Weak response of nuclear matter

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Outline

- Motivations
- The neutrino-nucleus cross section within Nuclear Many Body Theory (NMBT)
- Charged current transitions at low momentum transfer ($\sim$ 10 MeV)
- Neutrino-nucleus cross section at $E_\nu \sim 1$ GeV
- Summary and Outlook

Motivations

- The weak nuclear response at $E_\nu \sim 1$ GeV is needed for the analysis of many neutrino oscillation experiments, which use nuclei as detectors

- Neutrino interactions with nuclear matter are critical in determining supernovæ evolution and neutron star cooling

Neutrino-nucleus cross section

- Charged current neutrino-nucleus interaction

$$\nu_\ell + A \to \ell^- + p + (A-1)$$

- Nuclear cross section

$$d\sigma_A \propto L_{\mu\nu} W_A^{\mu\nu}$$

$$L_{\mu\nu} = 8[k'_\mu k_\nu - (kk')g_{\mu\nu} + k'_\nu k_\mu - i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta]$$

$$W_A^{\mu\nu}(q) = \sum_n \langle 0 | J_A^{\mu\dagger}(q) | n \rangle \langle n | J_A^\nu(q) | 0 \rangle \delta^{(4)}(p_0 + q - p_n)$$

- Need to develop a consistent theoretical description of

$$|0\rangle \ \ , \ \ |n\rangle \ \ , \ \ J_A^\mu$$

- At low $\mathbf{q}(\sim 10$ MeV) calculations are feasable within non relativistic Nuclear Many Body Theory (NMBT)
- At large $\mathbf{q}(\sim 1$ GeV) Impulse Approximation

$$J^\mu \to \sum_i j_i^\mu \qquad , \qquad |f\rangle = |A-1\rangle \otimes |\mathbf{p}_f\rangle$$

Nuclear Many Body Theory

- Our starting point : the nuclear Hamiltonian

$$H = H_0 + V = \sum_i t_i + \sum_{i,j} v_{ij}$$

- v_{ij} phenomenological $\longrightarrow$ spin-isospin dependent, non central and strongly repulsive at short distance

- Problem: V cannot be treated as perturbation using the basis of eigenstates of H_0.

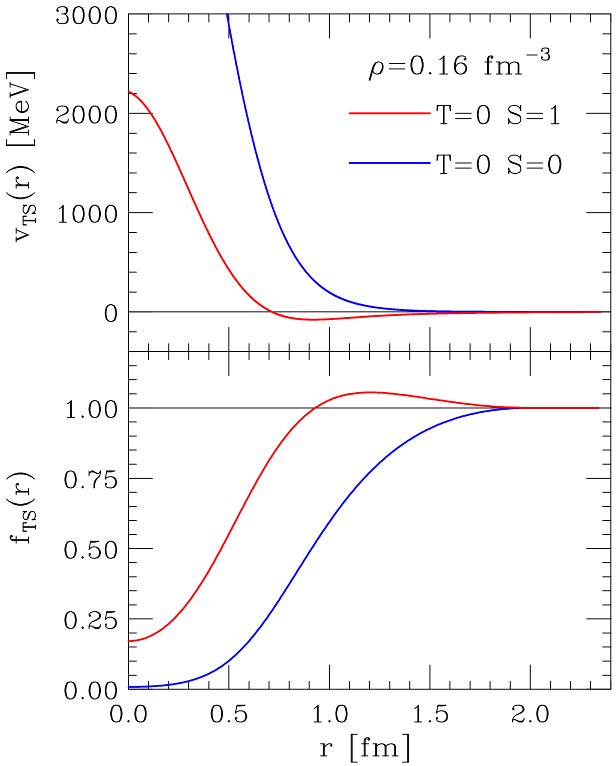
- Possible solution: Change basis states in such a way as to incorporate the non-perturbative effects of the interactions

Correlated Basis Formalism (CBF)

- Basis states are built from Fermi Gas states. In uniform nuclear matter

$$|n\rangle = F|n_{FG}\rangle = \mathcal{S}\prod_{j>i} f_{ij}|n_{FG}\rangle$$

- The shape of f_{ij} reflects the structure of the interaction



- At long distances, v_{ij} goes to zero $\longrightarrow$ f_{ij} goes to 1

- At short relative distances, in $T = 0, S = 0$ channel, v_{ij} is very large $\longrightarrow$ f_{ij} tends to zero

The Effective Interaction

- In principle one can define an effective interaction

$$\langle n_{FG}|F^\dagger H F|n_{FG}\rangle = E_0 + \langle n_{FG}|V_{eff}|n_{FG}\rangle$$

$$H_0|n_{FG}\rangle = E_0|n_{FG}\rangle$$

- However there is a price to pay going from FG to CBF. Calculation of matrix elements with correlated states is not feasable for $A \gtrsim 10$
- Cluster Expansion

