Saturation in Deep Inelastic Scattering from ADS/CFT

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with M.S.Costa and J. Penedones

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Introduction

- High energy phenomena probe relevant features of interactions
- In flat space string theory they are controlled by the leading Regge trajectory of states
- In perturbative YM theory (QCD and susy completions) dominated by Pomeron exchange
- High energy strings in holographic backgrounds will interpolate between these two regimes
  - At large 't Hooft coupling, analyze gravi-Reggeon exchange in AdS and recover flat space interactions
  - Focus of this talk will be at weak 't Hooft coupling. Use holography to correctly analyze known pQCD results

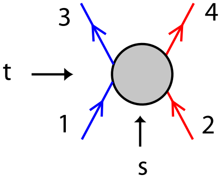
High Energy Correlators in CFT

- Consider

$$\langle \mathcal{O}_1(q_1)\,\mathcal{O}_1(q_3)\,\mathcal{O}_2(q_2)\,\mathcal{O}_2(q_4)\rangle_{\text{CFT}_4}$$

with

$$t \;,\; q_i^2 \qquad \text{fixed}$$
$$s \to \infty$$



- S–matrix in AdS_5

$$-\frac{dx^+dx^-}{\rho^2}+\underbrace{\frac{d\mathbf{x}\cdot d\mathbf{x}+\rho^2}{\rho^2}}_{\text{transverse } H_3}$$

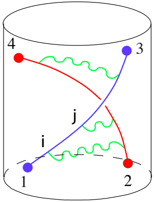
- External states

$$e^{i\sqrt{s}x^-}\cdot e^{i\mathbf{q}_1\cdot\mathbf{x}}\cdot f_1(\rho) \qquad\qquad \left(\rho^2\sim 1/q_1^2\right)$$

- High energy states follow null geodesics in AdS parameterized by affine parameter x^+ and labelled by x^- and a point $\mathbf{x}, \rho$ in H_3

- Propagator

$$\frac{1}{\sqrt{s}}\theta\left(x_j^+ - x_i^+\right) \cdot$$
$$\cdot\, \delta\left(x_j^- - x_i^-\right) \delta_{H_3}\left(\mathbf{x}_j, \rho_j | \mathbf{x}_i, \rho_i\right)$$



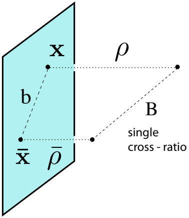
- Eikonal resummation leads to impact parameter representation

$$s \int \frac{d\rho d^2\mathbf{x}}{\rho^3} \cdot f_1(\rho) f_3(\rho) e^{i\mathbf{q}\cdot\mathbf{x}} \cdot \int \frac{d\bar{\rho} d^2\bar{\mathbf{x}}}{\bar{\rho}^3} \cdot f_2(\bar{\rho}) f_4(\bar{\rho}) e^{-i\mathbf{q}\cdot\bar{\mathbf{x}}} \cdot e^{2i\Delta(S,B)}$$

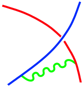
- Phase shift Δ depends on

$$S = s\rho\bar{\rho}$$

$$\cosh B = \frac{1}{2\rho\bar{\rho}}\left[\rho^2 + \bar{\rho}^2 + (\mathbf{x} - \bar{\mathbf{x}})^2\right]$$



- For graviton exchange phase given by tree level interaction between geodesics

$$\Delta(S, B) = i \int dx^+ dx^-$$  $$\simeq \frac{G}{\ell^3}\, S\, \Pi_2(B)$$

with $\Pi_2(B)$ propagator in H_3 of scalar of dimension 3

Impact Parameter Representation and Unitarity

- Final answer

$$s \int d^2\mathbf{b}\ e^{i\mathbf{q}\cdot\mathbf{b}}\ e^{2i\delta(s,\mathbf{b})}$$

with

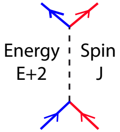
$$e^{2i\delta(s,\mathbf{b})} = \int \frac{d\rho}{\rho^3} f_1(\rho)\, f_3(\rho) \int \frac{d\bar{\rho}}{\bar{\rho}^3} f_2(\bar{\rho})\, f_4(\bar{\rho}) \quad \cdot e^{2i\Delta(S,B)}$$

