



UNIVERSITY OF TURKU, FINLAND

CHARACTERIZING, QUANTIFYING, AND CONTROLLING NON-MARKOVIAN QUANTUM DYNAMICS

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Non-Markovian Processes and Complex Systems Group**

In collaboration with:

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Experiments: Chuan-Feng Li, Guang-Can Guo et al. (USTC, Hefei, China)



1. A stochastic description of open system dynamics:

Non-Markovian Quantum Jumps

Piilo, Maniscalco, Härkönen, Suominen: Phys. Rev. Lett. 100, 180402 (2008).

2. Quantifying and controlling non-Markovianity:

-Trace distance based measure

-Controlling by reservoir engineering

Breuer, Laine, Piilo: Phys. Rev. Lett. 103, 210401 (2009).

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo: Nature Physics 7, 931-934 (2011).

3. Nonlocal memory effects:

-Initial correlations between local reservoirs

Laine, Breuer, Piilo, C.-F. Li, Guo: Phys. Rev. Lett. 108, 210402 (2012)

Liu, Cao, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo:

"Photonic realization of nonlocal memory effects and non-Markovian quantum probes",
submitted for publication.



I. Stochastic descriptions: Non-Markovian quantum jumps

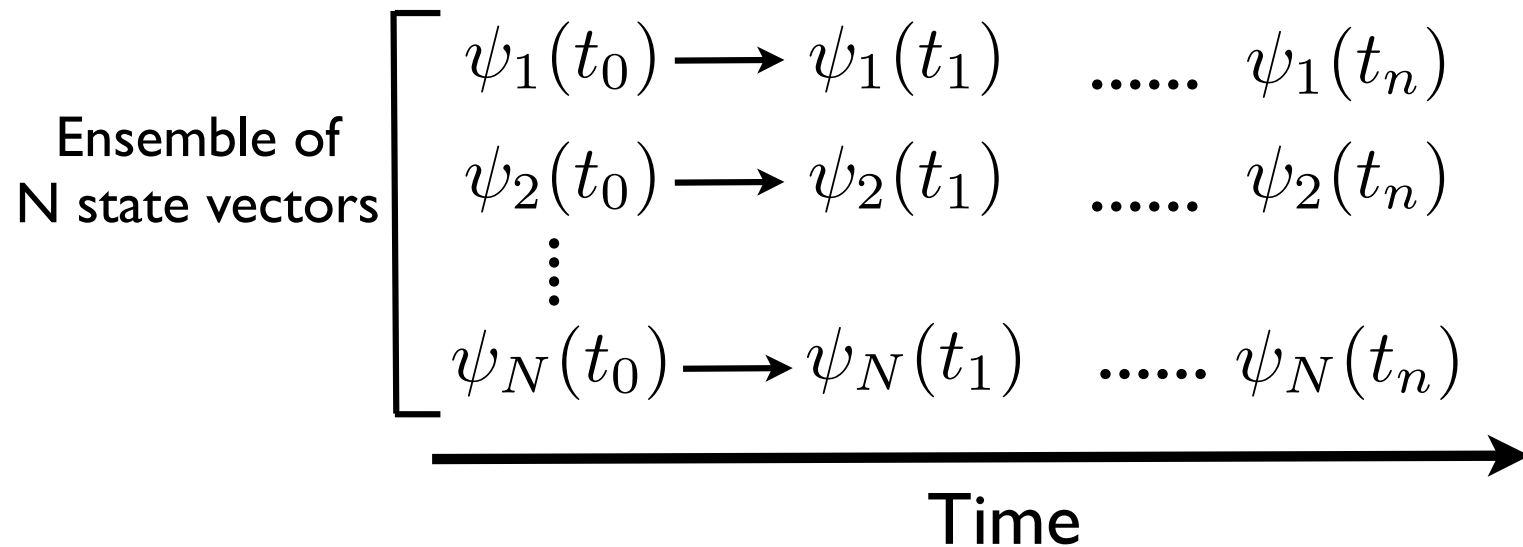


A master equation for the density matrix ρ :

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar} [H_S, \rho] + \sum_m \Gamma_m C_m \rho C_m^\dagger - \frac{1}{2} \sum_m \Gamma_m (C_m^\dagger C_m \rho + \rho C_m^\dagger C_m)$$

Also: at each point of time, density matrix ρ as ensemble of state vectors:

$$\rho(t) = \sum_{\alpha} \frac{N_{\alpha}(t)}{N} |\psi_{\alpha}(t)\rangle \langle \psi_{\alpha}(t)|$$



The time-evolution of each Ψ_i contains continuous or discontinuous stochastic element...



Simple classification of Monte Carlo methods

	Markovian	non-Markovian
Quantum Jump methods:	MCWF (Dalibard, Castin, Molmer) Quantum Trajectories (Zoller, Carmichael)	Fictitious modes (Imamoglu) Pseudo modes (Garraway) Doubled H-space (Breuer, Petruccione) Triple H-space (Breuer) NMQJ
Diffusion methods:	QSD (Diosi, Gisin, Percival...)	Non-Markovian QSD (Strunz, Diosi, Gisin) Stochastic Schrödinger equations (Bassi)

Plus: Wiseman, Gambetta, Budini, Gaspard, Lacroix...and others
(not comprehensive list, apologies for any omissions)

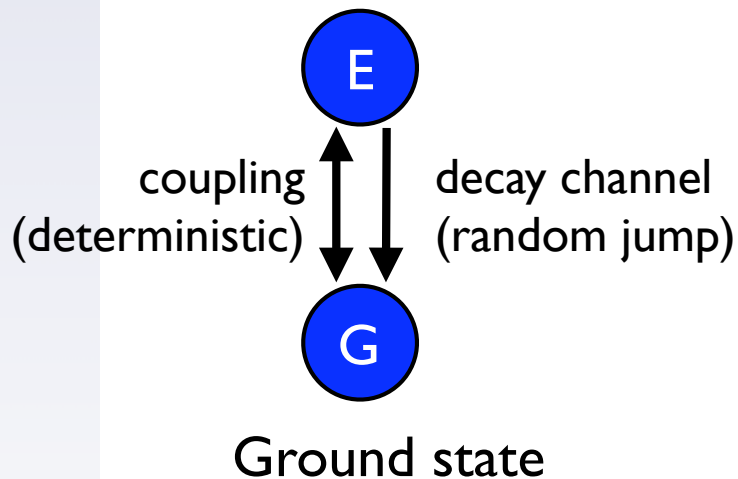


Markovian Monte Carlo wave function method, example

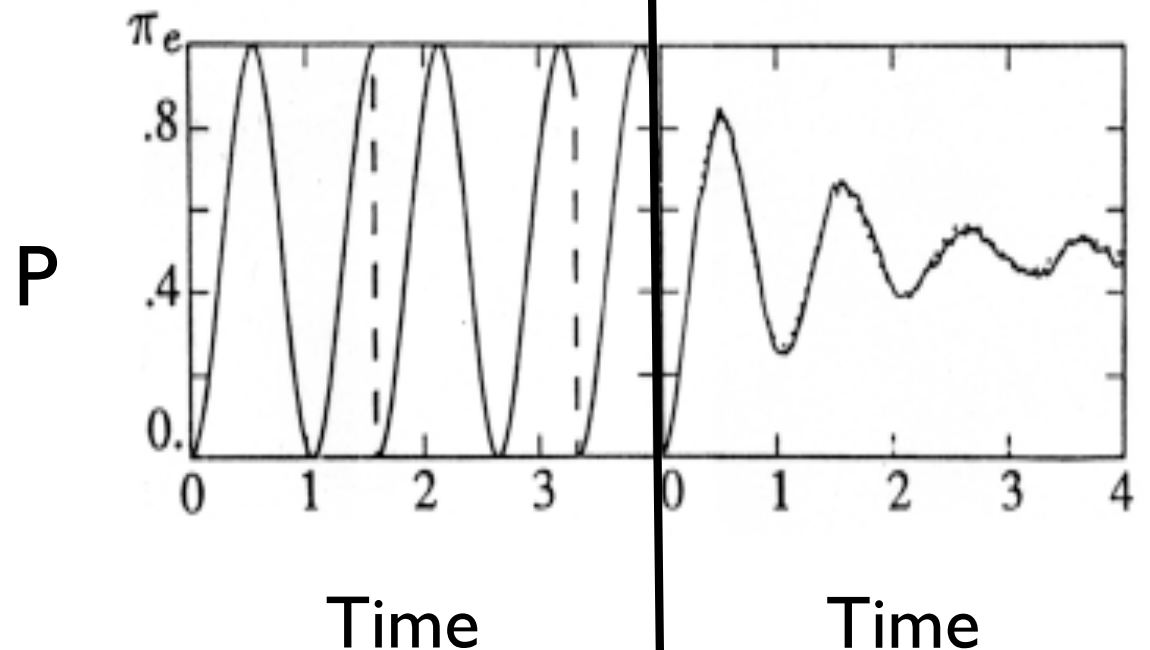
Quantum jump: Discontinuous stochastic change of the state vector.

