Competition between Superconductivity and Charge Density Waves: the Role of Disorder

PhD Student: **Alessandro Attanasi**[1]

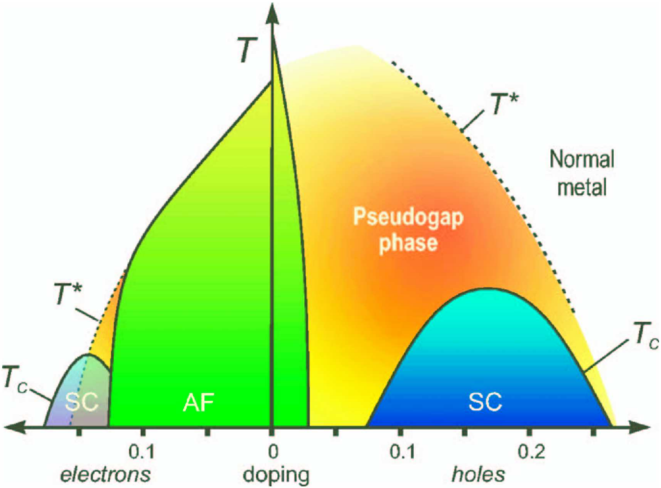
Supervisors: J. Lorenzana[2] A. Cavagna[2] C. Di Castro[1]

[1] Physics Department–University of Rome "Sapienza"

[2] Centre for Statistical Mechanics and Complexity–INFM and ISC–CNR

21 October 2008

Where Are We ?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electrons
doping
holes

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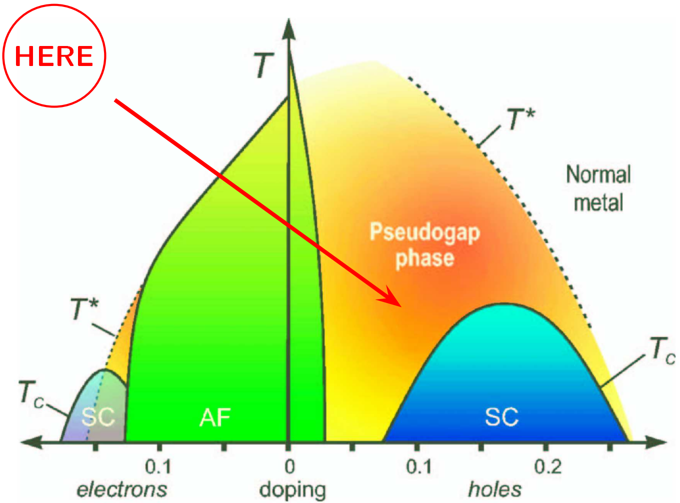
HERE
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electrons
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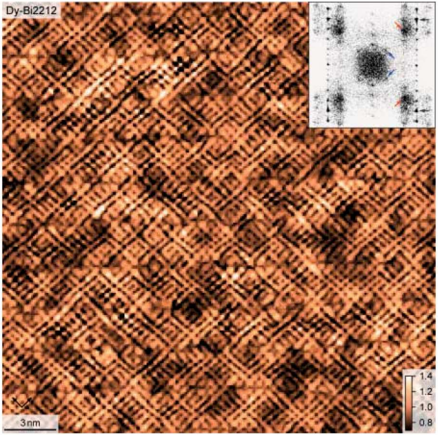
Just Two Important Experiments

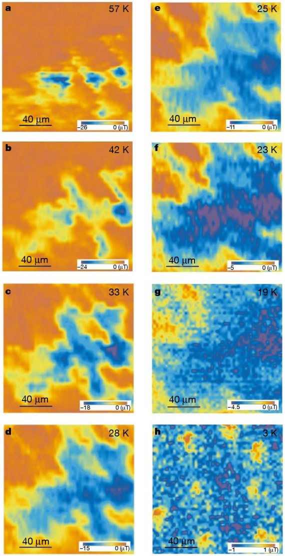
- Glassy-like CDW configurations



Kohsaka et al., Science, **315**, 2007

Just Two Important Experiments

- Diamagnetic effects above T_c



Iguchi et al, Nature, **412** 2001

Our Target

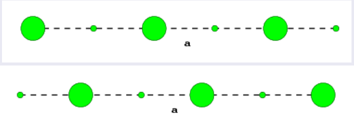
- Build a simplified model trying to explain the above experiments.

