SIGNATURES OF NON-MARKOVIANITY IN OPEN-SYSTEM DYNAMICS

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"Open Problems in Quantum Mechanics"

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Outline

- **Introduction**
- **Classical non-Markov processes**
- **Non-Markovianity in quantum dynamics**

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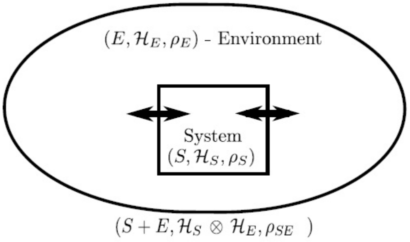
Open quantum systems: reduced dynamical maps



$$
\begin{array}{ccc}
\rho_{SE} & \longrightarrow & \rho_{SE}(t) = U(t)\rho_{SE}U^\dagger(t) \\
\downarrow & & \downarrow \\
\rho_S & \longrightarrow & \rho_S(t) = \mathrm{Tr}_E[U(t)\rho_{SE}U^\dagger(t)]
\end{array}
$$

IF the initial total state is $\rho_{SE} = \rho_S \otimes \rho_E$ and ρ_E is **fixed**

One parameter family of CPT *reduced dynamical maps* $\{\Lambda(t)\}_{t\geq 0}$

$$\mathcal{S}(\mathcal{H}_S) \longrightarrow \mathcal{S}(\mathcal{H}_S)$$

$$\rho_S \longrightarrow \rho_S(t) =: \Lambda(t)\rho_S = \mathrm{Tr}_E\left[U(t)\rho_S \otimes \rho_E U^\dagger(t)\right]$$

$$= \sum_\alpha M_\alpha(t)\rho_S M_\alpha^\dagger(t) \qquad \sum_\alpha M_\alpha^\dagger(t)M_\alpha(t) = \mathbb{1}$$

Completely positive quantum dynamical semigroups

- Which equations of motion provide a well-defined time evolution?

Semigroup composition law

$$\Lambda(t)\Lambda(s) = \Lambda(t+s) \;\; \forall t \geq s \geq 0$$

Theorem (Gorini-Kossakowski-Sudarshan (1976))

A linear operator L is the generator of a CPT quantum dynamical semigroup $\left\{\Lambda(t) = \mathrm{e}^{Lt}\right\}_{t\geq 0}$ iff it can be written as

$$L\rho = -i\,[\,H,\rho] + \sum_{\alpha=1}^{N^2-1} \gamma_\alpha \left(\sigma_\alpha \rho \sigma_\alpha^\dagger - \frac{1}{2}\left\{\sigma_\alpha^\dagger \sigma_\alpha, \rho\right\}\right)$$

$\gamma_\alpha \geq 0,\; H = H^\dagger$

- Extends to infinite dimensional Hilbert spaces for bounded generators [G. Lindblad (1976)]
- Master equation: $\mathrm{d}\rho(t)/\mathrm{d}t = L\rho(t)$

Memory effects in quantum dynamics

- Markov condition: $\tau_E \ll \tau_S$

  The influence that the open system has on the environment does not affect the open system back again.
  Memory effects in open-system dynamics can be neglected, as for classical Markov stochastic processes

Memory effects in quantum dynamics

◇ Markov condition: $\tau_E \ll \tau_S$

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- There are many physical systems where this condition is not satisfied: energy transfer in photosynthetic complexes, photonic band gaps, quantum dots,...
- One has to look for a more general description of the dynamics

The very notion of non-Markovianity for quantum dynamics has still to be cleared up and is a subject of intense debate

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Classical Markov processes

Discrete Markov process with values in $\{x_k\}_{k\in\mathbb{N}}$

$$p_{1|n}(x_n, t_n|x_{n-1}, t_{n-1}; \ldots ; x_0, t_0) = p_{1|1}(x_n, t_n|x_{n-1}, t_{n-1})$$

Chapman-Kolmogorov equation

$$p_{1|1}(x, t|y, s) = \sum_z p_{1|1}(x, t|z, \tau)\, p_{1|1}(z, \tau|y, s) \qquad t \geq \tau \geq s$$

- The definition involves the entire hierarchy of probability distributions
- It is of interest to find signatures of non-Markovianity in the dynamics of one-point probability distributions [B.Vacchini, A.Smirne, E.-M.Laine, J.Piilo, H.-P.Breuer (2011)]

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- It is of interest to find signatures of non-Markovianity in the dynamics of one-point probability distributions [B.Vacchini, A.Smirne, E.-M.Laine, J.Piilo, H.-P.Breuer (2011)]
  - Finite dimensional system $\mathbf{p}(t) = \Lambda(t,0)\,\mathbf{p}(0)$