- High energy unitarity

$$\operatorname{Im}\Delta(S,B) \geq 0$$

- Phase $\Delta(S,B)$ related to S–channel conformal partial wave decomposition for operators of large spin and energy

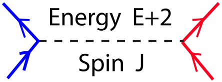
$$E = \sqrt{S}\cosh(B/2)$$
$$J = \sqrt{S}\sinh(B/2)$$



Regge Theory

- $\Delta(S,B)$ depends on $\bar{\alpha}_s$ and N
  For large B (at fixed S) $\Delta \to 0$ and planar graphs (string tree level) dominate Δ

- Focus on planar exchange of



- Euclidean OPE

$$S^{-2-E}$$

- Lorentzian correlator

$$\Delta(S,B) \sim \frac{1}{N^2}\, S^{J-1} \cdot \Pi_E(B)$$

$\left(\begin{array}{c} \Pi_E \text{ propagator in } H_3 \text{ of} \\ \text{scalar dimension } E+1 \end{array} \right)$

- General spin J planar exchange

$$\frac{1}{N^2}\, S^{J-1} \int d\nu\ \beta_J(\nu) \cdot \Omega_{i\nu}(B)$$

with

$$\left(\Box_{H_3} + \nu^2 + 1\right)\ \Omega_{i\nu} = 0$$

ν trasverse momentum transfer in H_3 radius units

- Resumming in J

$$\Delta \simeq \frac{1}{N^2} \int d\nu\ S^{j(\nu)-1}\ \beta(\nu)\ \Omega_{i\nu}(B)$$

where

$j(\nu)$ and $\beta(\nu)$ depend on $\bar{\alpha}_s$

- Trajectory of lowest twist operators of dim. $2 + i\nu$ and spin $j(\nu)$

Strong Coupling String Effects

- Regge trajectory of the graviton $j\left(\nu, \bar{\alpha}_s\right)$ with

$$\bar{\alpha}_s = \frac{\ell^4}{\left(2\pi\alpha'\right)^2} = \frac{g_{\mathrm{YM}}^2 N}{4\pi^2}$$

- Flat space limit

$$\lim_{\ell\to\infty} j\left(\ell q, \ell^4 / \left(2\pi\alpha'\right)^2\right) = 2 - \frac{\alpha'}{2} q^2$$

- Energy–momentum tensor $j\left(\pm 2i, \bar{\alpha}_s\right) = 2$
- Decreasing intercept

$$j\left(\nu, \bar{\alpha}_s\right) = 2 - \frac{4 + \nu^2}{4\pi\sqrt{\bar{\alpha}_s}} - \cdots$$

Weak Coupling Saturation

- BFKL trajectory

$$j(\nu) = 1 + o(\bar{\alpha}_s)$$
$$\Delta(S, B) \text{ and } \beta(\nu) \text{ imaginary}$$

- Vanishing momentum transfer $t = 0$

s	Q^2	$\bar{Q}^2$
	$\left(q_1^2 = q_3^2\right)$	$\left(q_2^2 = q_4^2\right)$

- Focus on cross section

$$\int \frac{d\rho}{\rho^3}\, f_1(\rho)\;\; f_3(\rho) \qquad \rho \sim 1/Q$$
$$\int \frac{d\bar{\rho}}{\bar{\rho}^3}\, f_2(\bar{\rho})\;\; f_4(\bar{\rho}) \qquad \bar{\rho} \sim 1/\bar{Q}$$
$$\int d^2\mathbf{b}\;\, \mathrm{Re}\left[1 - e^{2i\Delta(S,B)}\right] \qquad \sigma(s, Q, \bar{Q})$$

