Flavour and Mesons in the AdS/CFT correspondence

Johanna Erdmenger

Max Planck-Institut für Physik, München

Outline

I. Introduction to AdS/CFT

II. Adding Flavour

III. Chiral Symmetry Breaking and Mesons

IV. Finite Temperature and Density

AdS/CFT Correspondence

(Maldacena 1997, AdS: Anti de Sitter space, CFT: conformal field theory)

- Duality Quantum Field Theory $\Leftrightarrow$ Gravity Theory
- Arises from String Theory in a particular low-energy limit
- Duality: Quantum field theory at strong coupling

  $\Leftrightarrow$ Gravity theory at weak coupling
- Works for large N gauge theories at large 't Hooft coupling λ

Conformal field theory in four dimensions

$\Leftrightarrow$ Supergravity Theory on $AdS_5 \times S^5$

AdS/CFT correspondence

- Anti-de Sitter space is a curved space with constant negative curvature. It has a boundary.

  Metric: $ds^2 = e^{2r/R}\eta_{\mu\nu}dx^\mu dx^\nu - dr^2$

- Isometry group of $(d+1)$-dimensional AdS space coincides with conformal group in d dimensions ($SO(d,2)$).

- $SO(6) \simeq SU(4)$: Isometry of $S^5 \Leftrightarrow \mathcal{N}=4$ Supersymmetry

- Dictionary:

field theory operators $\Leftrightarrow$ supergravity fields

$$\mathcal{O}_\Delta \leftrightarrow \phi_m \ , \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + R^2 m^2}$$

AdS/CFT correspondence

- field-operator correspondence:

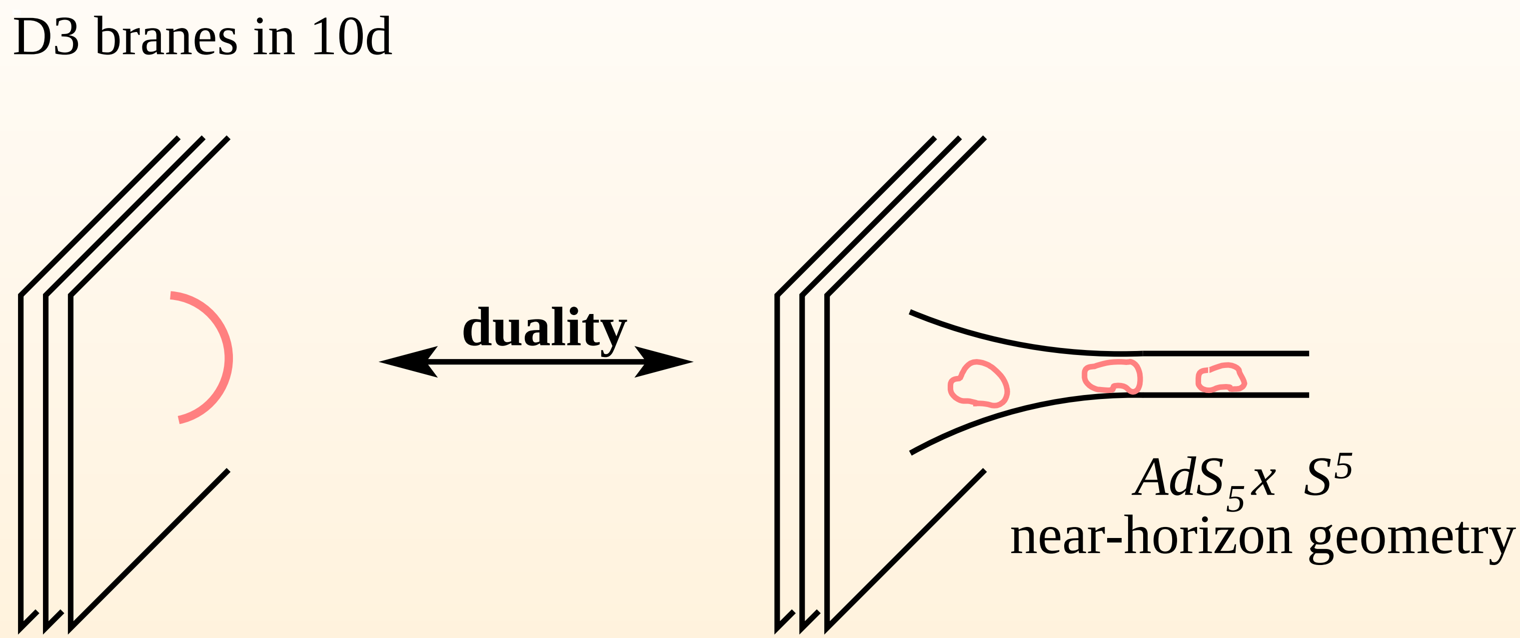
$$\langle e^{\int d^d x\, \phi_0(\vec{x})\mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{sugra}\Big|_{\phi(0,\vec{x})=\phi_0(\vec{x})}$$

Generating functional for correlation functions of particular composite operators in the quantum field theory

coincides with

Classical tree diagram generating functional in supergravity

String theory origin of AdS/CFT correspondence



$\Downarrow$ Low-energy limit

$\mathcal{N} = 4$ $SU(N)$ theory in four dimensions ($N \to \infty$)

Supergravity on $AdS_5 \times S^5$

$\mathcal{N} = 4\ SU(N)$ SUSY Gauge theory:

- $N \to \infty$
- Supersymmetry
- Conformal symmetry
- All fields in the adjoint representation of the gauge group

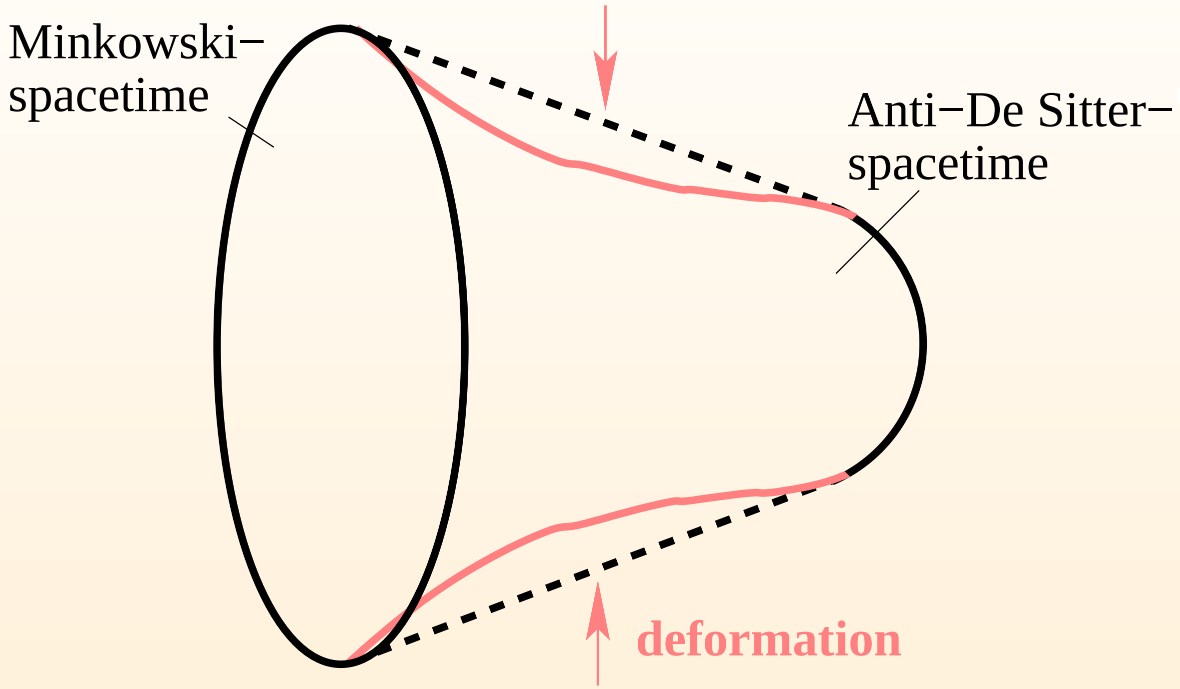
QCD:

- $N = 3$
- No supersymmetry
- Confinement
- Quarks in fundamental representation of the gauge group

Desirable extensions of AdS/CFT:

- Relax $N \to \infty$ limit ($1/N$ corrections)
  $\Leftrightarrow$ String theory instead of supergravity
- Break SUSY and conformal symmetry $\Leftrightarrow$ Deformation of AdS space
- Add quarks in fundamental representation of gauge group

Deformations of AdS space



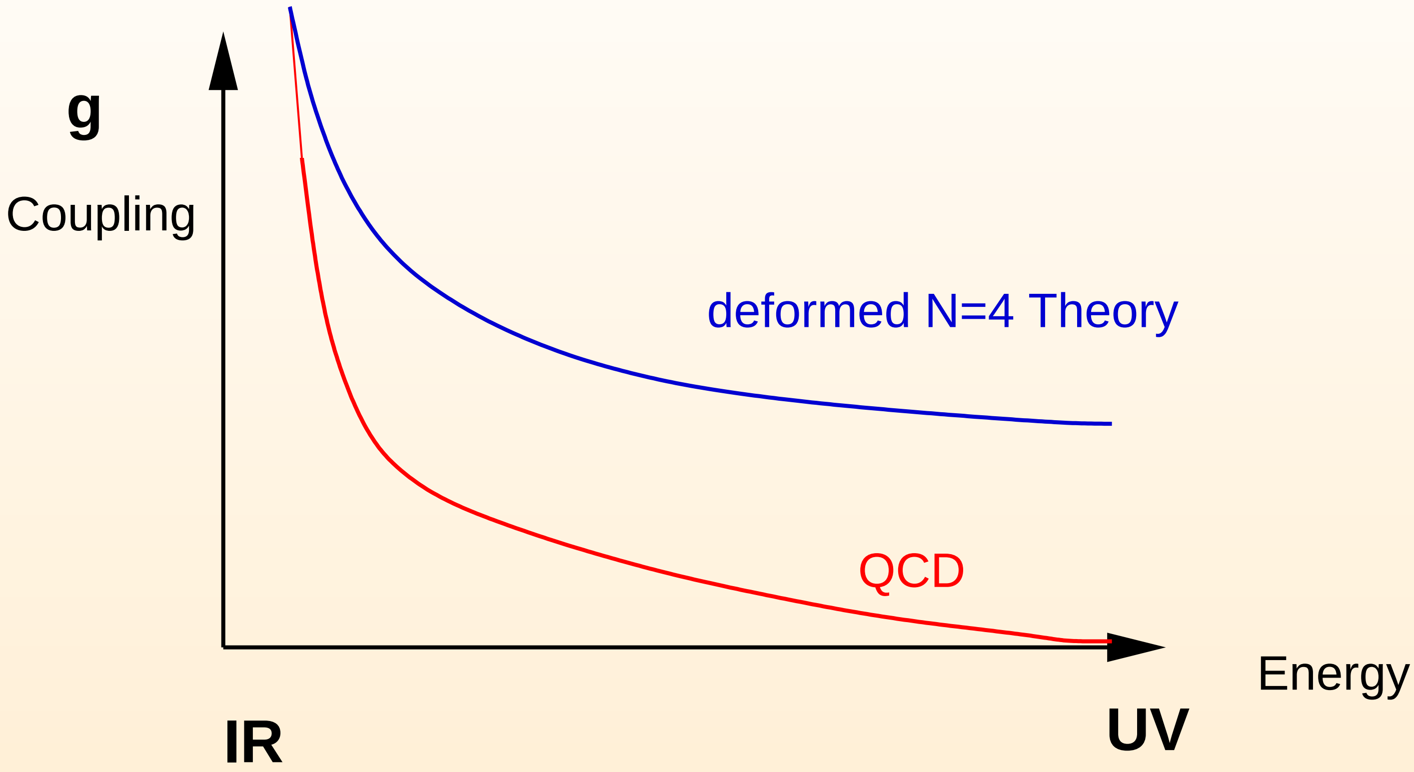
Fifth Dimension $\Leftrightarrow$ Energy scale

