Light mesons and glueballs in a bottom-up approach to holographic QCD

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Strings and Strong interactions

Holography and Beyond

Frascati, 18-19 September 2008

outline:

- QCD a candidate for holographic description
- AdS/QCD models: top-down vs bottom-up, hard-wall vs soft-wall
- light mesons in the soft-wall model
- glueballs
- comments on results, drawbacks, perpectives, …

AdS/CFT correspondence conjecture & QCD

- AdS/CFT correspondence involves conformal and supersymmetric field theories in M_4
- QCD is not conformal (neither supersymmetric): it has a mass gap Λ_{QCD}
- However, in the UV, neglecting quark mass and radiative effects, QCD is a nearly conformal theory

  - QCD counting rules, behaviour of form factors at large momentum transfer
  - light cone sum rule analyses

QCD a candidate for a description inspired by AdS/CFT

- Two conditions to be implemented:
  UV -> conformal behaviour -> AdS space
  IR -> modification (at least) of the AdS geometry of the bulk

AdS/QCD: extradimensional models, motivated by the AdS/CFT correspondence conjecture, developed to compute low energy QCD observables

Features of the models:

- large N_c limit of QCD implemented
- spontaneous chiral symmetry breaking
- match IR to UV in the best possible way
  (reminding the Migdal's program for the QCD correlation functions)

A few results seem universal (robust against variations of the model)
Others can be used to select the best model

top-down approach to AdS/QCD → Erdmenger's talk

start from a string theory in high dimensions and try to construct a low dimensional theory with features similar to QCD

At large N the theory has a weakly coupled dual description via AdS/CFT corr.

Gluonium (Csaki, Ooguri , Oz, de Mello, Mihailescu, Nunez, Russo, Terning, Minahan, Brower, Matur, Tan, ,...)

D3/D7 branes and flavour (Karch, Katz, Weiner, Sakai, Sonnenshein, Babington, Evans, Erdmenger, Guralnik, Kirsh, Nunez, Paredes, Ramallo, Sugimoto, Caseres, Paredes, Talavera, Vaman, ...)

bottom-up approach to AdS/QCD → Gursoy's talk

start from known QCD phenomenology and try to understand which feature its high dimension gravitational dual should have

- Select the geometry of the extradimensions
- Model chiral symmetry breaking pattern
- Model hadrons as KK modes in the extra dimensions

Low lying hadron spectra (Boschi-Filho, Braga, Brodsky, de Teramond, Yong, Strassler, Hirn, Sanz,...)

Light-front hadronic wave functions and form factors (Brodsky, de Teramond,, Grigoryan, Radyushkin, ...)

Structure functions (Polchinsky and Strassler,... Pire, Roisnel, Symanowski and Wallon)

$Q\bar{Q}$ potential (Andreev, Zakharov, ...)

QCD condensates (Hirn, Sanz, Andreev, Zakharov, ...)

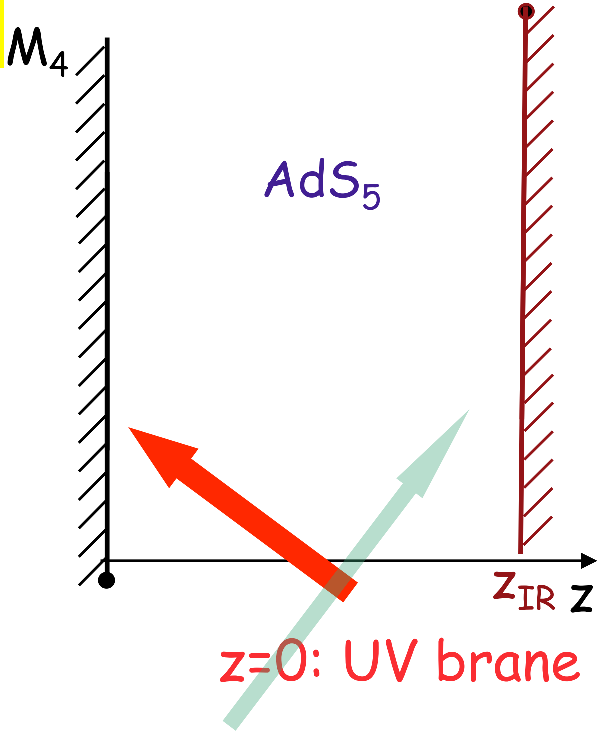
bottom-up approach to AdS/QCD : hard wall model

1: select geometry

$AdS_5 = M_4$ + radial (holographic) coordinate z

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\mu + dz^2)$$

$$\eta_{\mu\nu} = diag(- + + +)$$



$$x \to \lambda x$$
$$z \to \lambda z$$

maps scale transformations into z

if x is the length scale to which physics (hadrons) is examined, low values of distances correspond to low values of z (UV brane)

large distances correspond to large values of z (IR)

