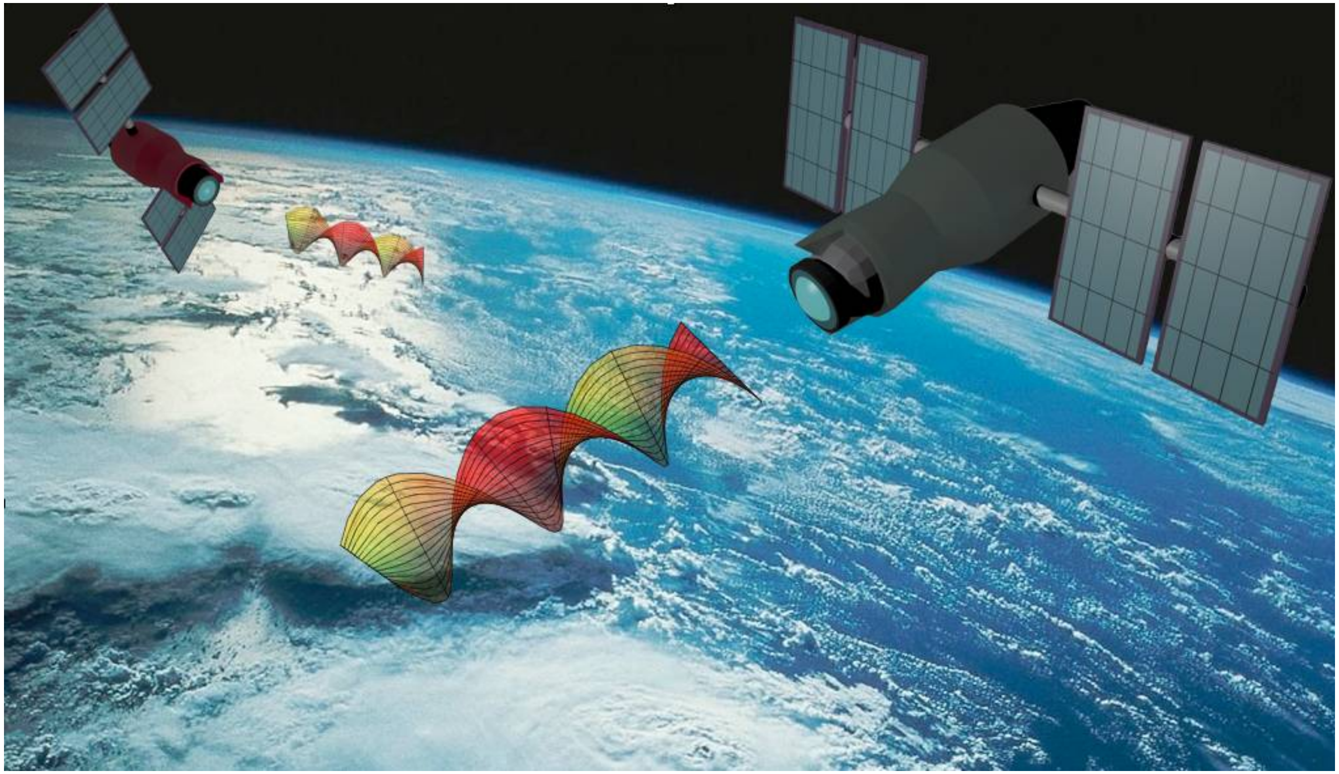
SAPIENZA UNIVERSITÀ DI ROMA

Quantum non-locality without reference frame

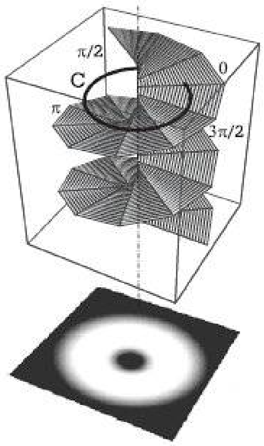
Fabio Sciarrino



The orbital angular momentum of light for quantum information processing

Quantum information ⇨ **qubit**

Different degrees of freedom:
- Polarization
- Linear Momentum
- Orbital Angular Momentum
- ....

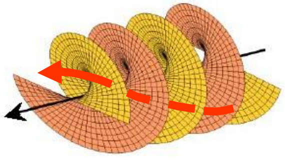


Orbital Angular Momentum (OAM)

⇩

Degree of freedom of light associated with rotationally structured transverse spatial modes

OAM ⇨ Infinite-dimensional degree of freedom ⇨ **Qudit (d > 2)** Multi-level quantum system



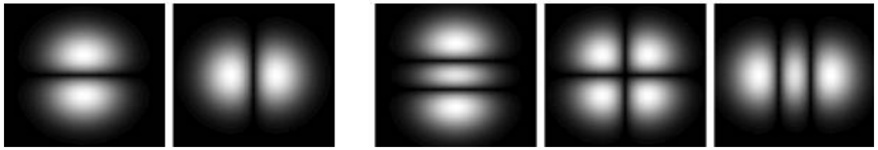
The orbital angular momentum of light

The dynamics of a light beam propagating along the z direction is described by the Helmoltz's equation in the paraxial approximation:

$$\frac{\partial^2 A(\vec{r})}{\partial x^2}+\frac{\partial^2 A(\vec{r})}{\partial y^2}=2ik\frac{\partial A(\vec{r})}{\partial z}$$

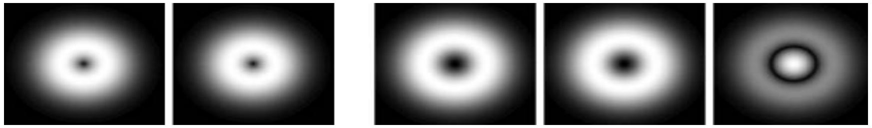
Cartesian coordinates

Hermite – Gauss modes



Cylindrical coordinates

Laguerre – Gauss modes

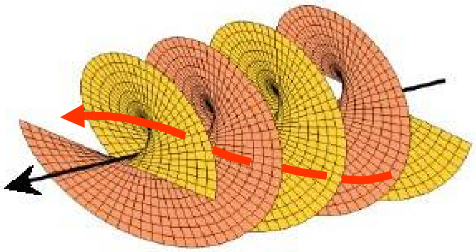


Laguerre-Gauss :

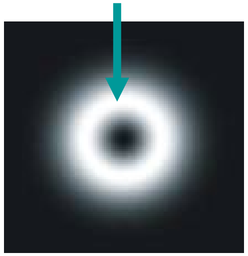
$$u_{p,l}(r,\varphi,z)\propto u(r,z)e^{-il\varphi}$$

Helicoidal phase front

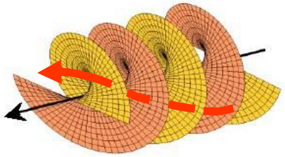
$l=0\ ,\ \pm 1\ ,\ \pm 2\ ,\dots$



Phase Singularity



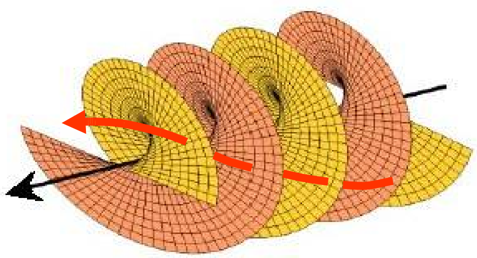
➡ Each photon carries OAM equal to $l\hbar$



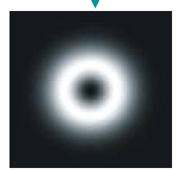
The orbital angular momentum of light

Laguerre-Gauss : $u_{p,l}(r,\varphi,z) \propto u(r,z)\,e^{-il\varphi}$

Helicoidal phase front
$l = 0\;,\;\pm 1\;,\;\pm 2\;,\ldots$



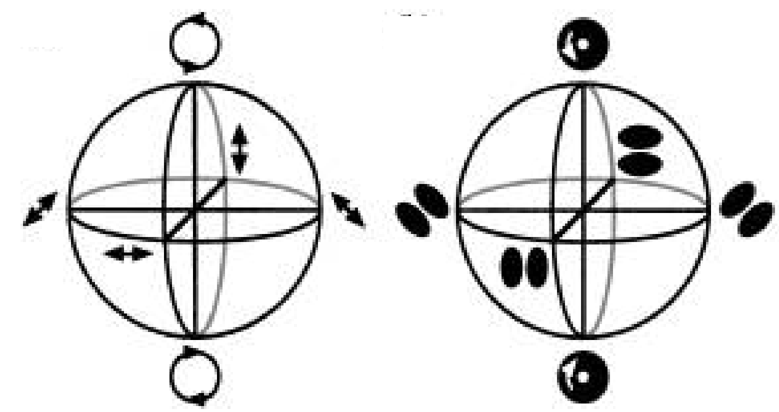
Phase Singularity



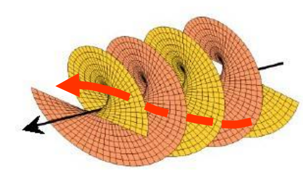
Observation: For a chosen OAM subspace $o_m=\{+m,-m\}$, it is possible to construct a sphere analogous to the Poincaré one, for superpositions of left- and right- handed LG modes.

ex. Subspace $o_2=\{+2,-2\}$

$$|d_{R,L}\rangle = \frac{1}{\sqrt{2}}\left(|+2\rangle \pm i|-2\rangle\right) \qquad |d_{\pm}\rangle = \frac{1}{\sqrt{2}}\left(|+2\rangle \pm |-2\rangle\right)$$



Polarization OAM

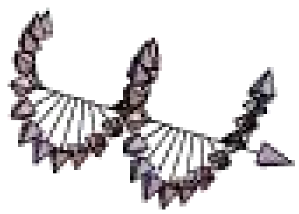


M. Padgett, and J. Courtial, *Optics Letters* **24**, 430 (1993)

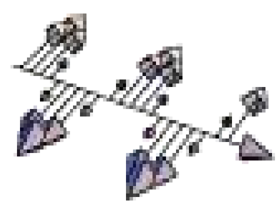
The polarization of light

SPIN

S = -1

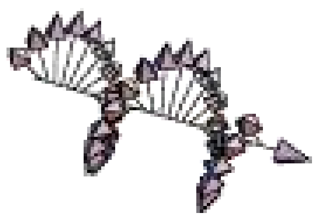


Left polarization



Horizzontal polarization

S = 1

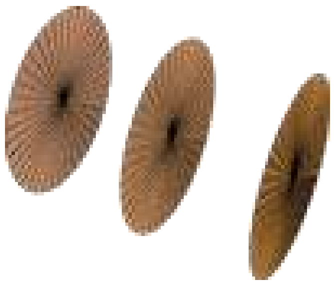


Right polarization

http://www.physics.gla.ac.uk/Optics/

The transverse profile of light: the Orbital Angular Momentum

L = 0

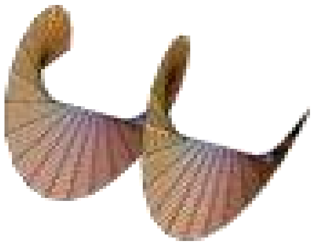


Laser beam
(laser pointer)

http://www.physics.gla.ac.uk/Optics/

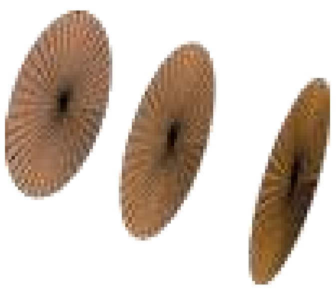
The transverse profile of light: the Orbital Angular Momentum

