Workshop: Is quantum theory exact? The endeavor for the theory beyond standard quantum mechanics

DISSIPATIVE EXTENSION OF THE GHIRARDI-RIMINI-WEBER MODEL

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MOTIVATION:
THE GRW MODEL AND THE ENERGY DIVERGENCE

Collapse models: challenging the superposition principle

- Unified description of microscopic and macroscopic systems

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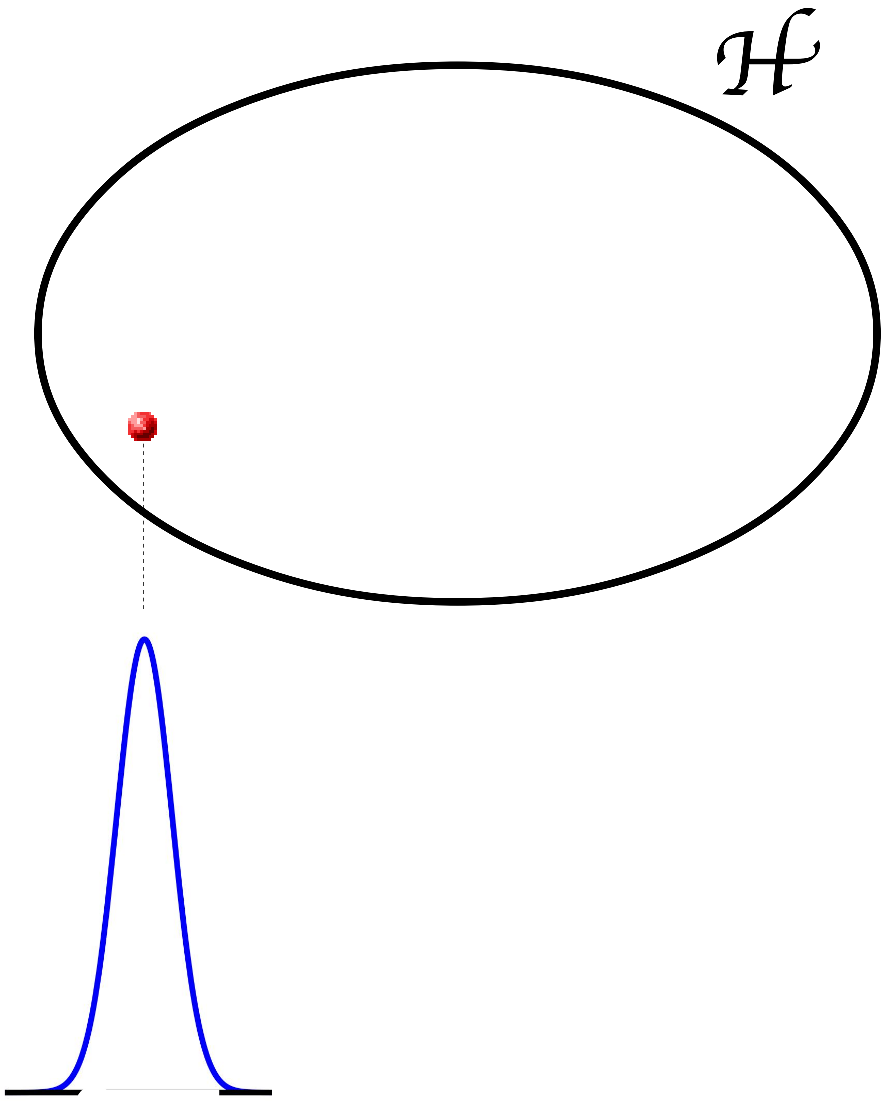
  No superposition of **macroscopic** systems

- Testable predictions different from standard QM

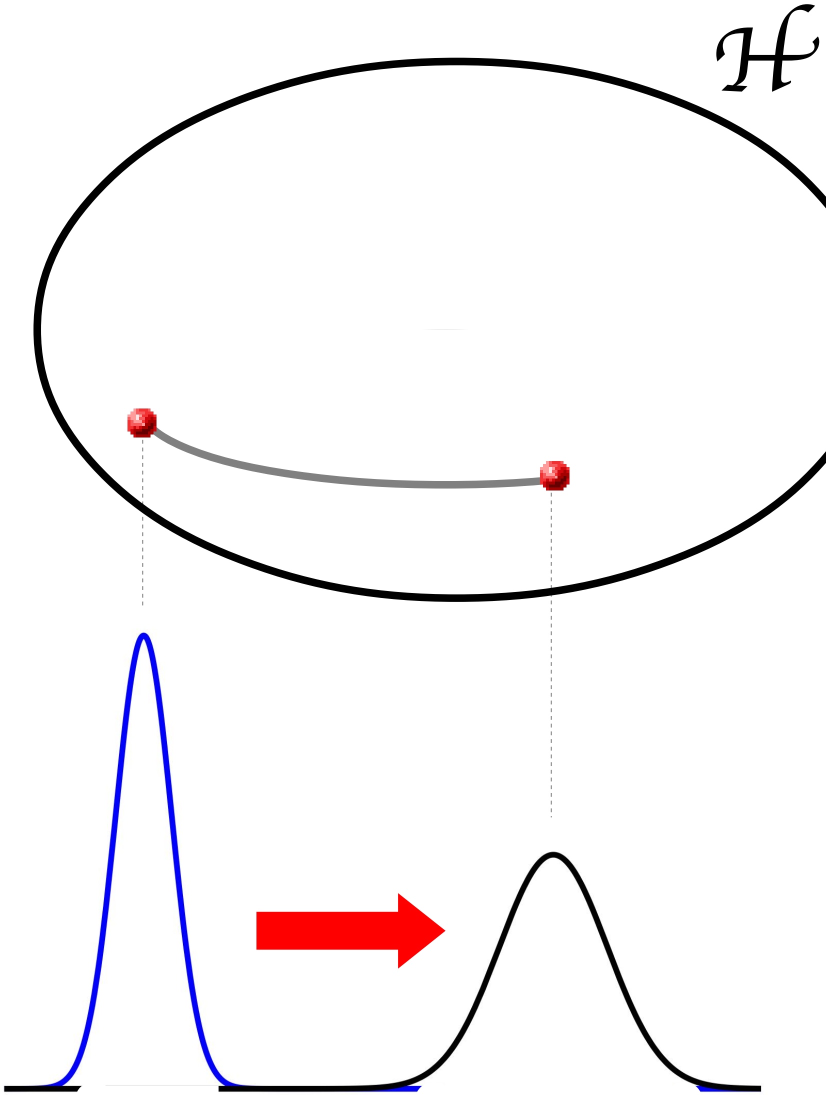
A. Bassi & G.C. Ghirardi, Phys. Rep. 2003
A. Bassi, K. Lochan, S. Satin, T.P. Singh & H. Ulbricht, Rev. Mod. Phys. 2013

