Optical interferometry in the presence of large phase diffusion

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theory → M. G. Genoni, M. G. A. Paris,

experiment → D. Brivio, S. Cialdi, D. Cipriani, A. Santamato, S. Vezzoli

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LiCHIS

Light Correlations for
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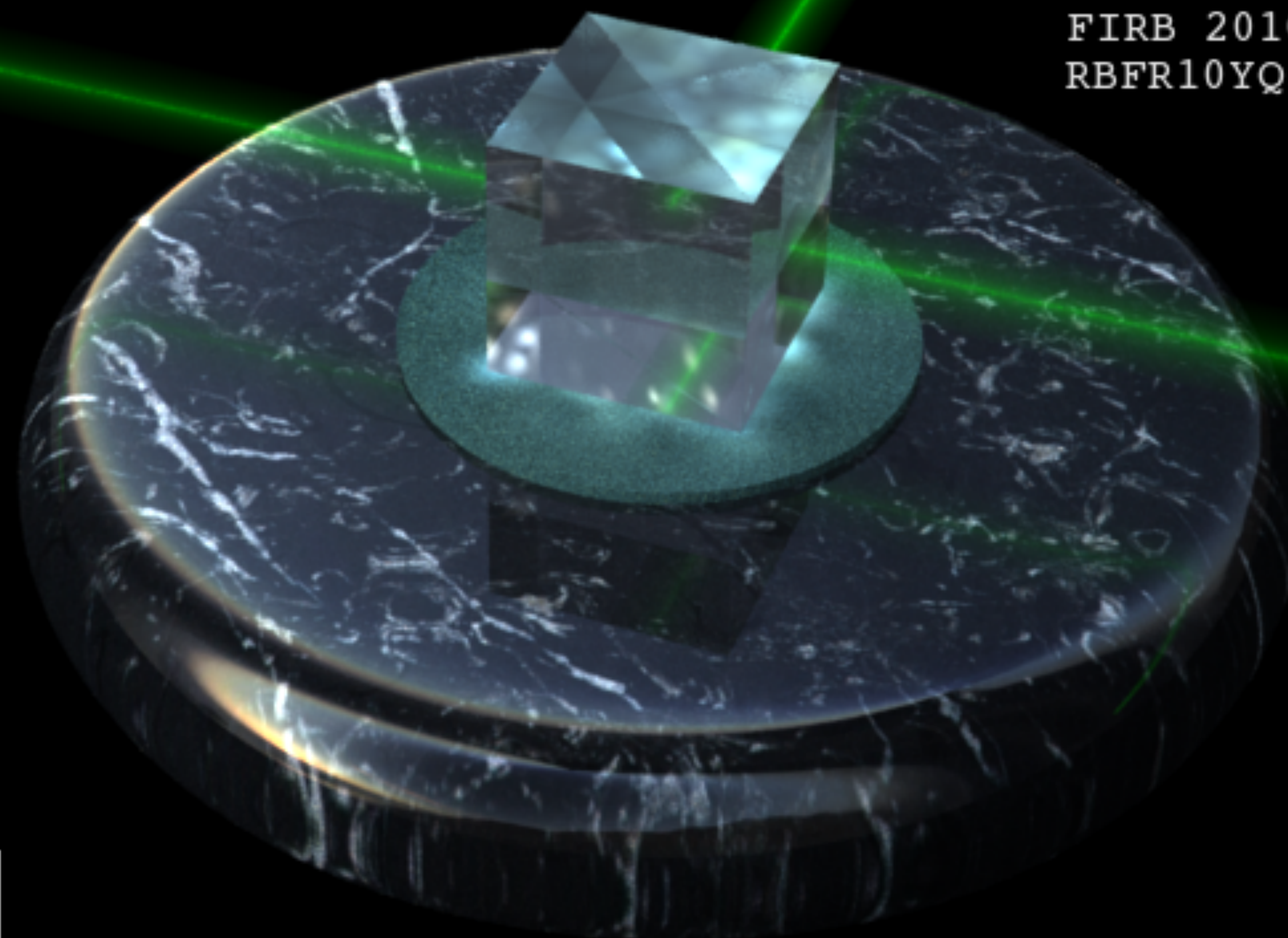

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Outline

一 Introduction

- *Motivations*
- *Review of classical & quantum estimation theory*

二 Optical phase estimation

- *With Gaussian states and in the presence of phase diffusion*

三 Experimental results

- *Phase estimation in the presence of large phase diffusion*
- *Bayesian strategy and optimality*

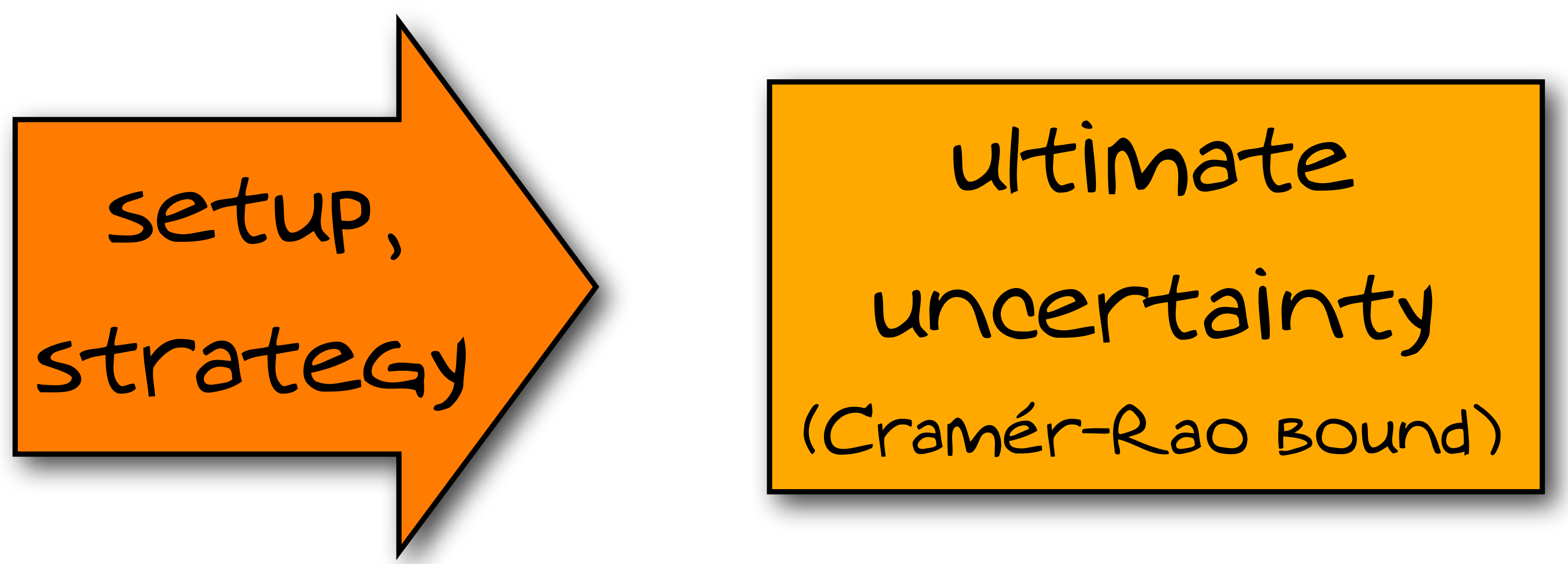
四 Concluding remarks

Motivations

- *Precise estimation of an optical phase shift is relevant for optical communication schemes where information is encoded in the phase of traveling pulses.*

- *In realistic conditions, one has to retrieve phase information that has been unavoidably degraded by different sources of noise.*

- *Phase-diffusive noise is the most detrimental for interferometry, and any signal that is unaffected by phase-diffusion is also invariant under a phase shift and thus totally useless for phase estimation.*



Single parameter estimation

When a physical parameter ϕ (e.g., phase, entropy, entanglement,...) is not directly accessible, one has to resort to indirect measurements.

$$\chi = (x_1, x_2, \ldots x_M)$$

sample

$$\hat{\phi} = \hat{\phi}(\chi)$$

estimator

minimize uncertainty ***reach minimum uncertainty***

Goals: optimal probe states & measurements

Classical estimation

Classically, optimal unbiased estimators saturate the Cramér-Rao inequality:

$$\mathrm{Var}(\phi) = E[\hat{\phi}^2] - E[\hat{\phi}]^2$$

$$\mathrm{Var}(\phi) \geq \frac{1}{M\, F(\phi)}$$

M is the statistical scaling due to the M outcomes.

$$F(\phi) = \int_\Omega dx\, \frac{[\partial_\phi\, p(x|\phi)]^2}{p(x|\phi)}$$

Fisher Information

$$\chi = (x_1, x_2, \ldots x_M)$$

sample

$$\hat{\phi} = \hat{\phi}(\chi)$$

estimator

conditional probability

Quantum estimation

Starting from the Born rule: $p(x|\phi) = \mathrm{Tr}[\Pi_x\, \varrho_\phi]$

We have the FI: $$F(\phi) = \int_\Omega dx\, \frac{\Re\mathrm{e}\left(\mathrm{Tr}[\varrho_\phi\, \Pi_x\, L_\phi]\right)^2}{\mathrm{Tr}[\varrho_\phi\, \Pi_x]}$$

$$2\,\partial_\phi \varrho_\phi = L_\phi \varrho_\phi + \varrho_\phi L_\phi$$ *symmetric logarithmic derivative (SLD)*

Maximizing over all the possible measurements (POVMs), we obtain the quantum Cramér-Rao bound:

$$\mathrm{Var}(\phi) \geq \frac{1}{M\, H(\phi)}$$

$$H(\phi) = \mathrm{Tr}[\varrho_\phi\, L_\phi^2] \geq F(\phi)$$

Quantum Fisher Information

The eigenstates of SLD operator correspond to the optimal POVM!

