Stochastic Schrödinger equations and non-Markovian open quantum systems with dissipation

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Outline

- Measurement problem and Collapse models
- Stochastic unraveling for open quantum systems
- Solution of a non-Markovian SSE with dissipation
- Conclusions & outlook

Measurement problem

- Measurement problem: QM → macroscopic superpositions
- Pragmatic solution: WPR postulate

Collapse models

Schrödinger evolution (linear, deterministic) ↘

WPR postulate (non-linear, stochastic) ↙

Stochastic Schrödinger equation (non-linear, stochastic)

Collapse Models

- Idea: "Spontaneous collapses occur more or less all the time, more or less everywhere" (J. Bell)

↓

The Schrödinger equation is modified, to include such effects which are non-linear and stochastic

- Constraints: → norm preserving equation; → no faster than light → structure almost uniquely defined

$$d|\psi\rangle_t = \left[-\frac{i}{\hbar} H\,dt + \sqrt{\lambda}(A - \langle A\rangle_t)dW_t - \frac{\lambda}{2}(A - \langle A\rangle_t)^2 dt \right] |\psi\rangle_t$$

$\langle A\rangle_t = \langle\psi_t|A|\psi_t\rangle$ ⟶ non-linear

dW_t → stochastic

Non-Markovian unravelling

- Stochastic unravelling for open quantum systems:

↓

the interaction with the environment is described by a noise

↙ ↘

Markovian: white noise

Non-Markovian: colored noise

No dissipation: infinite temperature

Dissipation: finite temperature

- Non-Markovian QSD: L. Diosi, W.T. Strunz, Phys. Lett. A 235 (1997).

$$\frac{d}{dt}\phi_t = \left[-\frac{i}{\hbar}H + \sqrt{\lambda}qw(t) - 2\sqrt{\lambda}q\int_0^t dsD(t,s)\frac{\delta}{\delta w(s)}\right]\phi_t \quad \longrightarrow \quad \rho_t = \mathbb{E}\left[|\phi_t\rangle\langle\phi_t|\right]$$

Non-Markovian QMUPL model with dissipation

- Non-Markovian collapse equation with dissipation:

$$\frac{d}{dt}\phi_t = \left[-\frac{i}{\hbar}\left(H + \frac{\lambda\mu}{2}\{q,p\}\right) + \sqrt{\lambda}\left(q + i\frac{\mu}{\hbar}p\right)w(t) - 2\sqrt{\lambda}q\int_0^t dsD(t,s)\frac{\delta}{\delta w(s)}\right]\phi_t$$

dissipative term

gaussian noise

correlation function

- Physical model: free particle, collapse in space, non-Markovian and dissipative

- Difficulties → functional derivative, integral term, general correlation function

Solution of the non-Markovian and dissipative free particle equation

$$\phi_t(x) = \int dx_0 \, G(x,t;x_0,0)\phi_0(x_0)$$

↓

Green's function

$$G(x,t;x_0,0) = \int_{q(0)=x_0}^{q(t)=x} \mathcal{D}[q] e^{\mathcal{S}[q]}$$

↓

non-standard action

$$\mathcal{S}[q] = \int_0^t ds \frac{i}{\hbar}\left[\frac{m}{2}\,\dot{q}^2(s) - m\lambda\mu\, q(s)\,\dot{q}(s) - \frac{m}{2}\Omega^2\, q^2(s) + m\sqrt{\lambda}\mu\,\dot{q}(s) - A(w,s)\,q(s) + q(s)\int_0^s dr\, B(r,s)q(r) - 2m\lambda\mu\, q(s)\int_0^s dr\, D(s,r)\dot{q}(r)\right]$$

↓

path-integration

Free particle trajectory

P → non-local term more difficult to treat → need to set up a new formalism

L. Ferialdi, A. Bassi
EPL 98, 30009 (2012).

Green's function:

$$G(x,t;x_0,0) = \sqrt{\frac{m}{2i\pi\hbar t u(t)}} exp\left[-\mathcal{A}_t x_0^2 - \tilde{\mathcal{A}}_t x^2 + \mathcal{B}_t x_0 x + \mathcal{C}_t x_0 + \mathcal{D}_t x + \mathcal{E}_t\right]$$

