Gravity-related spontaneous collapse in bulk matter

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Schrödinger Cats, Catness

Well-defined spatial mass distributions f_1, f_2

$$|\mathrm{Cat}\rangle = \frac{|f_1\rangle + |f_2\rangle}{\sqrt{2}}$$

Catness: squared-distance $\ell^2(f_1, f_2)$ [dim: energy]
Standard QM: Cat collapses immediately if we measure f
In "new" QM: we postulate spontaneous collapse

$$|\mathrm{Cat}\rangle \Longrightarrow \text{ either } |f_1\rangle \text{ or } |f_2\rangle \text{ with collapse rate } \ell^2/\hbar$$

Testable consequence: spontaneous decoherence (of $\hat{\rho}$)

$$|\mathrm{Cat}\rangle\langle\mathrm{Cat}| \Longrightarrow \frac{1}{2}|f_1\rangle\langle f_1| + \frac{1}{2}|f_2\rangle\langle f_2| \text{ with decoherence rate } \ell^2/\hbar$$

I discuss spontaneous decoherence (collapse would come easily).

Different catness in CSL and DP

$$\ell^2(f_1,f_2) = C_{11} + C_{22} - 2C_{12}$$

$$CSL:\ \ C_{ij} = \Lambda\int f_i(\mathbf{r})f_j(\mathbf{r})\mathrm{d}\mathbf{r} \qquad DP:\ \ C_{ij} = G\int\int f_i(\mathbf{r}_1)f_j(\mathbf{r}_2)\frac{\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2}{r_{12}}$$

Spatial cut-off σ is needed (by Gaussian g_σ of width σ):

$$f(\mathbf{r}) = m\sum_a g_\sigma(\mathbf{r}-\mathbf{x}_a)$$

$$CSL:\ \ \sigma = 10^{-5}cm, \quad D(P):\ \ \sigma = 10^{-12}cm$$

- DP: 'nuclear' σ, weak G; CSL: 'macroscopic' σ, strong Λ
- DP and CSL: same (similar) collapse for c.o.m. of a bulk
- DP: too much spontaneous heating, CSL: tolerable heating
- DP: significance for acoustic modes, CSL: no significance
- DP: large scale dominance; CSL: -

Master equation of spontaneous decoherence

$$\frac{d\hat{\rho}}{dt} = -\frac{\mathrm{i}}{\hbar}[\hat{H},\hat{\rho}] + \mathcal{D}\hat{\rho}$$

Key quantity: $\hat{f}(\mathbf{r}) = m\sum_a g_\sigma(\mathbf{r}-\hat{\mathbf{x}}_a)$
Dynamics of $\hat{\rho}$'s diagonalization in f at rate $\ell^2/\hbar$:

$$\mathcal{D}\hat{\rho} = -\frac{G}{2\hbar}\iint[\hat{f}(\mathbf{r}_1),[\hat{f}(\mathbf{r}_2),\hat{\rho}]]\frac{\mathrm{d}\mathbf{r}_1\mathrm{d}\mathbf{r}_2}{r_{12}}$$

$$\left[\ CSL : \mathcal{D}\hat{\rho} = -\frac{\Lambda}{2\hbar}\int[\hat{f}(\mathbf{r}),[\hat{f}(\mathbf{r}),\hat{\rho}]]\mathrm{d}\mathbf{r}\ \right]$$

Useful detailed Fourier form:

$$\mathcal{D}\hat{\rho} = -\frac{Gm^2}{2\hbar}\int\frac{4\pi\mathrm{e}^{-\mathbf{k}^2\sigma^2}}{k^2}\sum_{a,b}\left[\mathrm{e}^{\mathrm{i}\mathbf{k}\hat{\mathbf{x}}_a},\left[\mathrm{e}^{-\mathrm{i}\mathbf{k}\hat{\mathbf{x}}_b},\hat{\rho}\right]\right]\frac{\mathrm{d}\mathbf{k}}{(2\pi)^3}$$

Decoherence of acoustic d.o.f.

Elasto-hydrodynamics (acoustics) in homogeneous bulk
Displacement field $\hat{\mathbf{u}}(\mathbf{r})$, canonically conj. momentum field $\hat{\boldsymbol{\pi}}(\mathbf{r})$:

$$\hat{H} = \int \left(\frac{1}{2f^0} \hat{\boldsymbol{\pi}}^2 + \frac{f^0}{2} c_\ell^2 (\nabla \hat{\mathbf{u}})^2 \right) \mathrm{d}\mathbf{r},$$