Quantum estimation

If the probe state is a mixed state: $\varrho = \sum_n p_n |\psi_n\rangle\langle\psi_n|$ *statistical model*

and $\varrho_\phi = U_\phi\, \varrho\, U_\phi^\dagger$ *with* $U_\phi = \exp\{-i\phi\, G\}$

$$H = 2 \sum_{n\neq m} \frac{(p_n - p_m)^2}{p_n + p_m} |\langle\psi_n|G|\psi_m\rangle|^2$$

the QFI is independent of ϕ.

If the probe is a pure state:

$$\varrho = |\psi_0\rangle\langle\psi_0| \quad \Rightarrow \quad H = 4\langle\psi_0|\Delta G^2|\psi_0\rangle$$

namely, the QFI corresponds to the fluctuations of the generator G on the probe state and is independent on ϕ.

Optical phase estimation

In this case we have: $U_\phi = \exp\{-i\phi\, a^\dagger a\}$ *(phase shift)* $a^\dagger a|n\rangle = n|n\rangle$

Classical strategy *(the probe is a coherent state)*

$H \propto N$ *the QFI is proportional to the energy of probe state*

Quantum strategy

Bipartite entangled states & interferometric setup

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|N,0\rangle + |0,N\rangle\right) \quad \Rightarrow \quad H = N^2$$

P. Kok, *et al.*, PRA **63**, 063407 (2001)

NOON state *Heisenberg limit*

Phase est. with Gaussian states (GS)

Gaussian states (states with Gaussian Wigner function: coherent, thermal, squeezed states,...) play a major role in quantum information with continuous variables:

- *Generated and manipulated by means of linear and bilinear interactions of modes implemented in quantum optics laboratories.*

Pure single-mode GS:

$$D(\alpha)\,S(r)|0\rangle \qquad \beta = \frac{\sinh^2 r}{N}$$

squeezing fraction

squeezing photons: $\sinh^2 r$

total number of photons $N = \sinh^2 r + |\alpha|^2$

A. Monras, PRA **73**, 033821(2006)

$$S(r)|0\rangle \quad \Rightarrow \quad H = 8(N^2 + N)$$

Phase noise

Evolution of a light beam in a phase diffusing environment:

$$\dot{\varrho} = \Gamma \mathcal{L}[a^\dagger a]\varrho \qquad \mathcal{L}[O]\varrho = 2O\varrho O^\dagger - O^\dagger O \varrho - \varrho O^\dagger O$$

the solution reads:

$$\xi = N\,\Delta$$

$$\varrho(t) = \mathcal{N}_\Delta[\varrho(0)] = \sum_{n,m} e^{-\Delta^2 (n-m)^2} \varrho_{n,m}(0) |n\rangle\langle m|$$

$$\Delta = \Gamma t$$

or:

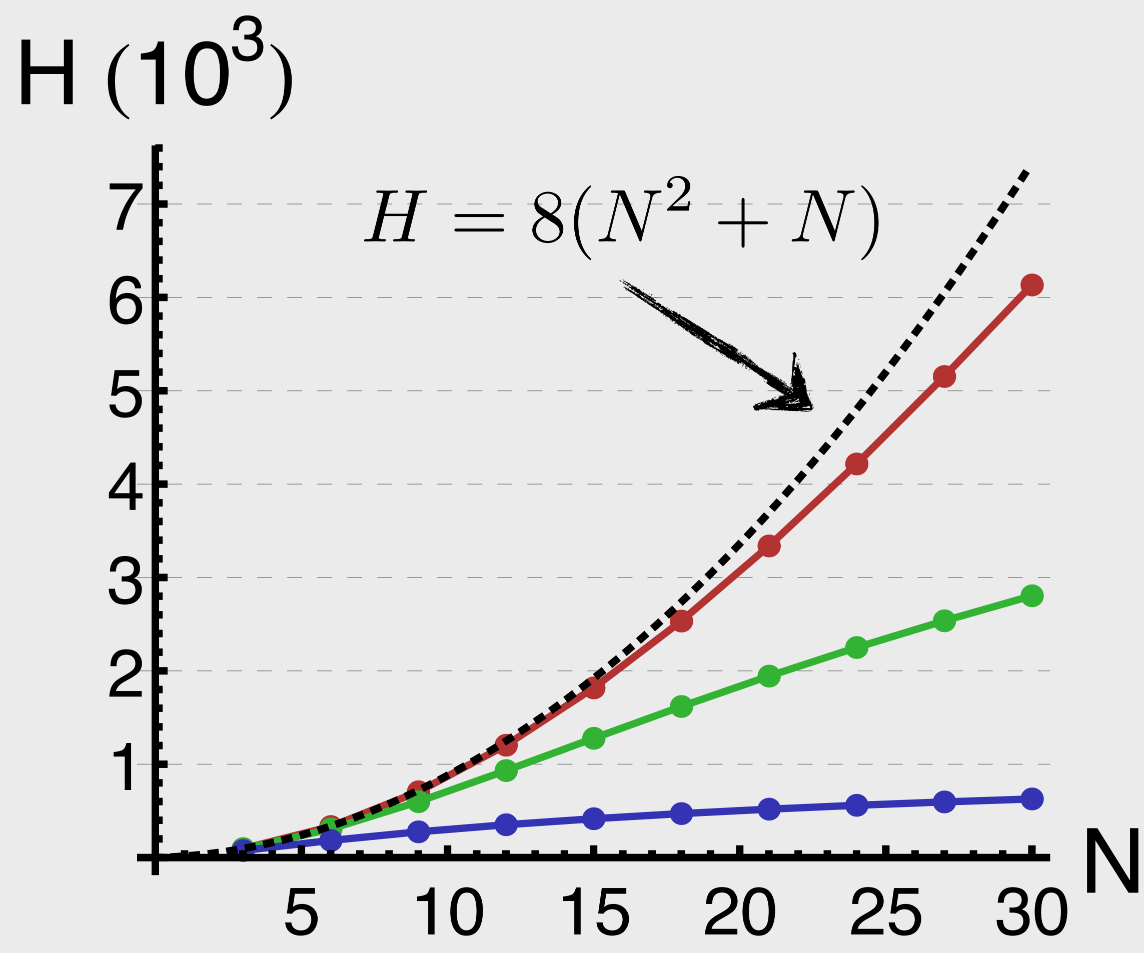
$$\mathcal{N}_\Delta[\varrho] = \int_{\mathbb{R}} d\varphi \, \frac{e^{-\varphi^2/(4\Delta^2)}}{\sqrt{4\pi\Delta^2}} U_\varphi \varrho U_\varphi^\dagger \qquad U_\beta = \exp(-i\varphi a^\dagger a)$$

Gaussian noise: useful representation for the experimental investigation...

Phase est., GS and phase noise

We evaluated numerically the QFI (H) for optimized pure Gaussian states.