Gaussian structure →

- same structure as the white noise case
- Gaussian wave functions preserve their form
- $\mathcal{A}_t,\ \tilde{\mathcal{A}}_t,\ \mathcal{B}_t$ are deterministic, $\mathcal{C}_t,\ \mathcal{D}_t,\ \mathcal{E}_t$ are stochastic

Free particle trajectory

- Parameters completely determined in terms of the solution of

$$\frac{im}{2\hbar}h''(s) + \lambda \int_0^t dr\, D(s,r)h(r) = \frac{\sqrt{\lambda}}{2}w(s)$$

→ solvable only for some $D(s,r)$

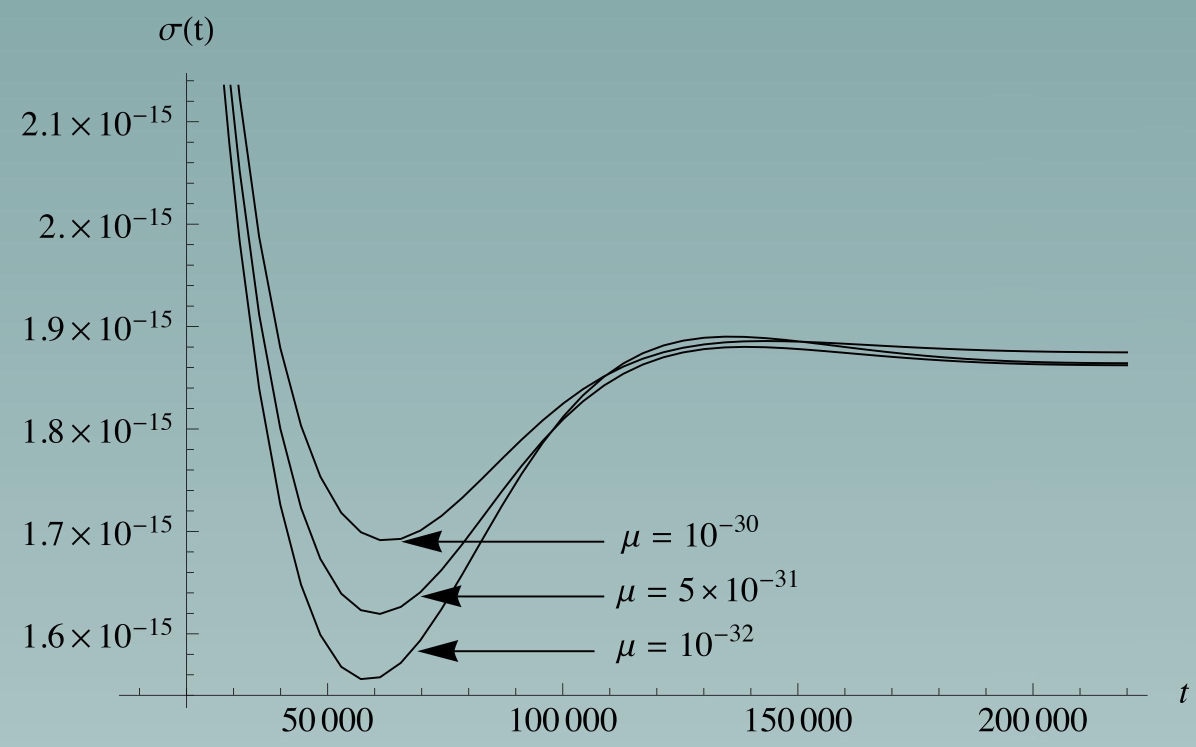
- Explicit solution for the exponential correlation function

$$D(t,s) = \frac{\gamma}{2}e^{-\gamma|t-s|}$$

- Second order integro-differential equation → Fourth order differential equation