Renormalization group flow from supergravity

$\Rightarrow$ 'holographic' Renormalization Group flow

SUSY broken by deformation of S^5

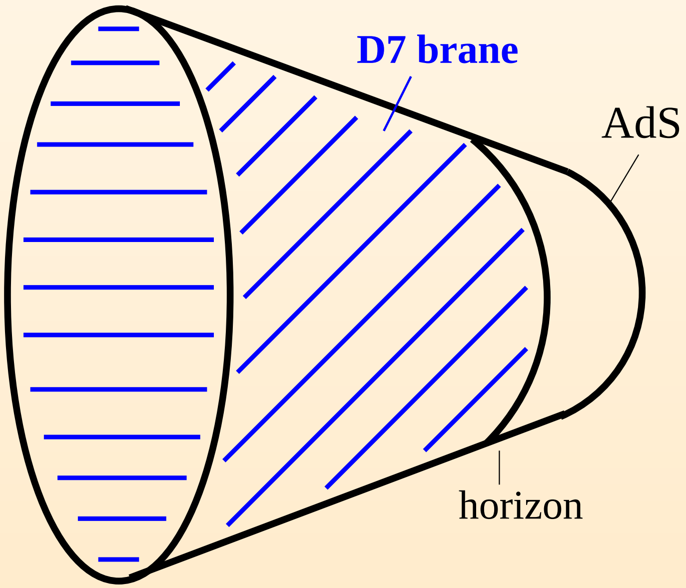
Running gauge coupling

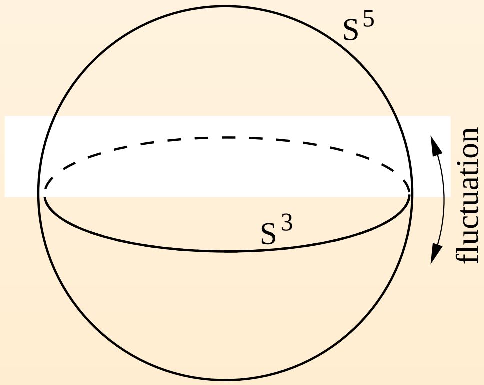


Quarks (fundamental fields) within the AdS/CFT correspondence

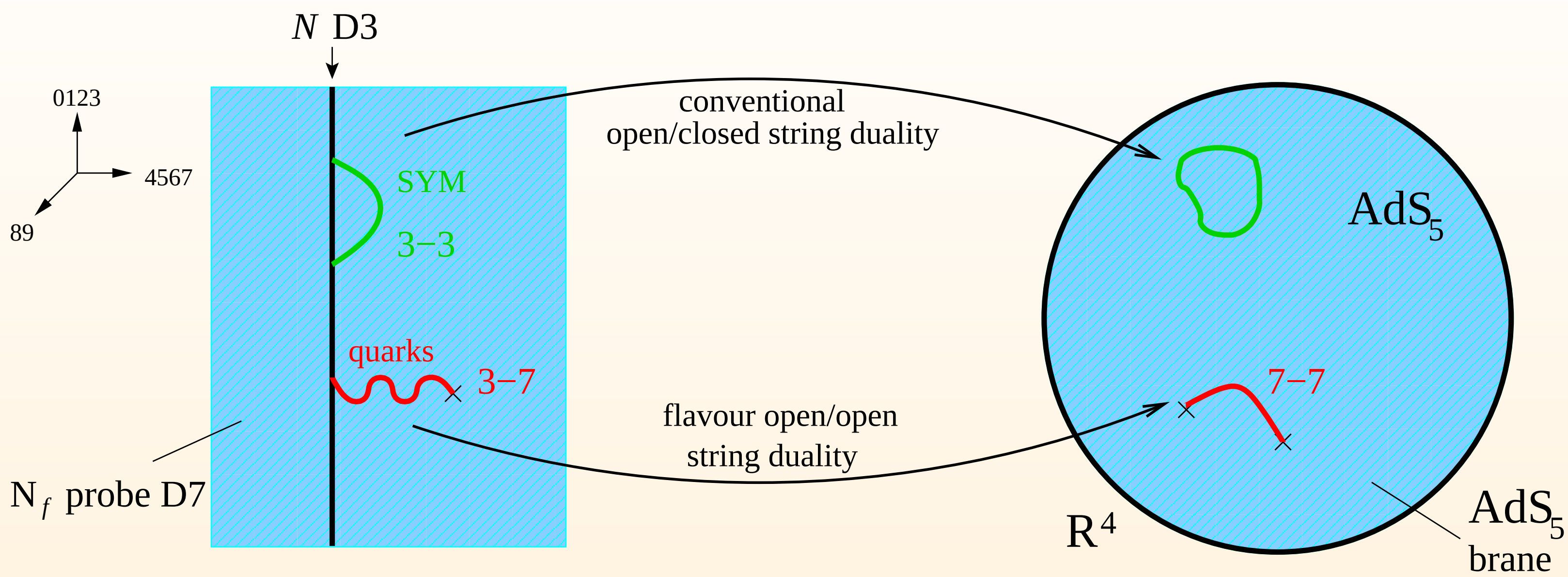
D7 brane probe:

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		





Quarks (fundamental fields) from brane probes



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N}=4$ SU(N) Super Yang-Mills theory coupled to $\mathcal{N}=2$ fundamental hypermultiplet	$\longleftrightarrow$	IIB supergravity on $AdS_5 \times S^5$ + Probe brane DBI on $AdS_5 \times S^3$

Karch, Katz 2002

Combine the deformation of the supergravity metric

with the addition of brane probes:

Dual gravity description of chiral symmetry breaking and Goldstone bosons

J. Babington, J. E., N. Evans, Z. Guralnik and I. Kirsch,

"Chiral symmetry breaking and pions in non-SUSY gauge/gravity duals"

hep-th/0306018

D7 brane probe in gravity backgrounds dual to

confining gauge theories without supersymmetry.

Example:

Constable-Myers background (particular deformation of $AdS_5 \times S^5$ metric)

- The deformation introduces a new scale into the metric.
- In UV limit, geometry returns to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$.

General strategy

1. Start from Dirac-Born-Infeld action for a D7-brane embedded in deformed background

2. Derive equations of motion for transverse scalars (w_5, w_6)

3. Solve equations of motion numerically using shooting techniques

   Solution determines embedding of D7-brane (e.g. $w_5 = 0, w_6 = w_6(\rho)$)

4. Meson spectrum:

   Consider fluctuations δw_5, δw_6 around a background solution obtained in 3.

   Solve equations of motion linearized in $\delta w_5, \delta w_6$

UV asymptotic behaviour of solutions to equation of motion:

$$w_6 \propto m\, e^{-r} + c\, e^{-3r}$$

Identification of the coefficients as in the standard AdS/CFT correspondence:

m quark mass, $c = \langle \bar{q} q \rangle$ quark condensate

Here:

$m \neq 0$: explicit breaking of $U(1)_A$ symmetry

$c \neq 0$: spontaneous breaking of $U(1)_A$ symmetry

The Constable-Myers deformation

$\mathcal{N} = 4$ super Yang-Mills theory deformed by VEV for $tr\, F^{\mu\nu} F_{\mu\nu}$
(R-singlet operator with $D = 4$) $\quad \rightarrow$ non-supersymmetric QCD-like field theory

The Constable-Myers background is given by the metric

$$ds^2 = H^{-1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1 \qquad (\Delta^2 + \delta^2 = 10)$$

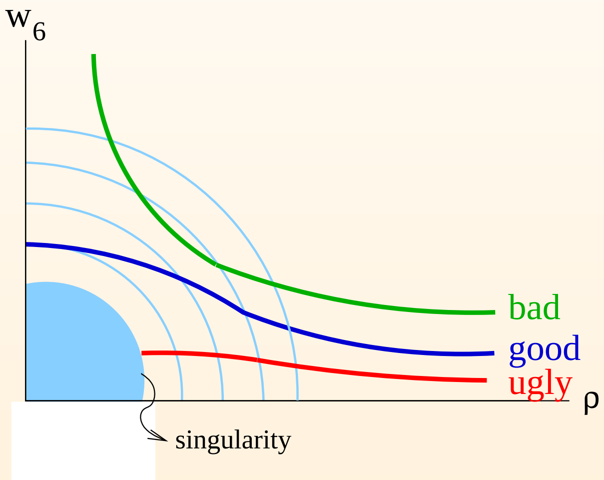
and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz$$

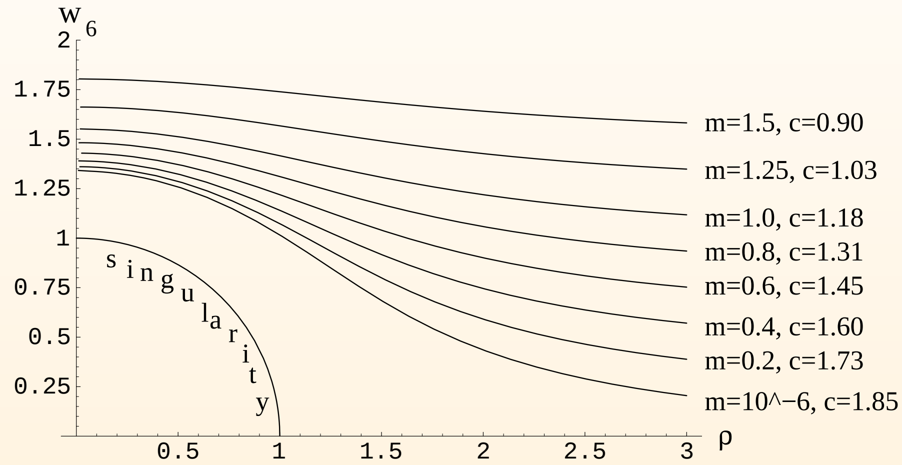
This background has a singularity at $w = b$

