Magnetic monopoles in high temperature QCD

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Strings and Strong Interactions 08, Frascati

Outline

Motivation

Abelian magnetic monopoles are candidates for explaining color confinement within the dual superconducting model of the QCD vacuum (confinement is induced by the breaking of a magnetic $U(1)$ symmetry via monopole condensation).

The magnetic component is supposed to be relevant (Chernodub & Zakharov '06, Liao & Shuryak '06 in explaining the physical properties of the Quark Gluon Plasma phase (above the transition).

It has been identified (Chernodub & Zakharov '06) with abelian magnetic monopoles "evaporating" from the condensate at $T > T_c$.

The Abelian Projection

How can we get abelian monopoles from a non abelian theory such as QCD?

- First we fix a gauge that leaves a $U(1)$ residual symmetry: in the Maximal Abelian Gauge we maximize $F_{\mathrm{MAG}} = \sum_{\mu,x} \mathrm{Re}\,\mathrm{tr}\left[U_\mu(x)\sigma_3 U_\mu^\dagger(x)\,\sigma_3\right]$
- Then we take the diagonal part of the links (Abelian Projection)

Possible dependence of the abelian observables on the gauge fixed prior the projection!!!

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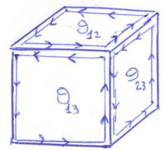
De Grand-Toussaint



De Grand elementary cube (in 3D)

On abelian projected configurations monopole currents are defined as
$m_\mu = \frac{1}{2\pi}\varepsilon_{\mu\nu\rho\sigma}\hat{\partial}_\nu\overline{\theta}_{\rho\sigma}$
where $\overline{\theta}_{\rho\sigma}$ is the compactified part of the abelian plaquette phase (De Grand & Toussaint '80).

- Quantization of charge
- Closure of monopole currents: $\hat{\partial}_\mu m_\mu = 0$

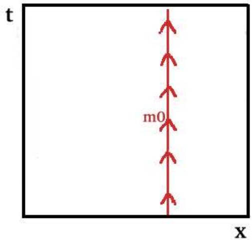
The thermal monopole density

At $T < T_c$ magnetic currents are virtual;
At $T > T_c$ currents and monopoles become real (magnetic currents percolate in temporal direction).

Real particle = wrapped trajectory on the compact t direction (Chernodub & Zakharov '07).

$$\rho = \frac{\sum_{\vec{x}} \left| N_{wrap}(m_0(\vec{x},t)) \right|}{V_s}$$

$m_0(\vec{x}, t)$ = magnetic trajectory in time direction



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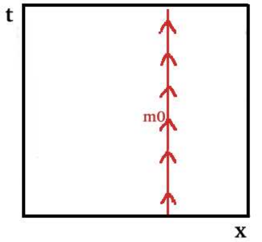

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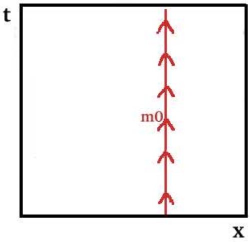

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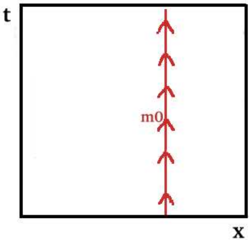


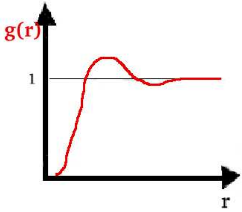
The monopole-(anti)monopole correlation function

$$g(r) = \frac{<\rho(0)\rho(r)>}{<\rho><\rho>} \text{ (monopole-monopole)}$$

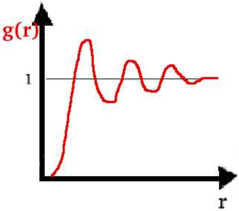
$$g(r) = \frac{<\rho^+(0)\rho^-(r)>}{<\rho^+><\rho^->} \text{ (monopole-antimonopole)}$$

$$g(r) = 1$$

g(r)-free gas



g(r)-liquid



g(r)-solid

- $g(r) = 1 \Rightarrow$ no interaction
- If the interaction potential $V(r)$ is weak we can extract it through $g(r) = exp(-V(r)/T)$.