Example: excited state probability P
for a driven 2-level atom

Unstable excited state



Markovian Monte Carlo
single realization **ensemble average**



damped Rabi oscillation
of the atom



Jump probability, example

Time-evolution of state vector Ψ_i :

At each point of time: decide if quantum jump happened.

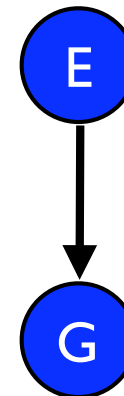
P_j : probability that a quantum jump occurs in a given time interval δt :

$$P_j = \delta t \Gamma p_e$$

time-step decay rate occupation probability of excited state

For example: 2-level atom

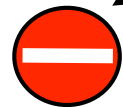
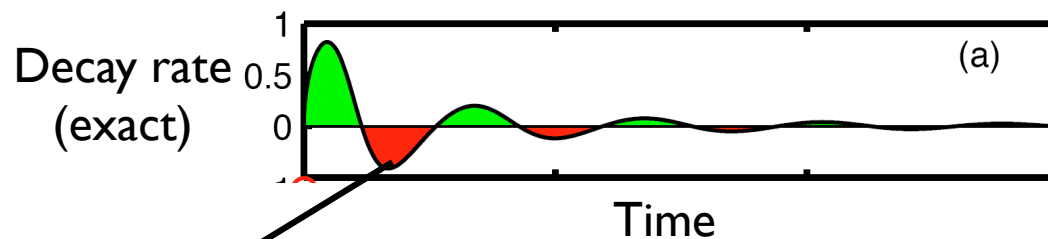
Probability for atom being transferred from the excited to the ground state and photon emitted.





What happens when the decay rate turns negative?

Example: 2-level atom in photonic band gap.



$$P_j = \delta t \Gamma p_e < 0$$

- Markovian description of quantum jumps fails, since gives negative jump probability.
For example: negative probability that atom emits a photon.



Markovian vs. non-Markovian evolution (2)

Markovian proof with the master equation:

$$\overline{\sigma_\alpha(t + \delta t)} = (1 - p_\alpha) \frac{|\phi_\alpha(t + \delta t)\rangle\langle\phi_\alpha(t + \delta t)|}{1 - p_\alpha} + \sum_j p_\alpha^j \frac{C_j |\psi_\alpha(t)\rangle\langle\psi_\alpha(t)| C_j^\dagger}{\langle\psi_\alpha(t)| C_j^\dagger C_j |\psi_\alpha(t)\rangle}$$

no jump probability deterministic part jump probabilities random jumps

- ⊙ Averaging over the deterministic and stochastic parts with the corresponding probabilities gives the master equation
- ⊙ ...for the non-Markovian case gives paths with weights > 1 and < 0 ...

What is the valid process corresponding to quantum jumps with negative rates?



Non-Markovian quantum jumps

Starting point:

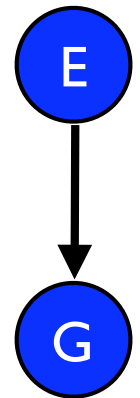
General non-Markovian master equation local-in-time:

$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar} [H_S, \rho] + \sum_m \Delta_m(t) C_m \rho C_m^\dagger - \frac{1}{2} \sum_m \Delta_m(t) (C_m^\dagger C_m \rho + \rho C_m^\dagger C_m)$$

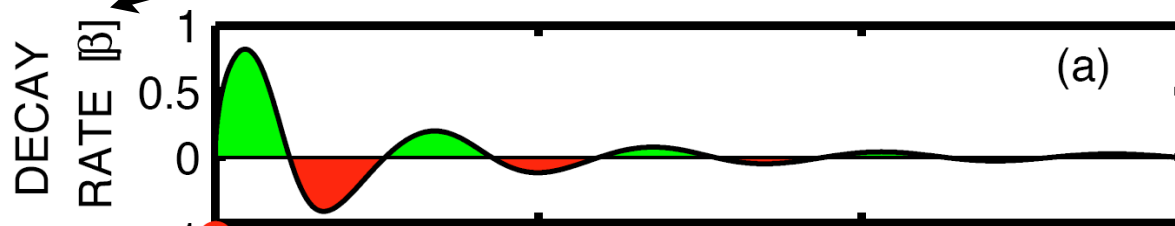
- Jump operators C_m
- Time dependent decay rates $\Delta_m(t)$.
- Decay rates have temporarily negative values.

Example: 2-level atom in photonic band gap.

Jump operator C for positive decay: $\sigma_- = |g\rangle\langle e|$



$$\frac{d\rho(t)}{dt} = \frac{1}{i\hbar} [H_S, \rho] + \Gamma(t) |g\rangle\langle e| \rho |e\rangle\langle g| - \frac{1}{2} \Gamma(t) (|e\rangle\langle e| \rho + \rho |e\rangle\langle e|)$$



Time



Non-Markovian quantum jumps

Quantum jump in negative decay region:

The direction of the jump process reversed

$$|\psi\rangle \xrightarrow{\text{green}} |\psi'\rangle = \frac{C_m |\psi\rangle}{\|C_m |\psi\rangle\|}, \quad \Delta_m(t) > 0$$

$$|\psi\rangle \xrightarrow{\text{red}} |\psi'\rangle = \frac{C_m |\psi\rangle}{\|C_m |\psi\rangle\|}, \quad \Delta_m(t) < 0$$

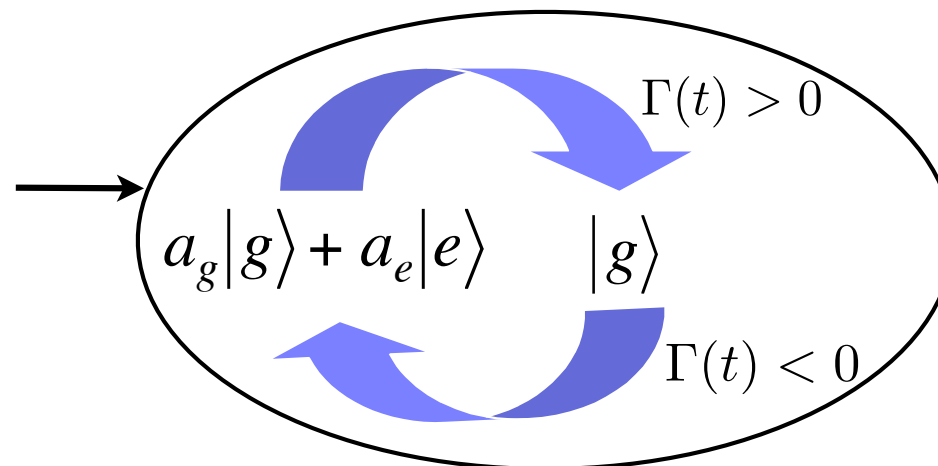
Jump probability:

$$P = \frac{N}{N'} \delta t |\Delta_m(t)| \langle \psi | C_m^\dagger C_m | \psi(t) \rangle$$

N: number of ensemble members in the target state
 N': number of ensemble members in the source state

The probability proportional to the target state!

For example:
 two-level atom
 $\sigma_- = |g\rangle\langle e|$

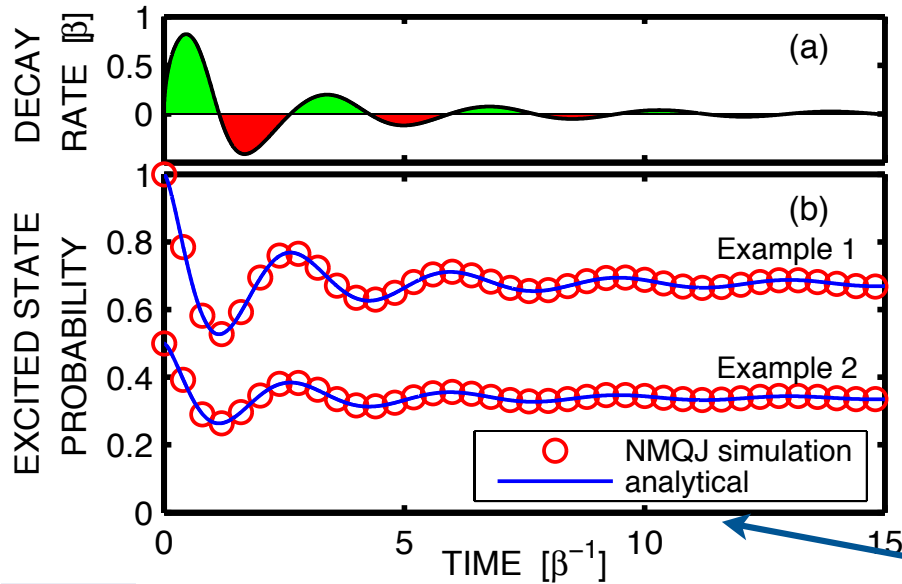


$$P = \frac{N_0}{N_g} \delta t |\Gamma(t)| |\langle \psi_0 | e \rangle|^2$$

The essential ingredient of non-Markovian system: memory.
 Characterizing feature I: Recreation of lost superpositions.



