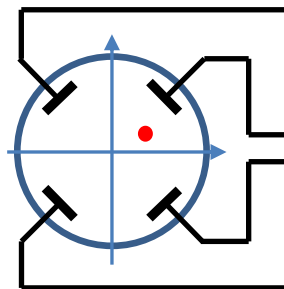


courtesy: M. Gasior (CERN)

● BPM building blocks

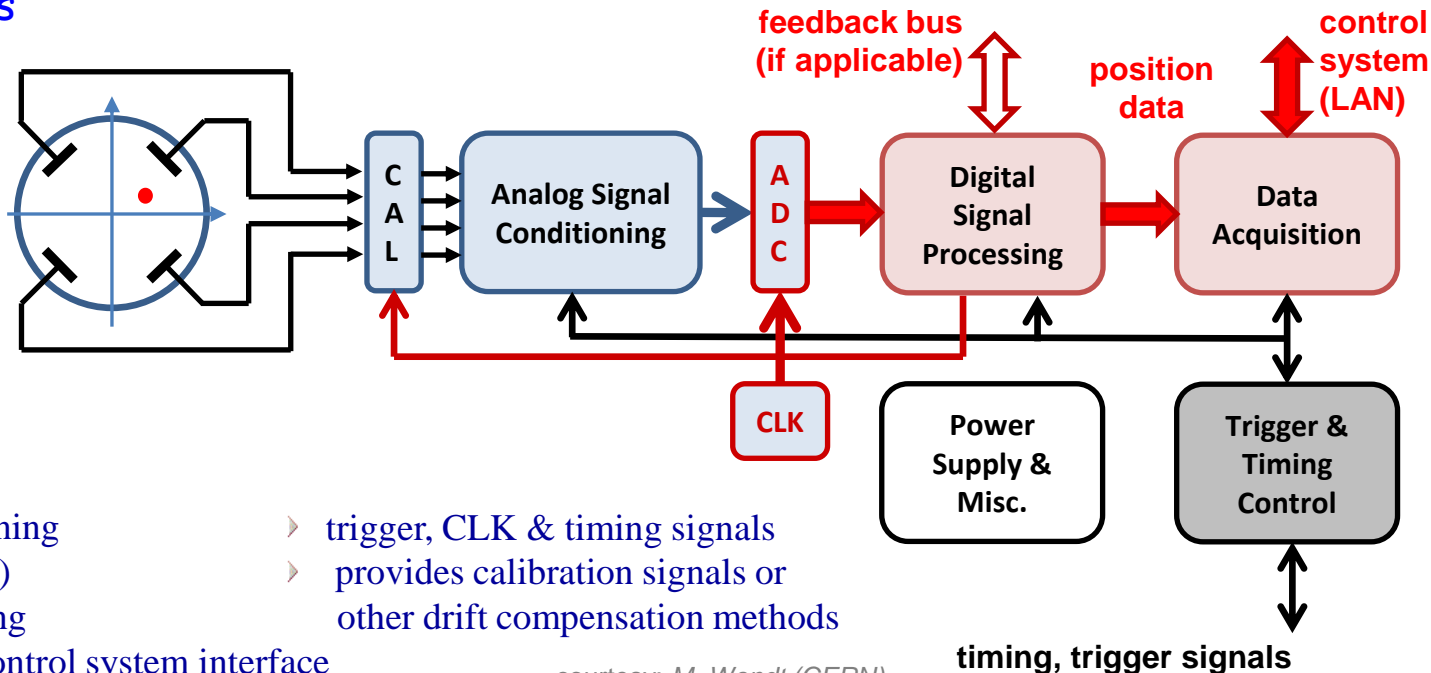
BPM Pickup

- RF device, EM field detection, center of charge
- symmetrically arranged electrodes or resonant structure



Read-out Electronics

- analog signal conditioning
- signal sampling (ADC)
- digital signal processing
- data acquisition and control system interface
- trigger, CLK & timing signals
- provides calibration signals or other drift compensation methods



courtesy: M. Wendt (CERN)

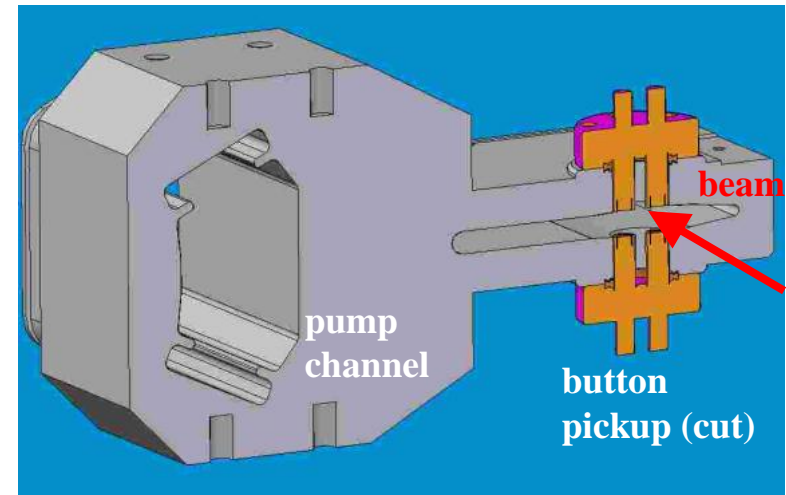
BPM Signals

● observation (2): nonlinearities

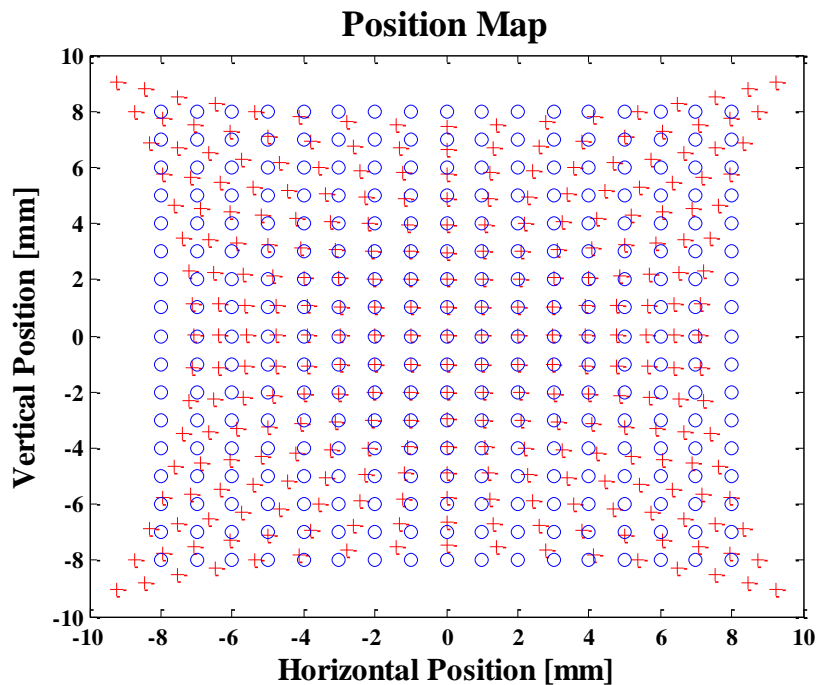
➔ especially BPMs for circular e-accelerators

- synchrotron radiation emission
 - pickups mounted **out of orbit plane**
- vacuum chamber not rotational symmetric
 - $\epsilon_{\text{hor}} \gg \epsilon_{\text{vert}}$ (SR emission in hor. plane)
 - injection oscillations due to off-axis injection (allows intensity accumulation)

courtesy: A.Delfs (DESY)



