Klaus Hornberger

Environmental distinction of pointer states

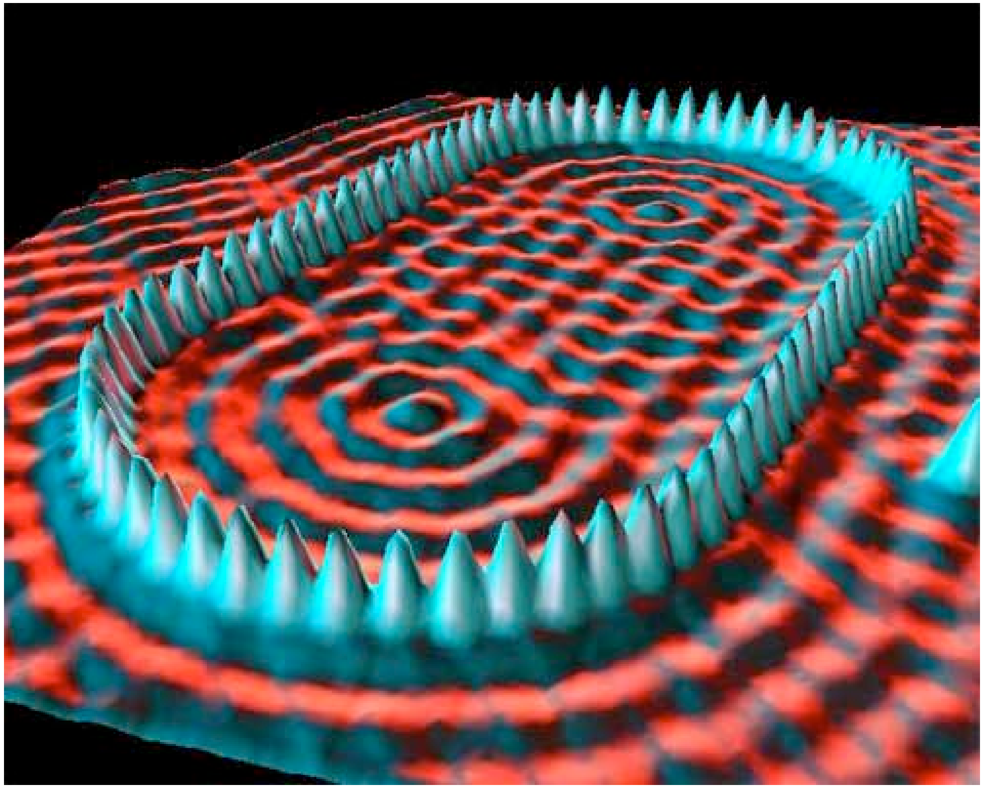
(joint work with M. Busse & J. Trost)

The quantum-classical divide

Classical world of our everyday experience

(Image: Spielzeug-Truhe.de)

Quantum world of elementary particles



(Image: Crommie, Lutz & Eigler; IBM)

Pointer states

“Into what mixture does
the wave packet collapse?” (Zurek 1981)

“Predictability sieve” (Zurek, Habib, Paz 1993)

“Hilbert-Schmidt robustness” (Gisin, Rigo 1995,
Diósi, Kiefer 2000)

Pointer states

Given a master equation $\partial_t \rho \; = \; \mathcal{L}\rho$
a set of projectors $\{P_j(t)\}$ may be called *pointer states* of $\mathcal{L}$
provided there is a decoherence time scale t_{dec}
such that for all ρ_0

$$\mathrm{e}^{\mathcal{L}t}\rho_0 \;\cong\; \sum_j \mathrm{Tr}[P_j(0)\,\rho_0]\; P_j(t) \qquad \text{for} \;\; t > t_{\mathrm{dec}}$$

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How do the statistical weights emerge?

How to identify the $\hat{P}_j$ and their equation of motion?

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Environmental distinction of pointer states

plan of the talk:

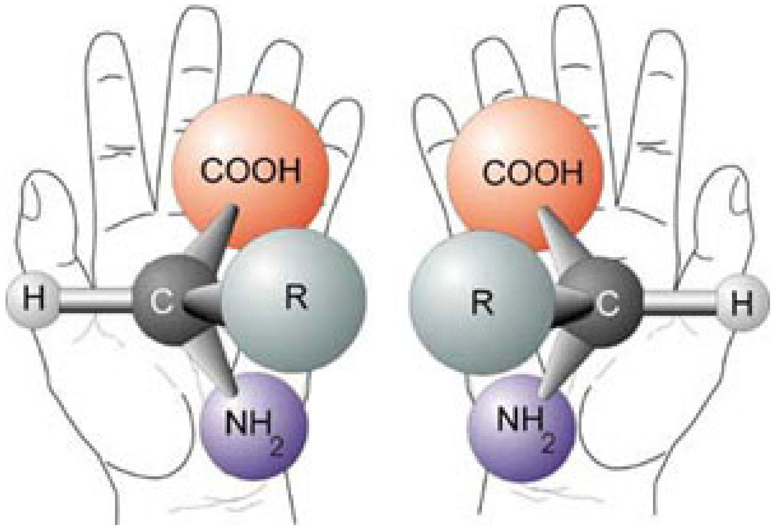
- Hund's paradox

  – super-selecting chiral molecular configuration states –

- Pointer states of motion

  – the pointer basis induced by collisional decoherence –

Hund's paradox of molecular chirality

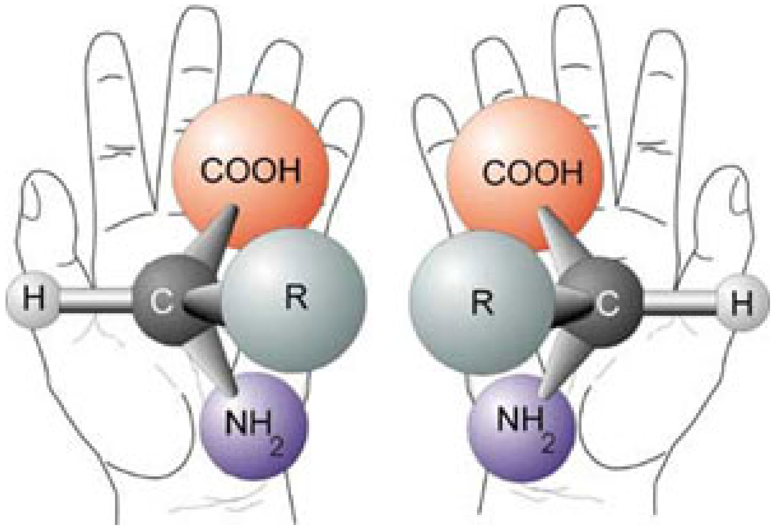


Friedrich Hund (1927)

Why are many molecules found in a chiral configuration?

—in spite of the parity invariance of their hamiltonian?