Example: 2-level atom in photonic band gap



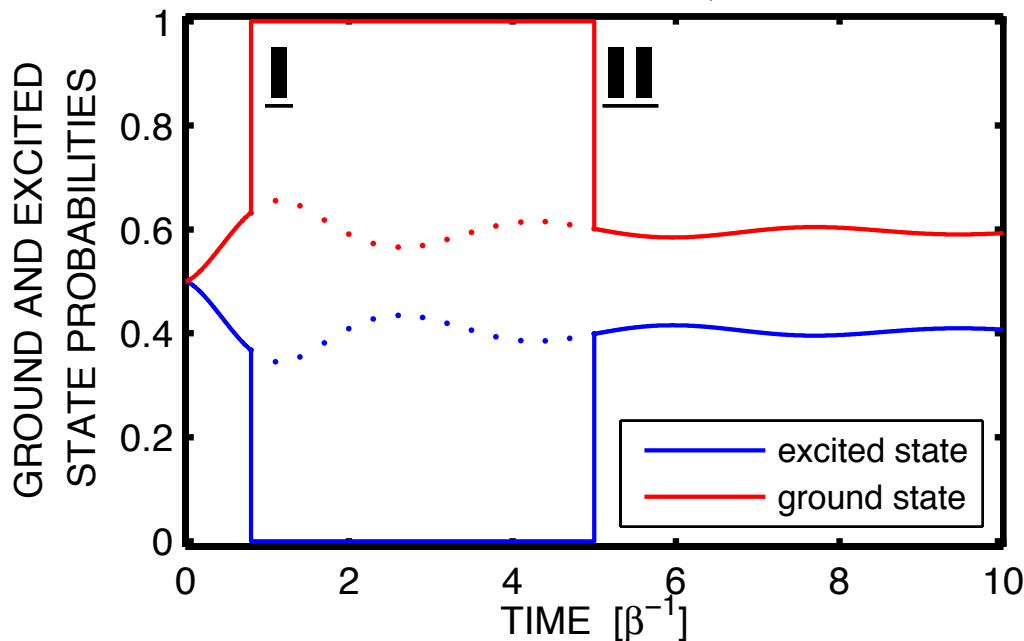
The simulation and exact results match.

Typical features of photonic band gap:

- Population trapping
- Atom-photon bound state.

Density matrix: average over the ensemble

Single state vector history



Example of one state vector history:

I: Quantum jump at positive decay region destroys the superposition.

II: Due to memory, non-Markovian jump recreates the superposition.



NMQJ: general algorithm

$$\begin{aligned}\frac{d}{dt}\rho &= -i[H(t), \rho] \\ &+ \sum_k \Delta_k^+(t) \left[C_k(t)\rho C_k^\dagger(t) - \frac{1}{2} \{C_k^\dagger(t)C_k(t), \rho\} \right] \\ &- \sum_l \Delta_l^-(t) \left[C_l(t)\rho C_l^\dagger(t) - \frac{1}{2} \{C_l^\dagger(t)C_l(t), \rho\} \right]\end{aligned}$$

$$\rho(t) = \sum_{\alpha} \frac{N_{\alpha}(t)}{N} |\psi_{\alpha}(t)\rangle \langle \psi_{\alpha}(t)| \quad \text{ensemble}$$

Deterministic evolution and positive channel jumps as before...

Negative channel with jumps

$$D_{\alpha \rightarrow \alpha'}^{j-}(t) = |\psi_{\alpha'}(t)\rangle \langle \psi_{\alpha}(t)|$$

where the source state of the jump is

$$|\psi_{\alpha}(t)\rangle = C_{j-}(t) |\psi_{\alpha'}(t)\rangle / \|C_{j-}(t) |\psi_{\alpha'}(t)\rangle\|$$

...and jump probability for the corresponding channel

$$P_{\alpha \rightarrow \alpha'}^{j-}(t) = \frac{N_{\alpha'}(t)}{N_{\alpha}(t)} |\Delta_{j-}(t)| \delta t \langle \psi_{\alpha'}(t) | C_{j-}^\dagger(t) C_{j-}(t) | \psi_{\alpha'}(t) \rangle.$$



Stochastic process description

Non-Markovian piecewise deterministic process.

A stochastic Schrödinger equation for non-Markovian open system:

$$d|\psi(t)\rangle = -iG(t)|\psi(t)\rangle dt \quad \leftarrow \text{Deterministic evolution}$$

$$\text{Positive channels} \quad + \sum_k \left[\frac{C_k(t)|\psi(t)\rangle}{\|C_k(t)|\psi(t)\rangle\|} - |\psi(t)\rangle \right] dN_k^+(t)$$

$$\text{Negative channels} \quad + \sum_l \int d\psi' [|\psi'\rangle - |\psi(t)\rangle] dN_{l,\psi'}^-(t).$$

Poisson increments for positive and negative channels

Negative channel jump rate:

$$\Gamma_- = \Delta_l^- \frac{P[|\psi'\rangle] d\psi'}{\underbrace{P[|\psi\rangle] d\psi}} \langle \psi' | C_l^\dagger C_l | \psi' \rangle \delta \left(|\psi\rangle - \frac{C_l |\psi'\rangle}{\|C_l |\psi'\rangle\|} \right) d\psi.$$

- Denominator may go to zero (singularity), associated to lost positivity (the system tries to cancel something which never happened)

Characterizing feature 3:

Stochastic realizations depend on each other.



Information flow, preliminaries...

PHYSICAL REVIEW A 79, 062112 (2009)

Open system dynamics with non-Markovian quantum jumps

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In memoryless Markovian open systems, the environment acts as a sink for the system information. Due to the system-reservoir interaction, the system of interest loses information on its state into the environment and this lost information does not play any further role in the system dynamics. However, if the environment has a nontrivial structure, then the seemingly lost information can return to the system at a later time leading to non-Markovian dynamics with memory. This memory effect is the essence of non-Markovian dynamics.

How to understand and quantify the information flow...



2. Quantifying and controlling non-Markovianity

Breuer, Laine, Piilo
Phys. Rev. Lett. 103, 210401 (2009)

Liu, L. Li, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
Nature Physics 7, 931-934 (2011).



Motivation

- No universally agreed definition of non-Markovianity: anything not following Lindblad, non-exponential decay, memory-kernel, negative decay rates...
- Various theoretical frameworks: Nakazima-Zwanzig, memory-kernel equations, TCL, time-local master equations, stochastic descriptions (quantum jumps, diffusion), correlated projection operators...

Is it possible to define and quantify non-Markovianity

- Independently of the used mathematical formalism
- Intuitively clear interpretation
- Physically motivated approach (instead of formal mathematical approach)
- What does memory mean in quantum dynamics?

Our starting point: quantify information flow and its direction between the system and the environment...



Trace distance

Distance measure for two states ρ_1 and ρ_2 :
Trace distance D :

$$D(\rho_1, \rho_2) = \frac{1}{2} \text{Tr} |\rho_1 - \rho_2| \quad 0 \leq D \leq 1$$

- For identical states $D = 0$, for orthogonal states $D = 1$
- Physical interpretation: measure of distinguishability
The max probability to distinguish the two states is equal to $\frac{1}{2}(1 + D)$
- In terms of information:

The larger D , the higher the probability to distinguish,
more information which state we have

- Invariant under unitary transformations

- Contractive for all CPT-maps Φ

$$D(\Phi\rho_1, \Phi\rho_2) \leq D(\rho_1, \rho_2)$$



(Nielsen, Chuang)



Measure for non-Markovianity

Non-Markovianity: Backflow of information from the environment to the open system

A measure for non-Markovianity:

$$\mathcal{N}(\Phi) = \max_{\rho_{1,2}(0)} \int_{\sigma > 0} dt \sigma(t, \rho_{1,2}(0)).$$

