Is Quantum Theory Exact? LNF 28.04.2014

Local quanta, unitary inequivalence, vacuum entanglement

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QUINFOG

Quantum Information and Foundations Group

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YouTube channel

Quinfog has now a YouTube channel

Local Quantum Theory

Quantum Information and Quantum Fields at Facultad de Físicas. 2nd, 3rd and 4th December!!

SUMMARY

IF IT IS NOT HERE, IS IT ZERO?

NO!

IF IT IS NOT HERE, THERE IS THE VACUUM

but……

This leaves no way for strict localization

Quantum springboards

$$H = \sum_i \tfrac{1}{2}(p_i^2 + q_i^2) + \lambda q_i q_{i+1} \quad \rightarrow \quad H = \sum_i \tfrac{1}{2}(P_i^2 + Q_i^2)$$

(q_i, p_i) local d.o.f. (Q_i, P_i) global d.o.f.

$$\hat{A}_N = \hat{Q}_N + i\hat{P}_N, \;\; \hat{A}_N^\dagger = \hat{Q}_N - i\hat{P}_N, \rightarrow H = \sum_N \hbar\Omega_N \hat{A}_N^\dagger \hat{A}_N$$

Particles $\longleftrightarrow$ elementary excitations of global oscillators

Vacuum $\longleftrightarrow$ ground state

All oscillators are present in the vacuum

Local excitations are not particles, Global are (standard Fock Space)

Vacuum entanglement: what you spot at q_i depends on q_j

Hegerfeldt Theorem

$$\psi_t = e^{-iHt}\psi_0 \qquad \hat{H} \geq c$$

$$\mathcal{P}_A(t) = \langle \psi_t | A | \psi_t \rangle \quad \hat{A} \geq 0$$

⇒

$$\begin{cases} Either\ \mathcal{P}_A(t) \neq 0 \,\forall\, t \in \mathbf{R} \\ or\ \mathcal{P}_A(t) = 0 \,\forall\, t \in \mathbf{R}. \end{cases}$$

R. Paley and N. Wiener Theorem XII

Take $A = \int_V |x\rangle\langle x|$ as $\langle x|e^{-iHt}|\psi\rangle$ analytic in $0 \geq \mathrm{Im}\, t$

$$Either\ \psi \text{ is in } V \text{ forever } (P_V(t) \neq 0 \,\forall\, t \in \mathbf{R})$$

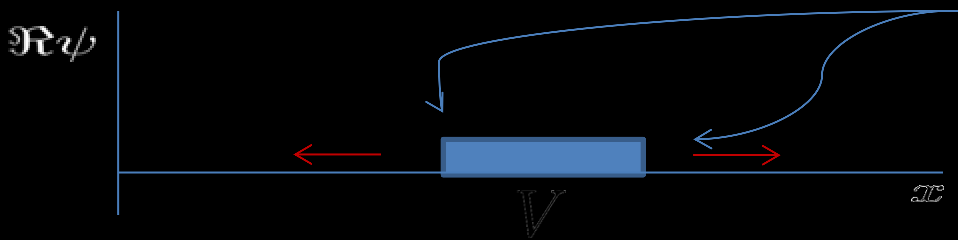
$$or\ \psi \text{ is never in } V \ (P_V(t) = 0 \,\forall\, t \in \mathbf{R})$$

Instantaneous spreading $\Longrightarrow$ causality problems in RQM and QFT

Prigogine: KG particle $$|\psi\rangle = \int_0^\infty d\omega_k \psi(\omega_k, x) a^\dagger(k)|0\rangle$$