Hund's paradox of molecular chirality



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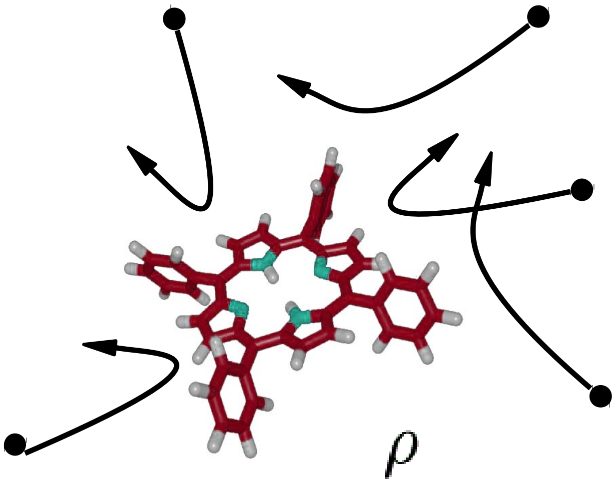
Why are many molecules found in a chiral configuration?

—in spite of the parity invariance of their hamiltonian?

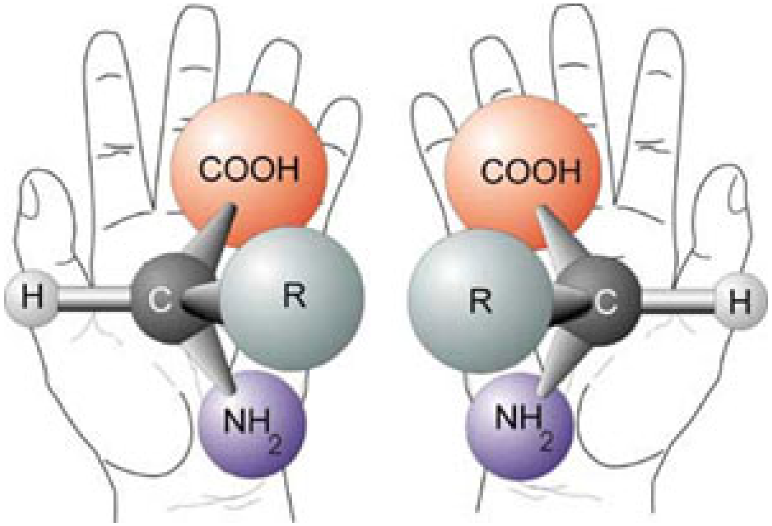
Effect of an *achiral* gas environment on the configuration & orientation state?

realistic master equation required !



Hund's paradox of molecular chirality

Effect of an *achiral* gas environment

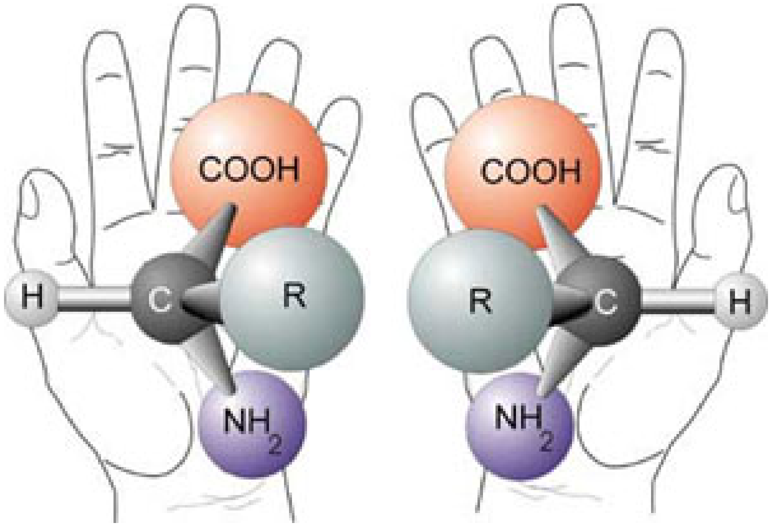


- $|L\rangle + \mathrm{e}^{i\varphi}|R\rangle$ decay with decoherence rate

$$\gamma \;=\; n_{\mathrm{gas}} \left\langle v \underbrace{\int \frac{\mathrm{d}\boldsymbol{n}\,\mathrm{d}\boldsymbol{n}_0}{8\pi} \left| f^{(L)}_{\alpha,\alpha_0}(v\,\boldsymbol{n}, v\,\boldsymbol{n}_0) - f^{(R)}_{\alpha,\alpha_0}(v\,\boldsymbol{n}, v\,\boldsymbol{n}_0) \right|^2}_{\text{“decoherence cross section”}} \right\rangle_{v,\alpha,\alpha_0}$$

Hund's paradox of molecular chirality

Effect of an *achiral* gas environment



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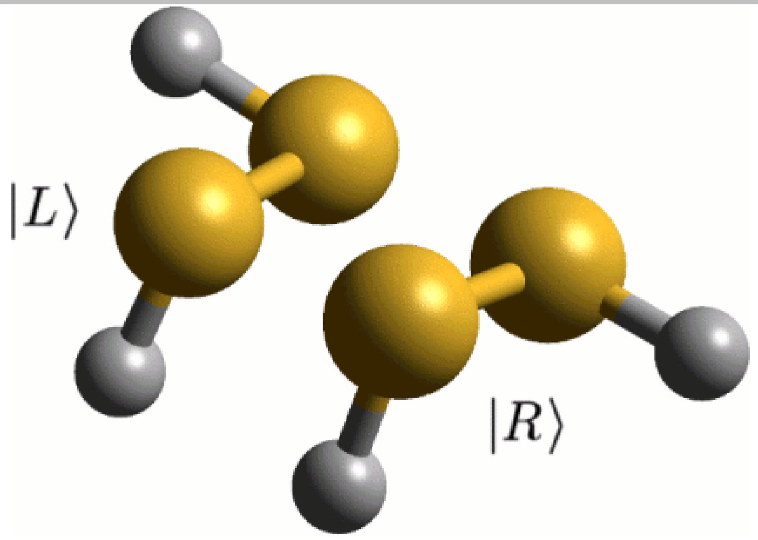
- *only* the chiral states $|L\rangle$ and $|R\rangle$ exhibit a quantum-Zeno-like stabilization $\sim\omega^2/\gamma$ against tunneling and decay if $\gamma \gg \omega$

Harris, Stodolsky (1978)