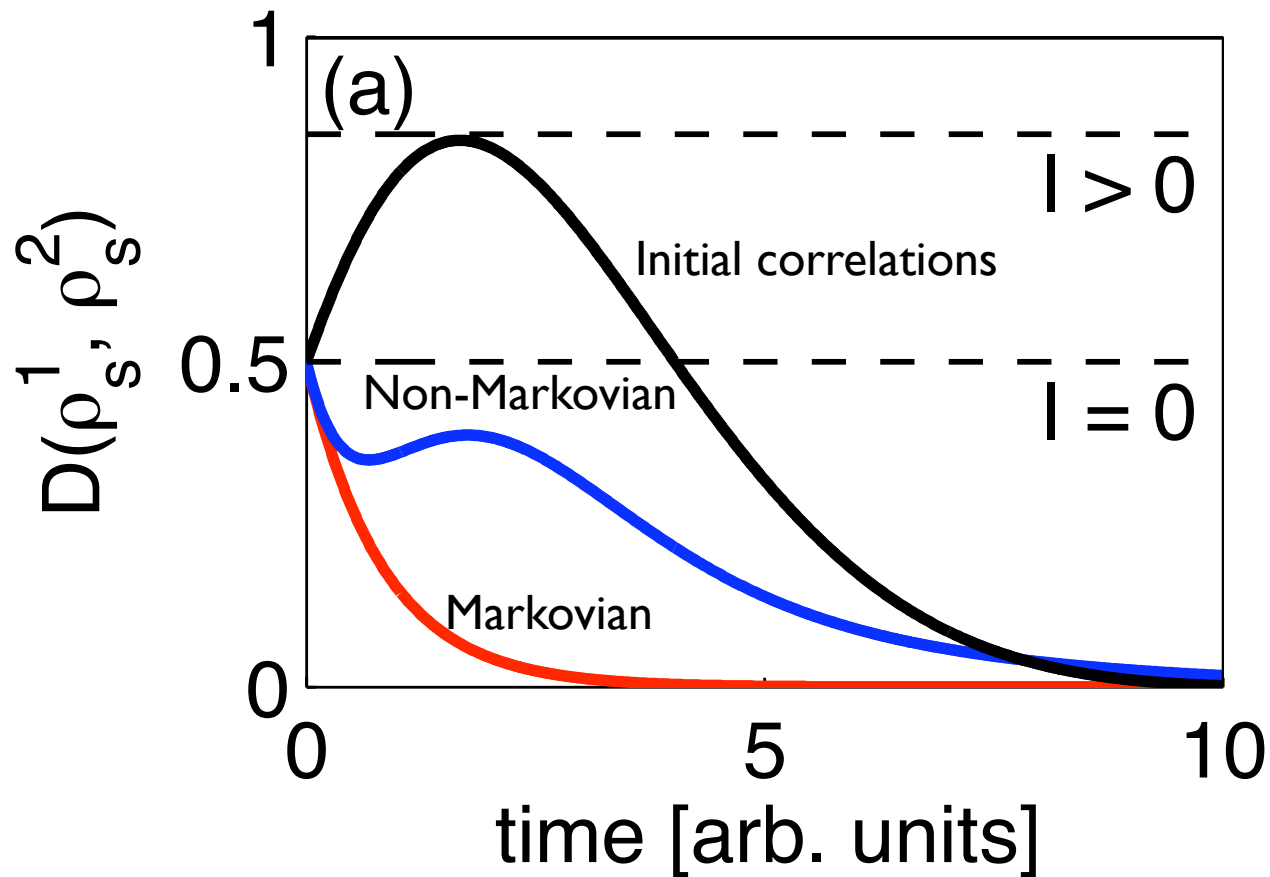
- Gives the total increase of the trace distance during the time evolution
- The total amount of information that has flown from the environment to the system during the time evolution.

General definition, independent of the used formalism to solve the open system dynamics*.



Markovian - non-Markovian - initial correlations

General classification based on the information flow

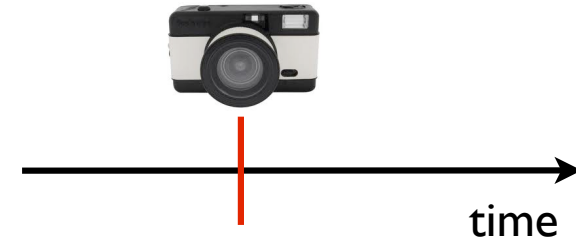




Other measures

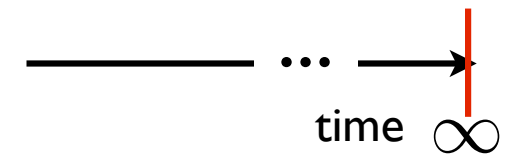
Wolf, Eisert, Cubitt, Cirac: PRL 2008

- Markovianity vs non-Markovianity from a snapshot of the evolution.



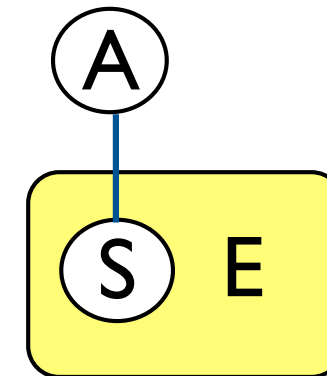
Chruscinski, Kossakowski, Pascazio: PRA 2010

- Asymptotic state dependence from initial conditions



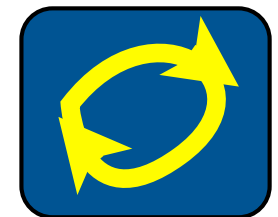
Rivas, Huelga, Plenio: PRL 2010

- Entanglement based measure (ancilla, system)
- Non-divisibility based measure



Our view: PRL 2009:

- Recycling of info between S and E, quantify backflow
- Memory is a feature of a system dynamics which has physical origin instead of being a mathematical property of the equation of motion





$$\frac{d\rho(t)}{dt} = -i\frac{\epsilon(t)}{2} [\sigma_z, \rho] + \frac{\gamma(t)}{2} (\sigma_z \rho \sigma_z - \rho)$$

- Time evolution and the map: two state system $|H\rangle, |V\rangle$

$$\rho_{H,H}(t) = \rho_{H,H}(0), \quad \rho_{V,V}(t) = \rho_{V,V}(0),$$

$$\rho_{H,V}(t) = \kappa^*(t)\rho_{H,V}(0), \quad \rho_{V,H}(t) = \kappa(t)\rho_{V,H}(0)$$

- Connection between the decoherence function $\kappa(t)$ and the rates in the master equation

$$\kappa(t) = \exp \left(- \int_0^t \gamma(t') + i\epsilon(t') dt' \right)$$

$$\epsilon(t) = -\Im \left[\frac{\dot{\kappa}(t)}{\kappa(t)} \right], \quad \gamma(t) = -\Re \left[\frac{\dot{\kappa}(t)}{\kappa(t)} \right]$$



Single photons in dephasing reservoir

- The system: polarization states of the photon: $|H\rangle$ $|V\rangle$

- Initial open system states

$$|\varphi_{1,2}\rangle = \frac{1}{\sqrt{2}} (|H\rangle \pm |V\rangle)$$

- Environment: frequency degrees of freedom $\{|\omega\rangle\}_{\omega \in \mathbb{R}}$ $|\omega_i\rangle$

- Initial environmental states $|\chi\rangle = \int d\omega \underbrace{f(\omega)}_{\text{Modified by the FP cavity}} |\omega\rangle$

Modified by the FP cavity

- Initial total system states

$$|\psi_{1,2}(0)\rangle = |\varphi_{1,2}\rangle \otimes |\chi\rangle$$

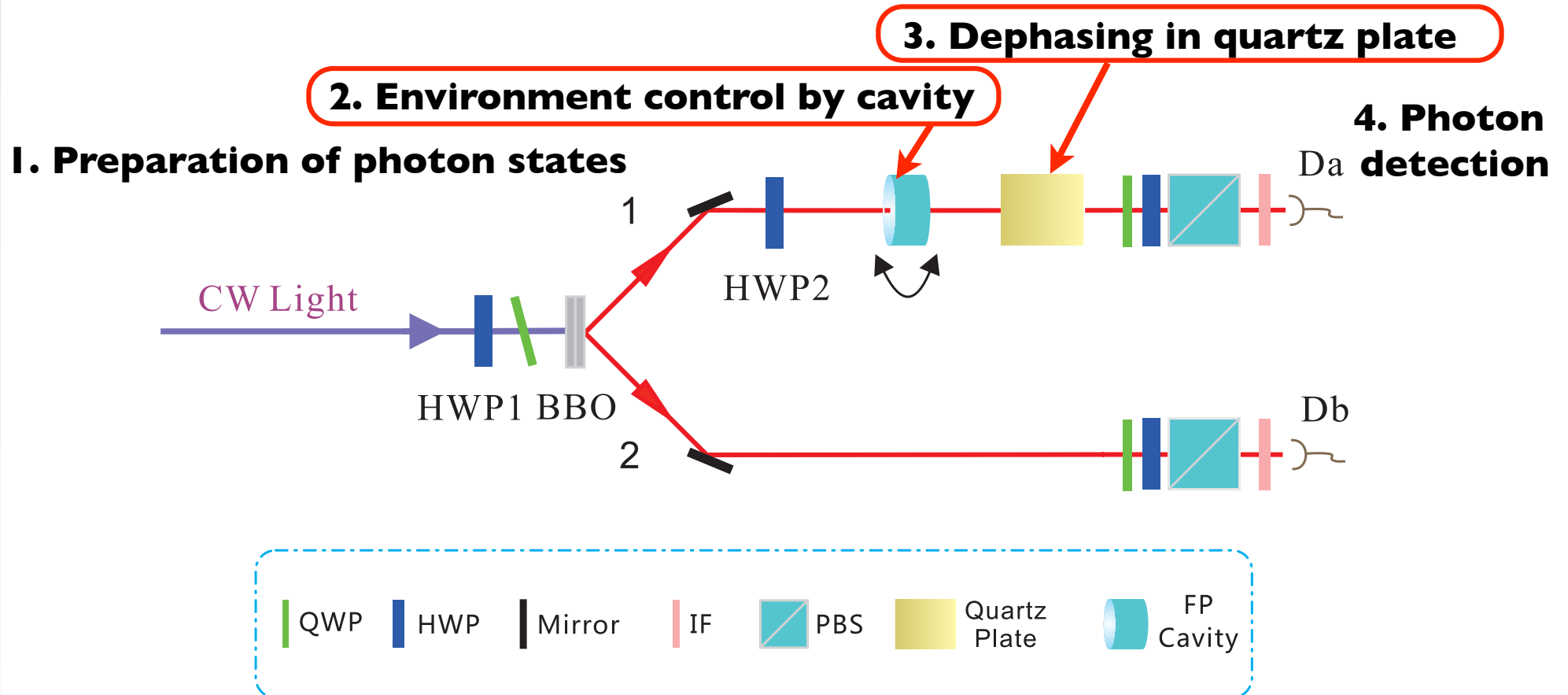
- Total system evolution in the quartz plate

