



Electron-Phonon Interaction and Charge Instabilities in Strongly Correlated Electron Systems

Rome, October 21, 2008

Ph-D Candidate - Andrea Di Ciolo

`andrea.diciolo@roma1.infn.it`

Physics Department, University of Rome “Sapienza”

Piazzale A. Moro, 2 - 00185 Rome, Italy



- PH-D ADVISORS

Dr. J. G. Lorenzana

University of Rome “Sapienza”, SMC-INFM and ISC-CNR, Italy

Prof. M. Grilli

University of Rome “Sapienza” and SMC-INFM, Italy

- COLLABORATOR

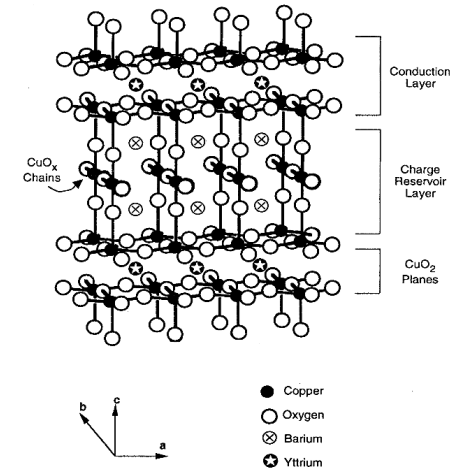
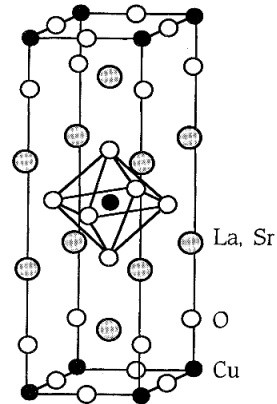
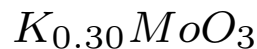
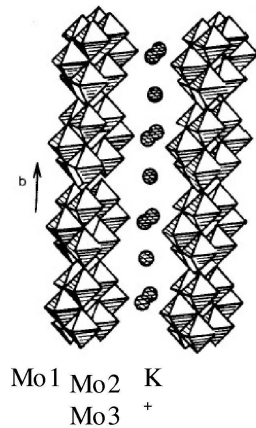
Prof. G. Seibold

Institut für Physik, BTU Cottbus, Germany



- Charge Density Waves + Anomalous Phonon Softening in the Cuprates
- Hubbard-Holstein Model + Gutzwiller-RPA Method
- Homogeneous Metal \implies Charge Inhomogeneities
- Stripes \implies Anomalous Phonon Softening
- Conclusions

Charge Density Wave (CDW) Materials



● CLASSICAL CDW's

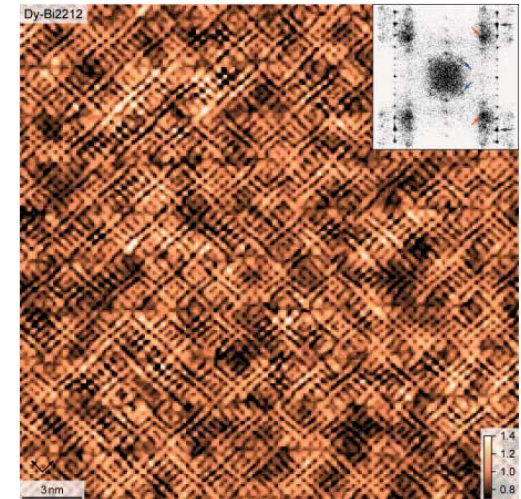
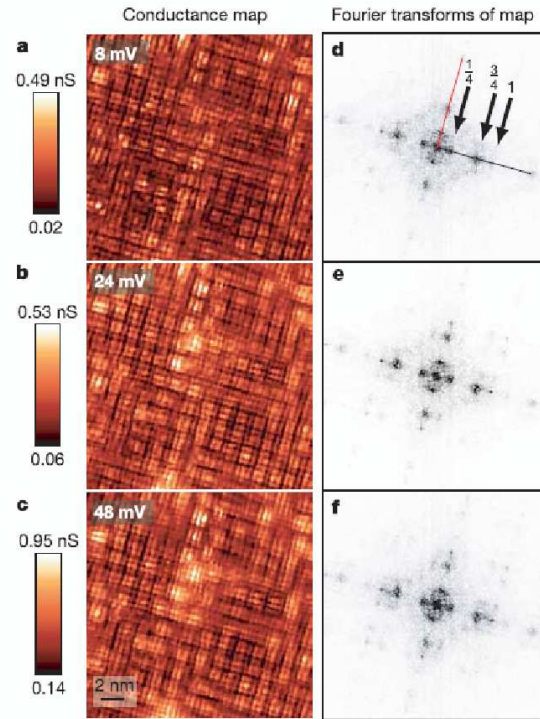
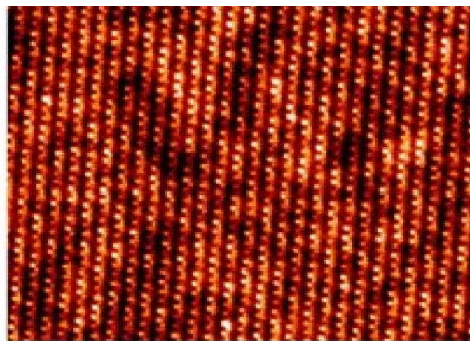
1D-blue bronzes ($A_{0.30}MoO_3$ with $A = K, Rb$);

2D-dichalcogenides (MC_2 with $M=Ta, Ti, Nb, Mo$ and $C=S, Se$) ...

● STRONGLY CORRELATED CDW's

cuprates, manganites, nickelates, cobaltites ...