- finding the minimum uncertainty -



QFI of optimized pure input Gaussian states

$\Delta^2 = 0$

$\Delta^2 = 5.0 \times 10^{-6}$

$\Delta^2 = 5.0 \times 10^{-5}$

$\Delta^2 = 5.0 \times 10^{-4}$

M. G. Genoni, S. Olivares and M. G.A. Paris, PRL **106**, 153603 (2011)

Phase est., GS and phase noise

We evaluated numerically the QFI (H) and investigated the role of

$$\xi = N\,\Delta$$

We observe that a law of scale holds:

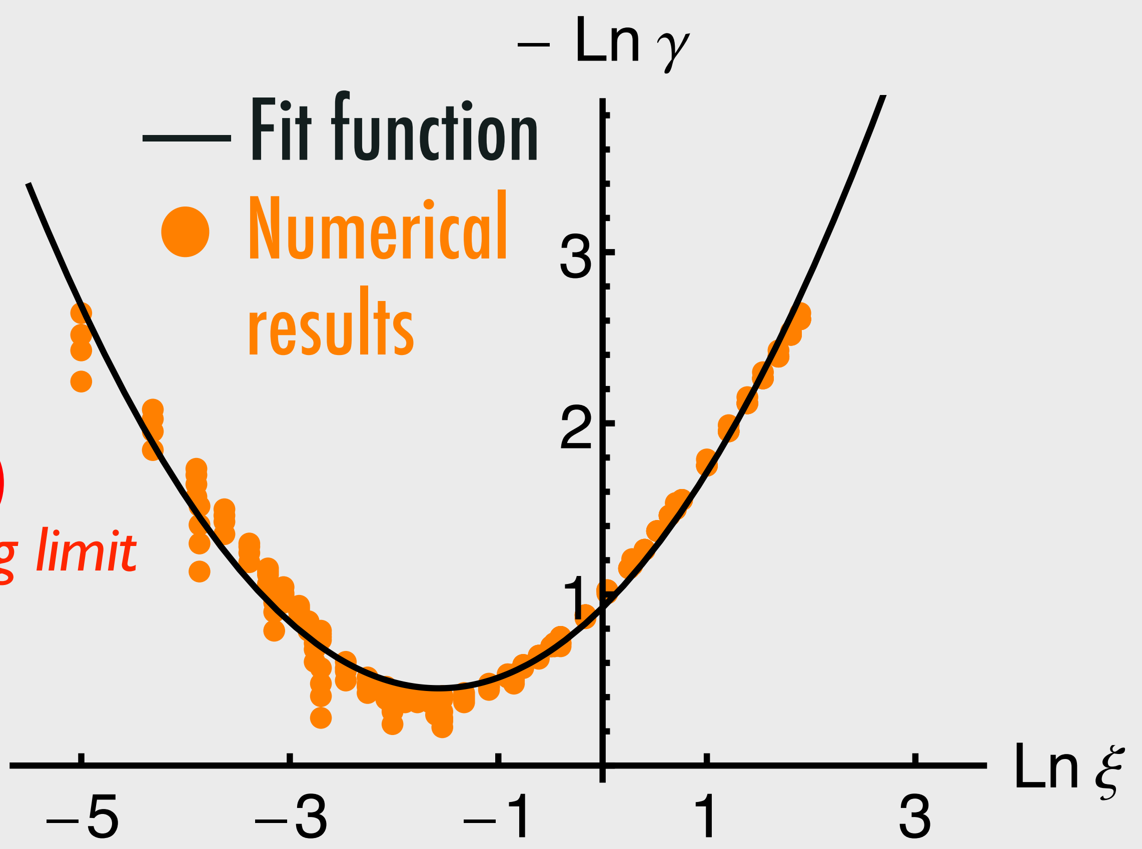
$N \leq 30, \quad 10^{-3} \leq \Delta \leq 1$

$$H(N,\Delta) \approx k^2 H(N/k, k\Delta)$$

that is

$$H(N,\Delta) \approx N^2 \frac{\gamma(\xi)}{\xi} \approx \mathcal{O}(1)$$

Heisenberg limit



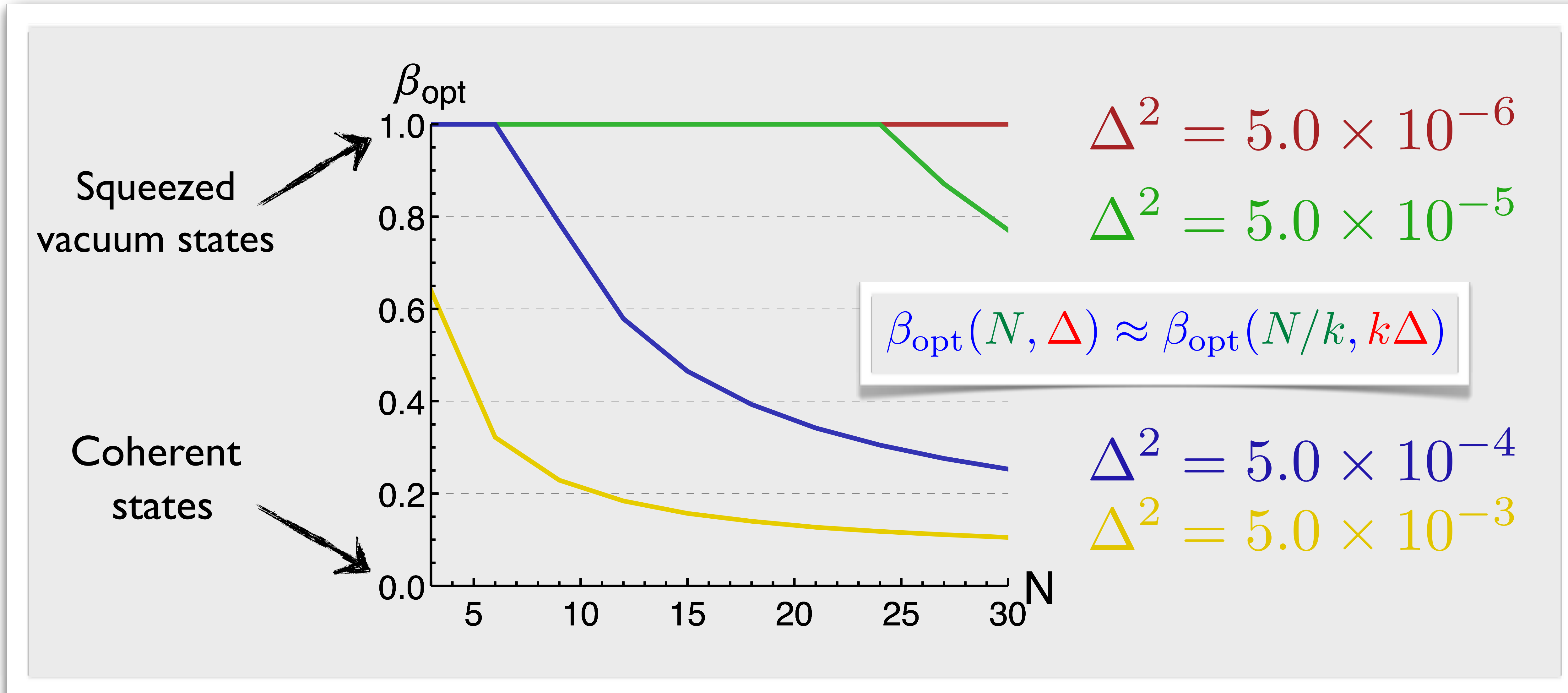
$$\gamma(\xi) \approx \xi^{-b} \exp(-a \ln^2 \xi)$$