initially localized in $V, \;\; \psi(\omega_k, x) = 0 \text{ if } x \notin V$

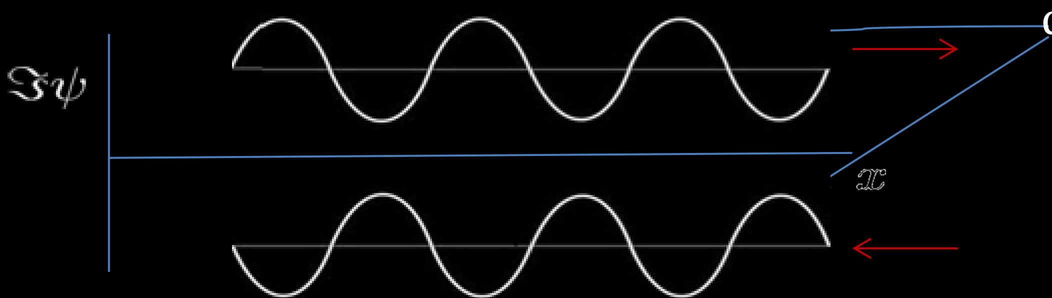
box splits and expand at the speed of light



$$\Box\psi(t,x) = 0$$

$$\psi(t,x) = \Re\psi(t,x) + i\Im\psi(t,x)$$

destructive interference at $t = 0$



$\Im\psi(t,x)$ strictly non local

Antilocality of $\hat{\omega}_k : \psi(t,x) = 0, \text{ and } \hat{\omega}_k\psi(t,x) = 0 \forall x \in I \Rightarrow \psi(t,x) \equiv 0$

A simplified version of Ree-Schlieder th.

Even if $\psi(t=0,x) = 0, \; \forall x \notin V, \;$ necessarily $\dot{\psi}(t=0,x) \neq 0$ for $x \notin V$

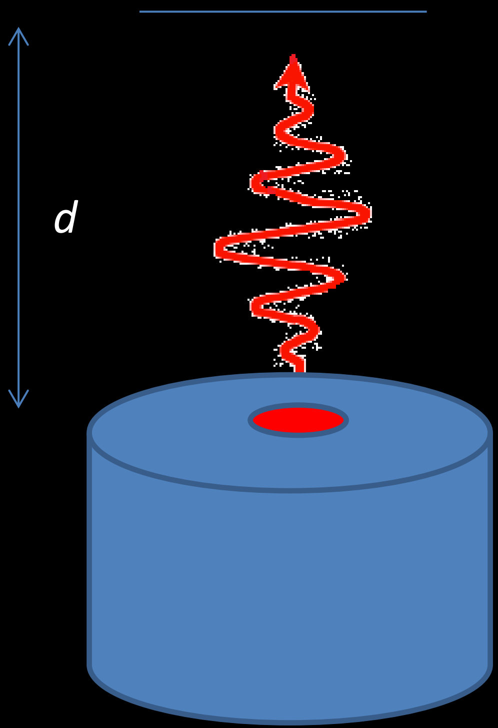
We localized a Fock state just to discover that It instantaneously spreads everywhere

Wave function and its time derivative

vanishing outside a finite region

requires of positive and negative frequencies

What happens when a photon,
produced by an atom inside a cavity,
escapes through a pinhole?



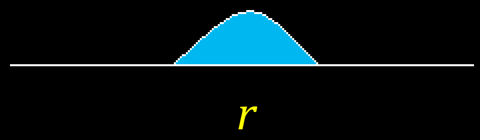
Eventually the photon will impact on a screen at $t = d/c$

But at *t*=0 $\phi \neq 0$ only at the pinhole ,
and the photon energy is positive (*back to Prigogine*)

According to Hegerfeld + antilocality
the photon will spread everywhere almost
instantaneously

As this is not the case,

we have to abandon Fock space for describing the photon through the pinhole



Cauchy surface t=0

Local quanta given by $\phi = 0 = \dot{\phi}$ out of r

Modes $u_l(x,t)$ initially localized in r $u_l(x,0) = 0 \quad x \notin r$

Operators $a_l, \quad a_l^\dagger, \quad [a_l, a_m^\dagger] = \delta_{lm}$

Two types of quanta

global	$\Omega_N > 0$	eigenstates	cannot vanish outside finite intervals
local		both frequencies	localized within finite intervals