Hund's paradox of molecular chirality

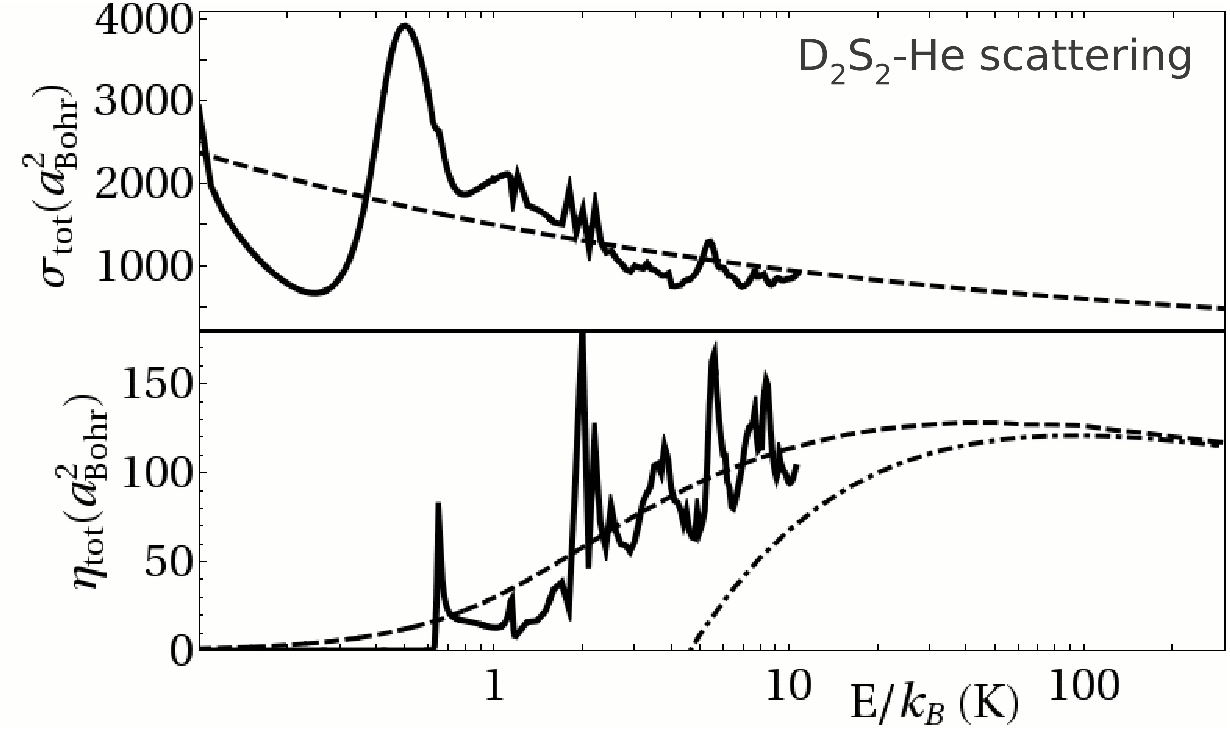


D_2S_2 tunnels with 28 Hz in vacuum

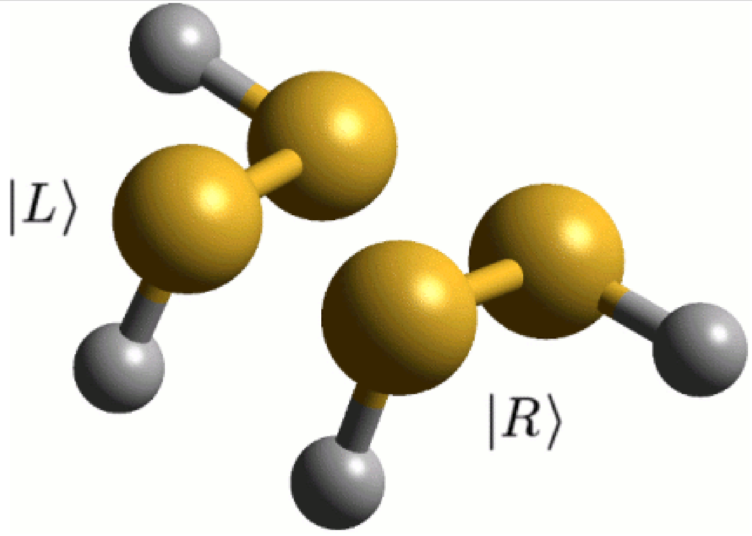
The stabilization effect is dominated by a higher order contribution to the van der Waals interaction described by the EQED tensor $A_{j,k\ell}(i\omega)$

scattering cross section

decoherence cross section



Hund's paradox of molecular chirality



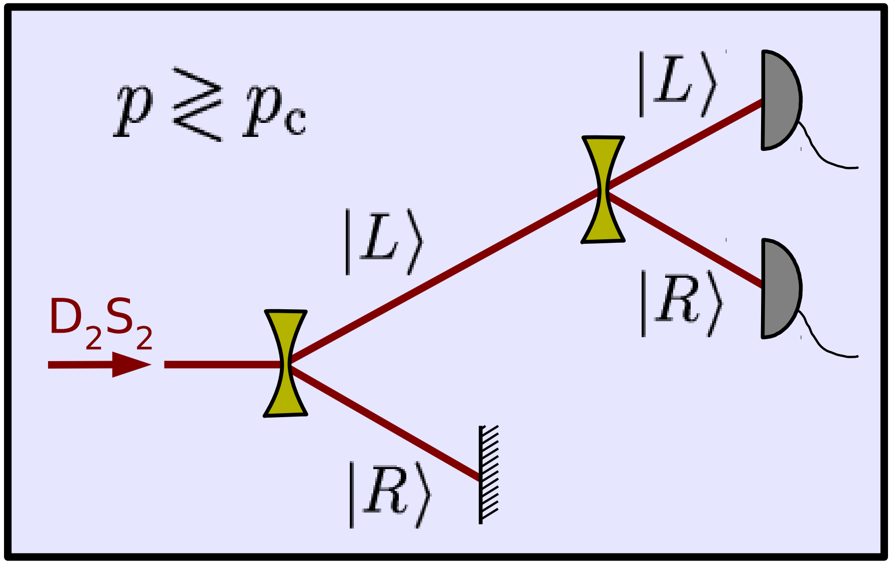
D_2S_2 tunnels with 28 Hz in vacuum

The stabilization effect is dominated by a higher order contribution to the van der Waals interaction described by the EQED tensor $A_{j,k\ell}(i\omega)$

critical pressure in 300K He atmosphere:

$$p_c = 1.6\times 10^{-5}\,\text{mbar}$$

... allows one to observe the chiral stabilization in an optical Stern-Gerlach type setup [e.g. Li, Bruder, Sun: PRL 2007]



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Environmental distinction of pointer states

plan of the talk:

- Hund's paradox

  – super-selecting chiral molecular configuration states –

- Pointer states of motion

  – the pointer basis induced by collisional decoherence –

reminder: definition of Pointer states

Given a master equation $\partial_t \rho \;=\; \mathcal{L}\rho$
a set of projectors $P_j(t)$ may be called *pointer states* of $\mathcal{L}$
provided there is a decoherence time scale t_{dec}
such that for all ρ_0