maximum separation -> maximum value of z: z_{IR}

non conformal metric: AdS slice Polchinski, Strassler

↓ confinement

↓ $z_{IR} \sim 1/\Lambda_{QCD}$

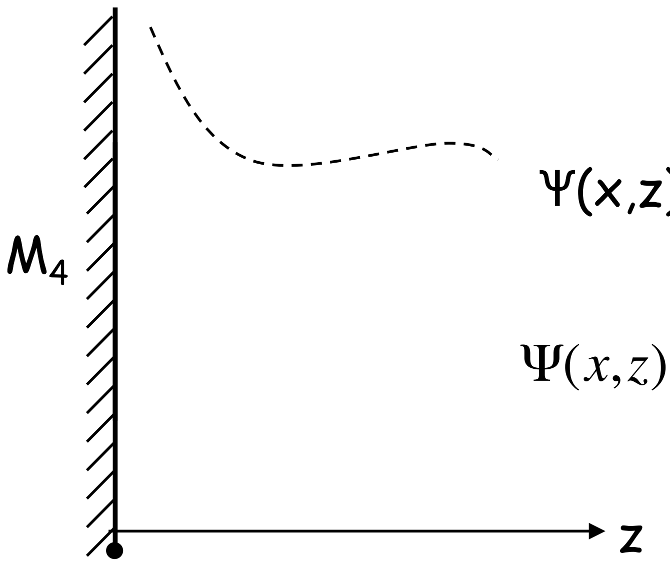
AdS/QCD translator

4D	5D
gauge invariant QCD operator O(x) (p form)	field Ψ(x,z)
hadrons with quantums numbers of O	normalizable modes $\psi_n(x,z)$
hadron mass^2	eigenvalue of a 5D wave eq.
conformal dimension Δ	5D mass m_5
	$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4)$
small distances, large momenta	small z
mass gap Λ_{QCD}	z_{IR}, background dilaton
decay constant $\langle 0 \vert O \vert \text{hadron} \rangle$	$\left.\frac{\psi'(z)}{z^n}\right\vert_{z\to 0}$

following AdS/CFT correspondence

Maldacena,
Gubser, Klebanov and Polyakov
Witten

$\Psi(x,0) = \Psi_0(x)$ boundary value



$\Psi(x,z)$ bulk field
obeying an eq.of motion in AdS_5

$$\Psi(x,z) = \int_{M_4} d^4y\, K(x,z;y)\, \Psi(y,0)$$

↑ bulk-to-boundary propagator

$\Psi_0(x)$ coupled to the QCD operator $O(y)$ in M_4 via $\int_{M_4} d^4x\, \Psi_0(x)\, O(x)$

$$\left\langle \exp\, i \int_{M_4} \Psi_0 O \right\rangle_{CFT(QCD)} \quad \text{from} \quad \exp\left(iS_{AdS}(\Psi)\right)$$

AdS/QCD hard wall model (Erlich, Katz, Son and Stephanov, Da Rold and Pomarol, Brodsky and de Teramond, ...)

- AdS_5 metric $$ds^2 = \frac{R^2}{z^2}(dx^2 + dz^2)$$ slice $0 < z < z_{IR}$

- Chiral symmetry: - two sets of conserved currents in QCD : $J^\mu_{L,R} = \bar{q}\gamma^\mu \frac{1 \mp \gamma_5}{2} q$
  - two bulk gauge fields in 5D : $A_{L,R}{}^M (x,z)$

- Chiral symmetry breaking:
  - in QCD characterized by m_q and the condensate $\langle \bar{q}^\alpha_R q^\beta_L \rangle \propto \delta^{\alpha\beta}$ $(\alpha,\beta = 1\ldots N_f)$
  - a massive scalar bulk field in 5D
    $X^{\alpha\beta}$ bifundamental with respect to $SU(N_f)_L x SU(N_f)_R$

$X = X_0\, e^{2i\Pi(x,z)}$ with $X_0(z) \to m_q\, z + \langle \bar{q}q \rangle\, z^3$ $(z \to 0)$

- action :
$$S_{AdS} = -\frac{1}{k}\int d^5x\sqrt{|g|}\left[|DX|^2 + m_5^2 X^2 + \frac{1}{4g_5^2} Tr\left(F_L^2 + F_R^2\right)\right]$$

$$D_M X = \partial_M X - iA_{LM} X + iXA_{RM}$$

- boundary conditions at $z = z_{IR}$: $D_z X = 0$; $F_{zM} = 0$

Define vector and axial fields

$V=(A_L + A_R)/2$

$A=(A_L - A_R)/2$

Equation for the vector field:

$$\partial_z\left(\frac{1}{z}\partial_z V(q,z)\right)-\frac{q^2}{z}V(q,z)=0$$ + bc -> spectrum (ρ mass)

Quadratic action for the axial field

$$S^{A(2)}=-\frac{1}{k}\int d^4x dz Tr\left[\frac{X_0^2(z)}{z^3}\left(A_M-\partial_M\pi\right)^2+\frac{1}{4g_5^2 z}\left(F_{(A)MN}F_{(A)}{}^{MN}\right)\right]$$

results

Erlich, Katz,
Son and Stephanov

Observable	Measured (MeV)	Model (MeV)
m_π	139.6	141
m_ρ	775.8	832
m_{a_1}	1230	1220
f_π	92.4	84.0
$F_\rho^{1/2}$	345	353
$F_{a_1}^{1/2}$	433	440
$g_{\rho\pi\pi}$	6.03	5.29
m_{f_2}	1275	1236

parameters
z_{IR} =1/(346 MeV)
$\langle qq \rangle$ (-308 MeV)^3
m_q =2.3 MeV

- some hadronic parameters reproduced
- GMOR relation obtained
- dependence:
  $m^2_n \sim n^2$

