FLUCTUATIONS IN DEFORMING MATERIALS

PhD in Physics – XXI cycle – October 2008

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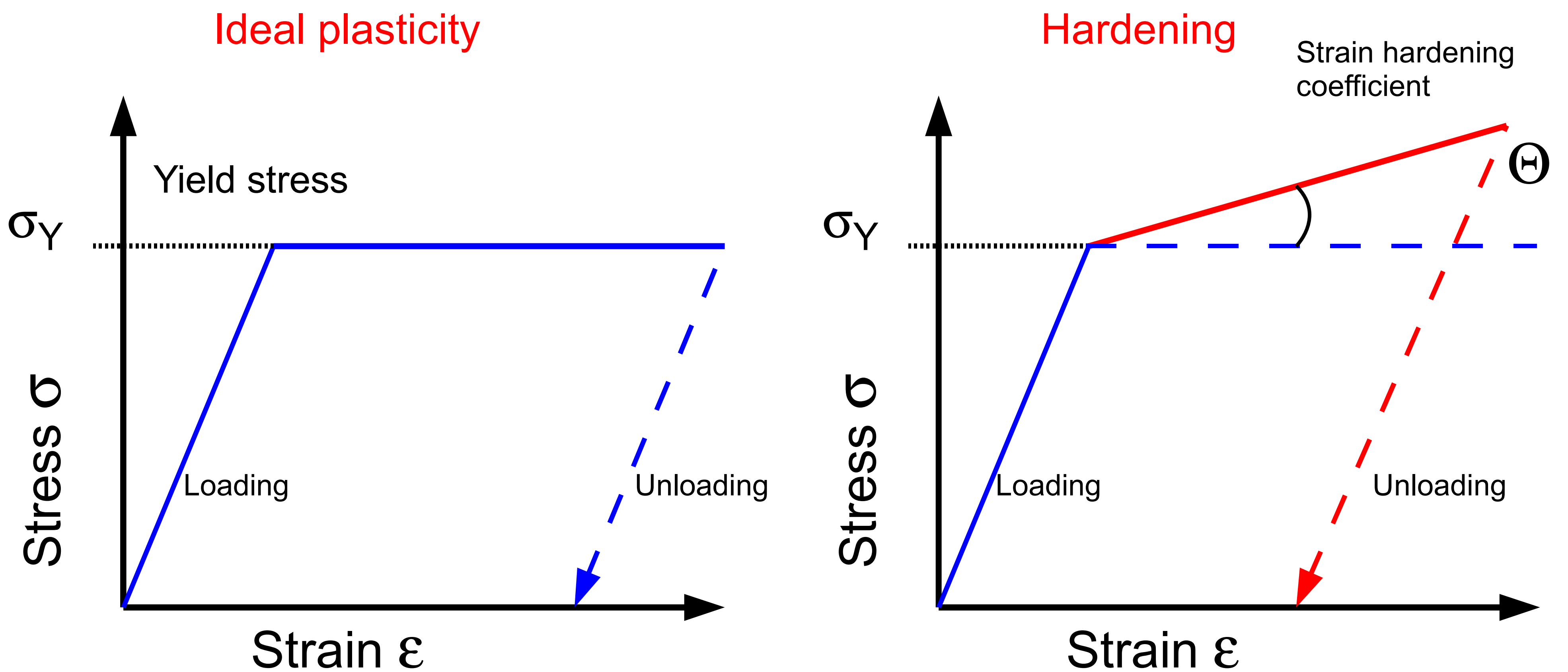
Plasticity

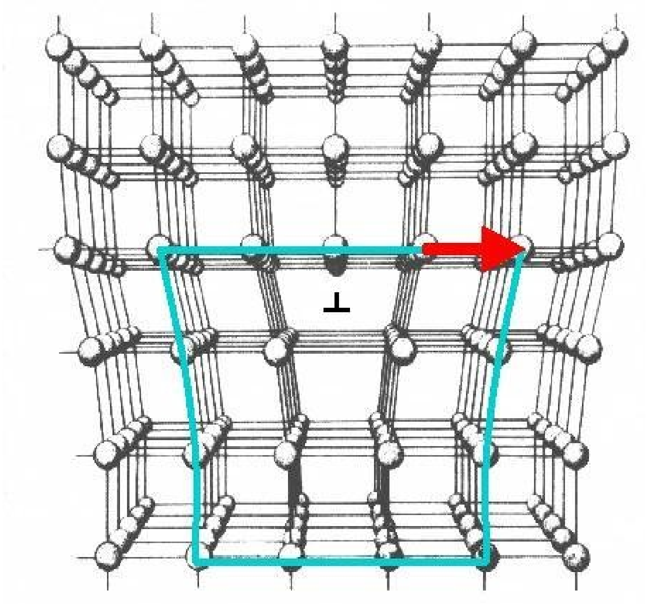
[Continuum macroscopic plasticity]