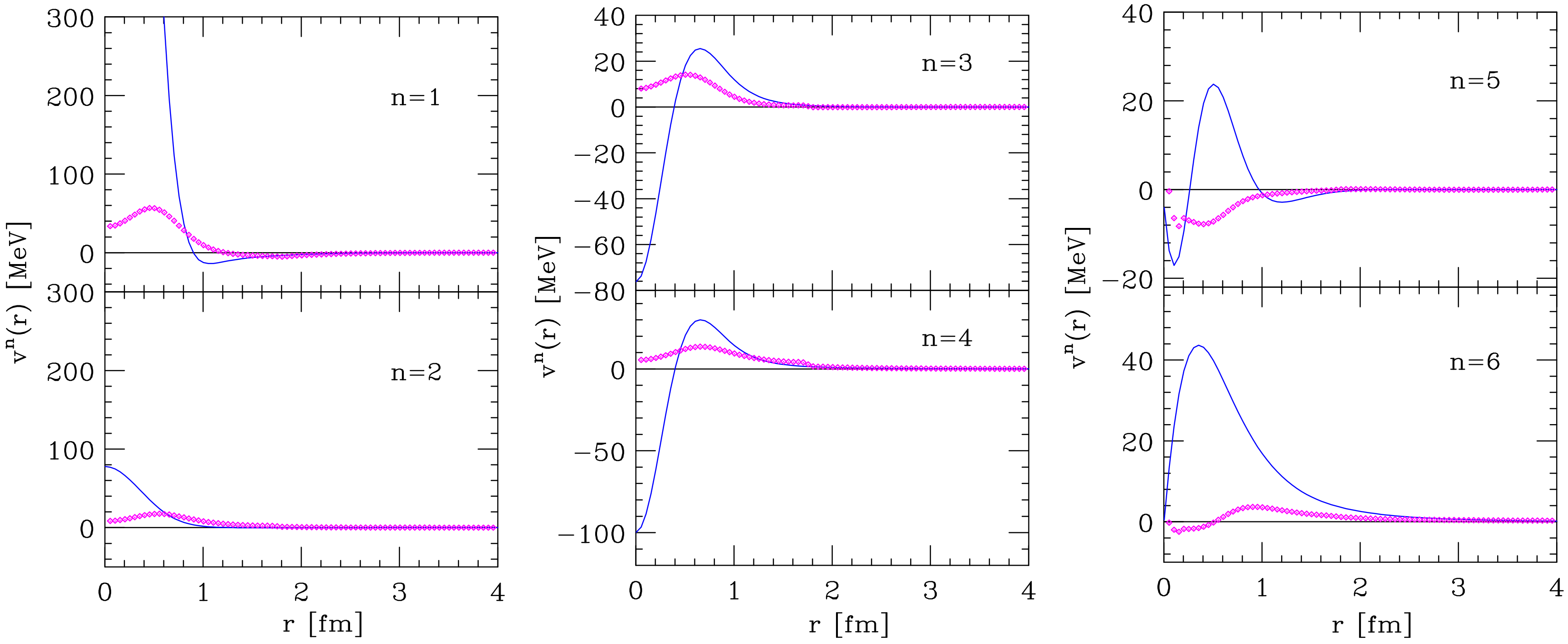
$$M_n = \langle n_{FG}|F^\dagger O F|0_{FG}\rangle = O_0 + \sum_n (\Delta O)_n$$

$$O_0 = \langle n_{FG}|O|0_{FG}\rangle$$

- For example

$$V_{eff} = \sum_{j>i} f^\dagger_{ij}\left[-\frac{1}{m}\sum_i \left(\nabla^2 f_{ij} + 2\boldsymbol{\nabla} f_{ij}\cdot\boldsymbol{\nabla}\right) + v_{ij} f_{ij}\right]$$

Effective interaction versus bare interaction



The effective interaction does not have a strong repulsive core → perturbation theory is applicable.

