Non-commutative fields in semiclassical gravity, anomalous diffusion and deformed Fock space

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MARIE CURIE ACTIONS

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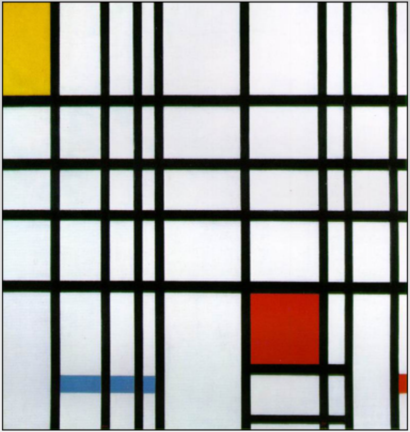
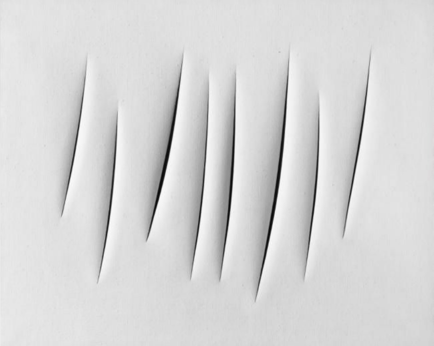
Beyond local QFT



vs.



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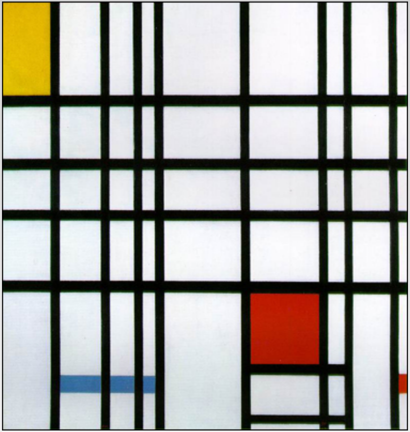

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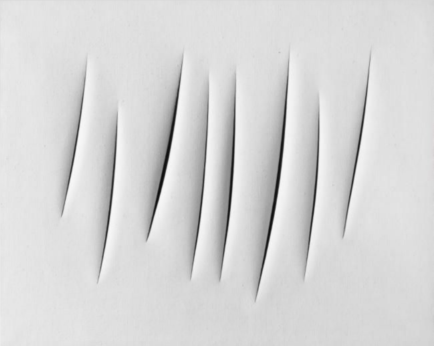

What's the difference in their phase space, in the associated field theories and what happens after quantization?

Outline

- **“Bending” phase space in 3d gravity: group valued momenta and NC-fields**
- **NC heat kernel: running spectral dimension**
- **4d case: κ-Poincaré, κ-Minkowski**
- **κ-Fock space: “hidden entanglement” at the Planck scale**

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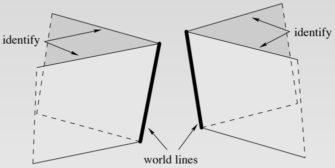

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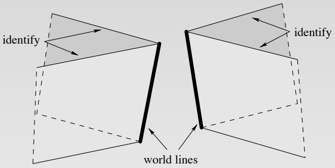

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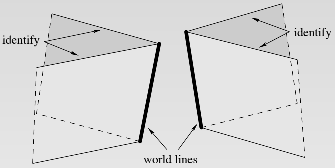

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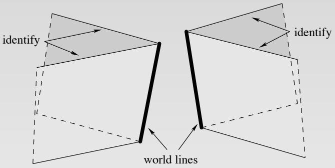

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- *Next step:* characterize the **phase space** of such *topologically gravitating* particle

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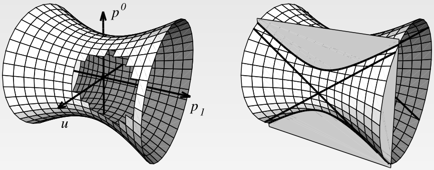
- **Velocity:** *matching condition* at location of particle $\Longrightarrow$ $\dot{\mathbf{x}}(\tau) \equiv \mathbf{P}^{-1}\dot{\mathbf{x}}(\tau)\mathbf{P}$

$$\Downarrow$$

momenta are proportional to the *projection* of $\mathbf{P} \in SL(2,\mathbb{R})$ on its Lie algebra $\mathfrak{sl}(2)$

$$\mathbf{P} = u\mathbb{1} + 4\pi G\vec{p}\cdot\vec{\gamma} \quad \text{with} \quad u^2 - 16\pi^2 G^2 \vec{p}^{\,2} = 1$$