universal fit function

M. G. Genoni, S. Olivares and M. G.A. Paris, PRL **106**, 153603 (2011)

Phase est., GS and phase noise

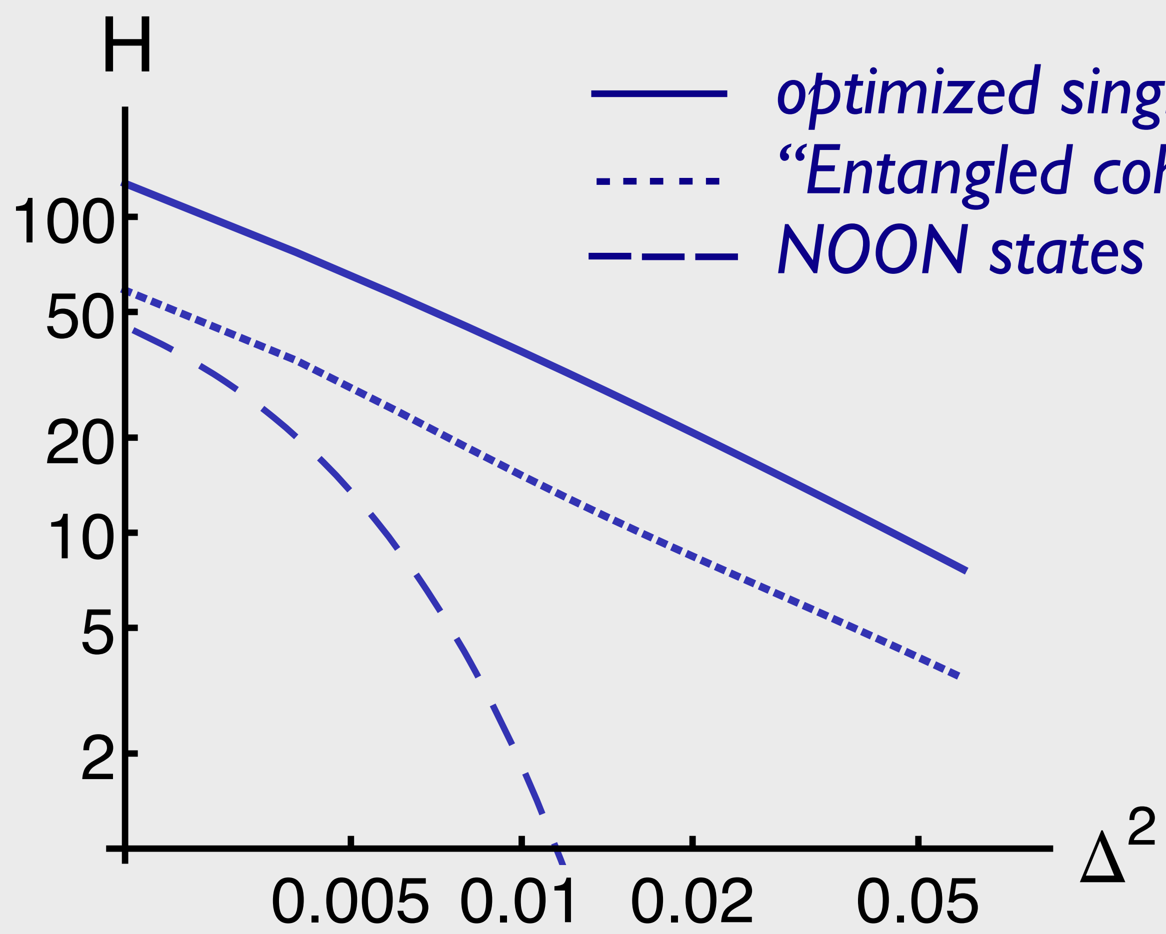
We evaluated numerically the QFI and optimized the squeezing fraction at fixed total number of photons N and at fixed noise Δ.



M. G. Genoni, S. Olivares and M. G.A. Paris, PRL **106**, 153603 (2011)

Phase est., GS and phase noise

Comparison between optimized single-mode GS and entangled probe states.



At fixed number of photons single-mode Gaussian states are the more robust against phase noise!

M. G. Genoni, S. Olivares and M. G. A. Paris, PRL **106**, 153603 (2011)

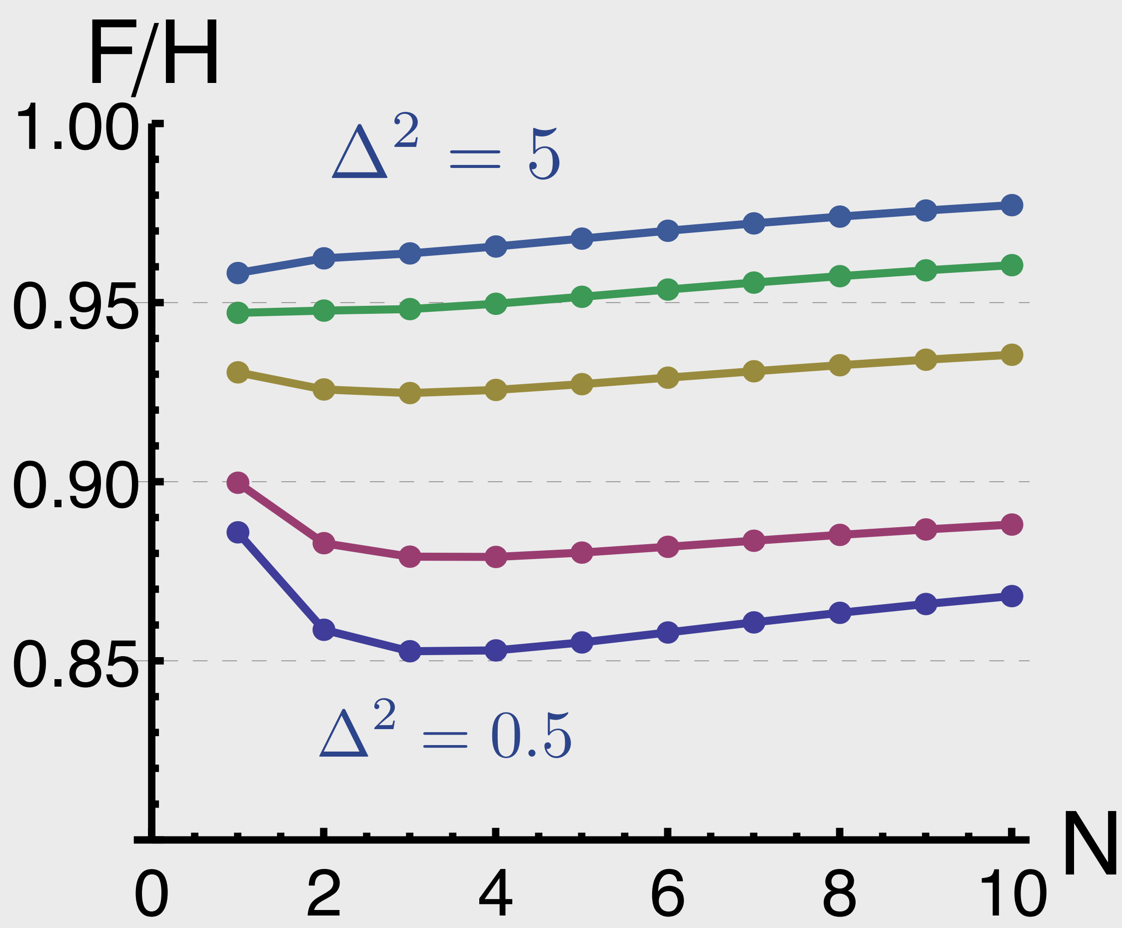
Phase est., GS and phase noise

... and the optimal measurement? Homodyne detection?

M. G. Genoni, S. Olivares and M. G. A. Paris, PRL **106**, 153603 (2011)

Phase est., GS and phase noise

... and the optimal measurement? Homodyne detection?
Ratio between homodyne FI with coherent probe states and (optimized) QFI.