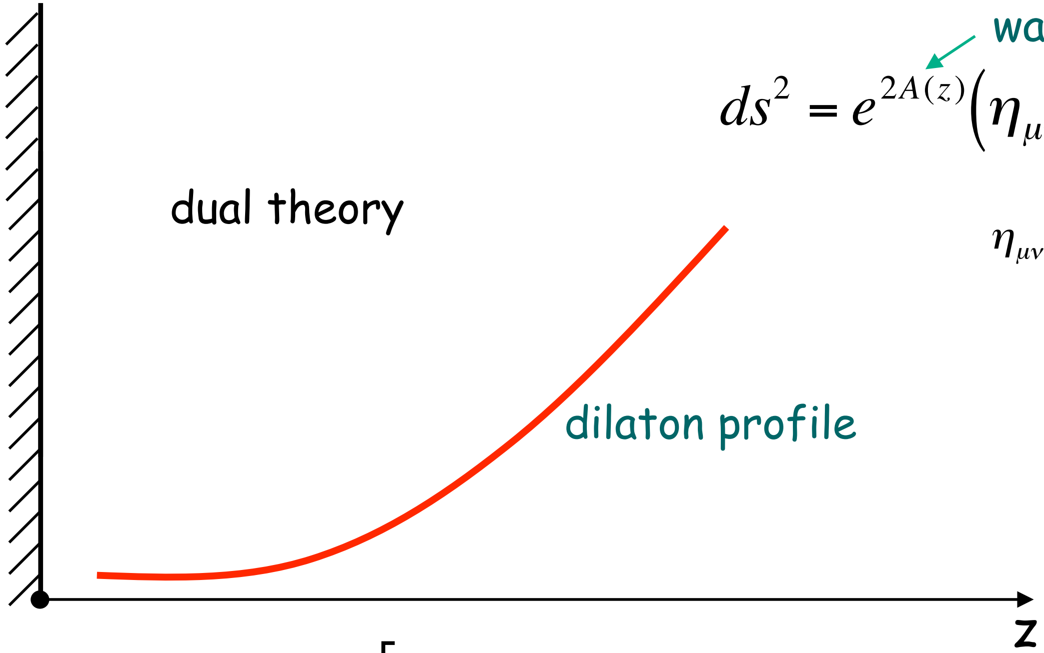
experiment: linear Regge trajectories
$m^2_n \sim n$

AdS/QCD soft wall model (Karch, Katz, Son and Stephanov, Andreev, Radyushkin, ...)

- Conformal invariance broken by a background dilaton field in the bulk: $\Phi(z)$

(soft-wall)

QCD in M^4



warp factor

$$ds^2 = e^{2A(z)}\left(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

$$\eta_{\mu\nu} = diag(-,+,+,+)$$

$$S = -\frac{1}{k}\int d^5x\sqrt{|g|}\; e^{-\varphi(z)}\left[|DX|^2 + m_5^2 X^2 + \frac{1}{4g_5^2}Tr\left(F_L^2 + F_R^2\right)\right]$$

$$\varphi(z) - A(z) \underset{z\to\infty}{\to} c^2 z^2$$

$$\varphi(z) - A(z) \underset{z\to 0}{\to} -\ln\left(\frac{R}{z}\right)$$

$$A(z) \neq z^\beta \quad \text{with } \beta \geq 2$$

⟹ Regge behaviour for vector mesons

$$m_{\rho_n}^2 = c^2(4n+4)$$

Karch et al.

c parameter setting the scale of hadronic quantities

Light scalar mesons in the soft-wall (De Fazio, Giannuzzi, Nicotri, Jugeau, PC, PRD 2008)

$$S=-\frac{1}{k}\int d^5x\sqrt{|g|}\; e^{-\varphi(z)}\left[|DX|^2+m_5^2X^2+\frac{1}{4g_5^2}Tr\left(F_L^2+F_R^2\right)\right]$$

$X=(X_0(z)+S_1(x,z)+S_8(x,z))\,e^{2i\Pi(x,z)}$

mass spectrum

eom for each S: $$\partial_z\left(\frac{R^3}{z^3}e^{-\Phi(z)}\partial_z\tilde{S}\right)+3\frac{R^3}{z^5}e^{-\Phi(z)}\tilde{S}-q^2\frac{R^3}{z^3}e^{-\Phi(z)}\tilde{S}=0$$

admit normalizable solutions, corresponding to discrete values of q^2 : $m_{S_n}^2=-q_n^2$

$$m_{S_n}^2=c^2(4n+6)$$
$$m_{\rho_n}^2=c^2(4n+4)$$

← linear Regge trajectory also for scalars
same slope (as in constituent quark models)

↑ notice:
c sets the mass scale

$$R_{f_0(a_0)}=\frac{m_{f_0(a_0)}^2}{m_{\rho^0}^2}=\frac{3}{2}$$

$f_0(980)$ and $a_0(980)$ masses reproduced

$$R_{f_0}^{\exp}=1.597\pm0.033$$
$$R_{a_0}^{\exp}=1.612\pm0.004$$

Two point correlation function of the scalar operator

$$\Pi^{AB}_{\mathrm{QCD}}(q^2) = i \int d^4x e^{iq\cdot x} \langle 0 | T[\mathcal{O}^A_S(x)\mathcal{O}^B_S(0)] | 0 \rangle$$