L = -1



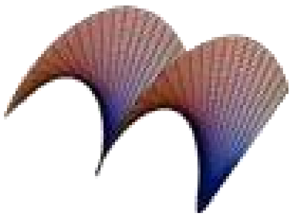
L = 1

L = 0

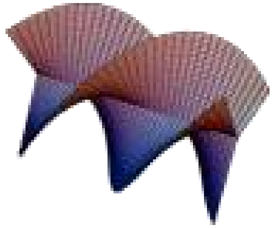


Laser beam
(laser pointer)

L = 2

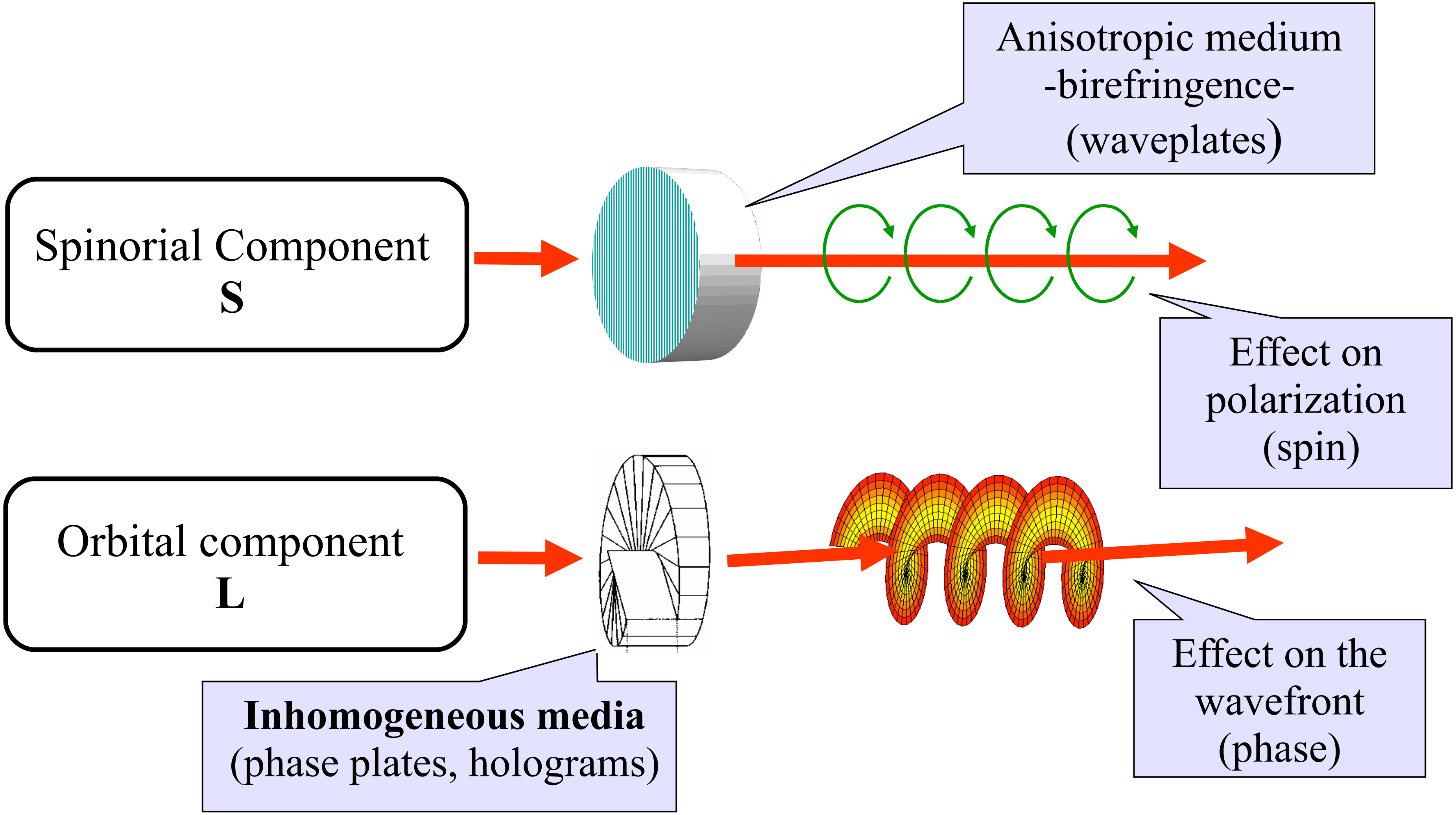


L = 3



http://www.physics.gla.ac.uk/Optics/

Manipulation of orbital angular momentum



The q-plate device (1/2)

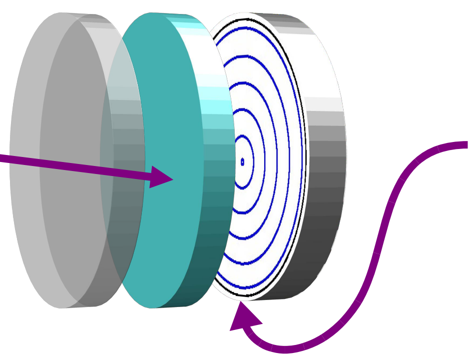
Angular Momentum
- **Spinorial ( )** → Anisotropic medium (birefringent)
- **Orbital (OAM)** → Inhomogenous medium

To couple and OAM: <u>Q-plate</u>
anisotropic and inhomogenous

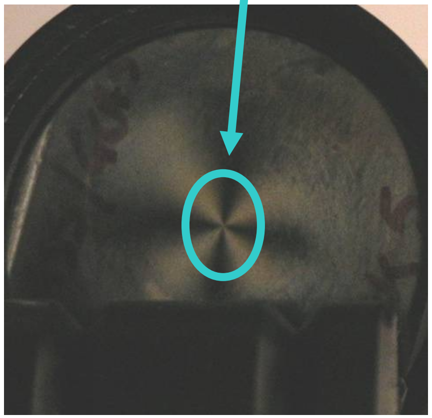
Topological charge q

$\Delta l = 2q\hbar$

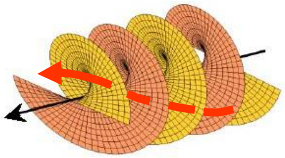
Nematic Liquid Crystal



Circular rubbing of one substrate for the NLC orientation



The q-plate thickness is chosen in order to have half-wave retardation depending on the working wavelength.



L. Marrucci, et al., *Physical Review Letters* **96**, 163905 (2006)

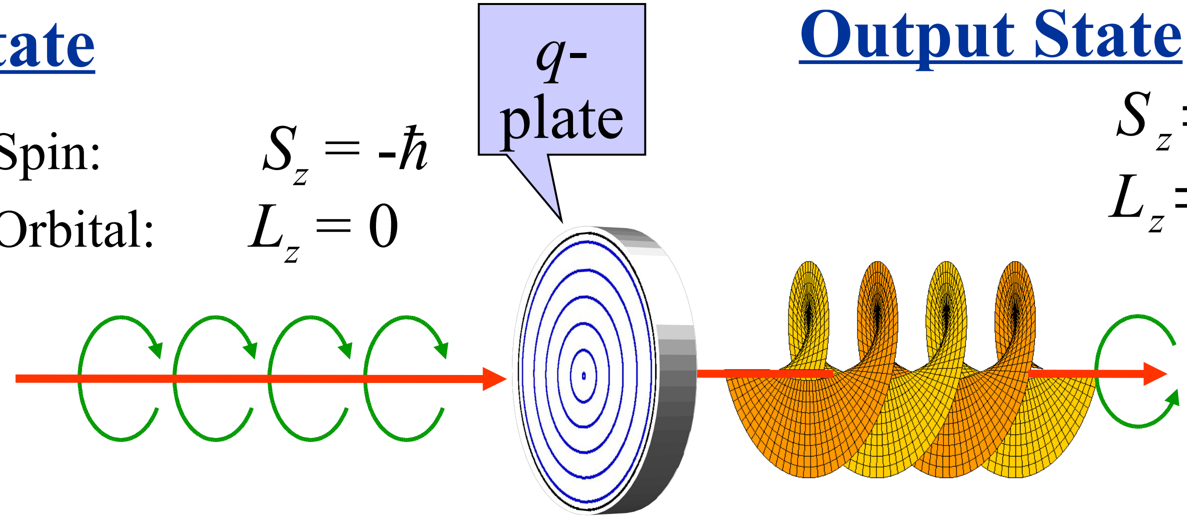
The q-plate device (2/2)

