The collapse of the wavefunction and the origin of cosmic seeds

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Outline

Basic ingredients of Big Bang Cosmology

- Cosmological Principle: At large scales (greater than 10^{30} m) the universe is homogeneous and isotropic.
- The metric of the universe can be described by the Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

- Einstein Equations:

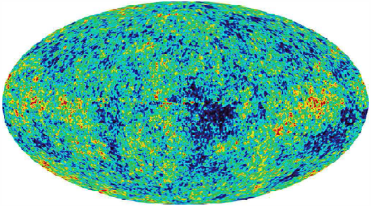
$$G_{ab} = 8\pi G T_{ab}$$

- Perfect Fluid:

$$T_{ab} = P g_{ab} + (\rho + P) u_a u_b$$

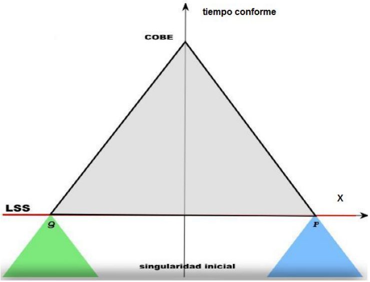
Successes of the Big Bang model

- Expansion of the universe (Hubble's Law, 1929).
- Predicts correctly the abundance of light elements (^{2}H, ^{3}He, ^{4}He and ^{7}Li) in the universe (Big Bang Nucleosynthesis).
- Predicts the existence of a cosmic radiation background (CMB) (discovered by Penzias and Wilson in 1964).



Problems of the Big Bang Model

- **Flatness Problem**: According to the Big Bang theory, we need very 'special' values in the initial conditions of the universe in order to predict a *nearly* flat universe as the observations suggest ($\Omega_0 = 1 \pm 0.058$), i.e. a fine-tuning problem.
- **Horizon Problem**: Distant regions in the cosmic radiation are causally disconnected, however its observed temperature is essentially the same (deviations of 10^{-5} K). Homogeneity must have been an initial condition.



Solution

- **What do we need?** [A. Guth, 1981] A mechanism that produces the special "initial conditions" of the Big Bang model, i.e. that reproduce an early universe such that it is practically flat and homogeneous.

$$\textbf{Inflation} \quad \Leftrightarrow \quad \ddot{a} > 0.$$

- An inflationary phase is accomplished by matter with the property $\rho + 3P < 0$.
- It is generally assumed that inflation started at energy scales near Planck energy ($E_P \simeq 10^{19}$ GeV). Since we do not have a fully developed theory of Quantum Gravity, we will use the formalism of Quantum Field Theory in curved space-times. The type of dominant matter during inflation is modeled by a scalar field (spin-0).
- The scalar field responsible for inflation is called: **inflaton**.

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Successes of the Inflationary Paradigm

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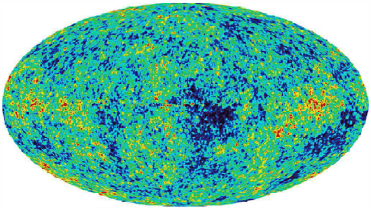
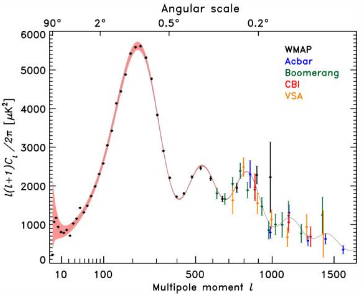
- The main success...
  Quantum Fluctuations of the inflaton results in theoretical predictions of the spectrum of primordial *inhomogeneities*. [Mukhanov, 1992]
- Observational Data in agreement with such predictions!