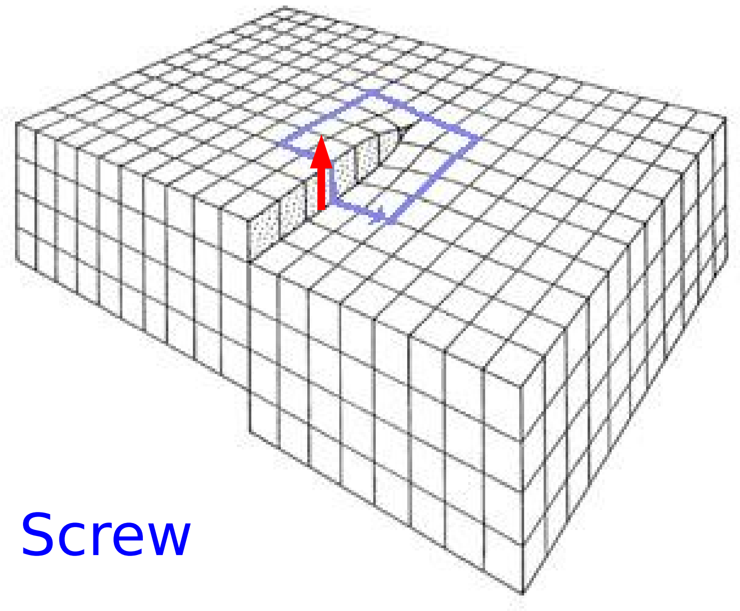
Crystal Dislocations

Burgers vector: *b*

$$\oint d\vec{u} = \vec{b}$$

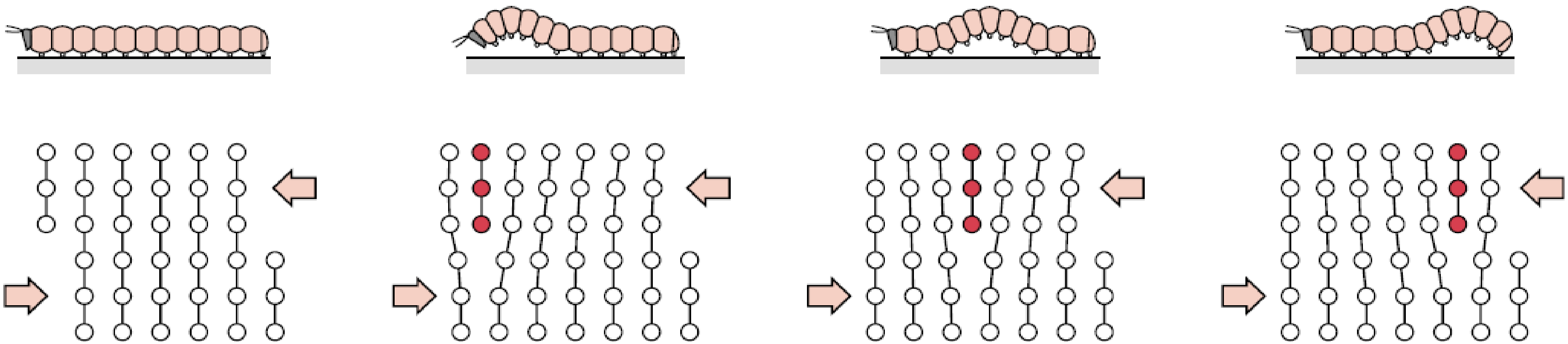


Edge



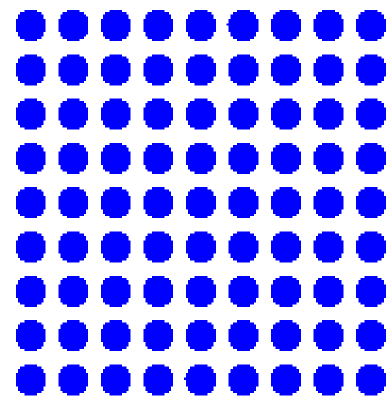
Screw

Dislocation Dynamics



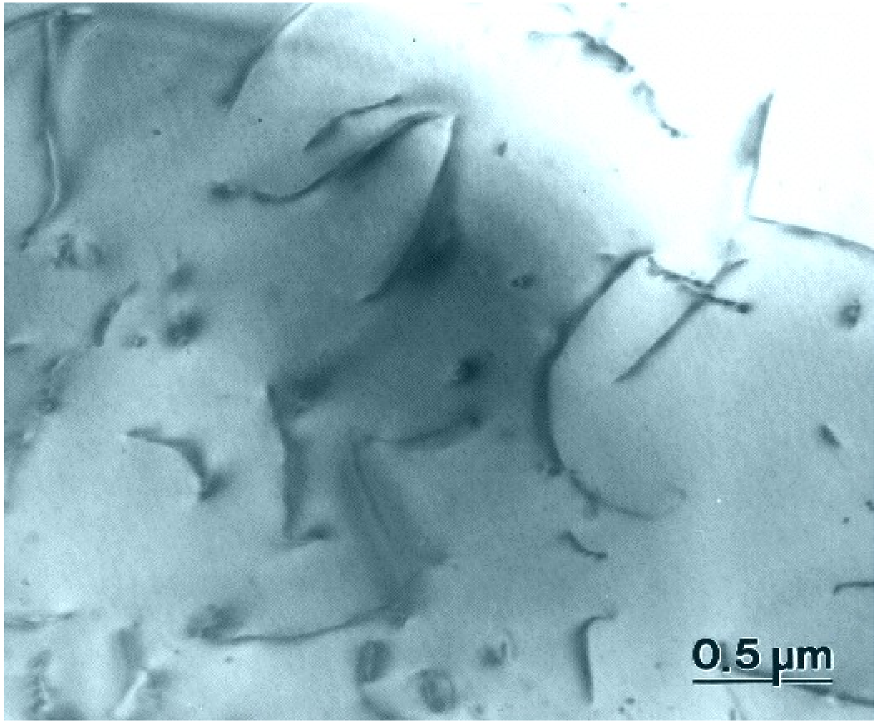
The force on a dislocation in a stress field σ (Peach-Koheler):

$$\boldsymbol{F}_{PK} = (\boldsymbol{b}\cdot\sigma)\times\hat{l}$$



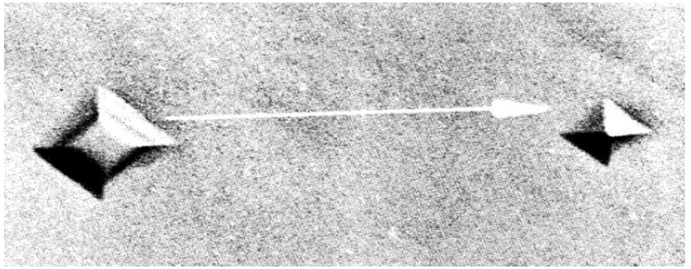
Experimental Observations

Transmission Electron Microscopy (TEM)

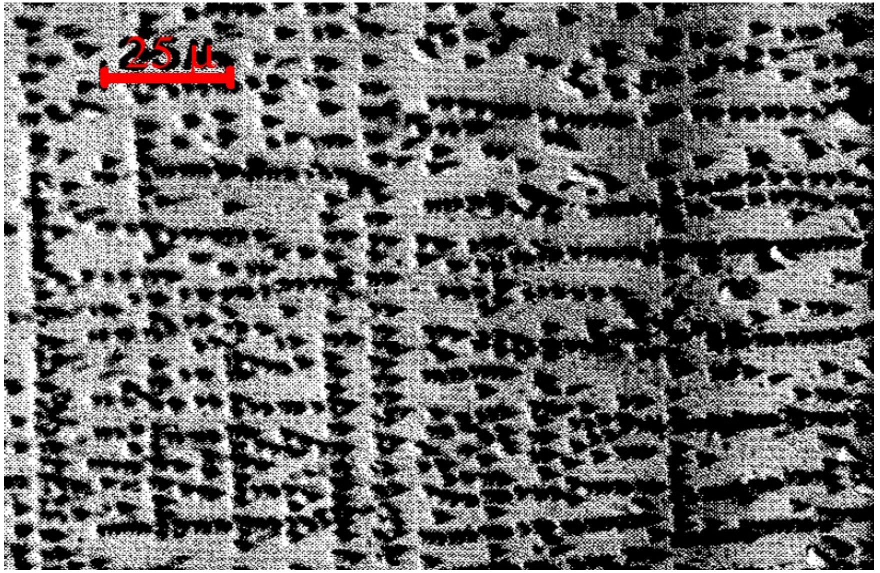


TEM image of the interior of a deformed quartz grain

Etch Pits



Dislocation motion as revealed by etch pits on a {100} surface of lithium fluoride

