

**OBSERVING THE DECAY
OF UNSTABLE STRINGS
IN $SU(2)$ YANG-MILLS THEORY**

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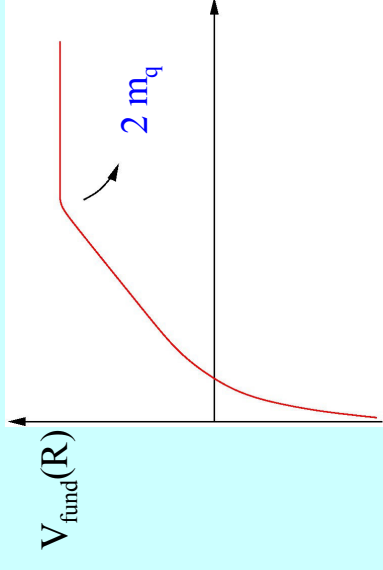
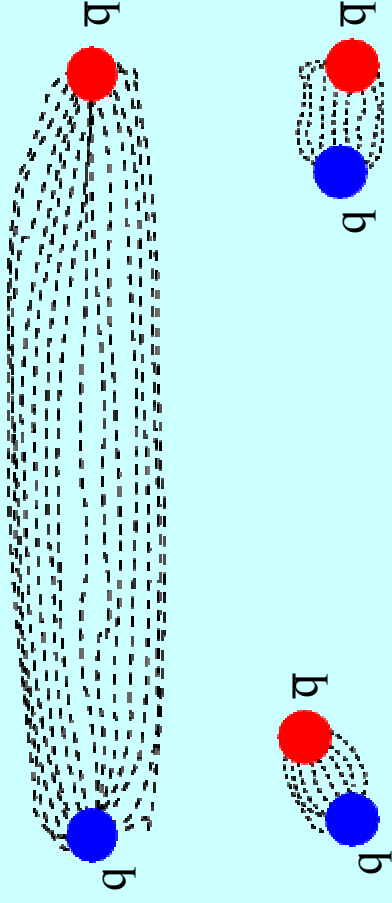
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PLAN OF THE TALK

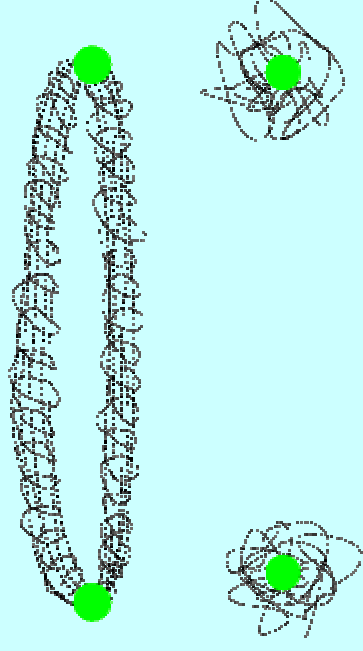
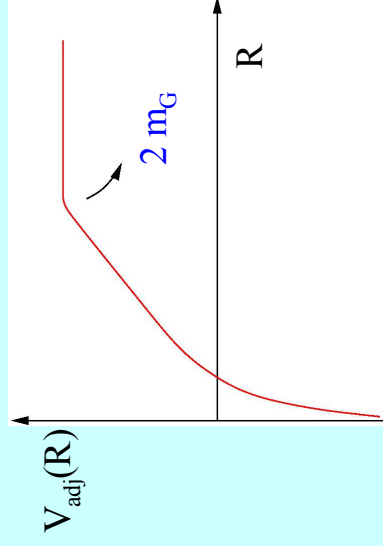
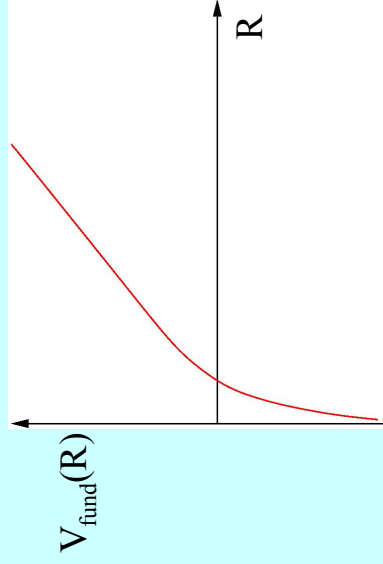
- Introduction
- String decay: a toy model
- Numerical results
- Conclusions

Introduction

At low temperature quarks are confined inside baryons



If $m_q \rightarrow \infty$ the string between two fundamental charges becomes stable
the string between two adjoint charges is still unstable by gluon emission