Charge Density Wave (CDW) Materials



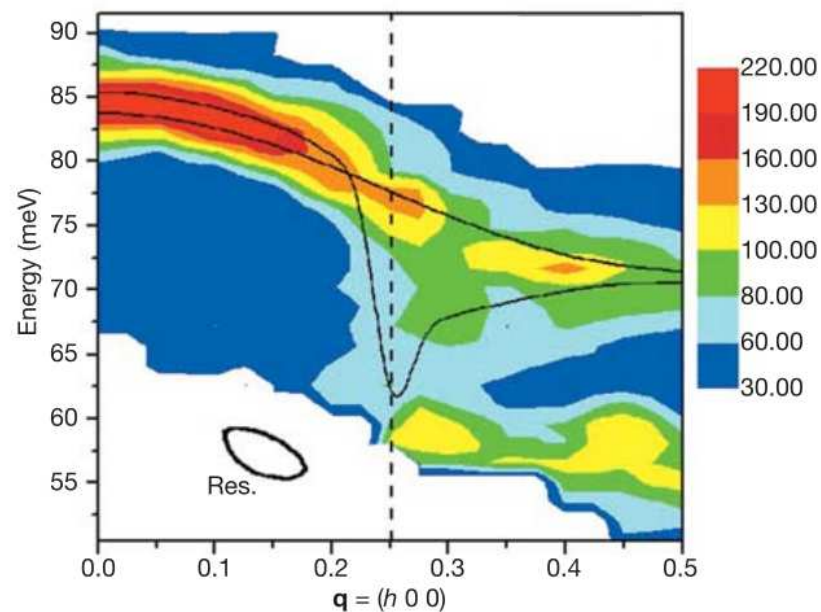
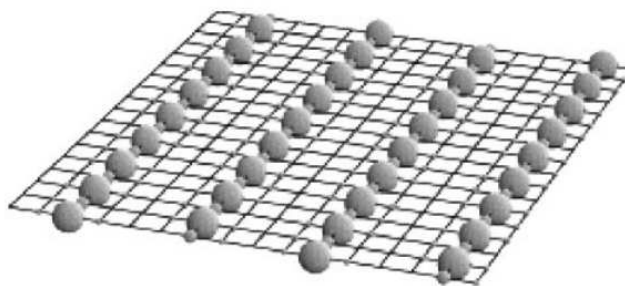
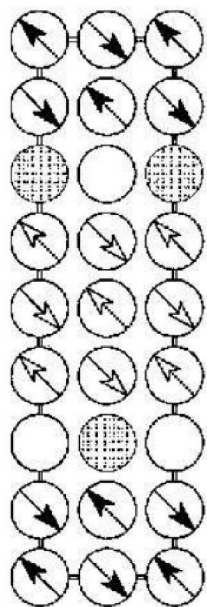
$Rb_{0.3}MoO_3$ [Brun05]

$Ca_{2-x}Na_xCuO_2Cl_2$ [Hanaguri04]

Dy-BSCCO [Kohsaka07]

CDW's in STM (Scansion Tunnelling Microscope) experiments

Anomalous Phonon Softening in the Cuprates



Stripe Phase[Tranquada96,Bosch01] Bond-Stretching Phonon Branch in LBCO[Reznik06]

OUR IDEA: STRIPES \Rightarrow Nearly 1D-metallic structures
 \Rightarrow [Kohn Anomaly] ANOMALOUS SOFTENING OF THE
BOND-STRETCHING PHONON BRANCH

Hubbard-Holstein Model + Gutzwiller-RPA Method



● $\sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} \implies$ ELECTRONIC KINETIC TERM

$\sum_i U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \implies$ E - E INTERACTIONS

$\sum_i \beta x_i (\hat{n}_i - n) \implies$ E - PH COUPLING

$\sum_i \left(\frac{1}{2M} P_i^2 + \frac{1}{2} K x_i^2 \right) \implies$ PHONONIC TERM

● **ADIABATIC LIMIT** ($M \rightarrow \infty$) $\implies \kappa_{\mathbf{q}}^{eph} = \frac{\kappa_{\mathbf{q}}}{1 - \lambda \kappa_{\mathbf{q}} / \chi_0^0}$ exact relation

$\lambda = \chi_0^0 \beta^2 / K \rightarrow$ e - ph coupling

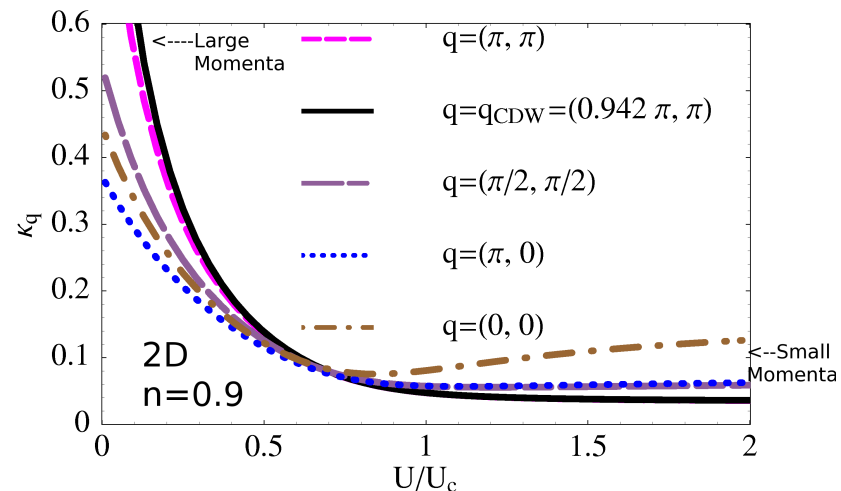
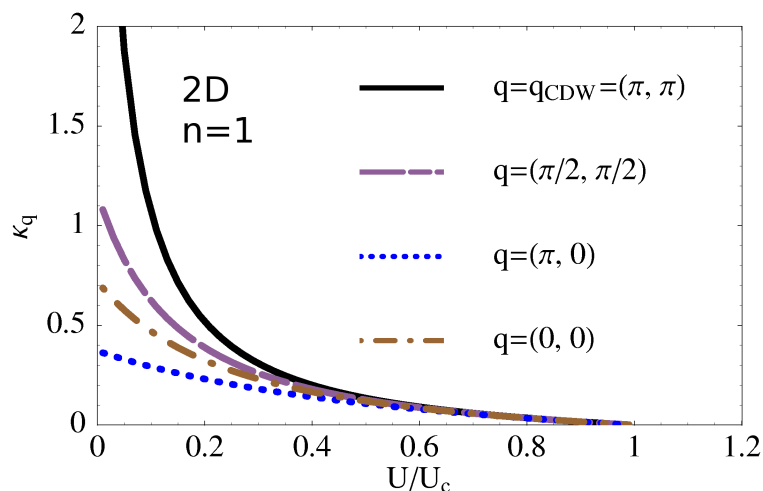
$\chi_0^0 \rightarrow$ density of states of the non-interacting system