The main ingredients

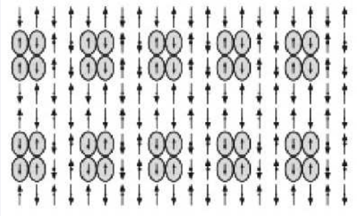
- An order parameter embodying CDW and SC
- A mechanism to produce disordered configurations

The CDW State

Two simple 1d CDW



A more complex stripe CDW



Translational and/or Rotational Symmetry Breaking

Complex CDW unit cell

More variants

The CDW State

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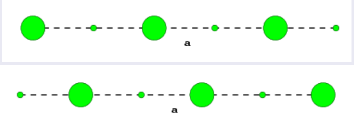

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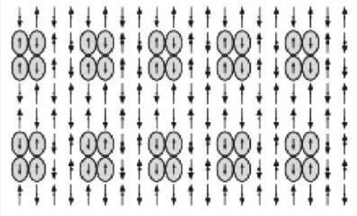


Translational and/or Rotational Symmetry Breaking

Complex CDW unit cell

More variants

A policristal/glass CDW

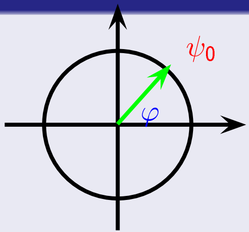
The SC State

The Ginzburg-Landau Functional

$$F[\psi] = \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m}\Big|\Big(-i\hbar\nabla - \frac{e^*\vec{A}}{c}\Big)\psi\Big|^2 + \frac{h^2}{8\pi}$$

The SC order parameter

$$\psi = \psi_0 e^{i\varphi}$$



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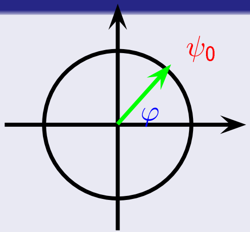
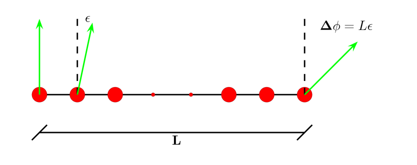


- Weak coupling $(\varphi = const) \implies T_c \propto \psi_0$
- Strong coupling $(\psi_0 = const) \implies T_c \propto \rho_s$

Stiffness ρ_s or Helicity Modulus Υ

$$\Upsilon \sim \frac{\partial^2 F}{\partial \Delta\Phi^2}\bigg|_{\Delta\Phi=0}$$



2d SC in strong coupling limit

$$F = -J_0 \int |\vec{\nabla}\phi|^2 d\,\vec{r} = -\frac{1}{2}\rho_s v_s^2 V$$

2d XY model

$$H = -J\sum_{\langle ij\rangle}(1 - \vec{S}_i \cdot \vec{S}_j) \approx -\frac{J}{2}\sum_{\langle ij\rangle}(\phi_i - \phi_j)^2$$

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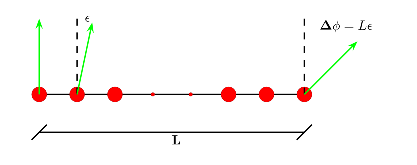

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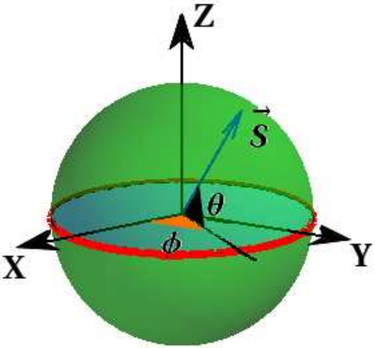
Kosterlitz and Thouless

$$\lim_{T \to T_c^-} \frac{\hbar^2}{m^2} \frac{\rho_s(T)}{K_B T} = \frac{2}{\pi}$$