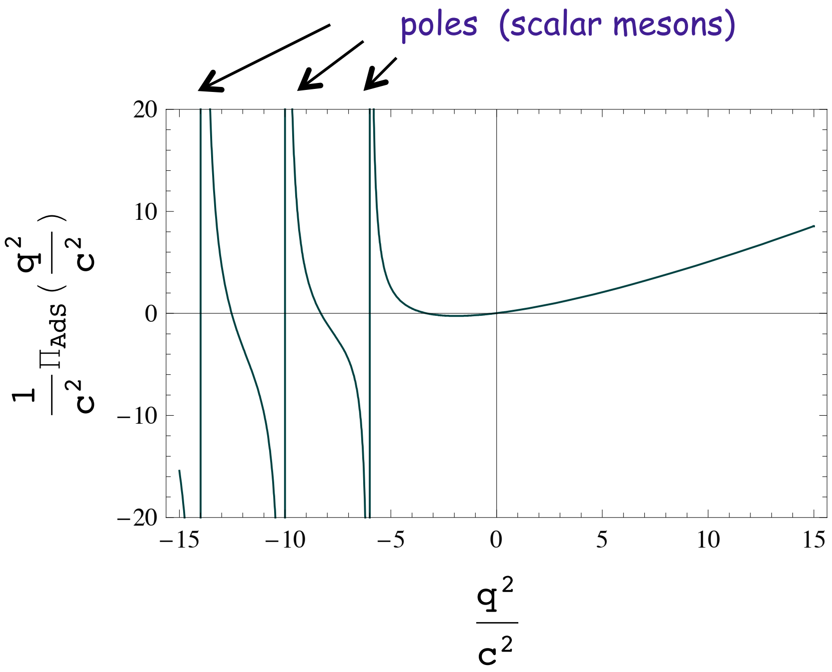
A,B flavor index

$$\Pi^{AB}_{\mathrm{AdS}}(q^2) = \delta^{AB} \frac{R^3 c^4}{k} S\left(\frac{q^2}{c^2}, \hat{z}^2\right) \frac{e^{-\Phi(\hat{z})}}{\hat{z}^3} \partial_{\hat{z}} S\left(\frac{q^2}{c^2}, \hat{z}^2\right) \Big|_{\hat{z}\to 0},$$

bulk to boundary propagator of the scalar field

$$\Pi^{AB}_{\mathrm{AdS}}(q^2) = \delta^{AB} \frac{4c^2 R}{k} \left[\left(\frac{q^2}{4c^2} + \frac{1}{2}\right) \ln(c^2 z^2) + \left(\gamma_E - \frac{1}{2}\right) + \frac{q^2}{4c^2}\left(2\gamma_E - \frac{1}{2}\right) + \left(\frac{q^2}{4c^2} + \frac{1}{2}\right)\psi\left(\frac{q^2}{4c^2} + \frac{3}{2}\right) \right] \Bigg|_{z=z_{\min}} \quad (32)$$

Euler function



residues: $F_n^2 = \frac{R}{k} 16 c^4 (n+1)$

$\frac{R}{k} = \frac{N_c}{16\pi^2}$ (next page)

$\longrightarrow$ $F_n^2 = \frac{N_c}{\pi^2} c^4 (n+1)$

c sets the mass scale

AdS/QCD $F_{a_0} = \frac{\sqrt{3}}{\pi} c^2 = 0.08 \text{ GeV}^2$

QCD estimate $F_{a_0} = \langle 0 | \mathcal{O}_S^3 | a_0(980)^0 \rangle = (0.21 \pm 0.05) \text{ GeV}^2$

short distance expansion:

$$\Pi^{AB}_{QCD}(q^2) \;=\; \frac{\delta^{AB}}{2}\left[\frac{3}{8\pi^2}\left(1+\frac{11\alpha_s}{3\pi}\right)q^2\ln(\frac{q^2}{\nu^2}) + \frac{3}{q^2}\langle m_q\overline{q}q\rangle + \frac{1}{8q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right.$$

$$+\;\frac{m_q g_s}{2q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)G^a_{\mu\nu}\rangle + \frac{\pi\alpha_s}{q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)^2\rangle$$

$$\left.+\;\frac{2\pi\alpha_s}{3q^4}\langle(\overline{q}\gamma_\mu\lambda^a q)\sum_{q=u,d}\overline{q}\gamma_\mu\lambda^a q\rangle + O(1/q^6)\right]$$

$$\Pi^{AB}_{AdS}(q^2) \;=\; \delta^{AB}\frac{R}{k}\left[q^2\ln(\frac{q^2}{\nu^2}) + q^2\left(2\gamma_E - \ln 4 - \frac{1}{2}\right) + 2c^2\left(\ln(\frac{q^2}{\nu^2}) - \ln 4 + 2\gamma_E + 1\right)\right.$$

$$\left.+\;\frac{2}{3}\frac{c^4}{q^2} + \frac{4}{3}\frac{c^6}{q^4} + O(1/q^6)\right] \qquad (34)$$

short distance expansion: UV matching

$$\Pi^{AB}_{QCD}(q^2) = \frac{\delta^{AB}}{2}\left[\frac{3}{8\pi^2}\left(1+\frac{11\alpha_s}{3\pi}\right)q^2\ln(\frac{q^2}{\nu^2}) + \frac{3}{q^2}\langle m_q\overline{q}q\rangle + \frac{1}{8q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right.$$

$$+ \frac{m_q g_s}{2q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)G^a_{\mu\nu}\rangle + \frac{\pi\alpha_s}{q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)^2\rangle$$

$$\left. + \frac{2\pi\alpha_s}{3q^4}\langle(\overline{q}\gamma_\mu\lambda^a q)\sum_{q=u,d}\overline{q}\gamma_\mu\lambda^a q\rangle + O(1/q^6)\right]$$