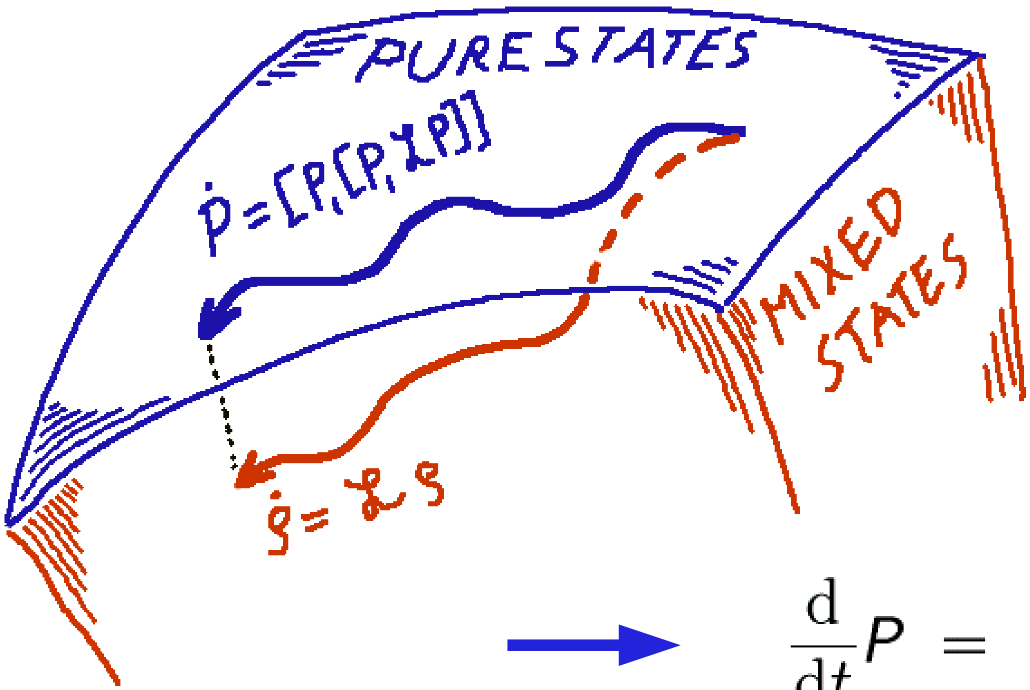
$$\mathrm{e}^{\mathcal{L}t}\rho_0 \;\cong\; \sum_j \mathrm{Tr}[P_j(0)\,\rho_0]\;P_j(t) \qquad \text{for} \;\; t > t_{\mathrm{dec}}$$

Continuous variable version

$$\mathrm{e}^{\mathcal{L}t}\rho_0 \;\cong\; \int \mathrm{d}\alpha\;\mathrm{prob}(\alpha\,|\,\rho_0) P_\alpha(t) \qquad \text{for} \;\; t > t_{\mathrm{dec}}$$

with $\int \mathrm{d}\alpha\;\mathrm{prob}(\alpha\,|\,\rho_0) = 1$

Nonlinear equation for candidate pointer states



among all e.o.m. propagating P within the manifold of pure states, choose the one minimizing $\|\dot{P} - \mathcal{L}\|^2_{\mathrm{HS}}$

$$\frac{\mathrm{d}}{\mathrm{d}t} P = [P, [P, \mathcal{L}P]]$$

(Rigo&Gisin 1995, Strunz 2002)

corresponds to the deterministic part of a particular unraveling of $\mathcal{L}$

In vector representation $P = |\psi\rangle\langle\psi|$

$$\partial_t |\psi\rangle = \frac{1}{i\hbar}(H - \langle H\rangle_\psi)|\psi\rangle + \sum_k \left\{ \langle L_k^\dagger\rangle_\psi (L - \langle L_k\rangle_\psi) - \frac{1}{2}\Big(L_k^\dagger L_k - \langle L_k^\dagger L_k\rangle_\psi \Big) \right\} |\psi\rangle$$

(Diosi 1986)

Orthogonal unraveling $\rho = \mathbb{E}[|\psi\rangle\langle\psi|]$

piecewise deterministic evolution

$$\partial_t|\psi\rangle \;=\; \frac{1}{i\hbar}(H - \langle H\rangle_\psi)|\psi\rangle + \sum_k \left\{\langle L_k^\dagger\rangle_\psi (L - \langle L_k\rangle_\psi) - \frac{1}{2}\Big(L_k^\dagger L_k - \langle L_k^\dagger L_k\rangle_\psi\Big)\right\}|\psi\rangle$$

interrupted by jumps to orthogonal states

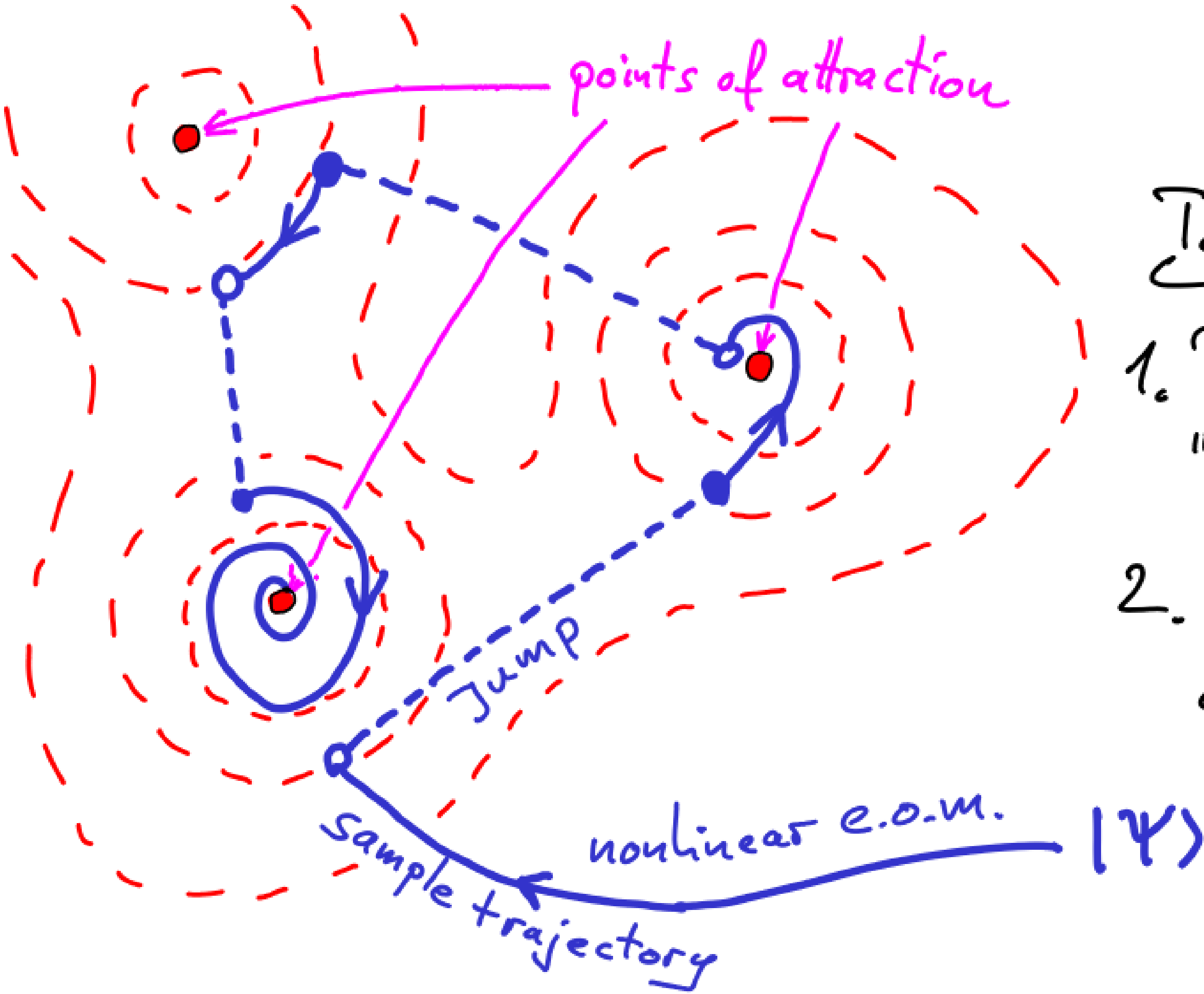
$$|\psi\rangle \;\to\; \frac{1}{\sqrt{r_k}}(L_k - \langle L_k\rangle_\psi)|\psi\rangle \qquad \text{with rate } r_k \;=\; \langle L_k^\dagger L_k\rangle_\psi - \langle L_k^\dagger\rangle_\psi\langle L_k\rangle_\psi$$