- Observing the decay of unstable strings is an important step in studying the phenomenology of confinement

- **SU(N) Yang-Mills**: a simplified framework where these effects can be studied with higher accuracy

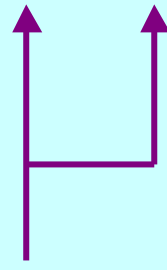
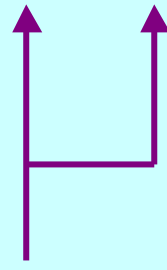
- SU(N) representations branch in N sectors (N-ality sectors)

$$\mathcal{R}_k \otimes \{\text{adj}\} \otimes \{\text{adj}\} \dots = \mathcal{R}'_k \oplus \dots \implies V_{\mathcal{R}_k} \sim V_{\mathcal{R}'_k}$$

by gluon emission

- For every N-ality sector there is a stable string; all other strings in that sector are unstable and decay into the stable as $R \rightarrow \infty$

- SU(2): 2 sectors

	$\{1\}$ sources of integer spin
	$\{-1\}$ sources of half-integer spin

- Very challenging numerically: only adjoint sources

Many groups:

C. Michael et al.,
 Bali et al., Sommer,
 Stephenson,
 Philipsen et al.,
 de Forcrand et al.,
 Gliozzi et al.,
 Vicari et al., ...



String decay: a toy model

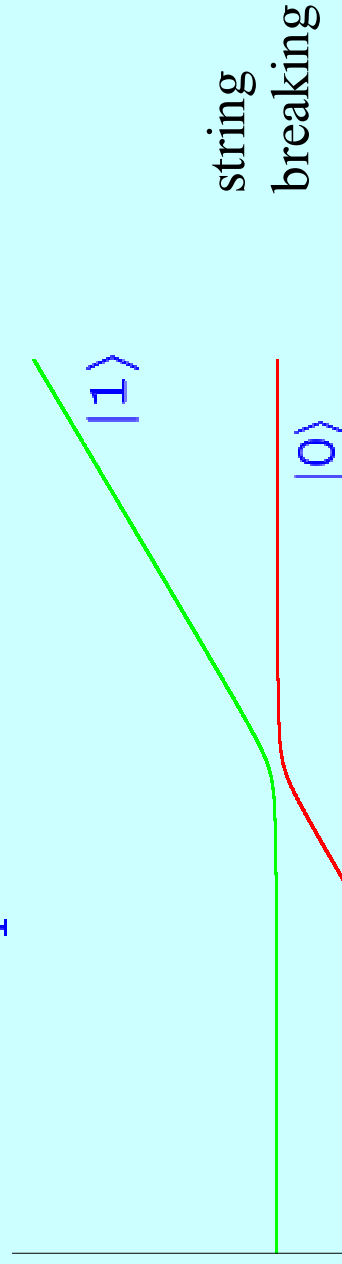
Consider a two-state quantum system described by the Hamiltonian

$$H_2 = \begin{pmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{pmatrix} \quad \text{in the basis} \quad |S\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |L\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

ground state: $V_0 \implies |0\rangle = a_0|S\rangle + b_0|L\rangle \quad a_0^2 + b_0^2 = 1$

excited state: $V_1 \implies |1\rangle = a_1|S\rangle + b_1|L\rangle \quad a_1^2 + b_1^2 = 1$

$|S\rangle \simeq$  $|L\rangle \simeq$  $E_1 = \sigma R$ $E_2 = 2m$

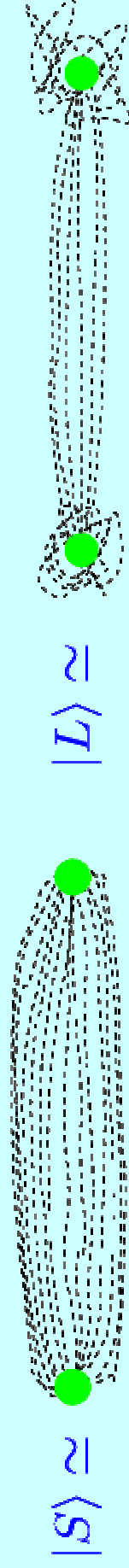


A different two-state quantum system described by a similar Hamiltonian

$$H_2 = \begin{pmatrix} E_1 & \epsilon \\ \epsilon & E_2 \end{pmatrix} \quad \text{in the basis} \quad |S\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |L\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