$f^0 = M/V$ is mass density; c_ℓ is (longitudinal) sound velocity.
Recall $\mathcal{D}$, insert $\hat{\mathbf{x}}_a = \bar{\mathbf{x}}_a + \hat{\mathbf{u}}\big(\bar{\mathbf{x}}_a\big)$; $\bar{\mathbf{x}}_a$ are fiducial positions.
Assume $\hat{\mathbf{u}}(\mathbf{r}) \ll \sigma$, $\exp[\mathrm{i}\mathbf{k}\hat{\mathbf{u}}(\bar{\mathbf{x}}_a)] \approx 1 + \mathrm{i}\mathbf{k}\hat{\mathbf{u}}(\bar{\mathbf{x}}_a)$; etc.

$$\mathcal{D}\hat{\rho} = -\frac{1}{2\hbar} f^0 (\omega_G^{\mathrm{nucl}})^2 \int [\hat{\mathbf{u}}(\mathbf{r}), [\hat{\mathbf{u}}(\mathbf{r}), \hat{\rho}]] \mathrm{d}\mathbf{r}.$$

$$\omega_G^{\mathrm{nucl}} = \sqrt{G f^{\mathrm{nucl}}} \sim 1\mathrm{kHz}$$

i.e.: frequency of Newton oscillator in density

$$f^{\mathrm{nucl}} = m/(4\pi\sigma^2)^{3/2} \sim 10^{12} \mathrm{g/cm}^3$$

Decoherence of acoustic modes

Fourier modes in rectangular bulk:

$$\hat{\mathbf{u}}(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} \hat{\mathbf{u}}_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\mathbf{kr}}, \qquad \hat{\boldsymbol{\pi}}(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} \hat{\boldsymbol{\pi}}_{\mathbf{k}} \mathrm{e}^{\mathrm{i}\mathbf{kr}}$$

Hamiltonian and decoherence:

$$\hat{H} = \frac{1}{2}\sum_{\mathbf{k}}\left(\frac{1}{f^0}\hat{\boldsymbol{\pi}}_{\mathbf{k}}^\dagger \hat{\boldsymbol{\pi}}_{\mathbf{k}} + f^0 c_\ell^2 k^2 \hat{\mathbf{u}}_{\mathbf{k}}^\dagger \hat{\mathbf{u}}_{\mathbf{k}}\right), \quad \mathcal{D}\hat{\rho} = \frac{-1}{2\hbar}\sum_{\mathbf{k}} f^0 (\omega_G^{\mathrm{nucl}})^2 [\hat{\mathbf{u}}_{\mathbf{k}}^\dagger, [\hat{\mathbf{u}}_{\mathbf{k}}, \hat{\rho}]]$$

Master equation of acoustic modes spontaneous decoherence:

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = \frac{1}{2\hbar}\sum_{\mathbf{k}}\left(\frac{-\mathrm{i}}{f^0}[\hat{\boldsymbol{\pi}}_{\mathbf{k}}^\dagger \hat{\boldsymbol{\pi}}_{\mathbf{k}}, \hat{\rho}] - \mathrm{i} f^0 c_\ell^2 k^2 [\hat{\mathbf{u}}_{\mathbf{k}}^\dagger \hat{\mathbf{u}}_{\mathbf{k}}, \hat{\rho}] - f^0 (\omega_G^{\mathrm{nucl}})^2 [\hat{\mathbf{u}}_{\mathbf{k}}^\dagger, [\hat{\mathbf{u}}_{\mathbf{k}}, \hat{\rho}]]\right)$$

Recall: summation over acoustic wave numbers $\mathbf{k}$.
Note: CSL would have $\mathcal{D}\hat{\rho} \sim \sum_{\mathbf{k}} k^2 [\hat{\mathbf{u}}_{\mathbf{k}}^\dagger, [\hat{\mathbf{u}}_{\mathbf{k}}, \hat{\rho}]]$.

Center of mass decoherence

C.o.m.dynamics: $\mathbf{k}=0$ acoustic mode

$$\hat{\mathbf{X}}=\frac{1}{\sqrt{V}}\hat{\mathbf{u}}_{\mathbf{0}},\qquad \hat{\mathbf{P}}=\sqrt{V}\hat{\boldsymbol{\pi}}_{\mathbf{0}}$$

(we set the fiducial c.o.m. to the origin)
Identify c.o.m. part in master equation:

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t}=\frac{1}{2\hbar}\sum_{\mathbf{k}}\left(\frac{-\mathrm{i}}{f^0}[\hat{\boldsymbol{\pi}}_{\mathbf{k}}^\dagger\hat{\boldsymbol{\pi}}_{\mathbf{k}},\hat{\rho}]-\mathrm{i}f^0c_\ell^2k^2[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]-f^0(\omega_G^{\mathrm{nucl}})^2[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger,[\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]]\right)$$

Get closed master equation for c.o.m.:

$$\frac{\mathrm{d}\hat{\rho}_{\mathrm{c.o.m.}}}{\mathrm{d}t}=\frac{-\mathrm{i}}{\hbar}\left[\frac{\hat{\mathbf{P}}^2}{2M},\hat{\rho}_{\mathrm{c.o.m.}}\right]-\frac{1}{2\hbar}M(\omega_G^{\mathrm{nucl}})^2[\hat{\mathbf{X}},[\hat{\mathbf{X}},\hat{\rho}_{\mathrm{c.o.m.}}]],$$

Full accordance with old derivations in DP-model.
Compare it to richness of acoustic mode spontaneous decoherence!