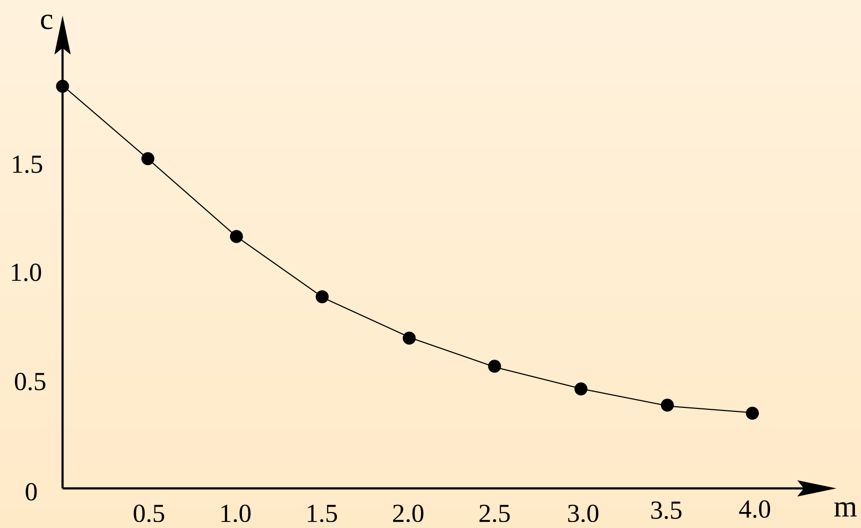
Chiral symmetry breaking

Solution of equation of motion for probe brane



Numerical Result:





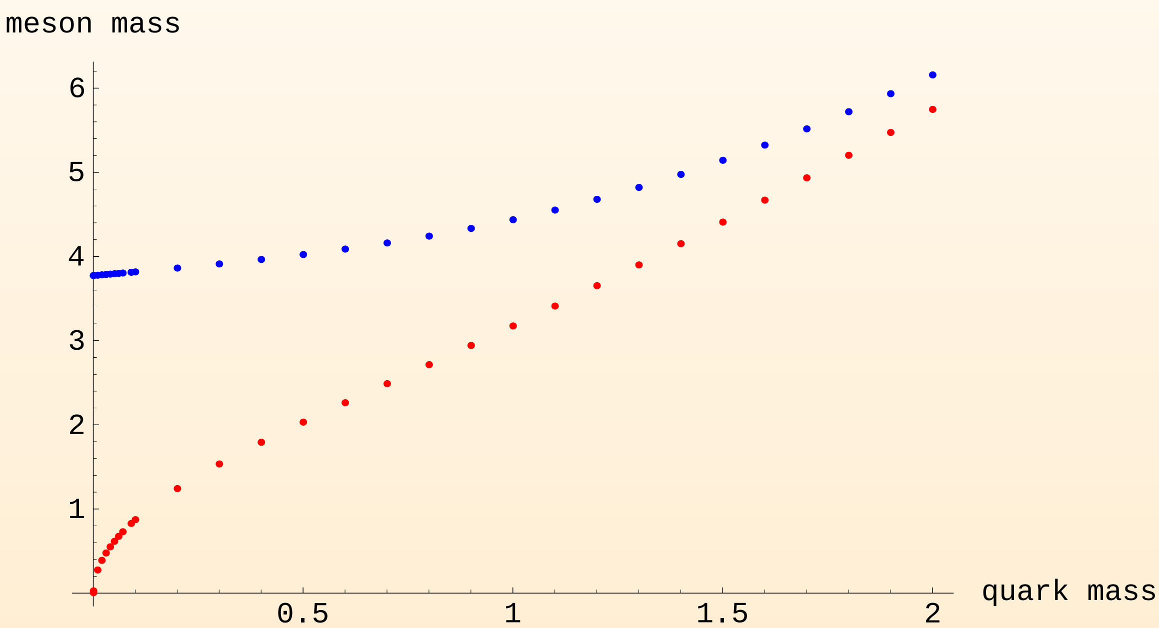
Result:

Screening effect: Regular solutions do not reach the singularity

Spontaneous breaking of $U(1)_A$ symmetry: For $m \to 0$ we have $c \equiv \langle \bar{\psi}\psi \rangle \neq 0$

From fluctuations of the probe brane

Ansatz: $\delta w_i(x,\rho) = f_i(\rho) sin(k \cdot x)\,, \qquad M^2 = -k^2$



Goldstone boson (η')