Our Model

Anisotropic Random Field Heisenberg Model

$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - G \sum_i (S_i^z)^2 + \frac{W}{2} \sum_i h_i S_i^z$$



Our Model: the microscopic derivation (part-one)

Attractive Hubbard model ($U < 0$)

$$H = \sum_{<i,j>} \sum_{\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} - \frac{1}{2}|U| \sum_{i,\sigma} n_{i\sigma} n_{i,-\sigma}$$

Possible ground states at half-filling
in the strong coupling limit

CDW

$$|\uparrow\downarrow\rangle \quad |0\rangle \quad |\uparrow\downarrow\rangle \quad |0\rangle$$

or

$$|0\rangle \quad |\uparrow\downarrow\rangle \quad |0\rangle \quad |\uparrow\downarrow\rangle$$

SC

$$\begin{pmatrix} |0\rangle \\ + \\ e^{+i\varphi}|\uparrow\downarrow\rangle \end{pmatrix} \begin{pmatrix} |0\rangle \\ - \\ e^{+i\varphi}|\uparrow\downarrow\rangle \end{pmatrix}$$

Our Model: the microscopic derivation (part-two)

The Attraction-Repulsion Transformation

$$|\uparrow\downarrow\rangle \quad \Longrightarrow \quad |\uparrow\rangle$$
$$|0\rangle \quad \Longrightarrow \quad |\downarrow\rangle$$

CDW

$$|\uparrow\downarrow\rangle \quad |0\rangle \quad |\uparrow\downarrow\rangle \quad |0\rangle$$

z-AntiFerromagnet

$$|\uparrow\rangle \quad |\downarrow\rangle \quad |\uparrow\rangle \quad |\downarrow\rangle$$

SC

$$\left(\begin{array}{c} |0\rangle \\ + \\ e^{+i\varphi}|\uparrow\downarrow\rangle \end{array} \right) \left(\begin{array}{c} |0\rangle \\ - \\ e^{+i\varphi}|\uparrow\downarrow\rangle \end{array} \right)$$

xy-AntiFerromagnet

$$\nearrow \quad \swarrow \quad \nearrow \quad \swarrow$$

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

Our Model: the microscopic derivation (part-three)

- The order parameter is the staggered magnetization
- At long wave lengths the staggered magnetization behaves classically
- AntiFerromagnetic to Ferromagnetic model

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \qquad \Longrightarrow \qquad H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

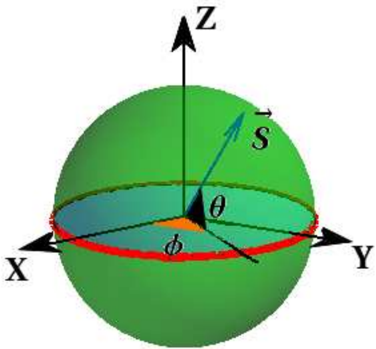
$$\uparrow \quad \downarrow \quad \uparrow \quad \downarrow \qquad\qquad\qquad \uparrow \quad \uparrow \quad \uparrow \quad \uparrow$$

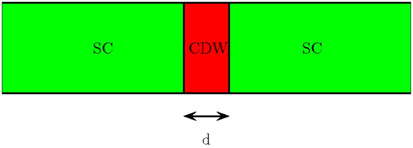
$$\nearrow \quad \swarrow \quad \nearrow \quad \swarrow \qquad\qquad\qquad \nearrow \quad \nearrow \quad \nearrow \quad \nearrow$$

Impurities favour locally one CDW variant

$$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - G \sum_i (S_i^z)^2 + \frac{W}{2} \sum_i h_i S_i^z$$