Localization of the wave function



- Usual Schrödinger evolution

Localization of the wave function



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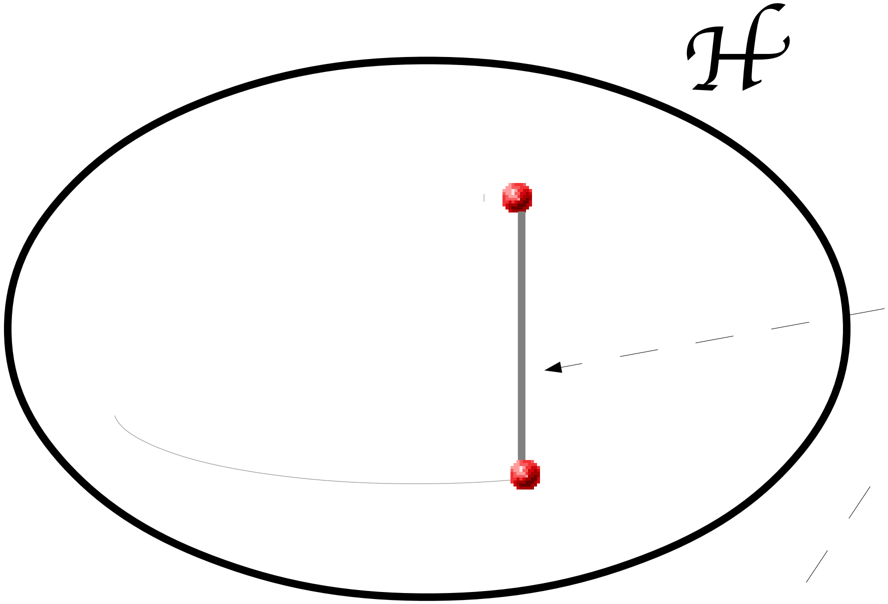
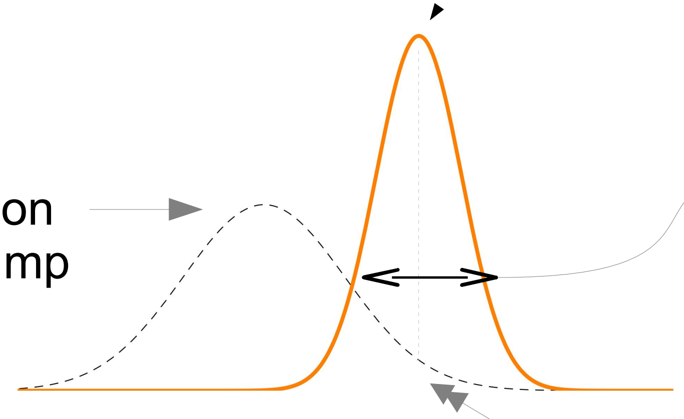