$$\Pi^{AB}_{AdS}(q^2) = \delta^{AB}\frac{R}{k}\left[q^2\ln(\frac{q^2}{\nu^2}) + q^2\left(2\gamma_E - \ln 4 - \frac{1}{2}\right) + 2c^2\left(\ln(\frac{q^2}{\nu^2}) - \ln 4 + 2\gamma_E + 1\right)\right.$$

$$\left. + \frac{2}{3}\frac{c^4}{q^2} + \frac{4}{3}\frac{c^6}{q^4} + O(1/q^6)\right] \qquad (34)$$

the matching fixes R/k independent of the mass scale c

$$\frac{R}{k} = \frac{N_c}{16\pi^2}$$

short distance behaviour only controlled by AdS

short distance expansion: power corrections and condensates

$$\Pi^{AB}_{QCD}(q^2) = \frac{\delta^{AB}}{2}\left[\frac{3}{8\pi^2}\left(1+\frac{11\alpha_s}{3\pi}\right) q^2 \ln(\frac{q^2}{\nu^2}) + \frac{3}{q^2}\langle m_q \overline{q} q\rangle + \frac{1}{8q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right.$$

$$+ \frac{m_q g_s}{2q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)G^a_{\mu\nu}\rangle + \frac{\pi\alpha_s}{q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)^2\rangle$$

$$\left. + \frac{2\pi\alpha_s}{3q^4}\langle(\overline{q}\gamma_\mu\lambda^a q)\sum_{q=u,d}\overline{q}\gamma_\mu\lambda^a q\rangle + O(1/q^6)\right]$$

d=4 gluon condensate term in the power expansion

$$\Pi^{AB}_{AdS}(q^2) = \delta^{AB}\frac{R}{k}\left[q^2\ln(\frac{q^2}{\nu^2}) + q^2\left(2\gamma_E - \ln 4 - \frac{1}{2}\right) + 2c^2\left(\ln(\frac{q^2}{\nu^2}) - \ln 4 + 2\gamma_E + 1\right)\right.$$

$$\left. + \frac{2}{3}\frac{c^4}{q^2} + \frac{4}{3}\frac{c^6}{q^4} + O(1/q^6)\right]$$

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = \frac{2}{\pi^2}c^4 \simeq 0.004 \text{ GeV}^4$$

commonly used value

$$\langle\frac{\alpha_s}{\pi}G^2\rangle \simeq 0.012 \text{ GeV}^4$$

short distance expansion: power corrections and condensates

$$\Pi^{AB}_{QCD}(q^2) = \frac{\delta^{AB}}{2}\left[\frac{3}{8\pi^2}\left(1+\frac{11\alpha_s}{3\pi}\right)q^2\ln(\frac{q^2}{\nu^2}) + \frac{3}{q^2}\langle m_q\bar{q}q\rangle + \frac{1}{8q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right.$$

$$+ \frac{m_q g_s}{2q^4}\langle(\bar{q}\sigma_{\mu\nu}\lambda^a q)G^a_{\mu\nu}\rangle + \frac{\pi\alpha_s}{q^4}\langle(\bar{q}\sigma_{\mu\nu}\lambda^a q)^2\rangle$$

$$\left.+ \frac{2\pi\alpha_s}{3q^4}\langle(\bar{q}\gamma_\mu\lambda^a q)\sum_{q=u,d}\bar{q}\gamma_\mu\lambda^a q\rangle + O(1/q^6)\right]$$

d=6 gluon condensate terms

$$\Pi^{AB}_{AdS}(q^2) = \delta^{AB}\frac{R}{k}\left[q^2\ln(\frac{q^2}{\nu^2}) + q^2\left(2\gamma_E - \ln 4 - \frac{1}{2}\right) + 2c^2\left(\ln(\frac{q^2}{\nu^2}) - \ln 4 + 2\gamma_E + 1\right)\right.$$

$$\left.+ \frac{2}{3}\frac{c^4}{q^2} + \frac{4}{3}\frac{c^6}{q^4} + O(1/q^6)\right]$$

differences in size and sign
with respect to common values

short distance expansion: power corrections and condensates

?

$$\Pi^{AB}_{QCD}(q^2) = \frac{\delta^{AB}}{2}\left[\frac{3}{8\pi^2}\left(1+\frac{11\alpha_s}{3\pi}\right)q^2\ln(\frac{q^2}{\nu^2}) + \frac{3}{q^2}\langle m_q\overline{q}q\rangle + \frac{1}{8q^2}\langle\frac{\alpha_s}{\pi}G^2\rangle\right.$$

$$+ \frac{m_q g_s}{2q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)G^a_{\mu\nu}\rangle + \frac{\pi\alpha_s}{q^4}\langle(\overline{q}\sigma_{\mu\nu}\lambda^a q)^2\rangle$$

$$\left.+ \frac{2\pi\alpha_s}{3q^4}\langle(\overline{q}\gamma_\mu\lambda^a q)\sum_{q=u,d}\overline{q}\gamma_\mu\lambda^a q\rangle + O(1/q^6)\right]$$

