New Hadronic Form Factors in Tauola

arXiv:1203.3955 [hep-ph]

Pablo Roig (IFAE, Barcelona)

Based on Collaborations with Tomasz Przedzinski, Olga Shekhovtsova, Z. Was **Tauola**
Daniel Gómez-Dumm, Antonio Pich, Jorge Portolés **RχT**

CABIBBO LAB
Laboratorio Nicola Cabibbo

SuperB

4th SuperB Collaboration Meeting - La Biodola (Isola d'Elba) Italy 31 May to 5 June 2012

CONTENTS

- Motivation
- Theoretical setting
- Theory at Work
- The project
- New Hadronic form factors
- Comparisons
- Conclusions

MOTIVATION

- Low energies (**Flavour factories**):
  Hadronic tau decays

a_μ, $\Delta\alpha(M_z^2)$ $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = -\left(\frac{\alpha M_Z^2}{3\pi}\right) \mathrm{Re} \int_{m_\pi^2}^{\infty} \mathrm{d}s \frac{R(s)}{s(s-M_Z^2-i\epsilon)}$

Davier, Eidelman, Höcker and Zhang '02,'03
Hagiwara, Martin, Nomura and Teubner '02,'03,'06
Jegerlehner '07
Jegerlehner and Nyffeler '09
Davier et. al. '10, '10
Hagiwara, Liao, Martin, Nomura and Teubner '11
BaBar Coll. '12

CC & NC Universality

~~CP~~ BaBar '11

Resonance Dynamics (NP QCD)

τ hadronic width $\Rightarrow \alpha_s(m_\tau^2) \rightarrow \alpha_s(M_z^2)$ Rodrigo, Pich, Santamaría '98

Baikov, Chetyrkin, Kuhn '08; Davier, Descotes-Genon, Malaescu, Zhang '08; Boito et. al. '11, '12

$m_s(m_\tau^2)$ & V_{us} Gámiz, Jamin, Pich, Prades, Schwab '02,'04

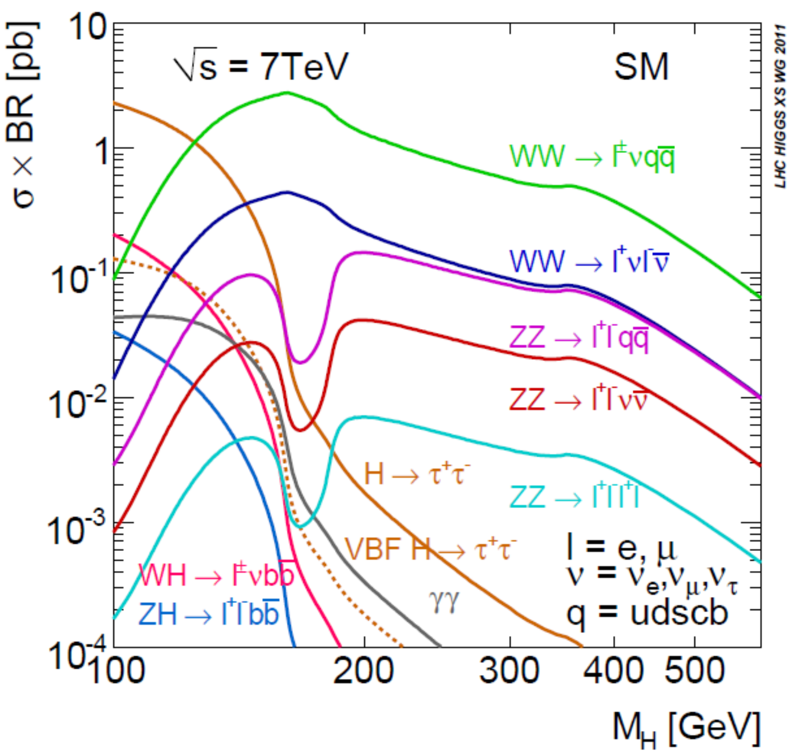
MOTIVATION

See last TAUOLA Workshop (Krakow, May 2012) devoted talks
http://indico.cern.ch/conferenceDisplay.py?confId=188018

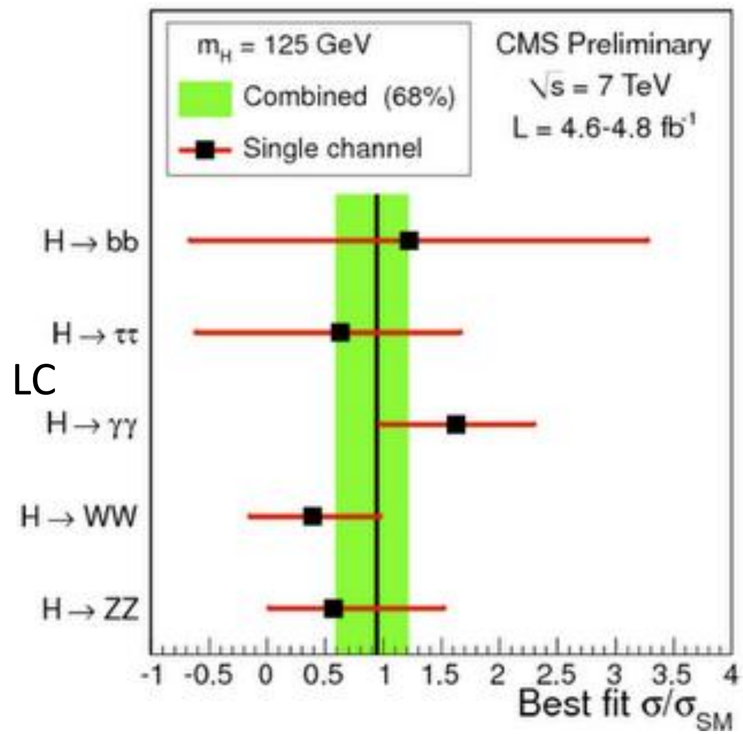
- High energies (**LHC**):
  Hadronic tau decays

Search of the scalar sector of the SM, origin of EWSB

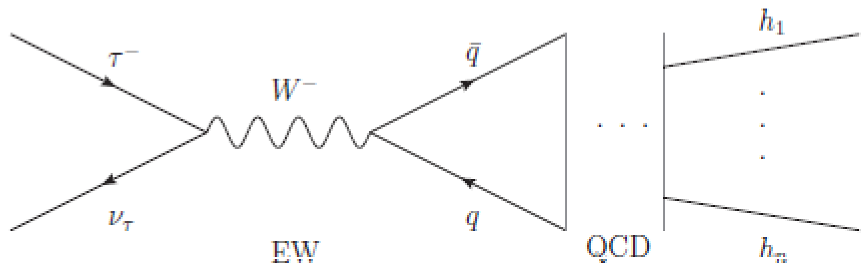
→ In the interesting low-E region the di-τ decay channel can give valuable information



Polarization, CP
Although better at LC



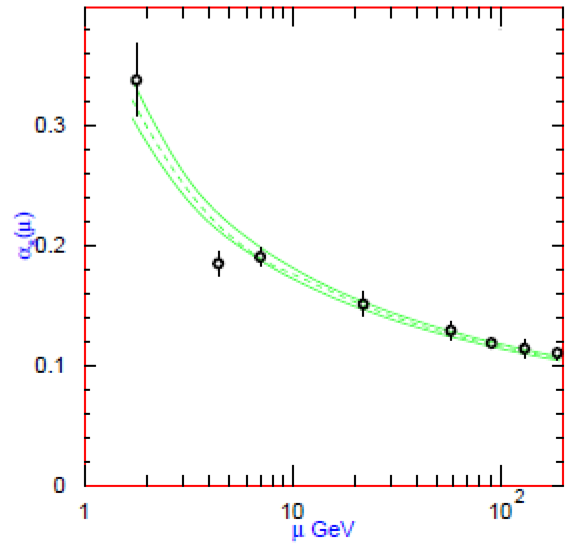
THEORETICAL SETTING



Controlled

Problem

Hadronization in $2m_\pi \leq E \leq M_\tau$



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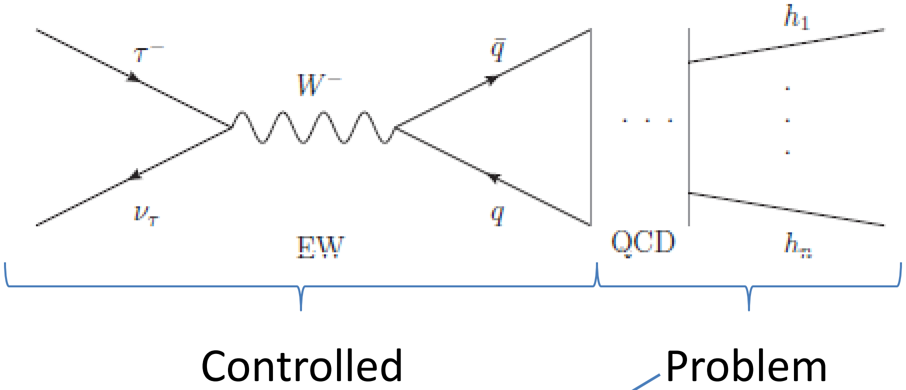

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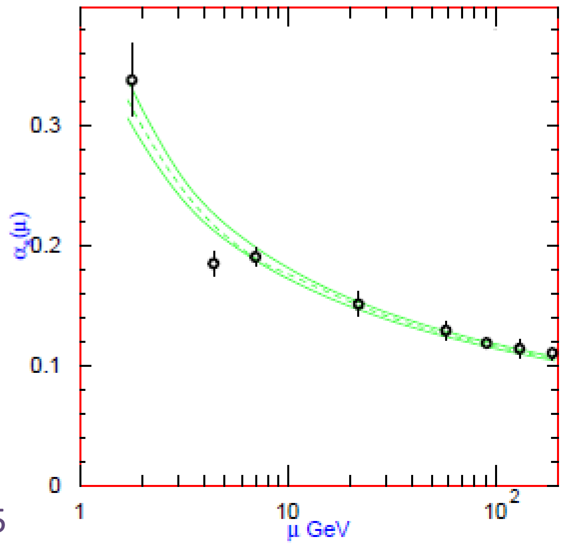

→ At very low energies, **χPT** is the **EFT** of QCD Gasser, Leutwyler '85

Only light quarks (u, d, s). In the massless limit $G \equiv SU(n_f)_L \otimes SU(n_f)_R$

Spontaneous Symmetry Breaking $SU(3)_L \otimes SU(3)_R \to SU(3)_V$

Approximately massless pGbs $\pi^\pm, \pi^0, \eta, K^\pm, K^0, \bar{K}^0$

Small masses ($m < M_\rho, M_{a_1}$) by explicit Symmetry Breaking

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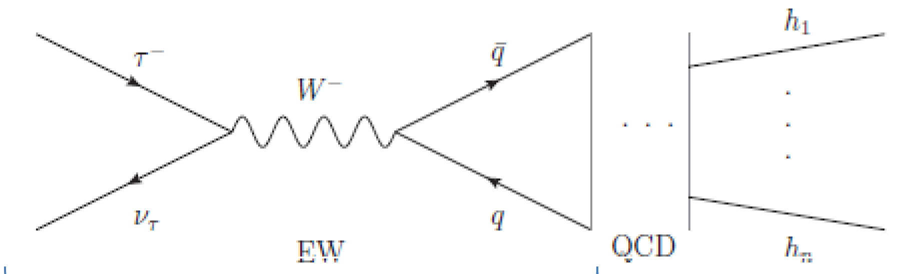

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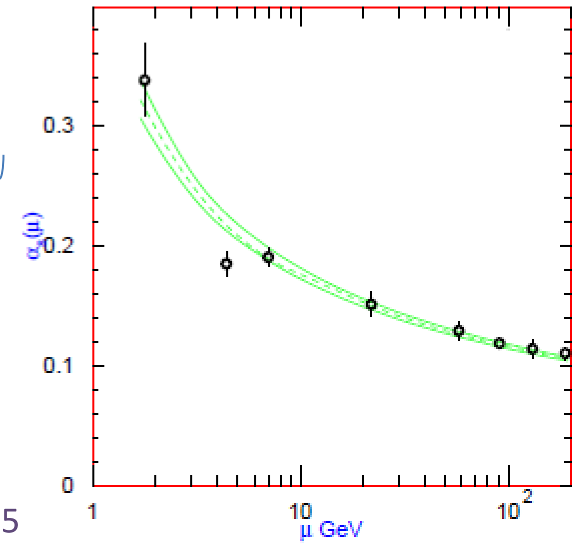

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EFT: $(p^2,m^2)/\Lambda^2$

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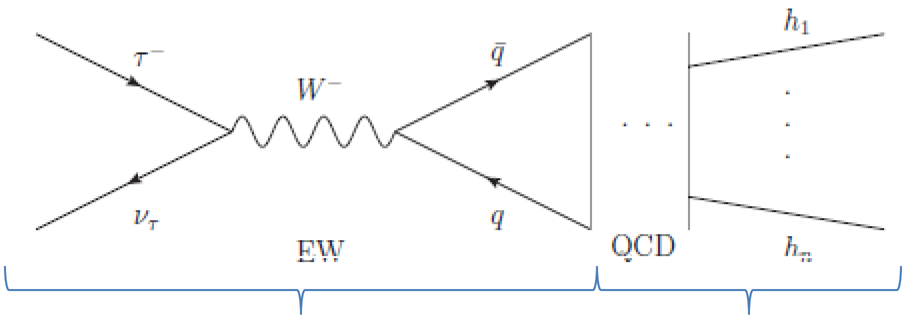

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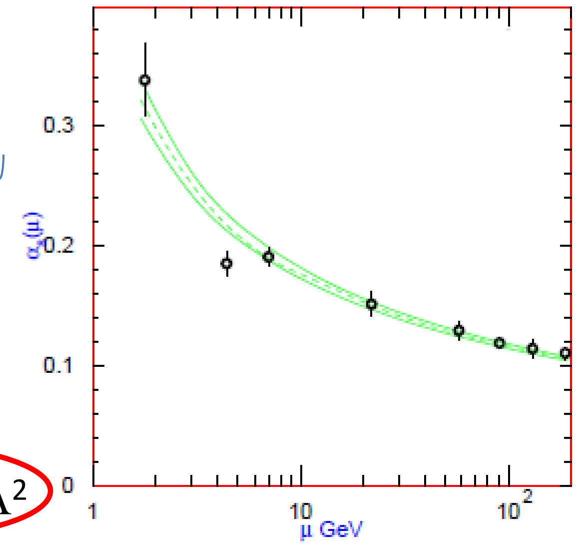


→ At very low energies, **χPT** is the **EFT** of **QCD** EFT: $(p^2,m^2)/\Lambda^2$

→ At larger energies the expansion breaks down and new dofs are needed

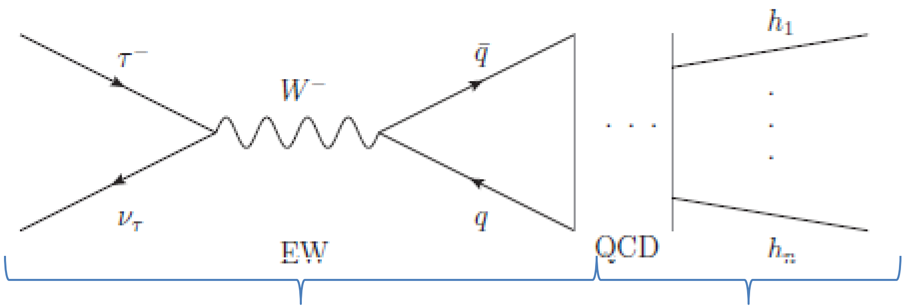
1/Nc 't Hooft '74 ⟷ $\rho, K^*, a_1, a_0, \ldots$

RχT Ecker et. al. '89

Simple description at LO

Preserves χPT results at low energies

THEORY AT WORK



Controlled Problem

$$\mathrm{M} = \frac{G_F}{\sqrt{2}} V_{CKM} \overline{u}(\nu_\tau)\gamma^\mu(1-\gamma_5)u(\tau)T_\mu$$

$$T_\mu = \langle \mathrm{Hadrons}|(V-A)_\mu \, e^{iS_{QCD}}|0\rangle = \Sigma_i(\text{Lorentz Structure})^i F_i(Q^2, s_j)$$

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\overline{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

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Different approaches to deal with the diverse energy regimes

- For $E<M_\rho \rightarrow \chi PT$ up to $O(p^6)$ Gasser, Leutwyler'85, Bijnens, Colangelo, Talavera '98, Bijnens, Talavera'02

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Guerrero, Pich '97

- For $M_\rho \leq E \leq 1$ GeV$\rightarrow$ Match χPT results to VMD using an Omnés solution for dispersion relation.

Omnés solution for dispersion relation Pich, Portolés '01

Unitarization approach Trocóniz, Ynduráin '01, Oller, Oser, Palomar '01

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•1 GeV $\leq$ E$\leq$ 2 GeV $\rightarrow$ Include ρ' through Schwinger-Dyson-like resummation.