$$U(t)|\lambda\rangle \otimes |\omega\rangle = e^{in_\lambda \omega t} |\lambda\rangle \otimes |\omega\rangle$$

- H and V acquire different phase due to the different refraction indices (birefringent quartz plate)



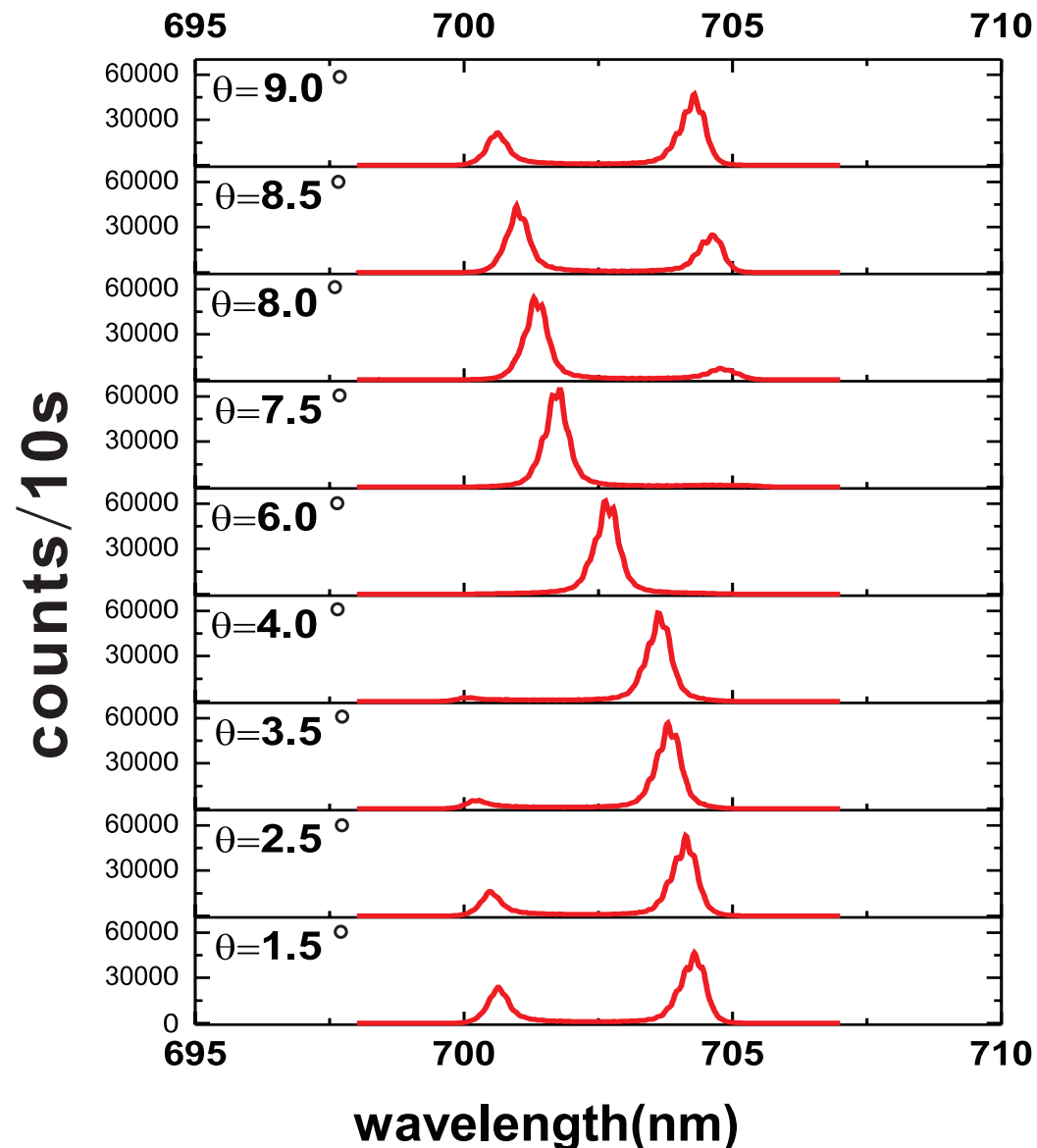
Experimental setup



- The tilting of the FP cavity modifies the frequency spectrum.



Environment control - frequency spectrum



Tilting of the cavity modifies the initial environmental state



Non-Markovianity

- The optimal trace distance $D(\rho_1(t), \rho_2(t)) = |\kappa(t)|$.

$$D(\rho_1(t), \rho_2(t)) = \sqrt{a^2 + |\kappa(t)b|^2}$$

$$b = \rho_1^{12}(0) - \rho_2^{12}(0)$$

$$a = \rho_1^{11}(0) - \rho_2^{11}(0)$$

- Two Gaussian peaks, relative weights

$$A_1 = \frac{1}{1+A}, \quad A_2 = \frac{A}{1+A}$$

- Decoherence function

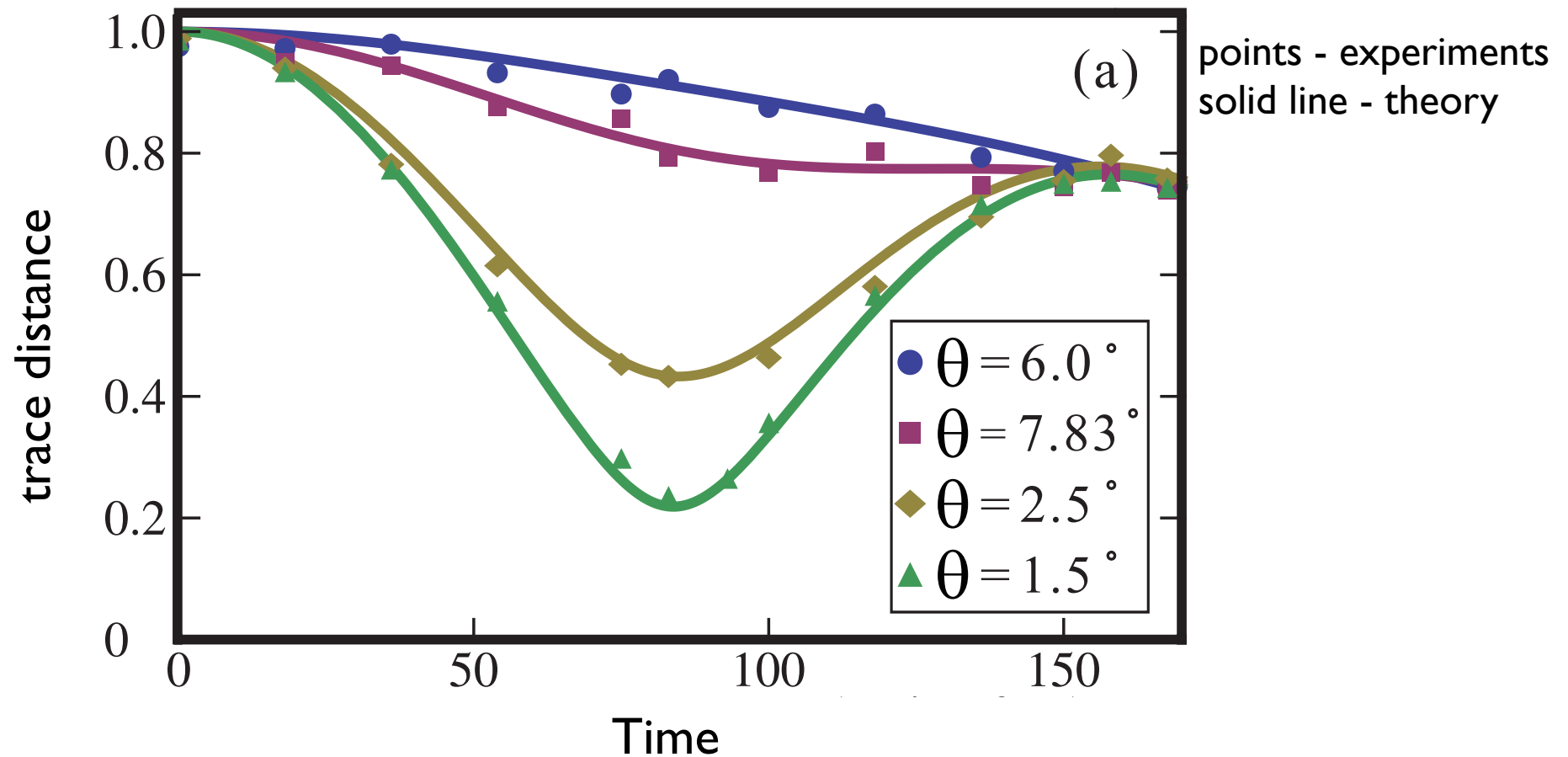
$$|\kappa(t)| = \frac{e^{-\frac{1}{2}\sigma^2(\Delta nt)^2}}{1+A} \sqrt{1 + A^2 + 2A \cos(\Delta\omega \cdot \Delta nt)}$$