D=2 condensate term
missing in the QCD expansion

$$\Pi^{AB}_{AdS}(q^2) = \delta^{AB}\frac{R}{k}\left[q^2\ln(\frac{q^2}{\nu^2}) + q^2\left(2\gamma_E - \ln 4 - \frac{1}{2}\right) + 2c^2\left(\ln(\frac{q^2}{\nu^2}) - \ln 4 + 2\gamma_E + 1\right)\right.$$

$$\left.+ \frac{2}{3}\frac{c^4}{q^2} + \frac{4}{3}\frac{c^6}{q^4} + O(1/q^6)\right] \qquad (34)$$

the existence of a d=2 condensate term in the QCD OPE old debated issue
(no local gauge invariant QCD operator of dimension two)
various possibilities considered (Zakharov, Narison, Schilcher, Dominguez,...)

strong coupling of scalar mesons to light pseudoscalars g_{SPP} obtained from a three-point correlation function

soft-wall: g_{SPP} related to the $X_0(z)$ field introduced to implement χSB
small value obtained for g_{SPP} (≅ 10 MeV)

$$g_{S_0PP} = \frac{\sqrt{N_c}}{4\pi}\frac{m_{S_0}^2}{f_\pi^2} Rc \int_0^\infty d\hat{z}\, e^{-\hat{z}^2} v(\hat{z}) \ .$$

v(z) (=$X_0(z)/2$) solution of

$$\partial_z\left(\frac{R^3}{z^3}e^{-\Phi(z)}\partial_z v(z)\right) + 3\frac{R^3}{z^5}e^{-\Phi(z)}v(z) = 0,$$

regular solution:

$$v(z) \underset{z\to 0}{\rightarrow} \frac{m_q z}{R} - \frac{c^2 m_q}{2R}\left(1 - 2\gamma_E - 2\ln(cz) - \psi(3/2)\right) z^3$$

coefficients of z and z^3 terms related ->
linear relation quark mass/quark condensate (not in QCD)

in QCD couplings expected to be large: $g_{a_0\eta\pi} = 12 \pm 6$ GeV

- soft wall model incomplete
  possible way out : add potential terms of the field X in the action

glueballs

(De Fazio, Nicotri, Jugeau, PC, PLB07)

model parameters + dilaton background fixed by the meson spectrum

glueball operators <-> bulk fields according to the AdS/CFT rules

Scalar glueball $J^{PC} = 0^{++}$

4D		5D
$O = Tr\,(F^2)$		$X(x,z)$
$\Delta = 4$		$m_5^2 = 0$
	AdS$_5$ metric	$g_{MN} = e^{2A(z)}\eta_{MN} \qquad A(z) = \ln\frac{R}{z}$
	back. dilaton	$\varphi(z) = c^2 z^2$
	action	$S = -\frac{1}{2k}\int d^5x\sqrt{\lvert g\rvert}\, e^{-\varphi(z)}\, g^{MN}\partial_M X \partial_N X$
	field eq.	$\partial_M\left[\sqrt{\lvert g\rvert}e^{-\varphi(z)}g^{MN}\partial_N X(x,z)\right] = 0$

Spectrum: $m_{G_n}^2 = 4c^2(n+2)$

Pseudoscalar glueball $J^{PC} = 0^{-+}$

4D	5D
$O = Tr(F\tilde{F})$ $\Delta = 4$	$Y(x,z)$ $m_5^2 = 0$

Spectrum: $m_{P_n}^2 = 4c^2(n+2)$

Vector glueball $J^{PC} = 1^{--}$

4D	5D
$O = Tr(F(DF)F)$ $\Delta = 7$ Landau Yung Pomeranchuk	$A_M(x,z)$ $m_5^2 = 24$

action $$S = -\frac{1}{2k}\int d^5x\sqrt{|g|}\,e^{-\varphi(z)}\left[\frac{1}{2}g^{MN}g^{ST}F_{MN}F_{ST} + m_5^2 g^{ST}A_S A_T\right]$$

Spectrum: $m_{V_n}^2 = 4c^2(n+3)$

all results together

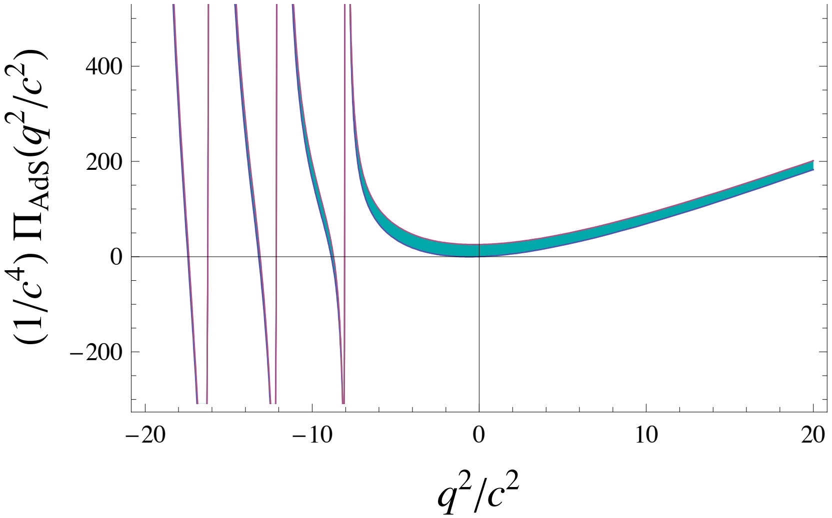
$J^{PC} = 0^{++}$	$m^2_{G_n} = 4c^2(n+2)$	$m^2_{0^{++}} = 2m^2_\rho$
$J^{PC} = 0^{-+}$	$m^2_{P_n} = 4c^2(n+2)$	$m^2_{0^{-+}} = m^2_{0^{++}}$
$J^{PC} = 1^{--}$	$m^2_{V_n} = 4c^2(n+3)$	$m^2_{1^{--}} = 3m^2_\rho$