- Integral over impact parameter

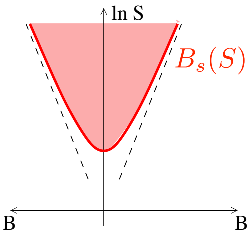
$$\sigma\left(s, Q, \bar{Q}\right) \sim \frac{1}{Q\bar{Q}} \int_{\left|\ln Q/\bar{Q}\right|}^{\infty} dB \;\sinh B \;\; \mathrm{Re}\left[1 - e^{2i\Delta(S,B)}\right]$$

with $S = s/Q\bar{Q}$

- For large B one has $|\Delta| \ll 1$. Phase shift ~ 1 along saturation line

$$\Delta \sim \int d\nu \; e^{\ln S \;\left(j(\nu)-1\right) - B(1+i\nu)}$$

so that

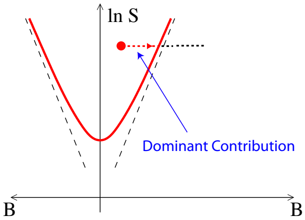


$$B_s\left(S\right) \sim \omega \ln S$$

$$\omega = 2.44\;\bar{\alpha}_s + \cdots$$
$$\sim 0.14 \quad \text{(exp. value)}$$

- Deep Saturation

$$|\ln Q/\bar{Q}| \lesssim B_s\,(s/Q\bar{Q})$$



- Approximate black disk

$$\frac{1}{Q\bar{Q}} \int_{|\ln Q/\bar{Q}|}^{B_s(S)} dB \;\; \sinh B \;\cdot\; 1$$

- When $B_s \sim \omega \;\ln S$ then

$$\frac{c}{Q\bar{Q}}\left[\left(\frac{s}{Q\bar{Q}}\right)^{\omega} + \left(\frac{s}{Q\bar{Q}}\right)^{-\omega}\right] - \frac{\tilde{c}}{Q\tilde{\Lambda}}\left[\frac{Q}{\tilde{\Lambda}} + \frac{\tilde{\Lambda}}{Q}\right]$$

with $c, \tilde{c}, \tilde{\Lambda}$ from $\rho, \bar{\rho}$ integrals

Applications to DIS

- $\mathcal{O}_1$ E&M current (photon)
  $\mathcal{O}_2$ proton

- Kinematics

$$s \simeq Q^2/x$$

$\bar{Q}$ related to confinement scale & mass of proton
(simulate confinement with wavefunction in $\bar{\rho}$)

- Cross section for small x is

$$Q^{-2}\ F_2(x, Q^2)$$

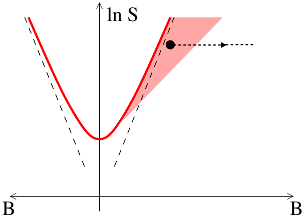
Geometric Scaling

- Near saturation

$$|\ln Q/\bar{Q}| \gtrsim B_s\,(s/Q\bar{Q})$$

cross section reads

$$\int d^2\mathbf{b}\ \operatorname{Im}\Delta\,(S,B) \simeq \frac{1}{Q\bar{Q}N^2}\int d\nu\ \beta\,(\nu)\ \ \left(\frac{s}{Q\bar{Q}}\right)^{j(\nu)-1}\ \ \left(\frac{Q}{\bar{Q}}\right)^{-i\nu}$$



- At saddle point

$$\sigma \sim \frac{1}{\bar{Q}^2}\ \tau^{-(1+i\nu_s)\frac{1-\omega}{2}}$$

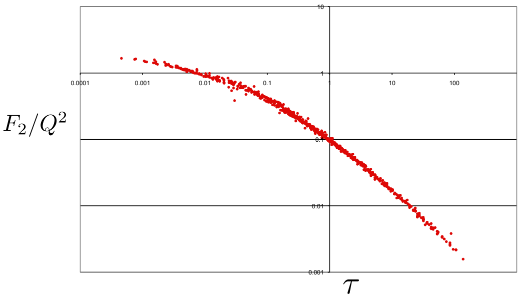
$$\tau = \left(\frac{Q}{Q_s}\right)^2 \qquad \text{with} \qquad Q_s = \bar{Q}^2\left(\frac{1}{x}\right)^{\frac{2\omega}{1-\omega}}$$

- In deep saturation

$$\sigma \sim \frac{1}{Q\bar{Q}} \left(\frac{Q}{x\bar{Q}} \right)^{\omega} \sim \frac{1}{\bar{Q}^2}\ \tau^{-\frac{1-\omega}{2}}$$