- Usual Schrödinger evolution
- ***Instantaneous*** jump

$$L_y(\hat{X}) = (\pi r_c^2)^{-1/4} e^{-\frac{(\hat{X}-y)^2}{2 r_c^2}}$$



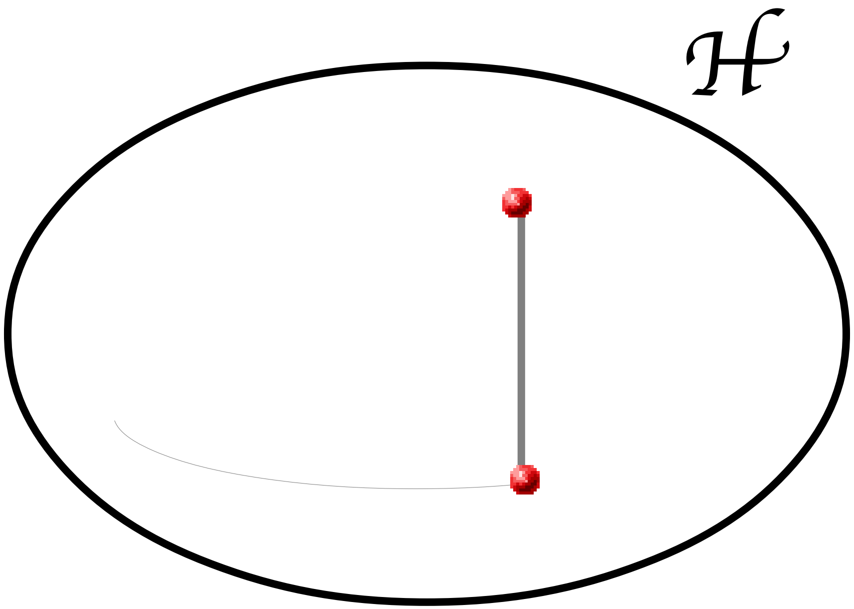
Wave function before the jump

Localization width

r_C ⚠ New parameter

Localization position y

Localization of the wave function



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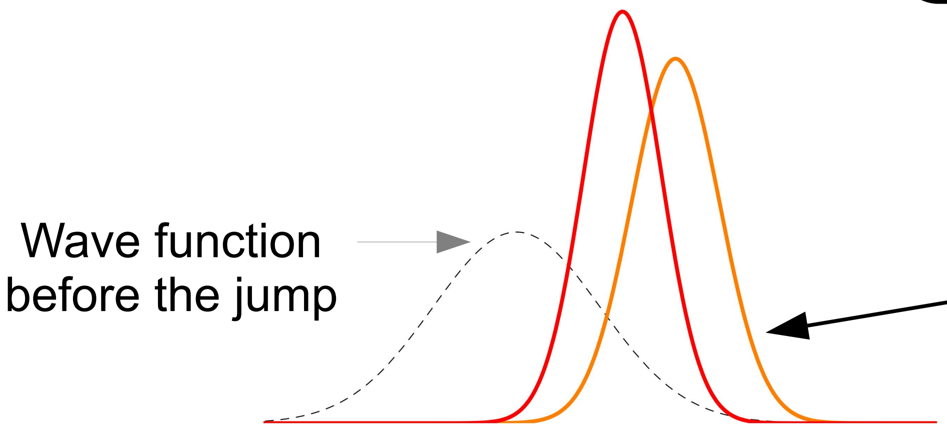


Wave function before the jump

Wave function localization!!

$$|\psi(t)\rangle \longrightarrow |\psi_y(t)\rangle \equiv \frac{L_y(\hat{X})|\psi(t)\rangle}{\|L_y(\hat{X})|\psi(t)\rangle\|}$$

G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. A 1986

Distribution of the jumps

- Probability distribution of the localization position:

$$p(y) = \|L_y(\widehat{X})|\psi(t)\rangle\|^2$$

→ Dynamical **derivation** of the Born rule

- Poisson time-distribution of the jumps

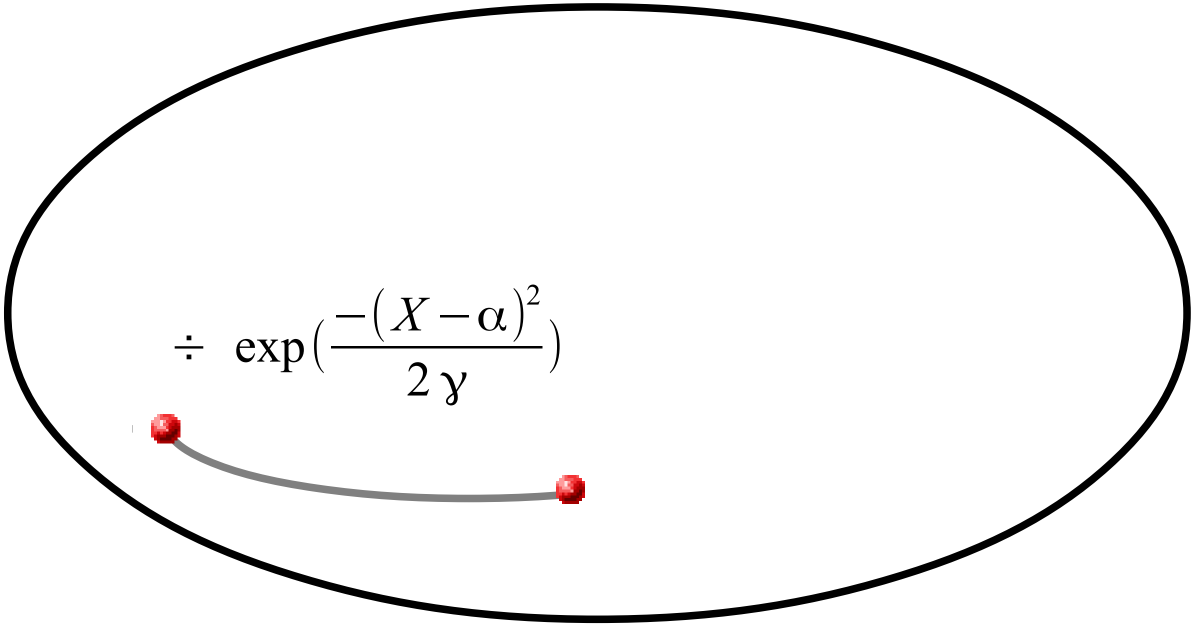
- Localization rate for one nucleon $\boxed{\lambda = 10^{-16}\, s^{-1}}$ New parameter

Microscopic systems are NOT affected by the localization mechanism!

- Localization rate for an N-particle system $\boxed{\lambda_{macro} = N\,\lambda}$

Amplification mechanism: macroscopic systems are strongly affected !

