Quantum non-contextuality as a generalisation of quantum theory

Petros Wallden*

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*Heriot-Watt University, Edinburgh
In collaboration with Fay Dowker and Joe Henson

Dowker, Henson, PW, New J. Phys. 16 (2014) 033033

Contents

Bounding Quantum Correlations

- QM allows correlations that violate Bell locality
- Physical principle "no-signalling" allows many unphysical supra-quantum correlations
- Need to characterise quantum correlations
- Need to provide a simple physical principle that limits the possible correlations
- These principles help: understand QM, reformulate, test, generalise, etc

Motivations from Quantum Gravity and Foundations

1. Quantum Gravity

- Can QM be generalised?

- Is there a causal principle including QM non-locality?

- Do histories allow more non-locality than QM?

2. Foundations

- What is special about QM?

- How to characterise quantum non-locality

- Simple principles bounding quantum non-locality?

• Characterising quantum non-locality ⇒ What histories allow

• Histories ⇒ Simple physical principle

Types of Scenarios

We examine Bell's scenarios: Experiments involving n-locations, each of them makes one of m measurements (settings) and each measurement has d possible outcomes. The behaviour is denoted (n, m, d)

- CHSH $(2, 2, 2)$: Two locations A, B. Measurement by A is denoted x and takes values $\{0, 1\}$, by B is denoted y and takes values $\{0, 1\}$. Possible outcomes for A are denoted a and for B are denoted b, and take values $\{+1, -1\}$

- The collection of probabilities $P_{xy}(a, b)$ is called a **behaviour**.

Classical Case - Fine's Trio

- Sample space of all counterfactual outcomes of local experiments

$$\begin{aligned} \Omega_J &= \{a_0 a_1 b_0 b_1\} \\ &= \{1,-1\} \times \{1,-1\} \times \{1,-1\} \times \{1,-1\} \end{aligned}$$

Classical Correlations are characterised by a trio of conditions

1. Non-contextuality. JPM $P_J(a_0 a_1 b_0 b_1)$. Experimental probabilities $P_{xy}(a_x b_y)$ arise as sums of counterfactual outcomes. e.g. $x = 0, y = 1$

$$P_{01}(a_0 b_1) = P_J(a_0 b_1) = \sum_{a_1 b_0} P_J(a_0 a_1 b_0 b_1)$$

2. Bell's locality (screening off model)

If one conditions in the common past c, different location probabilities become independent.

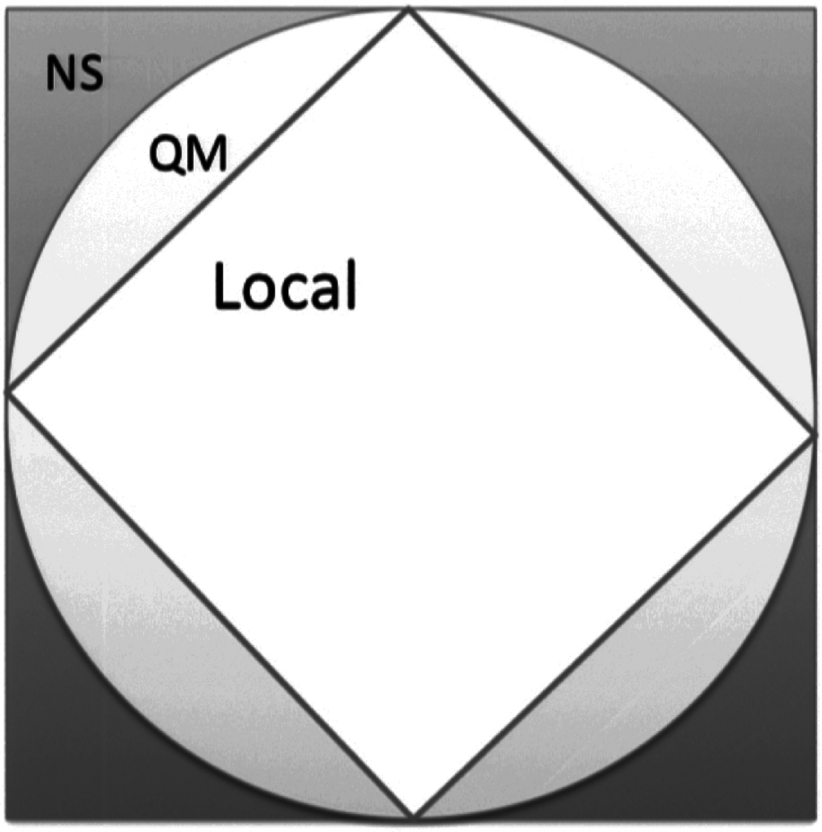
$$P_{xy}(a_x b_y|c) = P(a_x|c)P(b_y|c)$$

3. Satisfaction of all CHSH inequalities

$$\mathrm{CHSH} = |E_{00} + E_{10} + E_{01} - E_{11}| \leq 2$$

$$E_{xy} = \sum_{ab} ab P_{xy}(ab)$$

Possible Behaviours



- Every point is a particular $P_{xy}(ab)$

- NS = No-Signalling. Extremal points correspond to PR-boxes.