Photon emerging through the pinhole as a well posed Cauchy problem

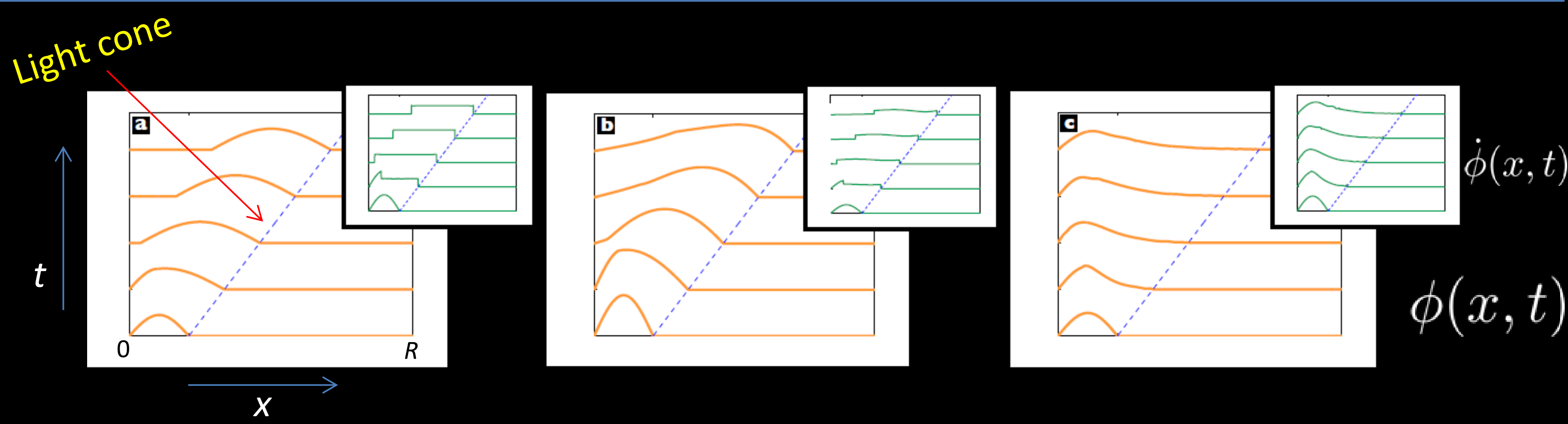
with initial values for ϕ and $\dot{\phi}$ vanishing outside the hole.

1+1 Dirichlet problem where the global space is a cavity $\{x \in [0, R]\}$

and the initial data vanish outside an interval of the cavit y $L = [0, r],\ r < R$

Cauchy data can be written as $\phi(x) = \sum_k c_k u_k(x), \quad \dot{\phi} = \sum_k \dot{c}_k \dot{u}_k(x)$

$$u_k(x) = \frac{1}{\sqrt{r\omega_k}} \sin \frac{\pi k x}{r}, \quad \dot{u}_k(x) = -i\omega_k \frac{1}{\sqrt{r\omega_k}} \sin \frac{\pi k x}{r}, k = 1, 2...$$



We follow a similar procedure for the case finite Cauchy data out of the interval:

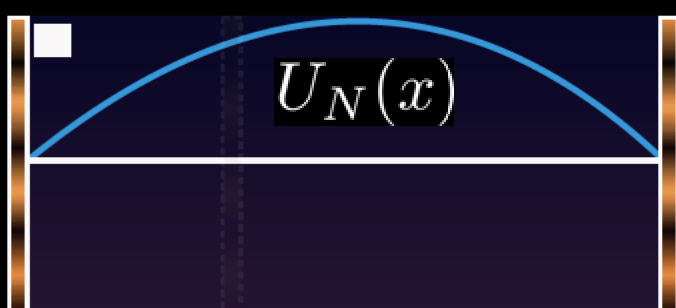
$$x \in \bar{L} = [r, R], \quad \bar{r} = R - r$$