$\mathbf{p}(t)$ probability vector, $\Lambda(t,0)$ stochastic matrix:

$$(\Lambda)_{jk} \geq 0 \qquad \sum_j (\Lambda)_{jk} = 1$$

P-divisibility and Chapman-Kolmogorov equation

P-divisibility of $\{\Lambda(t,0)\}_{t\geq 0}$

$$\Lambda(t,0) = \Lambda(t,s)\Lambda(s,0) \quad \forall t \geq s \geq 0 \qquad \Lambda(t,s) \;\; \text{stochastic}$$

- If $\Lambda(t,s) = \Lambda(t-s,0) \Rightarrow$ semigroup composition law

In general, $\Lambda(t,s) = \Lambda(t,0)\Lambda^{-1}(s,0)$, so that

$$\mathbf{p}\left(t\right) = \Lambda\left(t,s\right)\mathbf{p}\left(s\right) \quad \text{transition maps,}$$

but is $\Lambda\left(t,s\right)$ a stochastic matrix?

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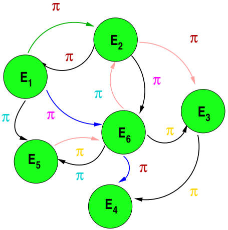
IF the process is Markovian $\Rightarrow (\Lambda(t,s))_{jk} = p_{1|1}(j,t|k,s) \Rightarrow$ P-div

In non-Markov processes

1. $(\Lambda(t,s))_{jk} \neq p_{1|1}(j,t|k,s)$
2. P-divisibility does not correspond to C-K equation
3. non-P-divisibility $\Rightarrow$ non-Markovianity, but *not* viceversa

Semi-Markov processes



- Markov chain: π_{jk}
- Renewal processes: $f(t)$

- *Uniquely* determined by $k \to j$ transition probabilities and w.t.d. $f_k(t)$
- Survival probabilities $g_k(t) = 1 - \int_0^t \mathrm{d}\tau f_k(\tau)$

  Integrodifferential equation for the one-point probability vector

$$\frac{\mathrm{d}}{\mathrm{d}t} p_k(t) = \int_0^t \mathrm{d}\tau \sum_j \left(W_{kj}(\tau) p_j(t-\tau) - W_{jk}(\tau) p_k(t-\tau) \right)$$

$$W_{jk}(t) = \pi_{jk} b_k(t) \qquad f_k(t) = \int_0^t \mathrm{d}\tau \, b_k(\tau) g_k(t-\tau)$$

- The process is Markovian iff $f_k(t) = \lambda_k \mathrm{e}^{-\lambda_k t}$

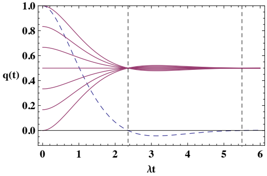
Explicit examples

Two-states system, state-independent w.t.d. and bistochastic matrix

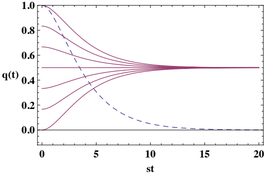
$$q(t) = \sum_{n=0}^{\infty} p(2n, t) - \sum_{n=0}^{\infty} p(2n+1, t)$$

P-divisibility is broken iff $|q(t)|$ increases

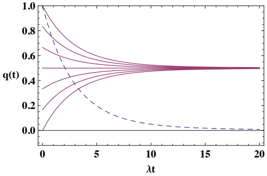
- $f = f * f$



- $f = f_1 * f_2$



- $f = \mu f_1 + (1-\mu) f_2$



In all these situations the process is non-Markovian, but P-divisibility still holds in some cases, depending on $f(t)$.

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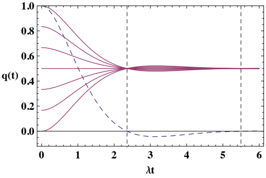
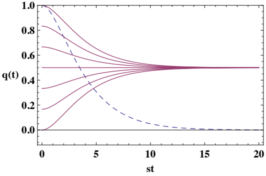


- $\mathsf{f} = f_1 * f_2$



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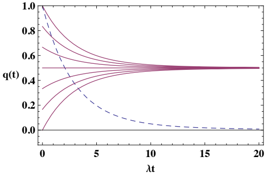

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Kolmogorov distance

It measures the distinguishability of classical probability distributions

$$D_K\left(\mathbf{p}^1(t),\mathbf{p}^2(t)\right)=\frac{1}{2}\sum_k\left|p_k^1(t)-p_k^2(t)\right|$$