Sanz-Cillero, Pich '03

Tower of resonances based on dual QCD

Domínguez '01, Bruch, Khodjamiriam, Kuhn '05

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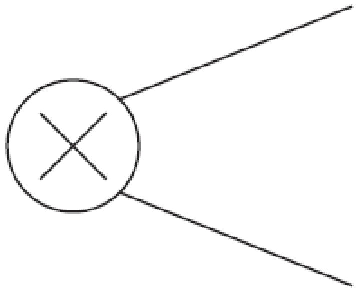

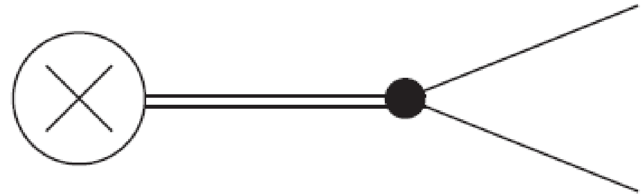

$$\mathcal{L}_\chi^{(2)} = \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{i\,G_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

$$u_\mu = i\left\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - il_\mu)u^\dagger\right\}$$

$$f_\pm^{\mu\nu} = u\,F_L^{\mu\nu}\,u^\dagger \pm u^\dagger\,F_R^{\mu\nu}\,u$$

$$u(\varphi) = \exp\left\{i\frac{\Phi}{\sqrt{2}\,F}\right\} \quad \Phi(x) \equiv \frac{1}{\sqrt{2}}\sum_{a=1}^{8}\lambda_a\,\varphi_a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

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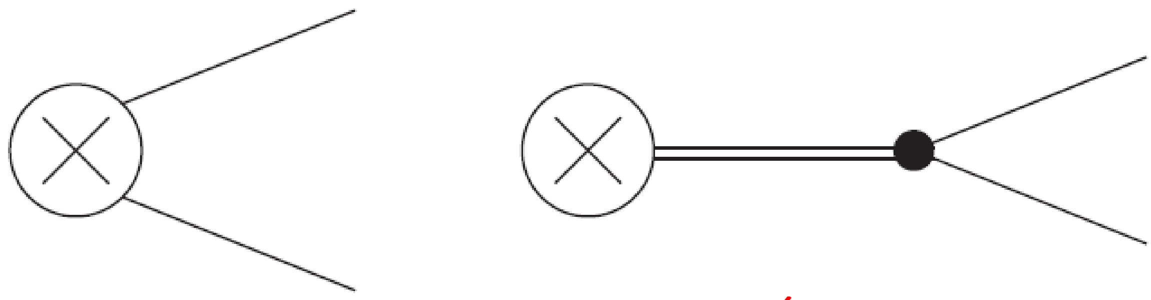
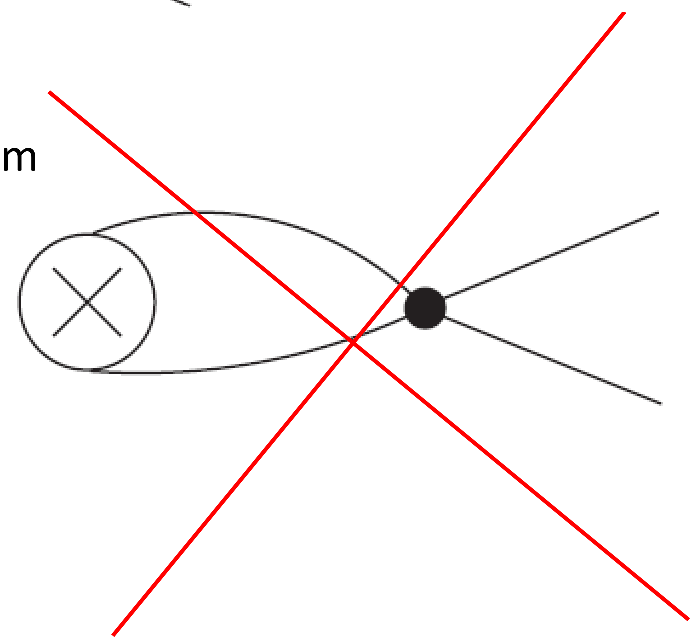


Antisymmetric tensor formalism for spin-one resonances

Ecker *et al.*

Phys.Lett.B223:425,1989

Nucl.Phys.B321:311,1989



To avoid double counting

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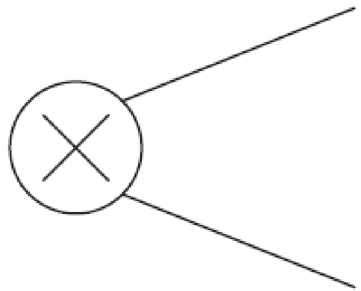

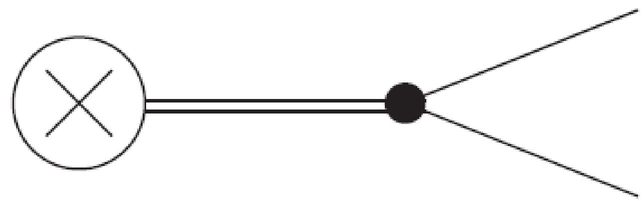

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$$F(s)^{V} = 1 + \frac{F_V G_V}{f_\pi^2}\frac{s}{M_\rho^2 - s}$$

Short-distance constraints

F(s) →0, for s→∞ $\Rightarrow F_V G_V / f_\pi^2 = 1$

$$\Downarrow$$

$$F(s)^{\mathrm{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

THEORETICAL SETTING: χPT, Large N_c, RχT

Portolés '10

Gasser, Leutwyler, '84, '85

- QCD has a well-defined expansion at low-energies that allows to build an **EFT**: **χPT**.

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Ecker, Gasser, Pich, De Rafael '89 Ecker, Gasser, Leutwyler, Pich, De Rafael '89

• Finally, QCD **high-energy** behaviour imposed to the **Green functions** or **form factors**.

Ruiz-Femenía, Pich, Portolés '03 Cirigliano, Ecker,Eidemüller, Pich, Portolés '04

Cirigliano, Ecker, Eidemüller, Kaiser, Pich, Portolés '05, '06

THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

- Tauola is the standard library for MC generation of tau lepton decays. Jadach, Kuhn, Was '90
  Jadach, Was, Decker, Kuhn '93
- Originally it included the hadronic currents at $O(p^2)$ in χPT. Kuhn, Santamaría '90
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The parametrizations used by experimental collaborations (based on 1997-1998 data):

1. Alain Weinstein : http://www.cithep.caltech.edu/~ajw/korb_doc.html#files *(cleo version)*
2. B. Bloch, private communication *(aleph version)* **MOST USED NOWADAYS**

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- 88% of tau hadronic width is covered: (π, K), 2π, 2K, Kπ, 3π, KKπ

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See Guo, Roig, '10 for the radiative decays and the definition of the one-meson decay width

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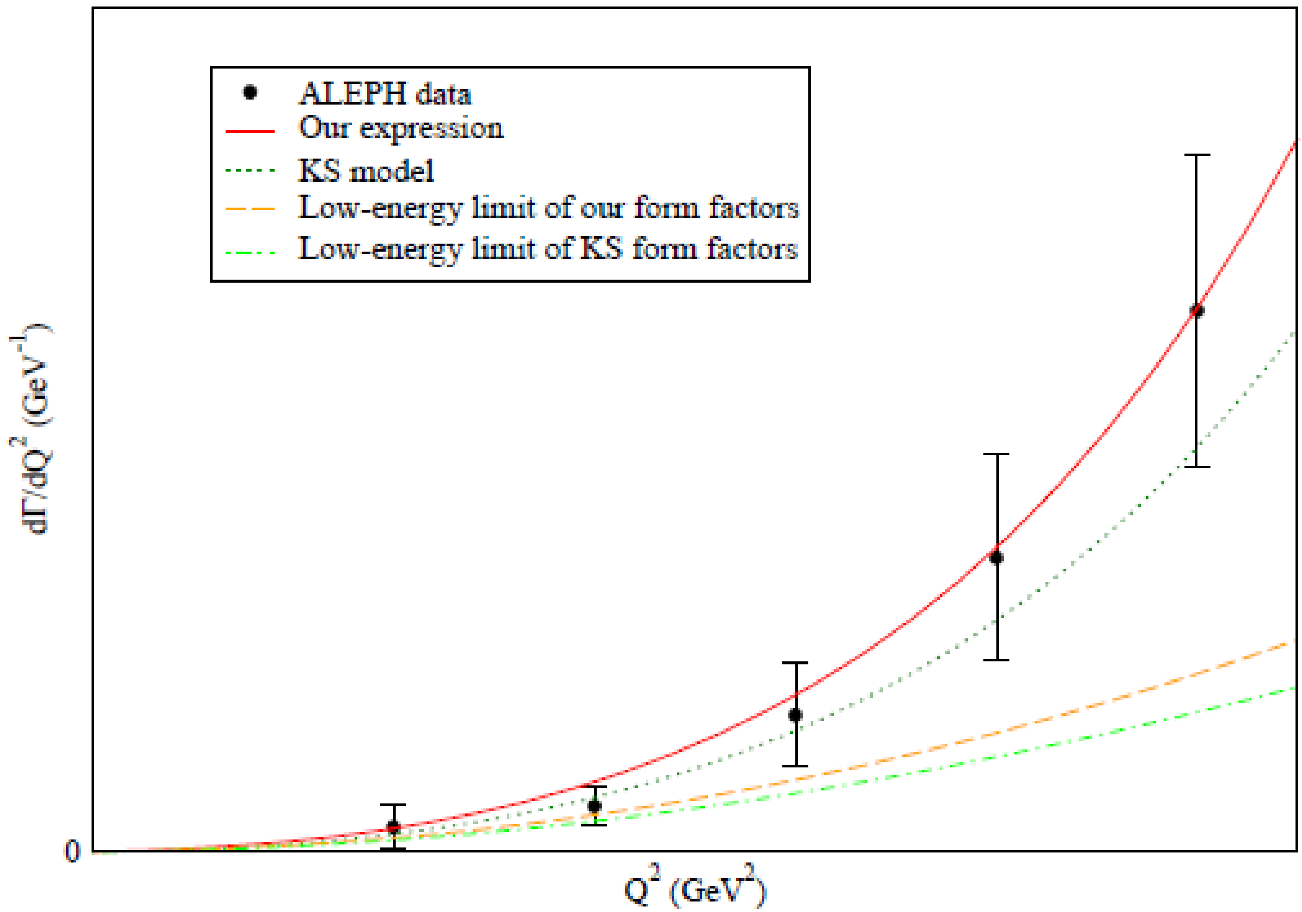
Is this important?

See Guo, Roig, '10 for the radiative decays and the definition of the one-meson decay width

THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

$\tau \longrightarrow \pi\pi\pi\nu_\tau$



THE PROJECT

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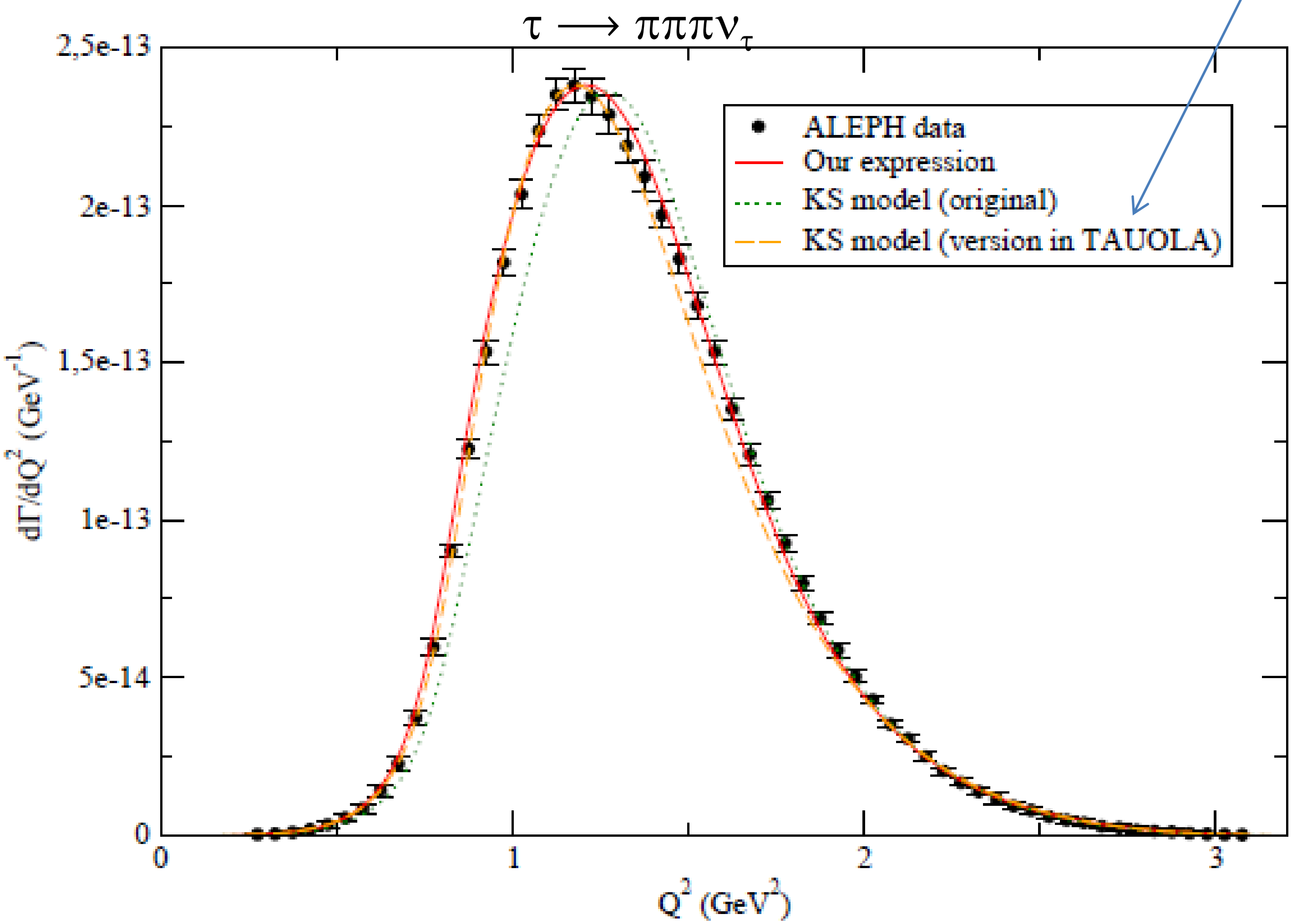
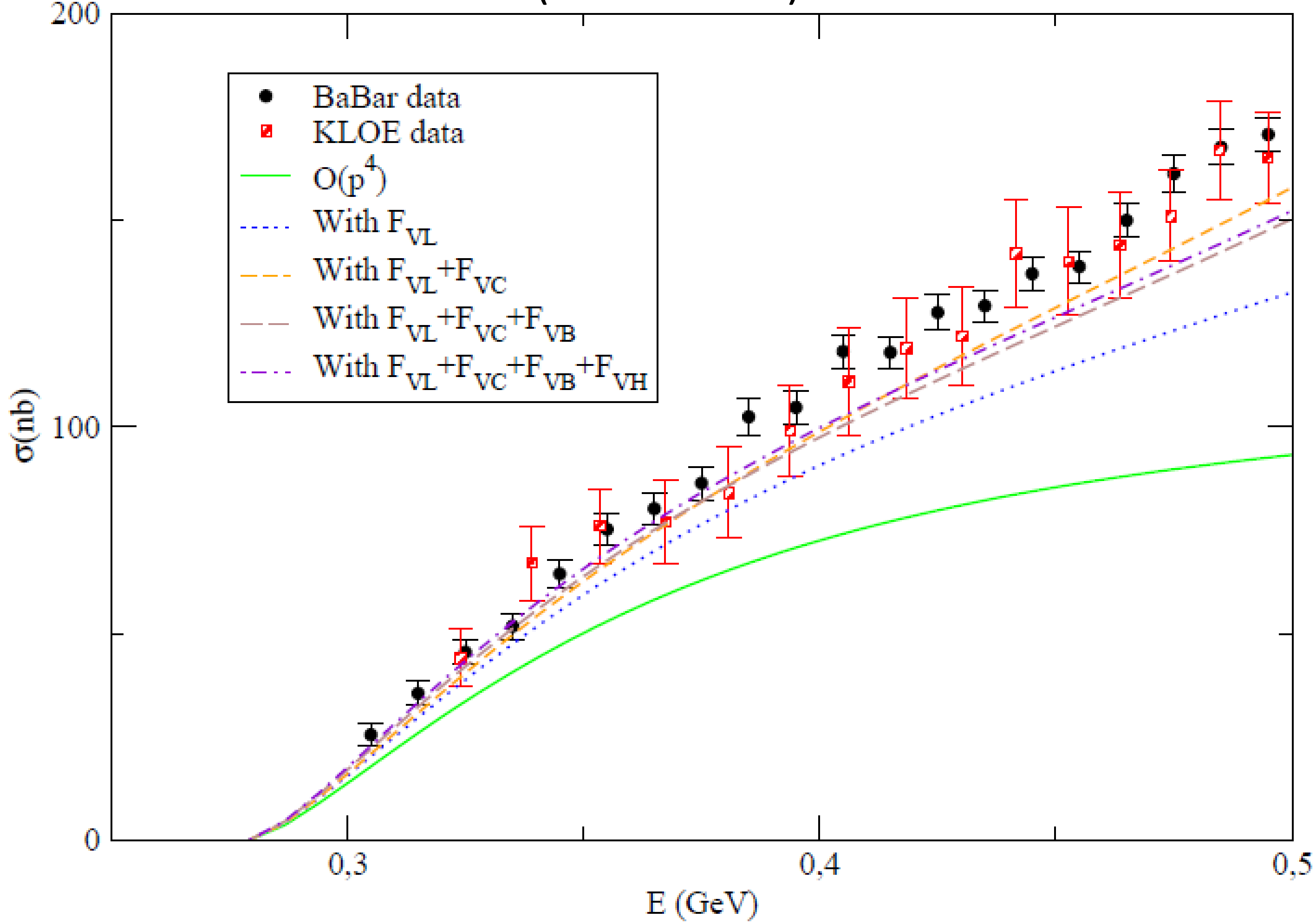
Private old version



THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

$\sigma(e^+e^- \longrightarrow \pi^+\pi^-)$



THE PROJECT

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$\tau \longrightarrow \pi\pi\pi\nu_\tau$ $\sigma(e^+e^- \longrightarrow \pi^+\pi^-)$

Chiral dynamics is important

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  Golonka, Kersevan, Pierzchala, Richter-Was, Was, Worek '03
- Later on, departures from data lead to private versions of the code.
- We plan to implement the most important hadronic currents for tau decay at (least at) $O(p^4)$ in χPT in a consistent way from a Lagrangian approach (RχT).
- 88% of tau hadronic width is covered: (π, K), 2π, 2K, Kπ, 3π, KKπ
- Common work with experimentalists: I. Nugent (BaBar), D. Epifanov, V. Cherepanov (Belle),…
  $\rightarrow$ parameter's fit

Released version: http://annapurna.ifj.edu.pl/~wasm/RChL/RChL.htm

THE PROJECT

Shekhovtsova, Przedzinski, Roig, Was

- Tauola is the standard library for MC generation of tau lepton decays. Jadach, Kuhn, Was ’90
  Jadach, Was, Decker, Kuhn ‘93
- Originally it included the hadronic currents at $O(p^2)$ in χPT. Kuhn, Santamaría ’90
  Decker, Mirkes, Sauer, Was ‘92
- Such approach was successful with LEP-I data.
  Golonka, Kersevan, Pierzchala, Richter-Was, Was, Worek ‘03
- Later on, departures from data lead to private versions of the code.
- We plan to implement the most important hadronic currents for tau decay at (least at) $O(p^4)$ in χPT in a consistent way from a Lagrangian approach (RχT).
- 88% of tau hadronic width is covered: (π, K), 2π, 2K, Kπ, 3π, KKπ
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  $\rightarrow$ parameter’s fit

NEW HADRONIC FORM FACTORS

T_μ=<Hadrons|(V-A)$_\mu$ $e^{iS_{QCD}}$|0>=Σ_i(Lorentz Structure)^i $F_i(Q^2,s_j)$

Two mesons $h_1(p_1)$, $h_2(p_2)$: $J^\mu = N\left[(p_1-p_2)^\mu F^V(s) + (p_1+p_2)^\mu F^S(s)\right]$ $s = (p_1+p_2)^2$

($T_\mu \sim J_\mu$)

Three mesons $h_1(p_1)$, $h_2(p_2)$, $h_3(p_3)$:

$$J^\mu = N \left\{T^\mu_\nu\left[c_1(p_2-p_3)^\nu F_1 + c_2(p_3-p_1)^\nu F_2 + c_3(p_1-p_2)^\nu F_3\right] + c_4 q^\mu F_4 - \frac{i}{4\pi^2F^2}c_5\epsilon^\mu{}_{\nu\rho\sigma}p_1^\nu p_2^\rho p_3^\sigma F_5\right\}.$$

$T_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$ $q^\mu = (p_1+p_2+p_3)^\mu$

$q^2 = (p_1+p_2+p_3)^2$ $s_1 = (p_2+p_3)^2$ $s_2 = (p_1+p_3)^2$ $s_3 = (p_1+p_2)^2$

More mesons ~ 4π (Fischer, Wess and Wagner '80; Bondar et. al. '02)

Form factors

NEW HADRONIC FORM FACTORS

$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

$$F(s)^{\text{VMD}} = \frac{M_\rho^2}{M_\rho^2 - s}$$

Guerrero, Pich '97

$$F(s)^{\text{ChPT}}_{O(p^4)} = 1 + \frac{2L_9^r(\mu)}{f_\pi^2}s - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/\mu^2) + \frac{1}{2}A(m_K^2/s, m_K^2/\mu^2)\right]$$

$$A(m_P^2/s, m_P^2/\mu^2) = \ln\left(m_P^2/\mu^2\right) + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3 \ln\left(\frac{\sigma_P+1}{\sigma_P-1}\right) \qquad \sigma_P \equiv \sqrt{1 - 4m_P^2/s}$$

NEW HADRONIC FORM FACTORS

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ChPT+VMD

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

NEW HADRONIC FORM FACTORS

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

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ChPT+VMD Guerrero, Pich '97

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Unitarity+Analiticity Omnés, '58

O(p²) result for δ^1_1 (s)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} \exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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ChPT+VMD Guerrero, Pich '97

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Guerrero, Pich '97

Gómez-Dumm, Pich, Portolés '00

$$\begin{aligned}\Gamma_\rho(s) &= \frac{M_\rho\, s}{96\pi f_\pi^2}\left\{\theta(s - 4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s - 4m_K^2)\,\sigma_K^3\right\} \\ &= -\frac{M_\rho\, s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\end{aligned}$$

NEW HADRONIC FORM FACTORS

ChPT+VMD Guerrero, Pich '97

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 f_\pi^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

Unitarity+Analiticity Omnés, '58

O(p^2) result for $\delta^1{}_1$ (s)

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s}\exp\left\{\frac{-s}{96\pi^2 f^2}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Guerrero, Pich '97

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi f_\pi^2}\left\{\theta(s-4m_\pi^2)\,\sigma_\pi^3 + \frac{1}{2}\theta(s-4m_K^2)\,\sigma_K^3\right\}$$

Gómez-Dumm, Pich, Portolés '00

$$= -\frac{M_\rho s}{96\pi^2 f_\pi^2}\,\mathrm{Im}\left[A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}A(m_K^2/s, m_K^2/M_\rho^2)\right]$$

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

NEW HADRONIC FORM FACTORS

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

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- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)
- Analiticity and unitarity constraints (NNLO)

Idea: Follow the approach of Jamin, Pich, Portolés '06 including excited resonances while retaining (some of) these nice properties

NEW HADRONIC FORM FACTORS

Starting point

Guerrero, Pich '97

Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2} \left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2) \right] \right\}$$

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
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- SU(2)
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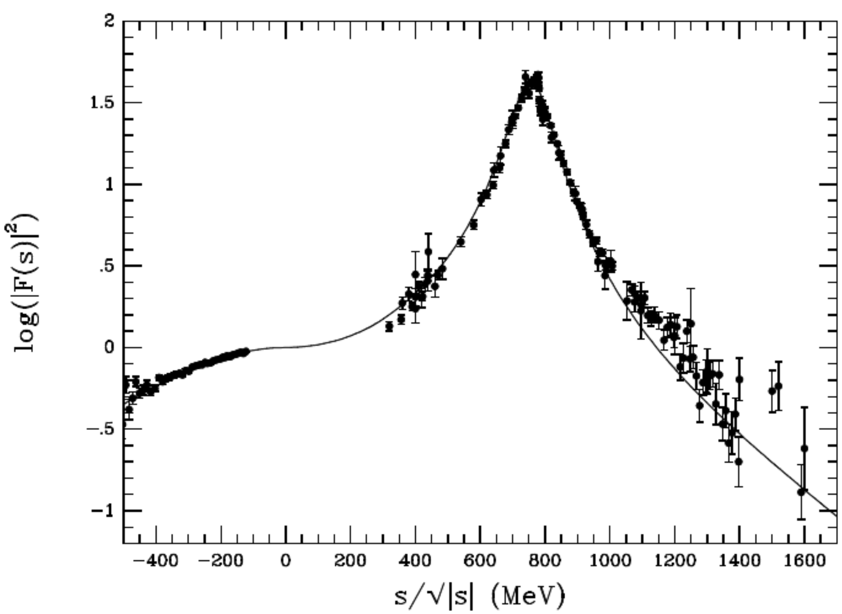

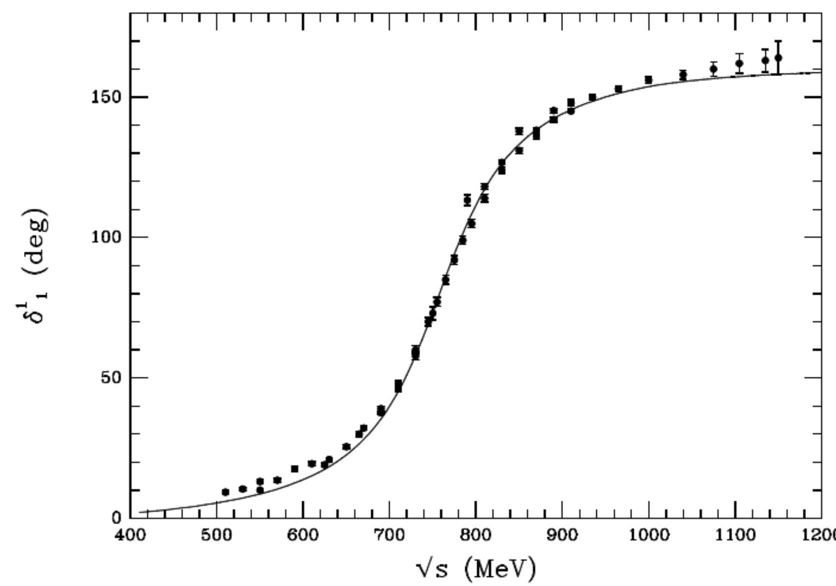

NEW HADRONIC FORM FACTORS

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Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right]$$
$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right]$$
$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right] .$$

- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)
- Analiticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of ρ' + ρ''

NEW HADRONIC FORM FACTORS

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Match χPT results to VMD using an Omnés solution for dispersion relation

$$F(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{ \frac{-s}{96\pi^2 f_\pi^2}\left[\mathrm{Re}A(m_\pi^2/s, m_\pi^2/M_\rho^2) + \frac{1}{2}\mathrm{Re}A(m_K^2/s, m_K^2/M_\rho^2)\right]\right\}$$

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$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right] - \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right] - \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right] .$$

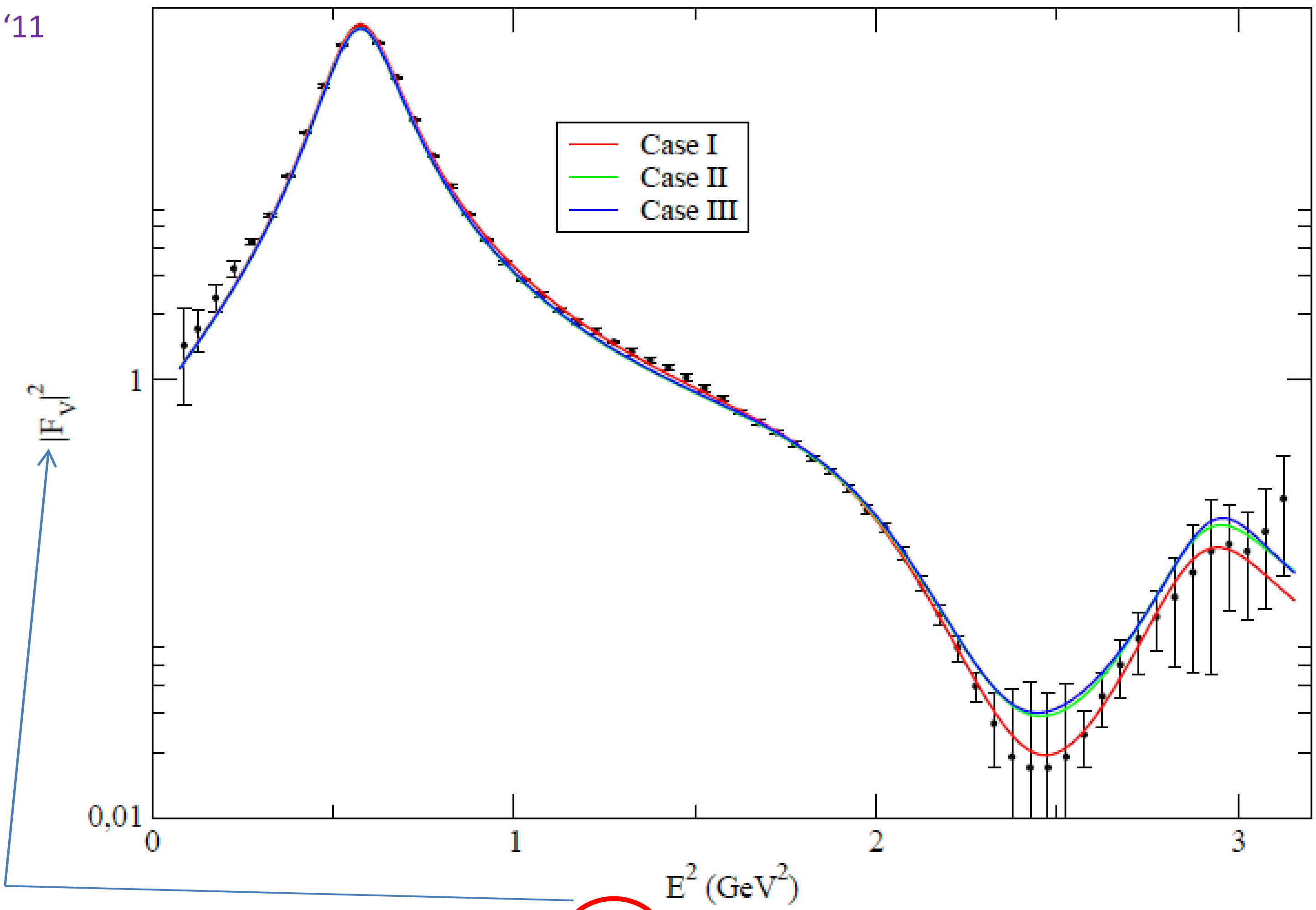
- χPT up to $O(p^4)$ and leading $O(p^6)$ contributions Guerrero '98
- Right fall-off at high energies
- SU(2)

- Analiticity and unitarity constraints (NNLO)
- (Phenomenological) contribution of ρ' + ρ''

This is what is included in TAUOLA right now

NEW HADRONIC FORM FACTORS

Roig '11



$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\overline{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

NEW HADRONIC FORM FACTORS

On the inclusion of excited resonances

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right]$$
$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3\sigma_\pi^3(M_{\rho'}^2)}\Re e A_\pi(s)\right]$$
$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3\sigma_\pi^3(M_{\rho''}^2)}\Re e A_\pi(s)\right].$$

$$\gamma \equiv -F'_V G'_V/F^2 \qquad \delta \equiv -F''_V G''_V/F^2 \qquad F_V G_V + F'_V G'_V + F''_V G''_V + \ldots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle$$

NEW HADRONIC FORM FACTORS

On the inclusion of excited resonances

Our formula

Roig '11

$$F_V^-(s) = \frac{M_\rho^2 + s(\gamma e^{i\phi_1} + \delta e^{i\phi_2})}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left[\frac{-s}{96\pi^2 F_\pi^2}\left(\Re e A_\pi(s) + \Re e A_K(s)/2\right)\right]$$
$$- \frac{\gamma s e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left[\frac{-s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \Re e A_\pi(s)\right]$$
$$- \frac{\delta s e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left[\frac{-s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \Re e A_\pi(s)\right].$$

$$\gamma \equiv -F'_V G'_V / F^2 \qquad \delta \equiv -F''_V G''_V / F^2 \qquad F_V G_V + F'_V G'_V + F''_V G''_V + \ldots = F^2$$

$$\mathcal{L}_2[V(1^{--})] = \frac{F_V}{2\sqrt{2}} \langle V_{\mu\nu} f_+^{\mu\nu} \rangle + \frac{iG_V}{2\sqrt{2}} \langle V_{\mu\nu} [u^\mu, u^\nu] \rangle$$

→ Easy to implement for two meson modes. For three meson modes a number of new couplings (involving new operator structures) appear. At which stage shall we include them?