PETRA-III BPM close to ID



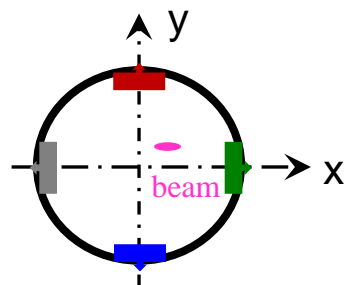
correction of strong non-linearities in beam position required

Position Reconstruction

two common monitor geometries

› difference in position reconstruction

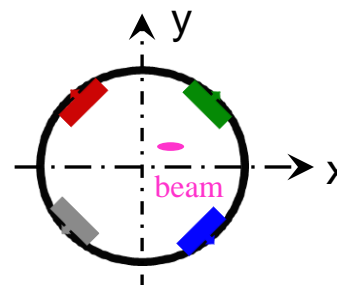
linac-type



$$x = K_x \frac{P_1 - P_3}{P_1 + P_3}$$

$$y = K_y \frac{P_2 - P_4}{P_2 + P_4}$$

storage ring-type



$$x = K_x \frac{(P_1 + P_4) - (P_2 + P_3)}{P_1 + P_2 + P_3 + P_4}$$

$$y = K_y \frac{(P_1 + P_2) - (P_3 + P_4)}{P_1 + P_2 + P_3 + P_4}$$

⇒ difference - over - sum or $\frac{\Delta}{\Sigma}$

position information

› requires knowledge of monitor constant K_x, K_y

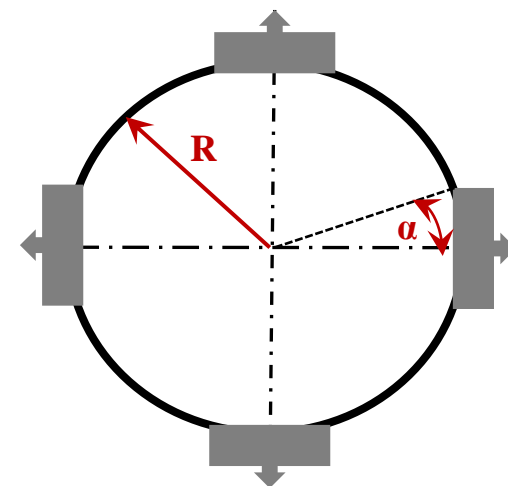
→ rule of thumb (circular duct)

$$K_{x,y} = \frac{R}{2} \frac{\alpha}{\sin \alpha}$$

linac-type

$$K_{x,y} = \frac{R}{\sqrt{2}} \frac{\alpha}{\sin \alpha}$$

storage ring-type

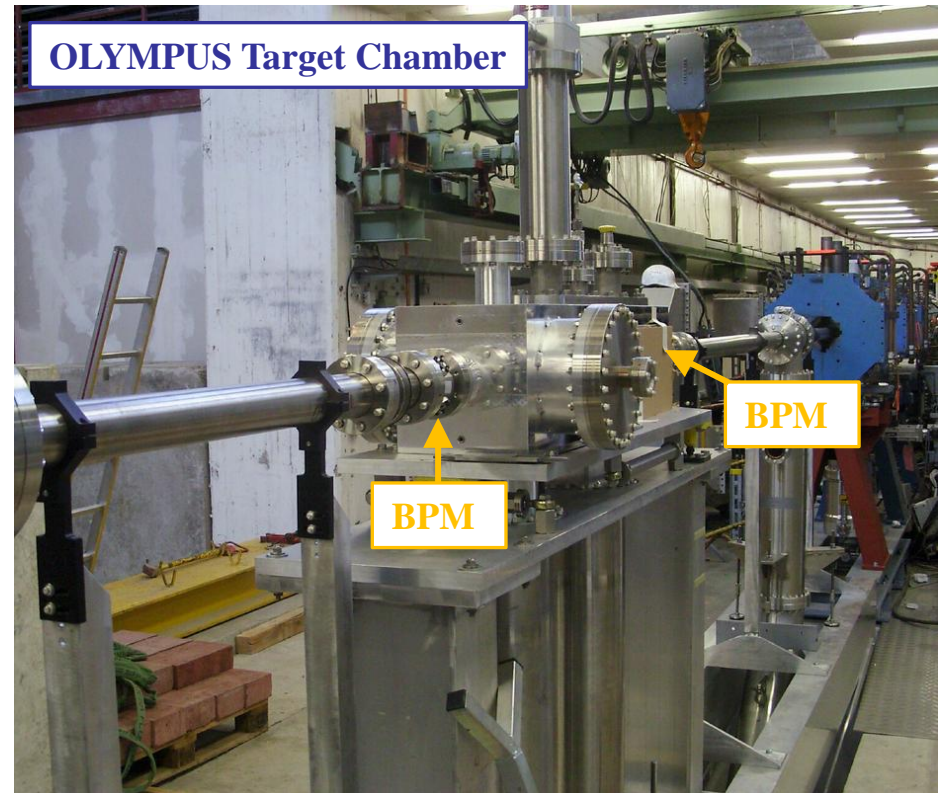
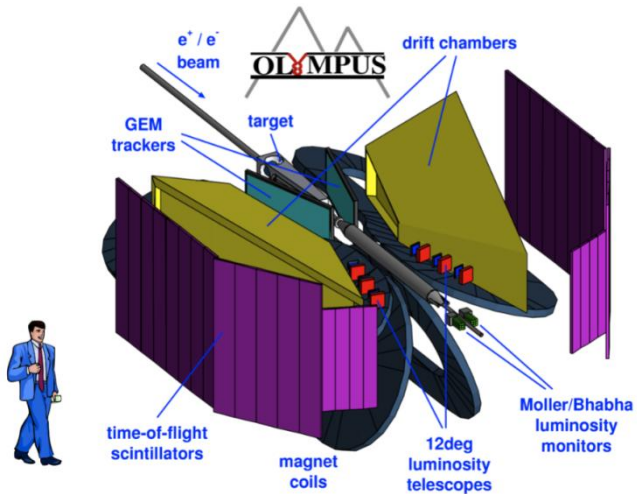


OLYMPUS @ DORIS (DESY)