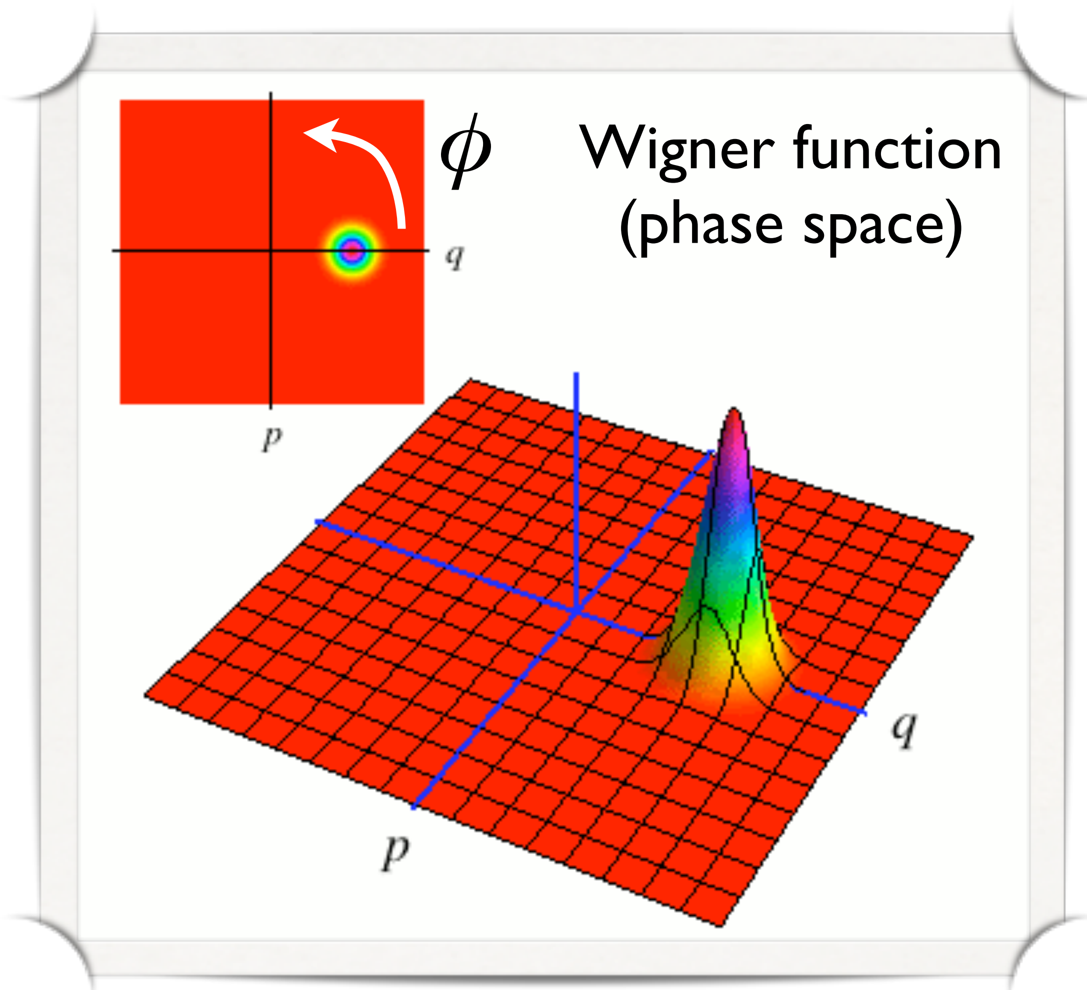


As the noise increases ($\Delta \gtrsim 1$) homodyne detection with coherent states is nearly optimal!

For small amount of noise ($\Delta \ll 1$) homodyne detection with squeezed vacuum states is the nearly optimal strategy.

M. G. Genoni, S. Olivares and M. G.A. Paris, PRL **106**, 153603 (2011)

Phase diffusion and coherent states



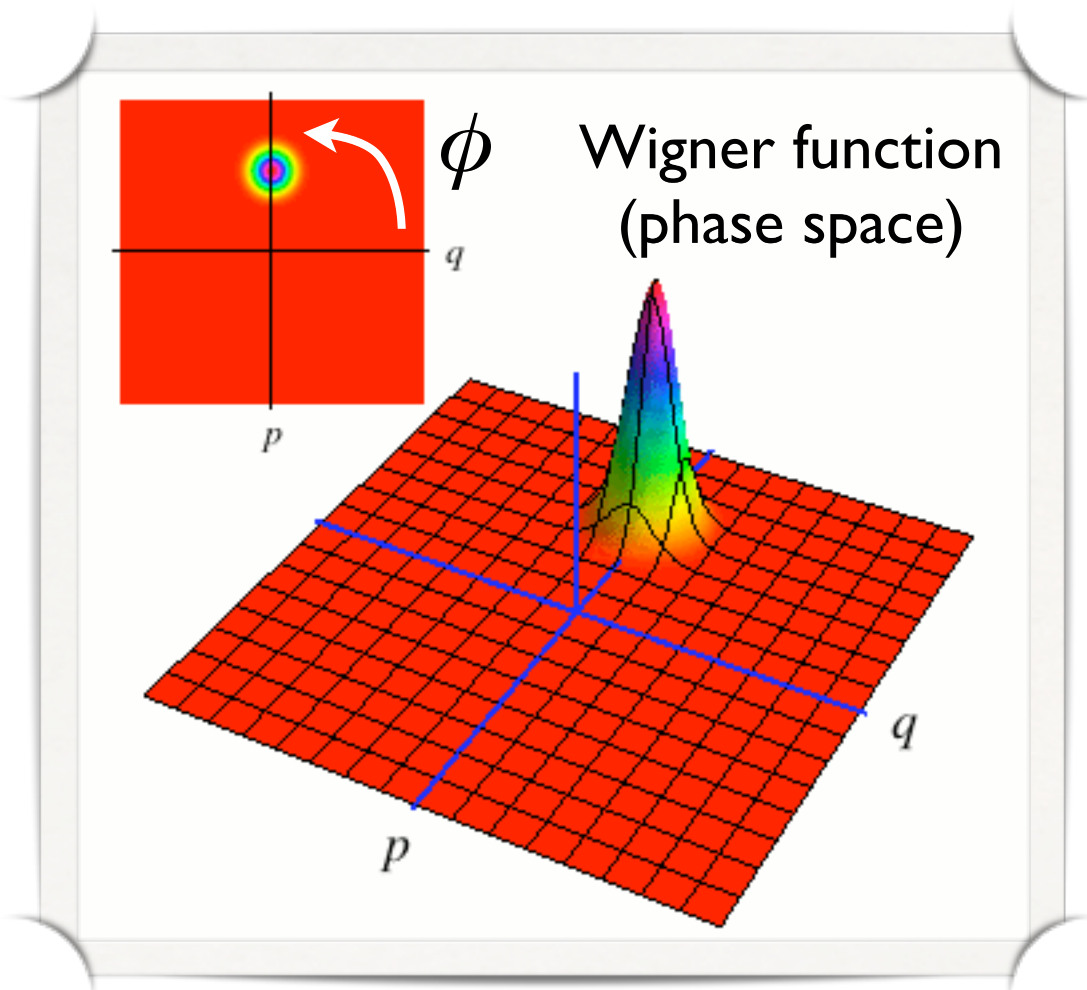
Coherent state:

$$|\alpha\rangle \qquad \alpha \in \mathbb{R}$$

Phase shift:

$$e^{-i\phi\, a^\dagger a}\,|\alpha\rangle = |\alpha\, e^{-i\phi}\rangle$$

Phase diffusion and coherent states



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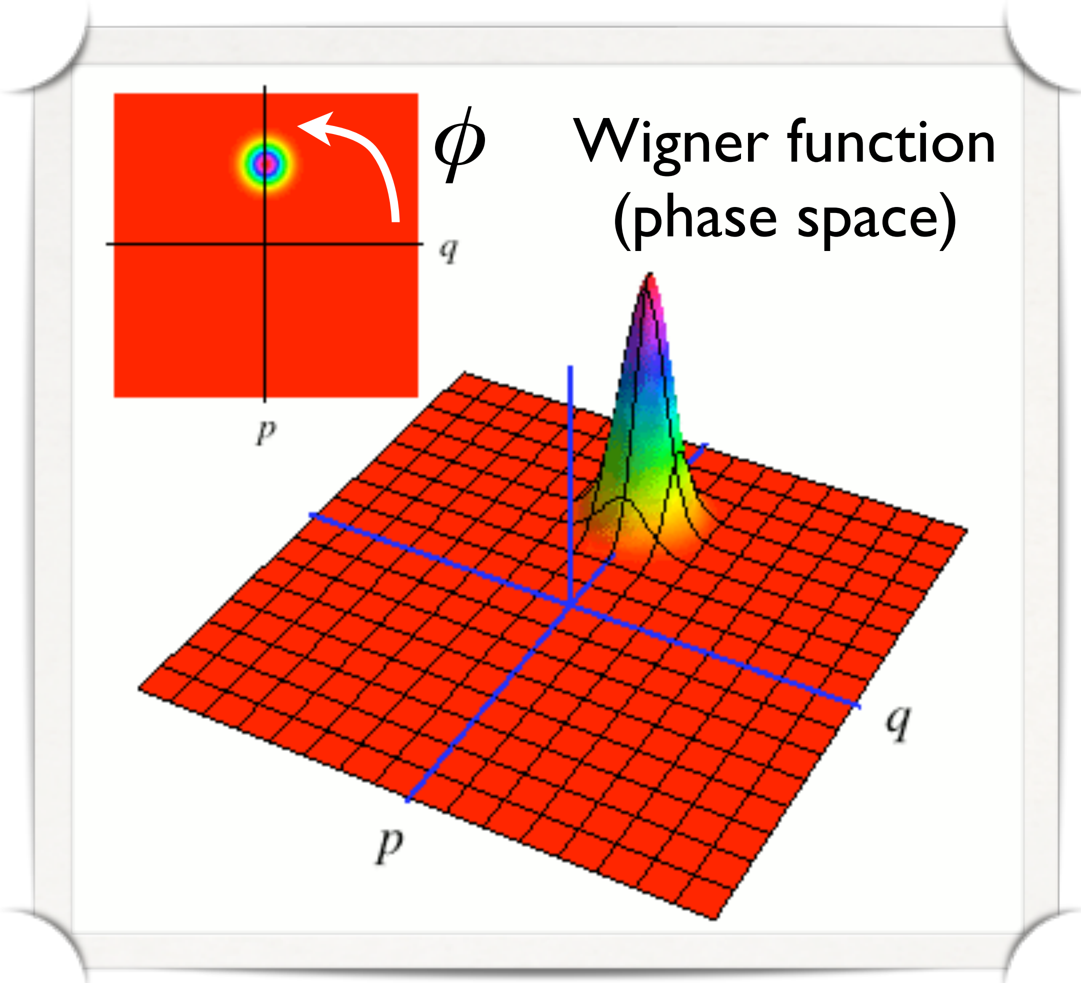