If $\Lambda(t,s)$ is a stochastic matrix $\left((\Lambda)_{jk}\geq 0\ \sum_j(\Lambda)_{jk}=1\right)$

$$\begin{aligned}D_K\left(\mathbf{p}^1(t),\mathbf{p}^2(t)\right) &= \frac{1}{2}\sum_j\left|\sum_k\Lambda(t,s)_{jk}\left(\mathbf{p}^1(s)-\mathbf{p}^2(s)\right)_k\right| \\ &\leqslant \frac{1}{2}\sum_j\sum_k\Lambda(t,s)_{jk}\left|\left(\mathbf{p}^1(s)-\mathbf{p}^2(s)\right)_k\right| \\ &= \frac{1}{2}\sum_k\left|\left(\mathbf{p}^1(s)-\mathbf{p}^2(s)\right)_k\right|=D_K\left(\mathbf{p}^1(s),\mathbf{p}^2(s)\right)\end{aligned}$$

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&\leqslant \frac{1}{2}\sum_j\sum_k\Lambda(t,s)_{jk}\left|\left(\mathbf{p}^1(s)-\mathbf{p}^2(s)\right)_k\right| \\
&= \frac{1}{2}\sum_k\left|\left(\mathbf{p}^1(s)-\mathbf{p}^2(s)\right)_k\right| = D_K\left(\mathbf{p}^1(s),\mathbf{p}^2(s)\right)
\end{aligned}$$

- Two signatures of non-Markovianity in the dynamics of $\mathbf{p}(t)$ have been introduced
- They are equivalent for the examples, but *not* in general
- They are sufficient but *not* necessary conditions to detect a non-Markov process

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The trace distance

The trace distance between two states ρ_1 and ρ_2

$$D(\rho_1,\rho_2) \;=\; \frac{1}{2}\|\rho_1-\rho_2\|_1 = \frac{1}{2}\mathrm{Tr}|\rho_1-\rho_2| \qquad |A| = \sqrt{A^\dagger A}$$

- $D(\rho_1,\rho_2) = \sum_i |x_i|/2$, with x_i eigenvalues of $\rho_1-\rho_2$
- $0 \leq D(\rho_1,\rho_2) \leq 1$

- all positive and trace preserving maps Λ are contractions for the trace distance

$$D(\Lambda\rho_1,\Lambda\rho_2) \;\leq\; D(\rho_1,\rho_2)$$

- The trace distance $D(\rho_1,\rho_2)$ quantifies the **distinguishability** between the two states ρ_1 and ρ_2
- The change in distinguishability of states of an open system S can be interpreted as an information flow between S and E

Measure of non-Markovianity based on trace distance

Reduced dynamics can be characterized by investigating the trace distance $D(\rho_S^1(t), \rho_S^2(t))$ between a pair of open-system states $\rho_S^1(t)$ and $\rho_S^2(t)$, which evolve from different initial conditions

- Markovian dynamics: the trace distance is monotonically non-increasing. Unidirectional information flow from S to E

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Trace-distance rate of change $\sigma(t, \rho_S^{1,2}(0)) = \frac{d}{dt} D(\rho_1(t), \rho_2(t))$
$\sigma > 0$ back-flow of information from E to S

Non-Markovianity measure [H.-P. Breuer, E.-M. Laine, J. Piilo (2009)]

$$\mathcal{N}(\Lambda) = \max_{\rho_S^{1,2}(0)} \int_{\sigma>0} dt \sigma(t, \rho_{1,2}^S(0))$$

$\mathcal{N}$ provides a clear criterion to experimentally detect non-Markovianity. Transition between Markovian and non-Markovian dynamics in an all-optical setting [B.-H. Liu, L. Li, Y.-F. Huang, C.-F. Li, G.-C. Guo, E.-M. Laine, H.-P. Breuer, and J. Piilo (2011)]

Measure of non-Markovianity based on CP-divisibility

- Different definition: Markovianity is identified with CP-divisibility [A. Rivas, S.F. Huelga, M.B. Plenio (2010)]

CP-divisibility of $\{\Lambda(t,0)\}_{t\geq 0}$

$$\Lambda(t,0) = \Lambda(t,s)\Lambda(s,0) \quad \forall t \geq s \geq 0 \qquad \Lambda(t,s) \;\; \mathrm{CP}$$