● **GUTZWILLER APPROXIMATION (GA)** \implies electronic GROUND STATE +
low-energy excitations (quasiparticles)

RANDOM PHASE APPROXIMATION (RPA) \implies FLUCTUATIONS \implies
dressed ELECTRONIC SUSCEPTIBILITY:

$\kappa_{\mathbf{q}}$ (without e - ph coupling) $\implies \kappa_{\mathbf{q}}^{eph}$ (with e - ph coupling)

Homogeneous Metal \implies Charge Inhomogeneities



$n=1, U=U_c \rightarrow$ Metal-Insulator Transition

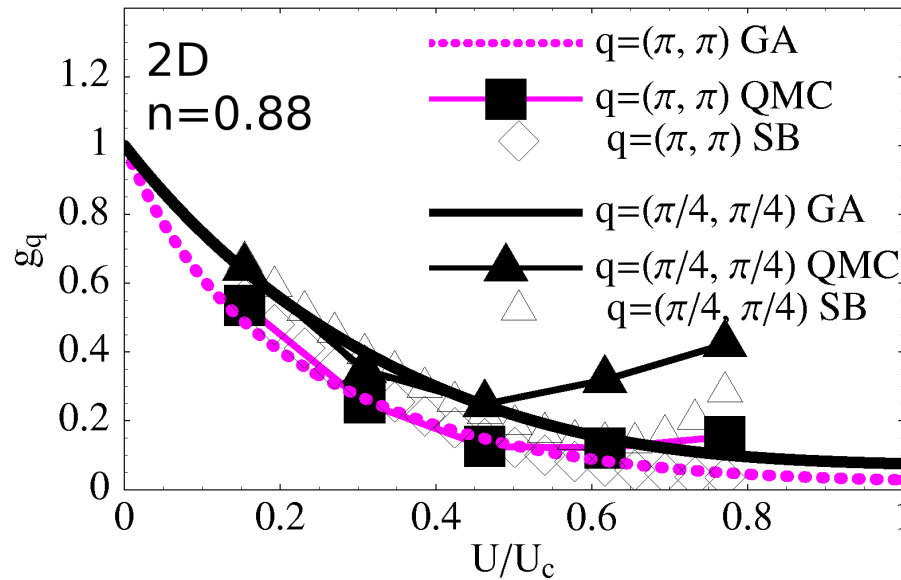
$$\kappa_{\mathbf{q}}^{eph} = \frac{\kappa_{\mathbf{q}}}{1 - \lambda \kappa_{\mathbf{q}} / \chi_0^0}$$

$U=0 \implies \kappa_{\mathbf{q}=2\mathbf{k}_F}$ maximum \implies strong phonon anomaly (KOHN ANOMALY)

SMALL $U \implies$ PEIERLS CDW

LARGE $U \implies$ PHASE SEPARATION (PS)