Low |q| regime: the non relativistic limit

- The hadronic current has vector and axial components

$$J^\mu = J^\mu_V + J^\mu_A$$

- In the non-relativistic limit, one expands the hadronic current matrix elements in powers of $|\mathbf{q}|/m$ and keeps the zeroth order terms

$$\langle p'|J^0_V|p\rangle \sim \chi_{\sigma'}\eta_{\tau'}O^i_F\chi_\sigma\eta_\tau \quad , \quad O^i_F(\mathbf{q}) = g_V e^{i\mathbf{q}\mathbf{r}_i}\tau^+_i$$

$$\langle p'|J^i_A|p\rangle \sim \chi_{\sigma'}\eta_{\tau'}\mathbf{O}^i_{GT}\chi_\sigma\eta_\tau \quad , \quad \mathbf{O}^i_{GT}(\mathbf{q}) = g_A e^{i\mathbf{q}\mathbf{r}_i}\sigma_i\tau^+_i$$

- The nuclear response becomes

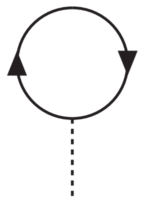
$$S_F(\mathbf{q},\omega) = \frac{1}{N}\sum_n |\langle 0|O_F(\mathbf{q})|n\rangle|^2\delta(\omega + E_0 - E_n)$$

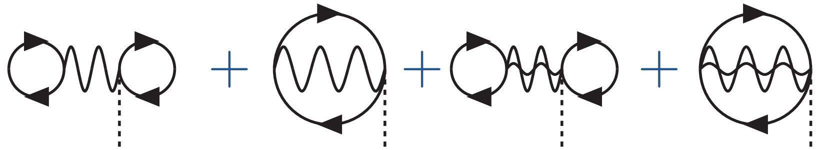
$$\mathbf{S_{GT}}(\mathbf{q},\omega) = \frac{1}{N}\sum_n |\langle 0|\mathbf{O_{GT}}(\mathbf{q})|n\rangle|^2\delta(\omega + E_0 - E_n)$$

Short range correlations

- Short range correlations affect the transition probabilities through modifications of both the initial and final states and the single particle spectrum: e.g. $|ph\rangle \rightarrow F|ph\rangle$, $\delta(\omega + E_0 - E_n) \rightarrow \delta(\omega + e_{\mathbf{h}} - e_{\mathbf{p}})$
- Weak matrix element at the two-body level