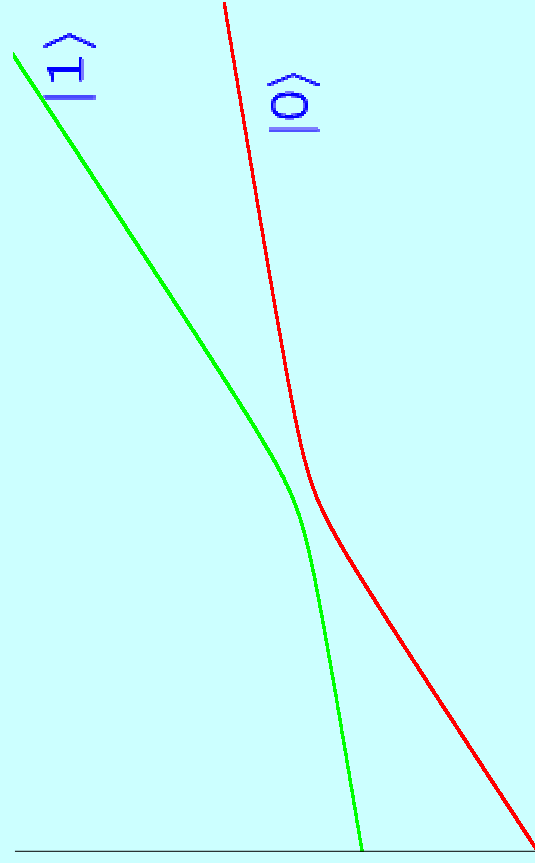
ground state: $V_0 \implies |0\rangle = a_0|S\rangle + b_0|L\rangle \quad a_0^2 + b_0^2 = 1$

excited state: $V_1 \implies |1\rangle = a_1|S\rangle + b_1|L\rangle \quad a_1^2 + b_1^2 = 1$



$$E_1 = \sigma_0 R$$

$$E_2 = \sigma_1 R + 2m$$



string decaying to
another string

Consider a three-state quantum system described by the Hamiltonian

$$H_3 = \begin{pmatrix} E_1 & \epsilon_1 & \epsilon_2 \\ \epsilon & E_2 & \epsilon_3 \\ \epsilon_2 & \epsilon_3 & E_3 \end{pmatrix} \text{ in the basis } |S\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |I\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |L\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

ground state: $V_0 \Rightarrow |0\rangle = a_0|S\rangle + b_0|I\rangle + c_0|L\rangle \quad a_0^2 + b_0^2 + c_0^2 = 1$

1st excited state: $V_1 \Rightarrow |1\rangle = a_1|S\rangle + b_1|I\rangle + c_1|L\rangle \quad a_1^2 + b_1^2 + c_1^2 = 1$

2nd excited state: $V_2 \Rightarrow |2\rangle = a_2|S\rangle + b_2|I\rangle + c_2|L\rangle \quad a_2^2 + b_2^2 + c_2^2 = 1$



string decaying and then breaking

“explicit mixing method”

We have a multi-state quantum system

$$|X_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle + \gamma_1|2\rangle + \dots$$

$$|X_2\rangle = \alpha_2|S\rangle + \beta_2|I\rangle + \gamma_2|L\rangle + \dots$$

...

$$\begin{pmatrix} \langle X_1|e^{-HT}|X_1\rangle & \langle X_1|e^{-HT}|X_2\rangle & \dots \\ \langle X_2|e^{-HT}|X_1\rangle & \langle X_2|e^{-HT}|X_2\rangle & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

$|X_1\rangle, |X_2\rangle, \dots$ chosen to enhance the overlap with $|0\rangle$ at every distance (small T)

“implicit mixing method”

$$\begin{aligned} \langle X|e^{-HT}|X\rangle &= \alpha^2 e^{-V_0 T} + \beta^2 e^{-V_1 T} + \gamma^2 e^{-V_2 T} + \dots \\ &= \alpha^2 e^{-V_0 T} \left(1 + \frac{\beta^2}{\alpha^2} e^{-(V_1 - V_0)T} + \frac{\gamma^2}{\alpha^2} e^{-(V_2 - V_0)T} + \dots \right) \end{aligned}$$