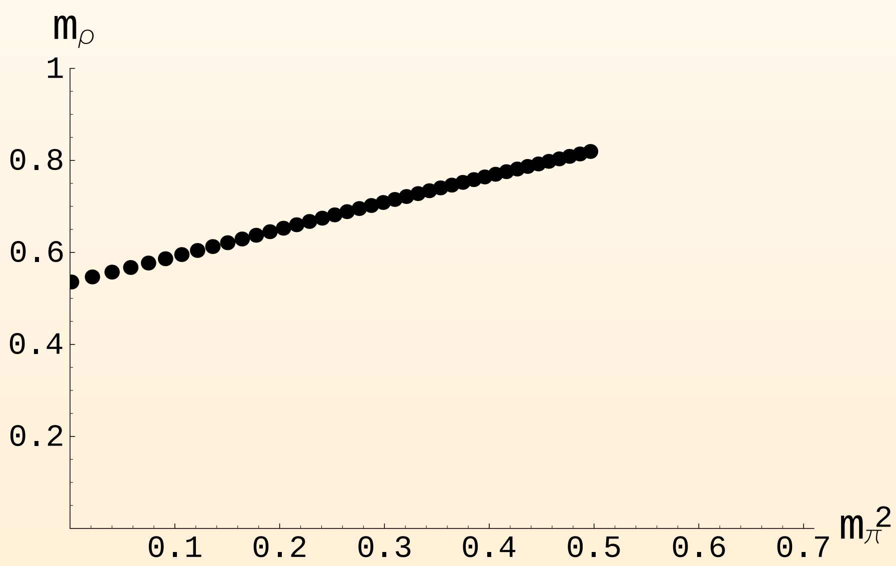
Gell-Mann-Oakes-Renner relation: $M_{Meson} \propto \sqrt{m_{Quark}}$

Comparison to lattice gauge theory

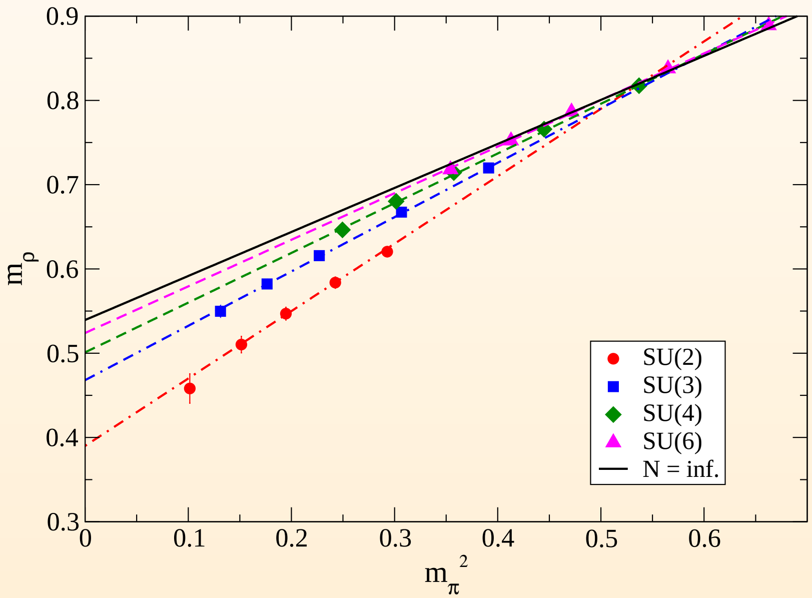
J.E., Evans, Kirsch, Threlfall 0711.4467, EPJA

m_ρ vs. ${m_\pi}^2$

(Lattice: Lucini, Del Debbio, Patella, Pica 0712.3036)



Slope: 0.57
Normalized to scale in metric



Slope: 0.52
Normalized to lattice spacing

(Similar results by Bali and Bursa)

Finite Temperature

$\mathcal{N} = 4$ Super Yang-Mills theory at finite temperature is dual to AdS black hole

$$ds^2 = \frac{1}{2}\left(\frac{\varrho}{R}\right)^2 \left(-\frac{f^2}{\tilde{f}}\mathrm{d}t^2 + \tilde{f}\mathrm{d}\vec{x}^2\right) + \left(\frac{R}{\varrho}\right)^2 (\mathrm{d}\varrho^2 + \varrho^2 \mathrm{d}\Omega_5^2)$$

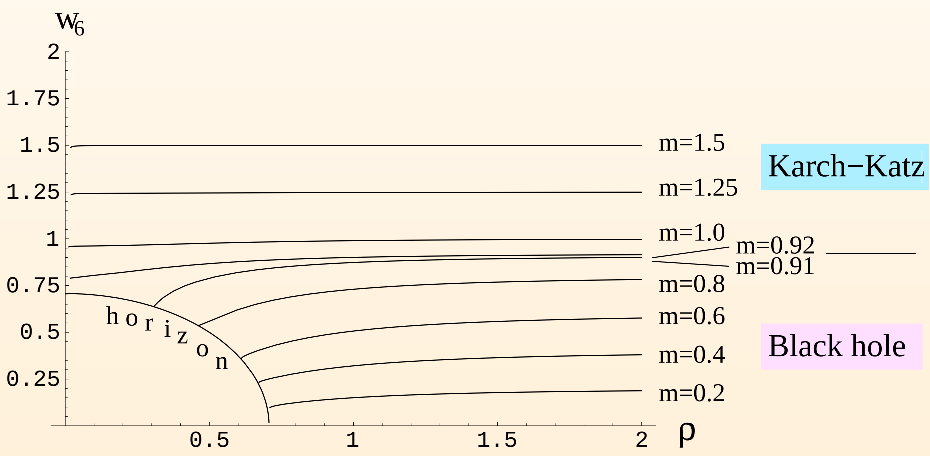
$$f(\varrho) = 1 - \frac{\varrho_H^4}{\varrho^4}, \quad \tilde{f}(\varrho) = 1 + \frac{\varrho_H^4}{\varrho^4}$$