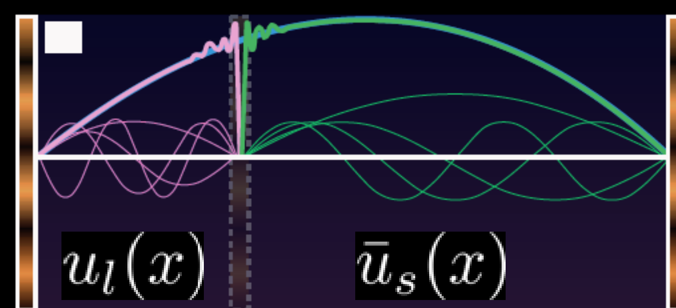
$$\bar{u}_k(x) = \frac{1}{\sqrt{\bar{r}\bar{\omega}_k}} \sin \frac{\pi k x}{\bar{r}}, \quad \dot{\bar{u}}_k(x) = -i\bar{\omega}_k \frac{1}{\sqrt{\bar{r}\bar{\omega}_k}} \sin \frac{\pi k x}{\bar{r}}, k = 1, 2...$$

Global modes for all times:

$$U_N(t,x) = U_N(x) e^{-i\Omega_N t} = \frac{1}{\sqrt{R\Omega_N}} \sin \frac{\pi N x}{R} e^{-i\Omega_N t}$$ *tionary modes*)

How sums of $u_l(x)$ and $\bar{u}_s(x)$ build up $U_N(x)$ at t=0



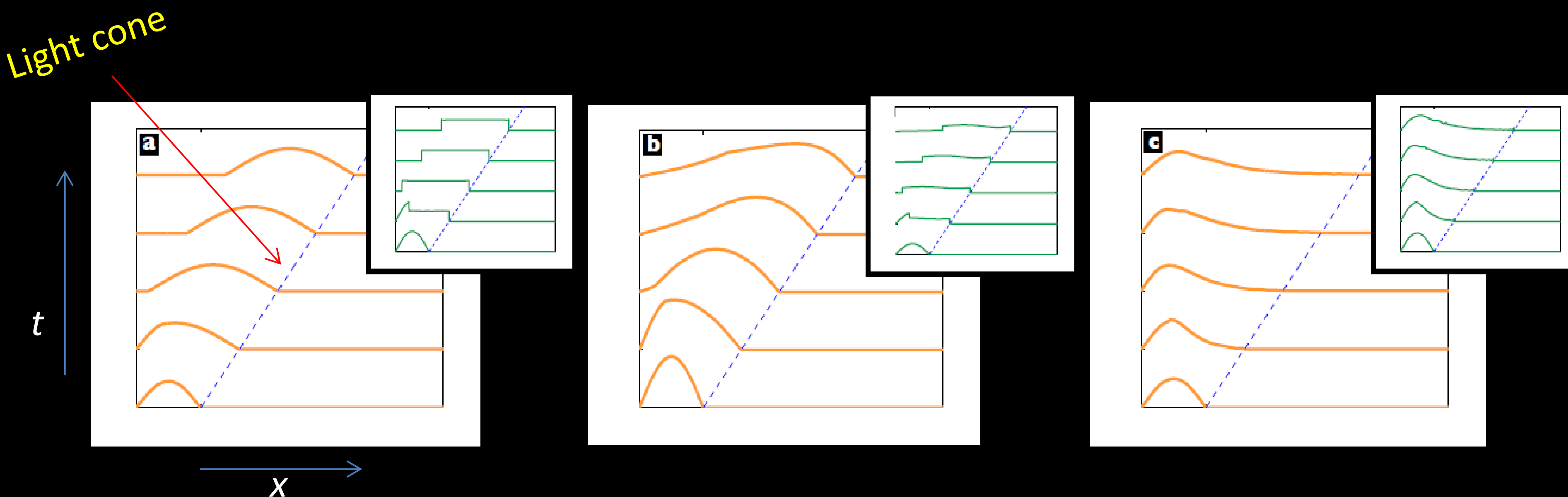


UPPER CASE *lower case* $\overline{\textit{lower case}}$

Local modes are superpositions of positive and negative frequencies

$$u_k(x,t) = \sum_N \left((\omega_k + \Omega_N) e^{-i\Omega_N t} - (\omega_k - \Omega_N) e^{i\Omega_N t} \right) \mathcal{U}_N(x) \mathcal{V}_{kN}$$