$$\xrightarrow{T \rightarrow \infty} \alpha^2 e^{-V_0 T}$$

large T: numerically challenging

Numerical results

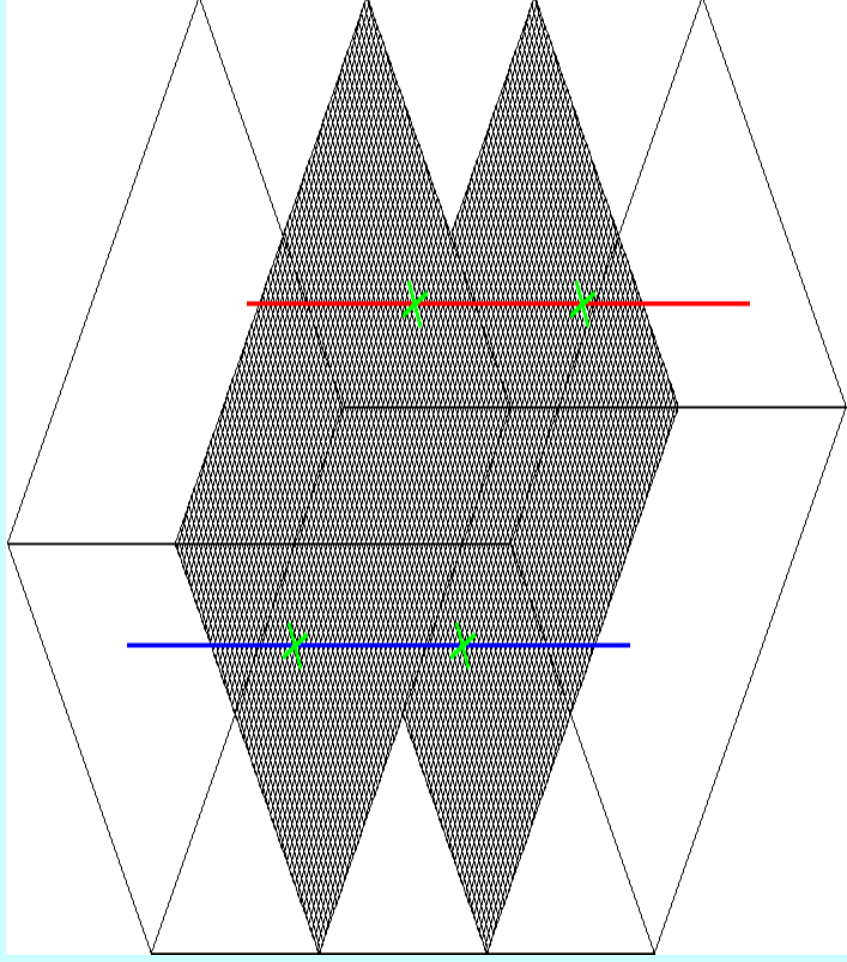
We consider the SU(2) Yang-Mills theory on the lattice in (2+1)-d and we use the “implicit method” to observe the string decay

$$\text{Wilson action: } S_{YM}[U] = -\beta \sum_P \text{Tr}(U_P)$$

$$32^2 \times 64 \quad \text{at } \beta = 6.0$$

$$\mathcal{O} = \frac{\int \mathcal{D}U \phi_{\mathcal{R}}(R) \phi_{\mathcal{R}}(0) e^{-S_{YM}}}{\int \mathcal{D}U e^{-S_{YM}}}$$

$$\phi_{\mathcal{R}}(\vec{x}) = \text{Tr}_{\mathcal{R}} \left[\prod_t U_4(\vec{x}, t) \right]$$