NEW HADRONIC FORM FACTORS

Similar philosophy for other two meson tau decay modes

$$F^V_{PQ}(s) = F^{VMD}(s)\,\exp\left[\sum_{P,Q} N^{PQ}_{loop}\frac{-s}{96\pi^2F^2}Re A_{PQ}(s)\right]$$

$$F^V_{KK}(s) = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{\frac{-s}{96\pi^2F^2}\left[Re A_\pi(s) + \frac{1}{2}Re A_K(s)\right]\right\}$$

Guerrero, Pich '97, Arganda, Herrero, Portolés '08

$$F_+^{K\pi}(s) = \left[\frac{M^2_{K^*}+\gamma\,s}{M^2_{K^*}-s-iM_{K^*}\Gamma_{K^*}(s)} - \frac{\gamma\,s}{M^2_{K^{*\prime}}-s-iM_{K^{*\prime}}\Gamma_{K^{*\prime}}(s)}\right]e^{\frac{3}{2}\mathrm{Re}[\tilde{H}_{K\pi}(s)+\tilde{H}_{K\eta}(s)]}$$

Jamin, Pich, Portolés '06

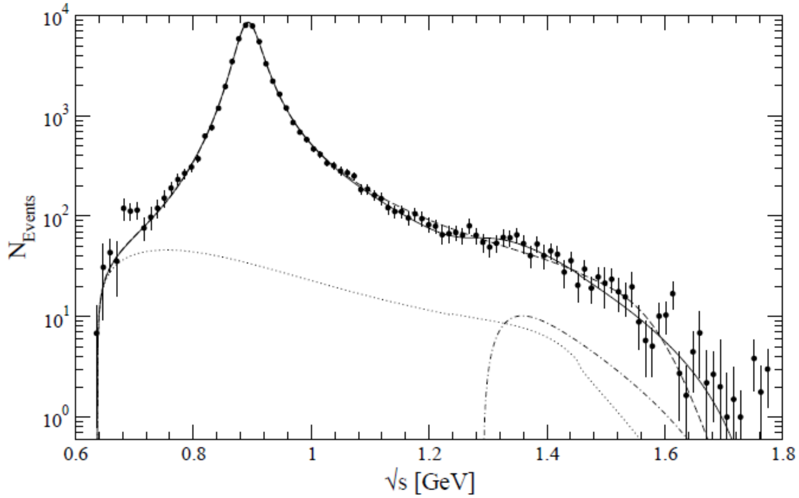
$$\Gamma_{K^*}(q^2) = \frac{M_{K^*}q^2}{128\pi F^2}\left[\lambda^{3/2}\left(1,\frac{m_K^2}{q^2},\frac{m_\pi^2}{q^2}\right)\theta(q^2-thr_{K\pi}) + \lambda^{3/2}\left(1,\frac{m_K^2}{q^2},\frac{m_\eta^2}{q^2}\right)\theta(q^2-thr_{K\eta})\right]$$

$$\langle\pi^-(p)|\bar{s}\,\gamma^\mu\,u|K^0(k)\rangle = \left[(k+p)^\mu - \frac{m_K^2-m_\pi^2}{q^2}(k-p)^\mu\right]F_+(q^2) + \frac{m_K^2-m_\pi^2}{q^2}(k-p)^\mu F_0(q^2)$$

Jamin, Oller, Pich '01, '06

NEW HADRONIC FORM FACTORS

Similar philosophy for other two meson tau decay modes



$$F_+^{K\pi}(s) = \left[\frac{M_{K^*}^2 + \gamma\, s}{M_{K^*}^2 - s - iM_{K^*}\Gamma_{K^*}(s)} - \frac{\gamma\, s}{M_{K^{*\prime}}^2 - s - iM_{K^{*\prime}}\Gamma_{K^{*\prime}}(s)}\right] e^{\frac{3}{2}\mathrm{Re}[\tilde{H}_{K\pi}(s)+\tilde{H}_{K\eta}(s)]}$$

Jamin, Pich, Portolés '06

$$\Gamma_{K^*}(q^2) = \frac{M_{K^*}q^2}{128\pi F^2}\left[\lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\pi^2}{q^2}\right)\theta(q^2 - thr_{K\pi}) + \lambda^{3/2}\left(1, \frac{m_K^2}{q^2}, \frac{m_\eta^2}{q^2}\right)\theta(q^2 - thr_{K\eta})\right]$$

$$\langle \pi^-(p)|\bar{s}\,\gamma^\mu\, u|K^0(k)\rangle = \left[(k+p)^\mu - \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu\right] F_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2}(k-p)^\mu F_0(q^2)$$

Jamin, Oller, Pich '01, '06

NEW HADRONIC FORM FACTORS

$$\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_+(0)}$$

With new improvements also *Exact analiticity and unitarity* Boito, Escribano, Jamin '08

$$\tilde{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*}\bar{A}_{K\pi}(0) + \gamma s}{D(m_{K^*},\gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*\prime}},\gamma_{K^{*\prime}})}$$

$$D(m_n,\gamma_n) = m_n^2 - s - \kappa_n \mathrm{Re}\bar{A}_{K\pi}(s) - i m_n \gamma_n(s)\,,$$

$$\gamma_{K^*}(s) = \gamma_{K^*}\frac{s}{m_{K^*}^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)}, \qquad \kappa_{K^*} = \frac{192\pi F F_K}{\sigma_{K\pi}(m_{K^*}^2)^3}\frac{\gamma_{K^*}}{m_{K^*}},$$

$$F_+^{K\pi}(0) = \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa\bar{A}_{K\pi}(0)}$$

$$\sigma_{K\pi}(s) = \sqrt{\left(1 - \frac{(m_K+m_\pi)^2}{s}\right)\left(1 - \frac{(m_K-m_\pi)^2}{s}\right)}\theta(s-(m_K+m_\pi)^2)$$

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right]/Re\left[F_V^{PQ}(s)\right]$$

$$F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

Other approaches:
- Jamin, Pich, Portolés '06, '08
- Boito, Escribano, Jamin '06
- Moussallam '07
- Bernard, Boito, Passemar '11

NEW HADRONIC FORM FACTORS

$$\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_+(0)}$$

With new improvements also
Exact analiticity and unitarity

Boito, Escribano, Jamin '08

$$\tilde{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*}\bar{A}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*\prime}}, \gamma_{K^{*\prime}})}$$

$$D(m_n, \gamma_n) = m_n^2 - s - \kappa_n \mathrm{Re}\bar{A}_{K\pi}(s) - i m_n \gamma_n(s)\,,$$

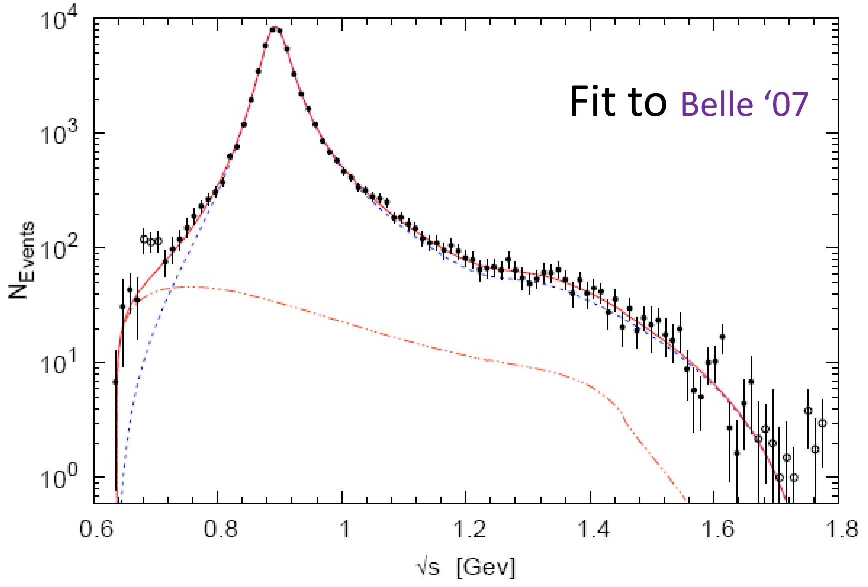
$$\gamma_{K^*}(s) = \gamma_{K^*}\frac{s}{m_{K^*}^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)}, \qquad \kappa_{K^*} = \frac{192\pi F F_K}{\sigma_{K\pi}(m_{K^*}^2)^3}\frac{\gamma_{K^*}}{m_{K^*}},$$

$$F_+^{K\pi}(0) = \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa\bar{A}_{K\pi}(0)}$$

$$\sigma_{K\pi}(s) = \sqrt{\left(1 - \frac{(m_K + m_\pi)^2}{s}\right)\left(1 - \frac{(m_K - m_\pi)^2}{s}\right)}\theta(s - (m_K + m_\pi)^2)$$

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right]/Re\left[F_V^{PQ}(s)\right]$$

$$F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s' - s - i\epsilon)}\right\}$$



Other approaches: Jamin, Pich, Portolés '06, '08
Boito, Escribano, Jamin '06
Moussallam '07
Bernard, Boito, Passemar '11

Pablo Roig

NEW HADRONIC FORM FACTORS

$$\tilde{F}_{+,0}(q^2) \equiv \frac{F_{+,0}(q^2)}{F_+(0)}$$

With new improvements also
Exact analiticity and unitarity

Boito, Escribano, Jamin ‘08

$$\tilde{F}_+^{K\pi}(s) = \frac{m_{K^*}^2 - \kappa_{K^*}\bar{A}_{K\pi}(0) + \gamma s}{D(m_{K^*}, \gamma_{K^*})} - \frac{\gamma s}{D(m_{K^{*\prime}}, \gamma_{K^{*\prime}})}$$

$$D(m_n, \gamma_n) = m_n^2 - s - \kappa_n \mathrm{Re}\bar{A}_{K\pi}(s) - i m_n \gamma_n(s)\,,$$

$$\gamma_{K^*}(s) = \gamma_{K^*}\frac{s}{m_{K^*}^2}\frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(m_{K^*}^2)}, \qquad \kappa_{K^*} = \frac{192\pi F F_K}{\sigma_{K\pi}(m_{K^*}^2)^3}\frac{\gamma_{K^*}}{m_{K^*}},$$

$$F_+^{K\pi}(0) = \frac{m_{K^*}^2}{m_{K^*}^2 - \kappa\bar{A}_{K\pi}(0)}$$

$$\sigma_{K\pi}(s) = \sqrt{\left(1 - \frac{(m_K + m_\pi)^2}{s}\right)\left(1 - \frac{(m_K - m_\pi)^2}{s}\right)}\theta(s - (m_K + m_\pi)^2)$$

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right] / Re\left[F_V^{PQ}(s)\right]$$

$$F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s' - s - i\epsilon)}\right\}$$

→ We will include also the dispersive approach for other two-meson decay channels

Other approaches: Jamin, Pich, Portolés ’06, ‘08

Boito, Escribano, Jamin ‘06

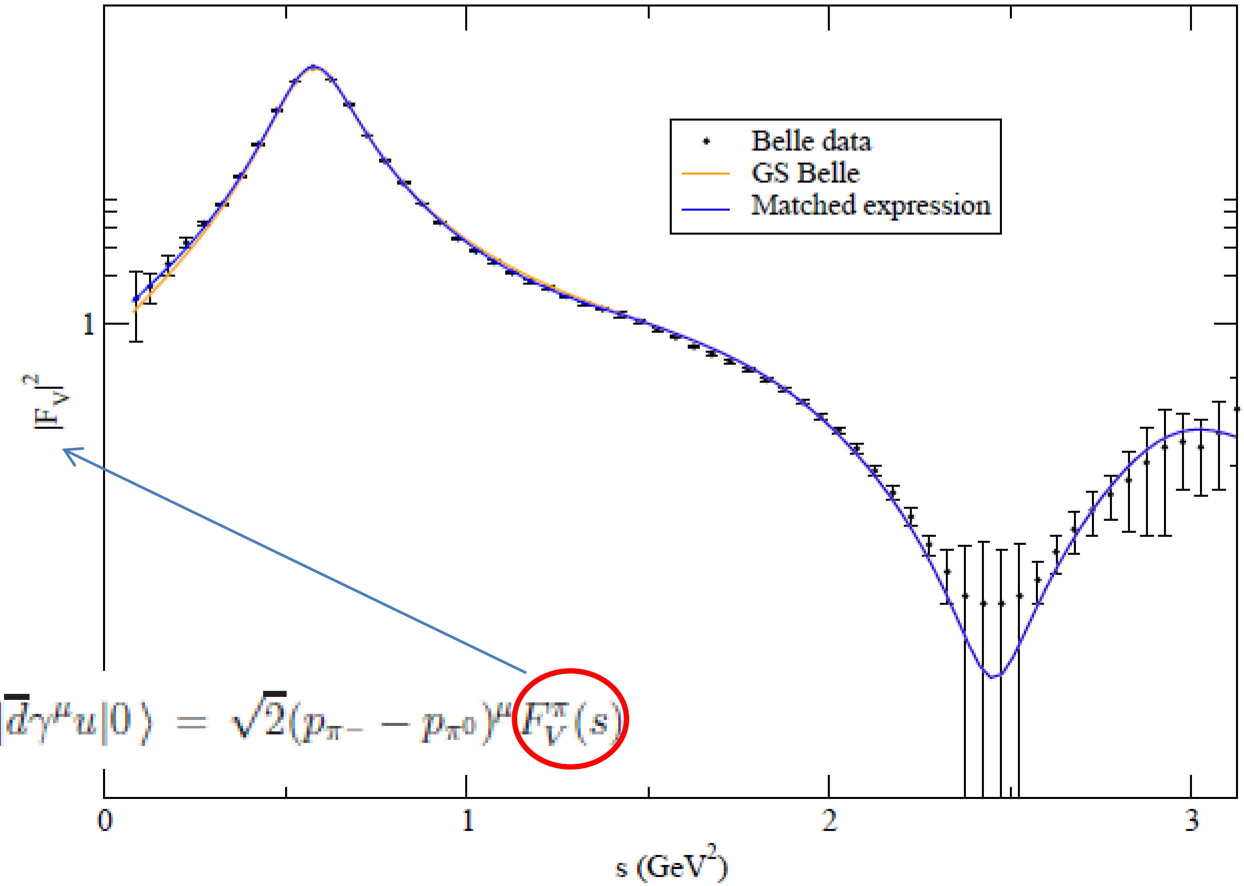
Moussallam ‘07

Bernard, Boito, Passemar ‘11

NEW HADRONIC FORM FACTORS

With new improvements also
Exact analiticity and unitarity

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right] / Re\left[F_V^{PQ}(s)\right] \longrightarrow F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s' - s - i\epsilon)}\right\}$$