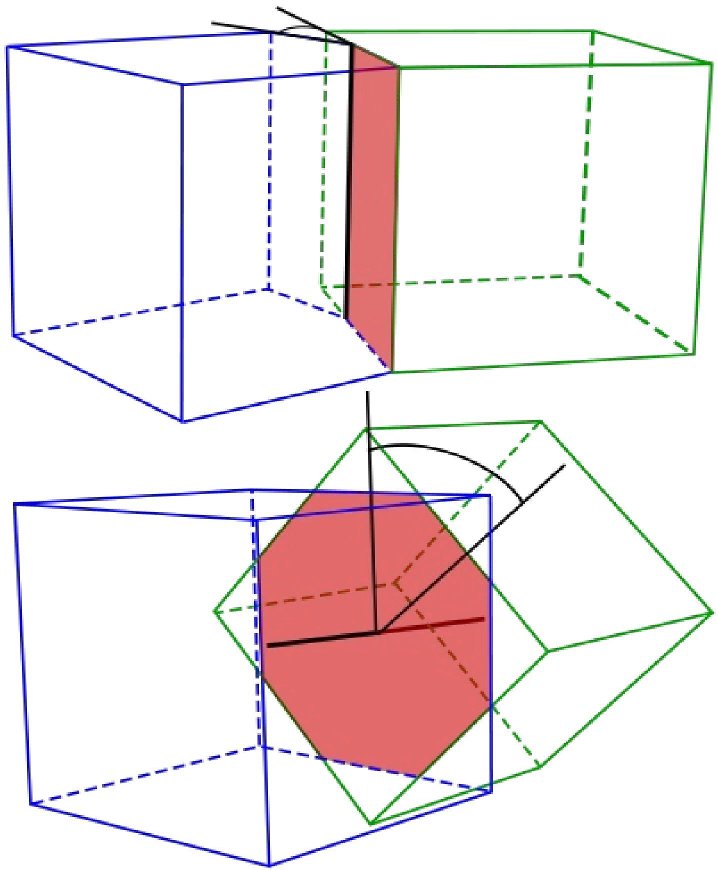


Etch pits in a silicon crystal

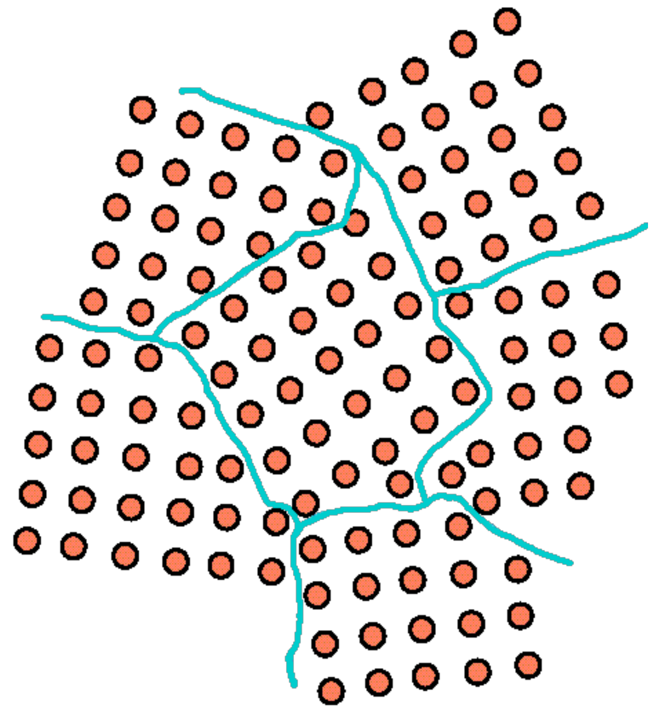
Grain Boundaries

Tilt
Boundary

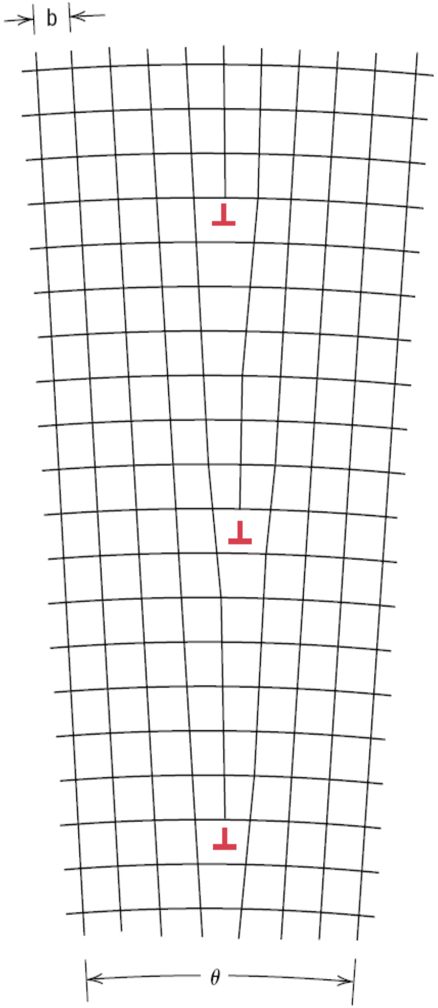
Twist
Boundary

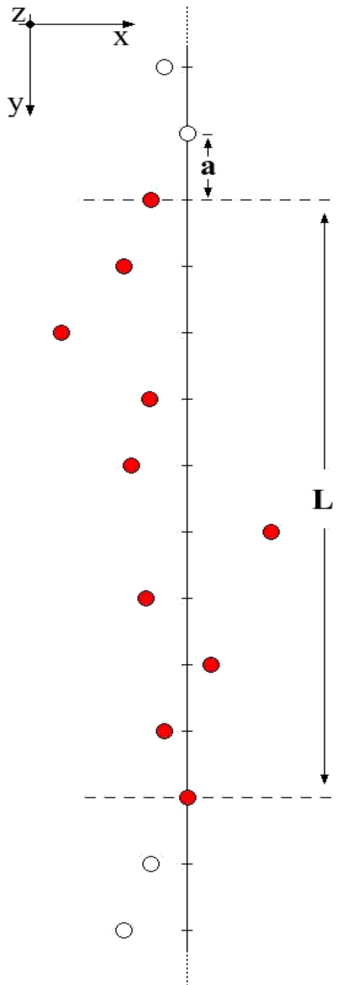


Polycrystal



Grain Boundary Dynamics





$$\gamma \dot{x}_i(t) = F_{PN}(x_i) + F_{PK}(x, y) + \eta_i(t)$$

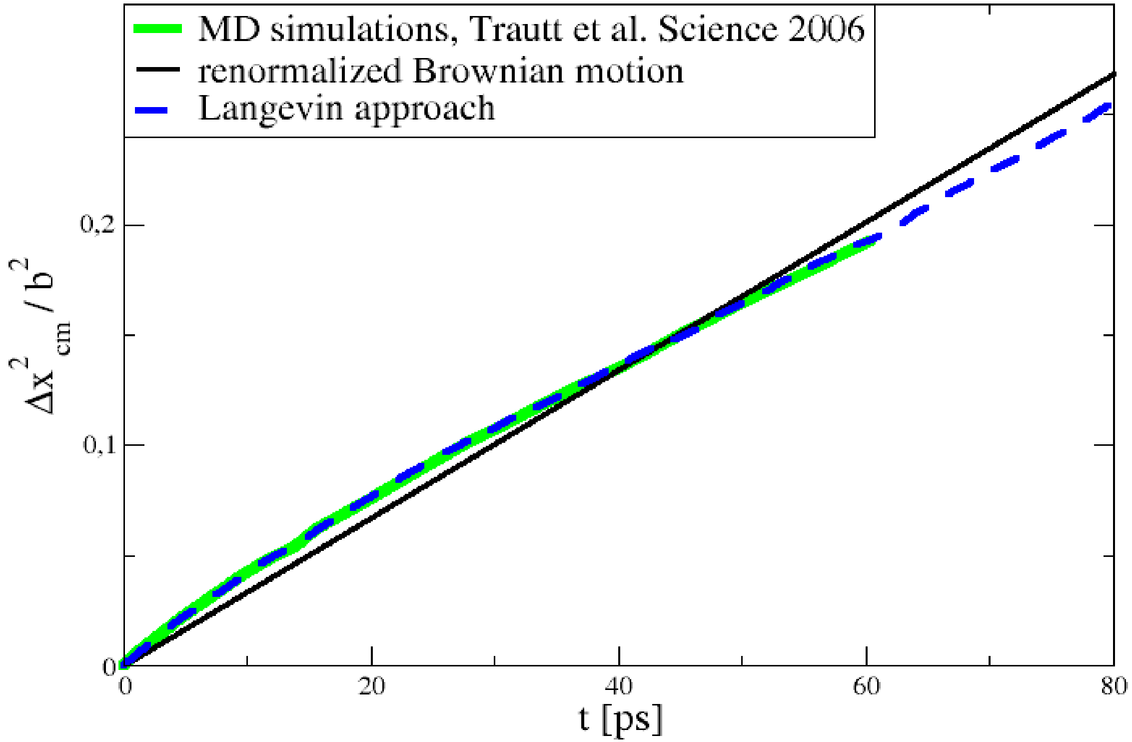
$$\begin{cases} F_{PN} = Peierls - Nabarro\ force \\ F_{PK} = Peach - Koehler\ force \\ \eta_i(t) = Gaussian\ white\ noise \end{cases}$$

Mean Square Displacement

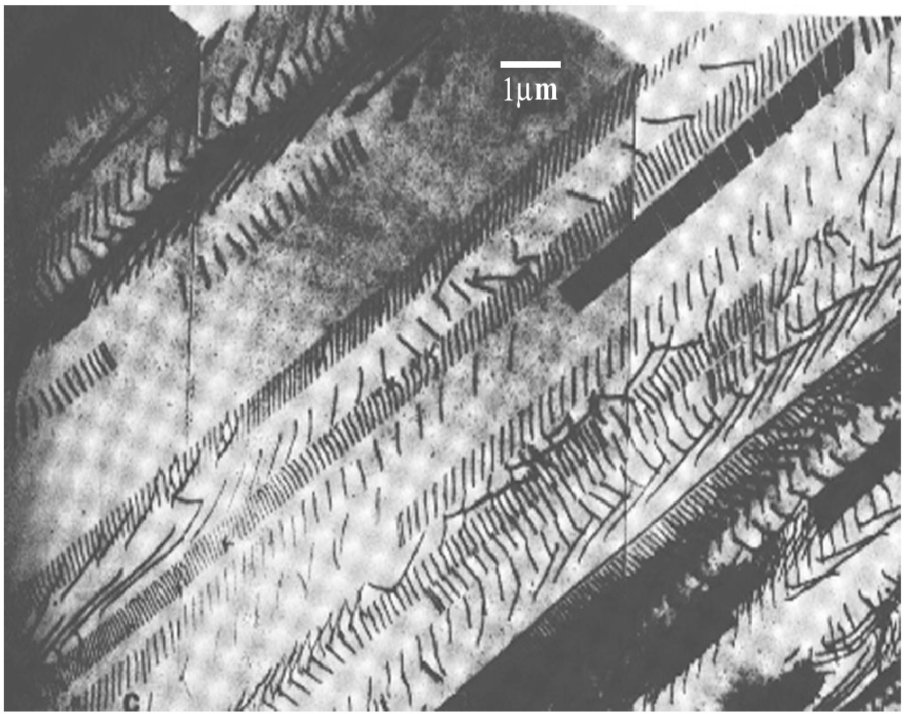
$$x_{cm}(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t)$$