Homogeneous Metal \implies Charge Inhomogeneities



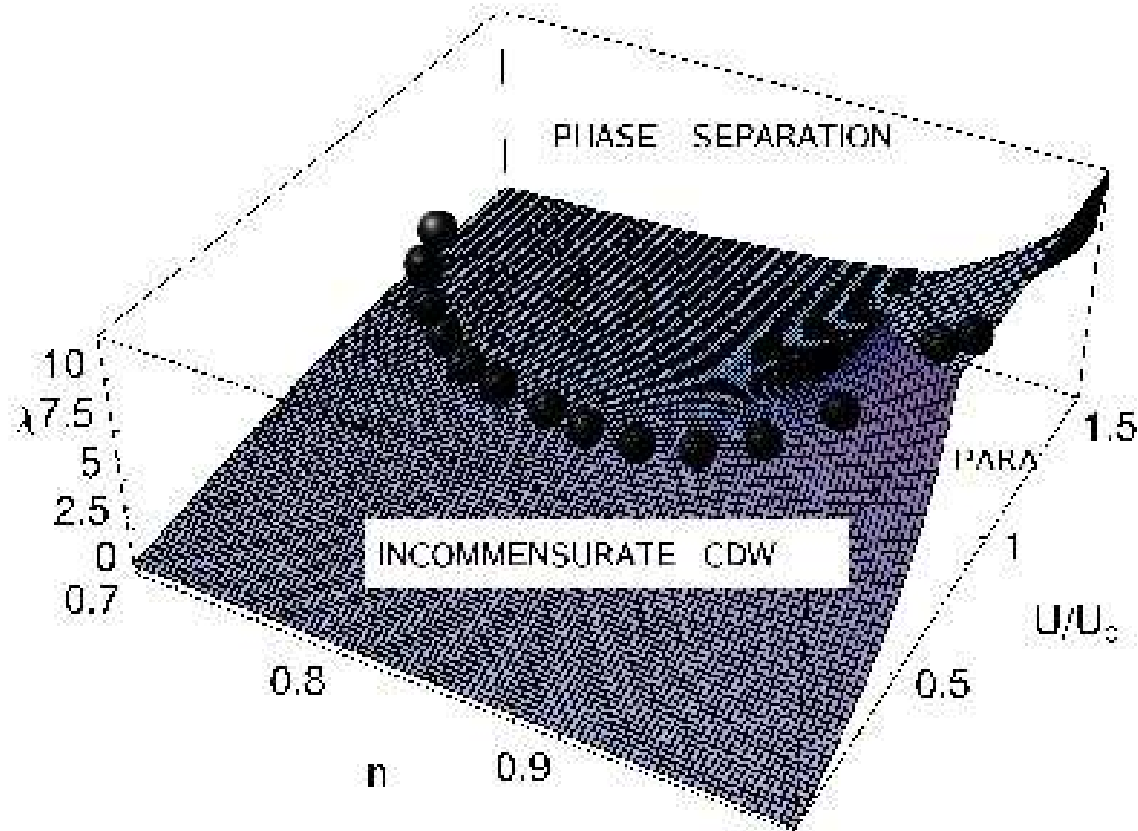
GA is accurate

U WEAKENS THE EFFECTIVE E - PH COUPLING

$$g_{\mathbf{q}} = z_0^2 \frac{\kappa_{\mathbf{q}}}{\kappa_{\mathbf{q}}^0} \quad z_0 \Rightarrow \text{GA renormalized hopping}$$