$$\bar{u}_k(x,t) = \sum_N \left((\bar{\omega}_k + \Omega_N) e^{-i\Omega_N t} - (\bar{\omega}_k - \Omega_N) e^{i\Omega_N t} \right) \mathcal{U}_N(x) \bar{\mathcal{V}}_{kN} .$$



Field expansions

$$\hat{\phi}(x,t) = \sum_{N=1}^{\infty} \left(U_N(x,t)\hat{A}_N + U_N^*(x,t)\hat{A}_N^\dagger \right), x \in R$$

$$\hat{\phi}(x,t) = \sum_l \left(u_l(x,t)\hat{a}_l + u_l^*(x,t)\hat{a}_l^\dagger \right), \quad x \in L$$

$$\hat{\phi}(x,t) = \sum_l \left(\bar{u}_l(x,t)\hat{\bar{a}}_l + u_l^*(x,t)\hat{\bar{a}}_l^\dagger \right), \quad x \in R - L$$

Bogoliubov transformations

$$a_m = \sum_N (u_m|U_N)A_N + (u_m|U_N^*)A_N^\dagger \qquad a_m^\dagger = \sum_N (U_N|u_m)A_N^\dagger + (U_N^*|u_m)A_N$$

$$\bar{a}_m = \sum_N (\bar{u}_m|U_N)A_N + (\bar{u}_m|U_N^*)A_N^\dagger \qquad \bar{a}_m^\dagger = \sum_N (U_N|\bar{u}_m)A_N^\dagger + (U_N^*|\bar{u}_m)A_N.$$

Canonical conmutation relations

$$[a_m, a_n^\dagger] = \delta_{mn} \qquad [a_m, \bar{a}_n^\dagger] = 0 \qquad [\bar{a}_m, \bar{a}_n^\dagger] = \delta_{mn}$$

Local quantization

$$a_m = \sum_N (u_m|U_N)A_N + (u_m|U_N^*)A_N^\dagger$$

$$\bar{a}_m = \sum_N (\bar{u}_m|U_N)A_N + (\bar{u}_m|U_N^*)A_N^\dagger$$

$$[a_m, \bar{a}_n] = 0, \quad [a_m, \bar{a}_n^\dagger] = 0$$

$$a_m|0_L\rangle = \bar{a}_m|0_L\rangle = 0 \ \forall m \in \mathbb{N}^+$$

Local vacuum

$$|n_1, n_2, \ldots\rangle = \prod_m \frac{(a_m^\dagger)^{n_m}}{\sqrt{n_m!}}|0\rangle_L, \qquad |\bar{n}_1, \bar{n}_2, \ldots\rangle = \prod_m \frac{(\bar{a}_m^\dagger)^{\bar{n}_m}}{\sqrt{\bar{n}_m!}}|0\rangle_L$$

Unitary Inequivalence

$$\sum_m \langle 0_G | n_m + \bar{n}_m | 0_G \rangle = \sum_N \langle 0_L | N_N | 0_L \rangle = \sum_{m,N} |(U_N^* | u_m)|^2 + |(U_N^* | \bar{u}_m)|^2$$

$$\langle 0_L | N_N | 0_L \rangle = \sum_m |(U_N^* | u_m)|^2 + |(U_N^* | \bar{u}_m)|^2 = \infty$$

global number operators N_N are ill defined in the local Fock space

$$\langle 0_G | n_m + \bar{n}_m | 0_G \rangle = \sum_N |(U_N^* | u_m)|^2 + |(U_N^* | \bar{u}_m)|^2 < \infty$$