Temperature and horizon related by

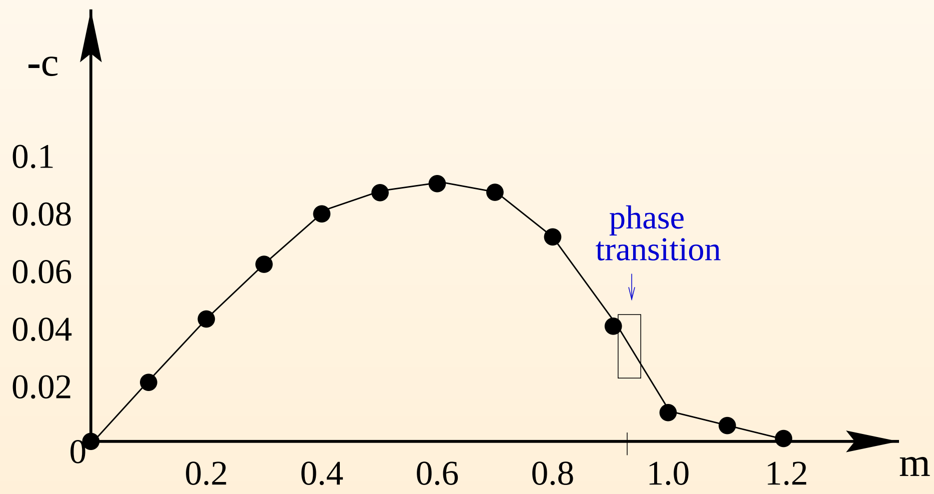
$$T = \frac{\varrho_H}{\pi R^2}$$

R AdS radius

D7 brane embedding in black hole background



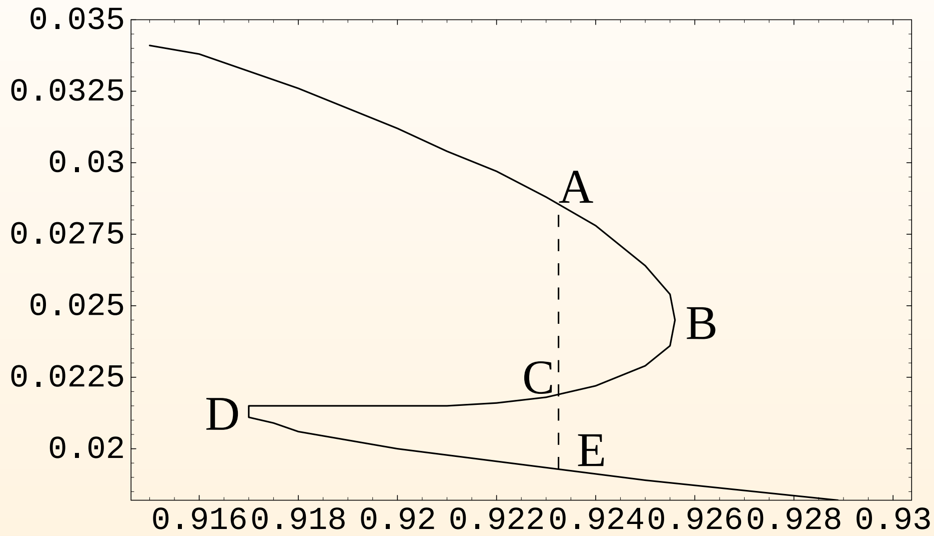
Condensate c versus quark mass m (c, m normalized to T)



Phase transition at $m/T \approx 0.92$

Babington, J.E., Evans, Guralnik, Kirsch 0306018

Phase transition



First order phase transition in type II B AdS black hole background

Ingo Kirsch, PhD thesis 2004

Finite $U(1)$ baryon density

Baryon density n_B and $U(1)$ chemical potential μ
from VEV for gauge field time component:

$$\bar{A}_0(\rho) \sim \mu + \frac{\tilde{d}}{\rho^2}, \qquad \tilde{d} = \frac{2^{5/2}}{N_f \sqrt{\lambda} T^3} n_B$$

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At finite baryon density, all embeddings are black hole embeddings