- One peak $A = 0$

$$|\kappa(t)| \quad \text{monotonically decreasing}$$

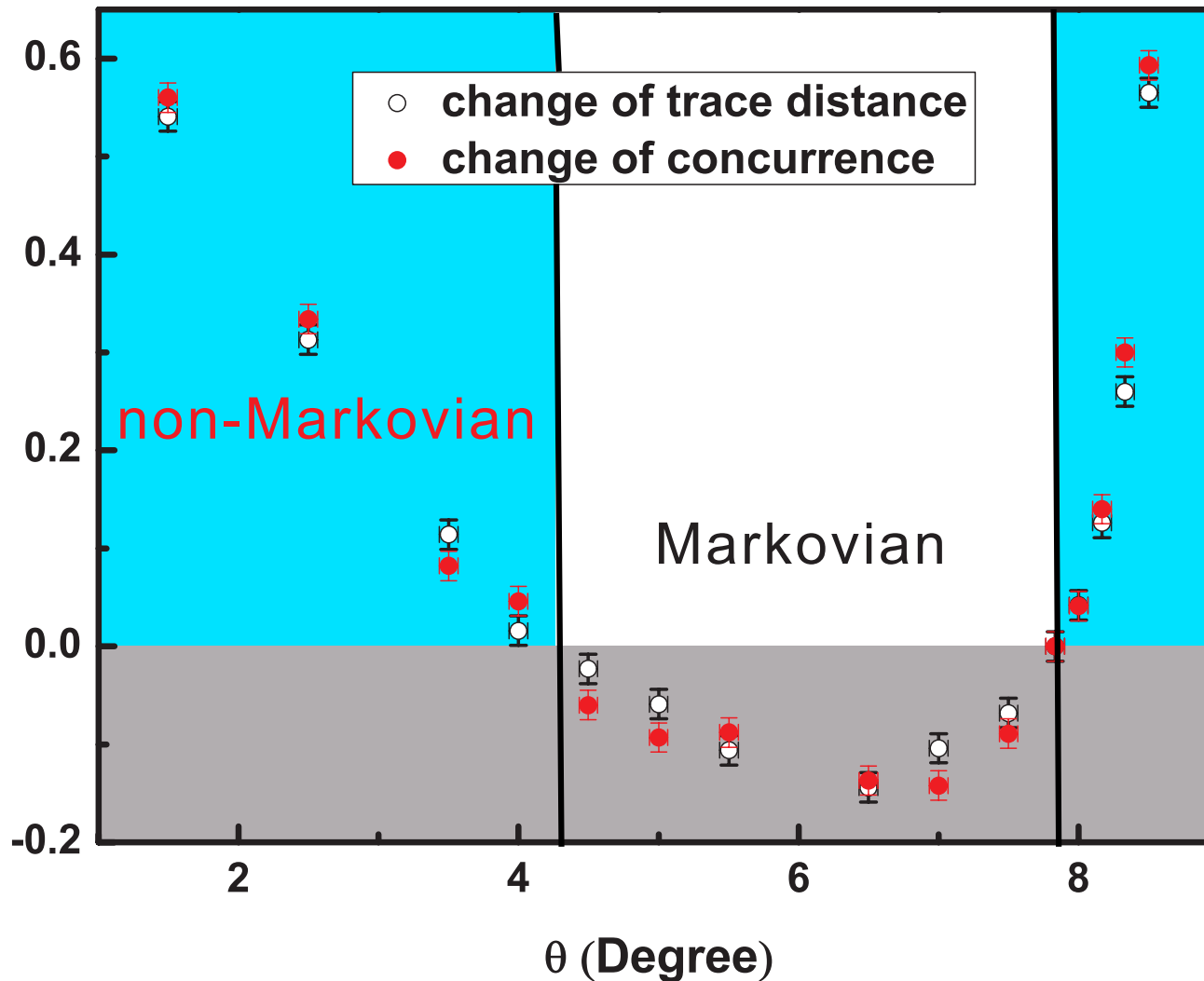


Trace distance dynamics - transition from monotonic to non-monotonic behavior





Markovian - non-Markovian transition



Experimental control on the amount and direction of the information flow between the system and the environment.



3. Nonlocal memory effects

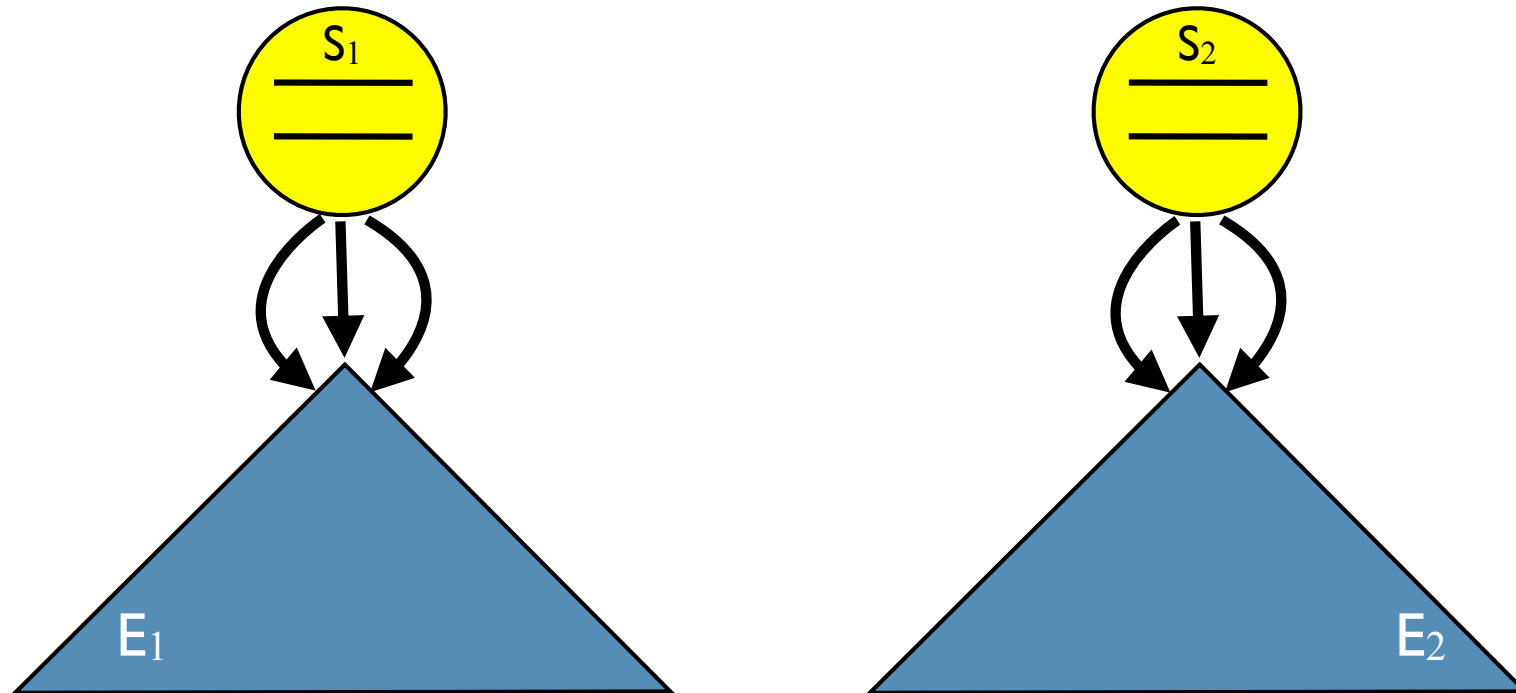
Laine, Breuer, Piilo, C.-F. Li, Guo,
Phys. Rev. Lett. 2012