Positivity of energy

$$H^G = H - \langle 0_G | H | 0_G \rangle \quad H^L = H - \langle 0_L | H | 0_L \rangle \qquad \mathcal{E} \equiv \langle 0_L | H^G | 0_L \rangle$$

$$\langle m_l, \bar{0} | H^G | m_l, \bar{0} \rangle = m_l \sum_N \Omega_N \left(|(u_l | U_N)|^2 + |(u_l^* | U_N)|^2 \right) + \mathcal{E}$$

$$\langle m_l, \bar{0} | H^L | m_l, \bar{0} \rangle = m_l \sum_N \Omega_N \left(|(u_l | U_N)|^2 + |(u_l^* | U_N)|^2 \right)$$

Energy of the local quanta: $\epsilon_l = \sum_N \Omega_N \left(|(u_l | U_N)|^2 + |(u_l^* | U_N)|^2 \right)$

Exciting the vacuum with local quanta $|0_G\rangle \rightarrow a_m^\dagger|0_G\rangle$

Normalized one-local quantum state $$|\psi\rangle = \frac{a_m^\dagger|0_G\rangle}{\sqrt{1+\langle 0_G|n_m|0_G\rangle}}$$

If $|\psi\rangle$ were strictly local

$$\langle\psi|\bar{n}_m|\psi\rangle - \langle 0_G|\bar{n}_m|0_G\rangle$$ should be zero

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N.B. in the local vacuum $$\hat{a}_l|0_L\rangle = 0$$

then the states $$|\phi_L\rangle = \hat{a}^\dagger|0_L\rangle$$

Become strictly local $$\langle\phi_L|\bar{n}_m|\phi_L\rangle - \langle 0_L|\bar{n}_m|0_L\rangle = 0$$