Time-local generator of a CP-divisible family of dynamical maps

$$L(t)\rho = -i\,[\,H(t),\rho] + \sum_{\alpha=1}^{N^2-1} \gamma_\alpha(t) \left(\sigma_\alpha(t)\rho\sigma_\alpha^\dagger(t) - \frac{1}{2}\left\{\sigma_\alpha^\dagger(t)\sigma_\alpha(t),\rho\right\} \right)$$

- $\gamma_\alpha(t) \geq 0$, $H(t) = H^\dagger(t)$; $\qquad \mathrm{d}\rho(t)/\mathrm{d}t = L(t)\rho(t)$

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- $\gamma_\alpha(t) \geq 0$, $H(t) = H^\dagger(t)$; $\qquad \mathrm{d}\rho(t)/\mathrm{d}t = L(t)\rho(t)$

$$\mathcal{I}(\Lambda) = \int_{\mathbb{R}_+} \mathrm{d}t \lim_{\epsilon\to 0} \frac{\frac{1}{2}\|\Lambda_{\text{Choi}}(t,t+\epsilon)\|_1 - 1}{\epsilon}$$

- Choi matrix $\Lambda_{\text{Choi}} = N\Lambda \otimes \mathbb{1}_N(|\phi\rangle_{ME}\langle\phi|)$

Quantum semi-Markovian dynamics

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \int_0^t \mathrm{d}\tau \mathcal{K}(\tau)\rho(t-\tau)$$ [H. P. Breuer and B. Vacchini (2008)]

$$K(\tau)\rho = -i\,[H(\tau),\rho] + \sum_\alpha k_\alpha(\tau)\left(\sigma_\alpha(\tau)\rho\sigma_\alpha^\dagger(\tau) - \frac{1}{2}\left\{\sigma_\alpha^\dagger(\tau)\sigma_\alpha(\tau),\rho\right\}\right)$$

- If $p_k(t) \equiv \langle k|\rho(t)|k\rangle$ obey a closed system of equations, this reads

$$\frac{\mathrm{d}}{\mathrm{d}t}p_k(t) = \int_0^t \mathrm{d}\tau \sum_j \left(W_{kj}(\tau)p_j(t-\tau) - W_{jk}(\tau)p_k(t-\tau)\right)$$

- $W_{kj}(t) \to b_k(t) = \sum_j W_{jk}(t) \to f_k(t) : \hat{f}_k(u) = \hat{b}_k(u)/(u + \hat{b}_k(u))$

  If $f_k(t)$ is a proper w.t.d. $\Rightarrow$ quantum semi-Markovian dynamics

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- Special case [A. Budini (2004)]

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = \int_0^t d\tau\, b(t-\tau)\,[\mathcal{E}-\mathbb{1}]\,\rho(\tau) \quad \mathcal{E}\ \text{CPT map}$$

If $f(t)$ is a w.t.d. $\rho(t) = \sum_{n=0}^\infty p(n,t)\mathcal{E}^n\rho(0)$ and CP is guaranteed

Dissipative dynamics

For a CPT map $\mathcal{E}\rho = \sigma_-\rho\sigma_+ + \sigma_+\rho\sigma_-$

$$\rho(t) = \begin{pmatrix} p_e(t)\rho_{11} + p_o(t)\rho_{00} & g(t)\rho_{10} \\ g(t)\rho_{01} & p_o(t)\rho_{11} + p_e(t)\rho_{00} \end{pmatrix}$$

$$\Lambda(t,0) = \mathrm{diag}\left(1, g(t), g(t), q(t)\right)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\rho = \gamma(t)\left(\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+, \rho\} + \sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\right) + \delta(t)\left(\sigma_z\rho\sigma_z - \rho\right)$$

- ◇ From time-local master equation CP-divisibility can be directly inferred

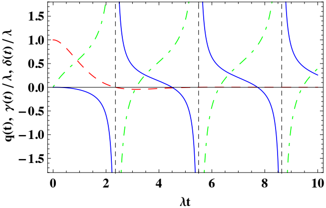
$$\gamma(t) = -\frac{1}{2}\frac{\dot{q}(t)}{q(t)}$$

$$\delta(t) = \frac{1}{2}\left(\frac{f(t)}{g(t)} - \gamma(t)\right)$$

- CP-divisibility and decrease of trace distance are not equivalent: $\delta(t)$ affects CP-divisibility, not trace distance monotonicity

Explicit examples for different w.t.d.