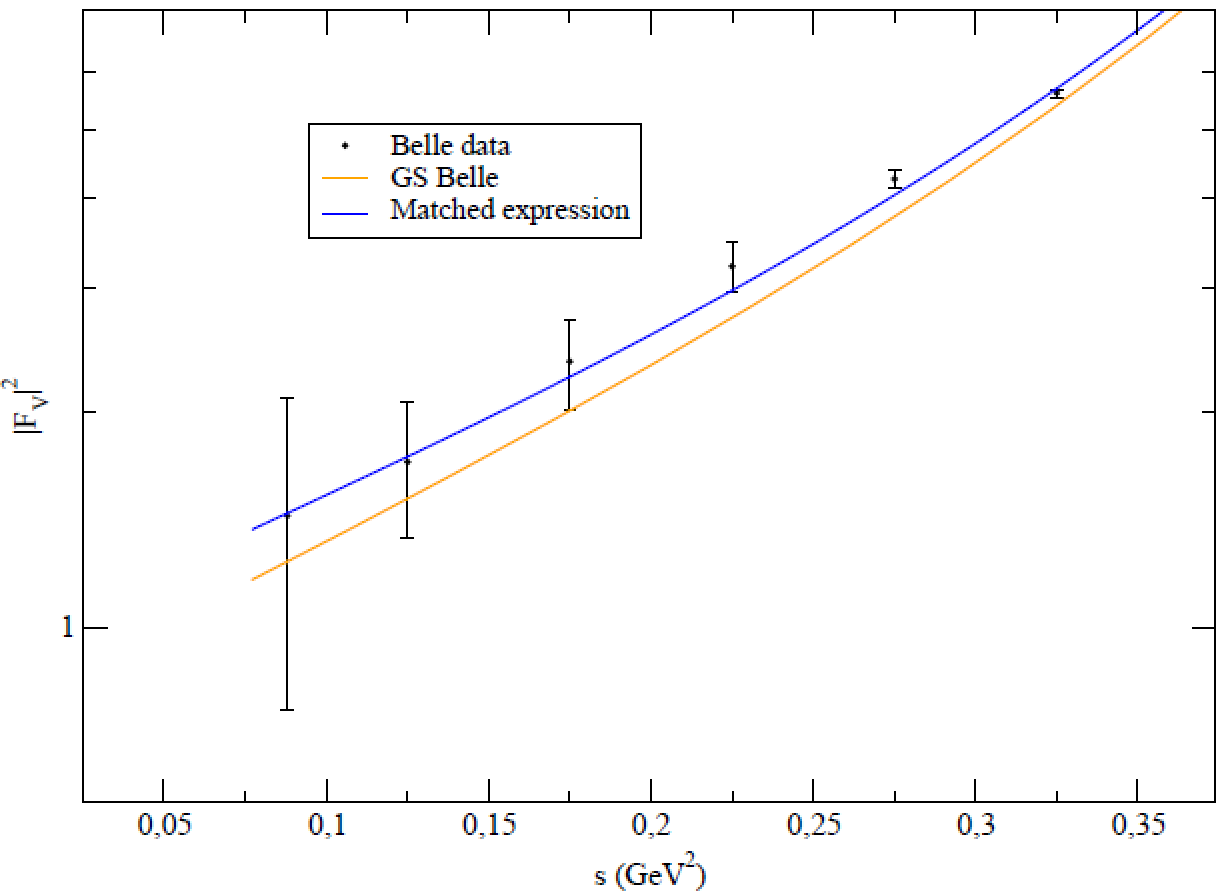


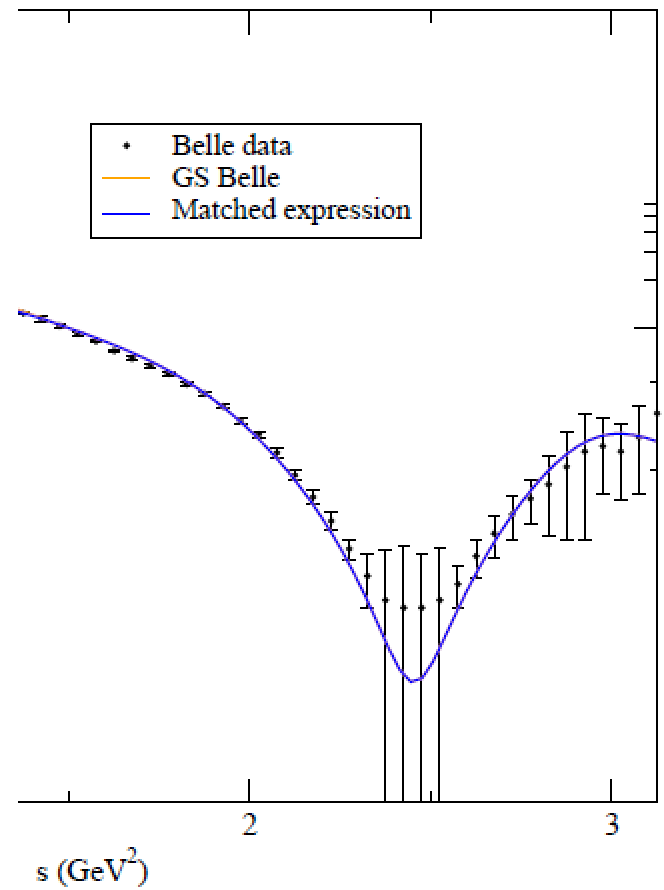
$$\langle \pi^-(p_{\pi^-})\pi^0(p_{\pi^0})|\bar{d}\gamma^\mu u|0\rangle = \sqrt{2}(p_{\pi^-} - p_{\pi^0})^\mu F_V^\pi(s)$$

NEW HADRONIC FORM FACTORS

With new improvements also
Exact analiticity and unitarity

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right]/Re\left[F_V^{PQ}(s)\right] \longrightarrow F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

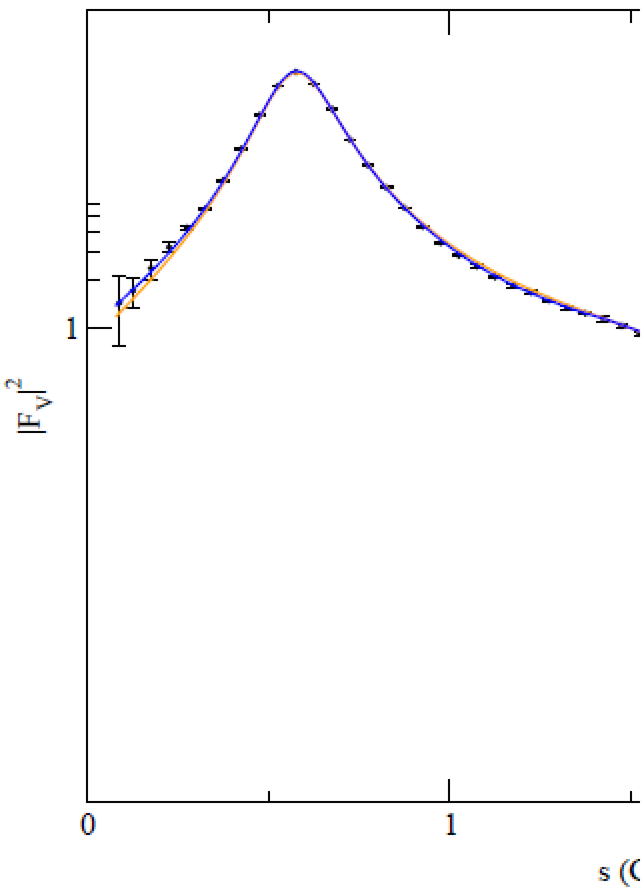


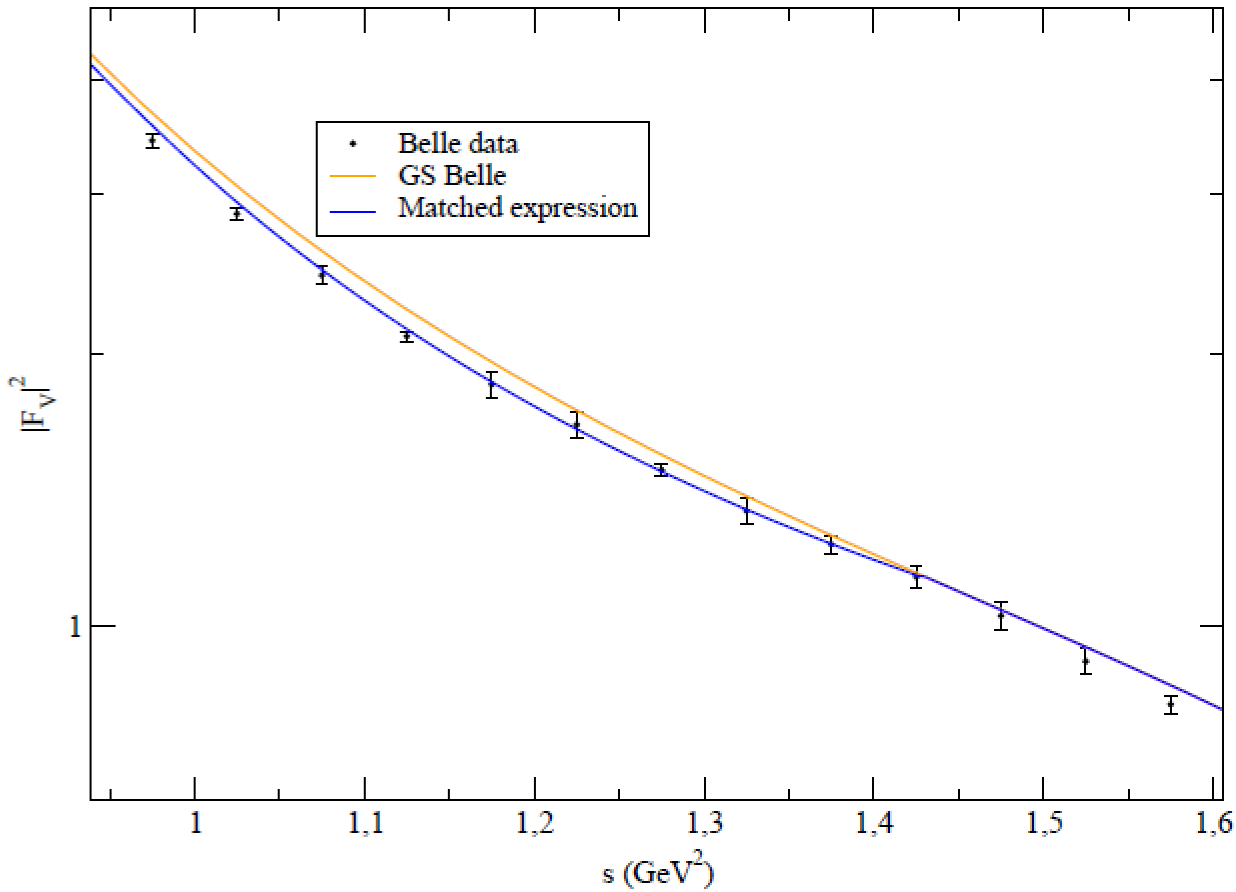


NEW HADRONIC FORM FACTORS

With new improvements also
Exact analiticity and unitarity

$$\delta^{PQ}(s) = Im\left[F_V^{PQ}(s)\right] / Re\left[F_V^{PQ}(s)\right] \longrightarrow F_V^{PQ}(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} ds' \frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

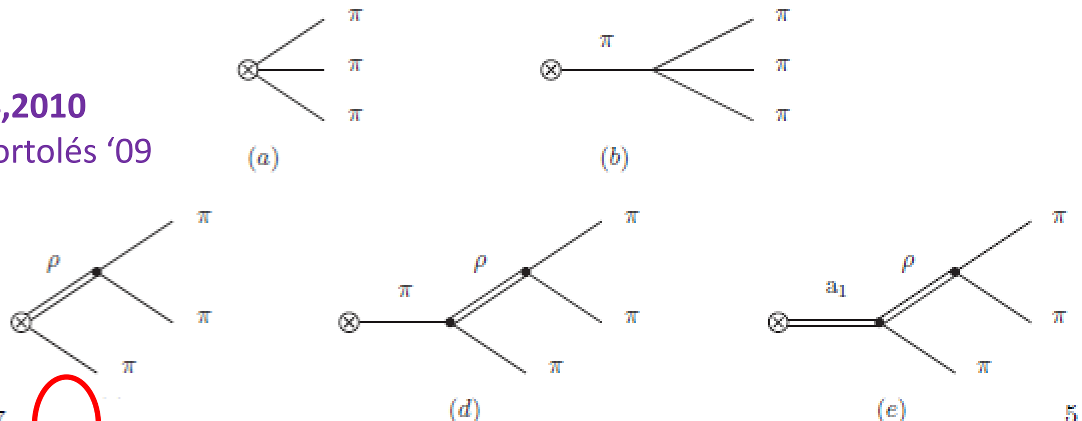




NEW HADRONIC FORM FACTORS

$\tau \longrightarrow \pi\pi\pi\nu_\tau$

Phys.Lett.B685:158-164,2010
Gómez-Dumm, Roig, Pich, Portolés ‘09

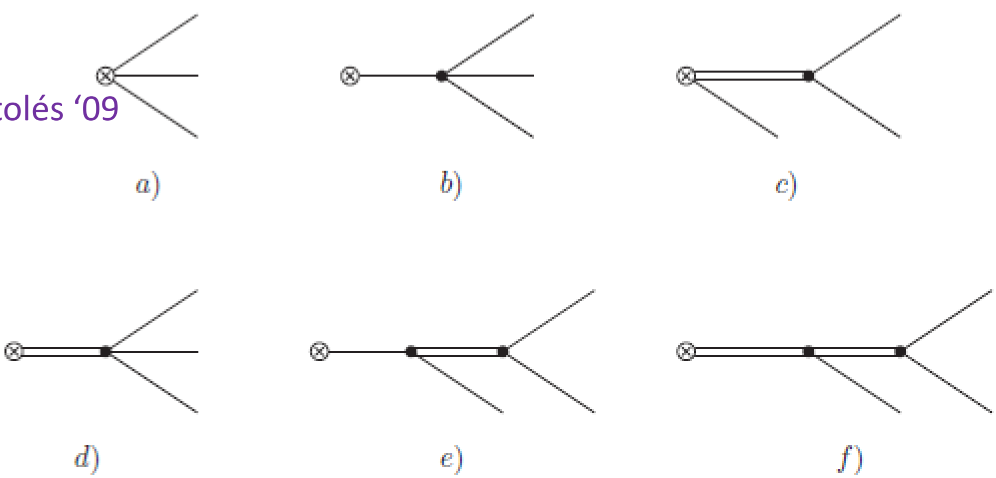


$$\mathcal{L}_4^{\rm V} = \sum_{i=1}^{5} \frac{g_i}{M_V} \mathcal{O}^i_{\rm VPPP} + \sum_{i=1}^{7} \frac{c_i}{M_V} \mathcal{O}^i_{\rm VJP}$$

$$\mathcal{L}_2^{\rm RR} = \sum_{i=1}^{5} \lambda_i \mathcal{O}^i_{\rm VAP} + \sum_{i=1}^{4} d_i \mathcal{O}^i_{\rm VVP}$$

$\tau \longrightarrow KK\pi\nu_\tau$

Gómez-Dumm, Roig, Pich, Portolés ‘09
Phys.Rev.D81:034031,2010



NEW HADRONIC FORM FACTORS

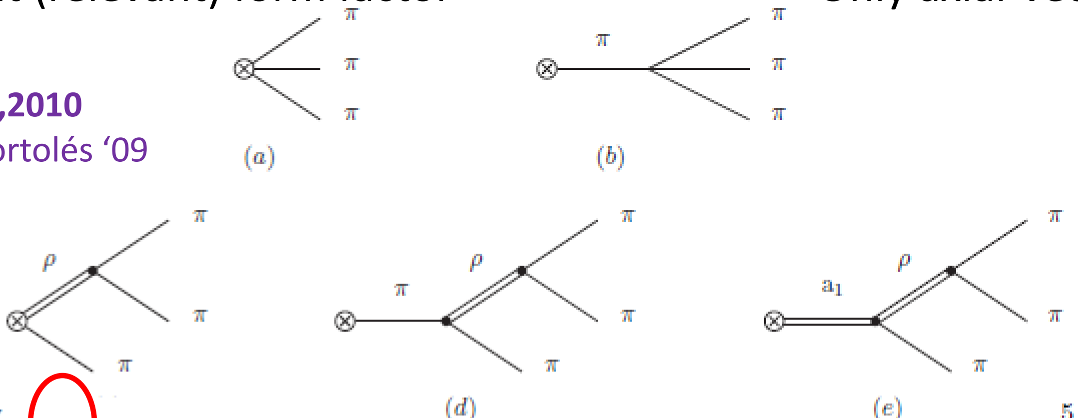
$\tau \longrightarrow \pi\pi\pi\nu_\tau$

Only one independent (relevant) form factor

Only axial-vector current

Phys.Lett.B685:158-164,2010

Gómez-Dumm, Roig, Pich, Portolés '09



$$\mathcal{L}_4^{\rm V} = \sum_{i=1}^{5} \frac{g_i}{M_V} \mathcal{O}^i_{\rm VPPP} + \sum_{i=1}^{7} \frac{c_i}{M_V} \mathcal{O}^i_{\rm VJP}$$

$$\mathcal{L}_2^{\rm RR} = \sum_{i=1}^{5} \lambda_i \mathcal{O}^i_{\rm VAP} + \sum_{i=1}^{4} d_i \mathcal{O}^i_{\rm VVP}$$

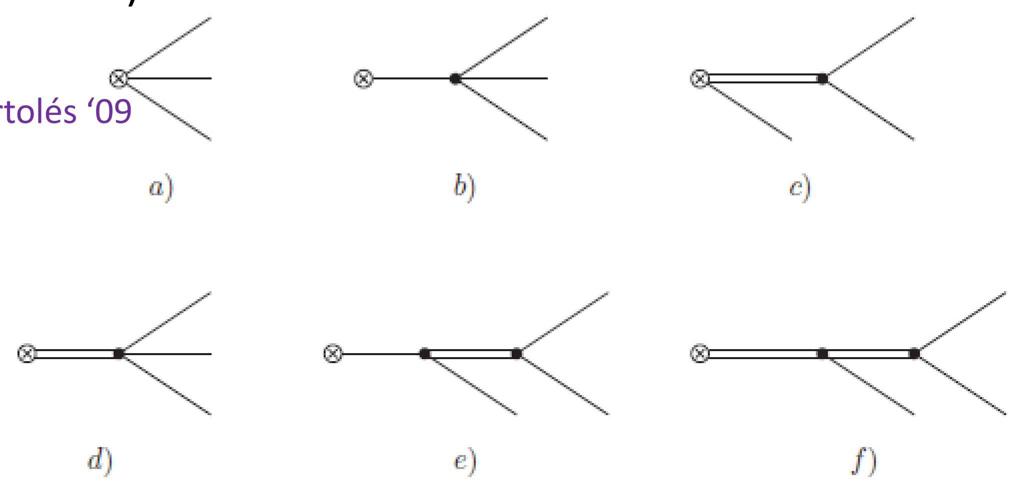
$\tau \longrightarrow KK\pi\nu_\tau$

Three independent (relevant) FFs

Both axial-vector and vector current

Gómez-Dumm, Roig, Pich, Portolés '09

Phys.Rev.D81:034031,2010



NEW HADRONIC FORM FACTORS

Phys.Rev.D69:073002,2004
Gómez-Dumm, Pich, Portolés '03

Phys.Lett.B685:158-164,2010
Gómez-Dumm, Roig, Pich, Portolés '09

$$F_{\pm i} = \pm (F_i^{\chi} + F_i^{\mathrm{R}} + F_i^{\mathrm{RR}}) \ , \qquad i = 1,2 \qquad\qquad F_2(Q^2,s,t) = F_1(Q^2,t,s)$$

$$F_1^{\chi}(Q^2,s,t) = -\frac{2\sqrt{2}}{3F}$$

$$F_1^{\mathrm{R}}(Q^2,s,t) = \frac{\sqrt{2}\,F_V\,G_V}{3\,F^3}\left[\frac{3\,s}{s-M_V^2} - \left(\frac{2G_V}{F_V}-1\right)\left(\frac{2Q^2-2s-u}{s-M_V^2}+\frac{u-s}{t-M_V^2}\right)\right]$$

$$F_1^{\mathrm{RR}}(Q^2,s,t) = \frac{4\,F_A\,G_V}{3\,F^3}\,\frac{Q^2}{Q^2-M_A^2}\left[-\,(\lambda'+\lambda'')\,\frac{3\,s}{s-M_V^2} + H(Q^2,s)\,\frac{2Q^2+s-u}{s-M_V^2} + H(Q^2,t)\,\frac{u-s}{t-M_V^2}\right]$$

$$H(Q^2,x) = -\,\lambda_0\,\frac{m_\pi^2}{Q^2} + \lambda'\,\frac{x}{Q^2} + \lambda''$$