● two-photon exchange in lepton scattering

- compare e^+p and e^-p elastic scattering

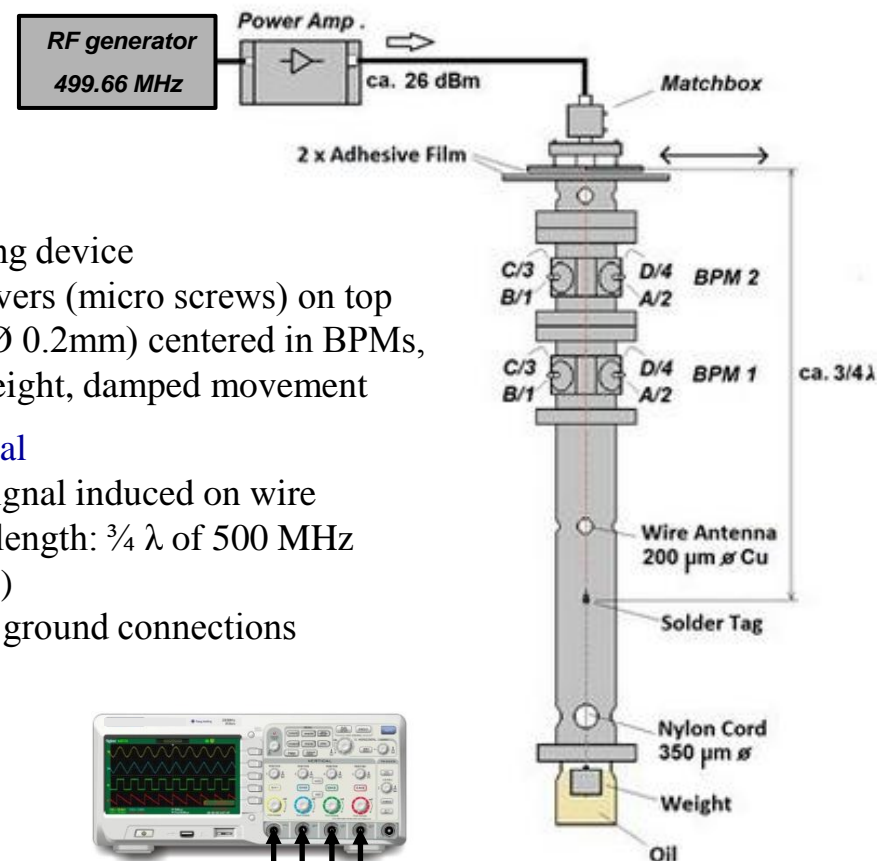
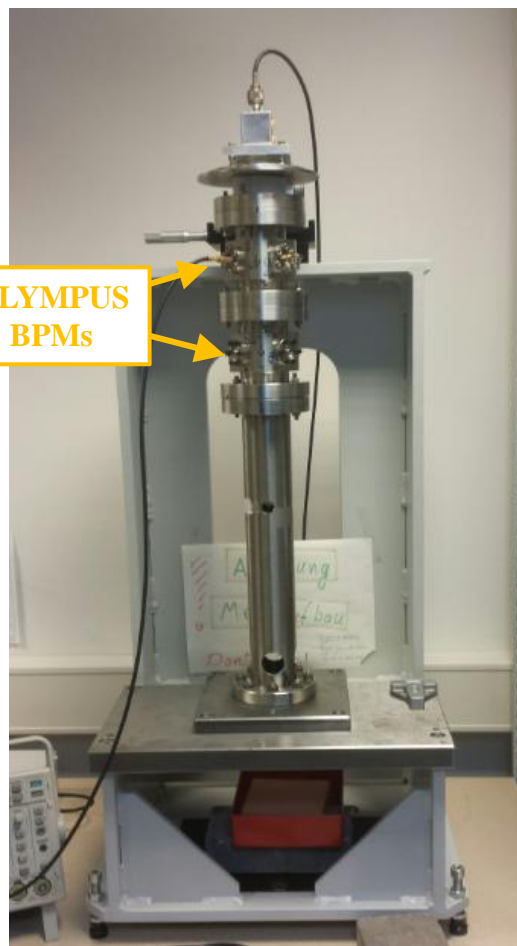
R. Milner et al., „The OLYMPUS experiment“, Nucl. Instrum. Methods **A741** (2014) 1



Monitor Constant Measurement

● BPM test stand

U. Schneekloth et al., Proc. IBIC 2014, Monterey (Ca), USA, (2014) 324



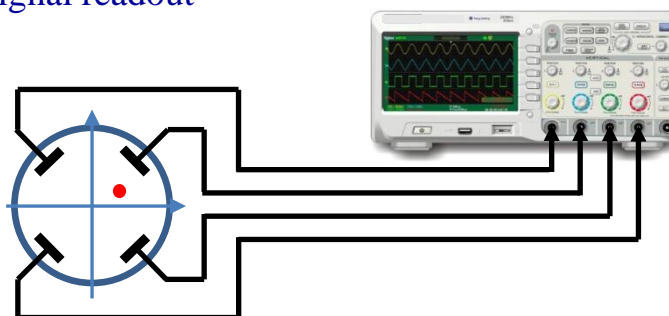
▶ test stand

- vertical scanning device
- 2 precision movers (micro screws) on top
- wire antenna (Ø 0.2mm) centered in BPMs, stretched by weight, damped movement

▶ electrical input signal

- 500 MHz cw signal induced on wire
- electrical wire length: $\frac{3}{4} \lambda$ of 500 MHz (standing wave)
- essential: solid ground connections

▶ signal readout



Tasks: BPMs

- calculate BPM signals using B.I.S.S
 - › get a first impression about BPM signal forms
 - chamber geometry influence
 - non-linearities
 - output impedance
- calculate monitor constants for OLYMPUS BPMs
 - › use rule-of-thumb formulae for both geometries
 - compare with simulation results
- measure OLYMPUS BPM monitor constants (both geometries)
 - › define electrical center of both BPM bodies (origin)
 - › perform 1-dim. scan along one axis → max. wire position: ± 15 mm (!!!)
 - measure signal amplitudes from each button
 - calculate Δ/Σ from measured signals
 - plot Δ/Σ versus wire position and compare with simulation results
 - › determine monitor constant from slope at origin
- (measure 2-dim. position map)

Transverse Phase Space: Beam Size and Emittance

Accelerator Key Parameters

● light source: spectral brilliance

- › measure for phase space density of photon flux

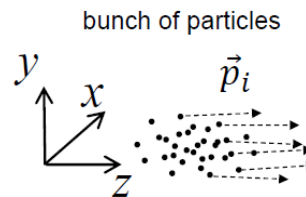
$$B = \frac{\text{Number of photons}}{[\text{sec}][\text{mm}^2][\text{mrad}^2][0.1\% \text{ bandwidth}]}$$

- › user requirement: high brightness
 - lot of monochromatic photons on sample
- › connection to machine parameters

$$B \propto \frac{N_\gamma}{\sigma_x \sigma_{x'} \sigma_z \sigma_{z'}} \propto \frac{I}{\varepsilon_x \varepsilon_z}$$

● requirements

- › design of small emittance machine
 - proper choice of magnet lattice
- › preserve small emittance
 - question of stability
 - require active feedback systems / careful design considerations



● collider: luminosity

- › measure for the collider performance

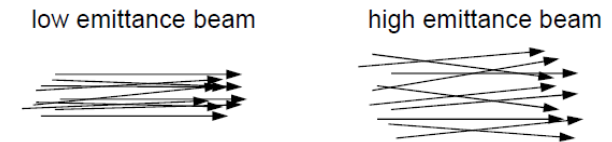
$$\dot{N} = L \cdot \sigma$$

relativistic invariant proportionality factor between cross section σ (property of interaction) and number of interactions per second