Specific dependence on the scaling variable

- Experimental evidence

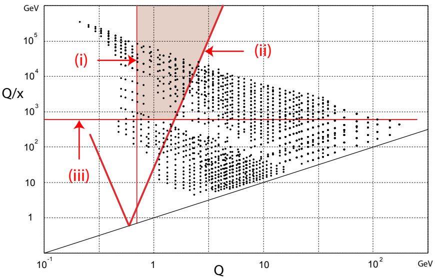


ω fixed experimentally to be $\omega \simeq 0.138 \pm 0.021$

Fit to DIS Data

- Expression for $F_2\left(Q^2, x\right)$

$$c\,\frac{Q}{\bar{Q}}\left[\left(\frac{Q}{x\bar{Q}}\right)^{\omega}+\left(\frac{Q}{x\bar{Q}}\right)^{-\omega}\right]-\tilde{c}\,\frac{Q}{\tilde{\Lambda}}\left[\frac{Q}{\tilde{\Lambda}}+\frac{\tilde{\Lambda}}{Q}\right]$$



(i) Weak coupling

$$Q > Q_{\text{min}} \sim 0.7 - 1\text{GeV}$$

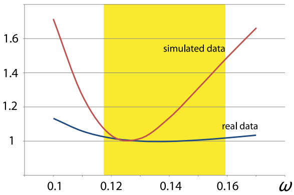
(ii) Inside saturation

$$\omega \ln \frac{Q}{x\bar{Q}} > \ln \frac{Q}{\bar{Q}} \qquad (\bar{Q} \sim 0.2 - 1 \text{ GeV})$$

(iii) Asymptotic linear regime

$$\frac{Q}{x\bar{Q}} \gtrsim 10^{\eta} \qquad (\eta \gtrsim 3)$$

- Minimize mean square deviation against experimental and simulated data



$\omega \simeq 0.126$

$c \simeq 0.13$

$\tilde{c} \simeq 0.14$

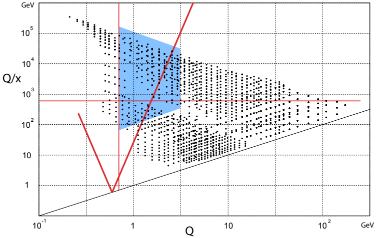
$\tilde{\Lambda} \simeq 1$ (GeV)

$\omega = 0.138 \pm 0.021$

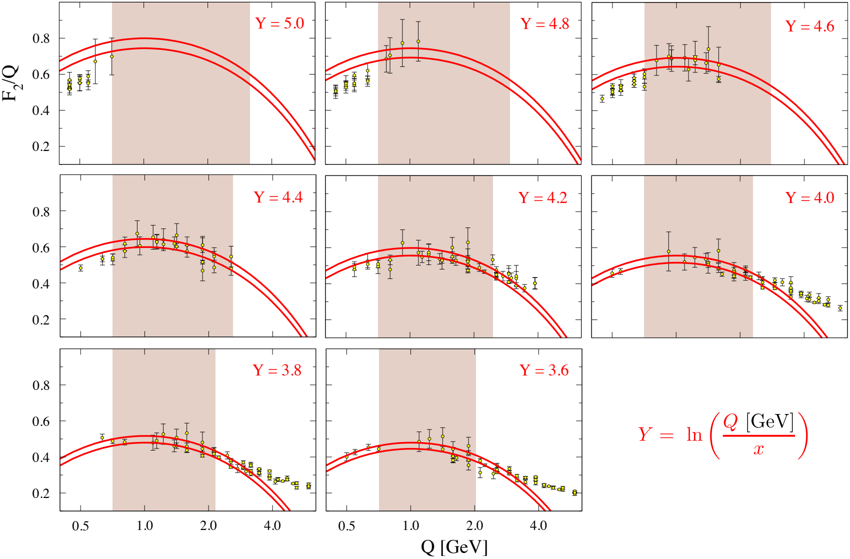
- Match experimental data in rather large kinematical range with 6% accuracy