total jump rate $\displaystyle\sum_k r_k \;=\; -\,\mathrm{Tr}(\rho\,\mathcal{L}\rho)$

entropy production rate!

If there are "points of attraction" with vanishing jump rate, an ensemble of (candidate) pointer states is naturally generated

Orthogonal unraveling – sample trajectory



To be shown:

1. Points of attraction "behave classically"
2. Arrival probabilities are given by Mr. Born

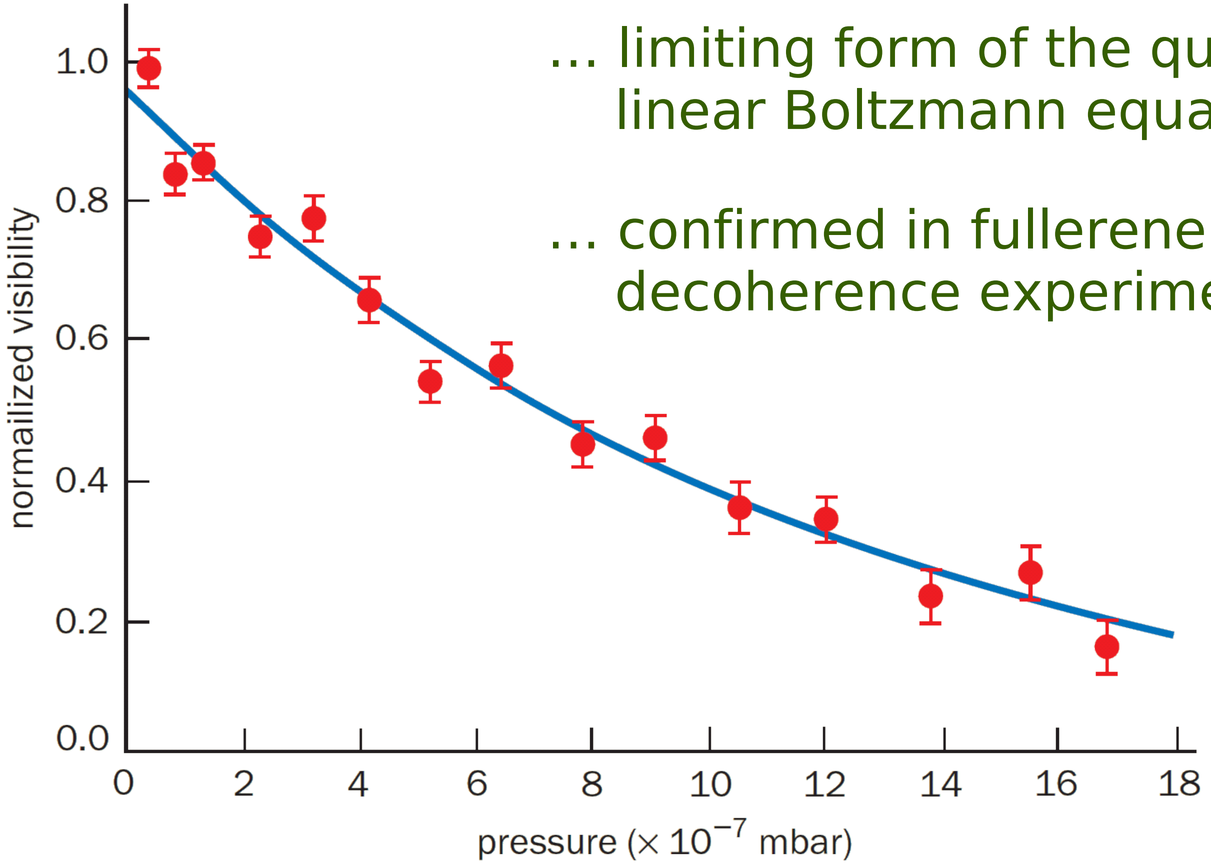
If there are "points of attraction" with vanishing jump rate, an ensemble of (candidate) pointer states is naturally generated

Collisional decoherence master equation

… describes particle "localization" by gas collisions

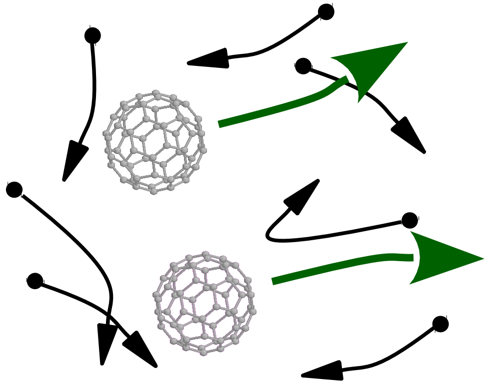
$$\mathcal{L}\rho = \frac{1}{i\hbar}[H,\rho] + \gamma \int \mathrm{d}q\, G(q)\left(\mathrm{e}^{ixq}\rho\,\mathrm{e}^{-ixq} - \rho\right)$$

G(q) : momentum exchange distribution



… limiting form of the quantum linear Boltzmann equation (Vacchini & KH, Phys Rep 2009)

… confirmed in fullerene buckyball decoherence experiments (KH et al, PRL 2003)



Nonlinear e.o.m. for collisional decoherence

$$\partial_t \psi(x) \;=\; -\frac{\hbar}{2m\,i}\partial_x^2 \psi(x) + \gamma\psi(x)\left(|\psi|^2 * \tilde{G}(x) - \int \mathrm{d}y\, |\psi(y)|^2 \left(|\psi|^2 * \tilde{G} \right)(y) \right)$$