$$\Delta x_{cm}^2 = \langle x_{cm}^2(t) \rangle - \langle x_{cm}(t) \rangle^2$$

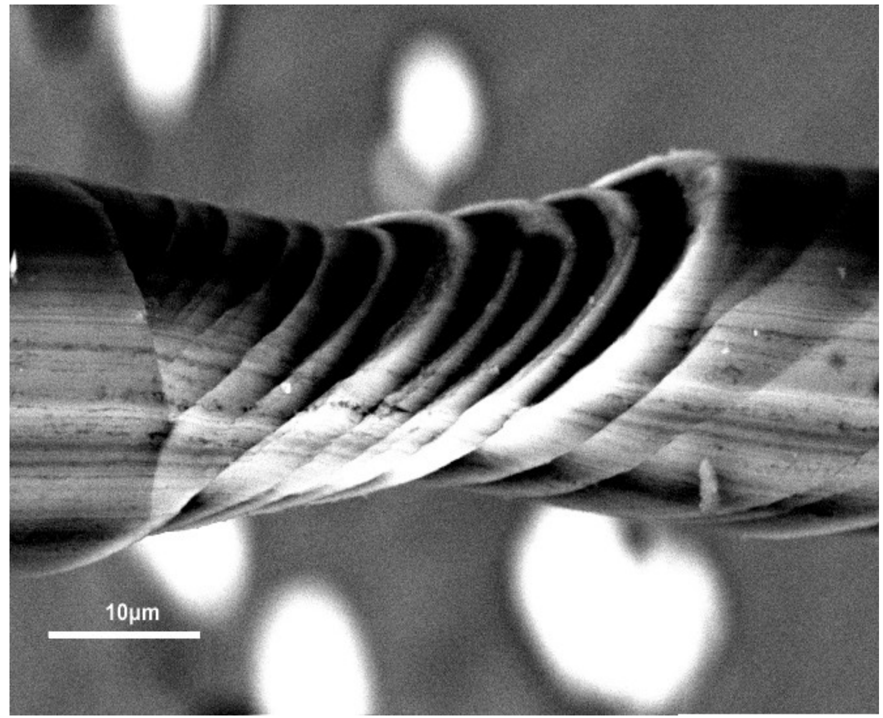
The GB Internal degrees of freedom are irrelevant for Δx_{cm}^2



Slip Line Formation

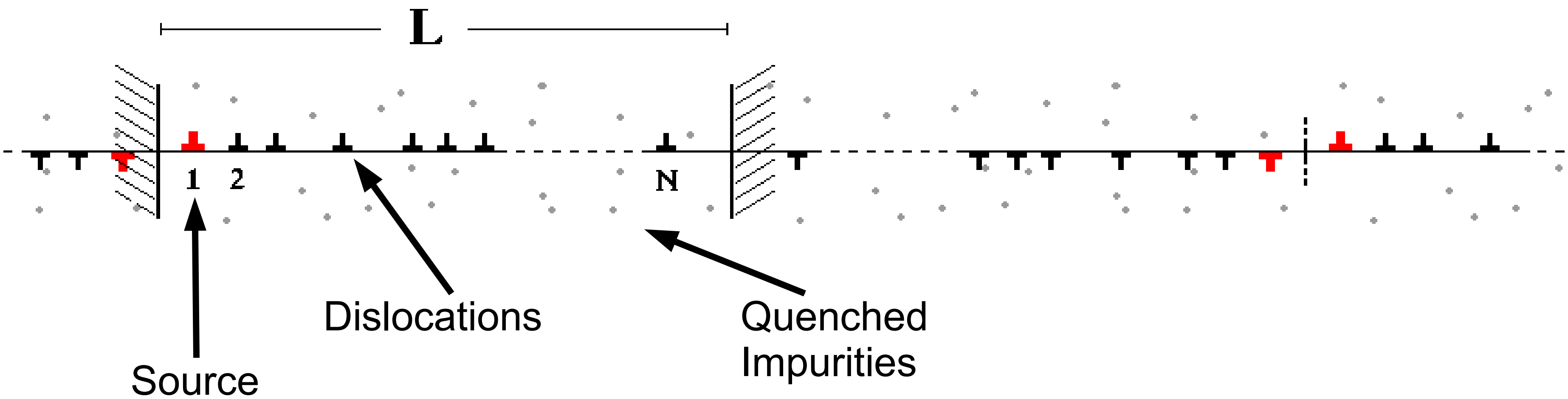


Dislocation groups in Cu-30% Zn observed in TEM



Localized deformation of a tensile tested bamboo structured aluminum wire of diameter $25\,\mu m$

Pile-up model for Slip Lines



$$\gamma \dot{x}_i = \sigma + \sum_{j=\text{۱}}^{N} \sigma_{ij}^{int} + \sigma_i^{img} + \sum_p f(x_i - x_p) \qquad (i=\text{۲},\dots,N)$$

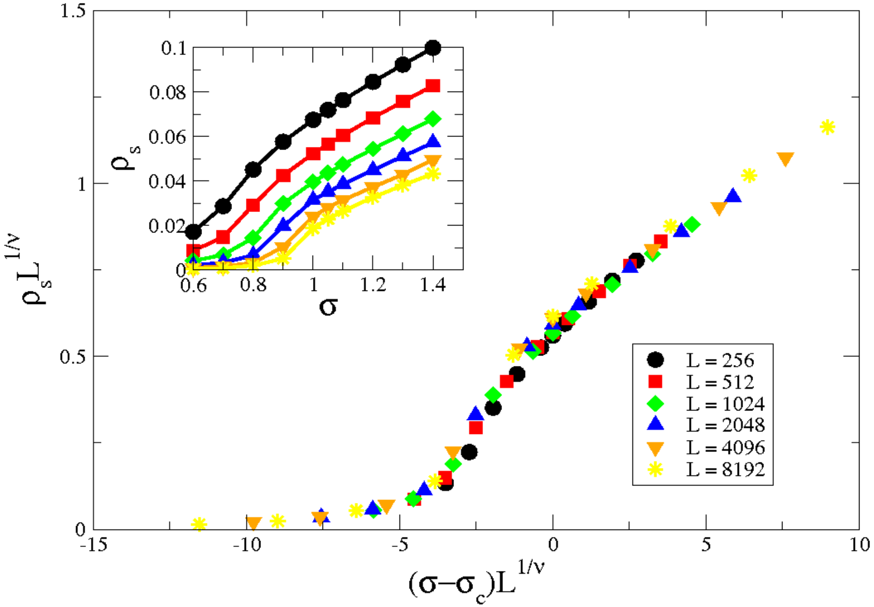
$$\Gamma \dot{b}_{\text{۱}}(t) = \theta(\sigma_{\text{۱}}^{eff})\,\sigma_{\text{۱}}^{eff}$$