Input State | **Output State**

TEM$_{00}$ with **right** circular polarization

Spin: $S_z = -\hbar$

Orbital: $L_z = 0$



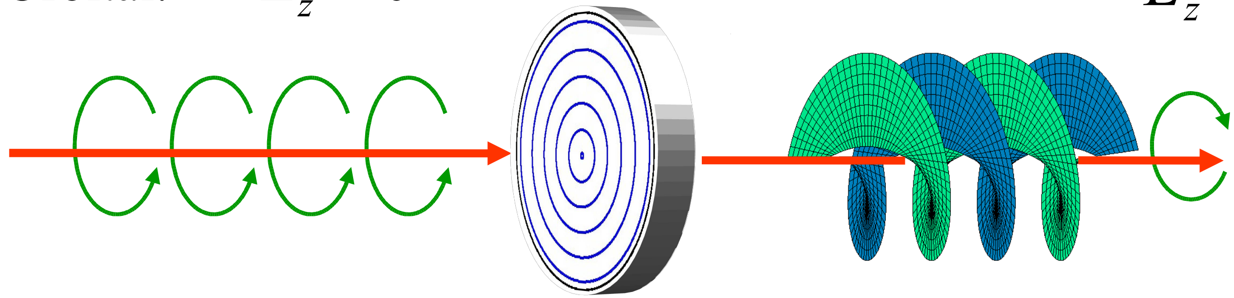
$S_z = +\hbar$

$L_z = -2\hbar q$

TEM$_{00}$ with **left** circular polarization

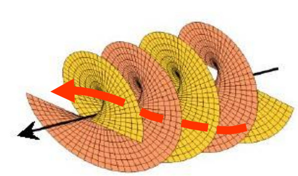
Spin: $S_z = +\hbar$

Orbital: $L_z = 0$

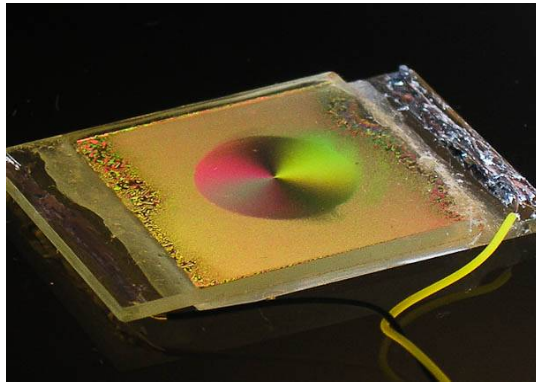


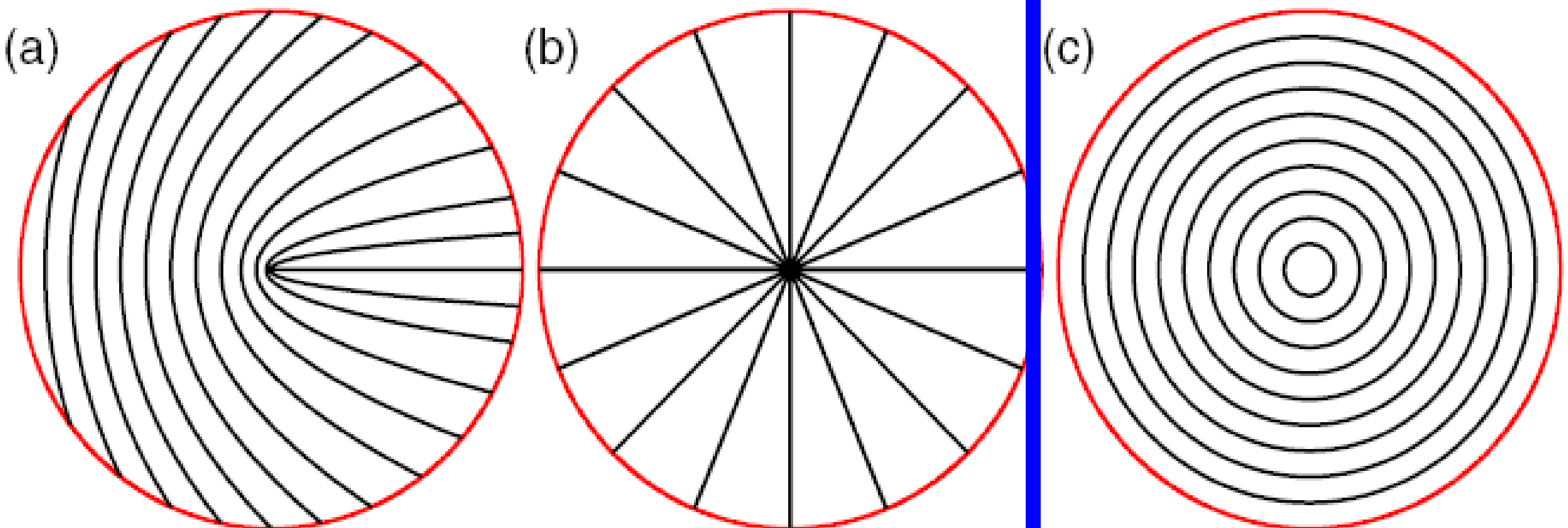
$S_z = -\hbar$

$L_z = +2\hbar q$



Different q-plate geometries





$q = ½$ $\qquad$ $q = 1$ $\qquad$ $q = 1$

Qplate in the quantum regime

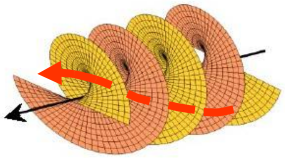
$$|L\rangle_\pi |m\rangle_o \xrightarrow{QP} |R\rangle_\pi |m+2\rangle_o$$

$$|R\rangle_\pi |m\rangle_o \xrightarrow{QP} |L\rangle_\pi |m-2\rangle_o$$

Unitary evolution on a generic input state

$$\alpha|L\rangle_\pi |m\rangle_o + \beta|R\rangle_\pi |m\rangle_o \xrightarrow{QP} \alpha|R\rangle_\pi |m+2\rangle_o + \beta|L\rangle_\pi |m-2\rangle_o$$

- The qplate: a quantum interface between polarization and OAM
  - Single photon entanglement between polarization and OAM
  - Quantum transferrer: polarization ⟶ OAM
  - Quantum transferrer: OAM ⟶ polarization



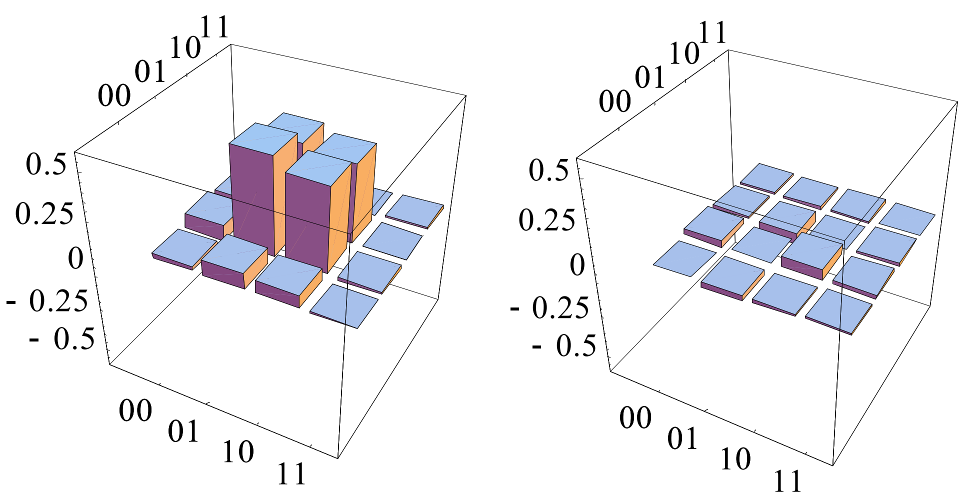
Single photon entanglement

The q-plate introduces a quantum correlation between the OAM and the polarization π degree of freedom

Single-photon entanglement

$$\begin{matrix} |H\rangle_\pi |0\rangle_o \\ |V\rangle_\pi |0\rangle_o \end{matrix} \xrightarrow{\textbf{QP}} \frac{1}{\sqrt{2}}\left[|L\rangle_\pi |-2\rangle_{o_2} \pm |R\rangle_\pi |+2\rangle_{o_2}\right]$$

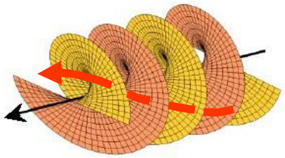
$\Re[\rho]$ $\Im[\rho]$



Input State $|H\rangle_\pi |0\rangle_o$

$$\rho_{\pi,o_2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = 0.95 \pm 0.02$$



E. Nagali, *et al., Physical Review Letters* **103,** 013601 (2009)

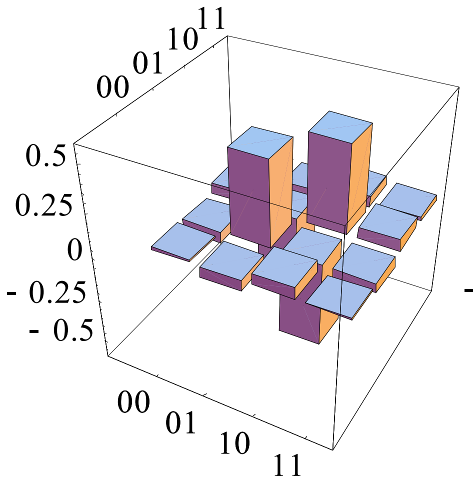
Single photon entanglement

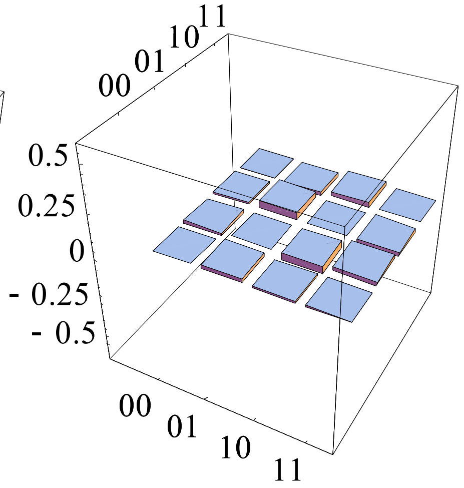
The q-plate introduces a quantum correlation between the OAM and the polarization π degree of freedom

Single-photon entanglement

$$\begin{matrix} |H\rangle_\pi |0\rangle_o \\ |V\rangle_\pi |0\rangle_o \end{matrix} \xrightarrow{\text{QP}} \frac{1}{\sqrt{2}}\left[|L\rangle_\pi |-2\rangle_{o_2} \pm |R\rangle_\pi |+2\rangle_{o_2}\right]$$

$\Re[\rho]$ $\Im[\rho]$

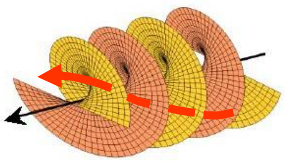




Input State $|V\rangle_\pi |0\rangle_o$

$$\rho_{\pi,o_2} = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = 0.97 \pm 0.02$$