$$M_{ph} = M_{ph}^{FG} + \Delta M_{ph}$$

$M_{ph}^{FG} =$ 

$\Delta M_{ph} =$  $+ 1 \leftrightarrow 2$

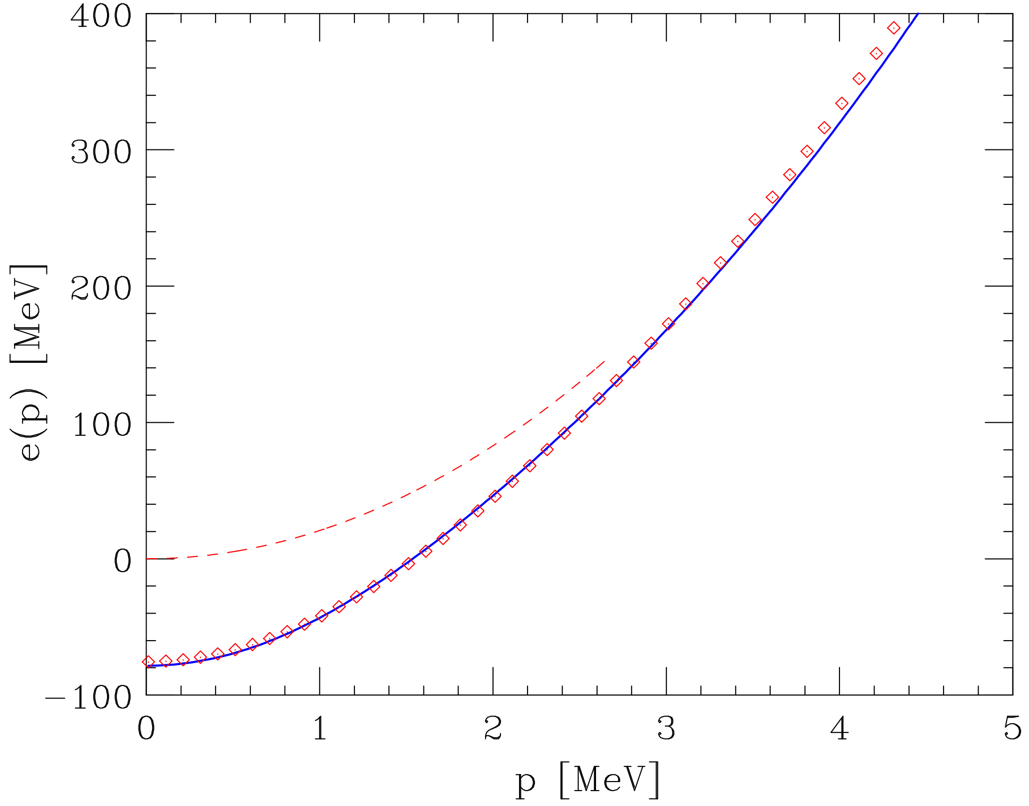
- Effective operator

$$M_{ph} = \langle ph_{FG}|O^{eff}| = |0_{FG}\rangle$$

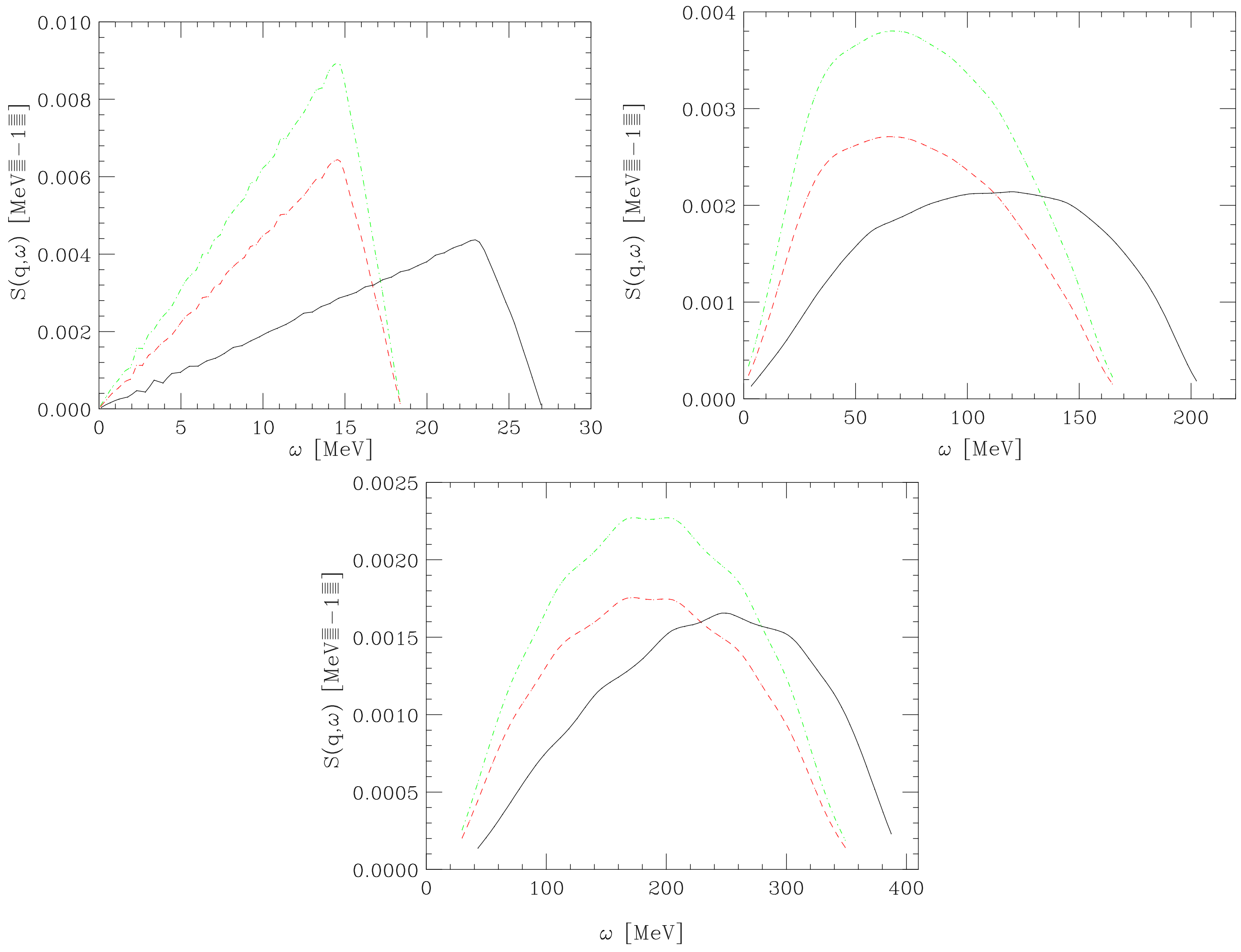
Energy spectrum in nuclear matter

- The single particle spectrum is defined through the effective interaction in Hartree-Fock approximation

$$e_{\mathbf{p}} = \frac{\mathbf{p}^2}{2m} + \sum_{\alpha} \langle \alpha p | V_{eff} | \alpha p \rangle_a$$



Results



- Nuclear response for Fermi transition, at $|\mathbf{q}| = .3 \text{ fm}^{-1}$ (upper left), $|\mathbf{q}| = 1.8 \text{ fm}^{-1}$ (upper right), and $|\mathbf{q}| = 3 \text{ fm}^{-1}$ (down). Calculations carried out on a discrete set of $\sim$ 10000 one particle-one hole states.