Energy divergence

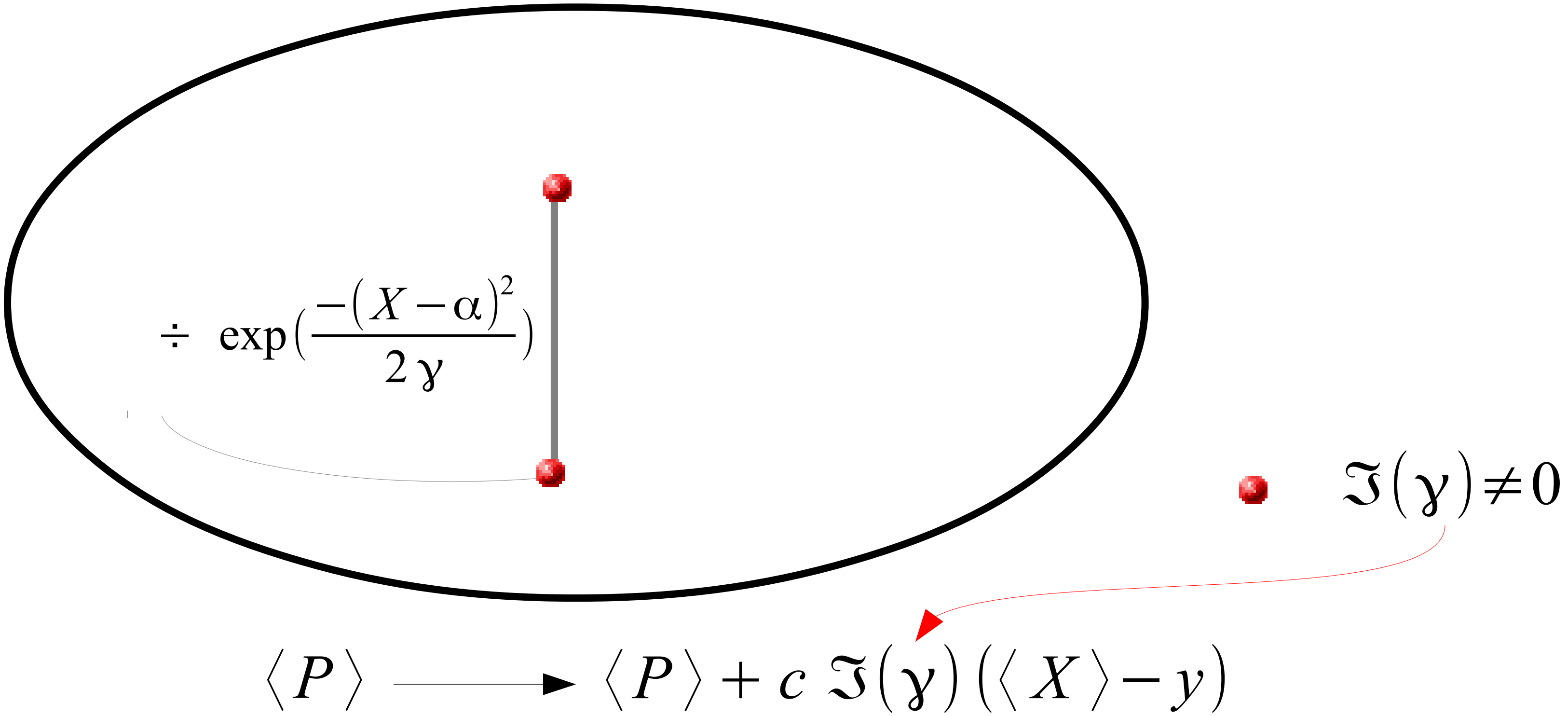


$$\div \ \exp\left(\frac{-(X-\alpha)^2}{2\gamma}\right)$$

- $\Re(\gamma)$ increases
- $\Im(\gamma)\neq 0$

$\langle P\rangle_t$ constant

Energy divergence

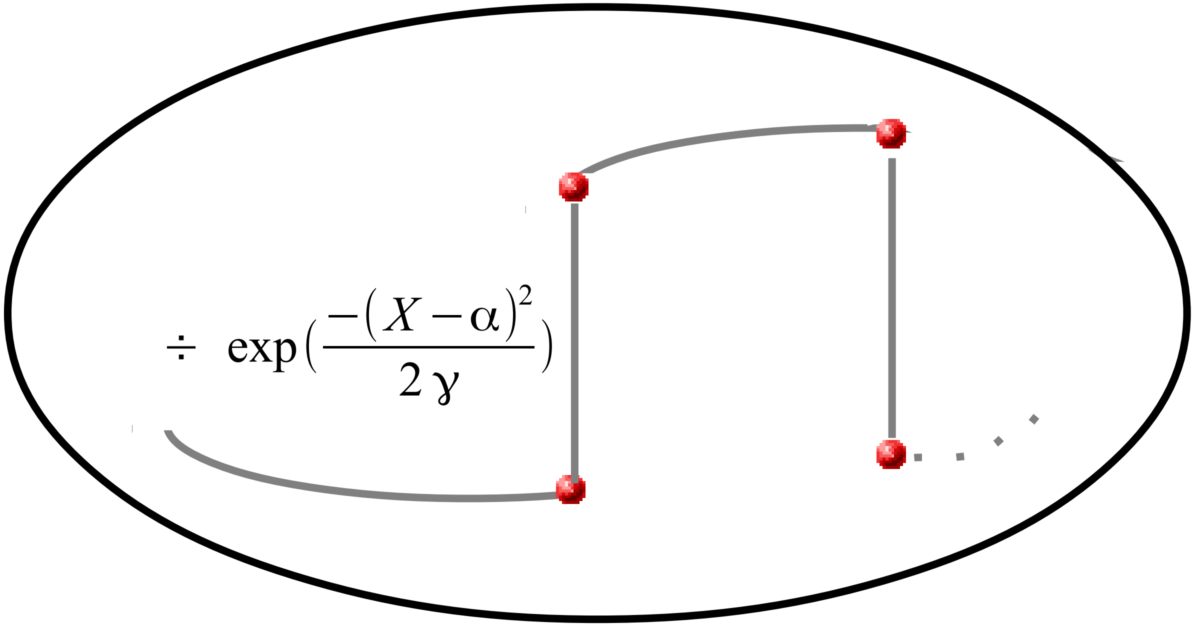


$$\div \; \exp\left(\frac{-(X-\alpha)^2}{2\gamma}\right)$$

$$\Im(\gamma) \neq 0$$

$$\langle P \rangle \xrightarrow{\qquad} \langle P \rangle + c\,\Im(\gamma)\,(\langle X \rangle - y)$$