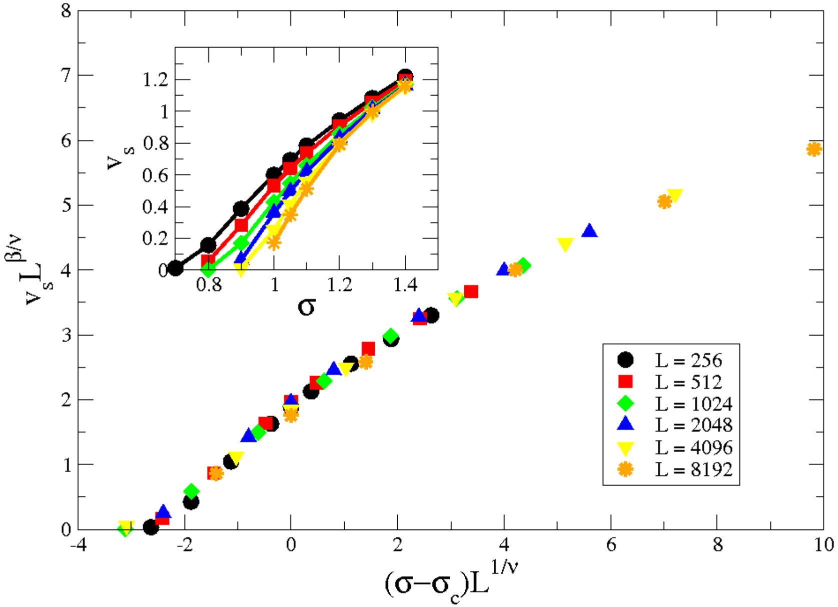
Finite Size Scaling

Yielding point as a Non-Equilibrium Phase Transition

$$\rho_s(L,\sigma) \sim L^{-\alpha/\nu} f[(\sigma-\sigma_c)L^{1/\nu}]$$

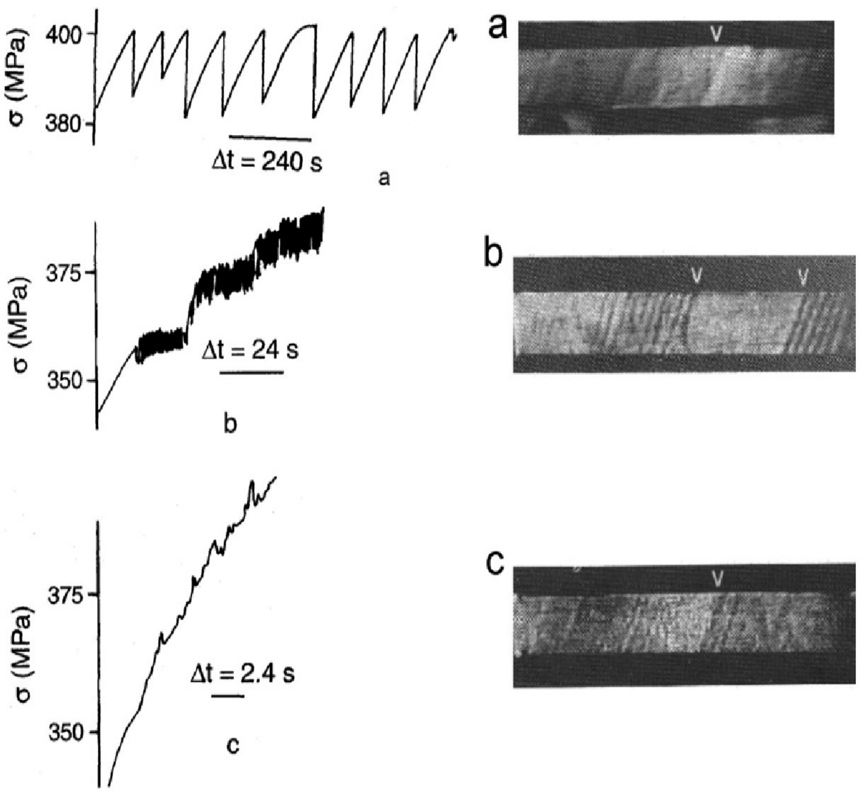
$$v_s(L,\sigma) \sim L^{-\beta/\nu} g[(\sigma-\sigma_c)L^{1/\nu}]$$



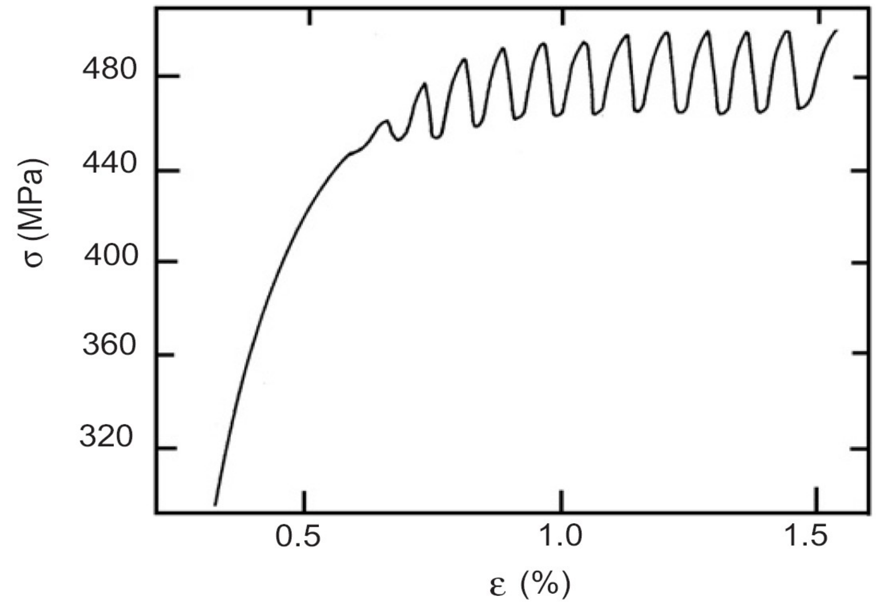


Portevin–Le Catelier (PLC) effect

Serration in the Stress-Time curve



Ananthakrishna's mesoscopic model

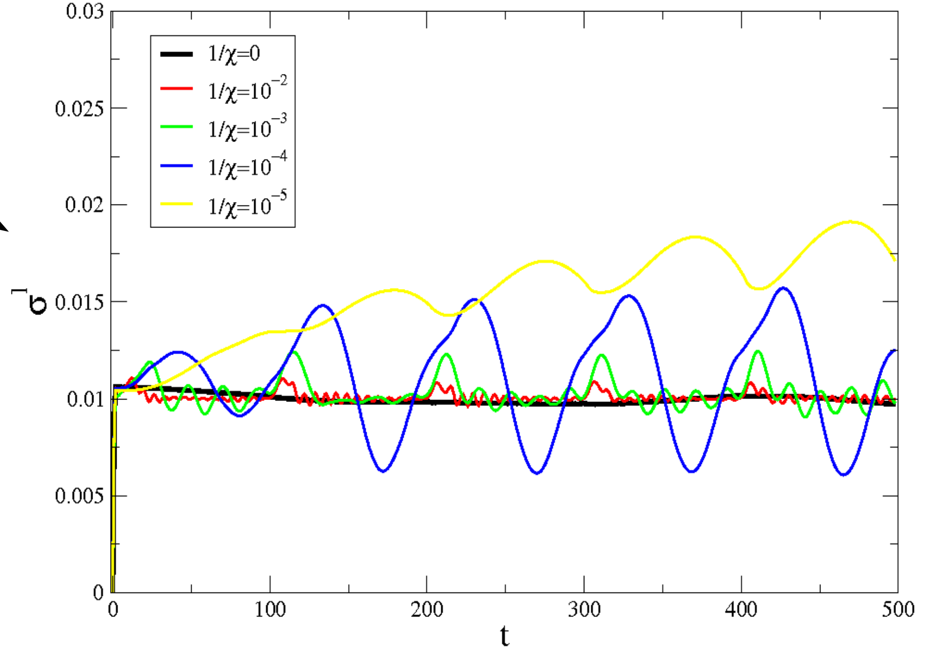


Orowan relation

$$\dot{\epsilon} = b\,\rho_m \langle v \rangle$$

PLC effect as collective motion

Serration in the stress-time curve



Mobile impurities

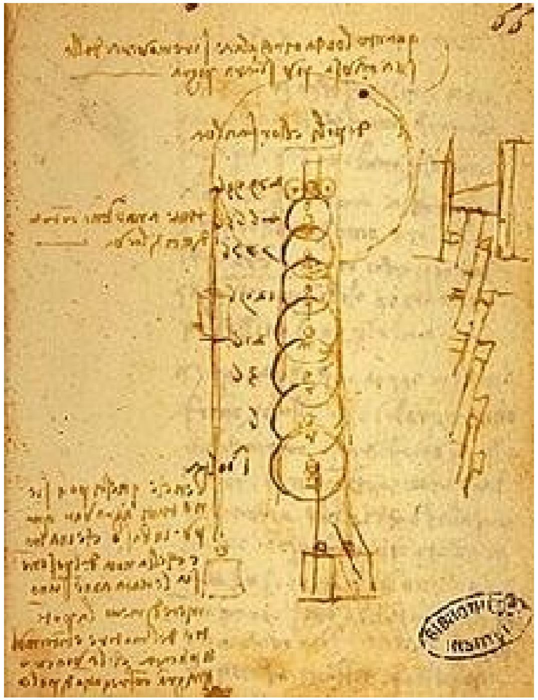
$$\chi \dot{x}_{s,p} = -\sum_{i=1}^{N} f(x_i - x_{s,p}) + \eta_p(t)$$