Universal dominance of spontaneous decoherence

Inspect long wavelength feature of master equation:

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t}=\frac{1}{2\hbar}\sum_{\mathbf{k}}\left(\frac{-\mathrm{i}}{f^0}[\hat{\boldsymbol{\pi}}_{\mathbf{k}}^\dagger\hat{\boldsymbol{\pi}}_{\mathbf{k}},\hat{\rho}]-\mathrm{i}f^0c_\ell^2k^2[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]-f^0(\omega_G^{\mathrm{nucl}})^2[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger,[\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]]\right)$$

Harmonic potential and decoherence terms: quadratic in $\hat{\mathbf{u}}_{\mathbf{k}}$.

Although structures are different, they compete, decoherence wins if:

$$c_\ell k\ll\omega_G^{\mathrm{nucl}}\sim 1\mathrm{kHz}\qquad\Longrightarrow 1/k\gg 1\mathrm{m}\ \text{(e.g. in solids)}$$

The master equation for these modes:

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t}=\frac{1}{2\hbar}\sum_{1/k\gg 1m}\left(\frac{-\mathrm{i}}{f^0}[\hat{\boldsymbol{\pi}}_{\mathbf{k}}^\dagger\hat{\boldsymbol{\pi}}_{\mathbf{k}},\hat{\rho}]-f^0(\omega_G^{\mathrm{nucl}})^2[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger,[\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]]\right).$$

Wavelength $\gg$1m: 'free motion' plus spontaneous decoherence.
Example: Bulk of rock as big as 100m, sub-volume about a few m's

$\Longrightarrow$ C.o.m. moves and decoheres like free-body.

Strong spontaneous decoherence at low heating

Side-effect of spontaneous decoherence: spontaneous warming up:

$$\frac{\mathrm{d}\hat{H}}{dt} = \mathcal{D}\hat{H} = N \times \dot{\epsilon} \qquad (N : \text{number of d.o.f.})$$

For a single acoustic mode $\hat{u}_{j\mathbf{k}} \equiv \hat{u}, \hat{\pi}_{j\mathbf{k}} \equiv \hat{\pi}$, heating rate:

$$\dot{\epsilon} = \mathcal{D}\frac{\hat{\pi}^\dagger\hat{\pi}}{2f^0} = \frac{-f^0}{2\hbar}(\omega_G^{\mathrm{nucl}})^2 \left[\hat{u}^\dagger, \left[\hat{u}, \frac{\hat{\pi}^\dagger\hat{\pi}}{2f^0}\right]\right] = \frac{1}{2}\hbar(\omega_G^{\mathrm{nucl}})^2 \sim 10^{-21}\text{erg/s}$$

In M=1g, the # of d.o.f. $N \sim 10^{23} \Longrightarrow N\dot{\epsilon} \sim 100$erg/s: far too much!
Refine DP-model: Spontaneous collapse for modes $1/k \gg \lambda$ only:

$$\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t} = \frac{-\mathrm{i}}{2\hbar}\sum_{\mathbf{k}}\left(\frac{1}{f^0}[\hat{\boldsymbol{\pi}}_{\mathbf{k}}^\dagger\hat{\boldsymbol{\pi}}_{\mathbf{k}},\hat{\rho}] + f^0 c_\ell^2 k^2[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]\right) - \frac{f^0}{2\hbar}(\omega_G^{\mathrm{nucl}})^2 \sum_{1/k\gg\lambda}[\hat{\mathbf{u}}_{\mathbf{k}}^\dagger,[\hat{\mathbf{u}}_{\mathbf{k}},\hat{\rho}]]$$

E.g.: λ=10^{-5}cm, # of d.o.f. $N \sim 10^{14} \Longrightarrow N\dot{\epsilon} \sim 10^{-7}$erg/s: fairly low!
DP-collapse of macroscopic acoustic modes (c.o.m., too) remains.

Concluding remarks

We

- killed Cats by collapse or just by decoherence
- compared spontaneous decoherence in DP and CSL
- derived G-related spontaneous decoherence of acoustic modes
- derived spontaneous decoherence master eq. for $\hat{\rho}$
- showed spontaneous DP-decoherence dominates at large scales
- reduced spontaneous heating in DP, kept macrosopic predictions
- spared spontaneous collapse stoch. eqs. for $|\psi\rangle$
- claimed spontaneous decoherence is the only testable local effect
- *claime spontaneous collapse is untestable global effect - for DP, CSL, GRW,...*
- *mention spontaneous collapse becomes testable in extended DP-model*

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