Coherent state:

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Phase shift:

$$e^{-i\phi\, a^\dagger a}\,|\alpha\rangle = |\alpha\, e^{-i\phi}\rangle$$

Phase diffusion:

$$\int_{\mathbb{R}} d\varphi\, \frac{e^{-\varphi^2/(4\Delta^2)}}{\sqrt{4\pi\Delta^2}}\, U_\varphi\, \varrho_\phi\, U_\varphi^\dagger$$

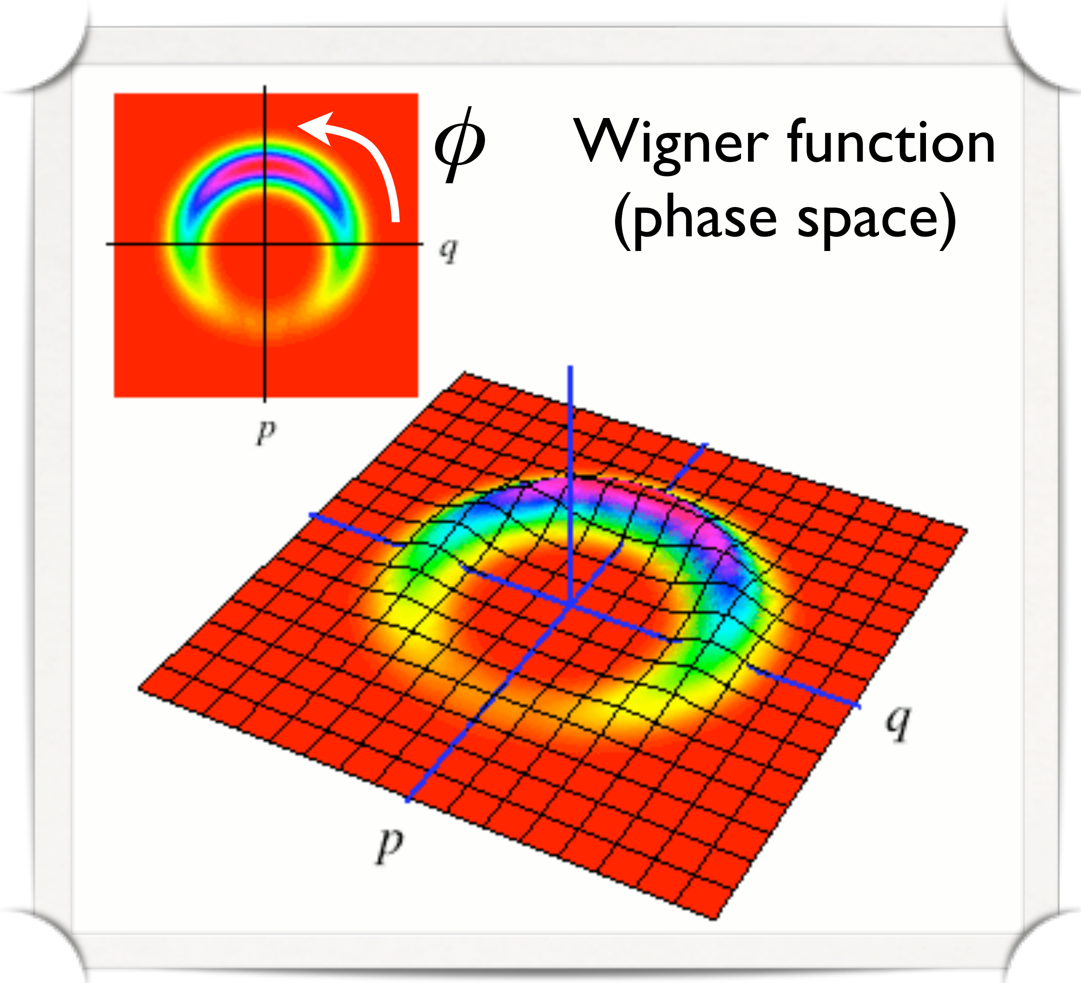
$$x_\theta = \frac{a^\dagger e^{i\theta} + a\, e^{-i\theta}}{2}$$

Homodyne detection

$$\chi = (x_1, x_2, \ldots x_M)$$

Data sample

Phase diffusion and coherent states



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Homodyne detection

$$\chi = (x_1, x_2, \ldots x_M)$$

Data sample

Bayesian estimation

$\chi = (x_1, x_2, \dots x_M)$

$\theta = 0 \quad \Rightarrow \quad x_0 = \dfrac{a^\dagger + a}{2}$

$$P(x|\phi) = \frac{e^{-2x^2}}{\pi\Delta} \int_{\mathbb{R}} d\varphi\, e^{-\frac{\varphi^2}{2\Delta^2} + 4\,\alpha x \cos(\varphi+\phi) - 2\,\alpha^2 \cos^2(\varphi+\phi)}$$

$$P(\chi|\phi) = \prod_k P(x_k|\phi)$$

sample probability (Likelihood)

Bayes' theorem

$$P(\phi|\chi) = \frac{1}{\mathcal{N}} \prod_k P(x_k|\phi)$$

Bayesian a posteriori probability

$$\overline{\phi} = \int_\Phi d\phi\, \phi\, P(\phi|X)$$

Bayesian estimator

Asympt. optimal

$M \propto ?$

S. Olivares and M. G. A. Paris, J. Phys. B: At. Mol. Opt. Phys. **42**, 055506 (2009)

Bayesian estimation

$$\chi = (x_1, x_2, \ldots x_M)$$

$$\theta = \arg(\alpha) + \phi + \pi/2$$

- adaptive method -

Optimal phase

$$P(x|\phi) = \frac{e^{-2x^2}}{\pi\Delta} \int_{\mathbb{R}} d\varphi \, e^{-\frac{\varphi^2}{2\Delta^2} + 4\,\alpha x \cos(\varphi+\phi) - 2\,\alpha^2 \cos^2(\varphi+\phi)}$$

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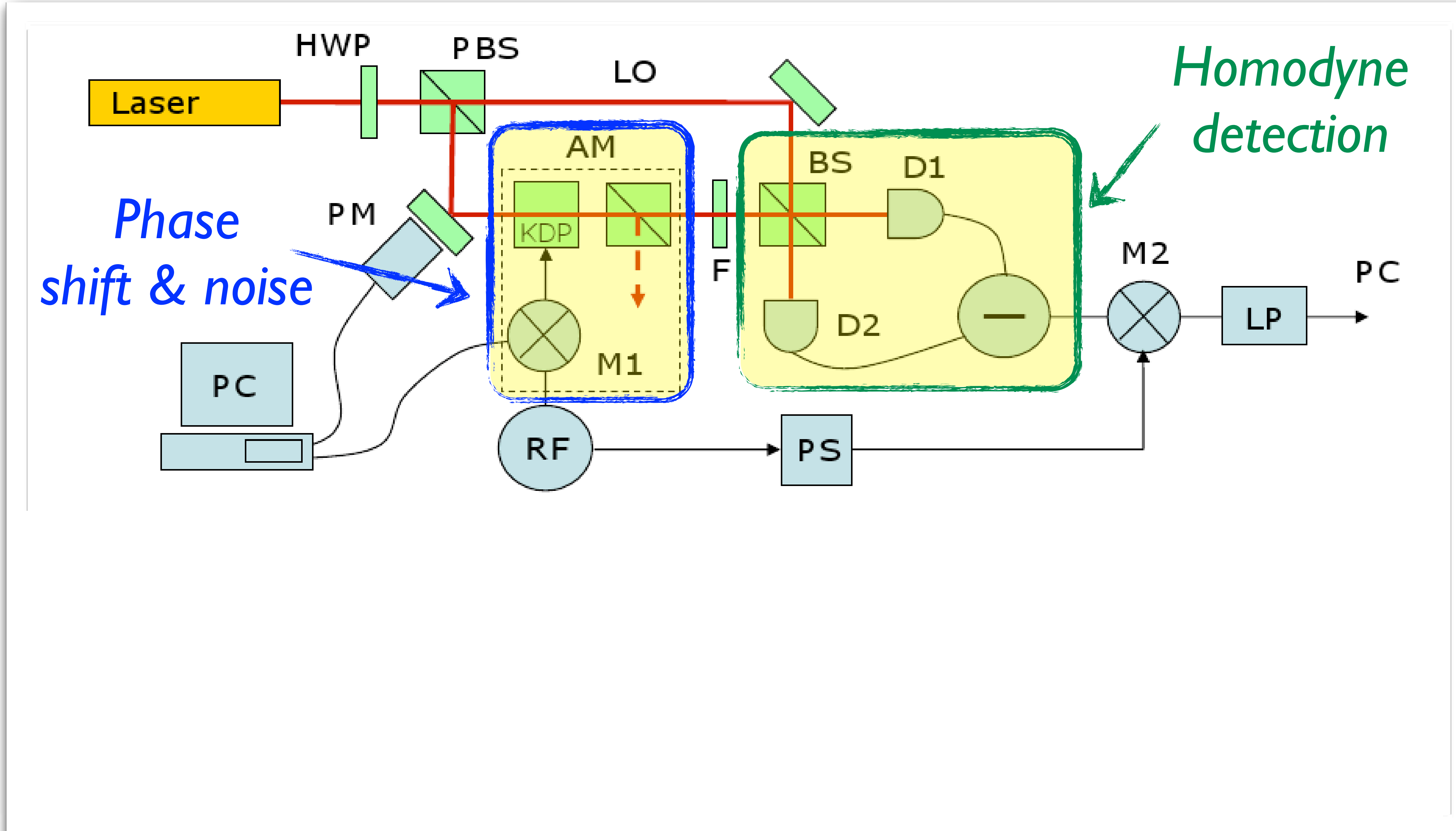
Experimental setup