Energy divergence



$$\div \ \exp\left(\frac{-(X-\alpha)^2}{2\gamma}\right)$$

● $\Im(\gamma) \neq 0$

$$\langle P \rangle \longrightarrow \langle P \rangle + c\,\Im(\gamma)\,(\langle X \rangle - \boxed{y})$$

Different spatial distributions of the jumps ➡ $\langle P \rangle_t \longrightarrow \pm\infty$

$$\langle H \rangle_t = ((\Delta_t P)^2 + \langle P \rangle_t^2)/(2M)$$

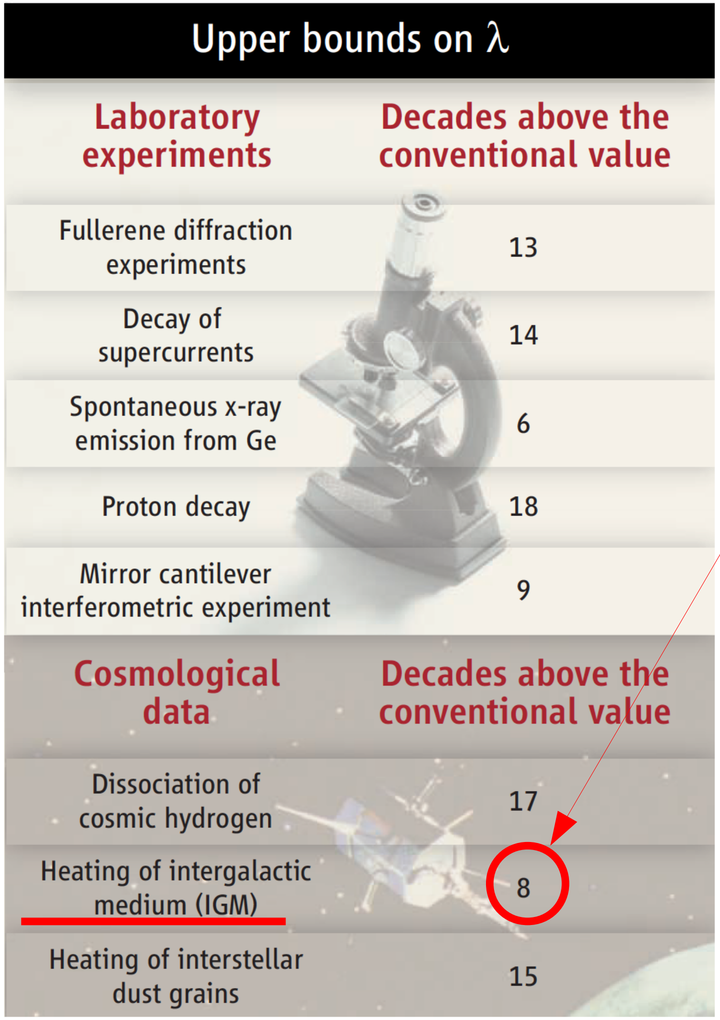
The average energy of the system diverges linearly in time, with a rate

$$\xi = \frac{\hbar^2 \lambda}{4 M r_c^2} \approx 10^{-25} eV\, s^{-1}$$

Why is it important to study this (very slow) energy increase ?

- Re-establishment of the energy conservation principle
- Possible modifications of the testable predictions of the model

- S. Adler & A. Bassi, Science, 325 (2009)
- S. Adler JPA, 40 (2007)



Upper bounds on λ

Laboratory experiments	Decades above the conventional value
Fullerene diffraction experiments	13
Decay of supercurrents	14
Spontaneous x-ray emission from Ge	6
Proton decay	18
Mirror cantilever interferometric experiment	9
Cosmological data	**Decades above the conventional value**
Dissociation of cosmic hydrogen	17
Heating of intergalactic medium (IGM)	8
Heating of interstellar dust grains	15

2nd strongest bound on λ !!

Secular energy increase compatible with **experimental data**

EXTENSION OF THE GRW MODEL: INCLUSION OF DISSIPATION

A. Smirne, B. Vacchini & A. Bassi, in preparation (2014)

Master equation

- Statistical operator $\hat{\rho}(t) \equiv \mathbb{E}[|\psi(t)\rangle\langle\psi(t)|]$

Lindblad equation:
Markovian dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{i}{\hbar}\left[\widehat{H}\,,\,\hat{\rho}(t)\right] + \lambda\left(\int \mathrm{d}y\, L_y(\widehat{X})\hat{\rho}(t)L_y(\widehat{X}) - \hat{\rho}(t)\right)$$

$$= -\frac{i}{\hbar}\left[\widehat{H}\,,\,\hat{\rho}(t)\right] + \underline{\lambda}\left(\int \mathrm{d}Q\; e^{\frac{i}{\hbar}Q\widehat{X}} G(Q)\,\rho(t)\,G(Q)\,e^{-\frac{i}{\hbar}Q\widehat{X}} - \rho(t)\right)$$