Nonlinear e.o.m. for collisional decoherence

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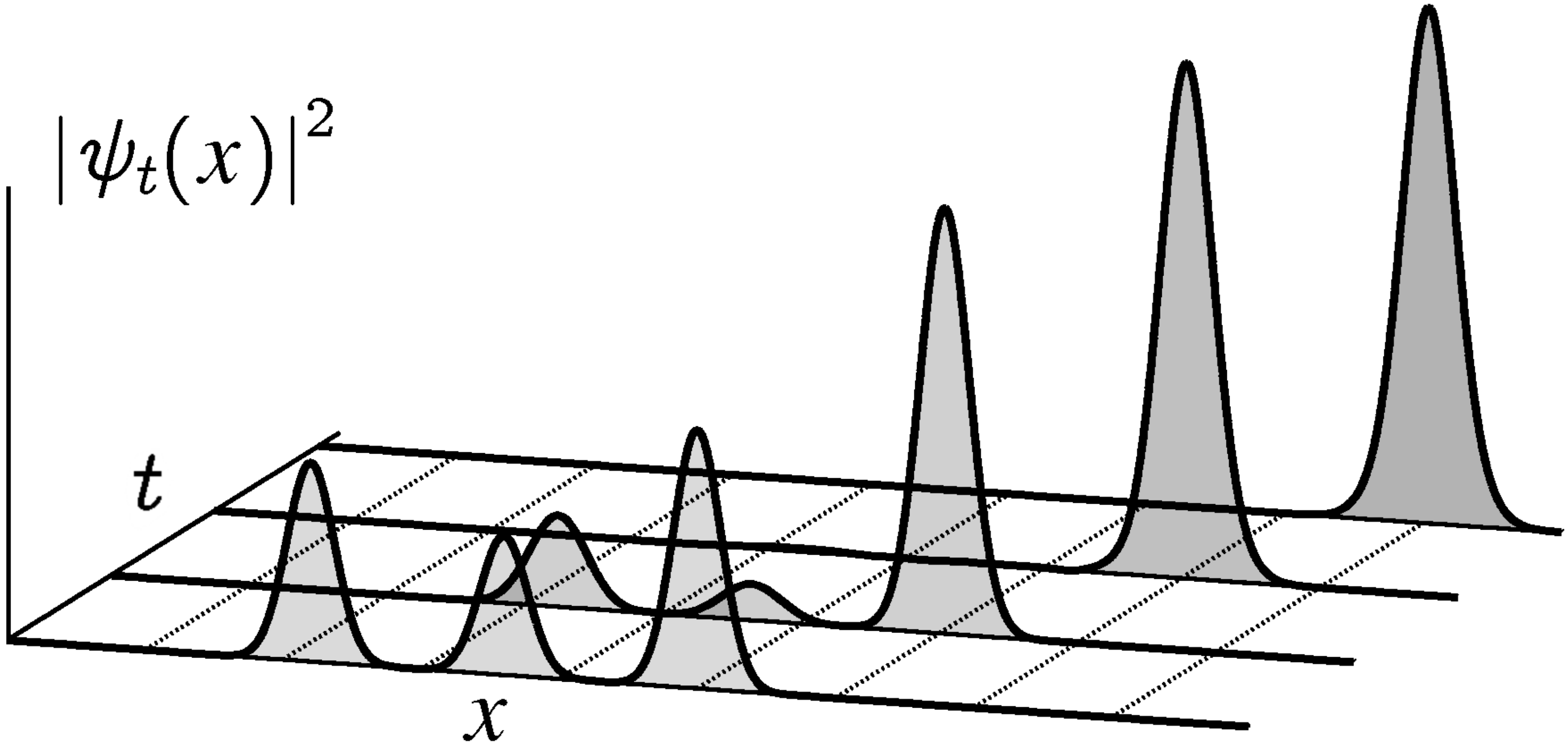
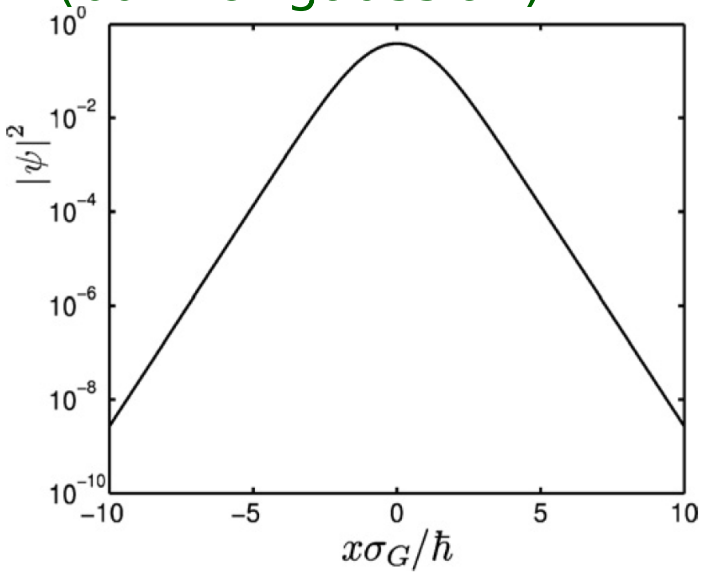
...exhibits *soliton-like solutions*, our candidate pointer states

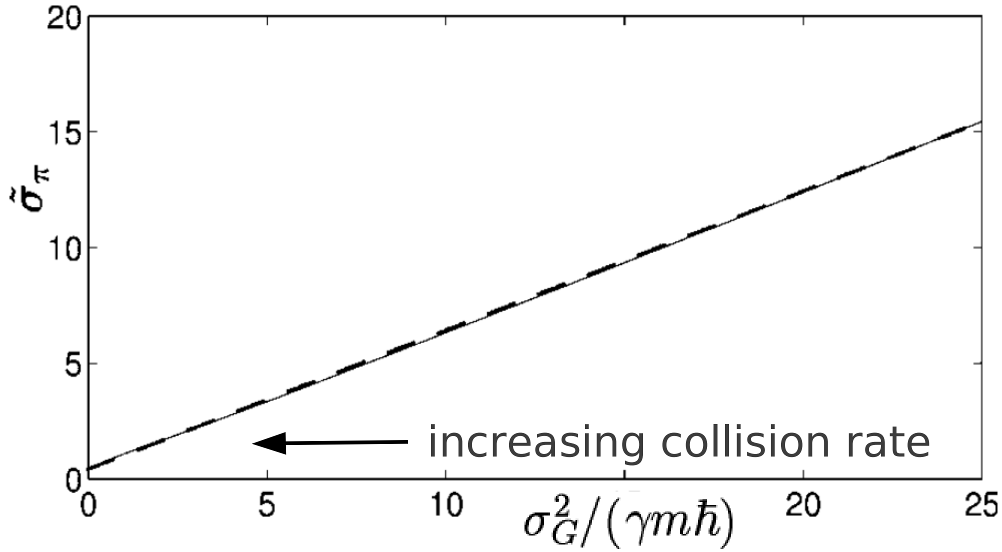


Properties of the (candidate) pointer states

exponentially localized
(but not gaussian)



increasing coupling
decreases their width



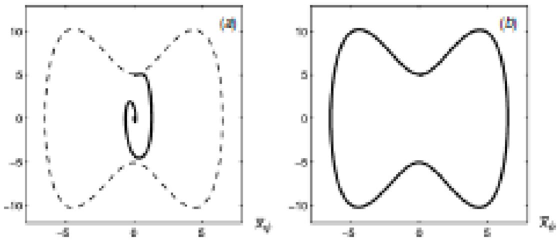
provide an
overcomplete basis

$$\int \mathrm{d}\Gamma\, I(\Gamma)\, P_\Gamma \;=\; I$$

(follows with covariance properties
of master the master equation)