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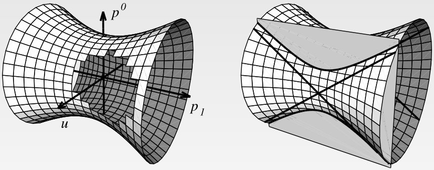

> The (extended) phase space manifold in the presence of "topological" gravitational backreaction becomes $\Upsilon_G = \mathbb{R}^3 \times SL(2,\mathbb{R})$

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Fourier transform maps fields *on the group manifold* to fields on a *dual "spacetime"*

$$\mathcal{F}(f)(x) = \int d\mu_H(\mathbf{P}) f(\mathbf{P})\, e_{\mathbf{P}}(x)\,,$$

where: $e_{\mathbf{P}}(x) = e^{\frac{i}{2\kappa}\mathrm{Tr}(\mathbf{xP})} = e^{i\vec{p}\cdot\vec{x}}$ with $\vec{p} = \frac{\kappa}{2i}\mathrm{Tr}(\mathbf{P}\vec{\sigma})$, $\mathbf{x} = x^i\sigma_i$ and $\kappa = (4\pi G)^{-1}$

Group-valued plane waves and deformed symmetries

...the group structure induces a non-commutative $\star$-**product** for plane waves

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i) differentiating both sides w.r.t. $\mathbf{P}_1$, $\mathbf{P}_2$ and setting momenta to zero

$$[x_i, x_j]_\star = i\kappa\epsilon_{ijk}\, x_k$$

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non-abelian composition of momenta = **non-trivial coproduct**

$$\Delta P_a = P_a \otimes \mathbb{1} + \mathbb{1} \otimes P_a + \frac{1}{\kappa}\,\epsilon_{abc} P_b \otimes P_c + \mathcal{O}(1/\kappa^2)$$

the *smoking gun* of symmetry deformation...P_a belong to a non-trivial Hopf algebra with κ as a **deformation parameter**!

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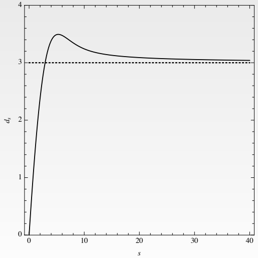
and calculate the *spectral dimension* $d_s = -2 \frac{\partial \log \tilde{T}r K}{\partial \log s} \ldots$ (plot for $G = 1$)



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    $$-\eta_0^2 + \eta_1^2 + \eta_2^2 + \eta_3^2 + \eta_4^2 = \kappa^2 \, ; \qquad \eta_0 + \eta_4 > 0$$

    with $\kappa \sim E_{Planck}$
  - dual Lie algebra "space-time" coordinates

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- consider a one-parameter group splitting of B, $0 \leq |\beta| \leq 1$

  $$e_p \equiv e^{-i\frac{1-\beta}{2}p^0 x_0} e^{ip^j x_j} e^{-i\frac{1+\beta}{2}p^0 x_0} \, .$$

  with momentum composition rules and "antipodes"

  $$p \oplus_\beta q = (p^0 + q^0;\, p^j \; e^{\frac{1-\beta}{2\kappa}q^0} + q^j \; e^{-\frac{1+\beta}{2\kappa}p^0}) \, , \qquad \ominus_\beta p = (-p^0;\, -e^{\frac{-\beta}{\kappa}p^0} p^i) \, .$$

  each choice of β corresponds to a *choice of coordinates* on the group manifold.

κ-Poincaré II

for $\beta = -1$ we have "flat slicing" coordinates

$$\begin{aligned}
\eta_0(p_0, \mathbf{p}) &= \kappa \sinh p_0/\kappa + \frac{\mathbf{p}^2}{2\kappa}\, e^{p_0/\kappa}, \\
\eta_i(p_0, \mathbf{p}) &= p_i\, e^{p_0/\kappa}, \\
\eta_4(p_0, \mathbf{p}) &= \kappa \cosh p_0/\kappa - \frac{\mathbf{p}^2}{2\kappa}\, e^{p_0/\kappa}.
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non-abelian composition of momenta: $p \oplus q = (p^0 + q^0;\ p^j\ e^{-\frac{q^0}{\kappa}} + q^j)$