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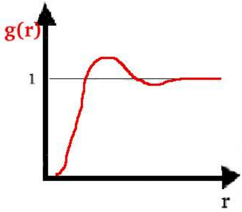

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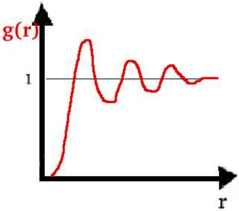

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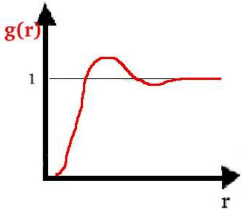

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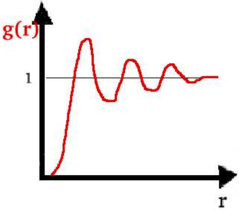

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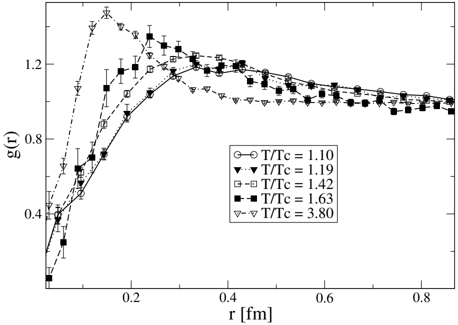

- Fit with screened Coulomb $V(r) = \alpha_M e^{-r/\lambda}/r$, $\lambda \sim 0.2$ fm;
- Liquid-like structure!!
  Stronger α_M coupling at high T (Liao & Shuryak '07);
- Agreement with MD simulation of std. EM plasma (Liao & Shuryak '07)

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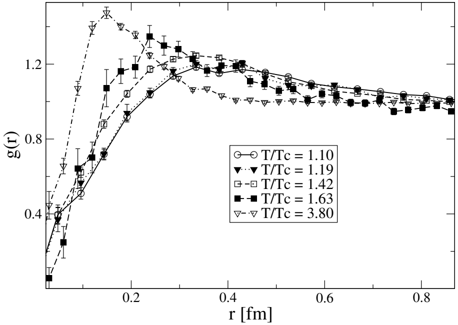

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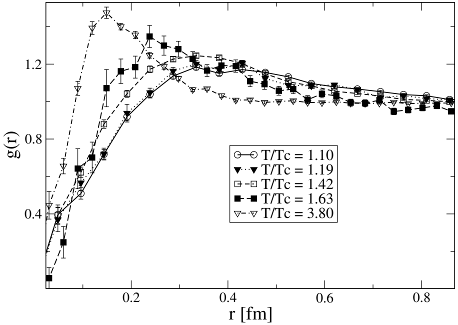

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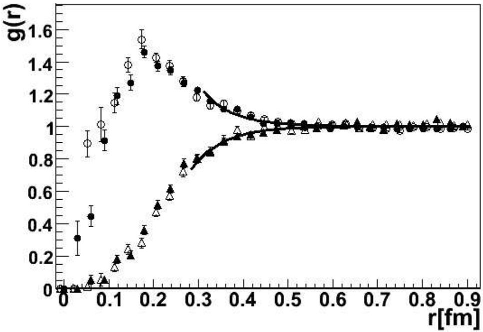

Monopole-monopole (triangles) Vs. Monopole-antimonopole (circles) at different β's

- Monopoles repel monopoles and attract antimonopoles;
- The scaling is good.