- ☺ model determined using parameters fixed by vector meson analysis
- ☺ Regge behaviour
- ☹ Regge trajectories of mesons and glueballs with the same slopes
- ☺ glueballs heavier than ρ meson
- ☹ scalar and pseudoscalar glueballs degenerate
- ☺ vector glueballs heavier than scalar
- ☹ glueball masses lighter than obtained by other methods (LQCD, QCDsr)

the two-point correlation function of scalar glueball operators shows features similar to meson operators

$$\Pi_{QCD}(q^2) = i \int d^4x\, e^{iq\cdot x} \langle 0|T[\mathcal{O}_S(x)\mathcal{O}_S(0)]|0\rangle$$

$$\Pi_{AdS}(q^2) = \frac{R^3}{8k}\left\{2\widetilde{B}\,c^4 - q^2(q^2+4c^2)\left(\ln(c^2\epsilon^2) + \psi\left(2+\frac{q^2}{4c^2}\right) + \gamma_E - 3\right)\right\}$$



Poles: $q_n^2 = -4c^2(n+2)$

residues (decay constants²) $F_n^2 = \frac{R^3}{k} 8c^6(n+1)(n+2)$

smaller than in other determinations (lattice)

low-energy theorem $\Pi_{QCD}(0) = -16\beta_1 \langle \mathcal{O}_4 \rangle$

At short distances:

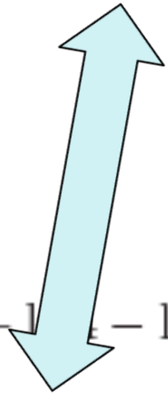
$$\Pi_{QCD}(q^2) \;=\; C_0\, q^4\left(-\ln\,(\frac{q^2}{\nu^2}) + 2 - \frac{1}{\varepsilon'}\right) \quad + \; C_4\langle\mathcal{O}_4\rangle + \frac{C_6}{q^2}\langle\mathcal{O}_6\rangle + \frac{C_8}{q^4}\langle\mathcal{O}_8\rangle$$

$$\begin{aligned}\Pi_{AdS}(q^2) \;=\; & \frac{R^3}{k}\Big\{q^4\cdot\frac{1}{8}\left[2-2\gamma_E+\ln 4-\ln(\frac{q^2}{\nu^2})\right] \\ & +q^2\left[-\frac{c^2}{2}\ln(\frac{q^2}{\nu^2})+\frac{c^2}{4}(1-4\gamma_E+2\ln 4)\right]+ \\ & +\frac{c^4}{6}(12\widetilde{B}-5)+\frac{2c^6}{3}\frac{1}{q^2}-\frac{4c^8}{15}\frac{1}{q^4}+O\left(\frac{1}{q^6}\right)\Big\}\end{aligned}$$

condensates

$$\begin{aligned}\langle\mathcal{O}_4\rangle &= \langle\frac{\alpha_s}{\pi}F^a_{\mu\nu}\,F^{\mu\nu a}\rangle \\ \langle\mathcal{O}_6\rangle &= \langle g_s f_{abc}\,F^a_{\mu\nu}\,F^b_{\nu\rho}\,F^c_{\rho\mu}\rangle \\ \langle\mathcal{O}_8\rangle &= 14\langle\Big(f_{abc}\,F^a_{\mu\alpha}\,F^b_{\nu\alpha}\Big)^2\rangle - \langle\Big(f_{abc}\,F^a_{\mu\nu}\,F^b_{\alpha\beta}\Big)^2\rangle\end{aligned}$$

$$\Pi_{QCD}(q^2) = C_0\, q^4\left(-\ln\left(\frac{q^2}{\nu^2}\right) + 2 - \frac{1}{\varepsilon'}\right) \,?\, + C_4\langle\mathcal{O}_4\rangle + \frac{C_6}{q^2}\langle\mathcal{O}_6\rangle + \frac{C_8}{q^4}\langle\mathcal{O}_8\rangle$$



D=2 condensate (absent in QCD)

$$\Pi_{AdS}(q^2) = \frac{R^3}{k}\Big\{ q^4 \cdot \frac{1}{8}\left[2 - 2\gamma_E + \text{[illegible]} - \ln\left(\frac{q^2}{\nu^2}\right)\right] + q^2\left[-\frac{c^2}{2}\ln\left(\frac{q^2}{\nu^2}\right) + \frac{c^2}{4}(1 - 4\gamma_E + 2\ln 4)\right] + \frac{c^4}{6}(12\widetilde{B} - 5) + \frac{2c^6}{3}\frac{1}{q^2} - \frac{4c^8}{15}\frac{1}{q^4} + O\left(\frac{1}{q^6}\right)\Big\}$$

condensates

$$\langle\mathcal{O}_4\rangle = \langle\frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{\mu\nu a}\rangle$$

$$\langle\mathcal{O}_6\rangle = \langle g_s f_{abc} F^a_{\mu\nu} F^b_{\nu\rho} F^c_{\rho\mu}\rangle$$

$$\langle\mathcal{O}_8\rangle = 14\langle\left(f_{abc} F^a_{\mu\alpha} F^b_{\nu\alpha}\right)^2\rangle - \langle\left(f_{abc} F^a_{\mu\nu} F^b_{\alpha\beta}\right)^2\rangle$$

stability with respect to perturbations of the geometry or of the dilaton

the background can be perturbed in different ways

$$\varphi = c^2 z^2 + \lambda z$$
$$A = \ln\left(\frac{R}{z}\right)$$

perturbing the dilaton

$$\varphi = c^2 z^2$$
$$A = \ln\left(\frac{R}{z}\right) - \lambda z$$

perturbing the metric

the results are different

Modifying the dilaton

0^+ glueballs

- $m_0^2 = 8 + \lambda \frac{3\sqrt{\pi}}{2}$
- $m_1^2 = 12 + \lambda \frac{27\sqrt{\pi}}{16}$
- $m_2^2 = 16 + \lambda \frac{237\sqrt{\pi}}{128}$