$$t \rightarrow tz_0^2$$

Homogeneous Metal \implies Charge Inhomogeneities

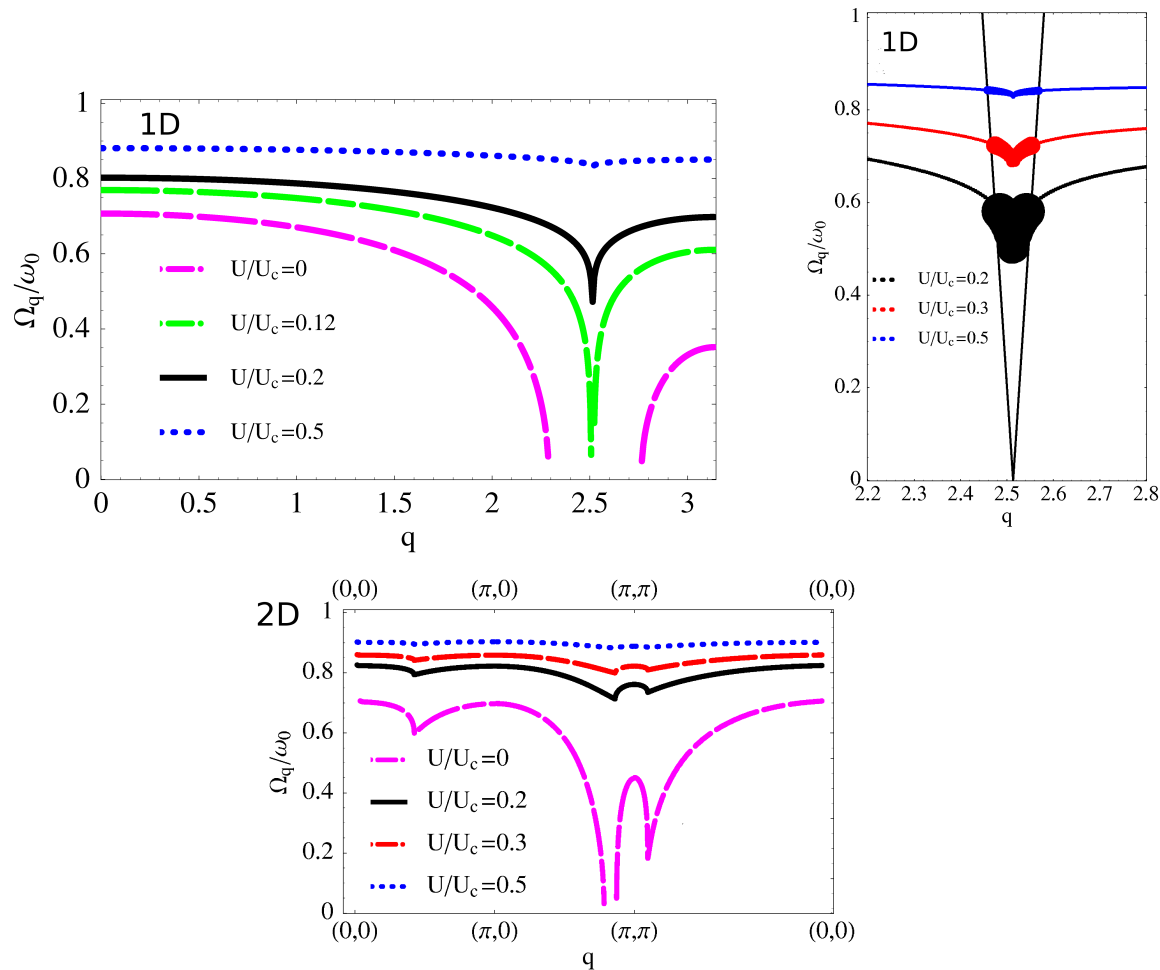


2D - PHASE DIAGRAM OF THE PARAMAGNETIC METAL

Kohn Anomaly in 1D and 2D

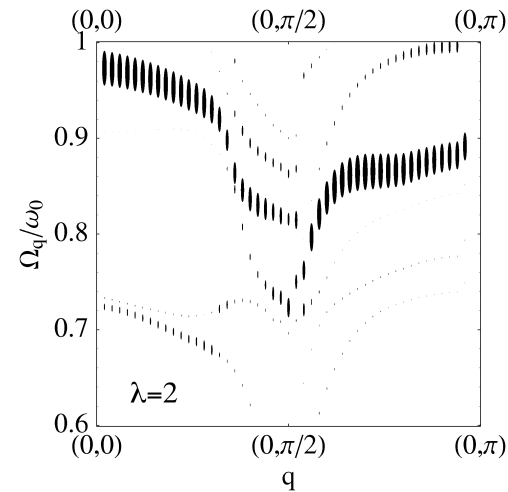
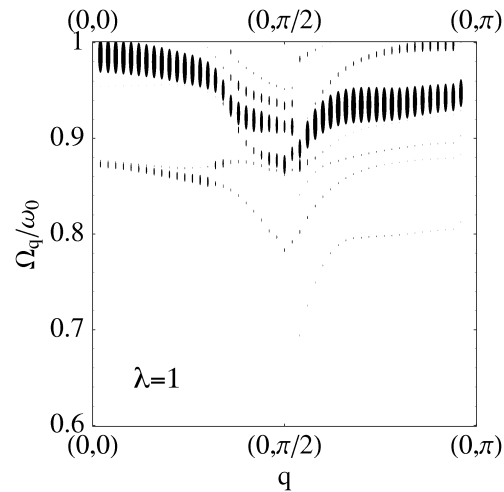
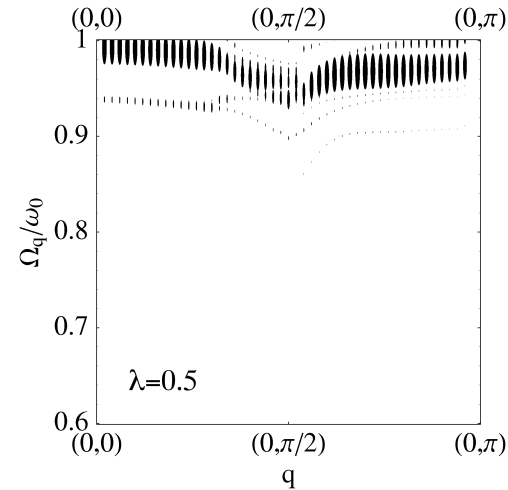
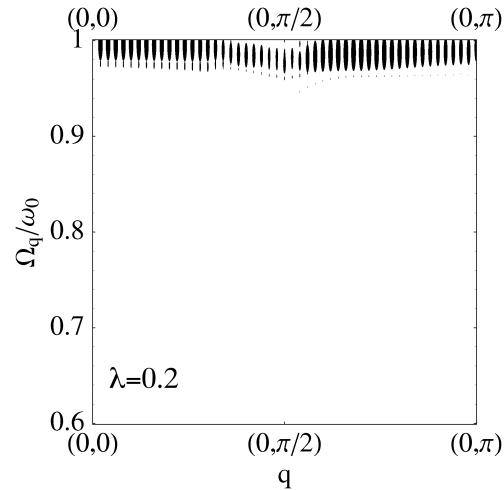


$\omega_0 \rightarrow$ bare phonon frequency



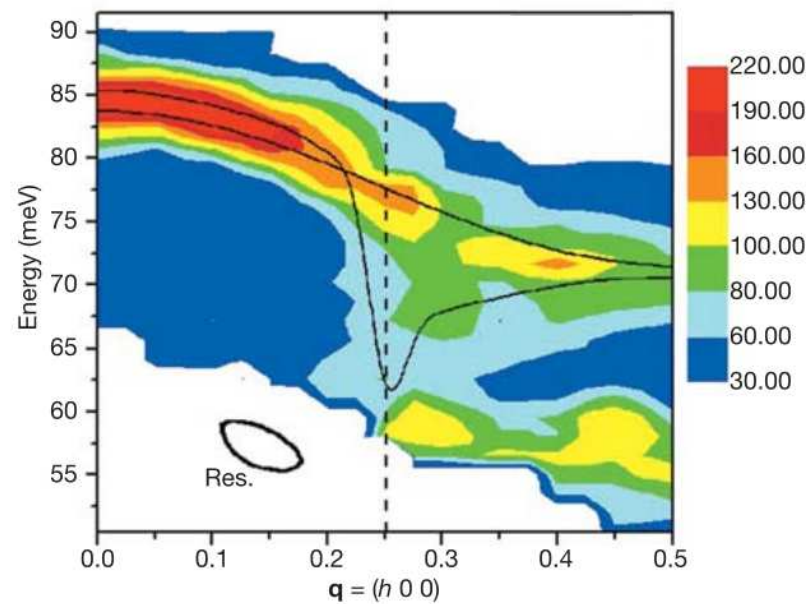
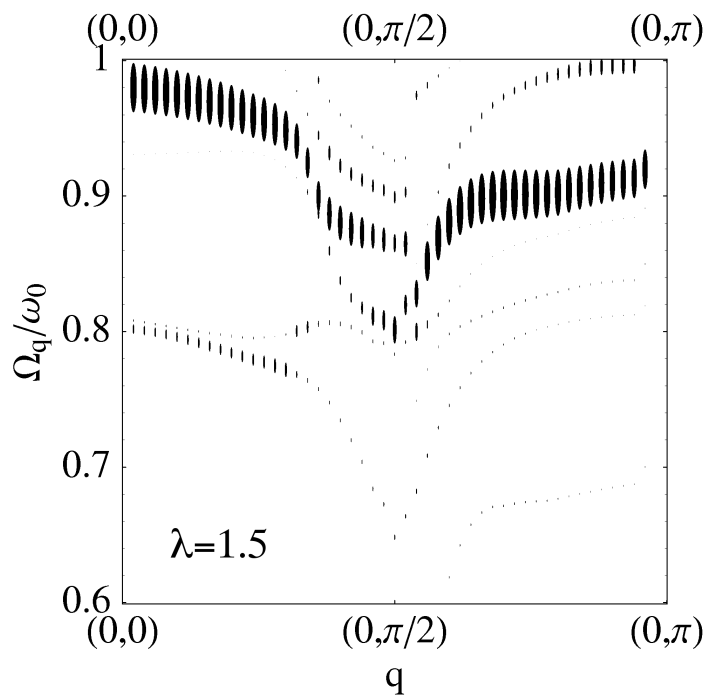
Results for $n = 0.8$ and $\lambda = 0.5$. **U SUPPRESSES THE KOHN ANOMALY.**