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- **deformed boost action**

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- **and co-products**

$$\begin{aligned} \Delta(N_j) &= N_j \otimes 1 + e^{-P_0/\kappa} \otimes N_j + \epsilon_{jkl}/\kappa\, P_k \otimes M_l \\ \Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0\,, \qquad \Delta(P_i) = P_i \otimes 1 + \exp(-P_0/\kappa) \otimes P_i \\ \Delta(M_i) &= M_i \otimes 1 + 1 \otimes M_i \end{aligned}$$

- **deformed mass Casimir** $\Rightarrow$ Lorentz invariant hyperboloid on B: $\eta_4 = \mathrm{const.}$

$$C_\kappa(P) = \left(2\kappa \sinh\left(\frac{P_0}{2\kappa}\right)\right)^2 - P_i P^i e^{P_0/\kappa}$$

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$$[N_j, P_l] = i\delta_{lj}\left(\frac{\kappa}{2}\left(1 - e^{-\frac{2P_0}{\kappa}}\right) + \frac{1}{2\kappa}\vec{P}^2\right) + \frac{i}{\kappa} P_l P_j$$

- **and co-products**

$$\begin{aligned}\Delta(N_j) &= N_j \otimes 1 + e^{-P_0/\kappa} \otimes N_j + \epsilon_{jkl}/\kappa\, P_k \otimes M_l \\ \Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0\,, \qquad \Delta(P_i) = P_i \otimes 1 + \exp(-P_0/\kappa) \otimes P_i \\ \Delta(M_i) &= M_i \otimes 1 + 1 \otimes M_i\end{aligned}$$

- **deformed mass Casimir** $\Rightarrow$ Lorentz invariant hyperboloid on B: $\eta_4 = \mathrm{const.}$

$$C_\kappa(P) = \left(2\kappa \sinh\left(\frac{P_0}{2\kappa}\right)\right)^2 - P_i P^i e^{P_0/\kappa}$$

in the limit $\kappa \longrightarrow \infty$ recover ordinary Poincaré algebra

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with $M^2(\mathbf{P}) = C_\kappa(\mathbf{P})\left(1 + \frac{C_\kappa(\mathbf{P})}{4\kappa^2}\right)$ and $d\mu(\mathbf{P})$ the left invariant Haar measure on $AN(3)$

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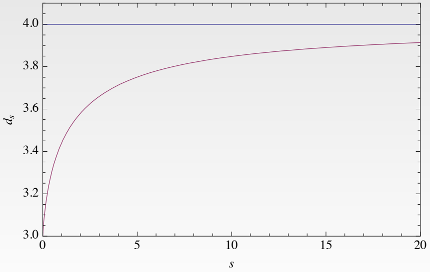
- calculate the *spectral dimension* $d_s = -2\frac{\partial \log \tilde{T}r K}{\partial \log s} \ldots$ (plot for $G = 1$)



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with **same energy** and different linear momentum

$$\mathbf{K_{12}} = \mathbf{k_1} \oplus \mathbf{k_2} = \quad \mathbf{k_1} + (1-\epsilon_1)\mathbf{k_2}$$

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given n-different modes one has $n!$ different n-particle states, one for each permutation of the n modes $\mathbf{k_1}\,, \mathbf{k_2} \ldots \mathbf{k_n}$

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with $\epsilon = \epsilon(\mathbf{k_1}) + \epsilon(\mathbf{k_2})$

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- e.g. the state superposition of two total "classical" energies $\epsilon_A = \epsilon(\mathbf{k}_{1A}) + \epsilon(\mathbf{k}_{2A})$ and $\epsilon_B = \epsilon(\mathbf{k}_{1B}) + \epsilon(\mathbf{k}_{2B})$ can be entangled with the additional hidden modes e.g.

$$|\Psi\rangle = 1/\sqrt{2}(|\epsilon_A\rangle \otimes |\uparrow\rangle + |\epsilon_B\rangle \otimes |\downarrow\rangle)$$

(MA., D. Benedetti, [arXiv:0809.0889 [hep-th]]. MA., A. Marciano, [arXiv:0707.1329 [hep-th]]. MA, A. Hamma, S. Severini, [arXiv:0806.2145 [hep-th]].)

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a new window to phenomenological effects??

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- Strong motivations to look at such deformations from **2+1 gravity coupled to relativistic particles**...application: appearance of *running spectral dimension*
- In **3+1 dimensions** the only known example of symmetry deformation with group valued momenta is κ-Poincaré: field theory exhibits similar features to the 2+1 case
- At the multiparticle level the non-trivial behaviour of field modes leads to a *fine structure* of Fock space: interesting **entanglement** phenomena can take place
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