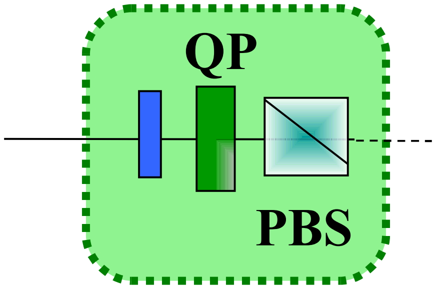


E. Nagali, *et al.*, *Physical Review Letters* **103,** 013601 (2009)

Quantum transferrers $\pi \leftrightarrow$ OAM

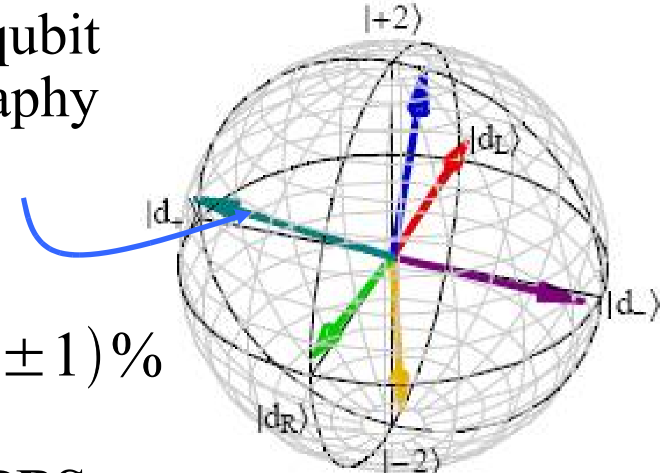
Transferrer $\pi \rightarrow 0_2$

$|\varphi\rangle_\pi |0\rangle_o$



$|H\rangle_\pi |\varphi\rangle_{o_2}$

Single qubit tomography

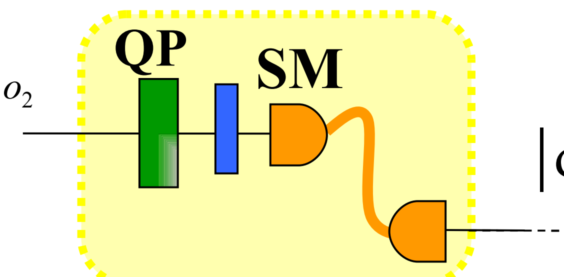


$F = (98 \pm 1)\%$

$$(\alpha|R\rangle_\pi + \beta|L\rangle_\pi)|0\rangle_{o_2} \xrightarrow{\text{QP}} \alpha|L\rangle_\pi|-2\rangle_{o_2} + \beta|R\rangle_\pi|+2\rangle_{o_2} \xrightarrow{\text{PBS}} |H\rangle_\pi(\alpha|-2\rangle_{o_2} + \beta|+2\rangle_{o_2})$$

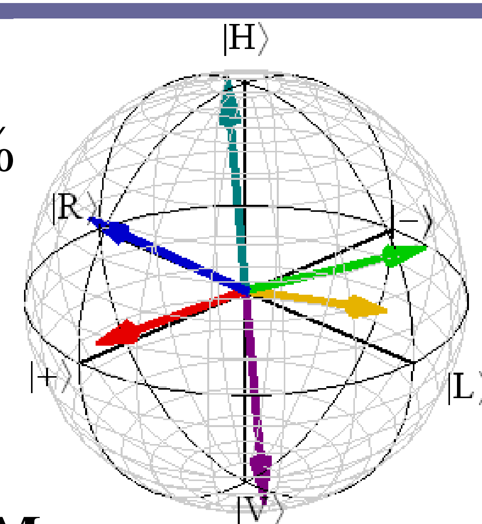
Transferrer $0_2 \rightarrow \pi$

$|H\rangle_\pi |\varphi\rangle_{o_2}$



$|\varphi\rangle_\pi |0\rangle_o$

$F = (97 \pm 1)\%$



$$|H\rangle_\pi(\alpha|-2\rangle_{o_2} + \beta|+2\rangle_{o_2}) \xrightarrow{\text{QP}} \alpha|R\rangle_\pi|0\rangle_o + \beta|L\rangle_\pi|0\rangle_o +$$

$$\alpha|R\rangle_\pi|-4\rangle_{o_4} + \beta|L\rangle_\pi|+4\rangle_{o_4} \xrightarrow{\text{SM}} (\alpha|R\rangle_\pi + \beta|L\rangle_\pi)|0\rangle_{o_2}$$

E. Nagali, *et al., Physical Review Letters* **103,** 013601 (2009); *Optics Express* **17,** 18745 (2009)

Orbital angular momentum....

An extra resource
for optical quantum information processing...

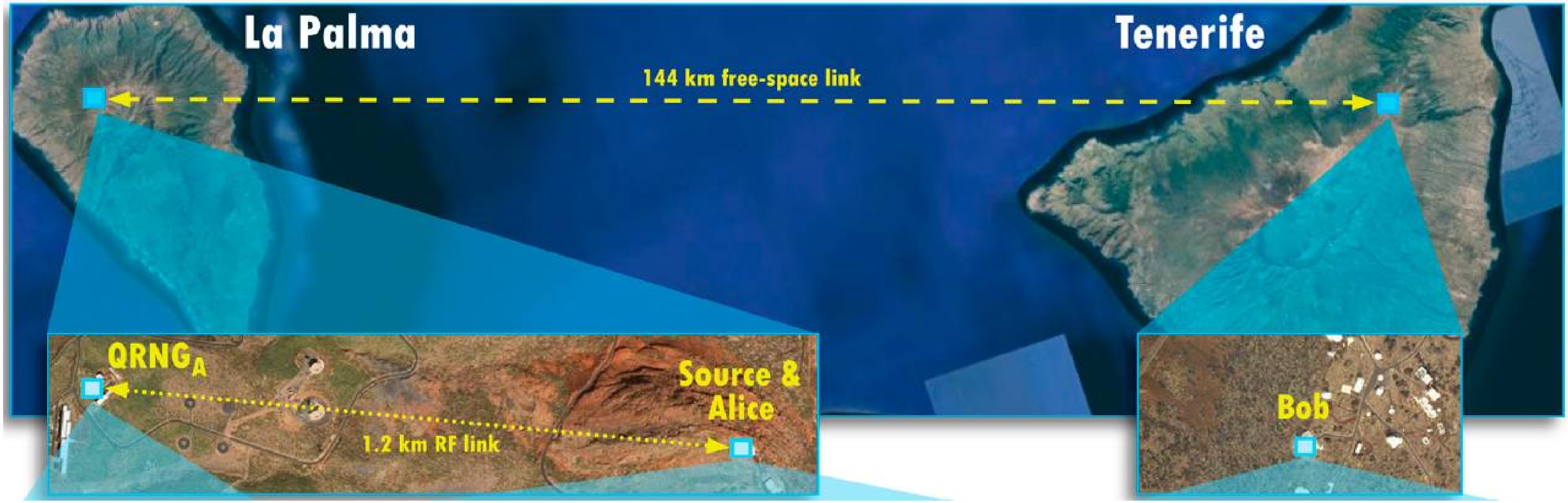
I) Quantum communication...

II) Higher dimensional quantum systems...

Free space quantum communication

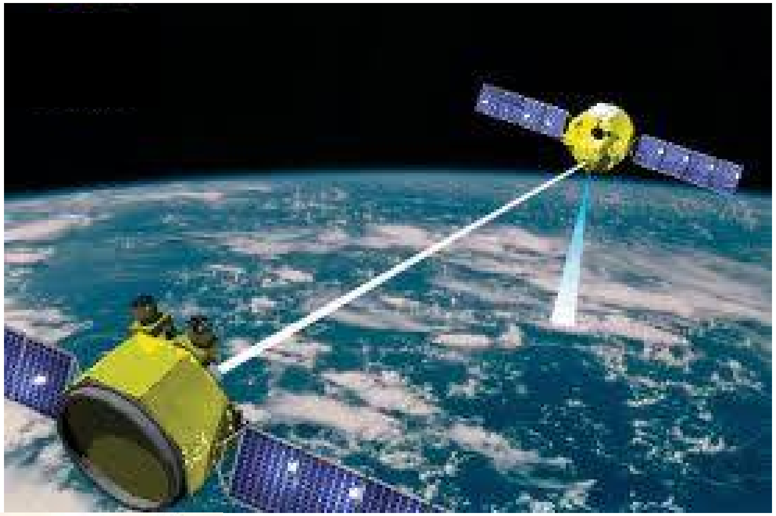
Violation of local realism with freedom of choice

Thomas Scheidl[a], Rupert Ursin[a], Johannes Kofler[a,b,1], Sven Ramelow[a,b], Xiao-Song Ma[a,b], Thomas Herbst[b], Lothar Ratschbacher[a,2], Alessandro Fedrizzi[a,3], Nathan K. Langford[a,4], Thomas Jennewein[a,5], and Anton Zeilinger[a,b,1]



Quantum cryptography over a distance of 144 km

Quantum Communication meets Satellite



Alice

Quantum communication (QC): transferring quantum states from one place to another using entanglement.

↓

Quantum mechanics ensures that QC is inherently secure from being hacked

Bob

Free-space and satellite quantum channels are possible ways to increase significantly the distance limit of current quantum communication systems.

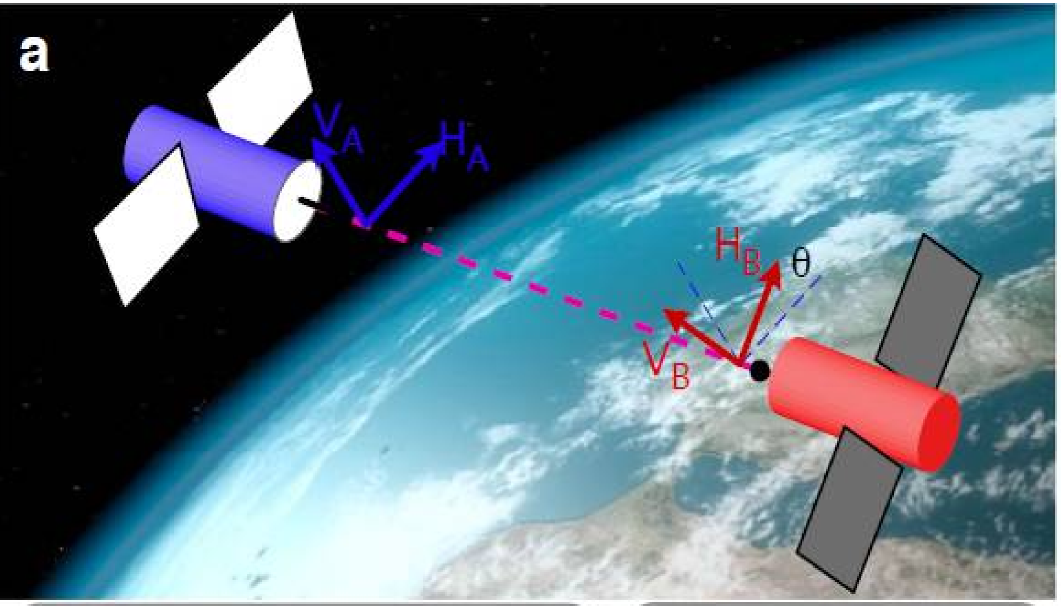
Problems:

- no optical medium to guide beams of light
- intense backlight
- diffusion and absorption
- atmospheric turbulence (distortion of the light's wavefront)

Quantum communication
+
Satellite

Long distance quantum cryptography, quantum teleportation, dense coding, ...

How to share a reference frame ?



Polarization based quantum communication:

- need to share a reference frame

How to deal with rotating physical systems ?