Fourier Transform: $L_y(\widehat{X}) = \int \frac{dQ}{\sqrt{2\pi\hbar}}\, e^{\frac{i}{\hbar}Q(\widehat{X}-y)}\, G(Q)$

Describes also recoil-free collisional decoherence

$M \longrightarrow \infty$

- $G(Q) = \sqrt{\frac{r_C}{\sqrt{\pi}\hbar}}\exp\left(-\frac{r_C^2 Q^2}{2\hbar^2}\right)$

Pure position-decoherence dynamics NO DISSIPATION

- To include dissipation $\boxed{G(Q) \longrightarrow G(Q\,,\widehat{P})}$

Dissipative jump operators

$$L_y(\hat{X},\hat{P}) = \int \frac{dQ}{\sqrt{2\pi\hbar}}\, e^{\frac{i}{\hbar}Q(\hat{X}-y)}\, G(Q,\hat{P})$$

$$\left(\frac{r_c}{\sqrt{\pi}\hbar} + \frac{1}{2\sqrt{\pi}Mv_\eta}\right)^{1/2} e^{-\frac{1}{2}\left(\left(\frac{r_c}{\hbar}+\frac{1}{2Mv_\eta}\right)Q+\frac{\hat{P}}{Mv_\eta}\right)^2}$$

⚠ New parameter

$$v_\eta = \frac{4k_B T r_C}{\hbar}$$

- Schrödinger evolution between the jumps
- Jumps given by $|\psi(t)\rangle \longrightarrow |\psi_y(t)\rangle \equiv \frac{L_y(\hat{X},\hat{P})|\psi(t)\rangle}{\|L_y(\hat{X},\hat{P})|\psi(t)\rangle\|}$
- Probability distribution of the jump position $p(y) = \|L_y(\hat{X},\hat{P})|\psi(t)\rangle\|^2$
- Time distribution of the jumps: Poisson distribution with rate λ

Dissipative jump operators

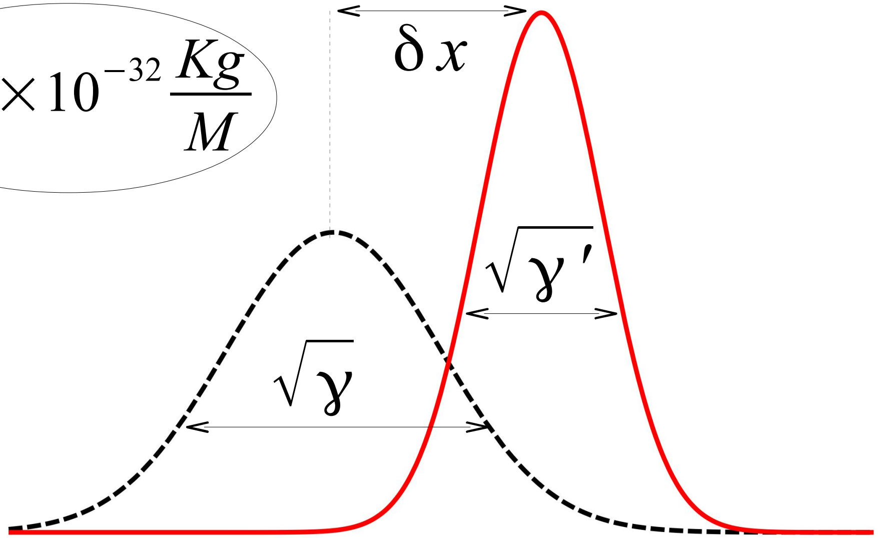
$$L_y(\hat{X},\hat{P})=\int\frac{dQ}{\sqrt{2\pi\hbar}}\;e^{\frac{i}{\hbar}Q(\hat{X}-y)}\,G(Q,\hat{P})$$

$$\left(\frac{r_c}{\sqrt{\pi}\hbar}+\frac{1}{2\sqrt{\pi}Mv_\eta}\right)^{1/2}\;e^{-\frac{1}{2}\left(\left(\frac{r_c}{\hbar}+\frac{1}{2Mv_\eta}\right)Q+\frac{\hat{P}}{Mv_\eta}\right)^2}$$

⚠ New parameter

$$v_\eta=\frac{4k_BTr_C}{\hbar}$$

$$k=5\times10^{-32}\frac{Kg}{M}$$



- $\delta x=(1-f_\gamma)(y-x)$

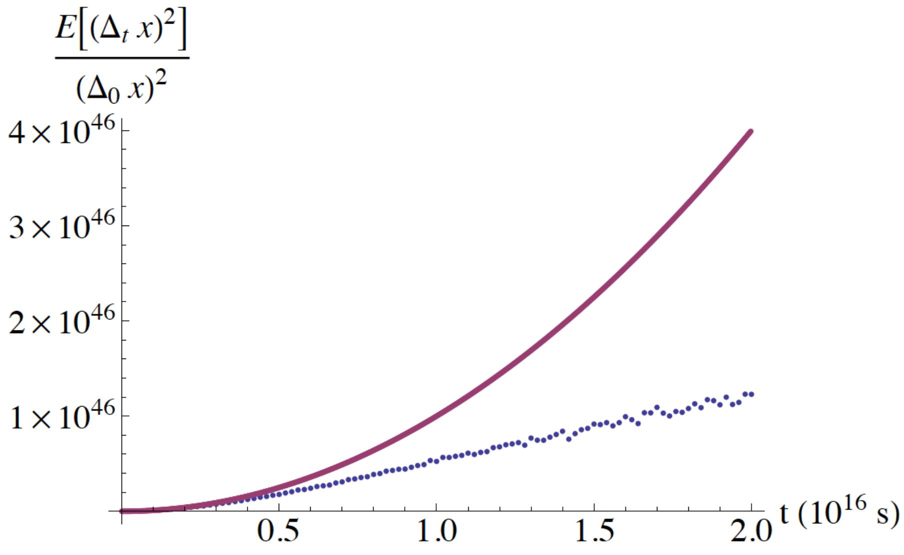
$$\left(\frac{\gamma}{r_c^2}+1\right)^{-1}\qquad\left(\frac{\gamma}{r_c^2(1-k^2)}+\frac{1-k}{1+k}\right)^{-1}$$