Liu, Cao, Huang, C.-F. Li, Guo, Laine, Breuer, Piilo
"Photonic realization of nonlocal memory effects and non-Markovian quantum probes",
submitted for publication



Cartoon of the system

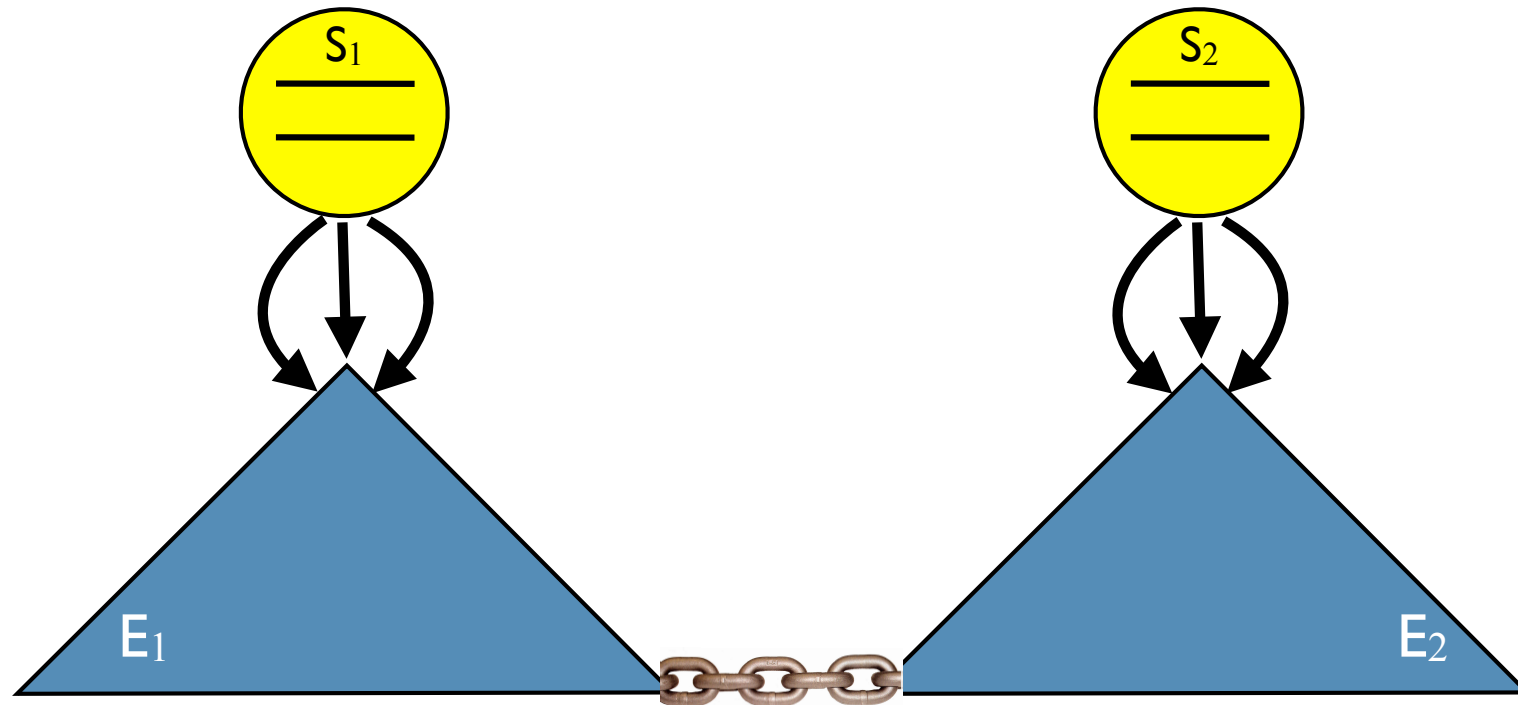
2-qubits interacting with their local environments





Cartoon of the system

2-qubits interacting with their local environments



What happens when the environments are initially correlated?

$$\begin{aligned}\rho_S^{12}(t) &= \Phi_{12}(t)(\rho_S^{12}(0)) \\ &= \text{tr}_E \left[(U_1(t) \otimes U_2(t)) \rho_S^{12}(0) \otimes \rho_E^{12}(0) (U_1^\dagger(t) \otimes U_2^\dagger(t)) \right]\end{aligned}$$



- Considered open system: two photons
 - polarizations H,V
 - initial pure state

$$|\psi_{12}\rangle = a|HH\rangle + b|HV\rangle + c|VH\rangle + d|VV\rangle$$

- Environment: frequency degrees of freedom

$$|\chi\rangle = \int d\omega_1 d\omega_2 g(\omega_1, \omega_2) |\omega_1, \omega_2\rangle$$

- joint probability distribution

$$P(\omega_1, \omega_2) = |g(\omega_1, \omega_2)|^2$$

- Initial system-environment product state

$$|\Psi(0)\rangle = |\psi_{12}\rangle \otimes \int d\omega_1 d\omega_2 g(\omega_1, \omega_2) |\omega_1, \omega_2\rangle$$



Nonlocal memory effects

- General dephasing for 2 qubits (photons, polarization)

$$\rho_S^{12}(t) = \begin{pmatrix} |a|^2 & ab^* \kappa_2(t) & ac^* \kappa_1(t) & ad^* \kappa_{12}(t) \\ ba^* \kappa_2^*(t) & |b|^2 & bc^* \Lambda_{12}(t) & bd^* \kappa_1(t) \\ ca^* \kappa_1^*(t) & cb^* \Lambda_{12}^*(t) & |c|^2 & cd^* \kappa_2(t) \\ da^* \kappa_{12}^*(t) & db^* \kappa_1^*(t) & dc^* \kappa_2^*(t) & |d|^2 \end{pmatrix}$$

- Local states for qubit 1 and 2 [trace out qubit 2 (I)]

$$\rho_1(t) = \begin{pmatrix} |a|^2 + |b|^2 & (ac^* + bd^*) \kappa_1(t) \\ (ca^* + db^*) \kappa_1^*(t) & |c|^2 + |d|^2 \end{pmatrix}$$

$$\rho_2(t) = \begin{pmatrix} |a|^2 + |c|^2 & (ab^* + cd^*) \kappa_2(t) \\ (ba^* + dc^*) \kappa_2^*(t) & |b|^2 + |d|^2 \end{pmatrix}$$

- Global** 2-qubit decoherence functions: $\kappa_{12}(t), \Lambda_{12}(t)$

- Local** decoherence functions: $\kappa_1(t), \kappa_2(t)$



Nonlocal memory effects

- ...however, the open system map is a product of local maps

$$\Phi_{12}(t) = \Phi_1(t) \otimes \Phi_2(t).$$

if and only if $\kappa_{12}(t) = \kappa_1(t)\kappa_2(t)$ $\Lambda_{12}(t) = \kappa_1(t)\kappa_2^*(t)$

- Local interaction Hamiltonian for photon i

$$H_i = - \int d\omega_i \omega_i [n_V |V\rangle\langle V| + n_H |H\rangle\langle H|] \otimes |\omega_i\rangle\langle\omega_i|.$$

$$H_{\text{int}}(t) = \chi_1(t)H_1 + \chi_2(t)H_2.$$

- Decoherence functions

$$\kappa_1(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n \omega_1 t_1}$$

$$\kappa_2(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n \omega_2 t_2}$$

$$\kappa_{12}(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n (\omega_1 t_1 + \omega_2 t_2)}$$

$$\Lambda_{12}(t) = \int d\omega_1 d\omega_2 P(\omega_1, \omega_2) e^{-i\Delta n (\omega_1 t_1 - \omega_2 t_2)}$$



Frequency correlations give
nonlocal map and
non-Markovian dynamics



Nonlocal memory effects

- Consider
 - Gaussian frequency distribution $P(\omega_1, \omega_2)$
 - Variance C
 - Correlation coefficient K
 - initial open system states

$$|\psi_{12}^{\pm}\rangle = (|HH\rangle \pm |VV\rangle) / \sqrt{2}$$

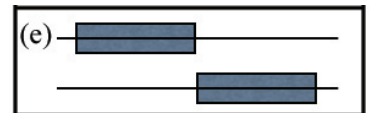
- ...trace distance dynamics of the open system

$$D(t) = \exp \left[-\frac{1}{2} \Delta n^2 C (t_1^2 + t_2^2 - 2|K|t_1 t_2) \right].$$

- ...and the non-Markovianity measure is

$$\mathcal{N} = e^{-\frac{1}{2} C_{11} (\Delta n T)^2} \left[e^{\frac{1}{2} C_{11} (\Delta n T)^2 K^2} - 1 \right]$$