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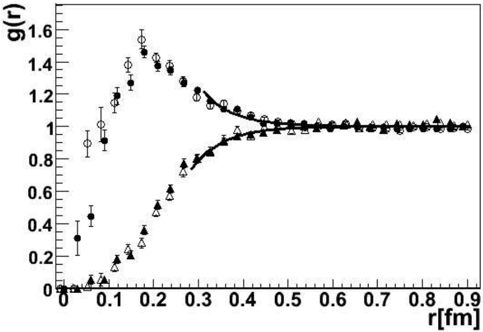

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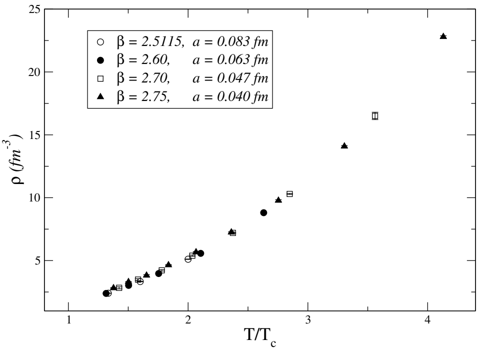

Monopole density (with MAG)

- $\rho \neq \frac{\zeta(3)}{\pi^2} T^3$ (free particles) $\Rightarrow$ interactions are important!!!
  Nice fit with $\rho \sim T^3/(\log(T/\Lambda_{eff}))^\alpha$ with $\alpha \sim 2-3$
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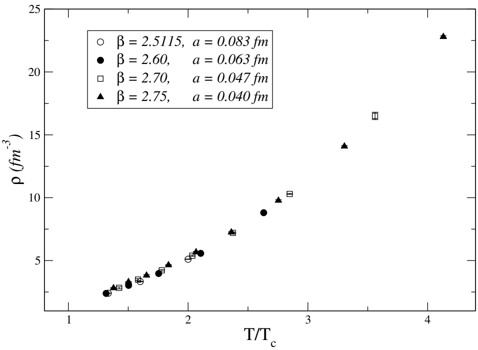

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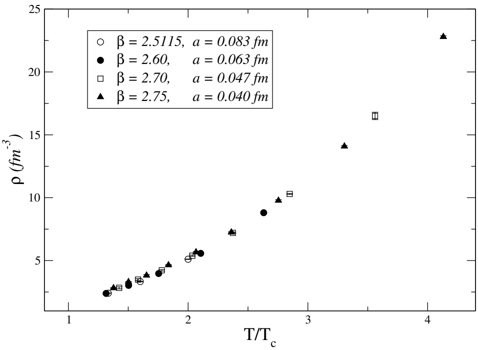

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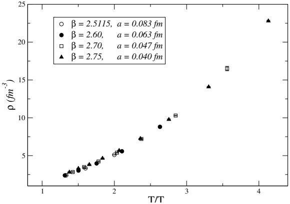
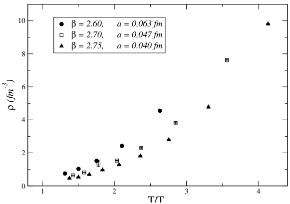
The gauge dependence problem

- In the Landau gauge, defined by maximizing $F_L = \sum_{\mu,x} \mathrm{Re}\,\mathrm{tr}\, U_\mu(x)$ before the Abelian projection, the monopole density is compatible with zero.

The Gribov ambiguity



Monopole density (MAG)



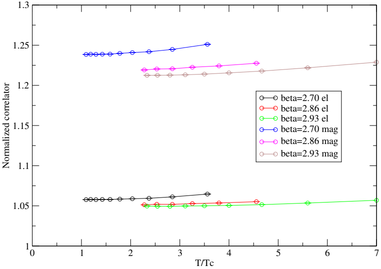
Monopole density (Landau preconditioning + MAG)

Within the same MAG gauge we start the gauge fixing iterative algorithm from a Landau gauged configuration: the density is now different and the scaling is lost. We are on a different local maximum of F_{MAG}.
A similar behavior was observed for vortices in center dominance studies (Bornyakov et al. '96, Kovacs & Tomboulis '99, Greensite et al. '01)