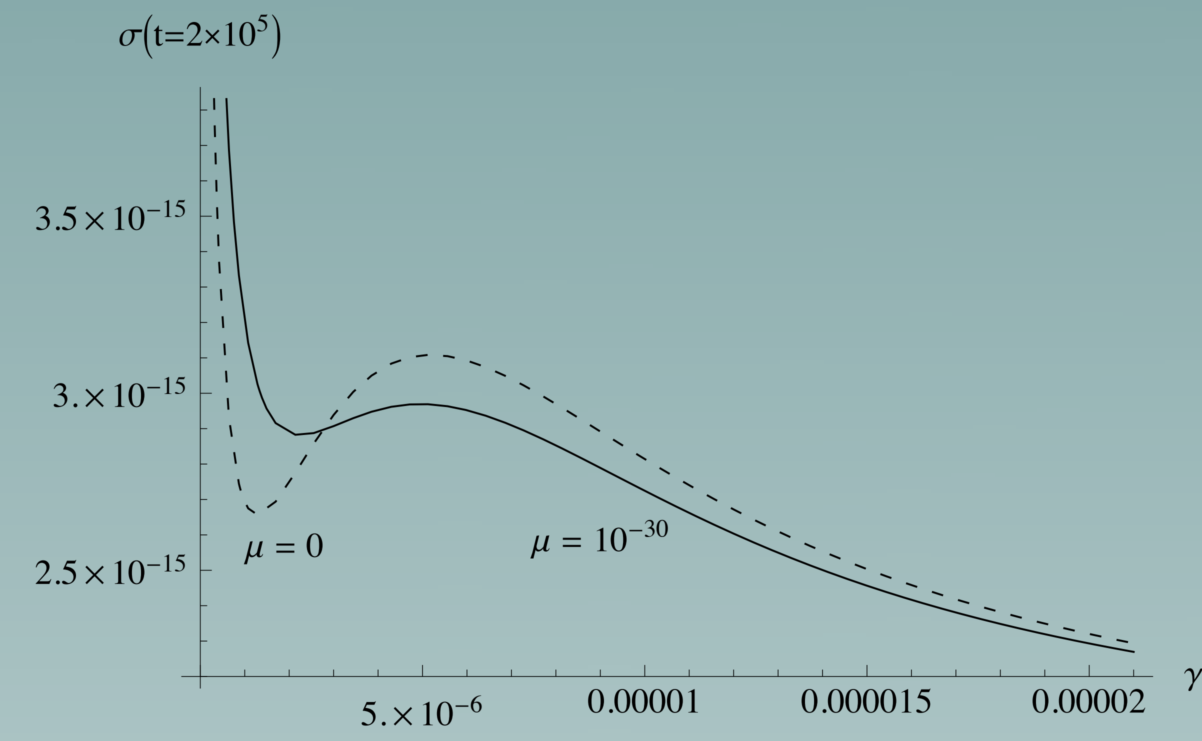
Behaviour of Gaussian wave functions

Spread evolution for different T



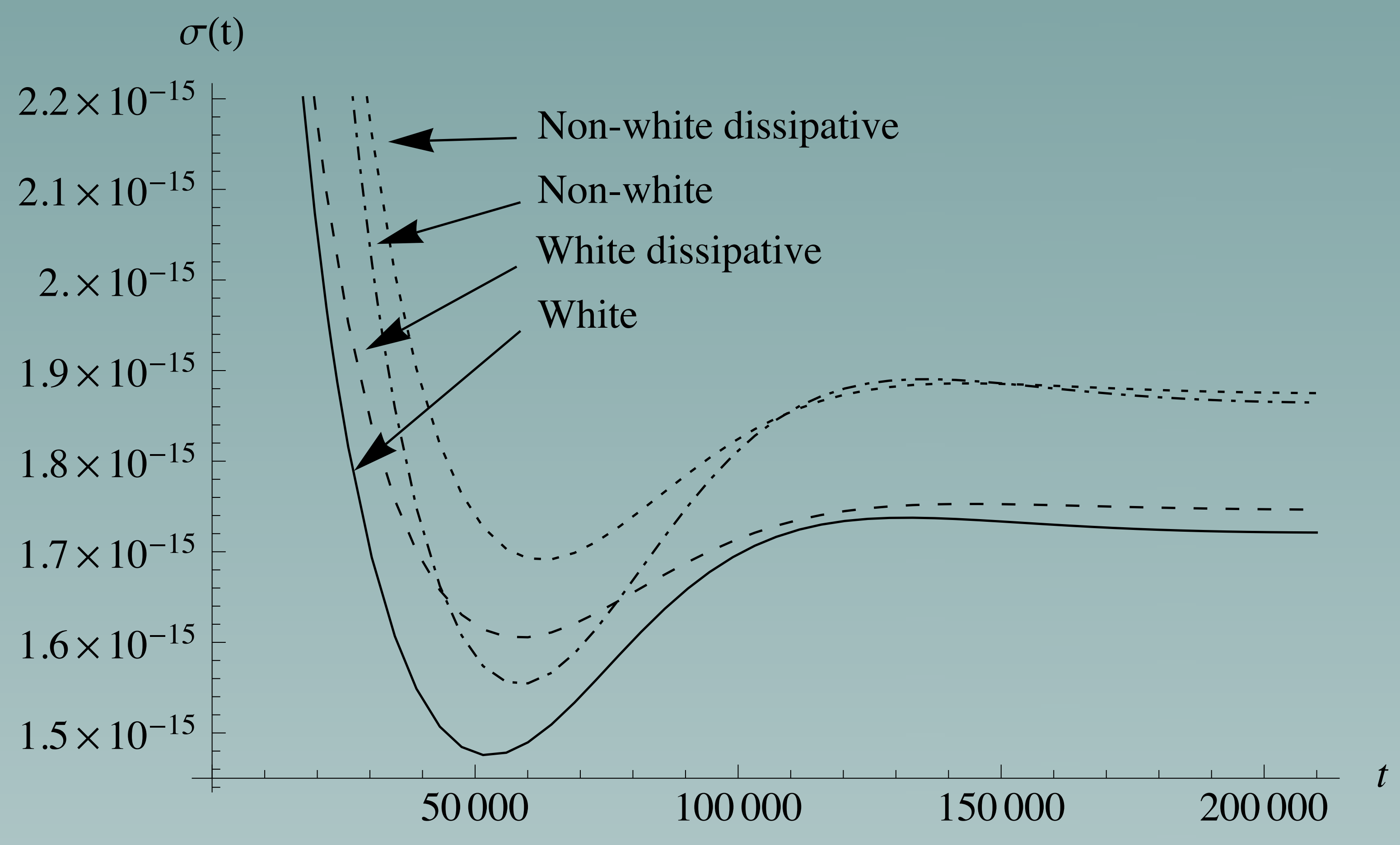
- The wave function shrinks in time.
- The stronger the dissipation, the lower the temperature, the weaker the collapse.
- The spread reaches an asymptotic finite value.

Spread vs γ



- The larger the value of γ, the faster the collapse of the wave function.
- The spread is not strictly decreasing with γ.
- The dissipation makes the transition smoother.

Behaviour of Gaussian wave functions



- The behavior is qualitatively the same for every version of the model: the spread decreases in time, reaching an asymptotic finite value.

L. Ferialdi, A. Bassi
Phys. Rev. Lett. 108, 170404 (2012).

- In the non-Markovian and in the dissipative models the collapse is slower than in the white noise case. When these two effects are combined the process is even slower.

in the Markovian model all frequencies contribute (in the non-Markovian high-frequencies are suppressed)

a finite temperature noise is less energetic than an infinite temperature noise.

Master equation

- $\rho_t = \mathbb{E}\left[|\phi_t\rangle\langle\phi_t|\right]$ → because of p standard techniques cannot be applied
  ↓
  need to set up a new formalism

- Infinite temperature model: master equation for the harmonic oscillator

$$\frac{d}{dt}\rho_{\rm S}(t) = -i[H_0, \rho_{\rm S}(t)] - \lambda[q,[q,\rho_{\rm S}(t)]]\int_0^t ds\, D(t,s)\cos\omega(t-s) + \frac{\lambda}{m\omega}[q,[p,\rho_{\rm S}(t)]]\int_0^t ds\, D(t,s)\sin\omega(t-s)$$

- Interesting to apply measures of non-Markovianity

Conclusions & future research

- SSEs are a powerful mathematical technique → analytical solution → numerical simulations

- Application to energy transfer phenomena and ultra-fast chemical reaction

- Ultimate goal: microscopic derivation for non-Markovian dynamics