MBER 8 PHYSICAL REVIEW LETTERS week
22 AUGU

Decoherence-Free Subspaces in Quantum Key Distribution

...uraddy, Alexander V. Sergienko, Bahaa E. A. Saleh, and Malvin C. Te...
...nt of Electrical & Computer Engineering, Boston University, 8 Saint Mary's...
Boston, Massachusetts 02215-2421, USA

...ed 26 March 2003; published 20 August 2003)

PHYSICAL REVIEW A **82**, 012304 (2010)

Reference-frame-independent quantum key distribution

Anthony Laing,[1,*] Valerio Scarani,[2,†] John G. Rarity,[1,‡] and Jeremy L. O'Brien[1,§]

[1]*Centre for Quantum Photonics, H. H. Wills Physics Laboratory & Department of Electrical and Electronic Engineering, University of Bristol, BS8 1UB, United Kingdom*

[2]*Centre for Quantum Technologies and Department of Physics, National University of Singapore, Singapore*

(Received 18 March 2010; published 7 July 2010)

...REVIEW LETTERS week ending
21 APRIL 2006

Experimental Quantum Communication without a Shared Reference Frame

Teng-Yun Chen,[1] Jun Zhang,[1] J.-C. Boileau,[2] Xian-Min Jin,[1] Bin Yang,[1] Qiang Zhang,[1,3] Tao Yang,[1,3] R. Laflamme,[2] and Jian-Wei Pan[1,3]

[1]*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*

[2]*Institute for Quantum Computing, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada*

Alignment-free qubits

Polarization

$$R_z(\theta)|R\rangle_\pi = e^{i\theta}|R\rangle_\pi$$

$$R_z(\theta)|L\rangle_\pi = e^{-i\theta}|L\rangle_\pi$$

OAM

$$R_z(\theta)|+1\rangle_{oam} = e^{-i\theta}|+1\rangle_{oam}$$

$$R_z(\theta)|-1\rangle_{oam} = e^{i\theta}|-1\rangle_{oam}$$

Alignment-free qubits

Polarization

$$R_z(\theta)|R\rangle_\pi = e^{i\theta}|R\rangle_\pi$$

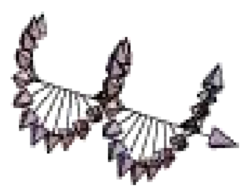
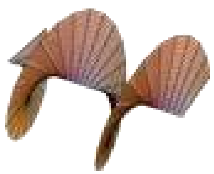
$$R_z(\theta)|L\rangle_\pi = e^{-i\theta}|L\rangle_\pi$$

OAM

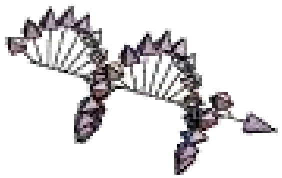
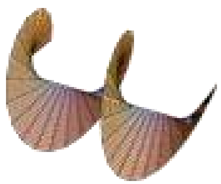
$$R_z(\theta)|+1\rangle_{oam} = e^{-i\theta}|+1\rangle_{oam}$$

$$R_z(\theta)|-1\rangle_{oam} = e^{i\theta}|-1\rangle_{oam}$$

$$R_z(\theta)|R\rangle_\pi|+1\rangle_{oam} = |R\rangle_\pi|+1\rangle_{oam}$$

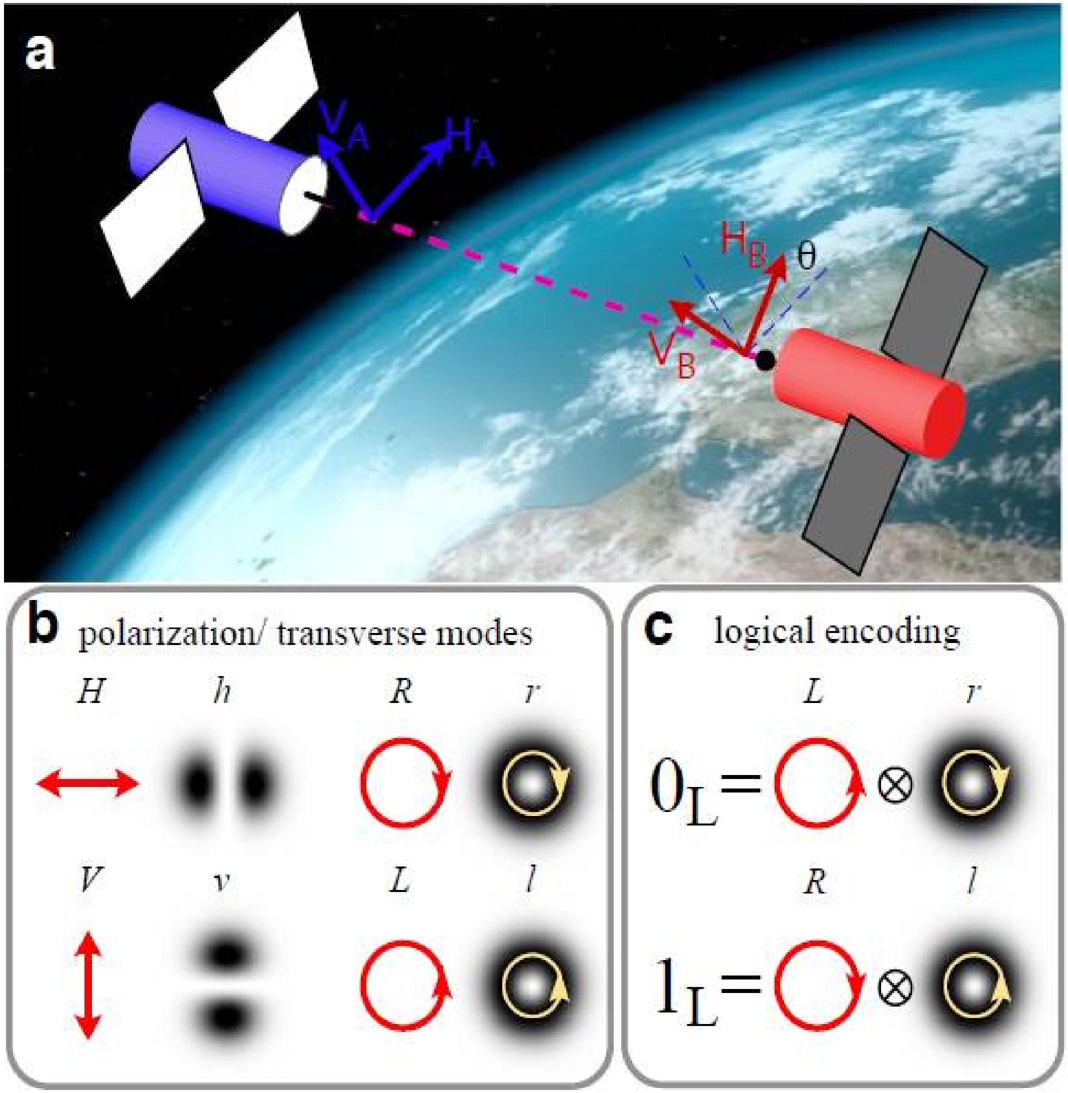
 x 

$$R_z(\theta)|L\rangle_\pi|-1\rangle_{oam} = |L\rangle_\pi|-1\rangle_{oam}$$

 x 

Invariant under arbitrary rotations along the z axis!!

Experimental reference frame-free quantum communication



Idea: adopt hybrid qubits encoded in the logical basis

$$|1\rangle_L = |R\rangle_p |l\rangle_o$$
$$|0\rangle_L = |L\rangle_p |r\rangle_o.$$

polarization OAM

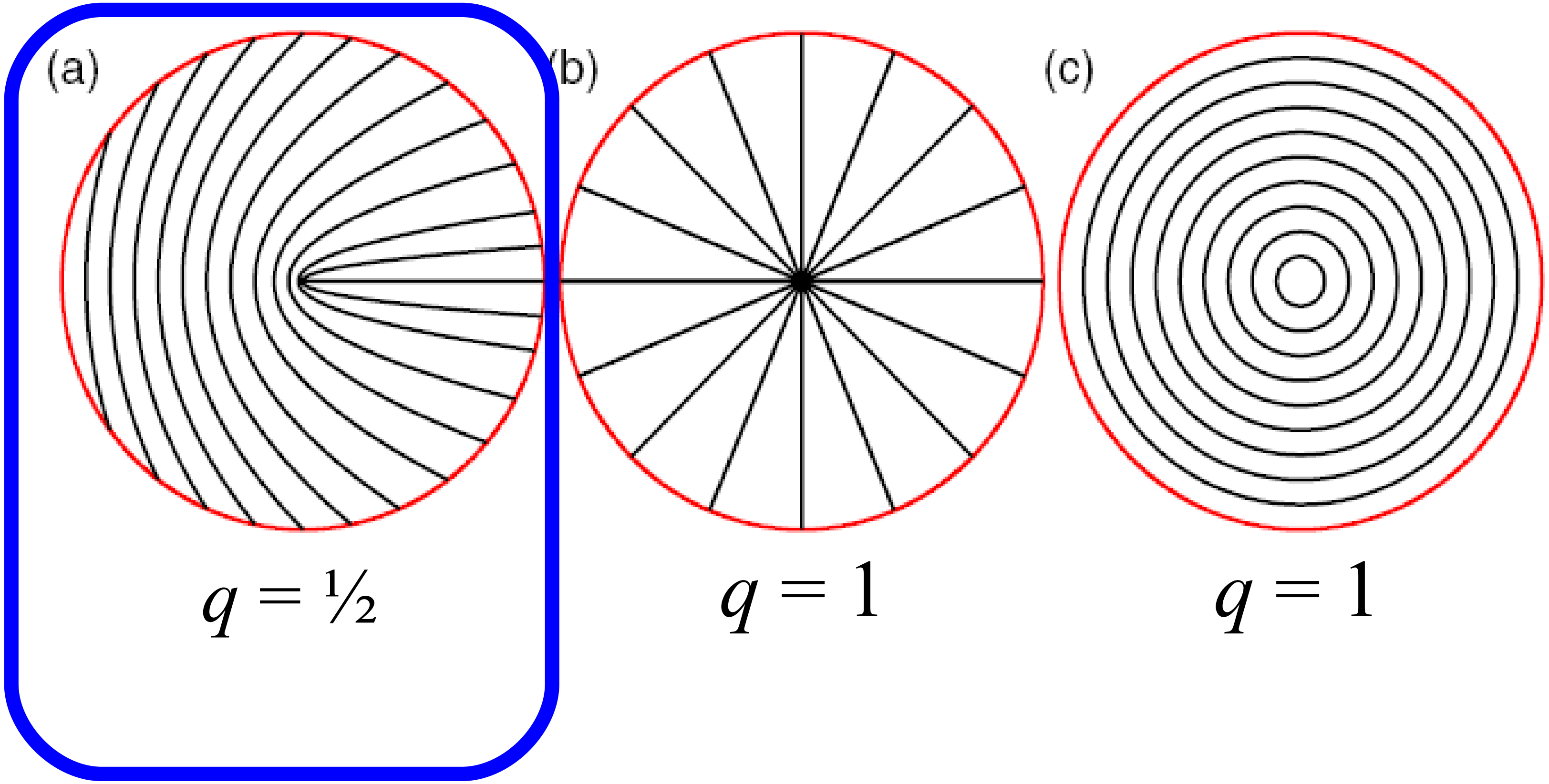
Under a rotation along the axis z:

$$|L/R\rangle_p \to e^{\mp i\theta}|L/R\rangle_p$$
$$|l/r\rangle_o \to e^{\mp i\theta}|l/r\rangle_o$$

➡ $|1\rangle_L$ and $|0\rangle_L$ invariant under arbitrary rotations along the propagtion axis!