$$\mathcal{O} \sim e^{-V_0 T} \sim (e^{-V_0 \tau})^{T/\tau}$$

$\mathcal{O} \rightarrow$ tensor product of
the two segments

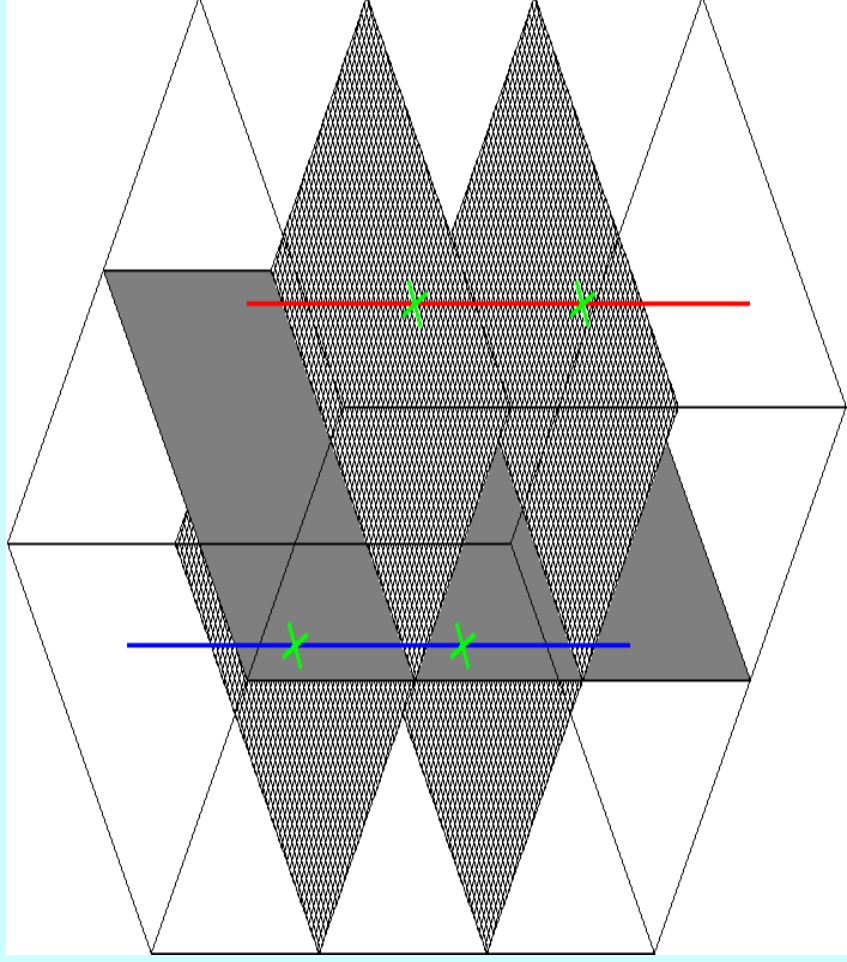
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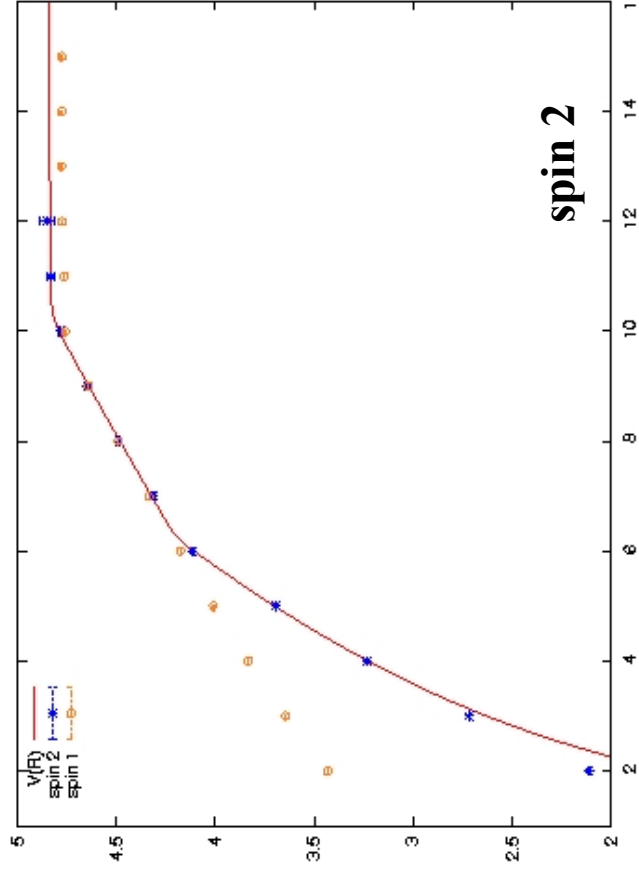