- $\gamma\,'=g_\gamma^{-1}$

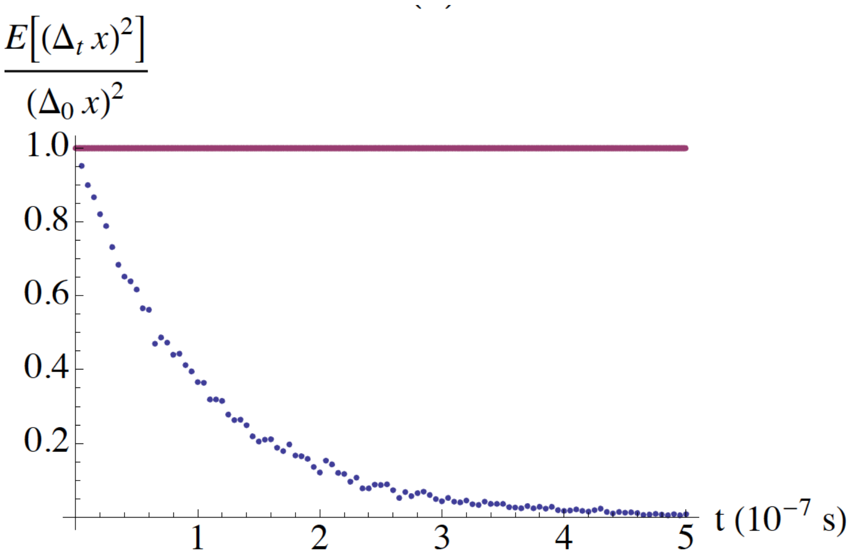
$$\frac{1}{\gamma}+\frac{1}{r_c^2}\qquad\frac{(1-k)^2}{\gamma(1+k)^2}+\frac{1}{r_c^2(1+k)^2}$$

Localization of the wave function (updated)

- The localization mechanism is left practically unchanged



$$M = 10^{-27} Kg \qquad \lambda = 10^{-16} s^{-1}$$



$$M = 10^{-3} Kg \quad \lambda_{\text{macro}} = N\lambda = 10^{7} s^{-1}$$

- Finite asymptotic value of position and momentum variance
- Lower threshold for the localization

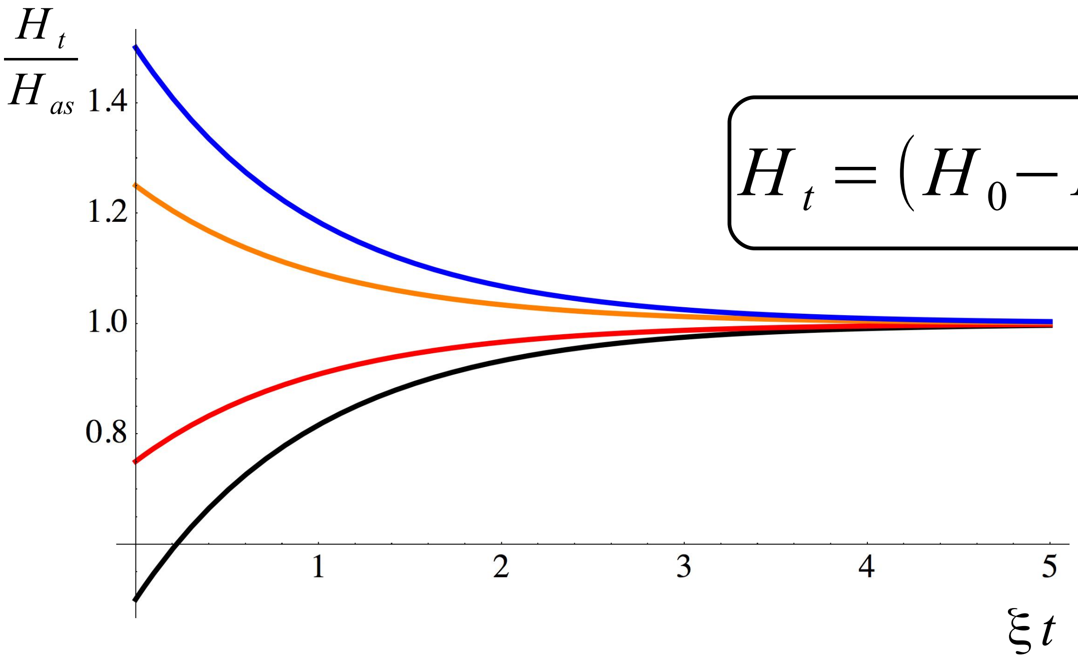
$$\Delta X_{thr} = 2 k r_c^2 = \left(10^{-45} \frac{Kg}{m}\right) \text{m}^2$$