How to manipulate hybrid qubit ?

Q-plate with charge q=1/2



$$|R\rangle_{\rm p}|0\rangle_{\rm o} \stackrel{QP}{\rightarrow} |L\rangle_{\rm p}|r\rangle_{\rm o} = |0\rangle_{\rm L}$$

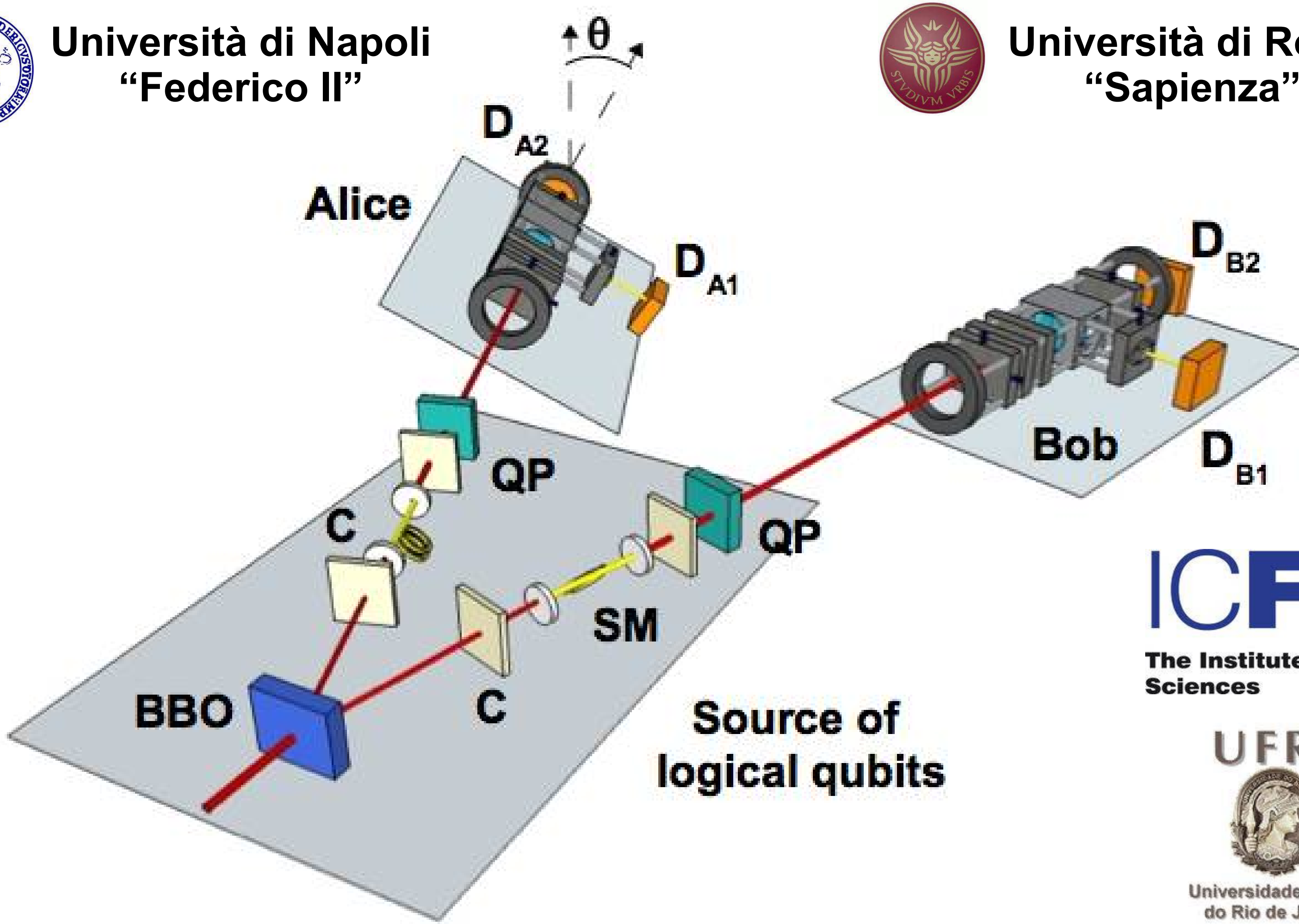
$$|L\rangle_{\rm p}|0\rangle_{\rm o} \stackrel{QP}{\rightarrow} |R\rangle_{\rm p}|l\rangle_{\rm o} = |1\rangle_{\rm L}$$

$$|\psi\rangle_{\rm p}|0\rangle_{\rm o} \stackrel{QP}{\rightarrow} \alpha|0\rangle_{\rm L} + \beta|1\rangle_{\rm L} = |\psi\rangle_{\rm L}$$

Experimental setup

Università di Napoli
"Federico II"

Università di Roma
"Sapienza"



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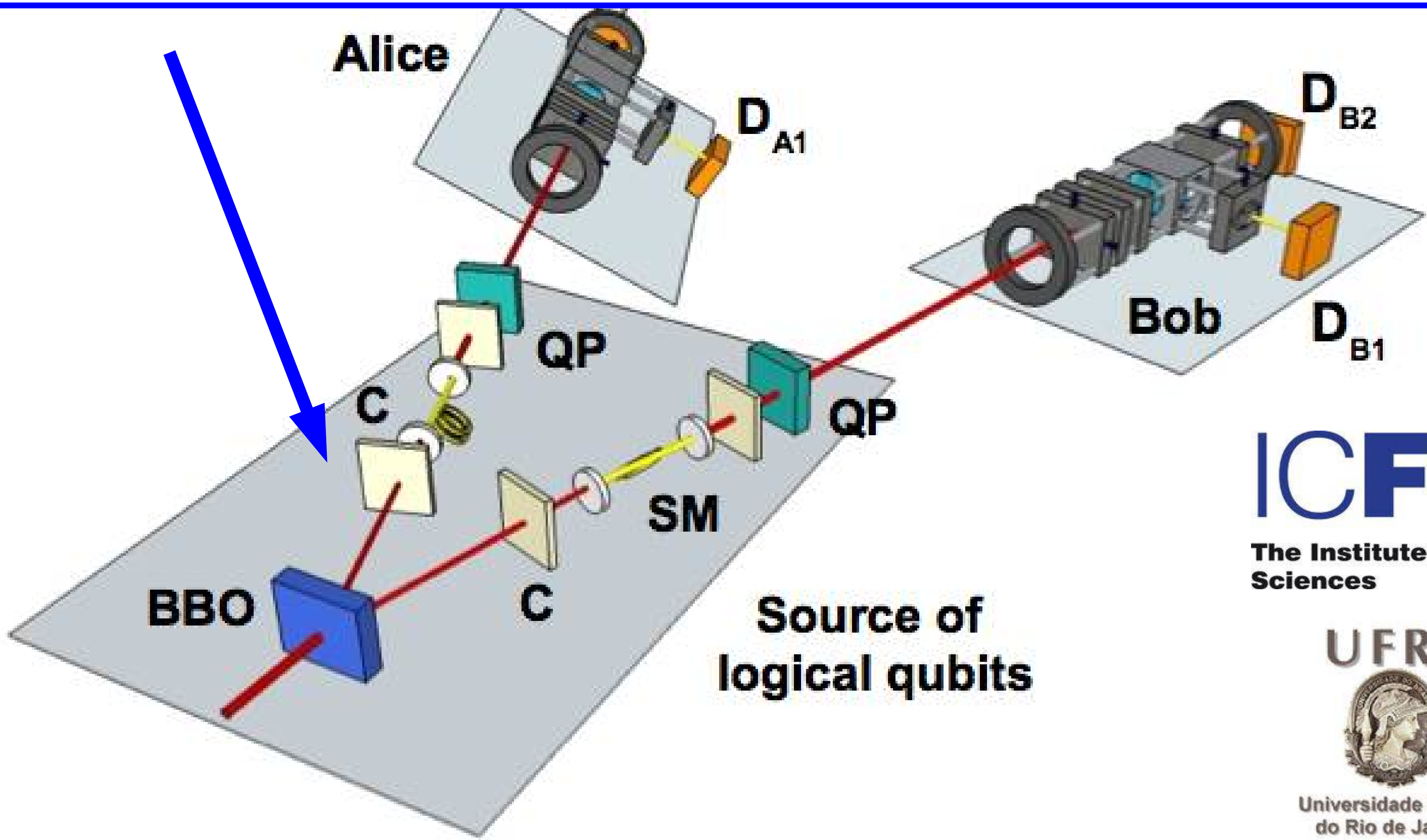
V. D'Ambrosio, E. Nagali, S. P. Walborn, L. Aolita, NS. Slussarenko, L. Marrucci, and F. Sciarrino, *Complete experimental toolbox for alignment-free quantum communication*, Nature Comm (in press)

Experimental setup

$$|\phi^-\rangle_{\mathrm{p}}^{AB} \overset{QPs}{\rightarrow} \frac{1}{\sqrt{2}}\left(|0\rangle_{\mathrm{L}}^{A}|0\rangle_{\mathrm{L}}^{B} - |1\rangle_{\mathrm{L}}^{A}|1\rangle_{\mathrm{L}}^{B}\right) = |\phi^-\rangle_{\mathrm{L}}^{AB}$$