Characterising Quantum Behaviours

Problem with two parts:

1. Given a behaviour can we decide whether it could have come from a quantum model?

Exists hierarchy of condition that a quantum behaviour should obey (NPA). e.g. Tsirelson's bound

$$\text{CHSH} = |E_{00} + E_{10} + E_{01} - E_{11}| \leq 2\sqrt{2}$$

General NS can go up to 4 (PR-boxes)

2. What physical principle restricts the allowed behaviours exactly to those present in nature?

(a) Does nature allow only quantum behaviours?
(no more, no less)

(b) Many possible principles:

Quantum non-contextuality

non-trivial communication complexity

no advantage for nonlocal computation

macroscopic locality

local orthogonality

information causality

Histories in QM

- Histories (path integral) formulations are closer to the spirit of GR because time does NOT appear with a distinguished status.

- Researchers in QG believe that the suitable formulation of QM for their purpose would be a histories formulation (e.g. Causal Sets can be quantised only using path integrals).

- In QM the probability for given sequence of outcomes is given

$P(\alpha_{t_1} \text{ at } t_1 \text{ and } \alpha_{t_2} \text{ at } t_2 \cdots \alpha_{t_n} \text{ at } t_n;\ \rho(t_0))=$

$\text{Tr}(\alpha_{t_n}(t_n)\cdots\alpha_{t_1}(t_1)\rho(t_0)\alpha_{t_1}(t_1)\cdots\alpha_{t_n}(t_n))$

Class operator: $C_{\underline{\alpha}} = \alpha_{t_n}(t_n)\cdots\alpha_{t_1}(t_1)$

Decohernece Functional (measures interference):

$\mathcal{D}(\underline{\alpha}, \underline{\alpha}') = \mathrm{Tr}(C_{\underline{\alpha}} \rho C^{\dagger}_{\underline{\alpha}'})$

Diagonal terms are positive but NOT probability measure:

$D(\{a, b\}, \{a, b\}) \neq D(a, a) + D(b, b)$

However it IS quantum measure (Sorkin)

Quantum Measure

Given sample space Ω we define quantum measure* μ a function such that:

1. Positivity $\mu(A) \geq 0$

2. Normalisation $\sum_{h \in \Omega} \mu(h) = 1$

3. No three-ways interference

$$\mu(A \sqcup B \sqcup C) = \mu(A \sqcup B) + \mu(A \sqcup C) +$$
$$+ \mu(B \sqcup C) - \mu(A) - \mu(B) - \mu(C)$$

- The diagonal parts of the decoherence functional of sequential measurements define a quantum meausre

$$\mu(A) = D(A, A)$$

*R. Sorkin Mod. Phys. Lett. A 9 (1994) 3119

- Can replace <u>Positivity</u> with <u>Strong Positivity</u>

$D(\gamma, \gamma')$ is a positive matrix

- Strong Positive Quantum Measure is composable

- More general than sequence of measurements in a Hilbert space

- Natural analogue of the classical non-contextuality.

Quantum Non-contextuality

- Require the existence of a Joint Quantum Measure over all the counterfactual outcomes

- Allows for non-composable systems. Instead we require Strongly Positive Joint Quantum Measure (SPJQM)

e.g. $(2,2,2)$ we require that there exists a positive matrix

$D(a_0a_1b_0b_1, a'_0a'_1b'_0b'_1)$ where for example

$P_{01}(a_0b_1) = \sum_{a_1,a'_1b_0,b'_0} D(a_0a_1b_0b_1, a_0a'_1b'_0b_1)$

Quantum and Almost Quantum Behaviours

- Quantum Behaviours:

Exist a state $|\psi\rangle$ in a Hilbert space and projections operators $[P_x^a, P_y^b] = 0$ such that

$P_{xy}(ab) = \langle\psi|P_x^a P_y^b|\psi\rangle$

- Almost Quantum Behaviours*:

Require $[P_x^a, P_y^b]|\psi\rangle = 0$ instead of $[P_x^a, P_y^b] = 0$

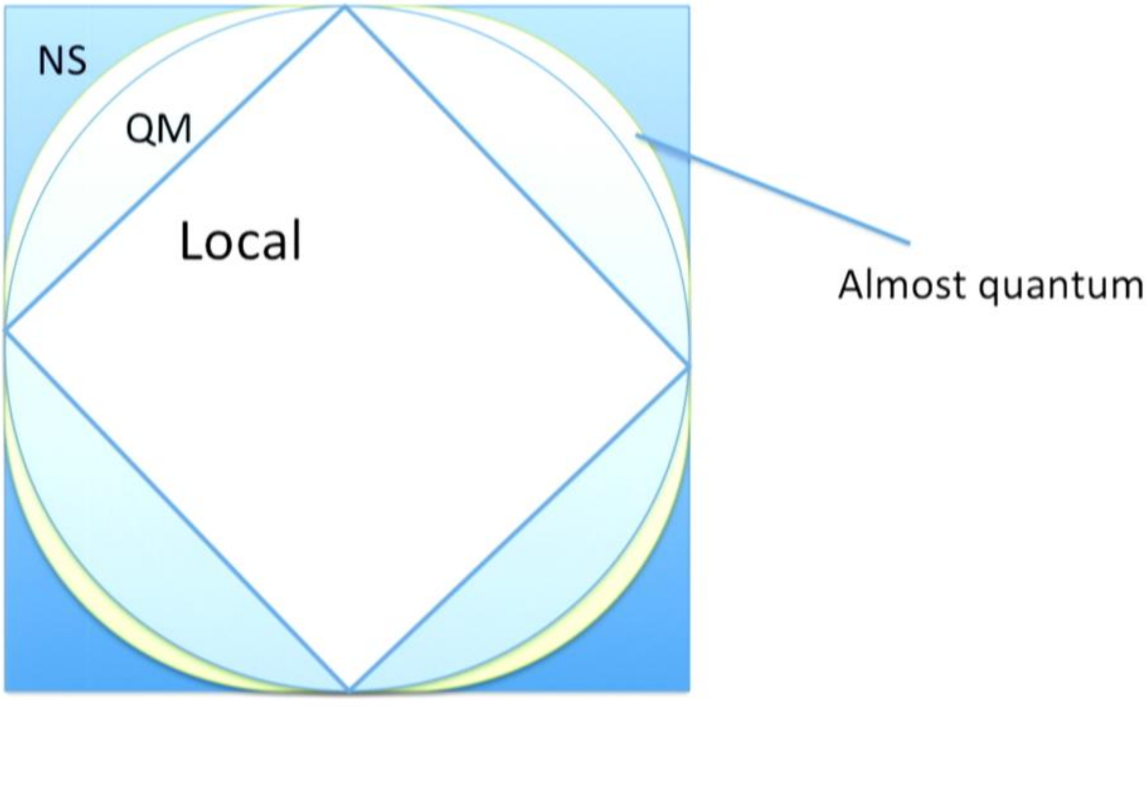
*Navascues, Guryanova, Hoban, Acin, arXiv:1403.4621

- These information theoretic principles follow:

Non-trivial communication complexity, no advantage for nonlocal computation, macroscopic locality, local orthogonality

- Numerical evidence for information causality

- For CHSH gives the quantum value $2\sqrt{2}$



SPJQM $\Leftrightarrow$ Almost Quantum

1. Almost Quantum $\Rightarrow$ SPJQM

Define:

$$D(a_0 a_1 b_0 b_1, a'_0 a'_1 b'_0 b'_1) = \langle\psi|P_0^a P_1^a P_0^b P_1^b P_0^{a'} P_1^{a'} P_0^{b'} P_1^{b'}|\psi\rangle$$

The marginals give the experimental probabilities, e.g.

$$\begin{aligned} P_{01}(a_0 b_1) &= \sum_{a_1, a'_1, b_0, b'_0} \langle\psi|P_0^a P_1^a P_0^b P_1^b P_0^a P_1^{a'} P_0^{b'} P_1^b|\psi\rangle \\ &= \langle\psi|P_0^a P_1^b|\psi\rangle \end{aligned}$$

2. SPJQM $\Rightarrow$ Almost Quantum

More complicated, see:

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- Therefore SPJQM implies all other information theoretic principles

- SPJQM is (a natural) generalisation of QM

Summary and Conclusion

- QG suggests histories formulations are closer to the spacetime spirit of GR
- For characterising non-locality this gives a natural physical principle in terms of the quantum measure
- This principle is "quantum" non-contextuality
- Need to see how tight does this principle bound the allowed non-locality. This would complete the Quantum Fine Trio
- Conventional contextuality present in QM (Kochen-Specker theorem) is still allowed by the joint quantum measure
- It is possible in principle to test. Can find scenario that violates most QM but there are difficulties to find a violating physical system.

Thanks for your attention!!

- Dowker, Henson, PW, New J. Phys. 16 (2014) 033033