Josephson junction geometry

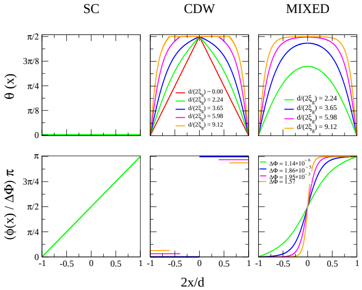


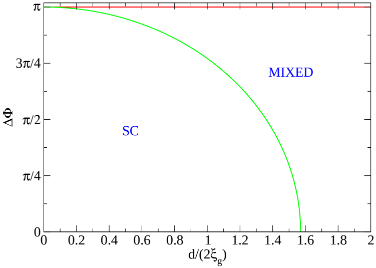


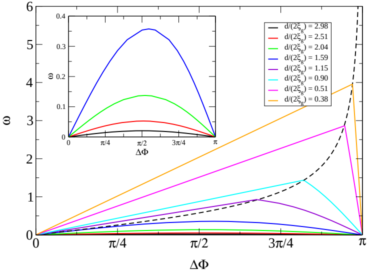
- Continuum limit, NO disorder, spherical coordinates

$$F[\theta(x), \phi(x)] = \int dx \left\{ \frac{\rho}{2} \left[\left(\frac{d\theta}{dx} \right)^2 + \cos^2\theta \left(\frac{d\phi}{dx} \right)^2 \right] - g \sin^2\theta \right\}$$

The Model in 1-dimension







$$I = \cos^2\theta \frac{d\phi}{dx} \equiv constant$$

$$\xi_g^2 \frac{d^2\theta}{dx^2} + \xi_g^2 I^2 \frac{\sin\theta}{\cos^3\theta} + \frac{1}{2}\sin(2\theta) = 0$$

$$\xi_g^2 = \frac{\rho}{2g}$$

The Model in 2-dimensions

Imry-Ma argument $\quad \text{RFIM} \implies d_l = 2$

$\text{RFHM} \implies d_l = 4$

Binder scale $\quad L^* \sim e^{(J/W)^2}$

Local SC—– in-plane magnetization

$$m_{xy} = \overline{\sqrt{\frac{1}{N}\sum_i^N \left[(S_x^i)^2 + (S_y^i)^2\right]}}$$

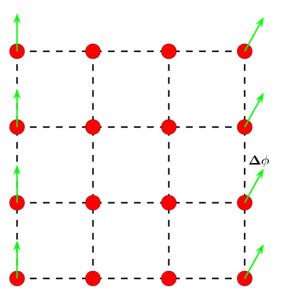
Global SC—— Stiffness

$$\Upsilon = ??$$

Stiffness Υ

$$H = -J \sum_{\langle i,j \rangle} \cos\theta_i \cos\theta_j \cos(\phi_i - \phi_j) - G \sum_i \sin^2\theta_i + \frac{W}{2} \sum_i h_i \sin\theta_i$$

$$H = -\sum_{\langle i,j \rangle} J_{ij} \cos(\phi_i - \phi_j)$$



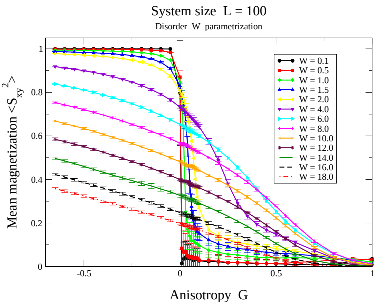
Random Conductance Network

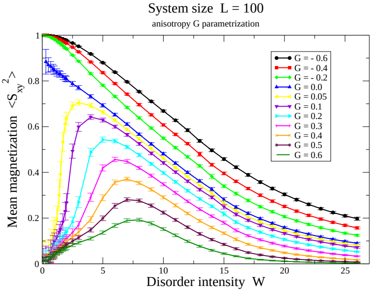
$$j_{ij} = J_{ij}(\phi_i - \phi_j)$$

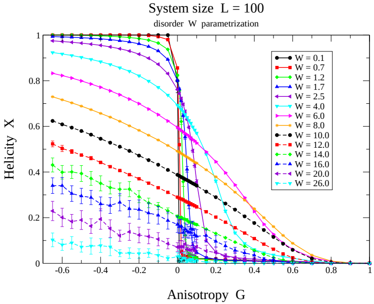
$$\sum_{k \in n.n.} j_{i,k} = 0 \qquad \forall\, i \in \mathcal{L}$$

