Quantisation of Gravity or Gravitisation of Quantum Mechanics?

Meanings of semi-classical gravity, the Schrödinger-Newton equation, and experimental tests

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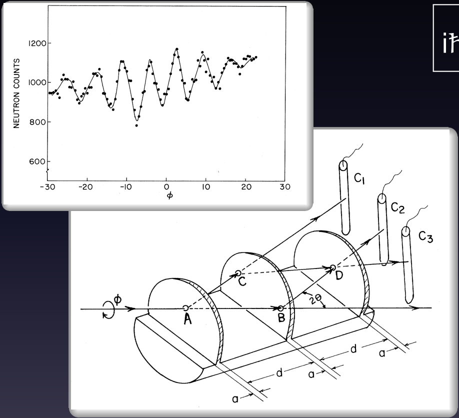
What is “Quantum Gravity” about?

Quantum Gravity is about finding a model to describe...

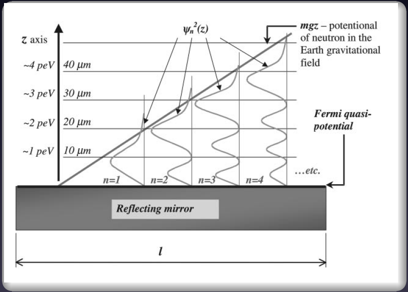
- quantum effects in systems where gravity is predominant (Black Holes, Cosmology, ...)
- gravitational effects in presence of quantum matter:
  - How does gravity change the hydrogen spectrum?
  - How does a quantum state evolve in a gravitational field?
  - What is the gravitational field of an atom or a molecule (felt by “classical” matter or other atoms)?
  - ...

“Known Physics” (so far)

$$i\hbar\,\dot{\psi} = -\frac{\hbar^2}{2m}\,\Delta\psi + m\,g\,z\,\psi$$



Colella, Overhauser, Werner
PRL 34 (1975) 1472



Nesvizhevsky *et al*
Nucl. Instr. Meth. Phys. Res. A 440 (2000) 754

Why quantise gravity?

- Quantum (field) theory is experimentally verified with astonishing precision: measurements of the fine structure constant, α, are in agreement with accuracy up to 10^{-8}.
- The gravitational field (space-time curvature) is just a dynamical field like all the others.
- Quantum theory teaches us that every dynamical field has quantum properties, i. e. it is described by a theory of linear operators

$\Rightarrow$ Alter gravity in order to obtain a linear quantum theory!

Why not quantise gravity?

- General Relativity is experimentally verified with astonishing precision: tests of the weak equivalence principle with a sensitivity up to 10^{-14}.
- General Relativity teaches us that Gravity is **not** a field but curvature of the space-time on which the other fields live.
- The curvature of space-time depends on the energy density (stress-energy-tensor) of matter fields.

$\Rightarrow$ Alter quantum theory (on curved space-time) in order to obtain a consistent expression for space-time curvature!

Semi-classical gravity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \langle \Psi | T_{\mu\nu} | \Psi \rangle$$

Weak-field non-relativistic limit: $\Delta U = 4\pi G \langle \Psi | m \hat{\psi}^\dagger \hat{\psi} | \Psi \rangle$

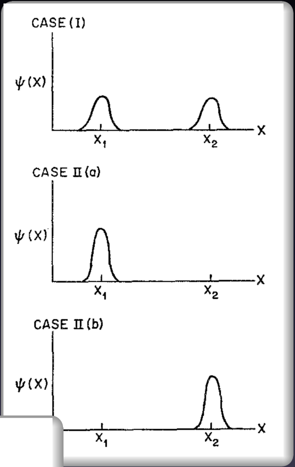
- **Semi-classical gravity is the mean-field limit of a quantised gravity whose low-energy limit is the quantised linearisation of General Relativity:**
  $\Rightarrow$ The equation only makes sense for states of a large number of particles
  $\Rightarrow$ In the one-particle sector gravitational self-interaction yields mass-renormalisation as in QED
- **Semi-classical gravity is fundamental:**
  $\Rightarrow$ In the one-particle sector $\langle \Psi | m \hat{\psi}^\dagger \hat{\psi} | \Psi \rangle = m |\psi|^2$
  $\Rightarrow$ One obtains the Schrödinger-Newton equation

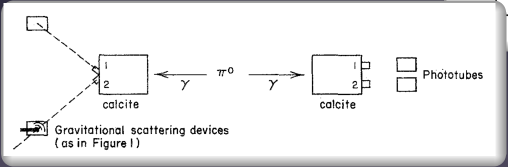
Is semi-classical gravity consistent?