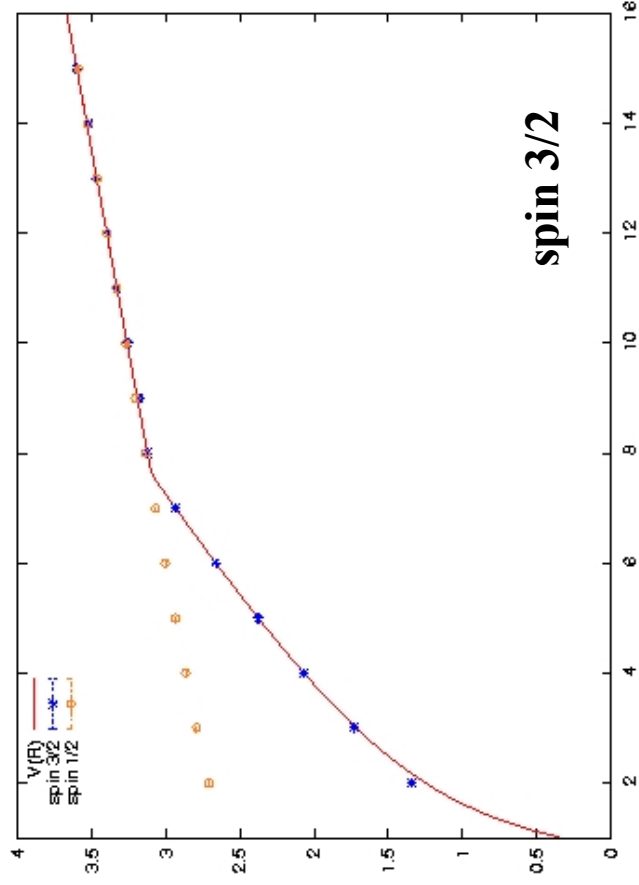
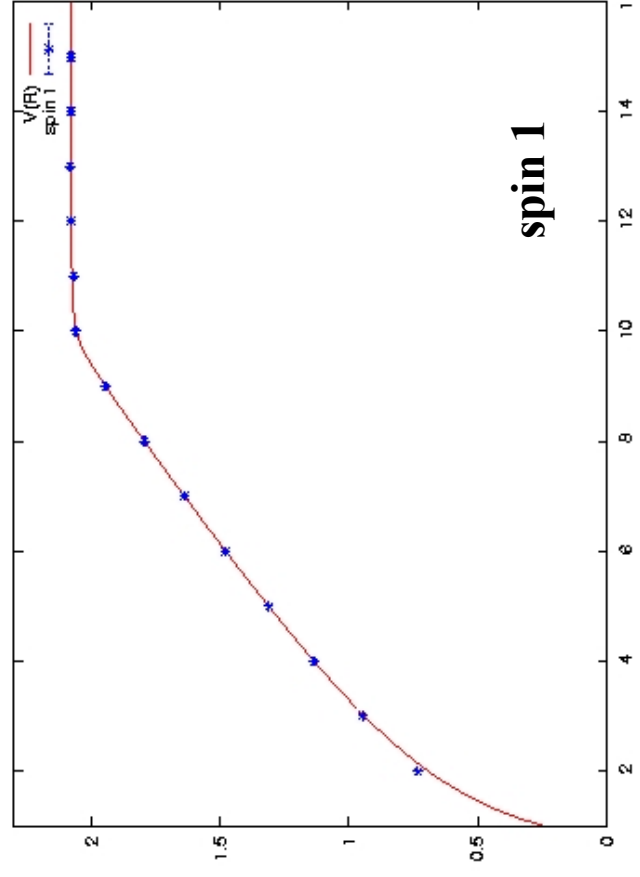
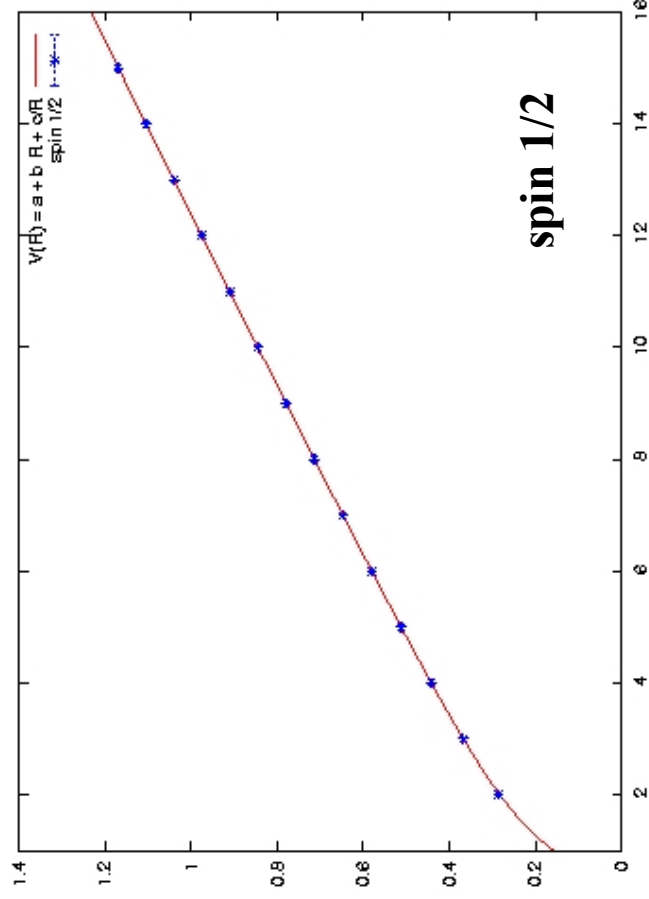
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$$e^{-V_0 \tau} \sim (e^{-V_0 \tau / 2})^2$$