Relations from short-distance QCD:

$$\left\{\begin{array}{l} F_V\,G_V = F^2 \\ F_V^2 - F_A^2 = F^2 \\ F_V^2\,M_V^2 = F_A^2\,M_A^2 \end{array}\right.$$

$$\lambda' = \frac{F^2}{2\sqrt{2}\,F_A\,G_V} = \frac{M_A}{2\sqrt{2}\,M_V} \ ,$$

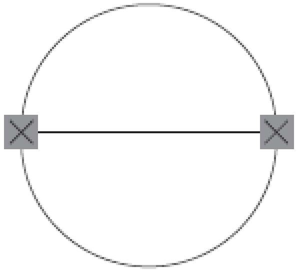
$$\lambda'' = \frac{2\,G_V - F_V}{2\sqrt{2}\,F_A} = \frac{M_A^2-2M_V^2}{2\sqrt{2}\,M_V\,M_A} \ ,$$

$$4\,\lambda_0 = \lambda'+\lambda'' = \frac{M_A^2-M_V^2}{\sqrt{2}\,M_V\,M_A} \ ,$$

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09

$\Gamma a_1(Q^2)$:



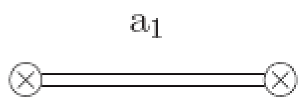
$$\Gamma_{a_1}(Q^2) \;=\; \Gamma^{\pi}_{a_1}(Q^2)\,\theta(Q^2-9m_\pi^2) \;+\; \Gamma^{K}_{a_1}(Q^2)\,\theta(Q^2-(2m_K+m_\pi)^2)\ ,$$

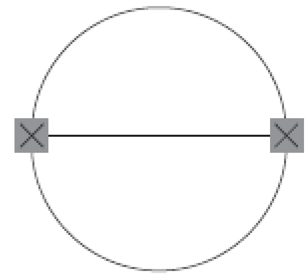
$$\Gamma^{\pi,K}_{a_1}(Q^2) \;=\; \frac{-S}{192(2\pi)^3F_A^2M_{a_1}}\left(\frac{M_{a_1}^2}{Q^2}-1\right)^2\int \mathrm{d}s\,\mathrm{d}t\; T^{\pi,K\mu}_{1^+}\,T^{\pi,K*}_{1^+\mu}\ .$$

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09

$\Gamma a_1(Q^2)$:





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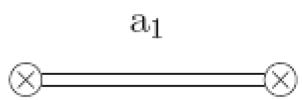
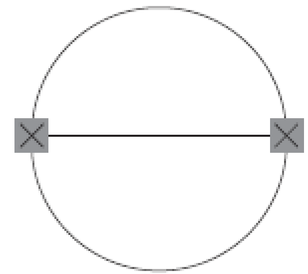
$$\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)} \longrightarrow \frac{1}{1+\beta_{\rho'}}\left[\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)}+\frac{\beta_{\rho'}}{M_{\rho'}^2-q^2-iM_{\rho'}\Gamma_{\rho'}(q^2)}\right]$$

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09

$\Gamma a_1(Q^2)$:





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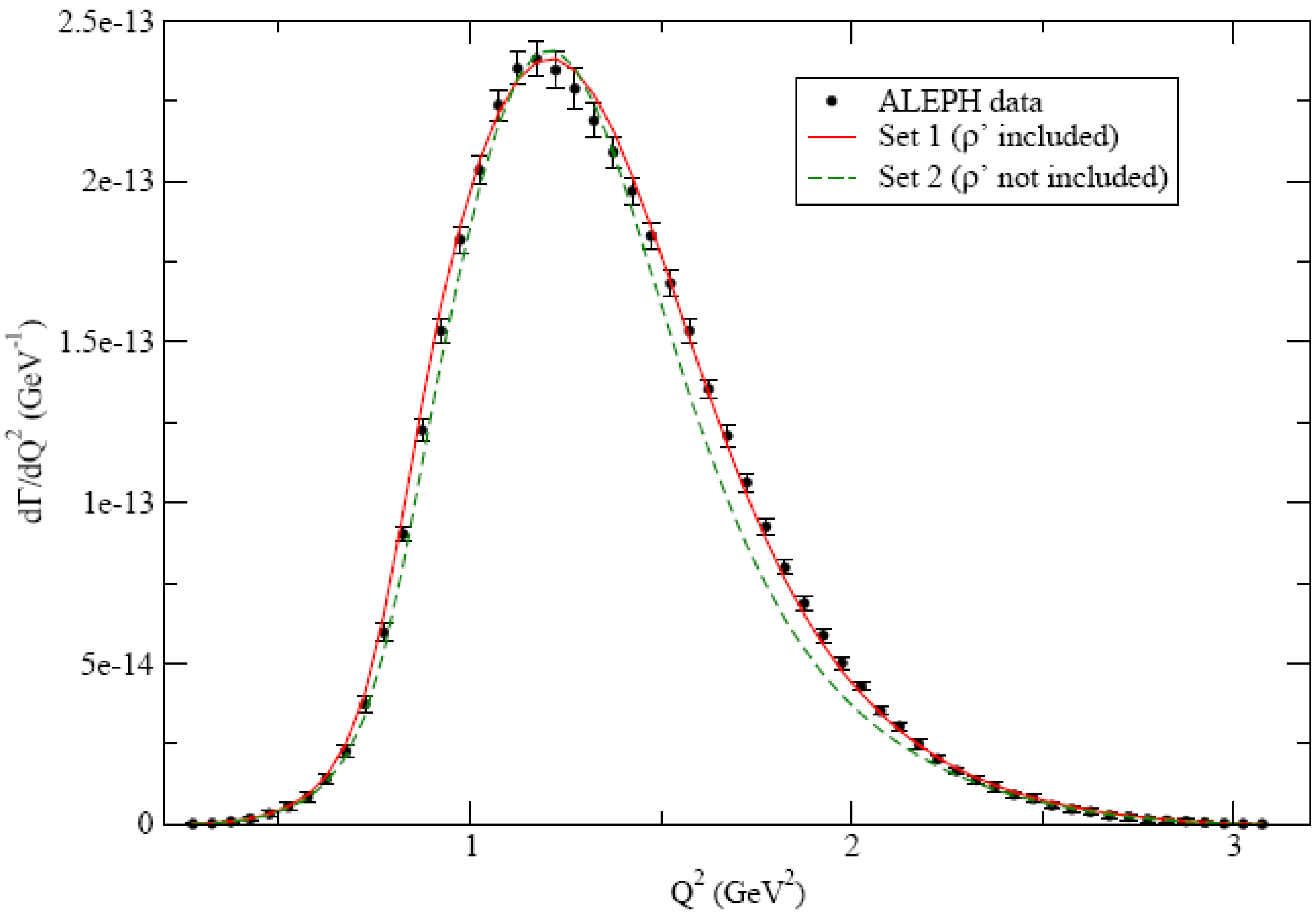
$$\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)} \longrightarrow \frac{1}{1+\beta_{\rho'}}\left[\frac{1}{M_\rho^2-q^2-iM_\rho\Gamma_\rho(q^2)}+\frac{\beta_{\rho'}}{M_{\rho'}^2-q^2-iM_{\rho'}\Gamma_{\rho'}(q^2)}\right]$$

→ Inclusion from a Lagrangian would imply 3 coups. instead of $\beta_{\rho'}$

F_V', G_V', F_A'

NEW HADRONIC FORM FACTORS

Gómez-Dumm, Roig, Pich, Portolés '09



NEW HADRONIC FORM FACTORS

Additional high-energy constraints are found on the **KKπ** channels:

$$
\begin{aligned}
c_1 - c_2 + c_5 &= 0\,, \\
c_1 - c_2 - c_5 + 2c_6 &= -\frac{N_C}{96\,\pi^2}\,\frac{F_V\,M_V}{\sqrt{2}\,F^2}\,, \\
d_3 &= -\frac{N_C}{192\,\pi^2}\,\frac{M_V^2}{F^2}\,, \\
g_1 + 2g_2 - g_3 &= 0\,, \\
g_2 &= \frac{N_C}{192\,\sqrt{2}\,\pi^2}\,\frac{M_V}{F_V}\,.
\end{aligned}
$$

•We do not have any hint on the value of two of the odd-intrinsic parity couplings.

• Up to now, excited resonances have not been implemented.

• Through the framework provided by TAUOLA, with the new currents from RχT installed, it will be much easier to learn about the unknown couplings and to estimate properly the size of the excited resonances contribution.

NEW HADRONIC FORM FACTORS

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•We do not have any hint on the value of two of the odd-intrinsic parity couplings.

• Up to now, excited resonances have not been implemented.

• Through the framework provided by TAUOLA, with the new currents from RχT installed, it will be much easier to learn about the unknown couplings and to estimate properly the size of the excited resonances contribution.

•**Essential to study the most relevant channels in unified framework for signal/background splitting and data analysis: New TAUOLA currents.**

NEW HADRONIC FORM FACTORS

Structure of *new-currents/RChL-currents*

Added to \tauola *cleo* version

- codes for currents
  - frho_pi.f pipi0 mode
  - fkk0.f kk0 mode
  - fkpipl.f kpi modes
  - f3pi_rcht.f 3 pion modes
  - fkkpi.f KKpi modes
  - fkk0pi0.f KK0pi0 mode
- library of functions used in the currents
  - funct_rpt.f Width of resonances etc
- code for a1 width as function of q^2
  - /tabler/a1/da1wid_tot_rho1_gauss.f
  - wid_a1_fit.f linear interpolation
- numerical values of fit parameters, dipswitches
  - value_parameter.f
- tests of MC results (for separate modes)
  /cross-check/check_analyticity_and_numer_integr

Every directories with own README

NEW HADRONIC FORM FACTORS

DIPSWITCH PARAMETERS

new-currents/RChL-currents/value_parameter.f

DIPSWITCH	VALUE	MEANING	MODE
FFVEC	0, 1*	FSI OFF, ON*	PIPI0, KPI, KK0
FFKPIVEC	0, 1*	FSI GS, EXPON	KPI
FFKKVEC	0*,1	RHOPR OFF*,ON	KK0

* default value

Input: parameters from fit etc.

NEW HADRONIC FORM FACTORS

Parameters to fit

new-currents/RChL-currents/value_parameter.f

1. Non - model parameters
(non resonance + narrow resonances)

PDG values

Parameter	Var. name	Default
m_τ	MTAU	1.777
m_{ν_τ}	MNUTA	0.001
$\cos\theta_{\mathrm{Cabibbo}}$	set in TAUOLA init.	0.975
G_F	set in TAUOLA init.	$1.166375 \cdot 10^{-5}$
$m_{\pi^\pm}$	mpic	0.13957018
m_{π^0}	mpiz	0.1349766
m_η	meta	0.547
$m_{K^\pm}$	mkc	0.493677
m_{K^0}	mkz	0.497648
M_ω	mom	0.78194
Γ_ω	gom	0.00843
M_ϕ	mphi	1.019
Γ_ϕ	gphi	0.0042

2. Parameters from vector currents (with F_5) in KKπ modes

3. Model (resonance) parameters (two mesons and axial-vector current for three mesons)

depend on the mode and dipswitches !!!

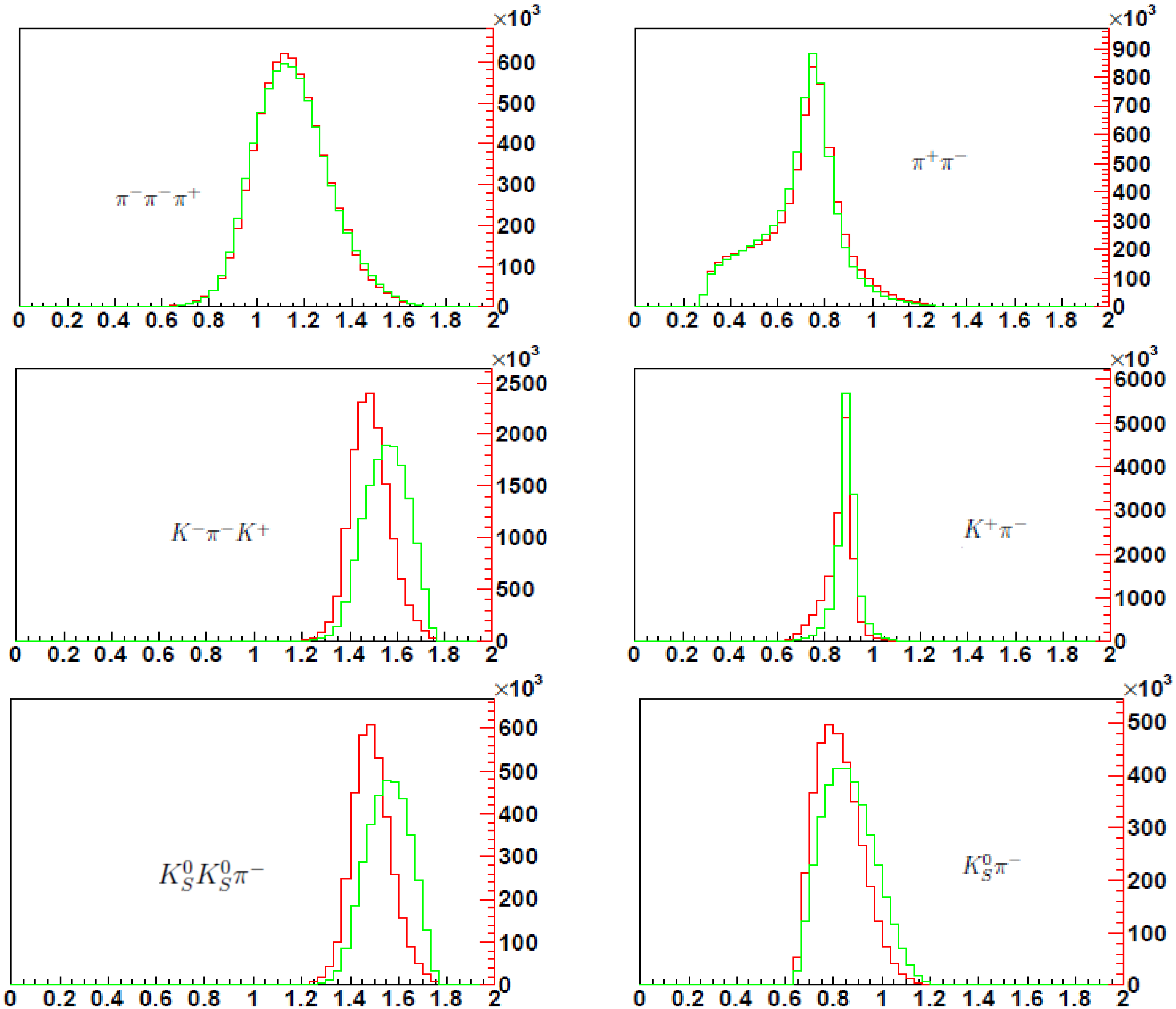
3.1. Model couplings, mixings and interferences