Stripes \implies Anomalous Phonon Softening



REALISTIC PARAMETERS $\implies n = 0.875$, $U/t = 8$, next-nearest hopping $t'/t = -0.2$

Stripes \implies Anomalous Phonon Softening



OUR OPTICAL PHONON BRANCHES VS EXPERIMENTAL BRANCHES

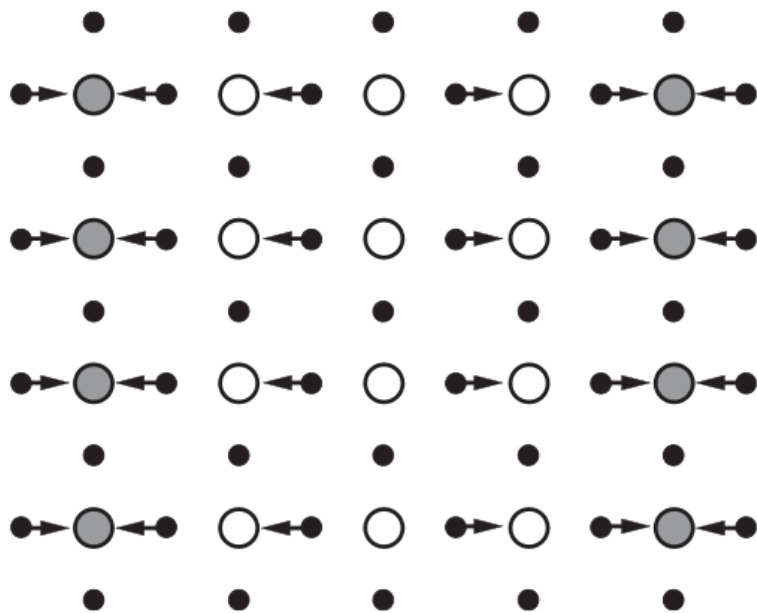
STRONG ASYMMETRY WELL REPRODUCED

NOT TRIVIAL RESULT

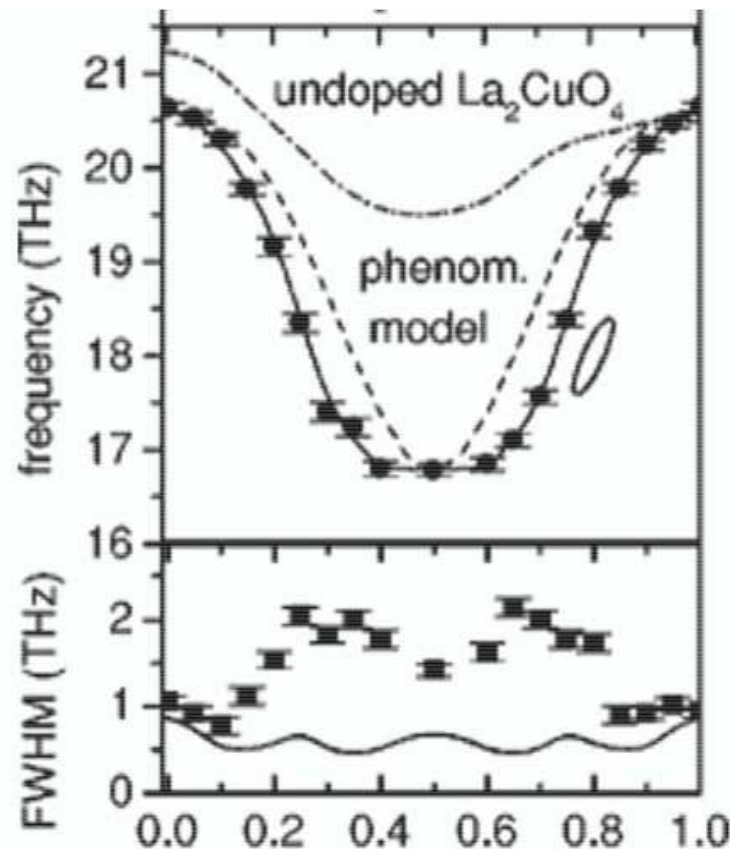


- e - ph coupling suppressed by e - e interaction
- Homogeneous Metal $\Rightarrow \lambda \Rightarrow$ Charge Inhomogeneities
- Peierls CDW $\Rightarrow U \Rightarrow$ PS
- Optical Phonons + Stripes $\Rightarrow U \Rightarrow$ Anomalous Phonon Softening

Anomalous Phonon Softening in the Cuprates



Half-Breathing Phonon Mode



Longitudinal Optical Branch [Pintschovius99]



The susceptibility $\kappa_{\mathbf{q}}$ has all the information on how $e-e$ interaction renormalizes $e-ph$ interaction:

$$\kappa_{\mathbf{q}}^{eph} = \frac{\kappa_{\mathbf{q}}}{1 - \lambda \kappa_{\mathbf{q}} / \chi_0^0} = \frac{\kappa_{\mathbf{q}}}{1 - \tilde{\lambda}_{\mathbf{q}}} , \quad \tilde{\lambda}_{\mathbf{q}} = \lambda \frac{\kappa_{\mathbf{q}}}{\chi_0^0}$$

- In the absence of other (first-order) instabilities, for $\lambda = \lambda_c = \chi_0^0 / \kappa_{\mathbf{q}_c}$ the system undergoes a transition to a CDW state with a typical \mathbf{q}_c .
- Increasing $U \Rightarrow \kappa_{\mathbf{q}}$ reduced with respect to $\kappa_{\mathbf{q}}^0 \Rightarrow \tilde{\lambda}_{\mathbf{q}}$ reduced \Rightarrow U WEAKENS the $e-ph$ coupling.
- The problem is then reduced to the study of the electronic susceptibility for $\lambda = 0$.