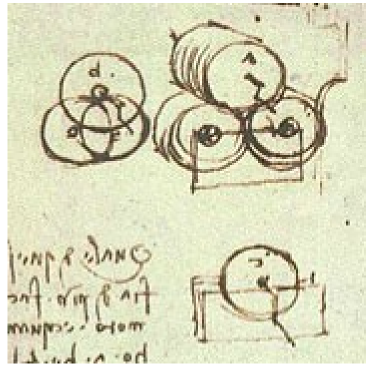
$$\gamma \dot{x}_i = k[Vt - x_i] + \sum_{j=1}^{N} \sigma_{ij}^{int} + \sigma_i^{img} + \sum_{p=1}^{N_p} f(x_i - x_{s,p})$$

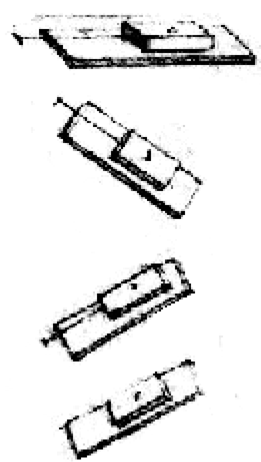
[Results from Dislocation Dynamics]

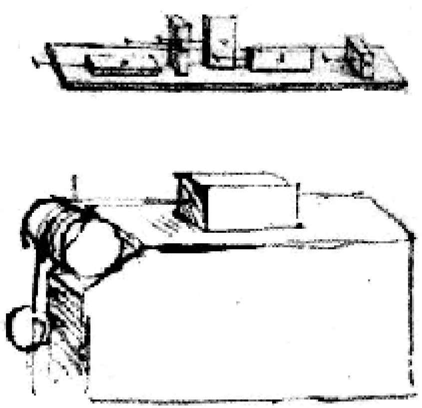
- The role of internal degrees and crystalline potential in Grain Boundary diffusion
- Yielding point as a Non-Equilibrium Phase Transition studied with the Pile-up model
- The PLC effect due to the collective motion of dislocations mediated by impurities

Friction





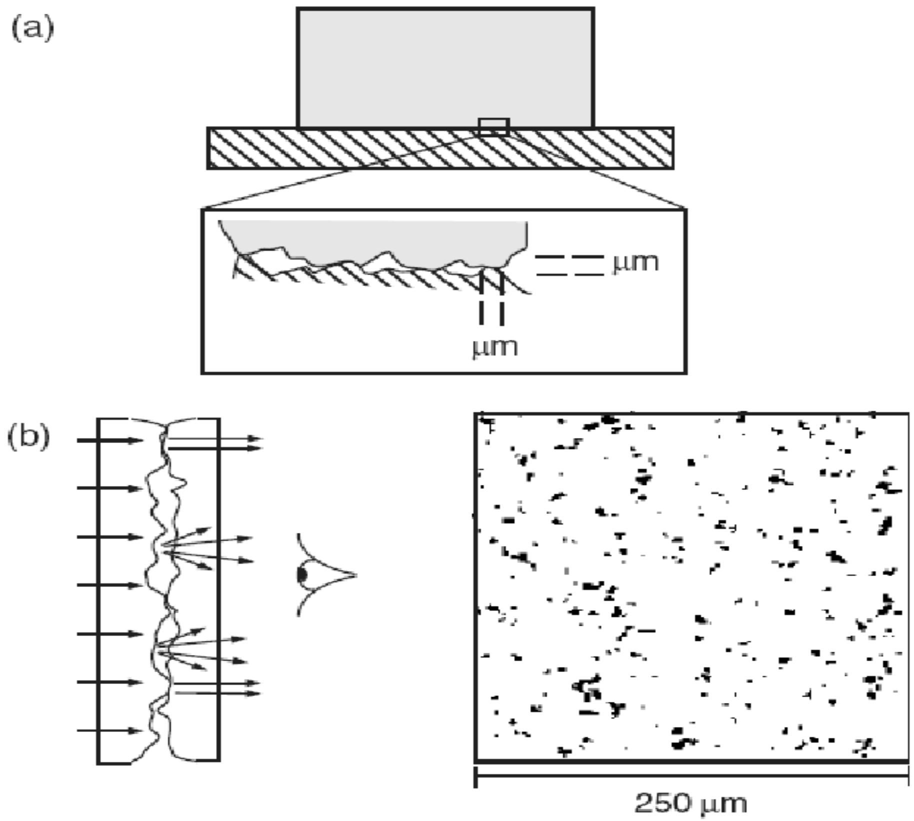




Rate and State Approach

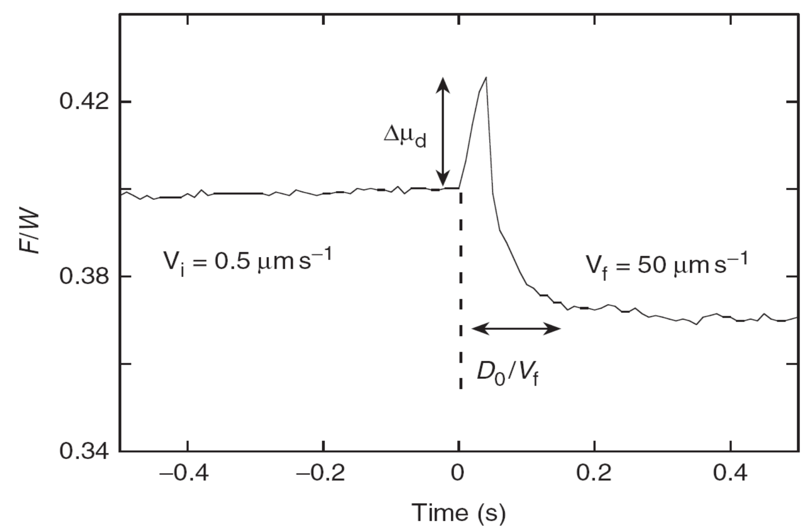
Tabor's decomposition of Friction: $F = \Sigma_r(\phi)\sigma_s(v)$

Real contact area: $\Sigma_r(\phi)$



Shear strength: $\sigma_s(v)$

Velocity jump experiments (Dieterich)

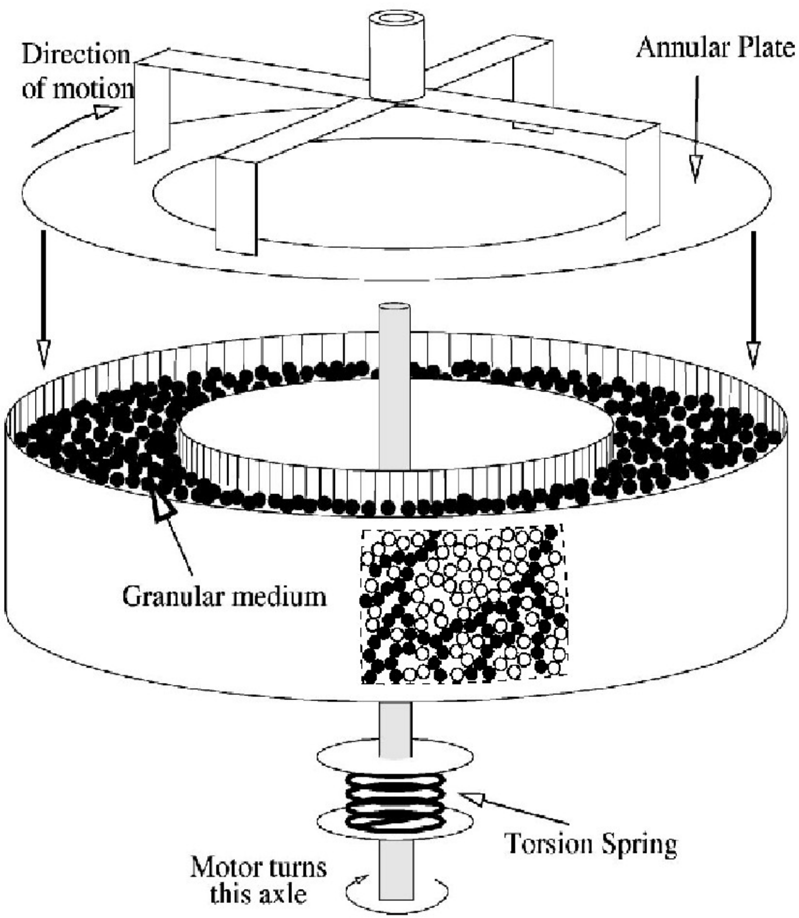


Friction in Granular Media (GM)