1^- glueballs

- $m_0^2 = 12 + \lambda \frac{189\sqrt{\pi}}{128}$
- $m_1^2 = 16 + \lambda \frac{105\sqrt{\pi}}{64}$
- $m_2^2 = 20 + \lambda \frac{14667\sqrt{\pi}}{8192}$

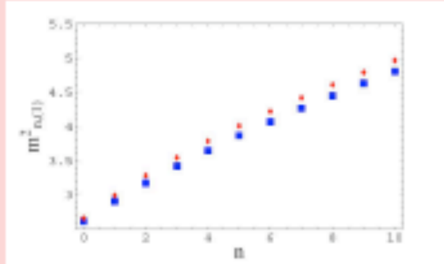


Figure: Mass shifts for scalar (red diamonds) and vector glueballs (blue boxes)

Modifying the geometry

0^+ glueballs

- $m_0^2 = 8 + \lambda \frac{9\sqrt{\pi}}{2}$
- $m_1^2 = 12 + \lambda \frac{81\sqrt{\pi}}{16}$
- $m_2^2 = 16 + \lambda \frac{711\sqrt{\pi}}{128}$

1^- glueballs

- $m_0^2 = 12 - \lambda \frac{1323\sqrt{\pi}}{128}$
- $m_1^2 = 16 - \lambda \frac{1239\sqrt{\pi}}{64}$
- $m_2^2 = 20 - \lambda \frac{74685\sqrt{\pi}}{8192}$

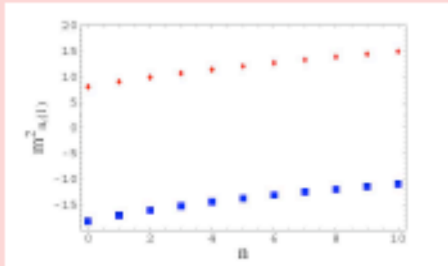


Figure: Mass shifts for scalar (red diamonds) and vector glueballs (blue boxes)

mass difference between scalar and vector glueballs
increased by modifying the geometry

- A few properties of the glueball sector reproduced

- Same discrepancies in the numerics for spectra

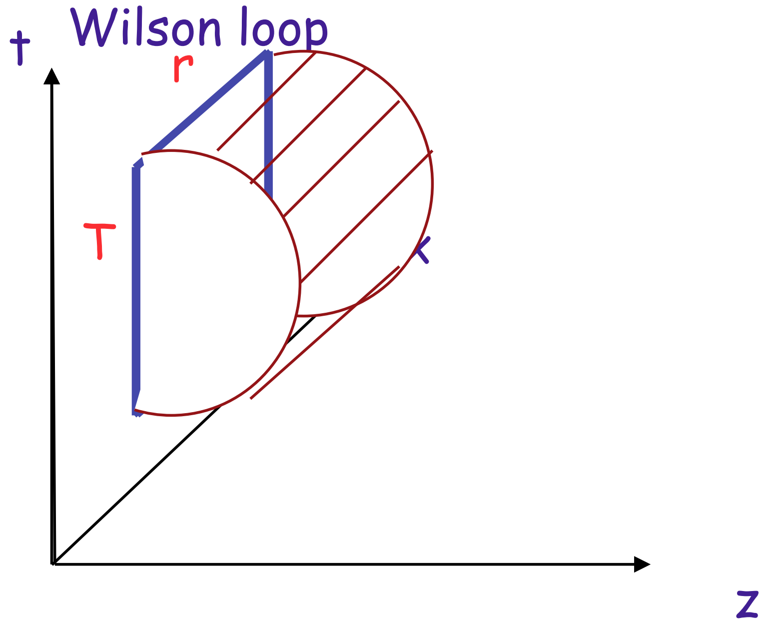
- The gluon condensate and the low energy theorem are not obtained in the standard approach

- Appearance of a D=2 condensates

- Indications for possible improvements (modify geometry and dilaton at intermediate z)

The AdS/QCD model with (quadratic) dilaton reproduces the Cornell potential

Andreev and Zakharov



Find the surface with minimal energy using the Nambu-Goto action

$$V(r) = -\frac{\alpha}{r} + const + \sigma^2 r$$

short distance term

string tension compatible with phenomenology

Conclusions & perspectives

- AdS/QCD models present some features of the old models of QCD
- Some hadronic quantities are reproduced (masses, decay constants)
- Some troubles in the power expansion of the two point correlation functions: extra terms, values of the condensates different from the conventional ones
- Soft-wall model: difficulties in correctly describing chiral symmetry breaking: possible improvement by adding terms in the action

New way of looking at the hadron physics

The features/drawbacks of currently used models show that they are incomplete -> improvement needed -> lot of activity foreseen in the future