Inflaton

- Action (Quantum Field Theory + General Relativity):

$$S = \int d^4x \quad \sqrt{-g}\left[\frac{1}{16\pi G}R[g] - \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V(\phi)\right] \tag{1}$$

- Separation (homogeneous-isotropic background + small fluctuations):

$$\phi(\mathbf{x},\eta) = \phi_0(\eta) + \delta\phi(\mathbf{x},\eta) \tag{2}$$

- Field perturbations induce metric perturbations:

$$ds^2 = a(\eta)^2[-(1+2\Psi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j] \tag{3}$$

- Introduce a new field variable $v = a\delta\phi + \Psi\phi_0' a'$
- After choosing a vacuum state, one proceeds to quantize the theory

$$\hat{v}(\eta, x) = \frac{1}{\sqrt{2}}\int \frac{d^3k}{(2\pi)^{3/2}}[v_k^\star(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}\hat{a}_k^- + v_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\hat{a}_k^+] \tag{4}$$

Theoretical Predictions

- Express $\hat{v}$ in terms of $\hat{\Psi}$ and calculate the 2-point correlation function in the vacuum state $|0\rangle$:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle = \int \frac{dk}{k}\frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \frac{\sin kr}{kr} \tag{5}$$

where $r \equiv |x-y|$.

- The predicted power spectrum of the metric perturbations is:

$$\mathcal{P}_\Psi(k,\eta) = \frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \tag{6}$$

- For the modes of observational interests (i.e. those with $k \ll aH$), one obtains $\Psi_k(\eta) \propto k^{-3}$, which implies that the predicted power spectrum is:

$$\mathcal{P}_\Psi(k,\eta) = A, \tag{7}$$

with $A \simeq 10^{-10}$ a constant (independent of k).

Something is not clear...

- It is not clear:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle = \overline{\Psi(\eta,x)\Psi(\eta,y)}$$

> *"These are quantum averages, not averages over an ensemble of classical field configurations...some sort of decoherence must set in...It is not apparent just how this happen..."*

[S. Weinberg, *Cosmology*; 2008]

- **Remarkable**: The universe was originally described by a space-time which is homogeneous and isotropic, and there is a scalar field (the inflaton) which is in a vacuum state also homogeneous and isotropic (there are some irrelevant deviations of this left from an imperfect inflation at the order of e^{-80}), but the universe ended up with inhomogeneities that fit the experimental data.
- **Question** How do we end up in a situation which is not symmetric (the symmetry being the homogeneity and isotropy) given that there is nothing in the dynamics that breaks such symmetries?

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The fundamental question

- **How did it happen?**

$$|\text{homogeneous}\rangle \quad \rightarrow \quad |\text{inhomogeneous}\rangle$$

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Is it "just philosophy?"

V. Mukhanov in his book *Physical Foundations of Cosmology* (2005), on the issue of how do the inhomogeneities arise:

> *"Quantum mechanical unitary evolution does not destroy translational invariance... However decoherence is not sufficient to explain the breaking of translational invariance. ... we have to appeal to either to Bohr's reduction postulate or to Everett's many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context."*

- There are of course other postures, as well. However, often these issues are resolved as "just philosophy", with no impact whatsoever on the predictions of the primordial spectrum and related issues.
- We will see that such preconception is mistaken

The transition and the collapse proposal

Our approach on attempting to answer the question: "How does it happen?
$|\text{homogeneous}\rangle \rightarrow |\text{in-homogeneous}\rangle$ is described as:

- Invoking the collapse of the wave function, one could break the symmetry of the original state.

$$|\text{homogeneous}\rangle \quad \rightarrow \textbf{Collapse} \rightarrow \quad |\text{inhomogeneous}\rangle$$

- According to Quantum Mechanics the collapse of the wave function (and therefore the onset of the asymmetry) occurs only as a result of a measurement. But, how do we apply this postulate in the cosmological setting?.
- Phenomenological Model: *Something* (possibly Quantum Gravity) intrinsic to the system occurred triggering the collapse of its wave function.
- Even if we do not know precisely the nature of the collapse mechanism, we can parameterize the collapse (e.g. by using expectation values in the post-collapse state) and obtain testable predictions.