Long Range Correlations

- Long range correlations are also important, (indeed dominant at very low $|\mathbf{q}|$), and need to be included
- Tamm Dancoff Approximation (TDA)

$$|ph\rangle = \sum_{i}^{N} c_i |p_i, h_i\rangle$$

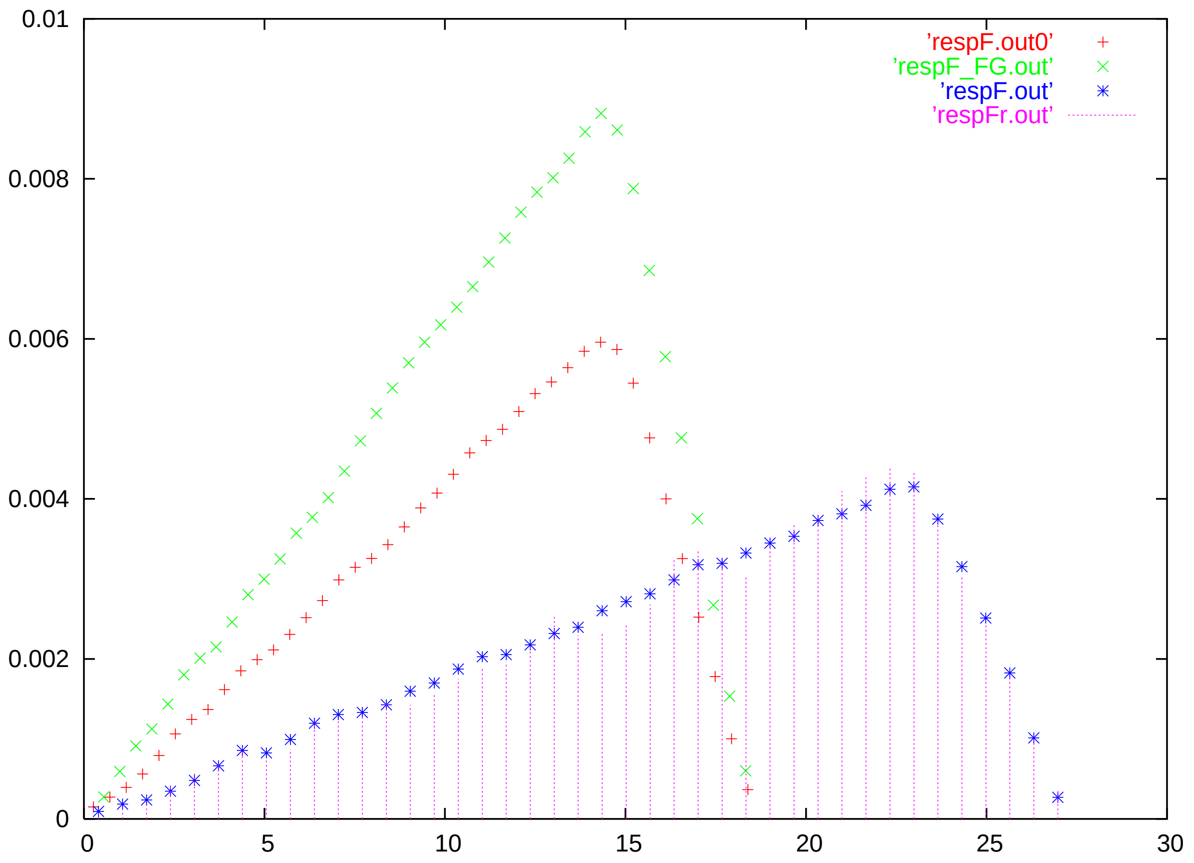
Diagonalization of H leads to

$$\sum_j [(e_{p_i} - e_{h_i})\delta_{ij} + \langle p_j, h_j|\widetilde{v}|p_i, h_i\rangle] c_j = \omega c_i$$

We obtain N eignvalues ω_n and response reads

$$S_{TDA}(\mathbf{q}, \omega) = \sum_n |\sum_i c_i \langle p_i, h_i|O^{eff}|0\rangle|^2 \delta(\omega - \omega_n)$$

Response in TDA



Preliminary, direct part only. $\sim$ 3000 basis states.