$$I\ddot{\theta}=k(\omega_D t-\theta)+F(t,\theta,\dot{\theta},...)$$

$$F(t,\theta,\dot{\theta},...)=F_v(\dot{\theta})+F_r(\theta)$$

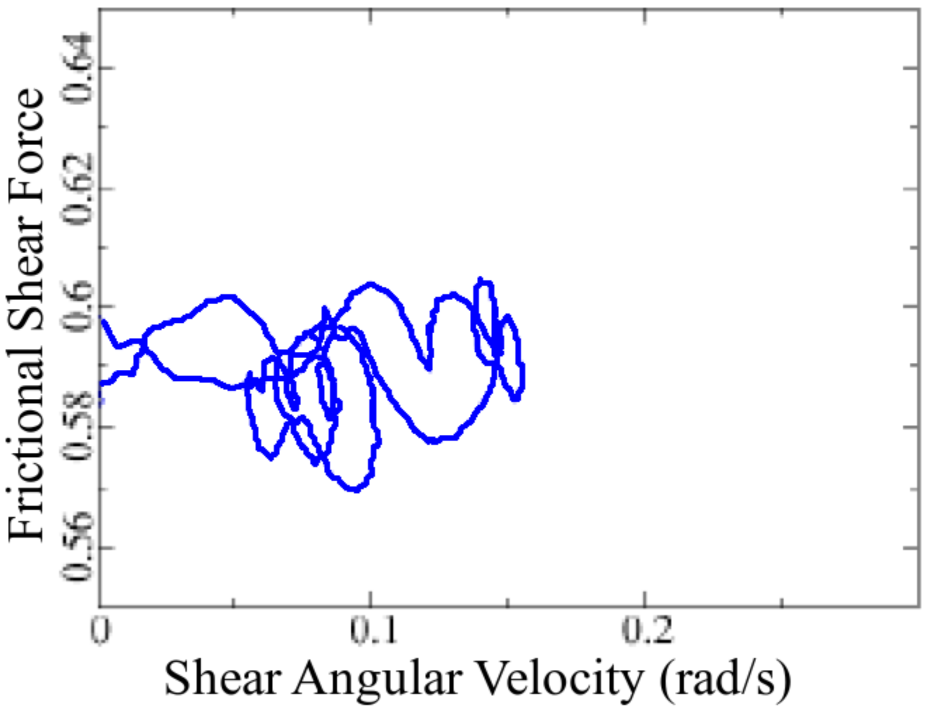
Baldassarri et al., PRL 2006



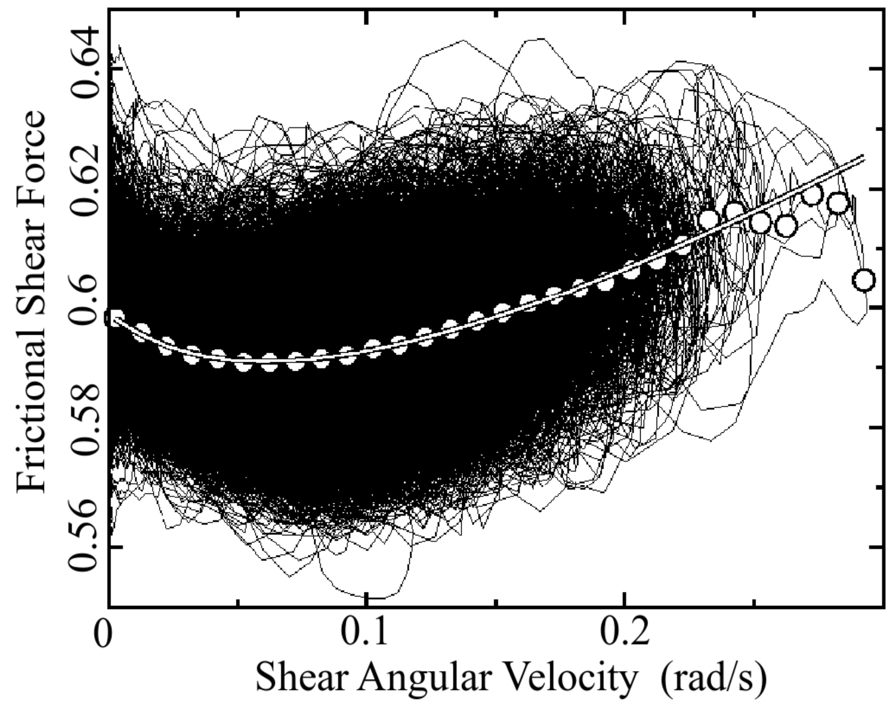
- Annular cell, sheared by an overhead plate driven by a motor via a torsion spring.
- 2 mm glass beads
- Spring: $0.1 < k < 1\ Nm/rad$
- Inertia: $0.015 < I < 0.02\ Kg\,m^2$
- Motor: $10^{-5} < \omega < 50\ rad/s$
- Free dilation of bed

Average Viscous Force

Single avalanche



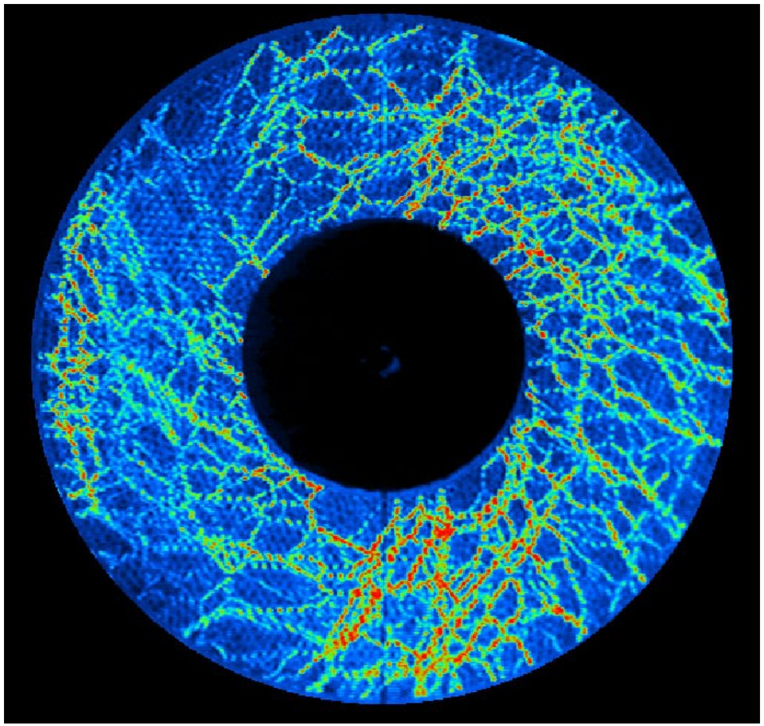
$$F_v(\dot{\theta}) = F_0 + \gamma[\dot{\theta} - 2v_0 \ln(1 + \dot{\theta}/v_0)]$$



Stress Chains in Granular Media

$$\frac{dF_r(\theta)}{d\theta} = -aF_r(\theta) + \eta(\theta)$$

$$\langle \eta(\theta)\eta(\theta')\rangle = D\,\delta(\theta-\theta')$$

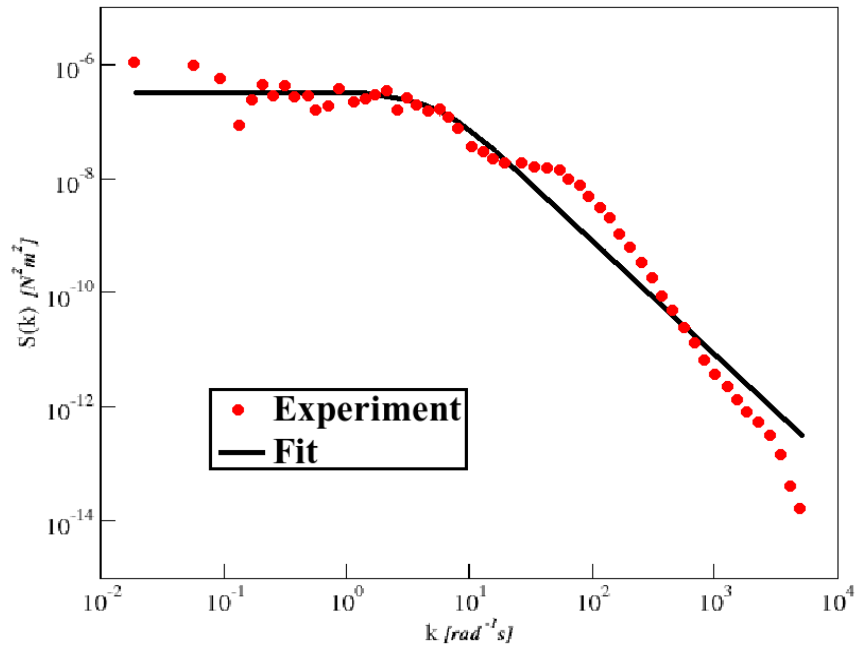


Howell at http://www.phy.duke.edu/~bob

Baldassarri et al., PRL 2006

Wiener-Kintchine theorem:

$$S(K) = \langle \left| \int d\theta\, F_r(\theta) e^{-i\theta K} \right|^2 \rangle = \frac{2D}{a^2+K^2}$$