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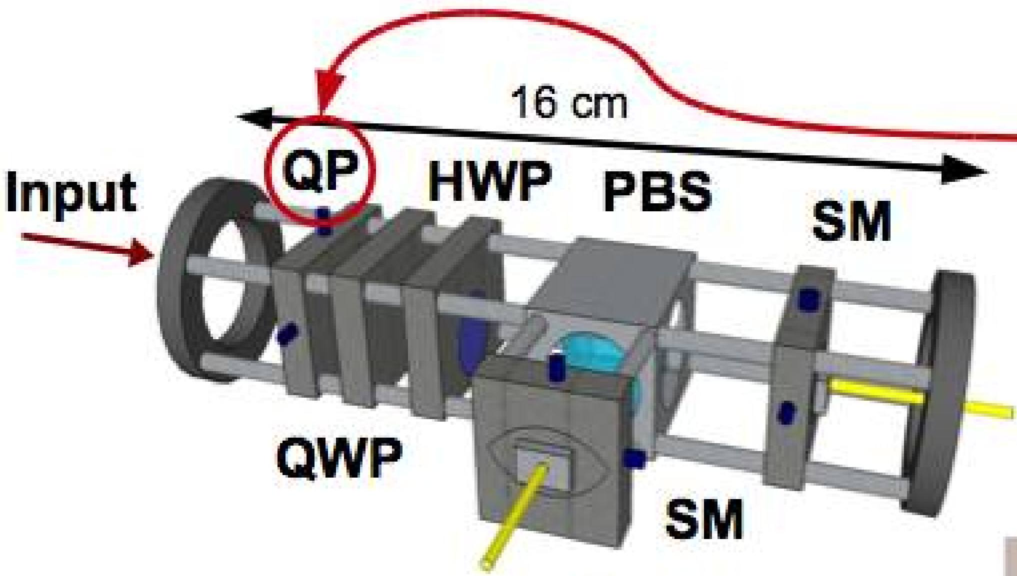
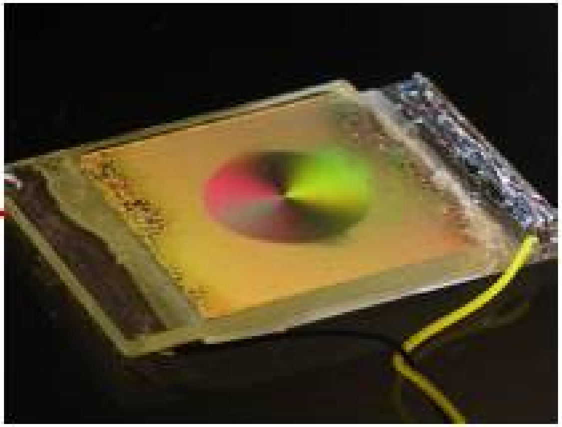
V. D'Ambrosio, E. Nagali, S. P. Walborn, L. Aolita, S. Slussarenko, L. Marrucci, and F. Sciarrino, *Complete experimental toolbox for alignment-free quantum communication*, [arXiv:1203.6417]

Rotational measurement kit

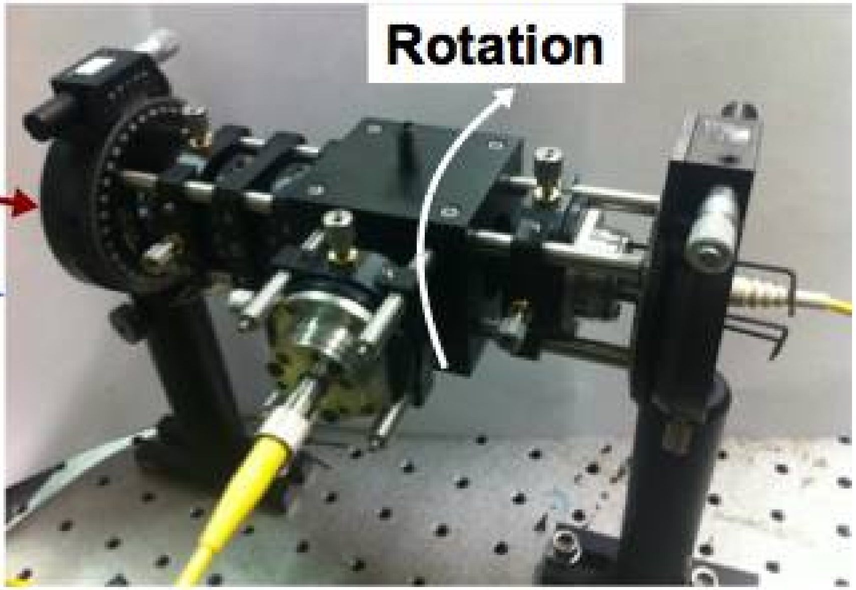




q = 1/2

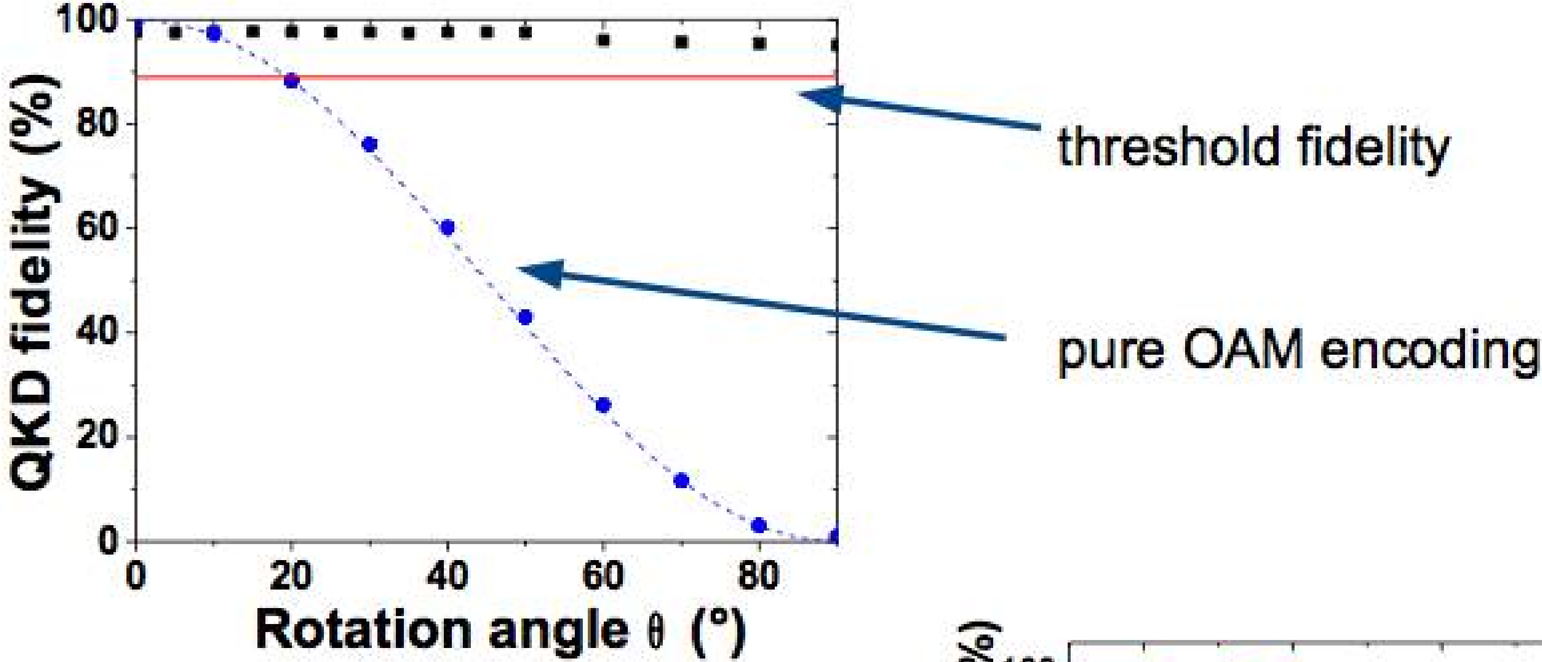
$$R\rangle_{\rm p}|0\rangle_{\rm o} \overset{QP}{\rightarrow} |L\rangle_{\rm p}|r\rangle_{\rm o} \;=\; |0\rangle_{\rm L}$$
$$|L\rangle_{\rm p}|0\rangle_{\rm o} \overset{QP}{\rightarrow} |R\rangle_{\rm p}|l\rangle_{\rm o} \;=\; |1\rangle_{\rm L},$$

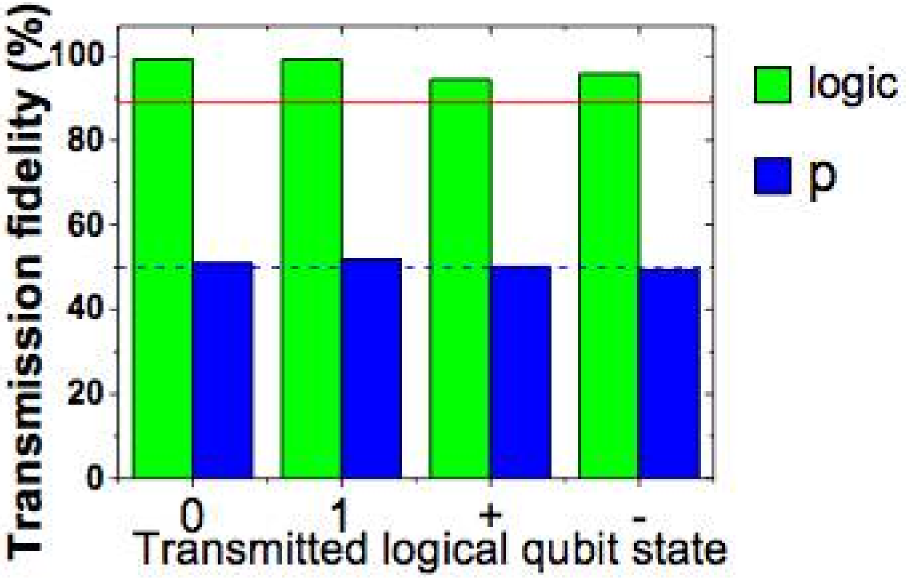
in the lab...



$$|\psi\rangle_{\rm p}|0\rangle_{\rm o} \overset{QP}{\rightarrow} \alpha|0\rangle_{\rm L} + \beta|1\rangle_{\rm L} = |\psi\rangle_{\rm L}.$$