- › user requirement: high luminosity
 - lot of interactions in reaction channel
- › connection to machine parameters

$$L \propto \frac{I_1 \cdot I_2}{\varepsilon}$$

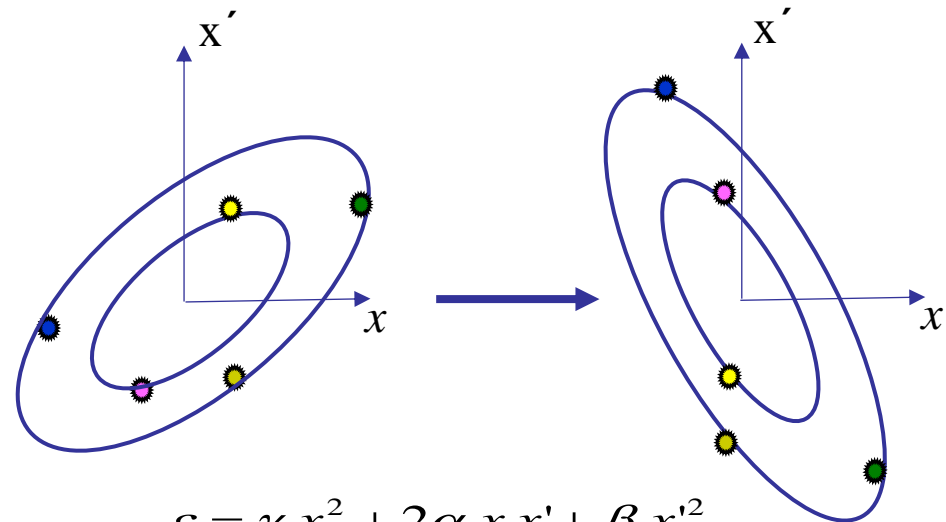
for two identical beams with emittances $\varepsilon_x = \varepsilon_z = \varepsilon$



- › **measure small emittance**

Transverse Emittance

- **projection of phase space volume**
 - separate horizontal, vertical and longitudinal plane
- **accelerator key parameter**
 - defines **luminosity** / **brilliance**
- **linear forces**
 - any particle moves on an ellipse in phase space (x, x')
 - ellipse rotates in magnets and shears along drifts
 - but area is preserved: **emittance**



$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

($\alpha, \beta, \gamma, \varepsilon$: Courant-Snyder or Twiss parameters)

- **transformation along accelerator**

‣ knowledge of the magnet structure (beam optics) → transformation from initial (i) to final (f) location

→ single particle transformation

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \underbrace{\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}}_R \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

→ transformation of optical functions

$$\begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_f = \begin{pmatrix} R_{11}^2 & -2R_{11}R_{12} & R_{12}^2 \\ -R_{11}R_{21} & 1+R_{12}R_{21} & -R_{12}R_{22} \\ R_{21}^2 & -2R_{21}R_{22} & R_{22}^2 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_i$$

Transverse Emittance Ellipse

propagation along accelerator

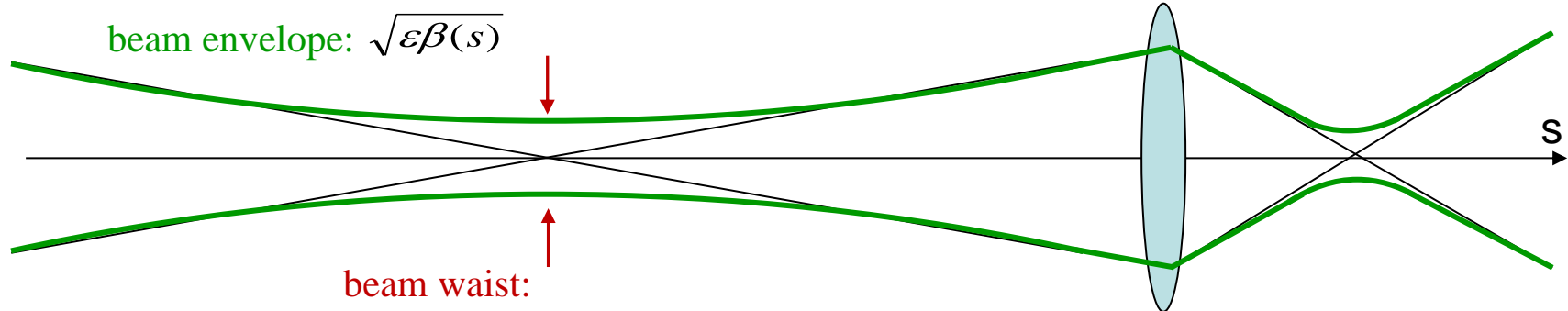
- change of ellipse shape and orientation → area is preserved

$$\alpha(s) = -\frac{\beta'(s)}{2}$$

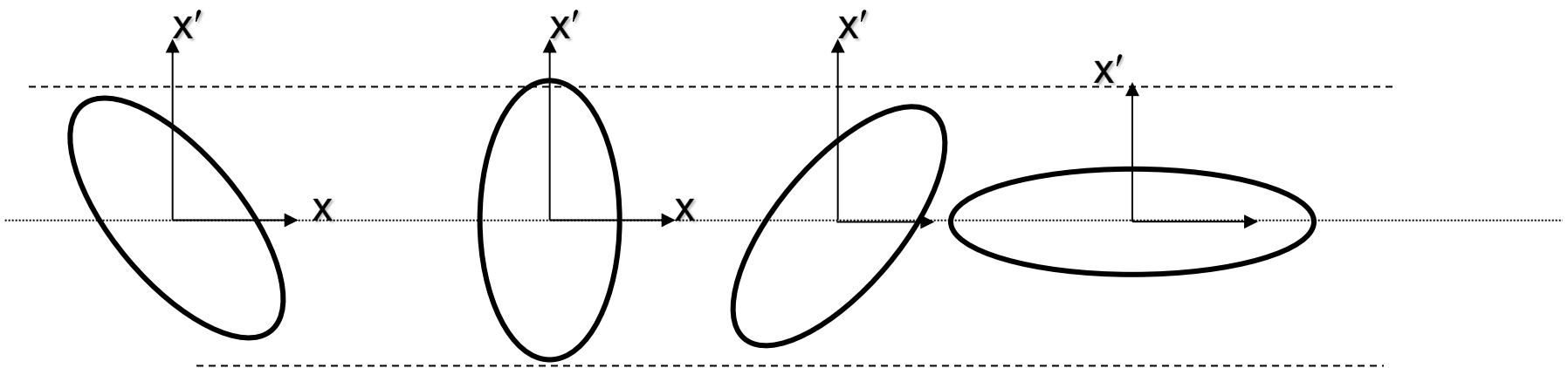
$$\gamma(s) = \frac{1 + \alpha^2(s)}{\beta(s)}$$

$$x(s) = \sqrt{\varepsilon\beta(s)} \cdot \cos[\Psi(s) + \Phi]$$

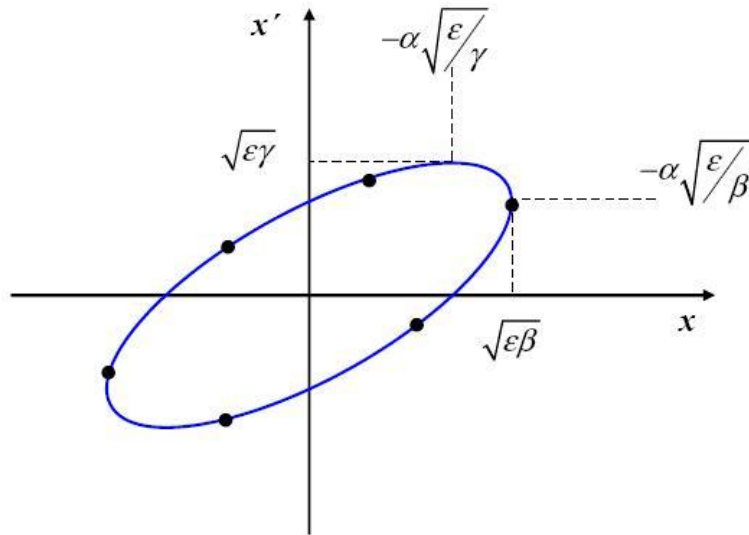
$$\varepsilon = \gamma(s) x(s)^2 + 2\alpha(s) x(s) x'(s) + \beta(s) x'(s)^2$$