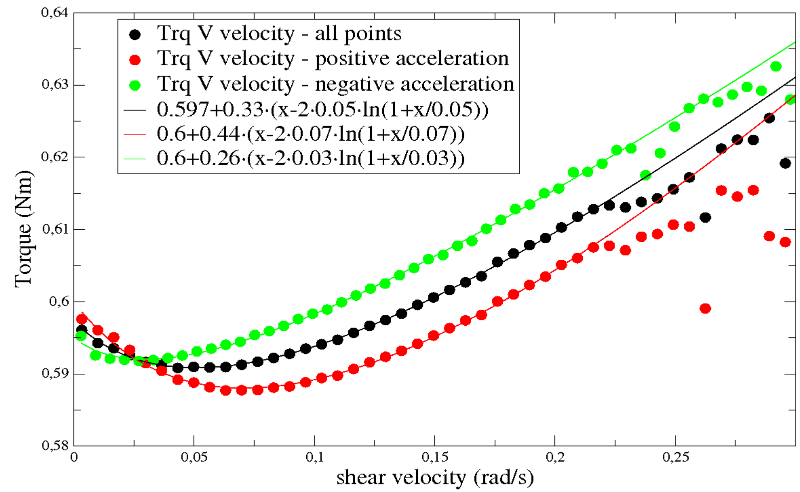


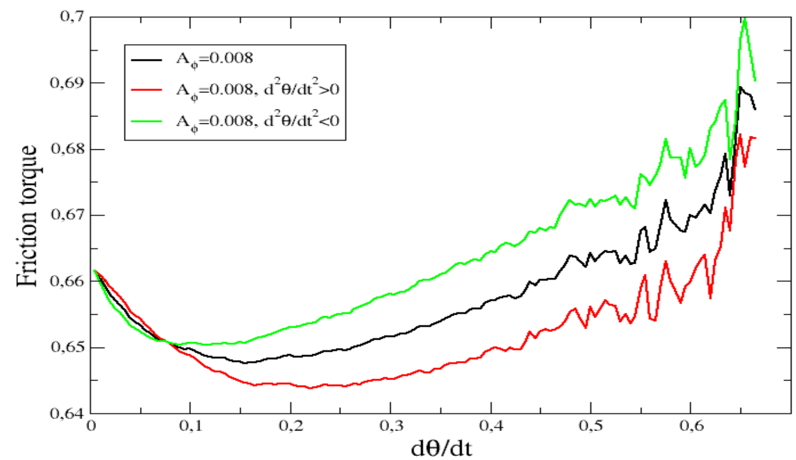
Rate and State friction in GM

$$F(t,\theta,\dot{\theta},\ldots)=F_v(\dot{\theta},\phi)+F_r(\theta)$$

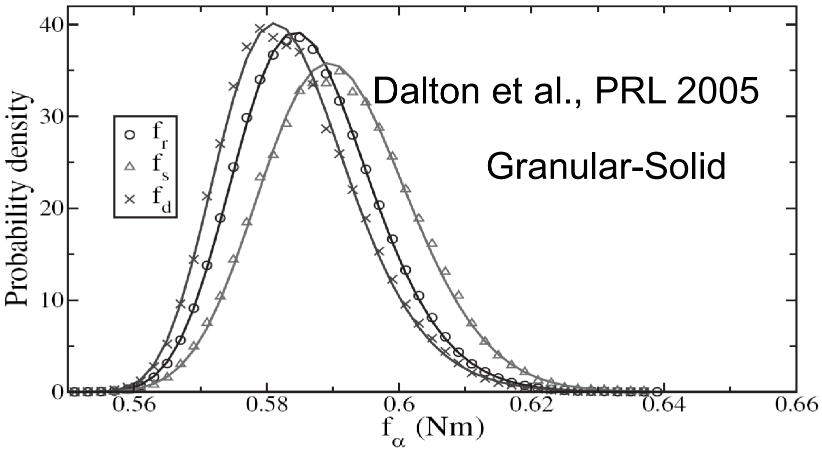
$$\begin{cases} F_v(\dot{\theta},\phi)=\left[1+\dfrac{\gamma}{A_0}\dot{\theta}\right]\left[A_0+A_\phi \ln\left(1+\dfrac{\phi}{\tau}\right)\right] \\ \dot{\phi}=1-\dfrac{\dot{\theta}\phi}{D_0} \end{cases}$$

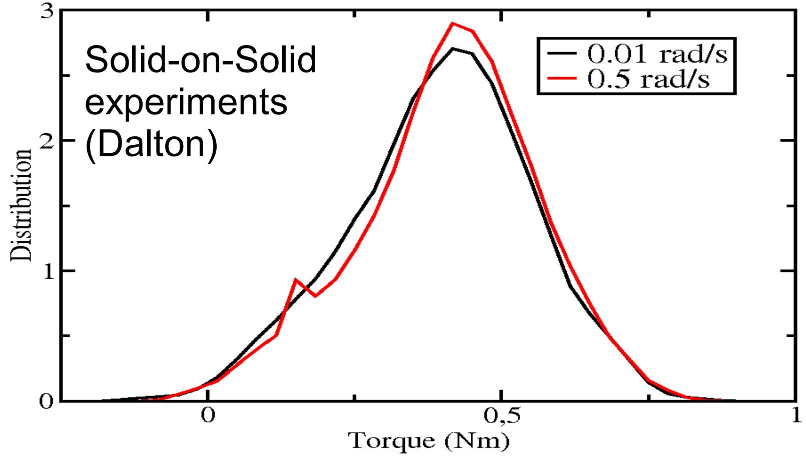
$$\begin{cases} \dfrac{dF_r(\theta)}{d\theta}=-aF_r(\theta)+\eta(\theta) \\ \langle \eta(\theta)\eta(\theta')\rangle=D\,\delta(\theta-\theta') \end{cases}$$



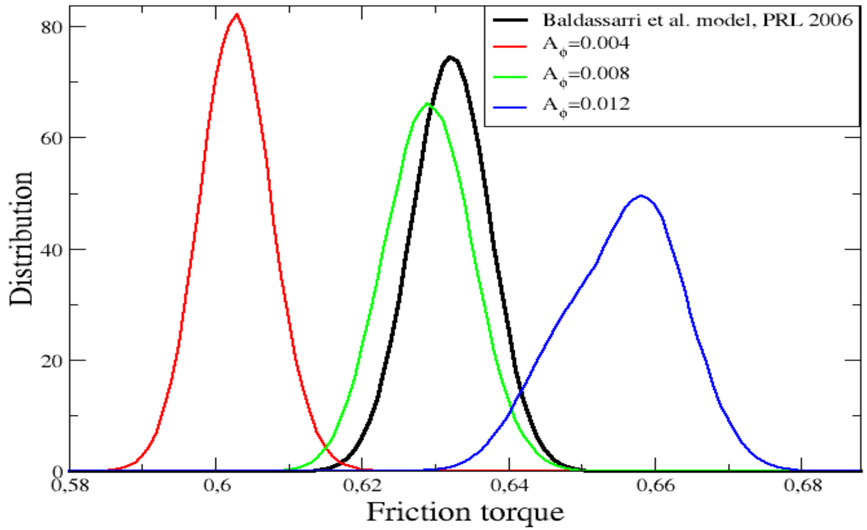


Friction Distribution Asymmetry





Increasing Aging effect: $A_\phi = 0.004 \rightarrow A_\phi = 0.008 \rightarrow A_\phi = 0.012$



Conclusions

- The role of disorder is essential to understand the microscopic mechanisms from which arise macroscopic properties
- Dislocation Dynamics and Sheared Granular Media can be viewed as Crackling systems

$$\sum_{k=-\infty}^{\infty} \frac{1}{x+kL} = \frac{\pi}{L}\cot(\pi x / L)$$