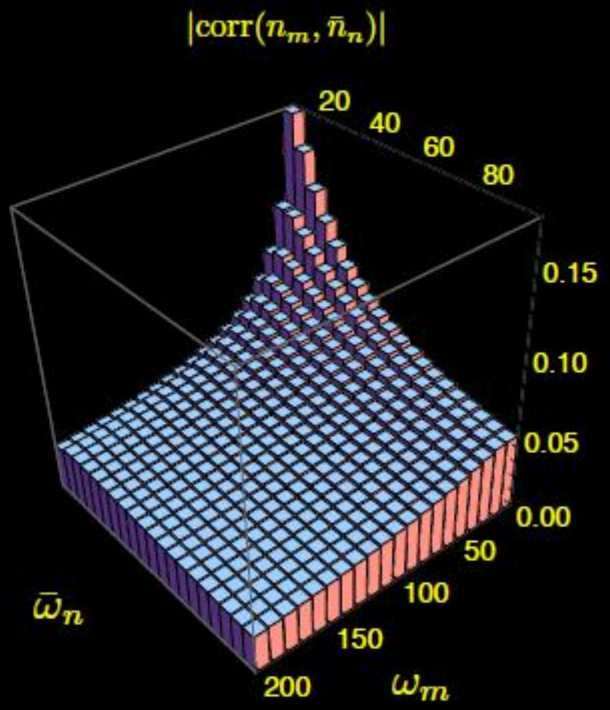
This is not the case in global vacuum

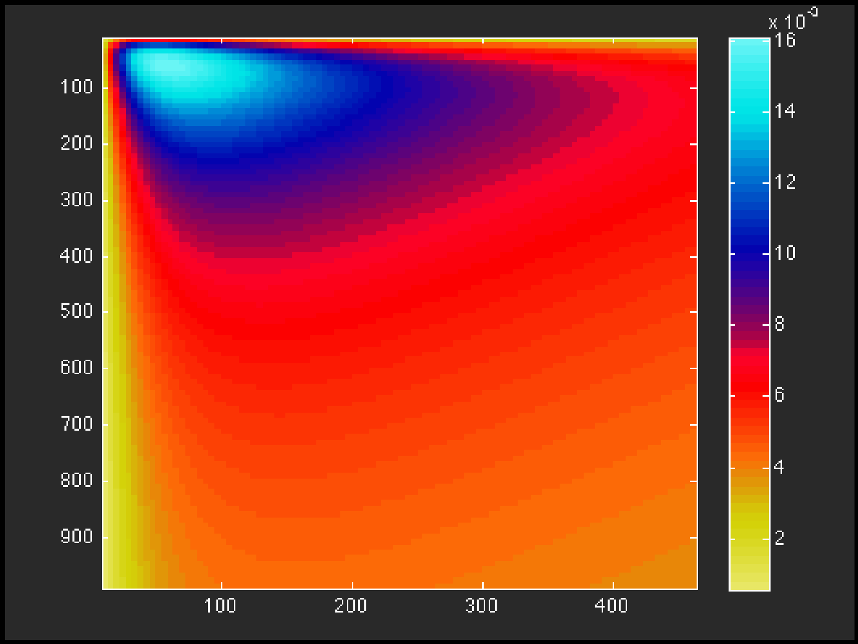
$$\langle\psi|\bar{n}_m|\psi\rangle - \langle 0_G|\bar{n}_m|0_G\rangle =$$

$$= \qquad\qquad = \frac{corr(n_l, \bar{n}_m)}{1 + \langle 0_G|n_l|0_G\rangle}$$

due to vacuum correlations

$$corr(n_m, \bar{n}_l) \equiv \langle 0_G|n_m\bar{n}_l|0_G\rangle - \langle 0_G|n_m|0_G\rangle\langle 0_G|\bar{n}_l|0_G\rangle$$

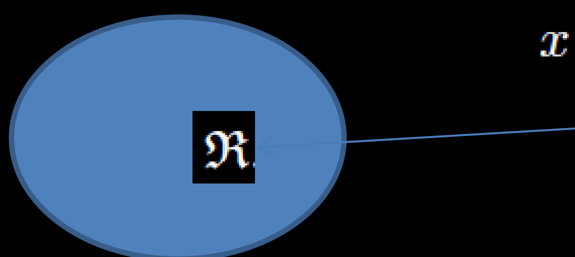




Strict localisation on the local vacuum

$|\psi\rangle$ is said to be *strictly localised* within a region of space $\mathfrak{R}$ if the expectation value of any local operator $\mathcal{O}(x)$ outside that region (i.e. $x \notin \mathfrak{R}$) is identical to that of the vacuum, i.e.

$$\langle\psi|\mathcal{O}(x)|\psi\rangle = \langle 0|\mathcal{O}(x)|0\rangle \text{ if } x \notin \mathfrak{R}.$$



$$|n_1, n_2, \ldots\rangle = \prod_m \frac{(a_m^\dagger)^{n_m}}{\sqrt{n_m!}} |0\rangle_L$$

$$\langle\psi|\mathcal{O}(\bar{a}_m, \bar{a}_m^\dagger)|\psi\rangle =$$

$$= \langle 0_L|\mathcal{O}(\bar{a}_m, \bar{a}_m^\dagger)|0_L\rangle$$

In QM there are conjugate operators Q, P and conjugate representations

They are unitarily equivalent (Stone Von-Neumann, Pauli).

In ∞QM there are conjugate operators ϕ(t,x), π(t,x) and conjugate representations

They may be unitarily inequivalent (infinite degrees of freedom) (Unruh, de Witt, Fulling)

Wigner representations ………………………………………….. global elementary exitations

Localized representations ………………………………………. local elementary excitations

Quantum vacuum is global ⟹ vacuum entanglement

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THANKS FOR YOUR ATTENTION!