→ minimum in envelope → minimum in β → $\beta' = 0$ → $\alpha = 0$



Emittance and Beam Matrix



- via Twiss parameters

$$\varepsilon = \gamma x^2 + 2\alpha x x' + \beta x'^2$$

- statistical definition

P.M. Lapostolle, IEEE Trans. Nucl. Sci. NS-18, No.3 (1971) 1101

$$\varepsilon_{rms} = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle^2}$$

2nd moment of beam distribution $\rho(x)$

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{\infty} dx x^2 \cdot \rho(x)}{\int_{-\infty}^{\infty} dx \rho(x)}$$

- beam matrix

$$\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle x x' \rangle \\ \langle x x' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11} \cdot \Sigma_{22} - \Sigma_{12}^2}$$

- transformation of beam matrix

$$\Sigma^1 = \mathbf{R} \Sigma^0 \mathbf{R}^T \quad \mathbf{R} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

- ε_{rms} is measure of spread in phase space

- root-mean-square (rms) of distribution

$$\sigma_x = \langle x^2 \rangle^{1/2}$$

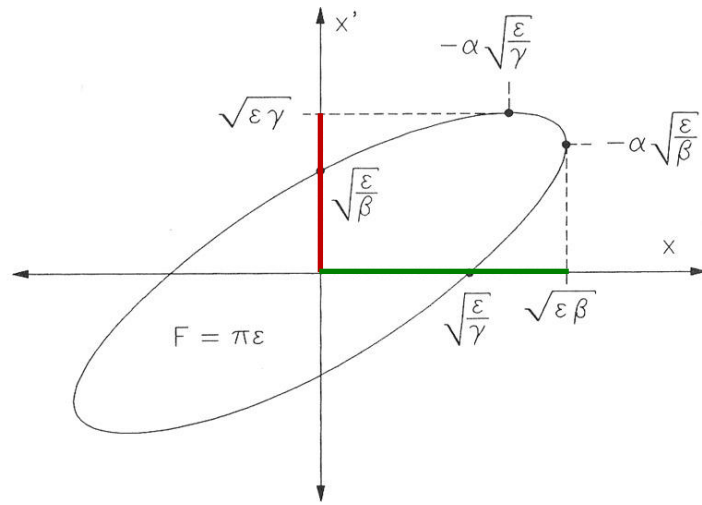
- ε_{rms} useful definition for non-linear beams

→ usually restriction to certain range

(c.f. 90% of particles instead of $[-\infty, +\infty]$)

Emittance Measurement: Principle

- emittance: projected area of transverse phase space volume
- not directly accessible for beam diagnostics



- measured quantity

- ▶ beam size $\sqrt{\Sigma_{11}} = \sqrt{\langle x^2 \rangle} = \sqrt{\epsilon \beta}$
- ▶ beam divergence $\sqrt{\Sigma_{22}} = \sqrt{\langle x'^2 \rangle} = \sqrt{\epsilon \gamma}$

- ▶ divergence measurements seldom in use

→ restriction to profile measurements

- measurement schemes

- ▶ beam matrix based measurements

→ determination of beam matrix elements:

$$\epsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11} \cdot \Sigma_{22} - \Sigma_{12}^2}$$

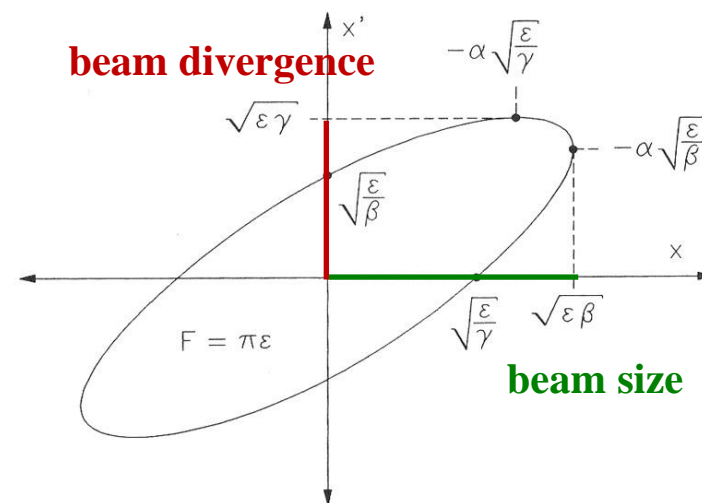
- ▶ ~~mapping of phase space~~

~~→ restrict to (infinitesimal) element in space coordinate, convert angles x' in position~~

Circular Accelerators

emittance diagnostics in circular accelerators

- ▶ circular accelerator: periodic with circumference L
 - one-turn transport matrix: $R(s+L) = R(s)$
 - Twiss parameters $\alpha(s)$, $\beta(s)$, $\gamma(s)$ uniquely defined at each location in ring
- ▶ measurement at one location in ring sufficient to determine ϵ
 - measured quantity: beam profile / angular distribution



classification

- ▶ imaging
 - beam size
- ▶ interference
 - beam size
- ▶ projection
 - beam divergence

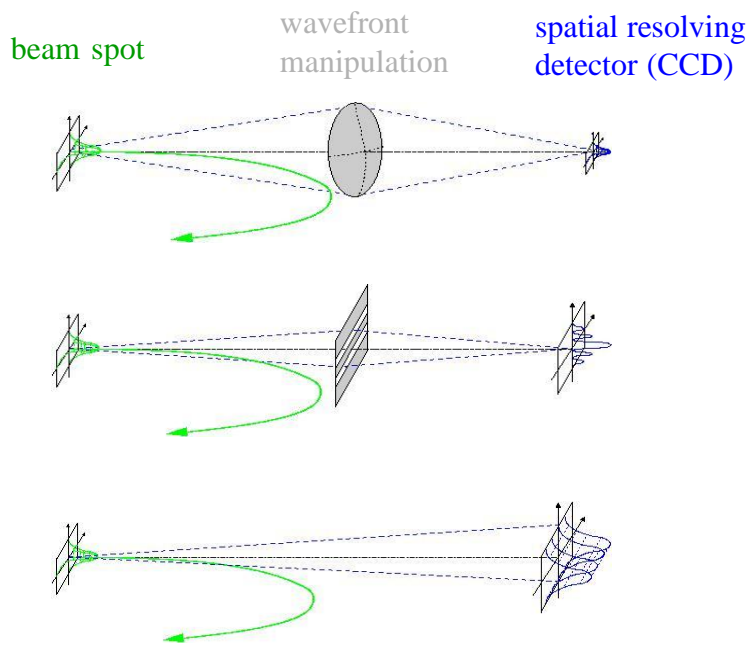
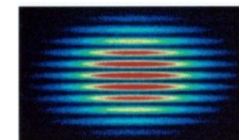
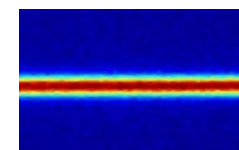


image size



interference pattern



angular distribution

Beam Matrix based Measurements

- starting point: beam matrix $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$