Potential

N-ality = {-1}

N-ality = {1}



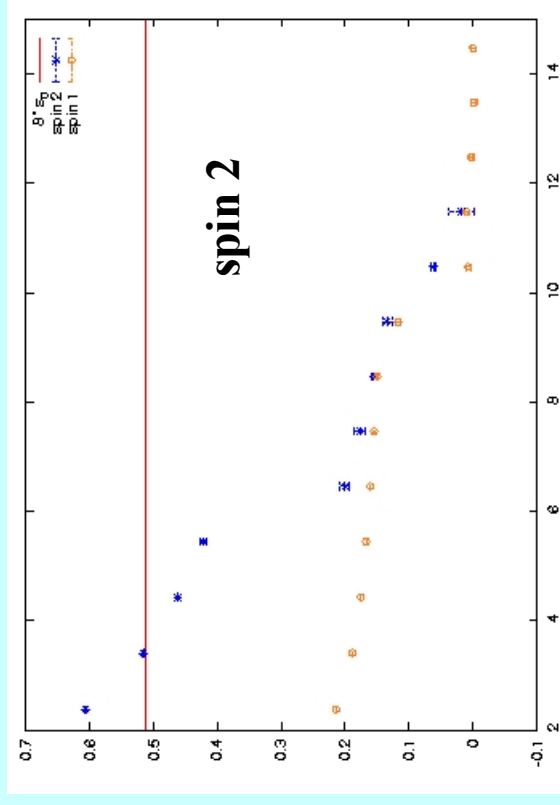
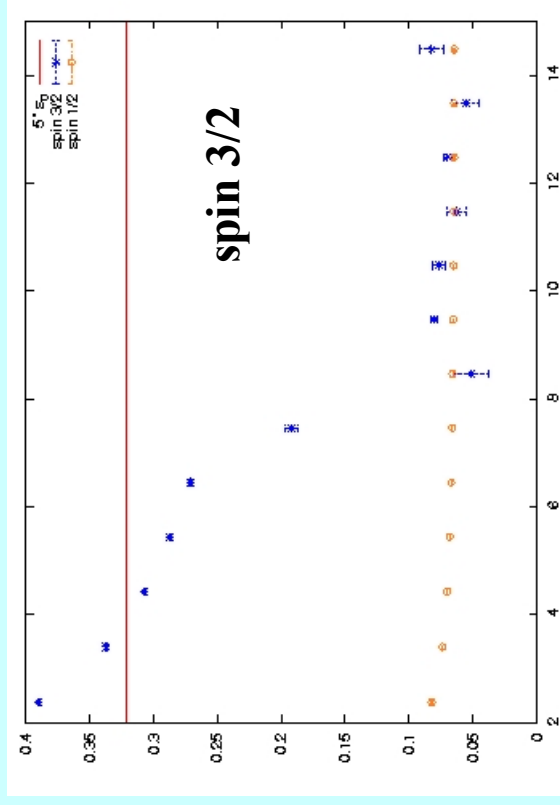
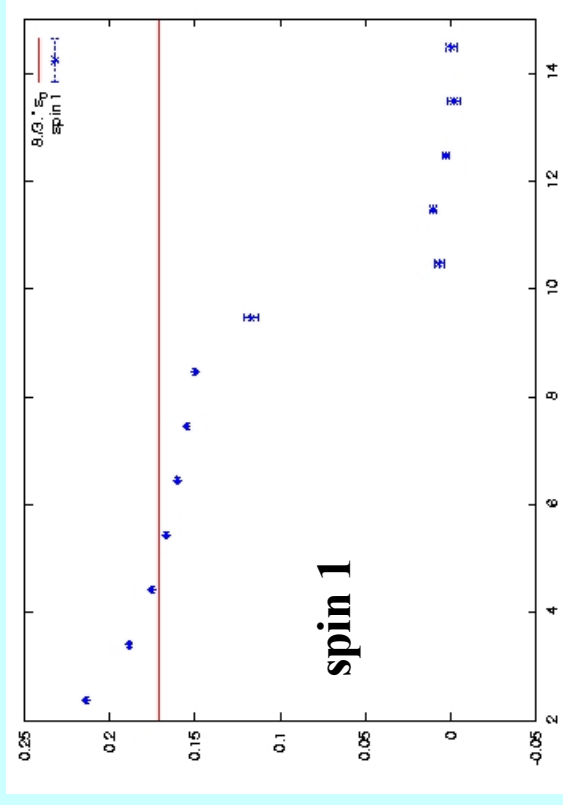
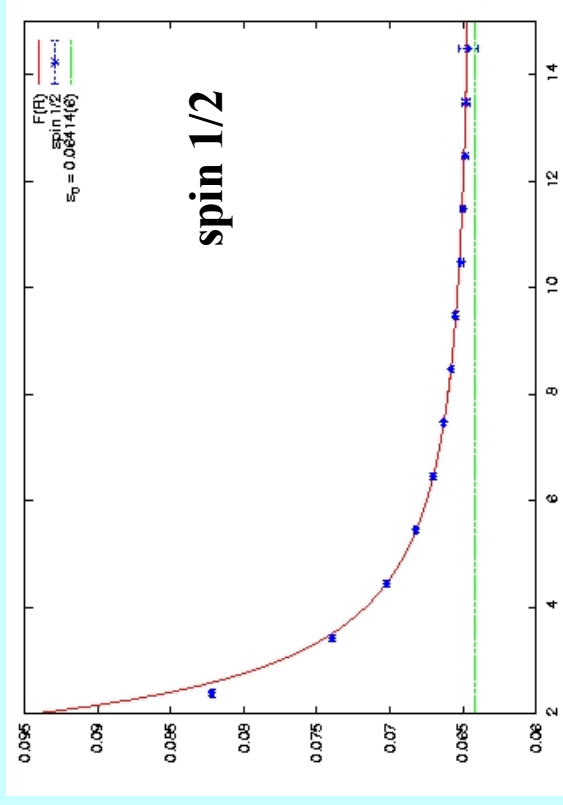
Force