$0.5 < Q^2 < 10$

$x < 10^{-2}$

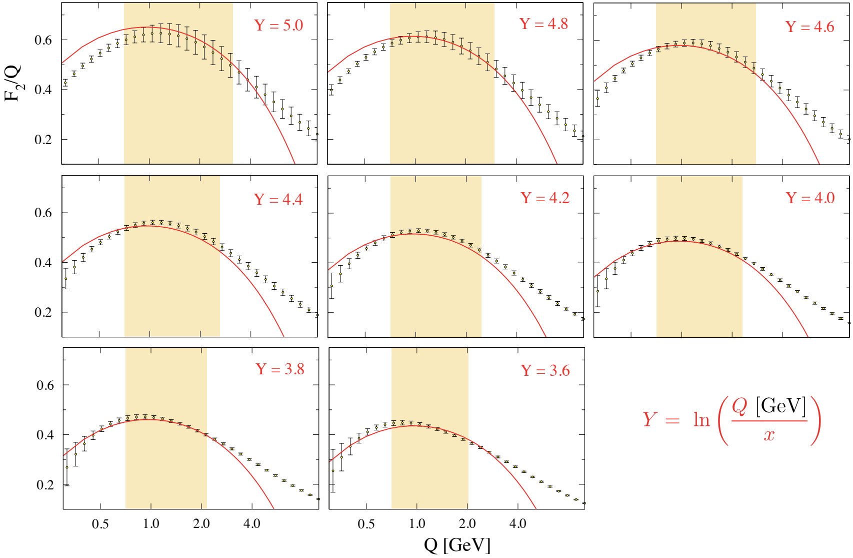


Real Data



$$Y = \ln\left(\frac{Q\ [\mathrm{GeV}]}{x}\right)$$

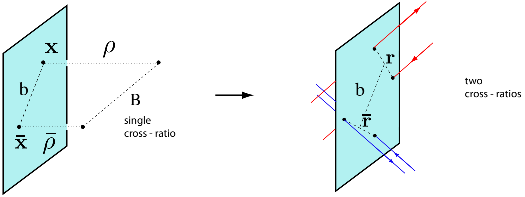
Simulated Data



Comments on Dipole Formalism

- Dipole phase shift

$$\Delta\left(s, \rho, \bar{\rho}, \mathbf{b}\right) \quad \longrightarrow \quad \Delta_D\left(s, \mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}\right)$$



- Representation of Δ_D

$$\int d\nu\ \alpha\left(\nu\right)\ s^{j(\nu)-1} \cdot \mathcal{T}_{i\nu}\left(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}\right)$$

where

$$\mathcal{T}_{i\nu}\left(\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}\right) \simeq \left(\left|\mathbf{r}\right|\left|\bar{\mathbf{r}}\right| / \mathbf{b}^2\right)^{1+i\nu} \qquad \text{for} \quad \left|\mathbf{r}\right|, \left|\bar{\mathbf{r}}\right| \ll \left|\mathbf{b}\right|$$

- For $\rho, \bar{\rho} \ll |\mathbf{b}|$

$$B \sim \ln \mathbf{b}^2 / \rho\bar{\rho}$$
$$\Delta \sim \int d\nu \; S^{j(\nu)-1} \cdot \; (\rho\bar{\rho}/\mathbf{b})^{1+i\nu}$$

- Δ_D does not satisfy unitarity constraints (no asymptotic dipole states)

- Even if one assumes saturation at $\operatorname{Im} \Delta_D \sim 1$, a simple exponential saddling for Δ_D is not possible for general $\mathbf{r}, \bar{\mathbf{r}}, \mathbf{b}$.

- For $|\mathbf{r}|, |\bar{\mathbf{r}}| \ll |\mathbf{b}|$ one obtains only the first term in σ. A pure black disk is then a poor approximation of experimental data

Future Directions

- Is deeply saturated DIS showing us a black disk in AdS ?
- Non conformal extension to holographic QCD models
- Role of strong coupling and of graviton exchange
- Role of non linear effects and relation to black hole formation
- More formal developments
  - Relation of closed BFKL trajectory to integrability of N=4 SYM and to open gluon trajectory
  - Relation of open and closed amplitudes
  - Flat space limit and S-duality
- Will LHC and future colliders give us precise data in the relevant kinematical window?