- $\mathrm{f} = f * f$



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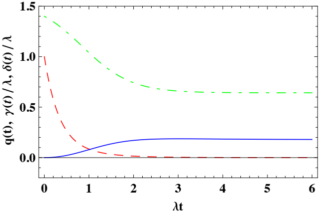


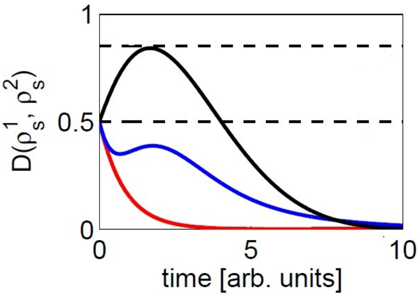
$$\mathcal{N}(\Lambda) = \int_{\Omega_+} \mathrm{d}t \frac{\mathrm{d}}{\mathrm{d}t} |q(t)| = \frac{1}{e^{\pi} - 1}$$ [Vacchini, Smirne, Laine, Piilo, Breuer (2011)]

$$\mathcal{I}(\Lambda) = \int_{\Omega_+} \mathrm{d}t \frac{\mathrm{d}}{\mathrm{d}t} \log |q(t)| = \infty$$

- For the convolution of a higher number of $f(t)$, $\mathcal{N}(\Lambda)$ detects an increasing non-Markovianity, while $\mathcal{I}(\Lambda)$ is always ∞
- For a proper f, $\mathcal{N}(\Lambda) = 0$, but $\mathcal{I}(\Lambda) > 0$
- Both measures can be zero also for non-exponential $f(t)$

The trace distance as a witness for initial correlations

- The assumption of a product total initial state is questionable in many physical systems, especially outside the weak coupling regime
- $\rho_S^1(t)$ evolved from $\rho_{SE}^1(0)$ and $\rho_S^2(t)$ evolved from $\rho_{SE}^2(0)$



- (red line) Markovian dynamics
- (blue line) Non-Markovian dynamics with initial product state
- (black line) Non-Markovian dynamics with initial correlations

The trace distance as a witness for initial correlations

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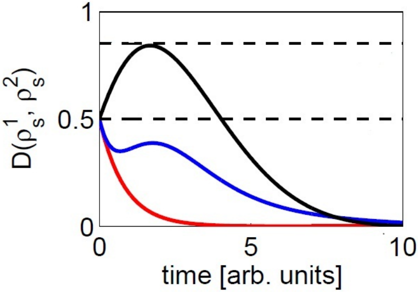

- Markovian dynamics (red line)
- Non-Markovian dynamics with initial product state (blue line)
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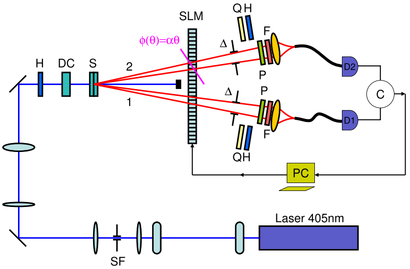
Fixed the environmental state, $D(\rho^1_S(t), \rho^2_S(t)) > D(\rho^1_S(0), \rho^2_S(0))$ means that at least one of the initial total states was correlated

Experimental realization

A. Smirne, D. Brivio, S. Cialdi, B. Vacchini, and M.G.A. Paris (2011)

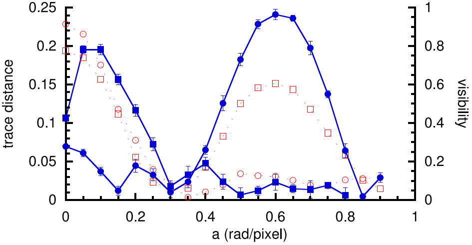


$$|\psi_{SE}(\alpha)\rangle = \frac{1}{\sqrt{2}} \int \mathrm{d}\theta_s \mathrm{d}\theta_i g(\theta_s) g(\theta_i)$$
$$\times \left(|H\theta_s\,\omega_s\rangle |H\theta_i\,\omega_i\rangle + e^{i(\alpha\theta_s + f(\theta_s))} |V\theta_s\,\omega_s\rangle |V\theta_i\,\omega_i\rangle \right)$$

- ● $f(\theta_s) = \sin(\lambda\theta)$
- ■ $f(\theta_s) = \tau\theta_s$

$\lambda = -0.6\,\mathrm{rad/pixel}$

$\tau = 0.1\,\mathrm{rad/pixel}$



Conclusions and outlook

- Differences between non-Markovian behavior in the state dynamics of a physical system and the notion of non-Markov process: *P-divisibility versus Chapman-Kolmogorov*
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◇ *Connection between non-Markovianity and correlations* due to system-environment interaction

◇ Dynamics of multi-time correlation functions: *beyond the quantum regression theorem*

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