- emittance determination

- measurement of 3 matrix elements $\Sigma_{11}, \Sigma_{12}, \Sigma_{22}$
- remember:** beam matrix Σ depends on location, i.e. $\Sigma(s)$

$$\varepsilon = \sqrt{\det \Sigma} = \sqrt{\Sigma_{11} \cdot \Sigma_{22} - \Sigma_{12}^2}$$

→ determination of matrix elements at same location required

- access to matrix elements

- profile monitor determines only $\sigma = \sqrt{\Sigma_{11}}$
- other matrix elements can be inferred from beam profiles taken under various transport conditions

→ knowledge of transport matrix R required

$$\Sigma^b = R \cdot \Sigma^a \cdot R^T \quad R = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}$$

- measurement of at least 3 profiles for 3 matrix elements

$$\Sigma_{11}^a$$

$$\Sigma_{11}^b = R_{11}^2 \cdot \Sigma_{11}^a + 2R_{11}R_{12} \cdot \Sigma_{12}^a + R_{12}^2 \cdot \Sigma_{22}^a$$

$$\Sigma_{11}^c = \bar{R}_{11}^2 \cdot \Sigma_{11}^a + 2\bar{R}_{11}\bar{R}_{12} \cdot \Sigma_{12}^a + \bar{R}_{12}^2 \cdot \Sigma_{22}^a$$

- measurement:** profiles $\sigma^{a,b,c} = \sqrt{\Sigma_{11}^{a,b,c}}$
- known:** transport optics R, \bar{R}
- deduced:** matrix elements $\Sigma_{11}^a, \Sigma_{12}^a, \Sigma_{22}^a$

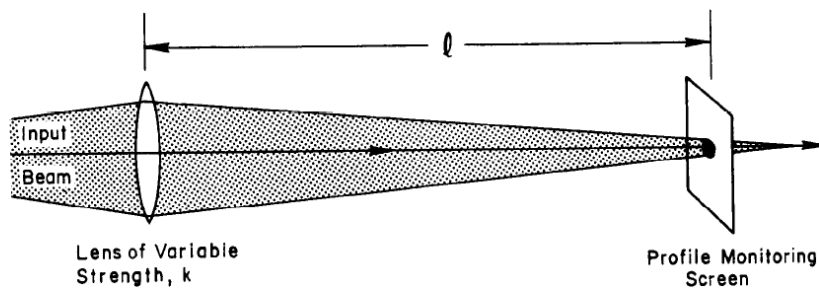
→ more than 3 profile measurements favourable, data subjected to least-square analysis

Beam Matrix based Measurements

• „quadrupole scan“ method

› use of variable quadrupole strengths

→ change quadrupole settings and measure beam size in profile monitor located downstream



Q ($f = 1/K$)

S (drift space)

quadrupole transfer matrix

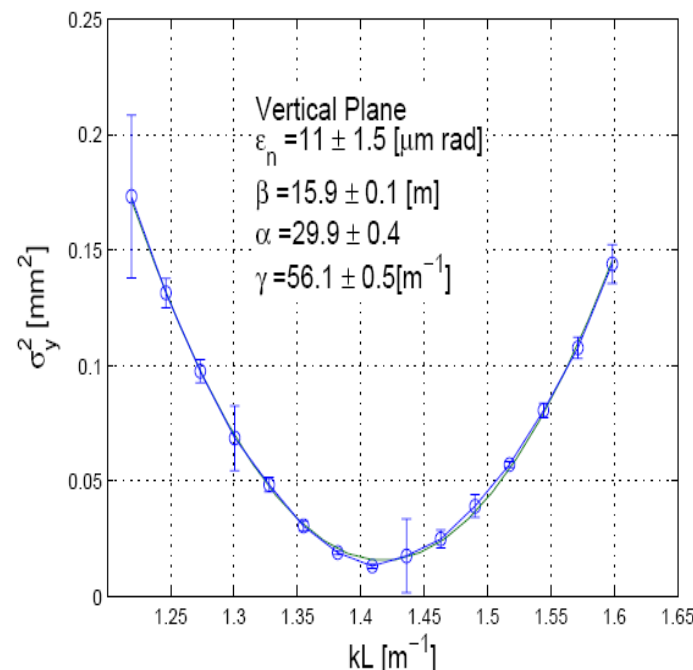
$$Q = \begin{pmatrix} 1 & 0 \\ \pm 1/f & 1 \end{pmatrix}$$

drift space transfer matrix

$$S = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

→ **R = SQ**

› Σ_{11} depends quadratically on quadrupole field strength



Beam Matrix based Measurements

- „multi profile monitor“ method

- fixed particle beam optics

→ measure beam sizes using multiple profile monitors at different locations

- example:

emittance measurement

setup at FLASH injector (DESY)

courtesy: K. Honkavaara (DESY)



4 monitor stations in a
FODO lattice

- task

- beam profile measurement

Storage Ring: Profile Measurement

- **circular accelerator**

- only non- or minimum-invasive diagnostics → otherwise beam loss after few turns

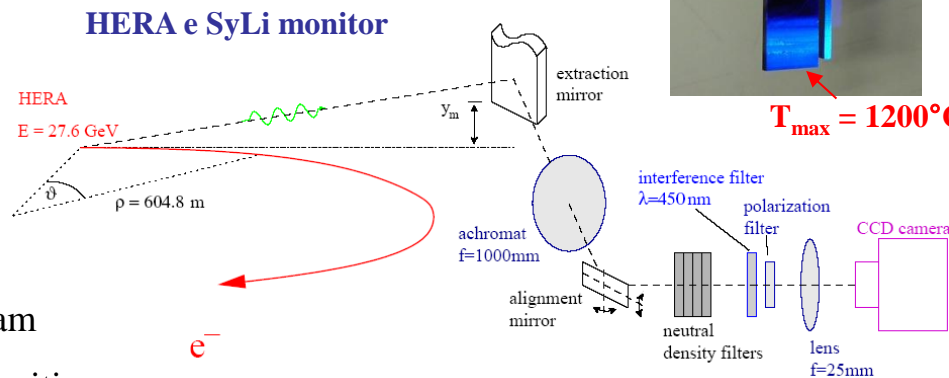
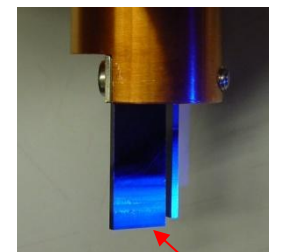
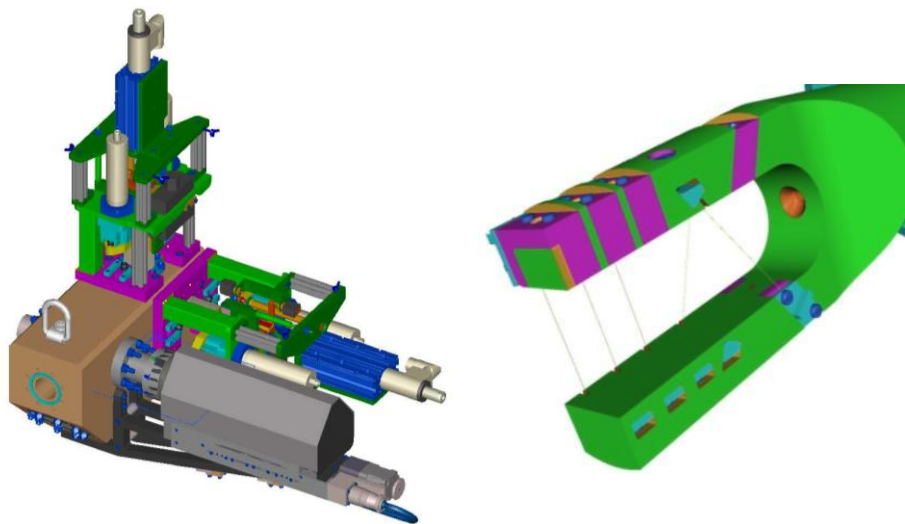
- **e⁻/e⁺ ring**