[Sudarsky, 2006]

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[Sudarsky, 2006]

Semiclassical Gravity

- Gravity and Matter are treated differently and are coupled to the inflaton according to:

$$G_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle$$

- This is supposed to hold at all times except when a quantum gravity induced collapse of the wave function occurs,
- At this *time of collapse* the excitation of the fundamental quantum-gravitational degrees of freedom must be taken into account and there is a breakdown in the semi-classical approximation.
- This possible breakdown is represented by the presence of a term Q_{ab} in the semi-classical Einstein's equation which is supposed to become non-zero **only** during the collapse of the quantum mechanical wave function.

$$G_{ab} + Q_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle$$

Detailed analysis of the collapse proposal

- Quantum field: $\hat{y} = a\hat{\delta\phi}$ (The metric perturbation Ψ is classical).
- Canonical conjugate momentum $\hat{\pi}_y = a\hat{\delta\phi}' = \hat{y}' - \hat{y}a'/a$.
- Einstein semiclassical equations:

$$G_{ab} = 8\pi G\langle \hat{T}_{ab}\rangle \qquad \Rightarrow \qquad \Psi_k(\eta) = \frac{-4\pi G\phi_0'}{ak^2}\langle \hat{\pi}_k(\eta)\rangle$$

- **Before** the collapse $|0\rangle$:

$$\langle \hat{y}_k\rangle_0 = 0; \qquad \langle \hat{\pi}_k\rangle_0 = 0 \qquad \Rightarrow \qquad \Psi_k(\eta) = 0$$

- **After** the collapse $|0\rangle \to |\Theta\rangle$

$$\langle \hat{y}_k\rangle_\Theta \neq 0; \qquad \langle \hat{\pi}_k\rangle_\Theta \neq 0 \qquad \Rightarrow \qquad \Psi_k(\eta) \neq 0$$

The space-time (always classical) is no longer homogeneous and isotropic.

Parametrization of the collapse

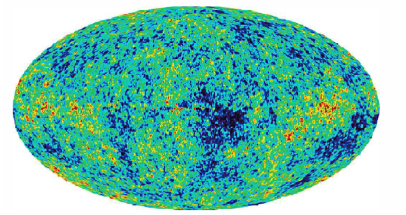
- The collapse proposal is based on the hypothesis that each mode would jump to a new state where the expectation value would be determined by both, the scale of the uncertainties and some random variable

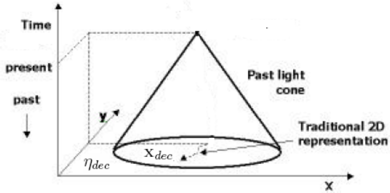
$$\langle \hat{y}_k^{R,I}(\eta_k^c) \rangle_\Theta = x_{k,y}^{R,I} \sqrt{(\Delta \hat{y}_k^{R,I})_0^2}, \tag{8}$$

$$\langle \hat{\pi}_k^{R,I}(\eta_k^c) \rangle_\Theta = x_{k,\pi}^{R,I} \sqrt{(\Delta \hat{\pi}_k^{R,I})_0^2}, \tag{9}$$

  where R, I denotes the real and imaginary parts of the field respectively.
- η_k^c is the conformal time of collapse.
- $(\Delta \hat{y}_k^{R,I})_0^2$, $(\Delta \hat{\pi}_k^{R,I})_0^2$ are the uncertainties of each mode of the field.
- $x_{k,y}^{R,I}$, $x_{k,\pi}^{R,I}$ are random variables with a distribution which in principle might be non-Gaussian, i.e. different modes could be correlated.

- The measured quantity is $\frac{\delta T}{T_0}(\theta, \varphi)$ which is related with Ψ





- The quantity of interest is:

$$a_{lm} = \int d\Omega \Psi(\eta_D, \mathbf{x}_D) Y^{\star}_{lm}(\theta, \varphi) \quad (10)$$

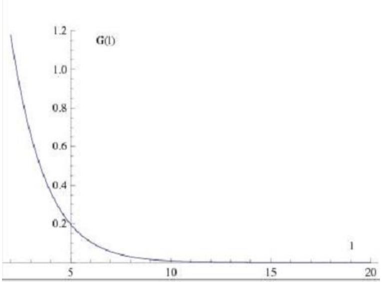
- One identifies the theoretical prediction of $|a_{lm}|^2$ with the observed value of $|a^{obs}_{lm}|^2$.