M. G. Genoni, *et al.*, PRA **85**, 043817 (2012)

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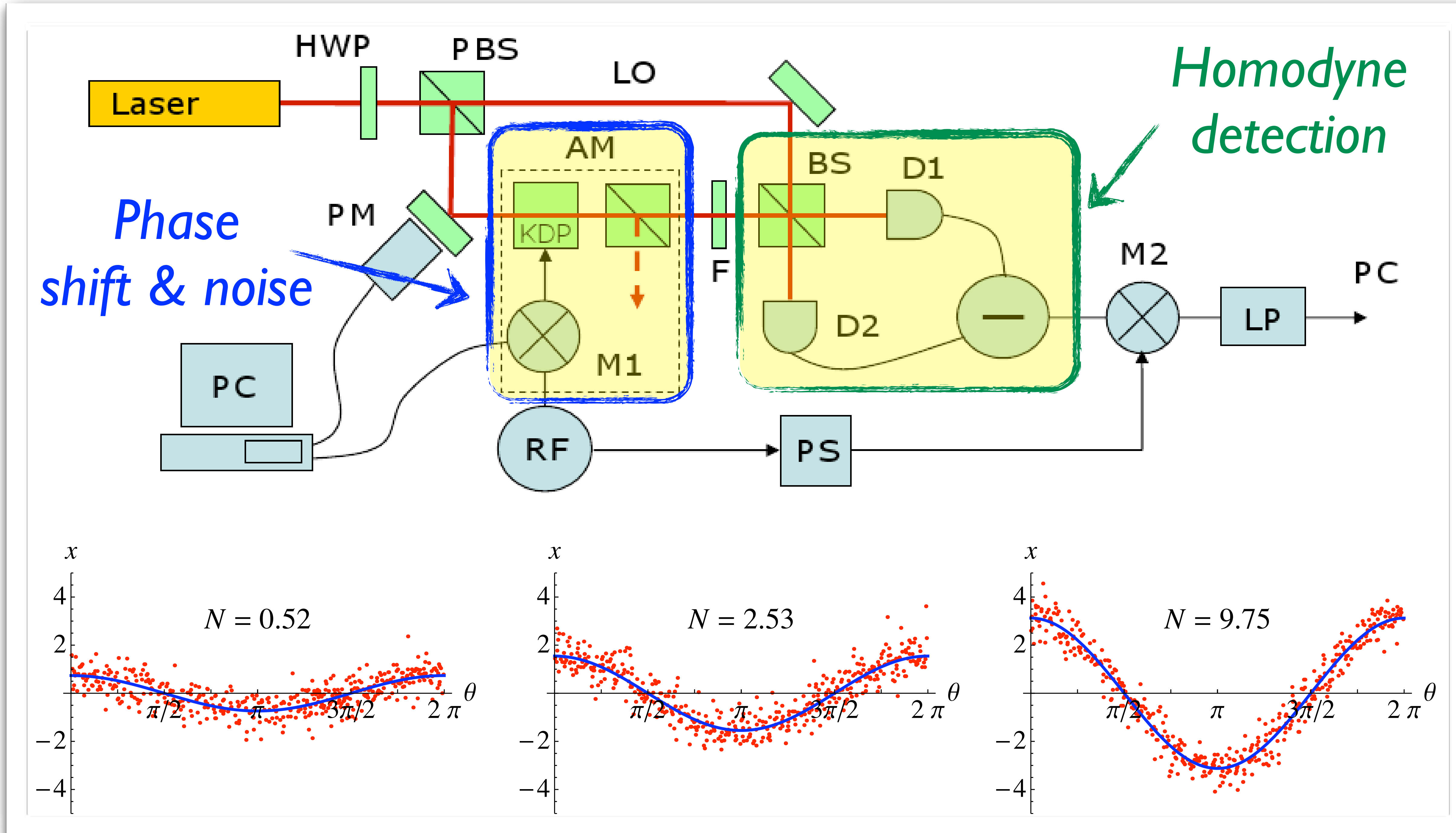
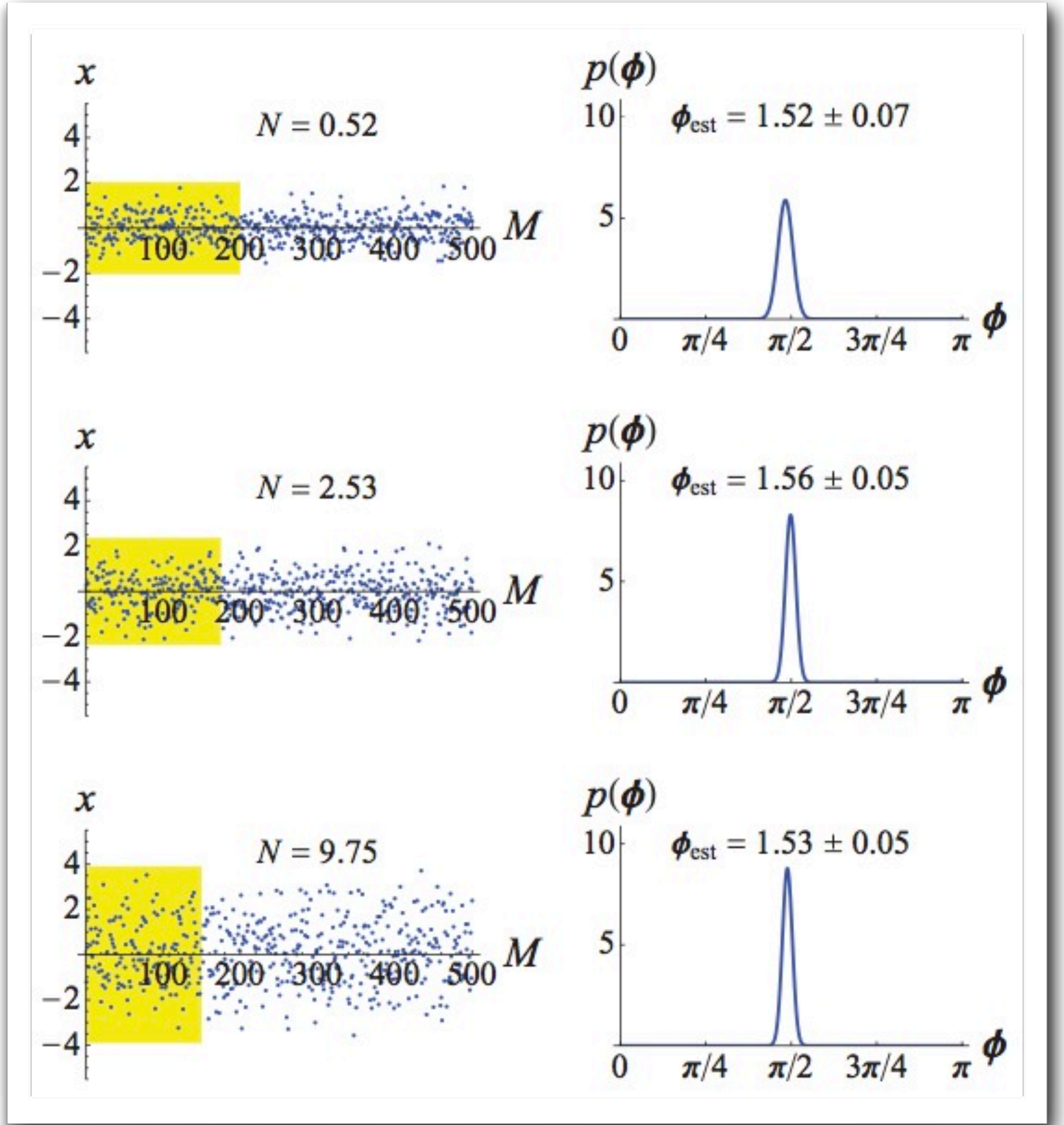