Energy regime of neutrino oscillation experiments

- Quantum mechanical phase difference developed by two neutrino mass eigenstates

$$\Delta\phi_{jk} = (E_k - E_j)L \sim \frac{\Delta m^2_{jk}}{2E_\nu} L$$

$$\Delta m^2_{12} \sim 6.9 \times 10^{-5}\mathrm{eV}^2 \;, \Delta m^2_{23} \sim 2.5 \times 10^{-3}\mathrm{eV}^2$$

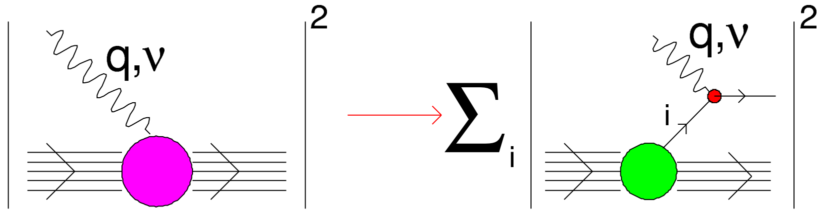
- ideal E_ν, once Δm^2_{23} is known dictated by

$$E_\nu = 0.60 [\frac{\Delta m^2_{23}}{3 \times 10^{-3}\mathrm{eV}^2}][\frac{L}{150\mathrm{Km}}]\mathrm{GeV}$$

- natural choice: $E_\nu \sim$ few GeV: the impulsive regime

The impulsive Regime

- In the Impulse Approximation scheme (IA) the scattering process off a nucleus reduces to the incoherent sum of the elementary processes involving individual nucleons:



The doubly differential cross section in IA is

$$\frac{d\sigma_A}{d\Omega_{\nu'} dE_{\nu'}} = \int d^4p \, P(p) \left(\frac{d\sigma_N}{d\Omega_{\nu'} dE_{\nu'}} \right)$$

- The first element of IA cross section calculation is the elementary $\nu + N \rightarrow \nu' + X$ cross section:

$$\frac{d\sigma_{\nu N}}{d\Omega_{\nu'} dE_{\nu'}} \propto L_{\mu\nu} w_N^{\mu\nu}$$

- The second is the target spectral function $P(\mathbf{p}, E)$: probability of removing a nucleon of momentum $\mathbf{p}$ from the target, leaving the residual spectator system with excitation energy E

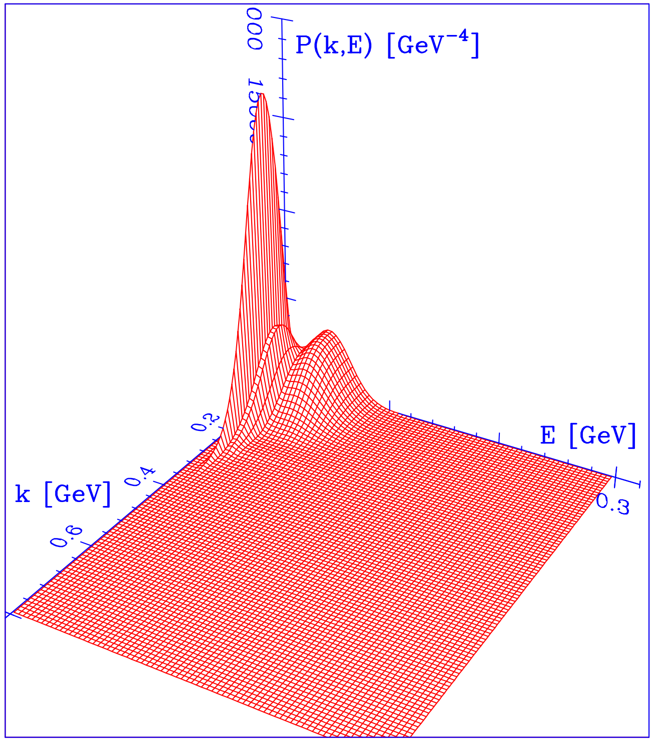
The Spectral Function

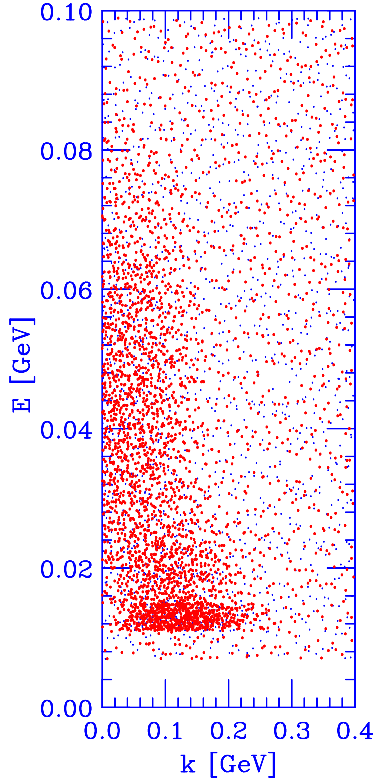
- Within IA, the final state can be written as $|f\rangle = |A-1\rangle \otimes |\mathbf{p}_f\rangle$: relativity only affects the fast struck particle
- The Spectral Function definition involves only properties of the target

$$P(\mathbf{p}, E) = \sum_n |\langle n_{(A-1)}(-\mathbf{p})|a_{\mathbf{p}}|0_A\rangle|^2 \delta(E - E_n + E_0)$$