- working horse: synchrotron radiation

- problem: heat load @ extraction mirror

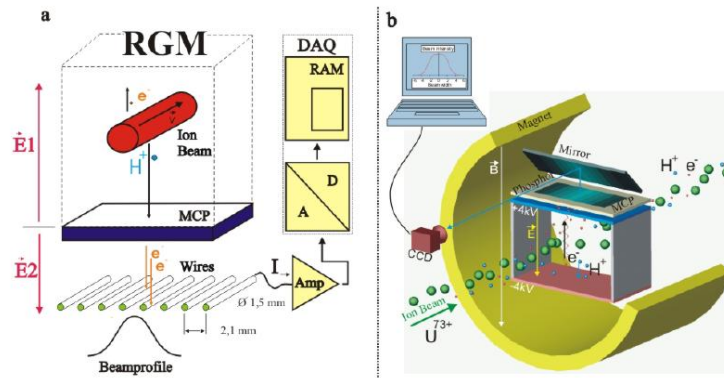
- **hadron ring**

- **wire scanners:** scan of thin wire across the beam
 - detect beam-wire interaction as function of wire position



- **residual gas monitor:**

- residual gas ionization / luminescence



T. Giacomini et al., Proc. BIW 2004, p.286

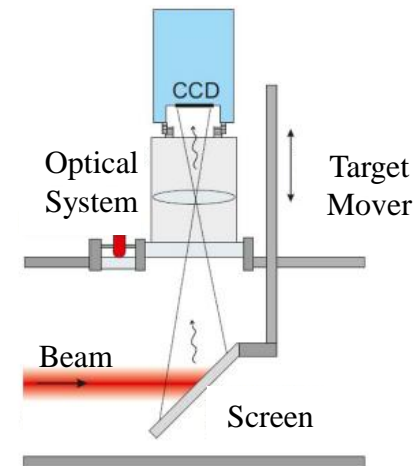
Linac or Transport Line: Profiles

linear machine

- single pass diagnostics → interaction with matter (care has to be taken)

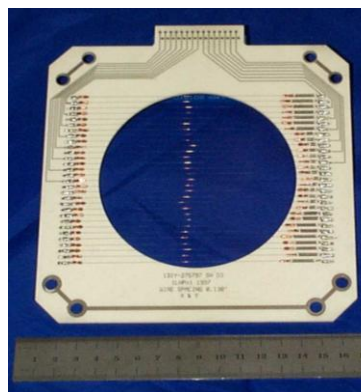
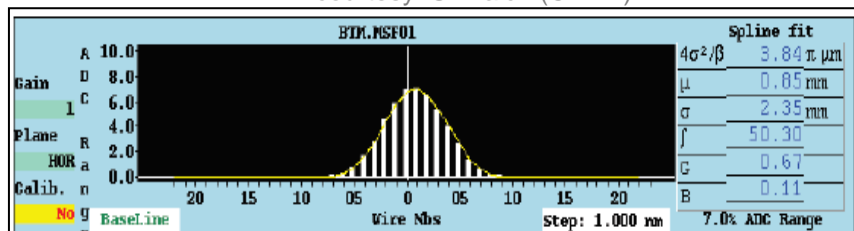
hadron accelerators

- working horse: screen monitors
 - scintillating light spot intensity corresponds to beam profile
- wire harp
 - extension of wire scanner



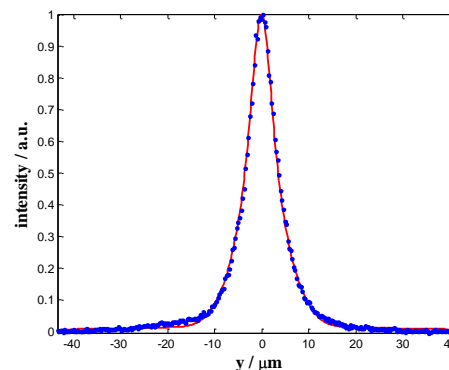
B. Walasek-Höhne et al., IEEE Trans. Nucl. Sci. **59** (2012) 2307

courtesy: U. Raich (CERN)



electron accelerators

- screen monitors → lower resolution (?)
- working horse: OTR monitors
 - even potential for sub-micron beams



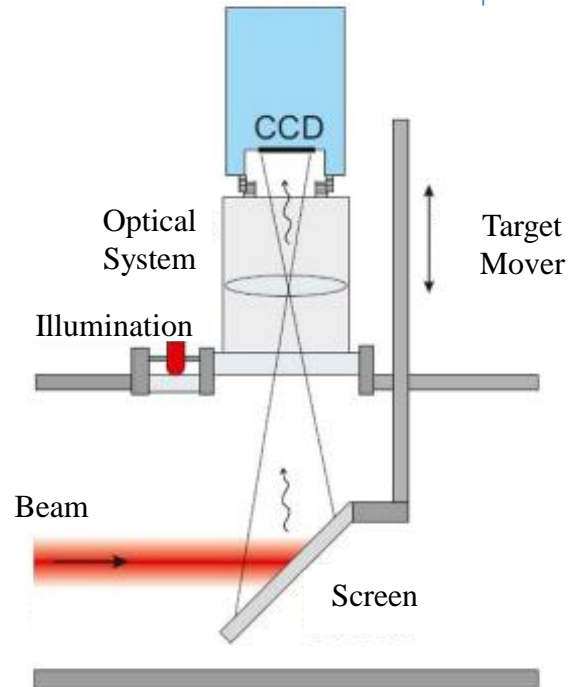
$\sigma = 1.44 \mu\text{m}$

G. Kube et al., Proc. IBIC 2015, Melbourne (Australia), TUPB012

Screen Monitors

principle

- › radiator
 - scintillator / OTR screen
 - generation of light spot: intensity distribution reflects particle beam density (i.e. linear light generation mechanism)
- › optical system / CCD
 - imaging / recording of light spot
- › target mover
 - move screen in / out of particle beam
- › illumination
 - check system performance



B. Walasek-Höhne et al., IEEE Trans. Nucl. Sci. **59** (2012) 2307



screen monitor setup

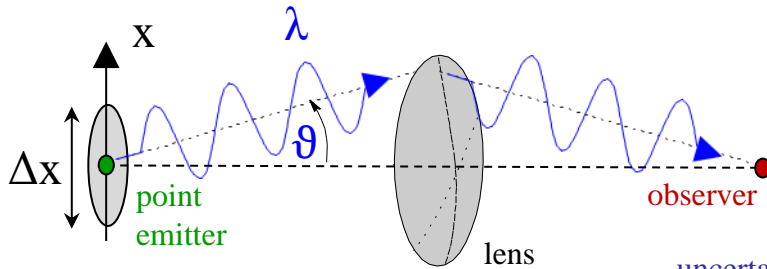
- › radiator → $\text{Al}_2\text{O}_3\text{:Cr}$ (Chromox) screen
(thickness 1.0 mm / 0.5 mm / 0.3 mm)
- › CCD → USB camera
- › optics → CCTV lens