M. G. Genoni, *et al.*, PRA **85**, 043817 (2012)

Experimental results



$$x_0 = \frac{a^\dagger + a}{2}$$

Homodyne samples @ fixed optimal θ and $\Delta = \pi/6$

&

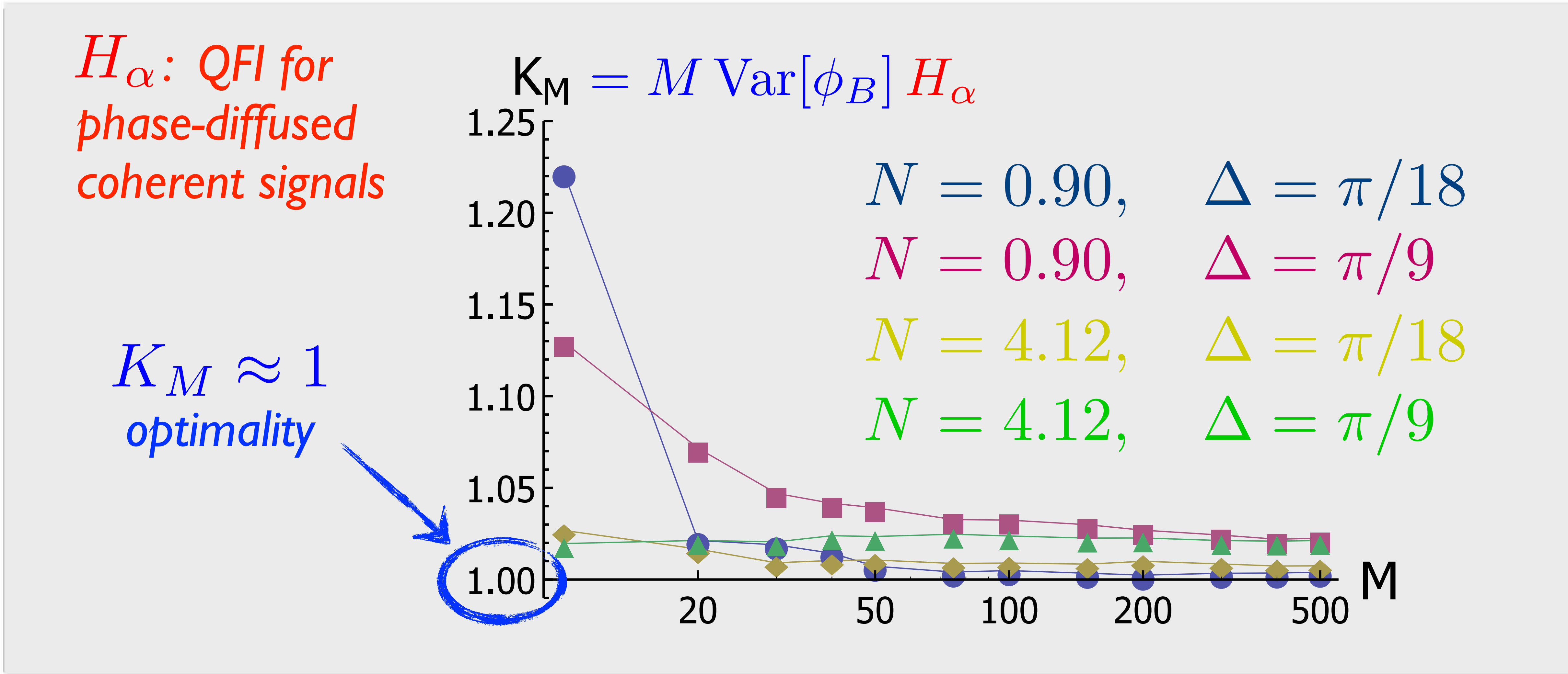
Bayesian a posteriori distribution for the phase shift

$M \propto 10^2$

yellow regions

M. G. Genoni, *et al.*, PRA **85**, 043817 (2012)

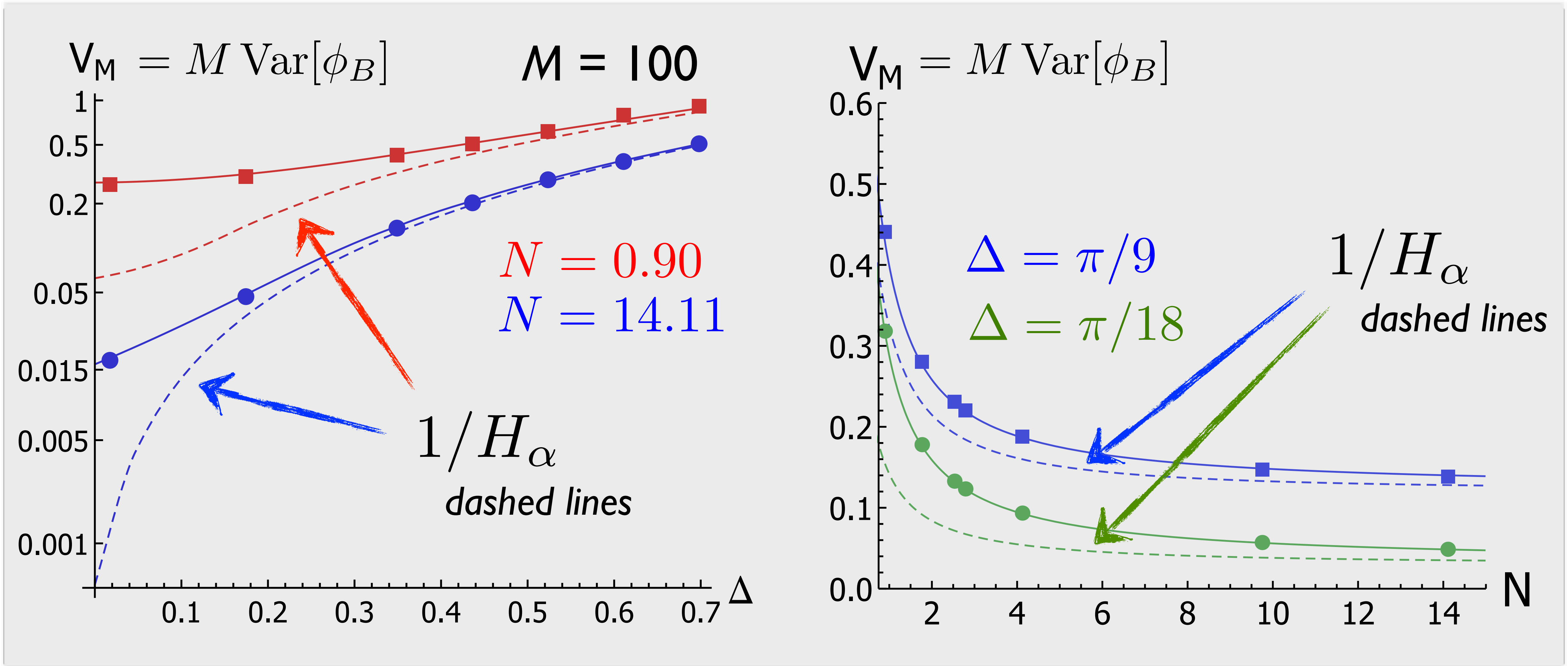
Experimental results



K_M *expresses the ratio between the actual precision of the interferometric setup and the quantum CR bound ($1/H_\alpha$).*

M. G. Genoni, *et al.*, PRA **85**, 043817 (2012)

Experimental results



Nearly optimal interferometric precision is achieved for increasing energy or phase diffusion.

M. G. Genoni, *et al.*, PRA **85**, 043817 (2012)

Concluding remarks & outlook

- *Evaluation of QFI for optical phase estimation under phase diffusive noise.*
- *Derivation of law of scales in terms of noise and number of photons (the "fundamental parameter" is $\xi = N\,\Delta$).*
- *Optimized single-mode GS are more robust than entangled probe states.*
- *Homodyne measurement with GS is optimal for small and large noise.*
- *Experimental results for coherent probe and homodyne detection.*

- *Which is the optimal measurement for intermediate values of noise?*
- *Optimized non-Gaussian resources?*

Thank you!