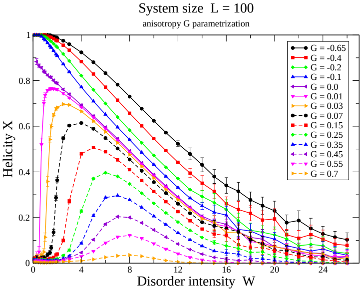
$$j_x^{TOT} = \Upsilon_x \Delta\Phi$$

Some results

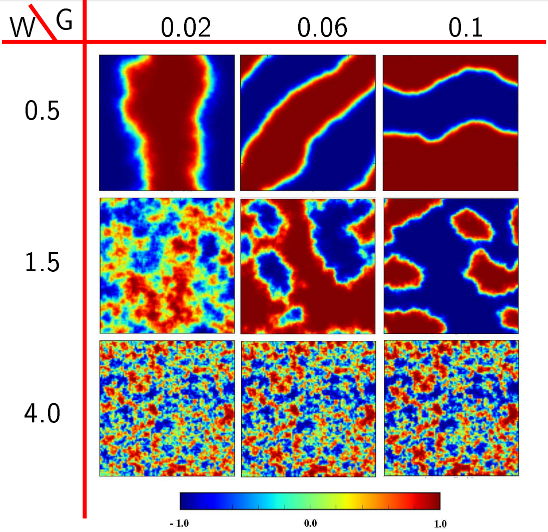






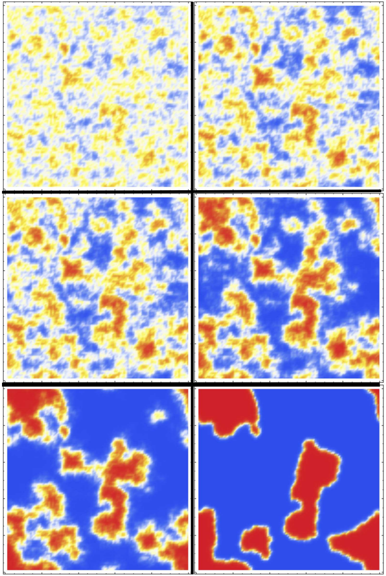


Some Configurations



Some Others Configurations

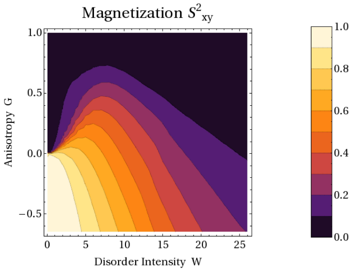
W=2.0

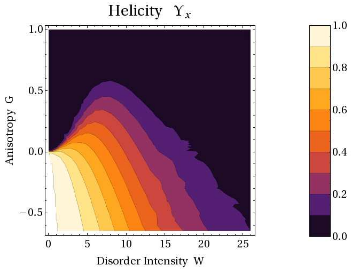


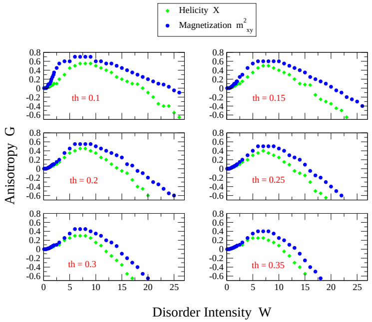
G

-0.1	0.0
0.03	0.07
0.1	0.4

A “Phase Diagram”







Conclusion

- Anomalous current-phase behaviour in 1d-Josephson junction
- Possible explanation of the Giant Proximity Effect
- Several orders give interestimg interplays. Starting from a CDW we can gain SC by the presence of disorder
- Global Stiffness *vs* Local Stiffness

Future Work

- Introduce Magnetic Field
- Introduce Temperature
- Study Percolating Properties