Linked to the
finite asymptotic value
of the energy

Energy relaxation

- Mean value of the kinetic energy

$$H_t \equiv \mathbb{E}[\langle\psi(t)|\frac{\widehat{P}^2}{2\mathrm{M}}|\psi(t)\rangle] = \mathrm{Tr}\left\{\hat{\rho}(t)\frac{\widehat{P}^2}{2\mathrm{M}}\right\}$$



$$\boxed{H_t = (H_0 - H_{as})e^{-\xi t} + H_{as}}$$

- Rate $\frac{4\lambda k}{(1+k)^2} \approx 10^{-20} s^{-1}$
- Asymptotic energy $\frac{\hbar^2}{16Mr_c^2k} \approx 10^{-4} eV$

Temperature of the noise

- $$\frac{\xi}{\lambda} = \frac{4k}{(1+k)^2} \ll 1$$

Collapse occurs on a time scale much shorter than dissipation

- Equipartition of the energy

$$\boxed{T = \frac{2H_{as}}{k_B} = \frac{\hbar v_{\eta}}{4k_B r_C} \approx 10^{-1} K}$$

It does not depend on the mass of the system

Full characterization of the noise:
some underlying theory beyond standard quantum mechanics

Amplification mechanism

- Center of mass of an N-particle system $\hat{\rho}_{\text{CM}}(t) = \text{Tr}_{\text{REL}}\left\{\hat{\varrho}(t)\right\}$
  - Center of mass coordinates $\widehat{X}_j = \widehat{X}_{\text{CM}} + \cancel{\sum_{j'=2}^{N} \Lambda^{-1}_{jj'}\widehat{r}_{j'}}$
  - Crucial **assumption**: rigid body $\widehat{P}_j \approx M_j \widehat{P}_{\text{T}}/M_{\text{T}}$

→ All the particles give the same contribution

→ Same master equation as the one-particle system with

$$\boxed{\begin{array}{ccc}\lambda & \longrightarrow & N\lambda \\ M & \longrightarrow & M_{\text{T}}\end{array}}$$

- Individual localization processes $L_y(\widehat{X}_1, \widehat{P}_1) \otimes \mathbb{1}_2$

  ❗ Incomplete description

Conclusions and outlooks

- We have extended the GRW model by means of position and **momentum dependent** jump operators, which induce **energy relaxation** to a finite asymptotic value

- Introduction of a **new parameter**, related to the temperature T of the noise, GRW recovered for T $\longrightarrow \infty$

- Toward the establishment of a **realistic phenomenological** model, with recovery of the energy conservation principle

- Detailed comparison with the **experimental bounds**, to be developed for the **CSL model**

Acknowledgments

cost
EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Angelo Bassi

University of Trieste

Fundamental Problems in Quantum Physics
(Action MP 1006)

nano QUESTFIT

SEVENTH FRAMEWORK PROGRAMME

EUROPEAN COMMISSION

Markus Arndt

University of Vienna

Testing quantum superposition
in a mass range so far unexplored

INFN
Istituto Nazionale
di Fisica Nucleare

I.S. BELL

Nino Zanghì

Università di Genova

UNIVERSITÀ
DEGLI STUDI DI TRIESTE

Threshold value and asymptotic energy

- Master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{i}{\hbar}\left[\widehat{H}\,,\,\hat{\rho}(t)\right] + \lambda\left(\frac{r_c(1+k)}{\sqrt{\pi}\hbar}\int \mathrm{d}Q\, e^{\frac{i}{\hbar}Q\widehat{X}} e^{-\frac{r_c^2}{2\hbar^2}\left((1+k)Q+2k\widehat{P}\right)^2}\hat{\rho}(t)e^{-\frac{r_c^2}{2\hbar^2}\left((1+k)Q+2k\widehat{P}\right)^2}e^{-\frac{i}{\hbar}Q\widehat{X}} - \hat{\rho}(t)\right)$$

- Mean value of any operator valued function of the momentum operator only

$$\ll O \gg_t \equiv \mathbb{E}[\langle O\rangle_t] = \mathrm{Tr}\left\{\hat{\rho}(t)\widehat{O}\right\}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \ll f(P) \gg_t \; = \lambda\frac{r_c(1+k)}{\sqrt{\pi}\hbar}\int \mathrm{d}Q \ll e^{-r_c^2((1+k)Q+2kP)^2/\hbar^2}\left(f(P+Q)-f(P)\right) \gg_t$$

$$[\widehat{H}\,,f(\widehat{P})]=0 \quad \longrightarrow$$ No contribution from the hamiltonian term

➡ Same predictions as for an ***"auxiliary"*** collapse model, with only jumps!!

$$(\Delta_t X)^2 \longrightarrow 2r_c^2 k \qquad\qquad (\Delta_t P)^2 \longrightarrow \frac{\hbar^2}{8r_c^2 k}$$

Threshold value and asymptotic energy

- Master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{i}{\hbar}\left[\widehat{H}\,,\,\hat{\rho}(t)\right] + \lambda\left(\frac{r_c(1+k)}{\sqrt{\pi}\hbar}\int \mathrm{d}Q\, e^{\frac{i}{\hbar}Q\widehat{X}} e^{-\frac{r_c^2}{2\hbar^2}\left((1+k)Q+2k\widehat{P}\right)^2}\hat{\rho}(t)e^{-\frac{r_c^2}{2\hbar^2}\left((1+k)Q+2k\widehat{P}\right)^2}e^{-\frac{i}{\hbar}Q\widehat{X}} - \hat{\rho}(t)\right)$$

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$$[\hat{H}\,,f\,(\hat{P})]=0$$ → No contribution from the kinetic energy

→ Same predictions as for an ***"auxiliary"*** collapse model, with only jumps!!

$$(\Delta_t P)^2 \longrightarrow \frac{\hbar^2}{8\,r_C^2\,k}$$

$$\boxed{H_t \longrightarrow \frac{\hbar^2}{16\,M\,r_C^2\,k} = H_{as}}$$