Reference frame-free QKD

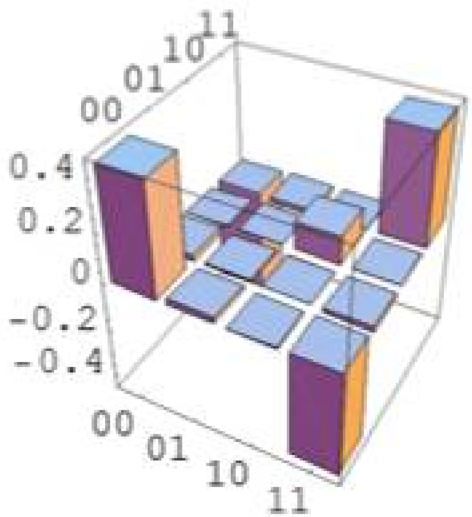




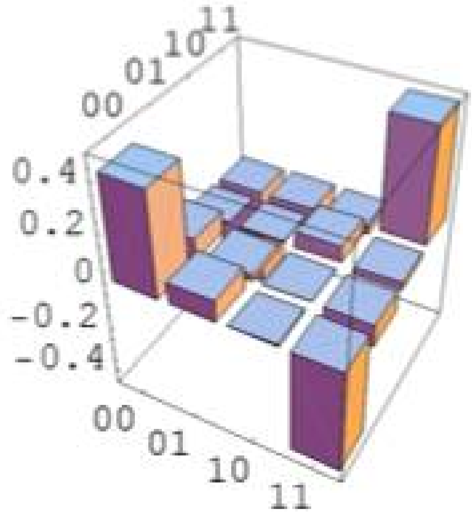
Experimental fidelities for each quantum state obtained by uniformly mixing the data over the measured angles

V. D'Ambrosio, E. Nagali, S. P. Walborn, L. Aolita, S. Slussarenko, L. Marrucci, and F. Sciarrino, *Nature Communication* (in press)

Reference frame-free quantum non-locality



@ 0°



@ 45°

$$|\phi^-\rangle_{\mathrm{p}}^{AB} \xrightarrow{QPs} \frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{L}}^{A}|0\rangle_{\mathrm{L}}^{B} - |1\rangle_{\mathrm{L}}^{A}|1\rangle_{\mathrm{L}}^{B}) = |\phi^-\rangle_{\mathrm{L}}^{AB}.$$

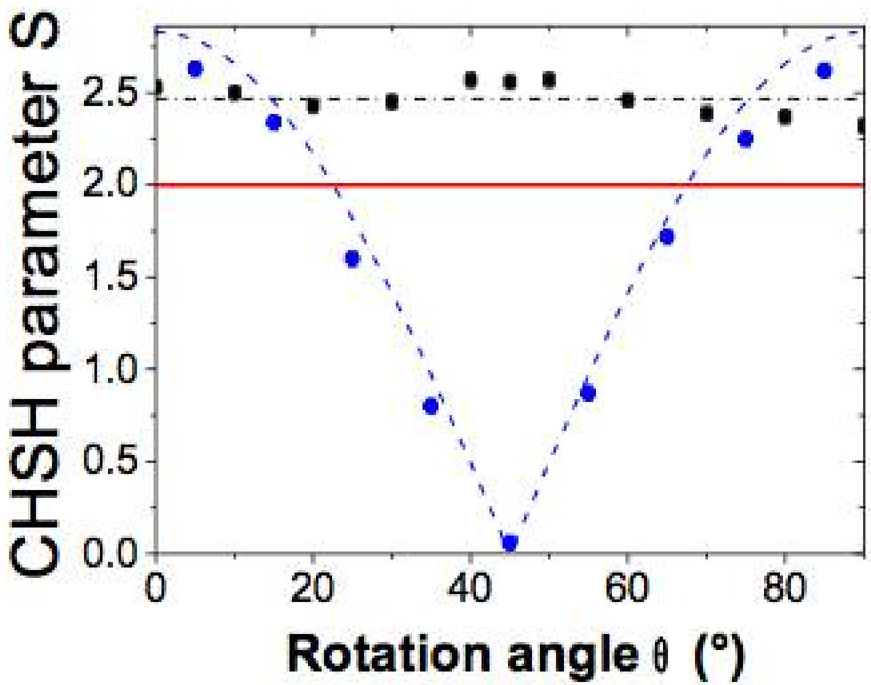
$$F_{45}(\rho_{\mathrm{L}}^{AB}, \rho_{\mathrm{p}}^{AB}) = (96 \pm 1)\%.$$

$$F_0(\rho_{\mathrm{L}}^{\dot{A}B}, \rho_{\mathrm{p}}^{AB}) = (93 \pm 1)\%.$$

$$S \leq 2$$

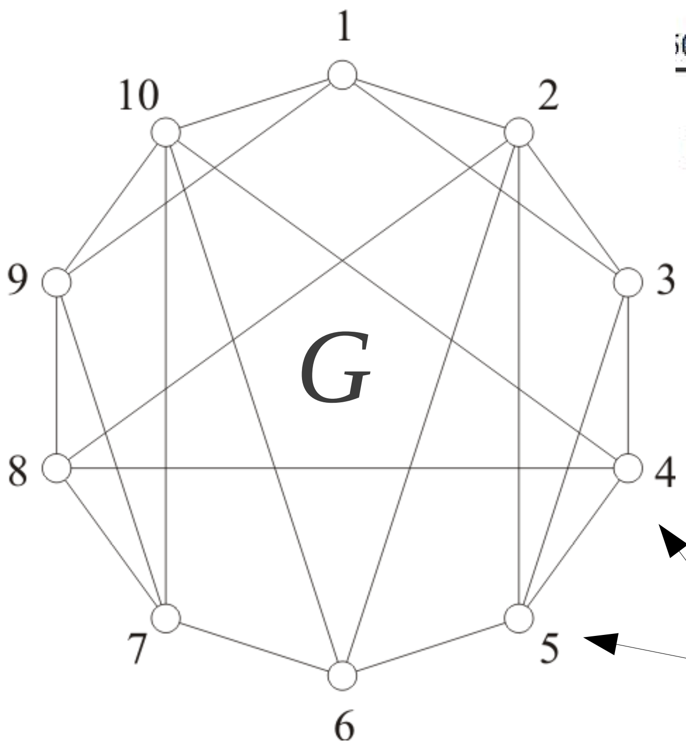
Experimentally

$$S = (2.47 \pm 0.01) > 2$$



V. D'Ambrosio, E. Nagali, S. P. Walborn, L. Aolita, S. Slussarenko, L. Marrucci, and F. Sciarrino, *Nature Communication* (in press)

Ququart: fundamental test and cryptography



Exclusive propositions

...01 (2012) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2012

Experimental Observation of Impossible-to-Beat Quantum Advantage on a Hybrid Photonic System

Eleonora Nagali,[1] Vincenzo D'Ambrosio,[1] Fabio Sciarrino,[1,*] and Adán Cabello[2,3,†]

[1]*Dipartimento di Fisica della "Sapienza" Università di Roma, Roma 00185, Italy*[‡]

[2]*Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain*

[3]*Department of Physics, Stockholm University, S-10691 Stockholm, Sweden*

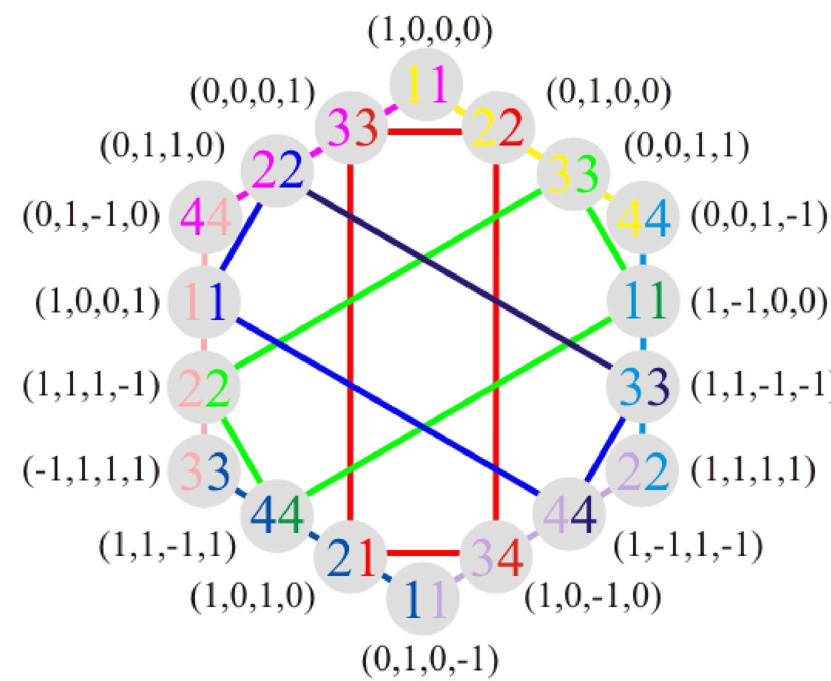
(Received 16 October 2011; published 27 February 2012)

Quantum mechanics reaches the maximum performances allowed by the laws of probability

Hybrid ququart-encoded quantum cryptography protected by Kochen-Specker contextuality

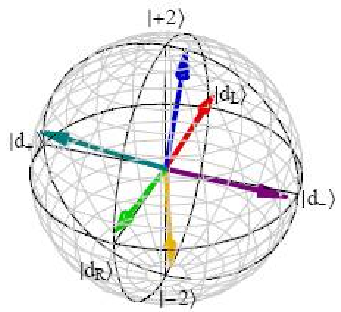
Qubits - Different bases are always disjoint

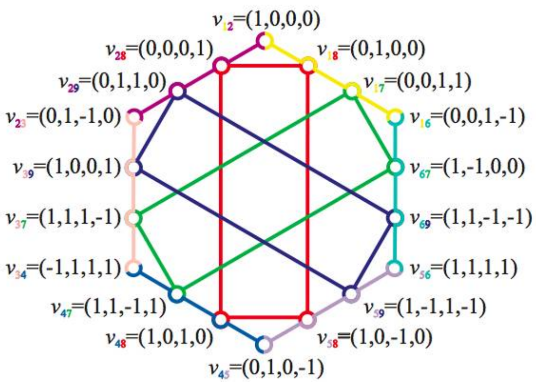
Ququarts - Different bases (colors) may share common elements

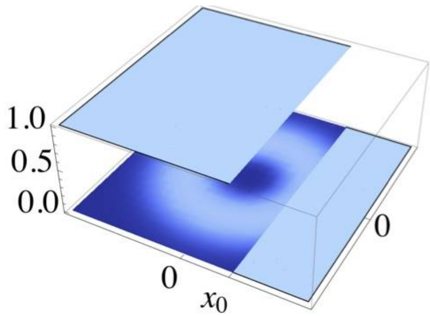


A. Cabello, V. D'Ambrosio, E. Nagali, F. Sciarrino, Phys. Rev. A 84, 030302 (2011).

Conclusions and perspectives







- Qplate: a reliable interface between OAM and polarization
  - Qubit transferrer from polarization to OAM and viceversa
  - Experimental implementation and manipulation of ququart states:
    - Optimal quantum cloning of ququart
    - Fundamental tests of quantum mechanics
    - Contextuality based QKD
  - NEXT STEPS: Higher dimensionality for fundamental test and protocols of quantum information

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