  A non-relativistic calculation within NMBT, which does not involve adjustable parameters, is possible
- Exact or highly accurate calculations are feasible for A=2,3,4 and ∞
- Spectral functions for Carbon, Oxygen, Iron and Gold can be obtained using CBF calculations and electron scattering data

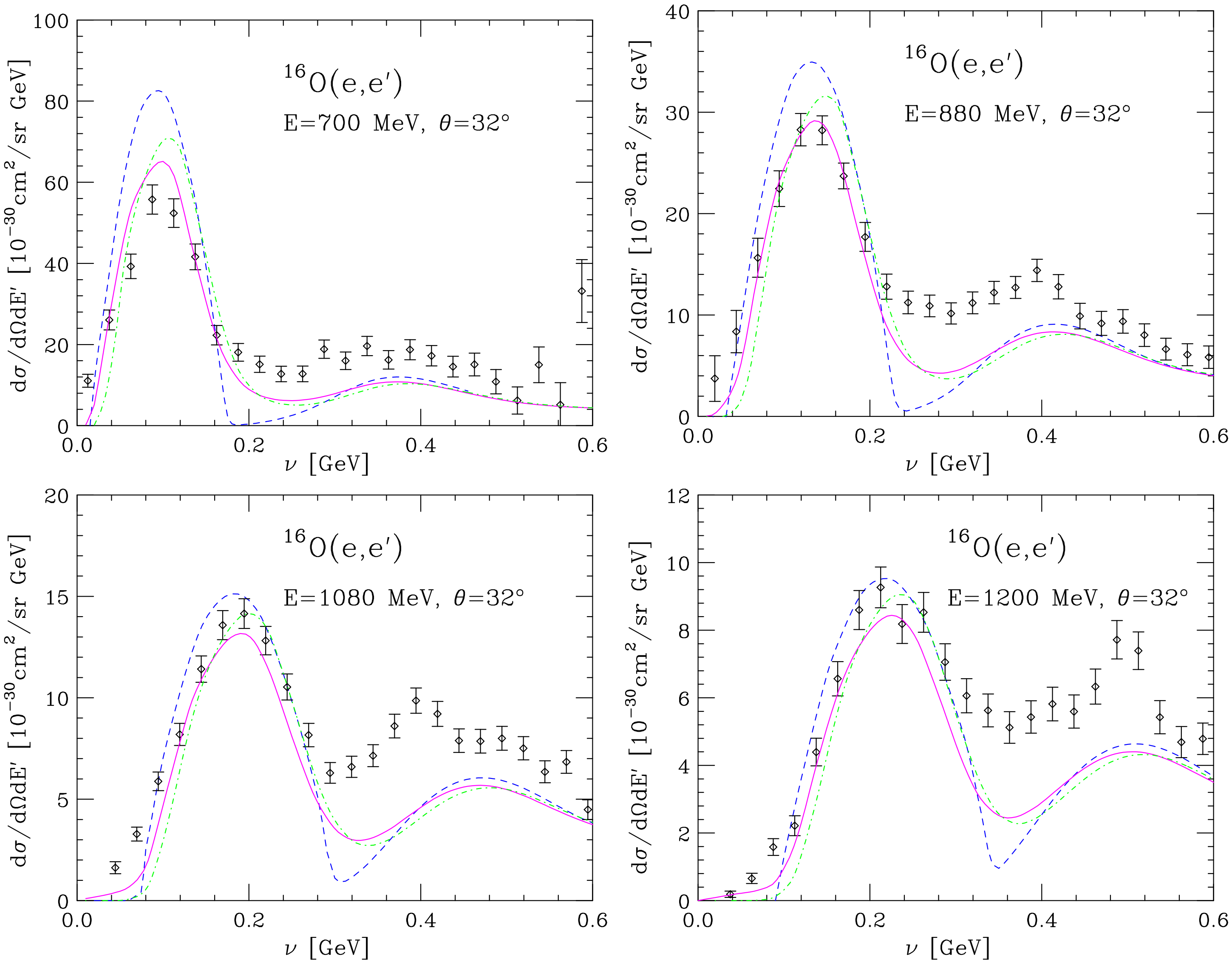
$$P(\mathbf{p}, E) = P_{MF}(\mathbf{p}, E) + P_{corr}(\mathbf{p}, E)$$