Size Measurement: Resolution

fundamental resolution limit

point observer detecting photons from point emitter

→ location of emission point ?



$$\Delta p_x = 2\hbar k \cdot \sin\vartheta \approx 2 \cdot \frac{h}{2\pi} \cdot \frac{2\pi}{\lambda} \cdot \sin\vartheta$$

NA = sinθ:
numerical aperture

uncertainty principle: $\Delta x \cdot \Delta p_x \approx h \Rightarrow$

$$\Delta x \approx \frac{\lambda}{2 \sin \vartheta}$$

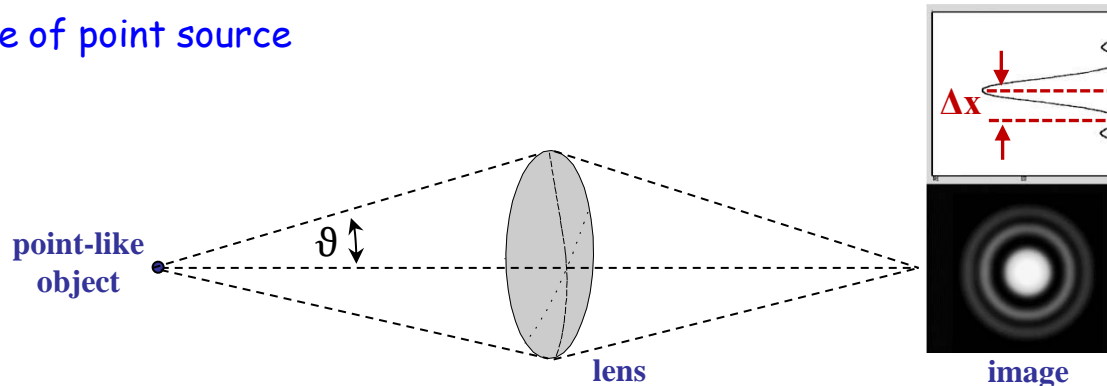


high resolution:

(i) **small λ**

(ii) **high NA**

image of point source



<http://www.astro.ljmu.ac.uk>

$$\Delta x = 0.61 \frac{M\lambda}{\sin\vartheta}$$

Airy pattern:
→ **Point Spread Function**

magnification M

resolution broadening: additional contributions

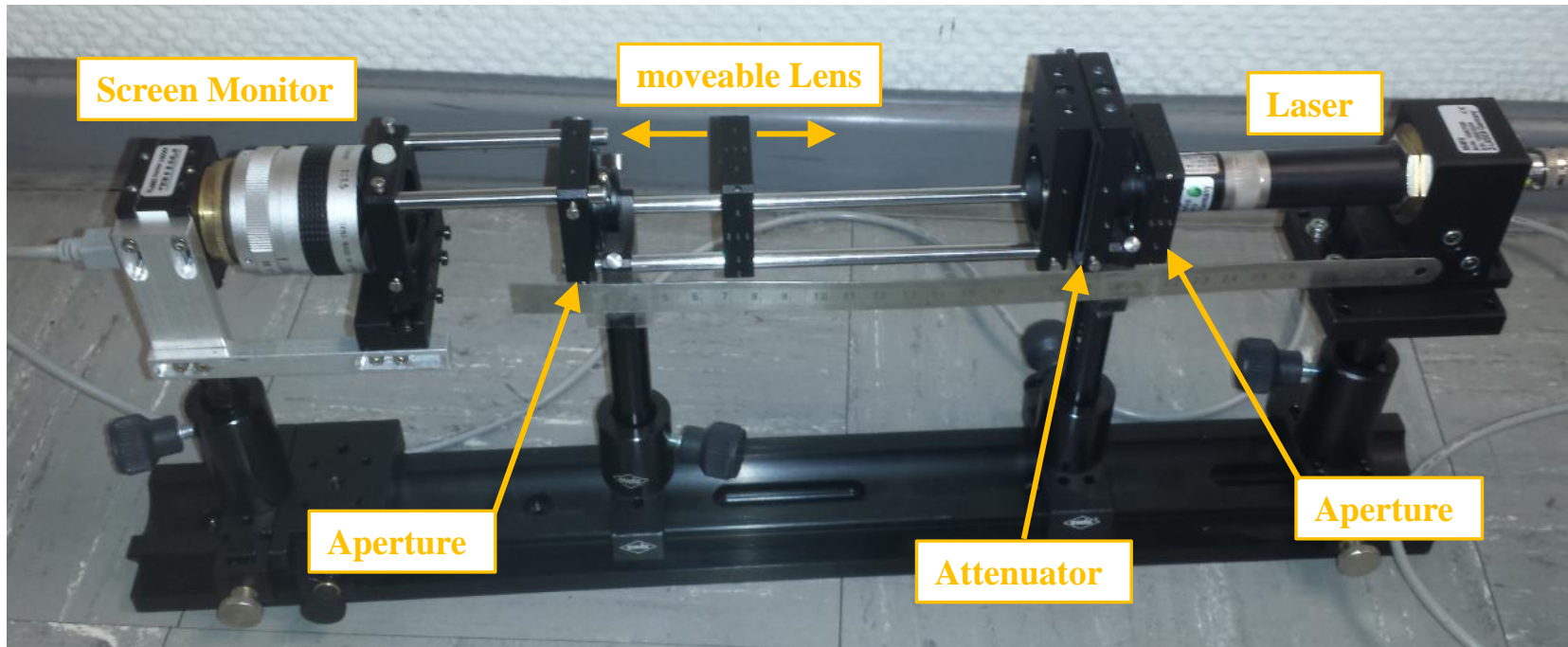
depth of field

→ mainly for synchrotron radiation based diagnostics

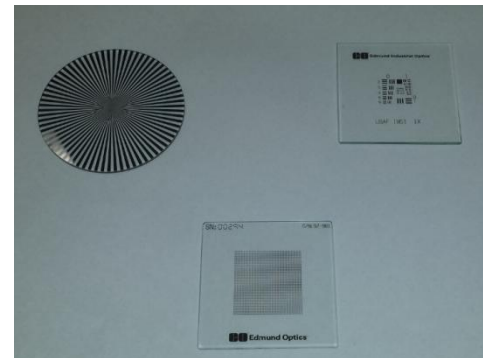
radius of curvature

Emittance Measurement Test Setup

- emittance of laser beam: „multi-profile monitor“ method



- calibration / resolution targets
 - › check system performance of detector system



Tasks: Emittance Diagnostics

- **estimate the image resolution for an optical synchrotron radiation profile monitor**
 - modern 3rd generation light source: $E = 6 \text{ GeV}, \lambda_{\text{obs}} = 500 \text{ nm}, \sigma_y = 10 \text{ }\mu\text{m}$
 - assume „self diffraction“, i.e. aperture limitation imposed by radiation angular distribution ($1/\gamma$)
- **derive the single particle transport matrix for a drift space**
 - assume paraxial approximation
 - $\sin(x') \approx x'$
- **calculate the evolution of the beam size after a drift space**
 - use the beam matrix transformation together with the transport matrix R for a drift space
- **investigate the performance of the CCD**
 - spatial calibration → dot grid target (0.5 mm spacing)
 - resolution → Siemens star, USAF 1951 target
- **measure the emittance of the laser beam**
 - measure spot sizes for different distances of the lens
 - analyse the horizontal profiles as function of the lens position
 - calculate the laser beam emittance → use the simplest way with only 2 values
 - (repeat with a different scintillator thickness)