The Hubbard model: $H_e = \sum_{ij\sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + c_{j\sigma}^\dagger c_{i\sigma}) + \sum_i U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}$

● GZW TRIAL STATE: $|\psi_G\rangle = \hat{P}_g |Sd\rangle \implies$

$|Sd\rangle \rightarrow$ SLATER DETERMINANT

$$\hat{P}_g = \prod_i [1 - (1 - g) \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}] = \prod_i [1 - (1 - g) \hat{D}_i]$$

For $g < 1$, \hat{P}_g suppresses double occupancies D_i .

For $g = 0$, all the configurations with $D_i \neq 0$ are ruled out.

● $E_e[\rho, D] = \langle \psi_G | H_e | \psi_G \rangle \simeq \sum_{ij\sigma} t_{ij} z_{i\sigma} z_{j\sigma} \rho_{ji\sigma} + \sum_i U D_i$

● $z_{i\sigma} [\rho_{ii\sigma}, \rho_{ii,-\sigma}, D_i] \rightarrow$ GZW-renormalized hopping factors

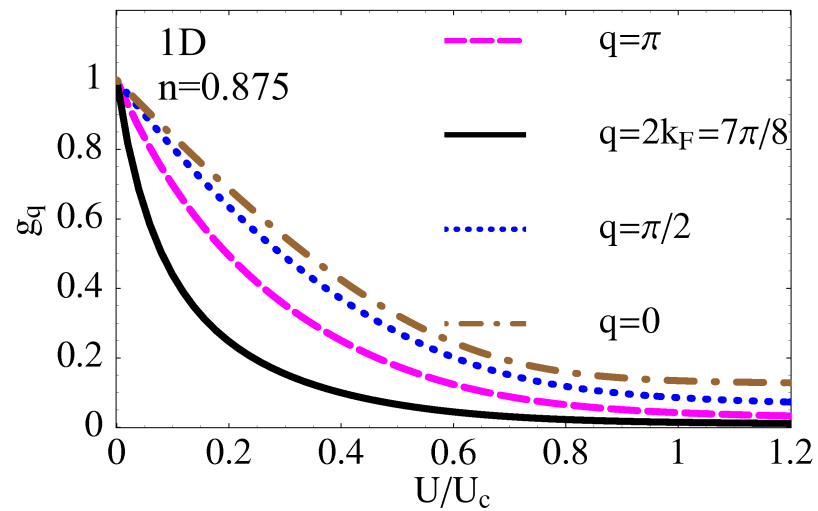
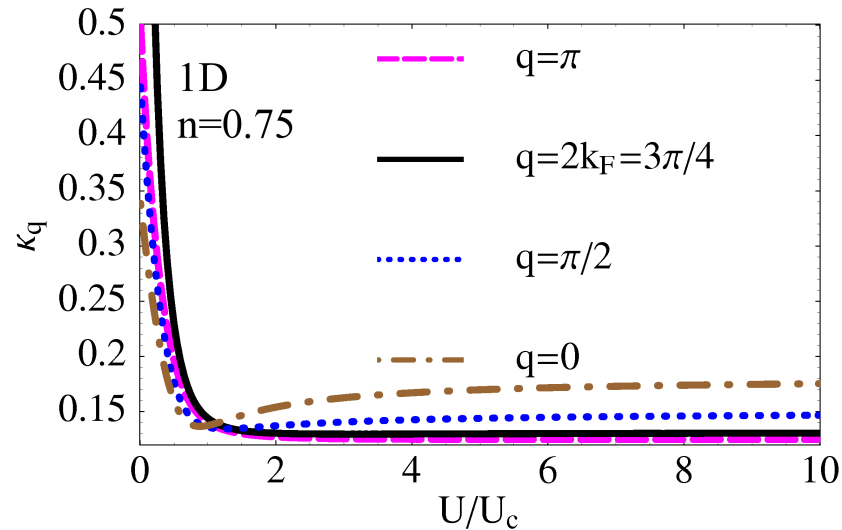
● $\rho_{ji\sigma} = \langle Sd | c_{i\sigma}^\dagger c_{j\sigma} | Sd \rangle \rightarrow$ uncorrelated single fermion