$$V(R) \sim \sigma R - \frac{c}{R} + k \implies$$

$$F(R) \sim \sigma + \frac{c}{R^2}$$

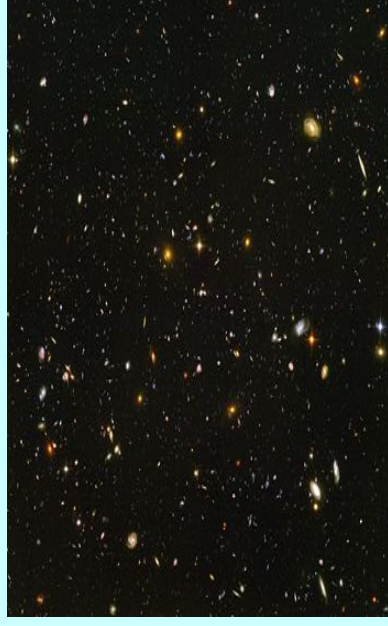
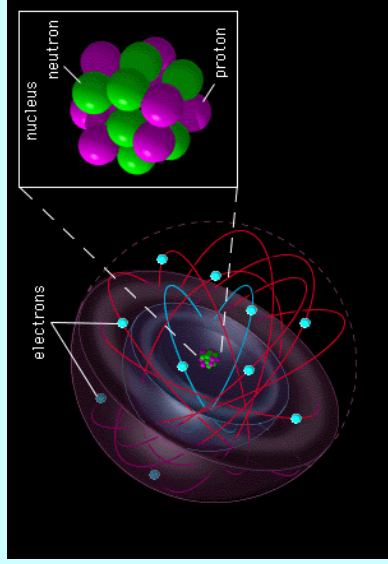
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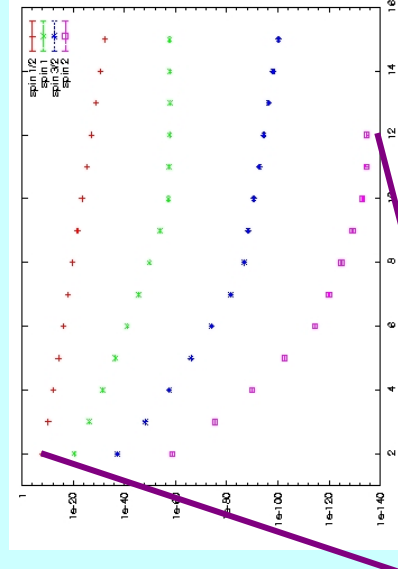


Conclusions

- We have observed the decay of unstable strings using a single observable in SU(2) Yang-Mills theory
- We have measured the 2-point function for the reps $\{2\}$, $\{3\}$, $\{4\}$, $\{5\}$: it is an important step in the study of the phenomenology of confinement
 - decay of $\{4\} \rightarrow \{2\}$ double decay $\{5\} \rightarrow \{3\} \rightarrow \{1\}$
- Casimir scaling for the string tensions is ruled out ((2+1)-d)
- Numerically very challenging: used the multilevel Lüscher-Weisz algorithm



nucleon radius \longleftrightarrow universe
40 orders of magnitude



$10^{-8} \rightarrow 10^{-135}$: ~ 130 orders
of magnitude!