3.2. Masses, widths of resonances

Numerical benchmarks of formfactor implementation: a_1 width

checks for every channel

Comparison between CLEO and TAUOLA2011



TAUOLA2011

CLEO

$\tau \rightarrow \pi^-\pi^-\pi^+\nu_\tau$

$\tau \rightarrow K^-\pi^-K^+\nu_\tau$

$\tau \rightarrow K^0\pi^-\bar{K}^0\nu_\tau$

http://annapurna.ifj.edu.pl/~wasm/RChL/RChL.htm

BABAR data: *Ian M. Nugent, (Victoria U.)*

SLAC-R-936, Dec16, 2009. Ph.D. Thesis

COMPARISONS

MC-Tester Davidson, Golonka, Przedzinski, Was '08

Shekhovtsova, Przedzinski, Roig, Was '12

σ

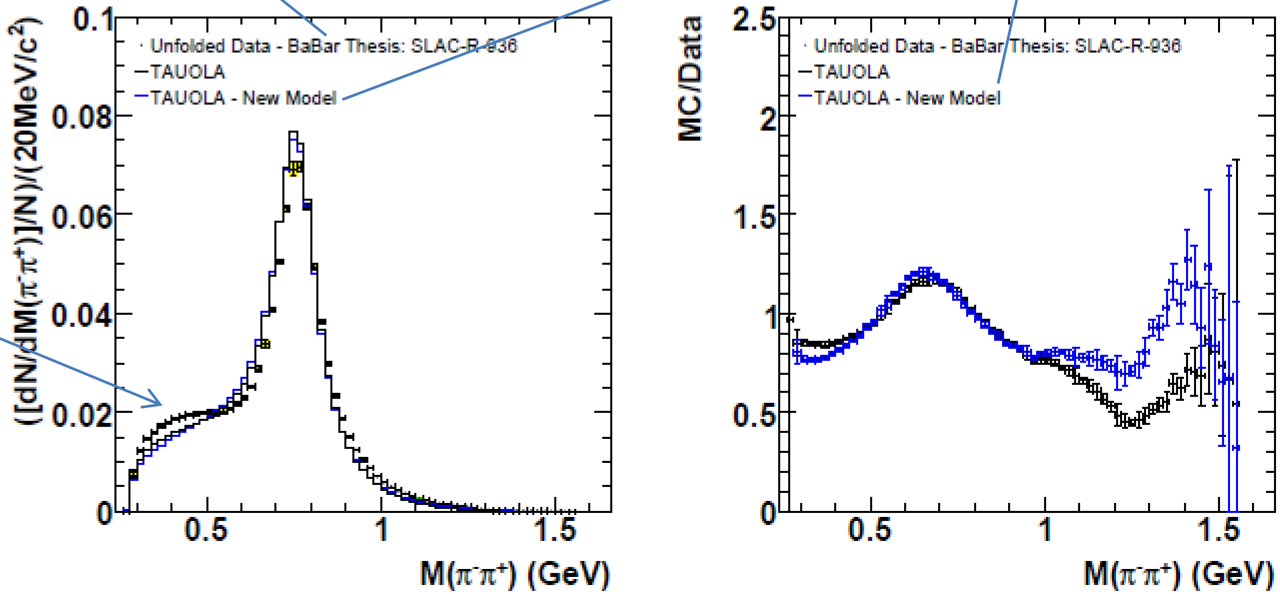
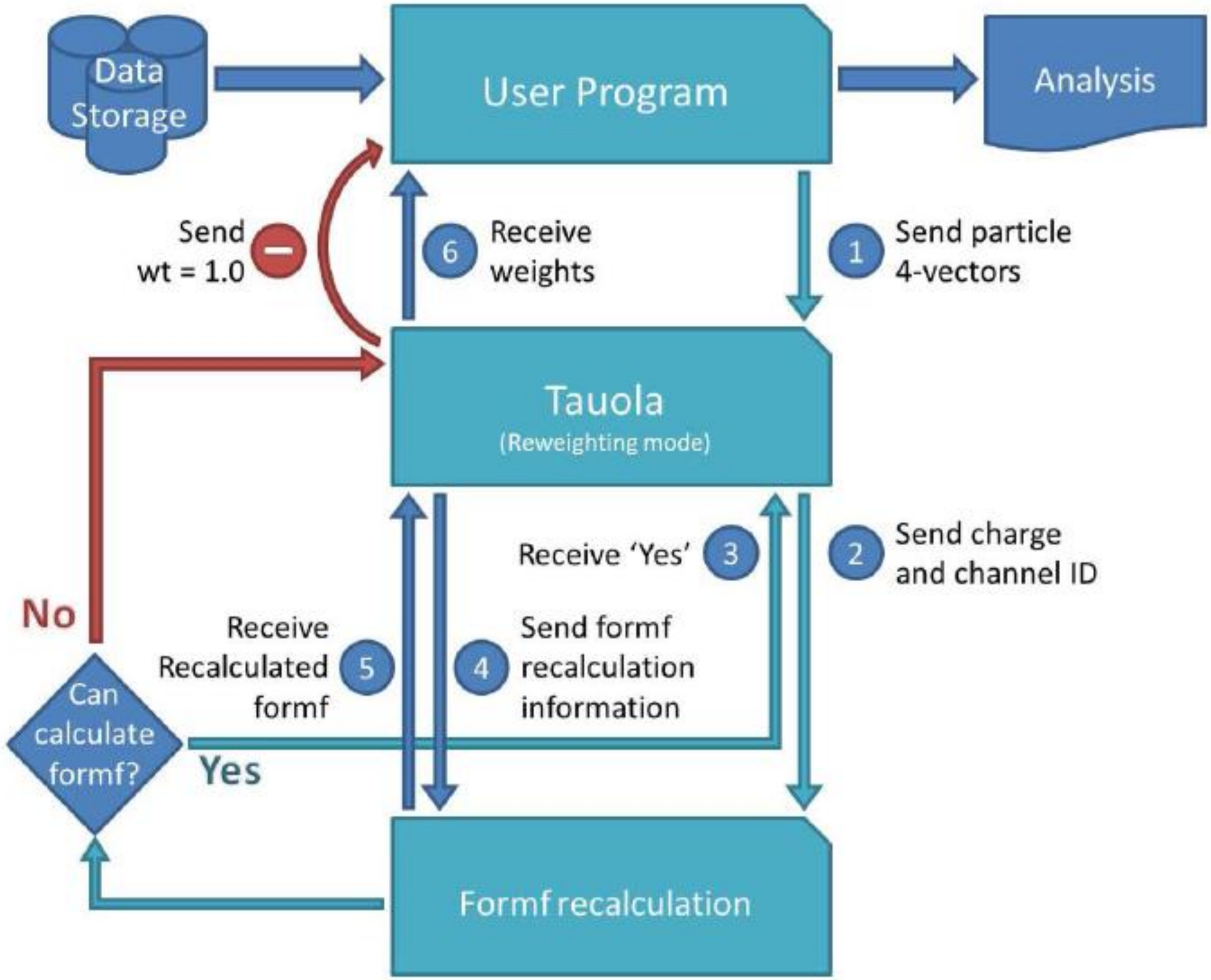


Figure 8: Invariant mass distribution of the $\pi^+\pi^-$ pair in $\tau \to \pi^+\pi^-\pi^-\nu_\tau$ decay. Lighter grey histogram is from our model, darker grey is from default parametrization of `TAUOLA cleo`. The unfolded BaBar data are taken from Ref. [57] . The plot on the left-hand side corresponds to the differential decay distribution, and the one on the right-hand side to plot ratios between Monte Carlo results and data. Courtesy of Ian Nugent.

http://annapurna.ifj.edu.pl/~wasm/RChL/RChL.htm

COMPARISONS

MC-Tester Davidson, Golonka, Przedzinski, Was '08



CONCLUSIONS

Shekhovtsova, Przedzinski, Roig, Was '12

Hadronic currents for the modes: $\pi^-\pi^0$, $(K\pi)^-$, K^-K^0, $(\pi\pi\pi)^-$, $(KK)\pi^-$, $K^-K^0\pi^0$ - from RχT have been implemented in TAUOLA (88% of hadronic decays of the tau lepton). They are ready for precise confrontation with data amassed at Belle and BaBar (and future Belle II & Frascati superB data). Collaboration with experimentalists is essential for the success of the project.

In order to obtain the maximum possible information from experiments, the theory input to the MC has to be as accurate as possible with known properties respected (χPT results at low energies, smooth behaviour of FF at short distances, unitarity, analiticity,…). That is why our effort is and will be worth.

There are improvements to be done in all modes…

Ongoing/Future improvements

- Resummation in the two meson modes. ε below 3%.
- FSI in three meson modes. Relevant in $d\Gamma/ds$ ($\sim$ 10%).
- SU(3) breaking terms in the Lagrangian. ($KK\pi$, $\sim$ 30%)
- Spin zero resonance contributions. ($\pi\pi\pi$)

Moussallam **Eur.Phys.J.C53:401-412,2008**

- Excited resonance contribution in $KK\pi$ channels.
- Complete $O(p^6)$ χPT, i.e. NNLO, in $\pi\pi$ channels.

Bijnens, Colangelo, Talavera **JHEP 9805:014,1998**

López Castro et. al. **Phys.Rev.D74:071301,2006**

- SU(2) breaking in $\pi\pi$ channels.

Cirigliano, Ecker, Neufeld **Phys.Lett.B513:361-370,2001** **JHEP 0208:002,2002**

- Some important remaining modes: $\pi\pi\pi\pi$, $K\pi\pi$, SFF in $K\pi$ ...
- Remaining two meson modes: $\pi\eta^{(\prime)}, K\eta^{(\prime)}$

Jamin, Oller, Pich '01,'06

Final words

• Historically, **TAUOLA** has benefited from feedback with experimental Colls. only when the experiments were closed and the lack of human power was pressing (as it is now the case with BaBar and it was before with CLEO and ALEPH).

Final words

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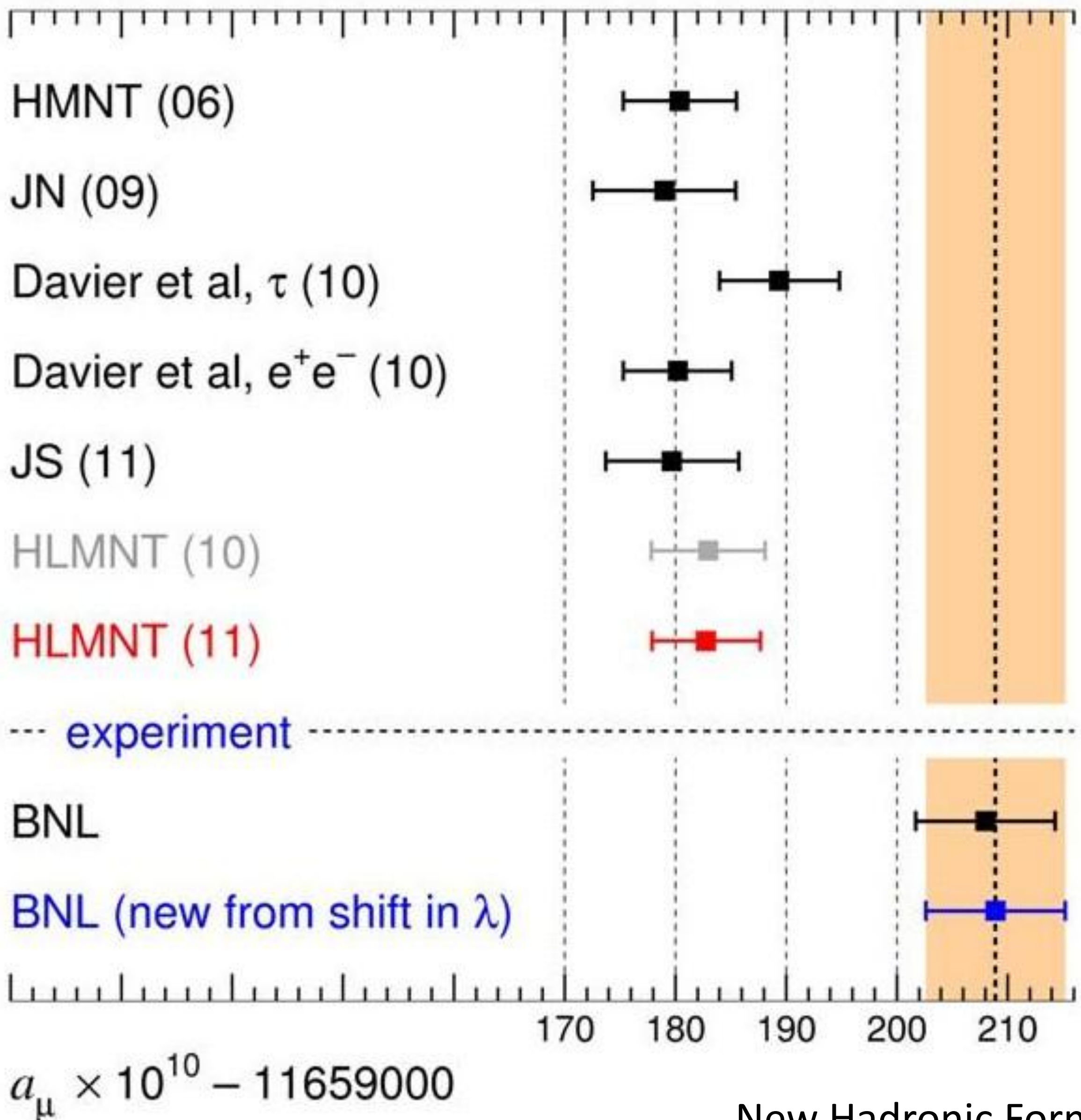
SKIPPED SLIDES

AMMM


HMNT (06)
JN (09)
Davier et al, τ (10)
Davier et al, e⁺e⁻ (10)
JS (11)
HLMNT (10)
HLMNT (11)
experiment
BNL
BNL (new from shift in λ)
170
180
190
200
210
$a_\mu \times 10^{10} - 11659000$

Parameters to fit

new-currents/RChL-currents/value_parameter.f

1. Non - model parameters
(non resonance + narrow resonance)

PDG values

Parameter	Var. name	Default
m_τ	MTAU	1.777
m_{ν_τ}	MNUTA	0.001
$\cos\theta_{\rm Cabibbo}$	set in TAUOLA init.	0.975
G_F	set in TAUOLA init.	$1.166375 \cdot 10^{-5}$
$m_{\pi^\pm}$	mpic	0.13957018
m_{π^0}	mpiz	0.1349766
m_η	meta	0.547
$m_{K^\pm}$	mkc	0.493677
m_{K^0}	mkz	0.497648
M_ω	mom	0.78194
Γ_ω	gom	0.00843
M_ϕ	mphi	1.019
Γ_ϕ	gphi	0.0042

2. Parameters from vector currents (with F_5) in KKpi modes

$$J^\mu = N\left\{\ldots - \frac{i}{4\pi^2 F^2} c_5 \varepsilon^{\mu\nu\rho\sigma} p_{1\nu} p_{2\rho} p_{3\sigma} F_5\right\}$$

$$\mathcal{L}_4^{\rm V} = \sum_{i=1}^{5} \frac{g_i}{M_V} \mathcal{O}^i_{\rm VPPP} + \sum_{i=1}^{7} \frac{c_i}{M_V} \mathcal{O}^i_{\rm VJP}$$

$$c_i, \quad d_i, \quad g_i$$

$$\mathcal{L}_2^{\rm RR} = \sum_{i=1}^{5} \lambda_i \mathcal{O}^i_{\rm VAP} + \sum_{i=1}^{4} d_i \mathcal{O}^i_{\rm VVP}$$

Correct high energy behaviour of vector form factor

$$\begin{aligned} c_1 - c_2 + c_5 &= 0\,, \\ c_1 - c_2 - c_5 + 2c_6 &= -\frac{N_C}{96\,\pi^2}\frac{F_V\,M_V}{\sqrt{2}\,F^2} \\ d_3 &= -\frac{N_C}{192\,\pi^2}\frac{M_V^2}{F^2}\,, \\ g_1 + 2g_2 - g_3 &= 0\,, \\ g_2 &= \frac{N_C}{192\,\sqrt{2}\,\pi^2}\frac{M_V}{F_V} \end{aligned}$$

$$H^3_{\mu\nu}(s,t,Q^2) \equiv T^3_\mu T^{3\,*}_\nu \qquad \int {\rm d}\Pi_3\, H^3_{\mu\nu}(s,t,Q^2) = (Q^2 g_{\mu\nu} - Q_\mu Q_\nu)\,\Pi_V(Q^2)$$

$$\Gamma(\omega \to \pi^+\pi^-\pi^0) \longrightarrow 2g_4 + g_5 = -0.60 \pm 0.02$$

P. Roig talk for errors/uncertainity

3. Model (resonance) parameters (two mesons and axial-vector current for three meson)

$\Gamma_{\rho'},\quad M_A,\quad M_\rho,\quad M_{\rho'},\quad F_V,\quad G_V,\quad F_A,\quad \beta_\rho,\qquad \Gamma_{K^*},\quad \Gamma_{K^{*\prime}},\quad M_{K^*},\quad M_{K^{*\prime}},\quad \gamma_{K\pi},\quad \gamma,\quad \delta$

depend on the mode and dipswicthes !!!