DENSITY MATRIX

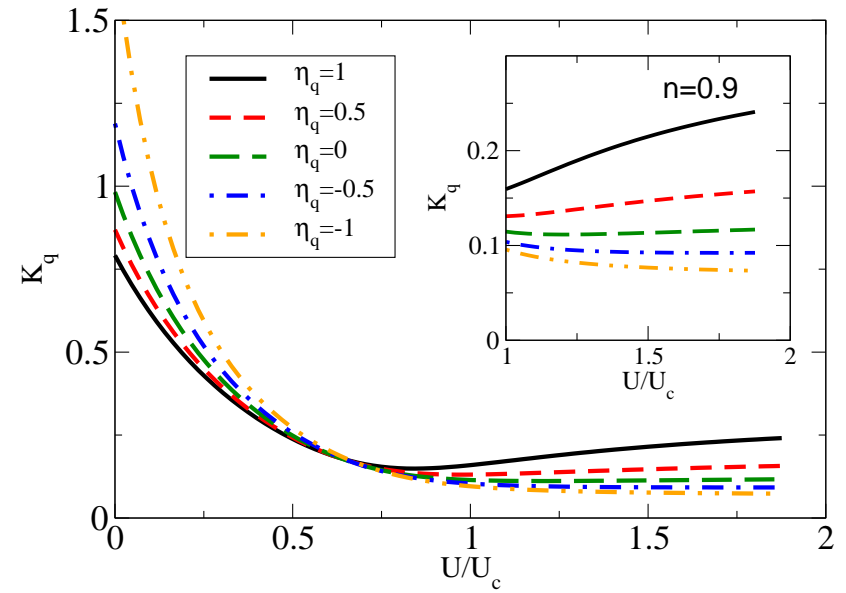
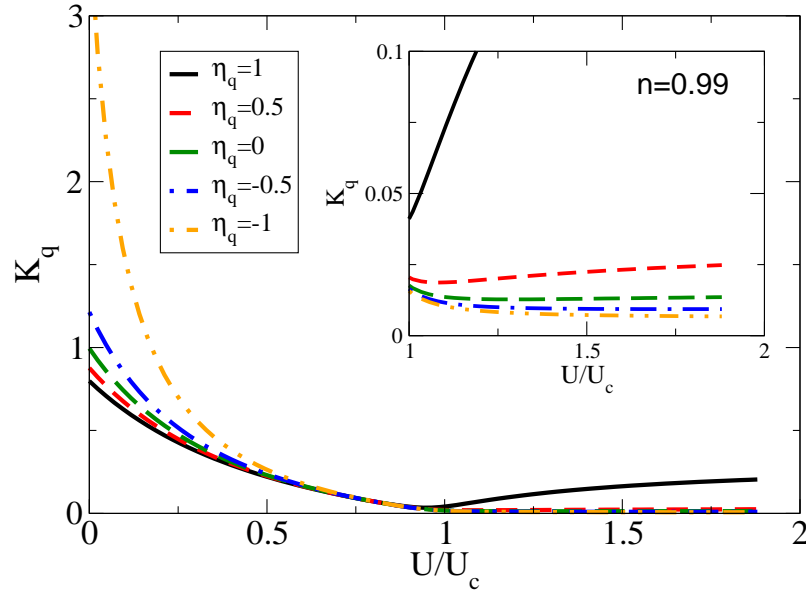


- Computations with the GA+RPA method [Seibold-Lorenzana0103], generalization of the HF+RPA technique [Ring80, Blaizot86]
- An external field induces small amplitude deviations δD and $\delta \rho$ around the GA saddle point \implies
$$E_e[\rho, D] = E_{e0} + Tr[h_0 \delta \rho] + \frac{1}{2} \delta \rho^\dagger L_0 \delta \rho + \delta D S_0 \delta \rho + \frac{1}{2} \delta D^t K_0 \delta D$$
with $h = \delta E_e / \delta \rho$.
- $\frac{\partial E}{\partial \delta D} = 0$ eliminates the δD deviations \implies
2nd order energy deviation: $\delta E_e = \frac{1}{2} \delta \rho^\dagger W \delta \rho$
- The interaction kernel W mediates local and intersite charge deviations.

Homogeneous Metal \implies Charge Inhomogeneities



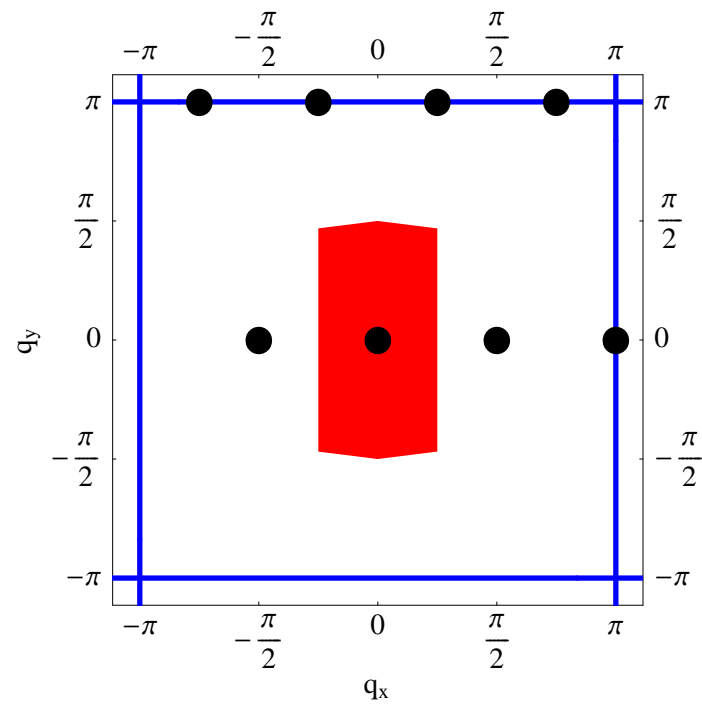
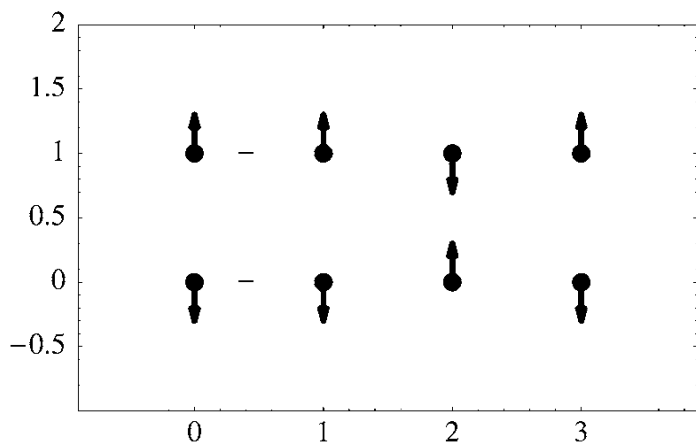
Homogeneous Metal \implies Charge Inhomogeneities



$$D = \infty$$

$$\eta_{\mathbf{q}} = \frac{1}{d} \sum_{\nu=1}^d \cos q_{\nu}$$

Stripes \implies Anomalous Phonon Softening



4×2 UNIT CELL \implies FIRST REDUCED BRILLOUIN ZONE