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} \langle\Psi|T_{\mu\nu}|\Psi\rangle$$

The Eppley and Hannah thought experiment:

- Case 1: gravitational wave leads to collapse of wave function
  $\Rightarrow$ violation of uncertainty relation
- Case 2: scattering is function of ψ
  $\Rightarrow$ violation of causality



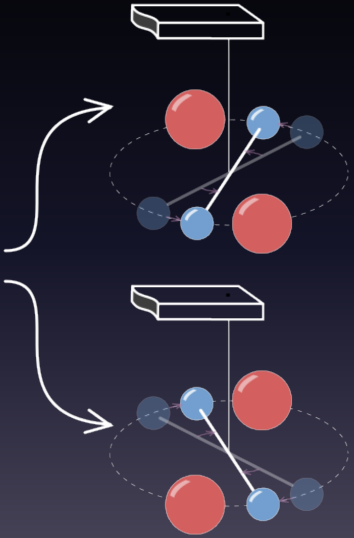


Eppley and Hannah
Found. Phys. 7 (1977) 51

The Page and Geilker[1] experiment

$\# = n_A$

$$\frac{n_A}{n_B} \gtrless \left\langle \frac{n_A}{n_B} \right\rangle \text{ ?}$$



$\# = n_B$

Images: Wikipedia (DMKTirpitz, Science Museum London)

[1] D. N. Page and C. D. Geilker. Indirect Evidence for Quantum Gravity. *Phys. Rev. Lett.*, 47:979–982, 1981

The Schrödinger-Newton equation

$$\begin{aligned} i\hbar\,\dot{\psi} &= -\frac{\hbar^2}{2m}\,\Delta\psi + m\,U\,\psi \\ \Delta U &= 4\pi\,G\,m\,|\psi|^2 \end{aligned}$$

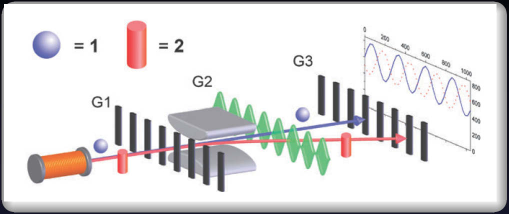
After integration of the potential U:

$$i\hbar\,\dot{\psi}\,(t,\vec{x}) = \left(-\frac{\hbar^2}{2m}\,\Delta - G\,m^2 \int \frac{|\psi(t,\vec{y})|}{|\vec{x}-\vec{y}|}\,d^3y\right)\,\psi(t,\vec{x})$$

The non-linear gravitational self-interaction also affects the centre-of-mass dynamics of many particle systems[2]

[2] D. Giulini and A. G. Centre-of-mass motion in multi-particle Schrödinger-Newton dynamics. arXiv: 1404.0624, 2014

Dispersion of a wave packet



Tüxen, Gerlich, Eibenberger, Arndt, Mayor
Chem. Commun. 46 (2010) 4145

Gaußian wave packet of width $a = 500\,\text{nm}$:

$$\psi(t=0, r) = (\pi a^2)^{-3/4} \exp\left(-\frac{r^2}{2a^2}\right)$$

Scaling behaviour of the SN equation:

With $\psi(t, \vec{x})$ for mass m, $\mu^{9/2}\psi(\mu^5 t, \mu^3 \vec{x})$ is a solution for mass $\mu \cdot m$

Dynamics of the wave packet

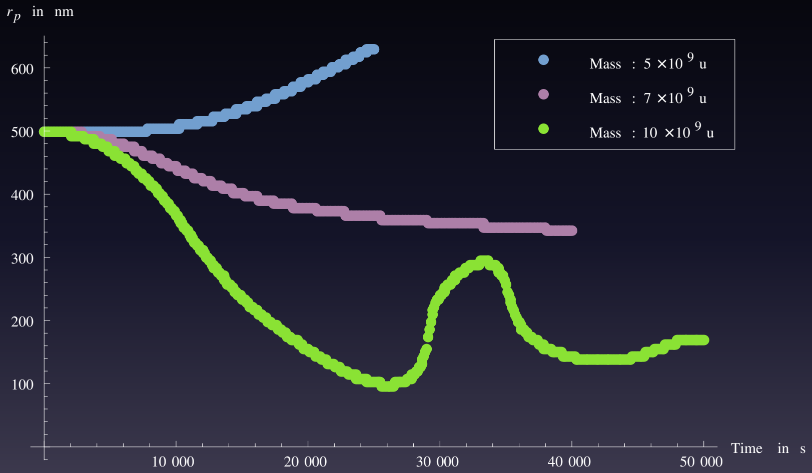


Figure: Maximum $r_p(t)$ of the radial probability density $4\pi r^2|\psi(r)|^2$

Conclusion

A quantised gravitational field and ”Gravitised Quantum Mechanics“ are both well justified approaches to consistently treat gravitational interaction in quantum systems.

It is for the experiment to decide which one is right!

The Schrödinger-Newton equation provides an experimentally falsifiable hypothesis that follows straightly from semi-classical (non-quantised) gravity.