3.1 . Model constants

1. Fixed F and F_k (FIXED!!!)

2. Correct high energy behaviour of pion FF

$$G_V = F^2/F_V$$

3. F_A , F_V fit Aleph 3 pion spectrum

4. beta_rho, gamma_rcht fit to Belle spectrum (hep-ph/ ....)

Parameter	Var. name	Default	[suggested range]
F	fpi_rpt	0.0924	$[0.0920, 0.0924]$
F_K	fk_rpt	1.198F	$[0.94F, 1.2F]$
F_V	fv_rpt	0.18	$[0.12, 0.24]$
G_V	gv_rpt	F^2/F_V	$[0.xxF^2/F_V, 1.xxF^2/F_V]$
F_A	fa_rpt	0.149	$[0.10, 0.20]$
β_ρ	beta_rho	-0.25	$[-0.36, -0.18]$
$\gamma_{K\pi}$	gamma_rcht	-0.043	$[-0.033, -0.053]$
$\gamma_{K\pi}$	gamma_rcht	-0.039	$[-0.023, -0.055]$
θ_V	THETA	$35.26°$	$[15^o, 50^o]$
γ	coef_ga	0.14199	$[0.077, 0.099]$
δ	coef_de	-0.12623	$[-0.035, -0.012]$
ϕ_1	phi_1	-0.17377	$[0.5, 0.7]$
ϕ_2	phi_2	0.27632	$[0.5, 1.1]$

4. ideal mixing angle for THETA $\theta_V = \tan^{-1}(1/\sqrt{2})$ (FIXED!!!)

3.2. Masses, widths of resonances

Parameter	Var. name	Default	[suggested range]
M_ρ	mro	0.77554	[0.770, 0.777]
M_ρ	mro	0.775	[0.770, 0.777]
M_{a_1}	mma1	1.12	[1.00, 1.24]
$M_{\rho'}$	mrho1	1.453	[1.44, 1.48]
$M_{\rho'}$	mrho1	1.465	[1.44, 1.48]
$\Gamma_{\rho'}$	grho1	0.50155	[0.32, 0.39]
$\Gamma_{\rho'}$	grho1	0.4	[0.32, 0.39]
$M_{\rho''}$	mrho2	1.8105	[1.68, 1.78]
$\Gamma_{\rho''}$	grho2	0.4178	[0.08, 0.20]
γ	coef_ga	0.14199	[0.077, 0.099]
δ	coef_de	-0.12623	[-0.035, -0.012]
ϕ_1	phi_1	-0.17377	[0.5, 0.7]
ϕ_2	phi_2	0.27632	[0.5, 1.1]
$M_{K^{*\pm}}$	mksp	0.89166	[0.891, 0.892]
$M_{K^{*0}}$	mks0	0.8961	[0.895, 0.897]
M_{K^*}	mkst	0.8953	[0.8951, 0.8955]
M_{K^*}	mkst	$(M_{K^{*\pm}} + M_{K^{*0}})/2$	
m_{K^*}	mkst	0.94341	[0.9427, 0.9442]
Γ_{K^*}	gamma_kst	0.0475	[0.047, 0.048]
γ_{K^*}	gamma_kst	0.06672	[0.0655, 0.0677]
$\Gamma_{K^{*\prime}}$	gamma_kstpr	0.206	[0.155, 0.255]
$\gamma_{K^{*\prime}}$	gamma_kstpr	0.240	[0.120, 0.380]
$M_{K^{*\prime}}$	mkstpr	1.307	[1.270, 1.350]
$m_{K^{*\prime}}$	mkstpr	1.374	[1.330, 1.450]

3 pions, Aleph

PDG
3 mesons

2 pions, Aleph fit

PDG

KKpi

Kpi modes
fit to Belle

Difference between (exact) models ~4%,
between GS and exponention ~15%

non-physical values for masses for GS in Kpi modes

Numerical benchmarks of formfactor implementation:

1. a1 width is tabulated to avoid problem with triple integration:

   Cross check with linear interpolation

2. Check of every channel: **/cross-check/check_analyticity_and_numer_integr**

   semi-analytical result (Gauss integration): comparison with linear interpolated spectrum

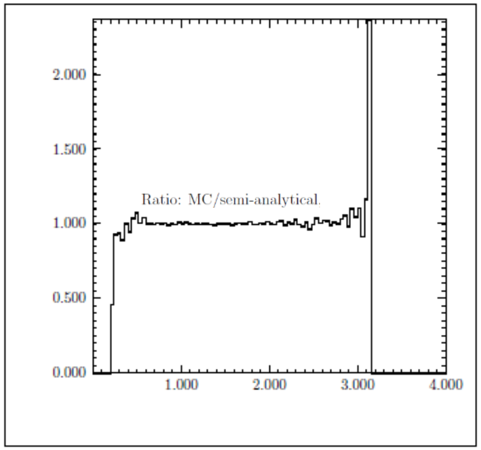
   ratio MC/semi-analytical of differential width (qq)

   comparison of analytical integration and MC for total width

   2 pion, 2 Kaon with physical mass of pions, Kaons

   others $m_\pi = (m_{\pi^0} + 2 \cdot m_{\pi^+})/3$ $m_K = (m_{K^0} + m_{K^+})/2$

An example: three pions ($\tau \to \pi^-\pi^-\pi^+\nu_\tau$):



- F_1 = F, F_{others} = 0 to check phase space
- F_1 = physical, F_{others} = 0
- F_{all} = physical

linear interpolation ~ 0.1% for whole spectrum except for ends

MC (6e6): (2.1013 0.016%)·10^{-13}GeV; semi-analyt(2.1007 0.02% %)·10^{-13}GeV

Comparison of semi-analytical integration and MC

3 pseudoscalars

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{ud}|^2}{128(2\pi)^5 M_\tau F^2}\left(\frac{M_\tau^2}{q^2}-1\right)^2 \int dsdt \left[W_{SA} + \frac{1}{3}\left(1+2\frac{q^2}{M_\tau^2}\right)(W_A + W_B)\right]$$

$$W_B = \frac{1}{64\pi^4 F^4}\left[stu + (m_{K,\pi}^2 - m_\pi^2)(q^2 - m_{K,\pi}^2)s + m_{K,\pi}^2(2m_\pi^2 - q^2)q^2 - m_{K,\pi}^2 m_\pi^4\right]|F_5|^2,$$

$$W_{SA} = q^2|F_4|^2. \qquad W_A = -(V_1^\mu F_1 + V_2^\mu F_2 + V_3^\mu F_3)(V_{1\mu}F_1 + V_{2\mu}F_2 + V_{3\mu}F_3)^*,$$

$$\int dsdt = \int_{4m_{K,\pi}^2}^{\left(\sqrt{q^2}-m_\pi\right)^2} ds \int_{t_-(s)}^{t_+(s)} dt$$

$$t_\pm(s) = \frac{1}{4s}\left\{(q^2-m_\pi^2)^2 - [\lambda^{1/2}(q^2,s,m_\pi^2) \mp \lambda^{1/2}(m_{K,\pi}^2, m_{K,\pi}^2, s)]^2\right\}$$

Two pions

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2|V_{ud}|^2 m_\tau^3}{384\pi^3}\left(1-\frac{q^2}{m_\tau^2}\right)^2 \lambda\left(1,\frac{m_{\pi^+}^2}{q^2},\frac{m_{\pi^0}^2}{q^2}\right)|F_\pi|^2$$

Channel	Analytical , GeV^{-1}	Monte Carlo , GeV^{-1}
pipi0	$(5.2431\pm0.02\%)\cdot10^{-15}$	$(5.2441\pm0.005\%)\cdot10^{-15}$
KK0	$(2.0863\pm0.02\%)\cdot10^{-15}$	$(2.0864\pm0.005\%)\cdot10^{-15}$
Kpi0	$(2.5193\pm0.02\%)\cdot10^{-14}$	$(2.5197\pm0.008\%)\cdot10^{-14}$
pipipi	$(2.1007\pm0.02\%)\cdot10^{-13}$	$(2.1013\pm0.016\%)\cdot10^{-13}$
K-pi-K+	$(3.7379\pm0.024\%)\cdot10^{-15}$	$(3.7383\pm0.02\%)\cdot10^{-15}$
K0pi-K0	$(3.7385\pm0.024\%)\cdot10^{-15}$	$(3.7383\pm0.02\%)\cdot10^{-15}$
Kpi0K0	$(2.7370\pm0.02\%)\cdot10^{-15}$	$(2.7367\pm0.02\%)\cdot10^{-15}$

$m_{\pi^\pm} = m_{\pi^0}$

$m_{K^\pm} = m_{K^0}$

Numerical results

Channel	Width, [GeV]		
	PDG	Equal masses	Phase space with masses
$\pi^-\pi^0$	$(5.778 \pm 0.35\%) \cdot 10^{-13}$	$(5.2283 \pm 0.005\%) \cdot 10^{-13}$	$(5.2441 \pm 0.005\%) \cdot 10^{-13}$
$\pi^0 K^-$	$(9.72 \pm 3.5\%) \cdot 10^{-15}$	$(8.3981 \pm 0.005\%) \cdot 10^{-15}$	$(8.5810 \pm 0.005\%) \cdot 10^{-15}$
$\pi^-\bar{K}^0$	$(1.9 \pm 5\%) \cdot 10^{-14}$	$(1.6798 \pm 0.006\%) \cdot 10^{-14}$	$(1.6512 \pm 0.006\%) \cdot 10^{-14}$
K^-K^0	$(3.60 \pm 10\%) \cdot 10^{-15}$	$(2.0864 \pm 0.007\%) \cdot 10^{-15}$	$(2.0864 \pm 0.007\%) \cdot 10^{-15}$
$\pi^-\pi^-\pi^+$	$(2.11 \pm 0.8\%) \cdot 10^{-13}$	$(2.1013 \pm 0.016\%) \cdot 10^{-13}$	$(2.0800 \pm 0.017\%) \cdot 10^{-13}$
$\pi^0\pi^0\pi^-$	$(2.10 \pm 1.2\%) \cdot 10^{-13}$	$(2.1013 \pm 0.016\%) \cdot 10^{-13}$	$(2.1256 \pm 0.017\%) \cdot 10^{-13}$
$K^-\pi^-K^+$	$(3.17 \pm 4\%) \cdot 10^{-15}$	$(3.7379 \pm 0.024\%) \cdot 10^{-15}$	$(3.8460 \pm 0.024\%) \cdot 10^{-15}$
$K^0\pi^-\bar{K}^0$	$(3.9 \pm 24\%) \cdot 10^{-15}$	$(3.7385 \pm 0.024\%) \cdot 10^{-15}$	$(3.5917 \pm 0.024\%) \cdot 10^{-15}$
$K^-\pi^0K^0$	$(3.60 \pm 12.6\%) \cdot 10^{-15}$	$(2.7367 \pm 0.025\%) \cdot 10^{-15}$	$(2.7711 \pm 0.024\%) \cdot 10^{-15}$

only ρ

with ρ' (parameters from pion mode) $(2.6502 \pm 0.008\%) \cdot 10^{-15}$ GeV

FSI effects

No.	Channel	Width [GeV] (FSI)	Width [GeV] (No FSI)
1.	$\pi^-\pi^0$	$5.2441 \cdot 10^{-13} \pm 0.005\%$	$4.0642 \cdot 10^{-13} \pm 0.005\%$
2.	$\pi^0 K^-$	$8.5810 \cdot 10^{-15} \pm 0.005\%$	$7.4275 \cdot 10^{-15} \pm 0.005\%$
3.	$\pi^-\bar{K}^0$	$1.6512 \cdot 10^{-14} \pm 0.006\%$	$1.4276 \cdot 10^{-14} \pm 0.006\%$
4.	K^-K^0	$2.0864 \cdot 10^{-15} \pm 0.007\%$	$1.2201 \cdot 10^{-15} \pm 0.007\%$

FSI **No FSI**

$14\% - 32\%$

FFVEC = 1 (FSI), 0 (no FSI)

Discussion on the errors

- $\varepsilon(1/N_c)\sim 1/3$?

't Hooft '74, Witten '79

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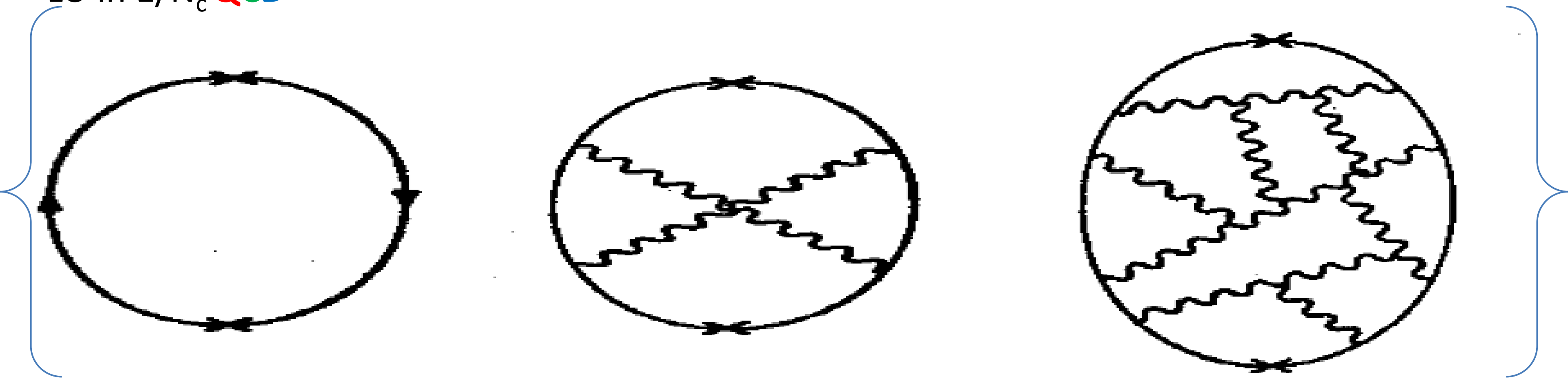
LO in $1/N_c$ **QCD**



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- $\varepsilon(1/N_c)<1/3$
- Resummation in the two meson modes

$$F_{PQ}^{V}(s) = F^{VMD}(s)\,\exp\left[\sum_{P,Q} N_{loop}^{PQ}\frac{-s}{96\pi^2 F^2} Re A_{PQ}(s)\right]$$

$$F(s)^{VMD} = \frac{M_V^2}{M_V^2 - s - iM_V\Gamma_V(s)}$$

In this way (exponentiation of Re $A_{PQ}(s)$) unitarity is violated at $O(p^6)$, i.e. NNLO

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Alternatively:

Exact Unitarity

$$F_V(s) = \frac{M_V^2}{M_V^2\left[1+\sum_{P,Q} N^{PQ}_{loop}\frac{s}{96\pi^2F^2}A_{PQ}(s)\right] - s}$$

$$\delta^{PQ}(s) = Im\left[F^{PQ}_V(s)\right]/Re\left[F^{PQ}_V(s)\right]$$

$$F^{PQ}_V(s) = \exp\left\{\alpha_1 s + \alpha_2 s^2 + \frac{s^3}{\pi}\int_{s_{thr}}^{s_{cut}} \mathrm{d}s' \frac{\delta^{PQ}(s')}{s'^3(s'-s-i\epsilon)}\right\}$$

Tiny differences in observables between both approaches

Jamin, Pich, Portolés '06, '08 Boito, Escribano, Jamin '07

Discussion on the errors

- $\varepsilon(1/N_c)<1/3$

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$$F_I^{\rm scal}(x) = F_I^{\rm resonant}(x) + R_I^{\rm scal}(x)$$

$$\begin{aligned} F_+ &= F_+^{\chi} + F_+^{R} + F_+^{RR} + \sqrt{2}\left[R_0^{\rm scal}(s) + R_0^{\rm scal}(t)\right] + R_2^{\rm scal}(s) + R_2^{\rm scal}(t)\,, \\ F_- &= -(F_+^{\chi} + F_+^{R} + F_+^{RR}) - \left[R_0^{\rm scal}(s) + R_0^{\rm scal}(t)\right] + \sqrt{2}\left[R_2^{\rm scal}(s) + R_2^{\rm scal}(t)\right] \end{aligned}$$

Isidori, Maiani, Nicolacci, Pacetti

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$$R_0(x) = \left\{\frac{\alpha_0}{Q^2} + \frac{\alpha_1}{Q^4}(x - M_{f_0}^2) + \mathcal{O}\left[(x - M_{f_0}^2)^2\right]\right\} e^{i\delta_0(x)}$$

Schenk **Nucl.Phys. B363 (1991) 97-116**

Colangelo, Gasser, Leutywler

Nucl.Phys. B603 (2001) 125-179

$$\tan\delta_I(x) = \sigma_\pi(x)\left(A_0^I + B_0^I q^2 + C_0^I q^4 + D_0^I q^6\right)\frac{4m_\pi^2 - x_0^I}{x - x_0^I}$$

We have to check if this approach is enough to confront the data successfully.

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Jamin, Oller, Pich '01,'06

Nucl.Phys.B622:279-308,2002 Phys.Rev.D74:074009,2006