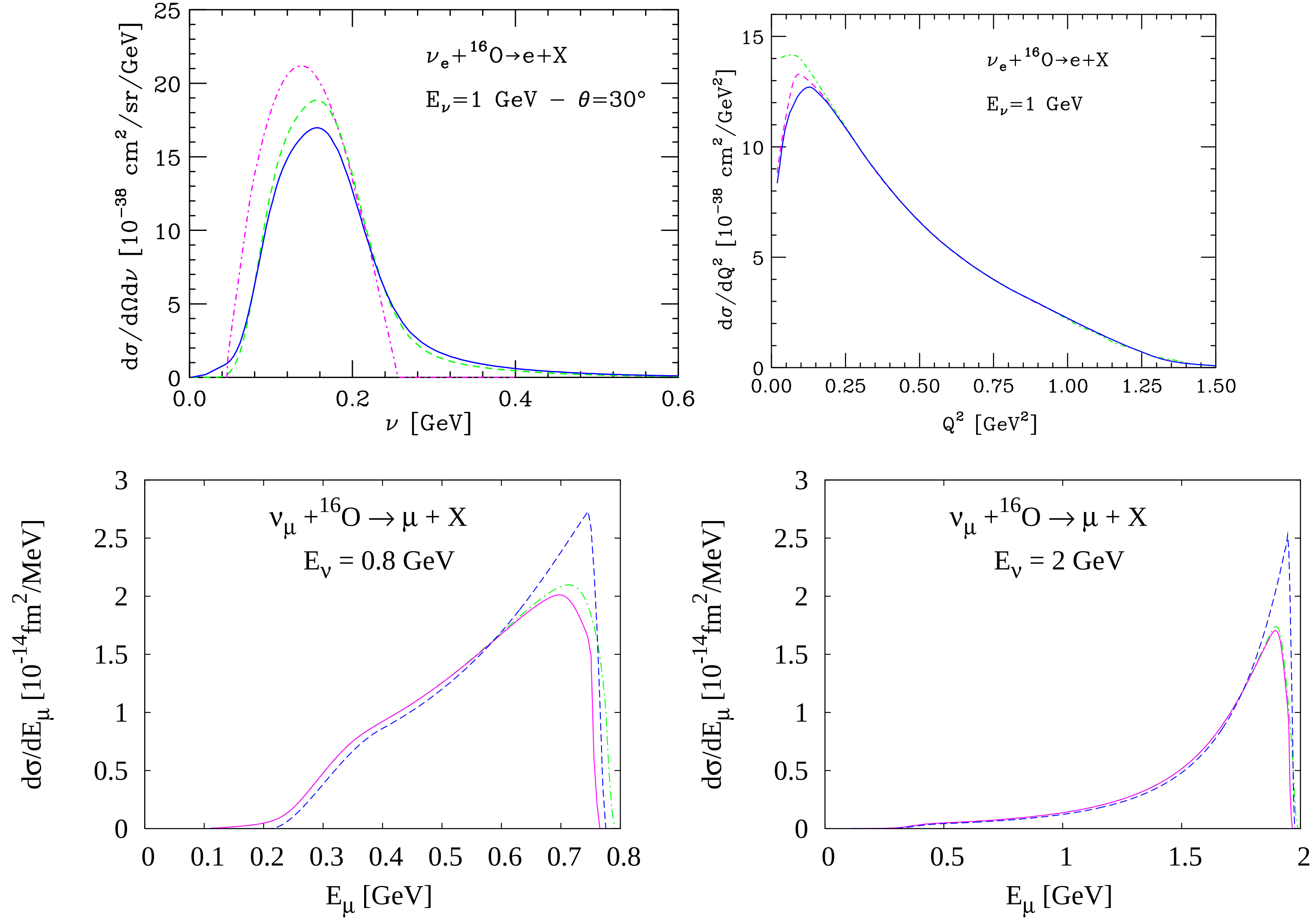


- The shell model contribution $P_{MF}(\mathbf{p}, E)$ accounts for $\sim$ 80% of the strenght
- The remaining $\sim$ 20%, accounted for by $P_{corr}(\mathbf{p}, E)$, is located at high momentum *and* large removal energy ($\mathbf{p} \gg p_F, E \gg e_F$)

Comparison to Frascati ^{16}O (e, e') data



Results for $^{16}O\ (\nu_e, e)$ scattering



Summary and Outlook

- We have devoloped a formalism allowing for a unified description of the weak nuclear response at low and high momentum transfer
- The results at low $\mathbf{q}$ show large correlation effects leading to an increase of the neutrino mean free path
- The results at large $\mathbf{q}$ indicate that the calculated cross sections are reaching the level of accuracy needed for the analysis of neutrino oscillation experiments
- Forthcoming extensions include the calculation of Gamow Teller and neutral current transition
- The formalism can be readily generalized to include finite temperature effects (at low $\mathbf{q}$) and deep inelastic contributions (at large $\mathbf{q}$)