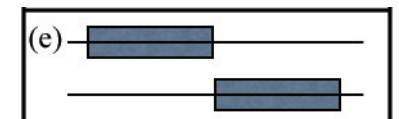
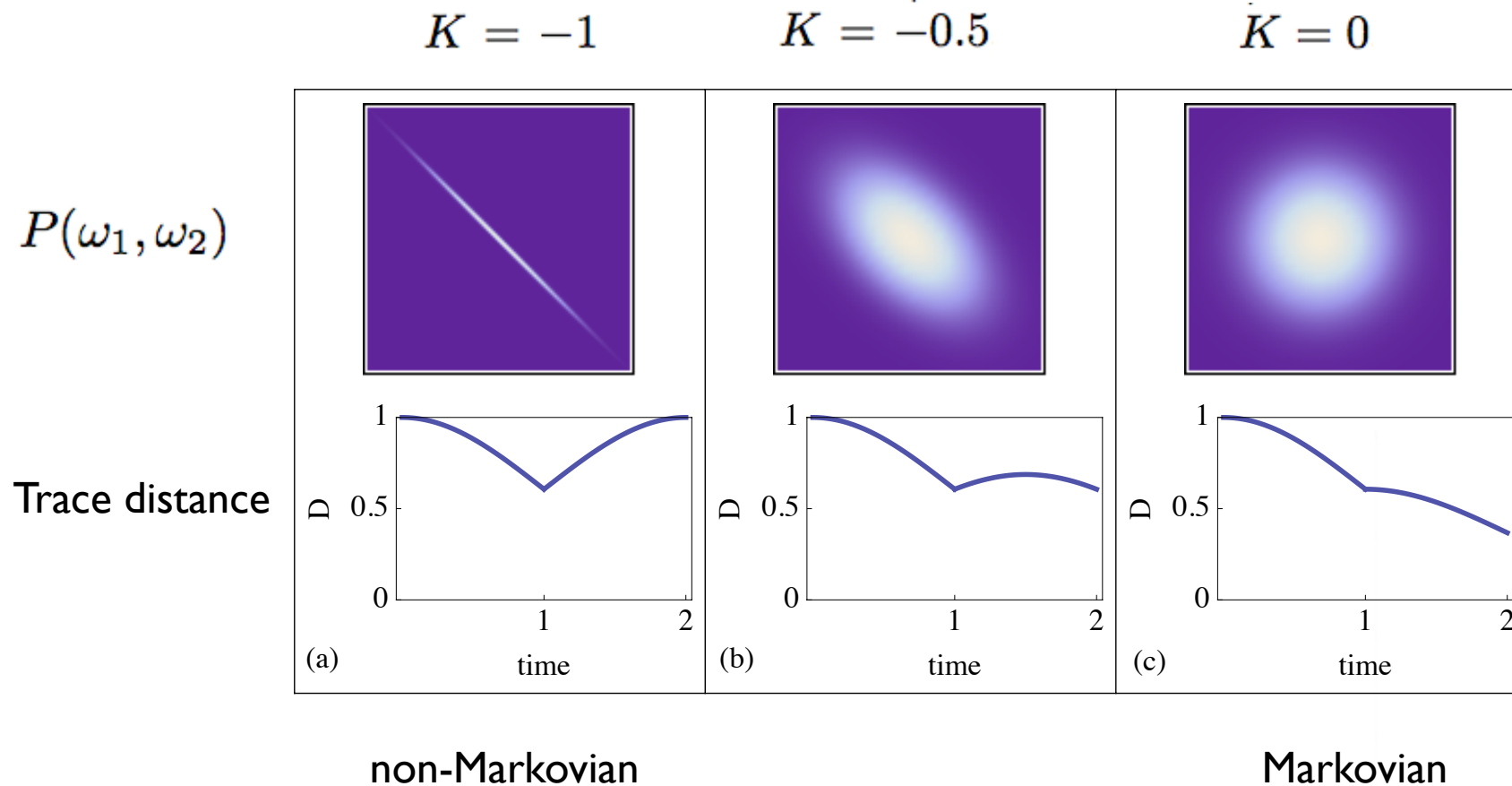
Direct connection between the amount of non-Markovianity of the open system and the correlations between the local environments





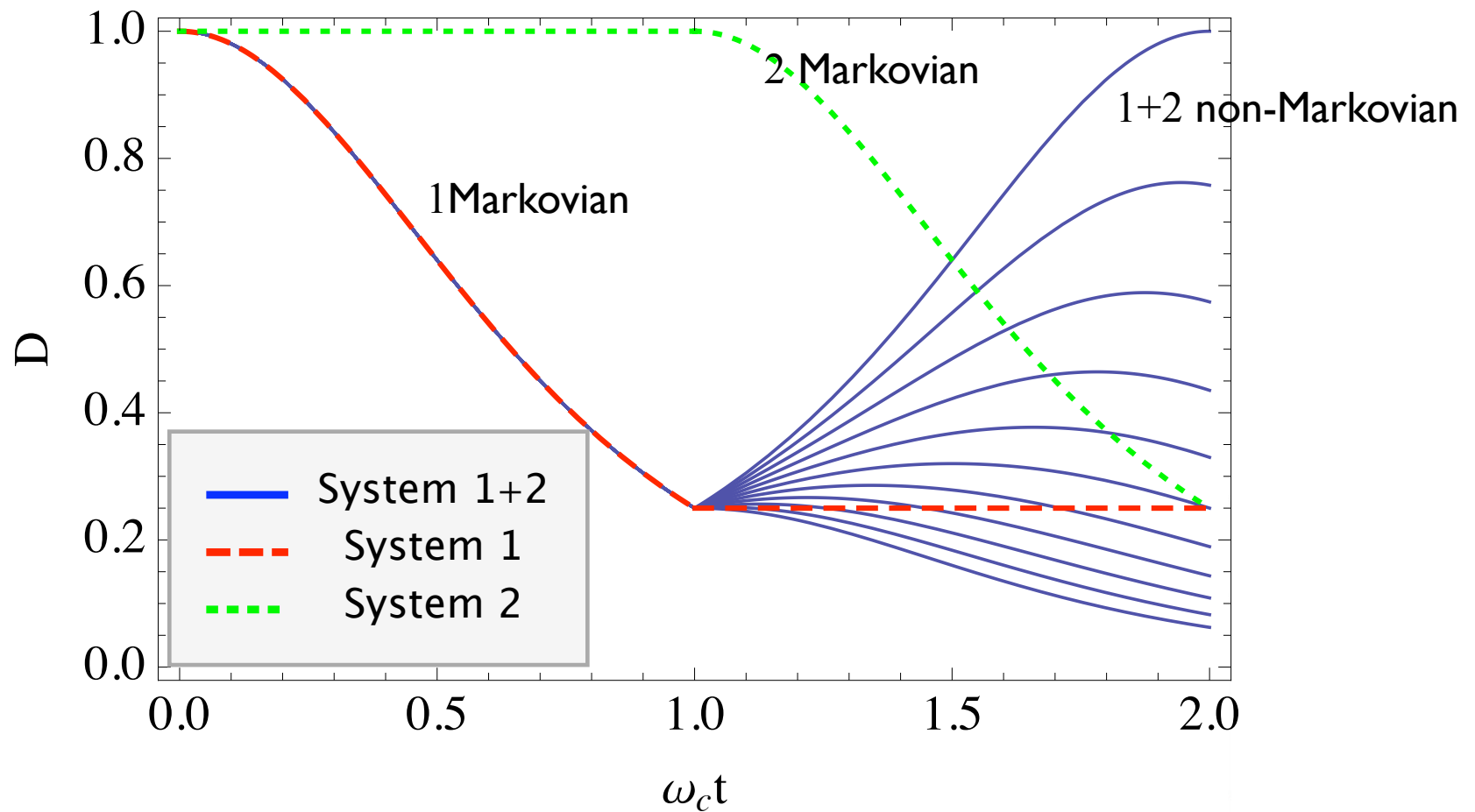
Nonlocal memory effects

Anticorrelated frequency distributions and non-Markovianity





Local Markovian dynamics - global non-Markovian dynamics





Conclusions

1: **Non-Markovian quantum jumps:**

Recreation of lost superpositions due to the memory effects

2: **Quantifying and controlling non-Markovianity:**

- trace distance based measure, information flow
- reservoir engineering and Markovian to non-Markovian transition

3: **Nonlocal memory effects:**

- nonlocal map by local interaction Hamiltonian
- correlations between local environments: a new source of non-Markovianity
- application: non-Markovian quantum probe: frequency correlation of two photons measured by their polarization state

- **Memory multifaceted phenomenon, stimulating recent theoretical progress by several groups**
- **Rigorous experimental tests becoming feasible**
- **Non-Markovian probes, as a resource, new diagnostic tools...**



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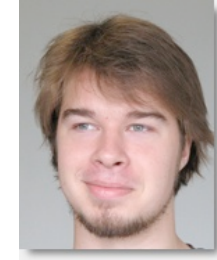
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