Work in progress I (also with M.Chernodub and V.Zakharov)

- Excess of chromoelectric/magnetic action around monopoles: monopoles are likely physical if they carry an excess of action (Bakker, Chernodub & Polikarpov '97)

$$\frac{<\sum_x \mathrm{Tr}(m_\mu(x) F^{el/mag}_{\mu\nu}(x))^2>}{<\sum_x m_\mu(x)^2><\sum_x \mathrm{Tr} F^{el/mag}_{\mu\nu}(x)^2>}$$



- Temporal-spatial asymmetry of monopole trajectories (how static are monopoles?)
- Distribution of the winding numbers of single trajectories

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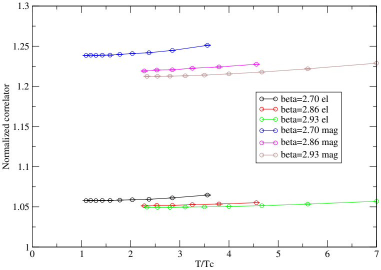

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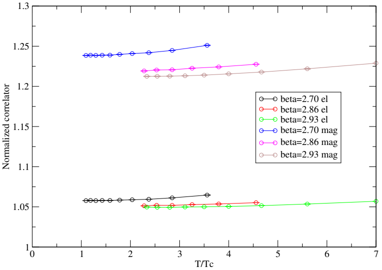

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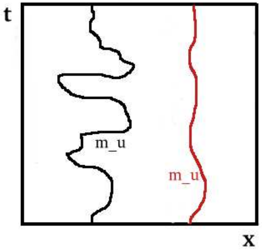
Work in progress II (following Shuryak and Liao's proposal)

- Monopole mass: Debye screening mass (varying from 0.5 to 5 GeV) from fits of g(r) $\neq$ dynamical mass, which is related to monopole current fluctuations in space.
  $T < \int dt (x(t) - x(0))^2 > \sim 1/(2mT)$ (free case)

T/T_c	m[GeV]
1.095	0.617(16)
2.848	4.856(11)
7.000	15.59(2)



fluctuations of a heavy/light monopole

Work in progress III

- Monopoles with the "gauge-independent" definition of Ito, Kato, Kondo, Murakami, Shibata, Shinohara (Phys.Lett.B645:67(2007)):
  1. Fix gauge on higher level ($SU(2) \times SU(2)/U(1) \rightarrow SU(2)$): $F_{\rm nMAG} = \sum_{\mu,x} \mathrm{Re}\,\mathrm{tr}\left[U_\mu(x)\vec{n}(x)\, U^\dagger_\mu(x)\, \vec{n}(x+\mu)\right]$ invariant under simultaneous $U_\mu(x)$ and $\vec{n}(x)$ (adjoint field) transformations
  2. CFN decomposition: $V_\mu = U_\mu + \vec{n}(x) U_\mu(x) \vec{n}(x+\mu)$ is the abelian part of U_μ for $\vec{n} = \sigma_3$
  3. m_μ from DeGrand cubes of $SU(2)$ gauge invariant plaquettes $\mathrm{Tr}[(1+\vec{n}(x))\, V_\mu(x)\, V_\nu(x+\mu)\, V^\dagger_\mu(x+\nu)\, V^\dagger_\nu(x)]$

As $F_{\rm nMAG} = F_{\rm MAG}$ for $\vec{n} = \sigma_3$ results are completely equivalent to MAG, as expected.

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Summary

- We measured the density of thermal monopoles in the deconfined phase. An interacting behavior is observed, as $\rho \not\propto T^3$ (precisely $\rho \sim T^3/(\log(T/\Lambda_{eff}))^\alpha$ with $\alpha \sim 2$, even 3 at high T)
- We observed the monopole-(anti)monopole correlation function. A liquid-like behavior is observed.
- A very good physical scaling is observed for monopoles obtained with the standard Maximal Abelian Gauge;
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