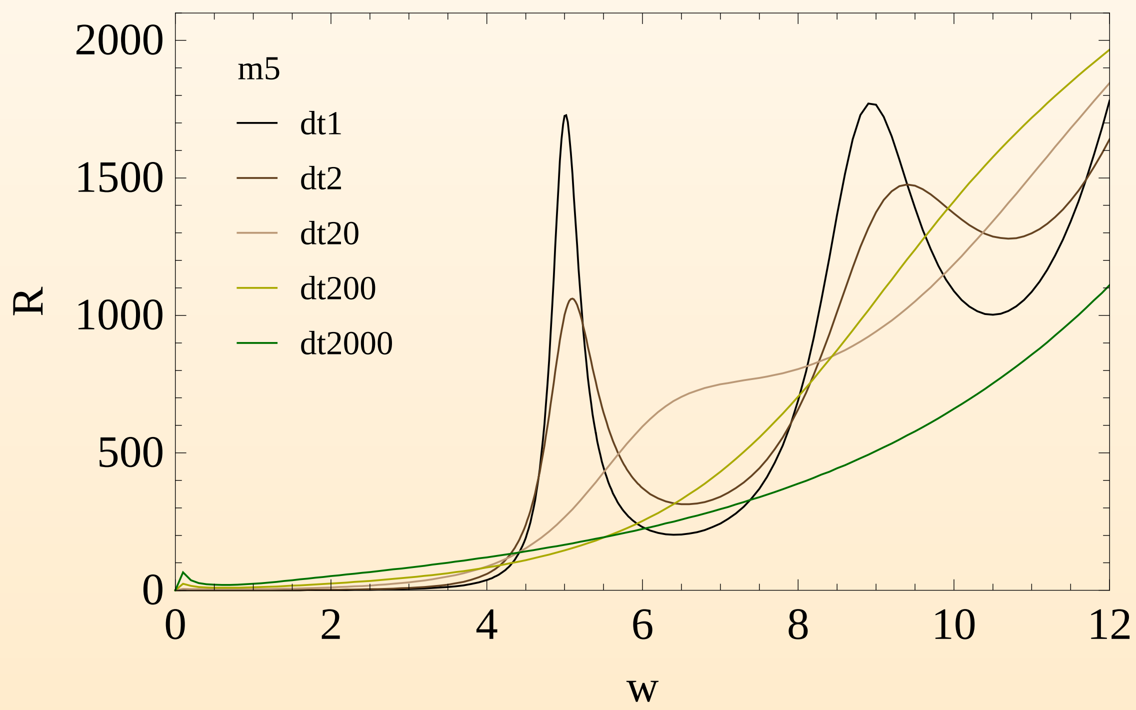
Mateos, Myers et al

Spectral functions for vector mesons

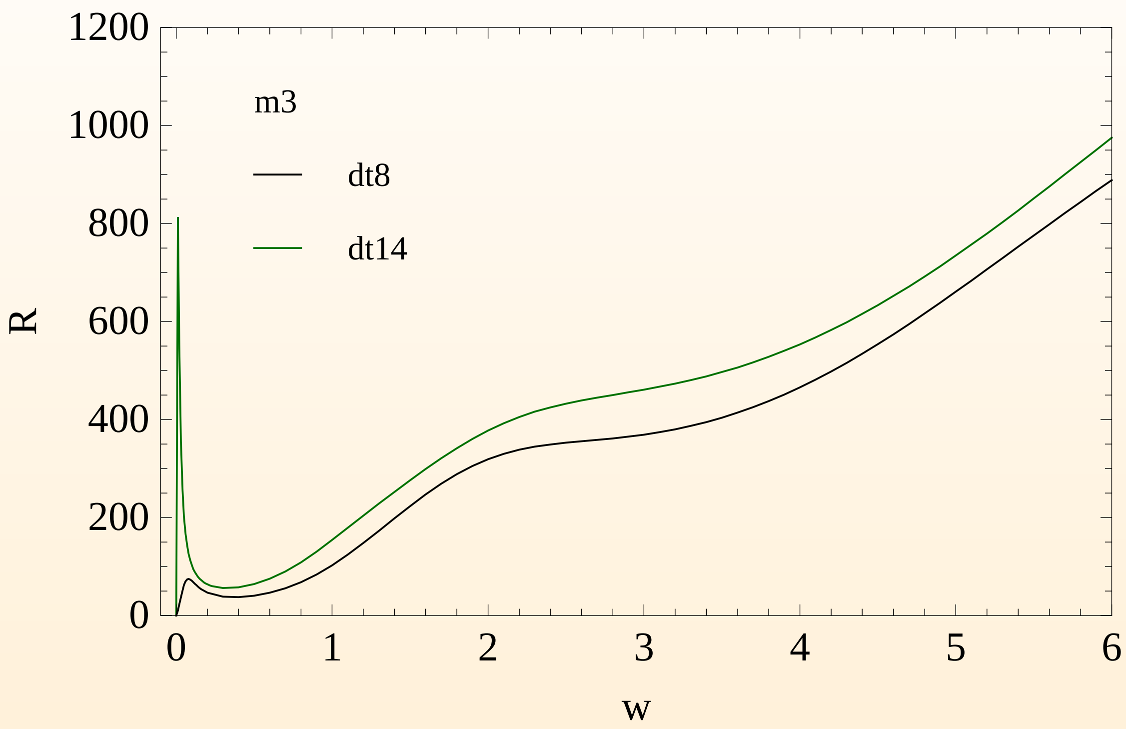
J.E., Kaminski, Kerner, Rust 0807.2663

From fluctuations of gauge field on the D7 brane

Meson melting for high densities



Spectral functions for mesons at finite isospin density



Vector meson condensation

Dusling, J.E., Kaminski, Rust, Teaney, Young 0808.0957

Consider heavy meson moving slowly through the plasma

Interaction with gluons

Dusling, J.E., Kaminski, Rust, Teaney, Young 0808.0957

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Interaction with gluons

Calculation of ratio $\kappa/\delta M^2$

κ: momentum diffusion, δM: in-medium mass shift

both at strong and at weak coupling in $\mathcal{N} = 4$ theory

Quarkonium transport in thermal AdS/CFT

Dusling, J.E., Kaminski, Rust, Teaney, Young 0808.0957

Consider heavy meson moving slowly through the plasma

Interaction with gluons

Calculation of ratio $\kappa/\delta M^2$

κ: momentum diffusion, δM: in-medium mass shift

both at strong and at weak coupling in $\mathcal{N} = 4$ theory

AdS/CFT result five times smaller than effective field theory result

Conclusion

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- New relation: String theory $\Leftrightarrow$ QFT, strong interactions

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- Numerous applications for specific problems:

  Chiral symmetry breaking

  Spectral functions and transport processes in quark-gluon plasma