Theoretical predictions under the collapse scheme

- Ignoring the physics of the plasma, the theoretical prediction for $|a_{lm}|^2$ given by our model is:

$$|a_{lm}|^2 = \mathcal{A}(1+\varepsilon G(l)); \qquad G(l) \equiv \frac{l(l+1)9\sqrt{\pi}\Gamma(l)}{2^{5/2+l}\Gamma(l+3/2)}\, {}_2F_1(l,-1/2;l+3/2,1/4). \quad (11)$$

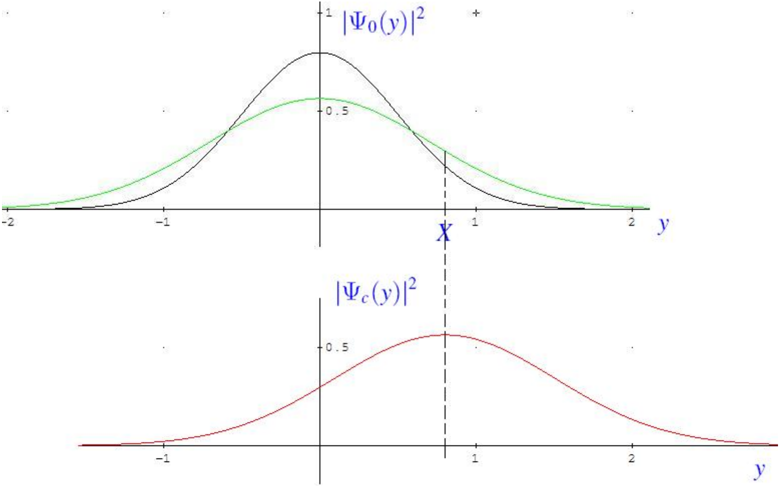
- $G(l)$ in the interval $l \in [2,20]$. The effect is stronger for low l and it decreases in a nearly exponential fashion for large l.



Conclusions

- The standard inflationary approach does not provides a satisfactory explanation on how does the universe evolves from a symmetrical state, corresponding to the primordial universe, to a non-symmetrical one.
- This work shows that, even though in principle we do not know precisely what is the nature of the physics behind what we call the collapse, we can obtain some insights on the "rules" that govern it.
- We have shown that the ideas tied to the collapse proposal are not 'just philosophy'. In particular, the standard flat spectrum is modified affecting only the lowest multipoles ($l \leq 20$). This signature of the collapse mechanism could be in principle searched for observationally.

Ref. G. León and D. Sudarsky, SIGMA 8 (2012), Special Issue on Loop Quantum Gravity and Cosmology, [arXiv:1109.0052].



$$\langle \hat{y}(\eta^c) \rangle_c = X = x\sqrt{(\Delta^2 \hat{y}(\eta^c))_0}$$

Our Approach

We have considered in detail one approach (inspired by Penrose's ideas): The standard paradigm + "self-induced collapse" hypothesis.

- The metric description of gravity is just an effective one. The fundamental theory describes gravity in terms of some more fundamental degrees of freedom. Quantum Mechanics is incomplete.
- A new ingredient brought into physics by quantum-gravity has an effective description as a self-induced collapse (which does not rely on an external agent to induce it).

Gravitational Waves from inflation

An important observation follows directly from the point of view adopted to relate the metric effective description of gravity with the quantum aspect of the matter field, i.e., $G_{ab} = 8\pi G\langle\hat{T}_{ab}\rangle$:

- The source of the fluctuations that lead to anisotropies and inhomogeneities lies in the quantum uncertainties for the scalar field, which collapses, due to some unknown quantum gravitational effect. Once collapsed, these density inhomogeneities and anisotropies feed into the gravitational degrees of freedom leading to nontrivial perturbations in the metric functions, in particular, the newtonian potential. However, the metric itself is not a source of the quantum gravitational induced collapse.
- Therefore, as the scalar field does not act as a source for the gravitational tensor modes -at least not at the lowest order considered here-, the tensor modes can not be excited. Thus, the scheme naturally leads to the prediction of a zero -or at least a strongly suppressed- amplitude of gravitational waves to the CMB.

References

- A. Perez, H. Sahlmann, and D.Sudarsky, *Classical and Quantum Gravity* **23** 2317, (2006) [arXiv: gr-qc/0508100].
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