move on the classical
Newtonian trajectories

...in the limit of strong coupling

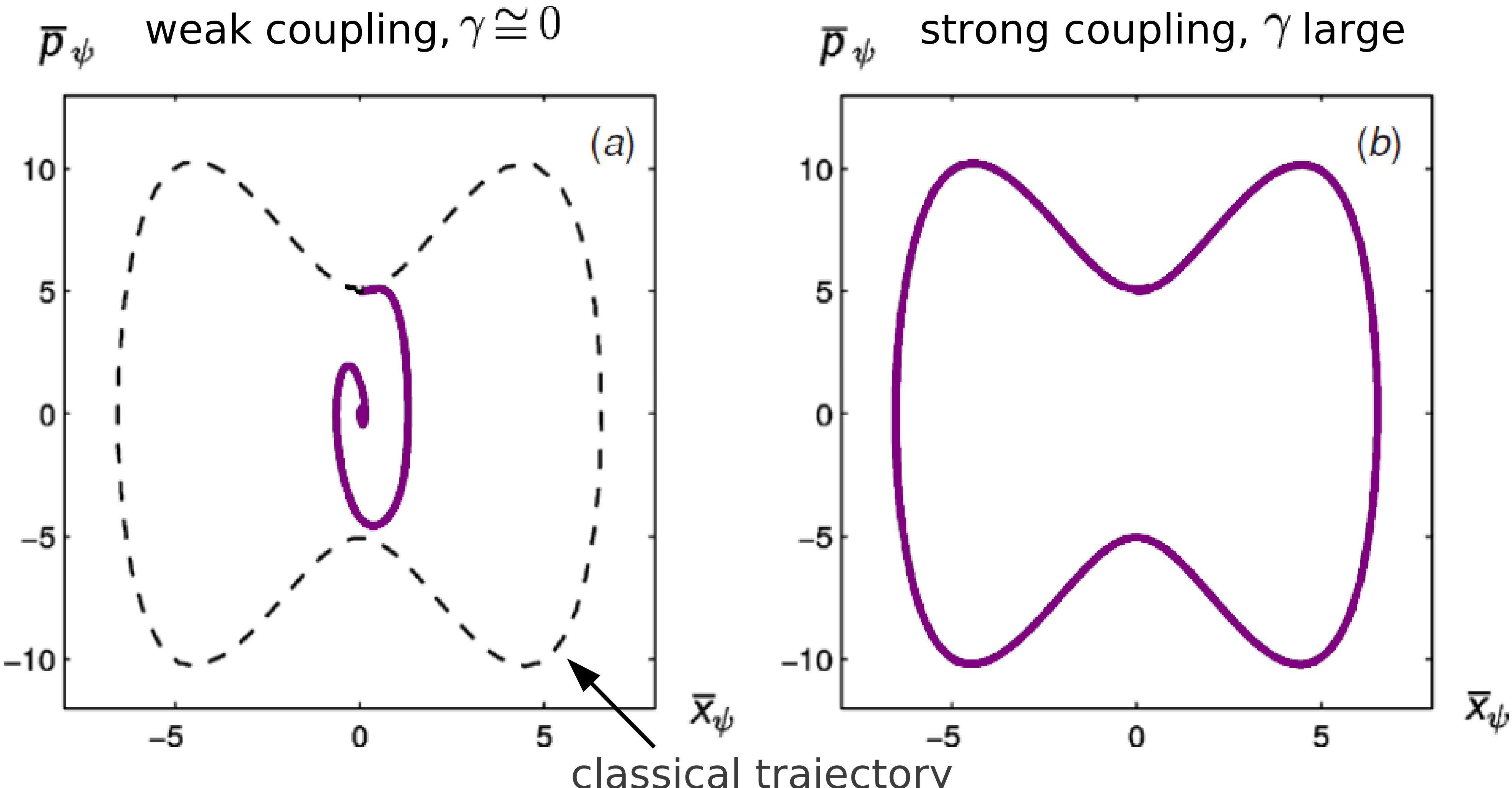


see
next
slide...

Properties of the (candidate) pointer states

phase space dynamics
in a quartic potential

$$V(x) = a\,x^4 + b\,x^2$$



M. Busse & K.H., JPA (2010)

The statistical weights

Superposing N spatially non-overlapping wave packtes,

$$|\psi_0\rangle = \sum_{i=1}^{N} c_i|\phi_i\rangle \qquad \phi_i(x)\,\phi^*_{j\neq i}(x) = 0$$

the stochastic process can be mapped to the coefficients $c_1, \ldots, c_N$

deterministic evolution:

$$\frac{\mathrm{d}}{\mathrm{d}t} c_i = -\left(\sum_{j=1}^{N} F_{ij}\,|\,c_j\,|^2 - \sum_{j,k=1}^{N} F_{jk}\,|\,c_j\,|^2\,|\,c_k\,|^2 \right) c_i$$

with localization rates $F_{ij} = \gamma\left\{1 - \tilde{G}\left(\langle x\rangle_{\phi_i} - \langle x\rangle_{\phi_j}\right)\right\}$

The statistical weights

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jumps:

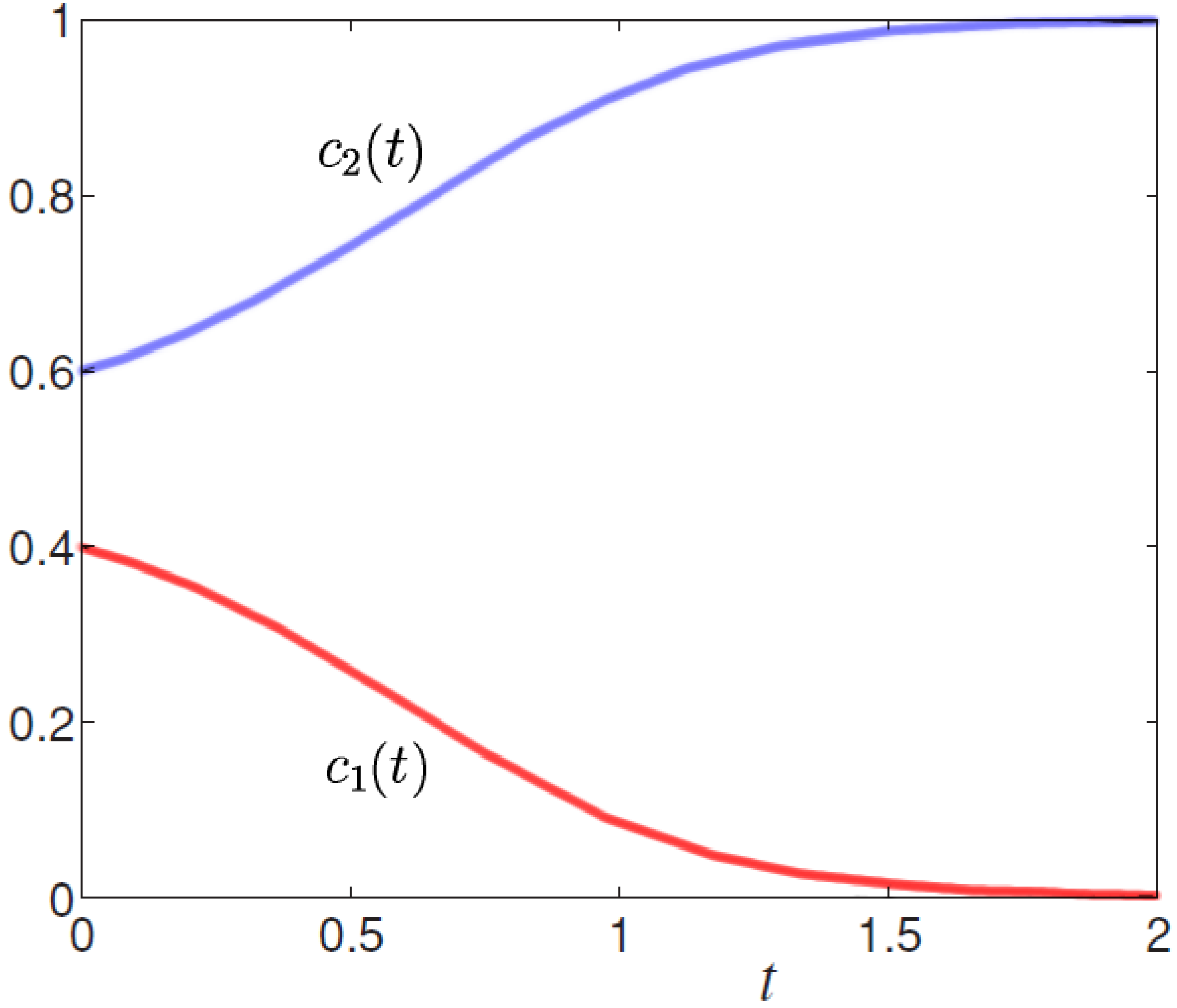
$$c_i^{(q)} \rightarrow \mathcal{N}_q \left(e^{iq\langle \mathsf{x}\rangle_{\phi_i}/\hbar} - \sum_{j=1}^{N} |\,c_j\,|^2\, e^{iq\langle \mathsf{x}\rangle_{\phi_j}/\hbar} \right) c_i$$

with localization rates $F_{ij} = \gamma\left\{1 - \tilde{G}\left(\langle \mathsf{x}\rangle_{\phi_i} - \langle \mathsf{x}\rangle_{\phi_j}\right)\right\}$

and jump rates $r^{(q)} = \gamma\, G\,(\,q\,)\left(1 - \sum_{i,j=1}^{N} |\,c_i\,|^2\,|\,c_j\,|^2 e^{iq\left(\langle \mathsf{x}\rangle_{\phi_i} - \langle \mathsf{x}\rangle_{\phi_j}\right)/\hbar} \right)$

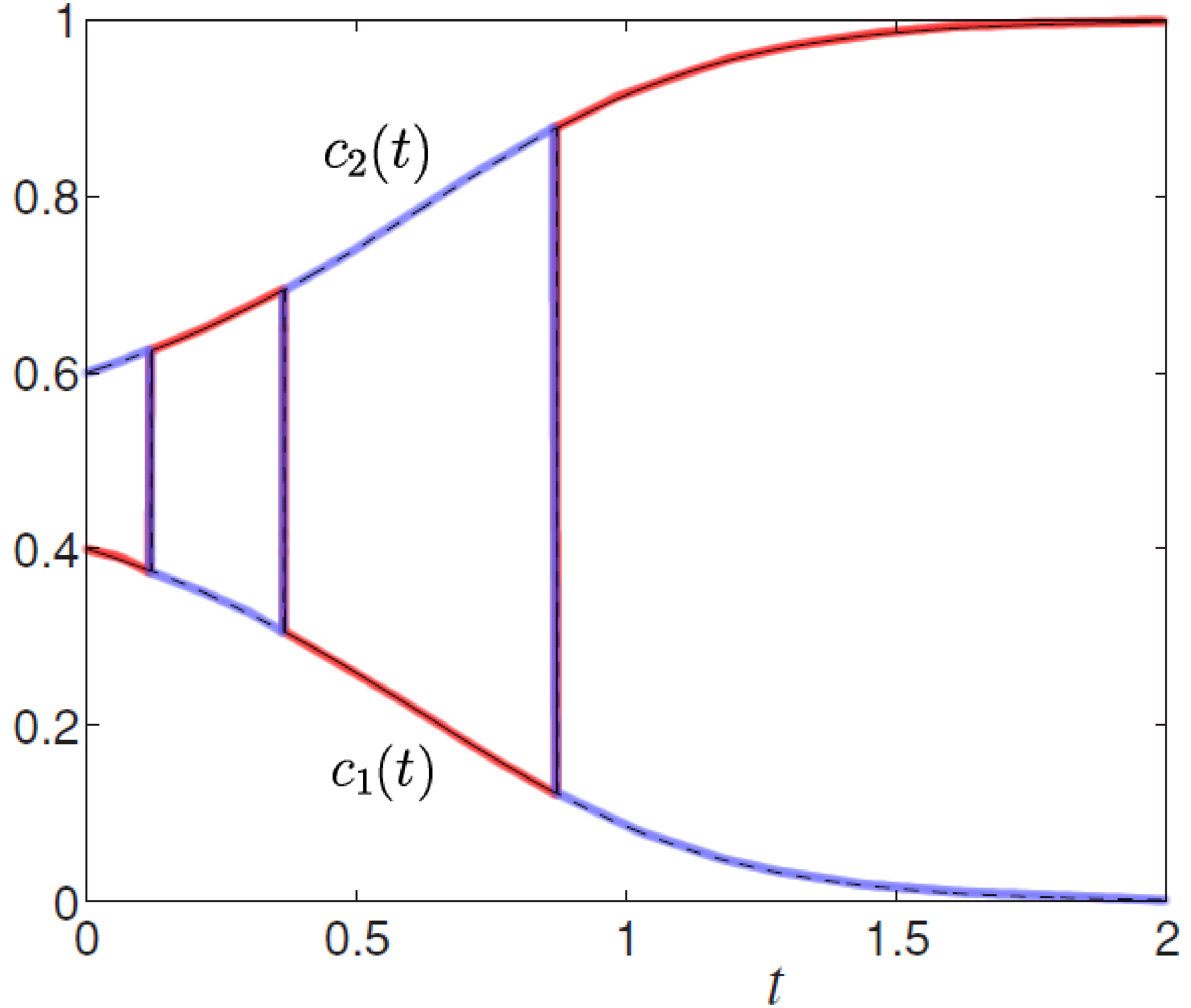
The statistical weights, N=2

deterministic evolution



The statistical weights, N=2

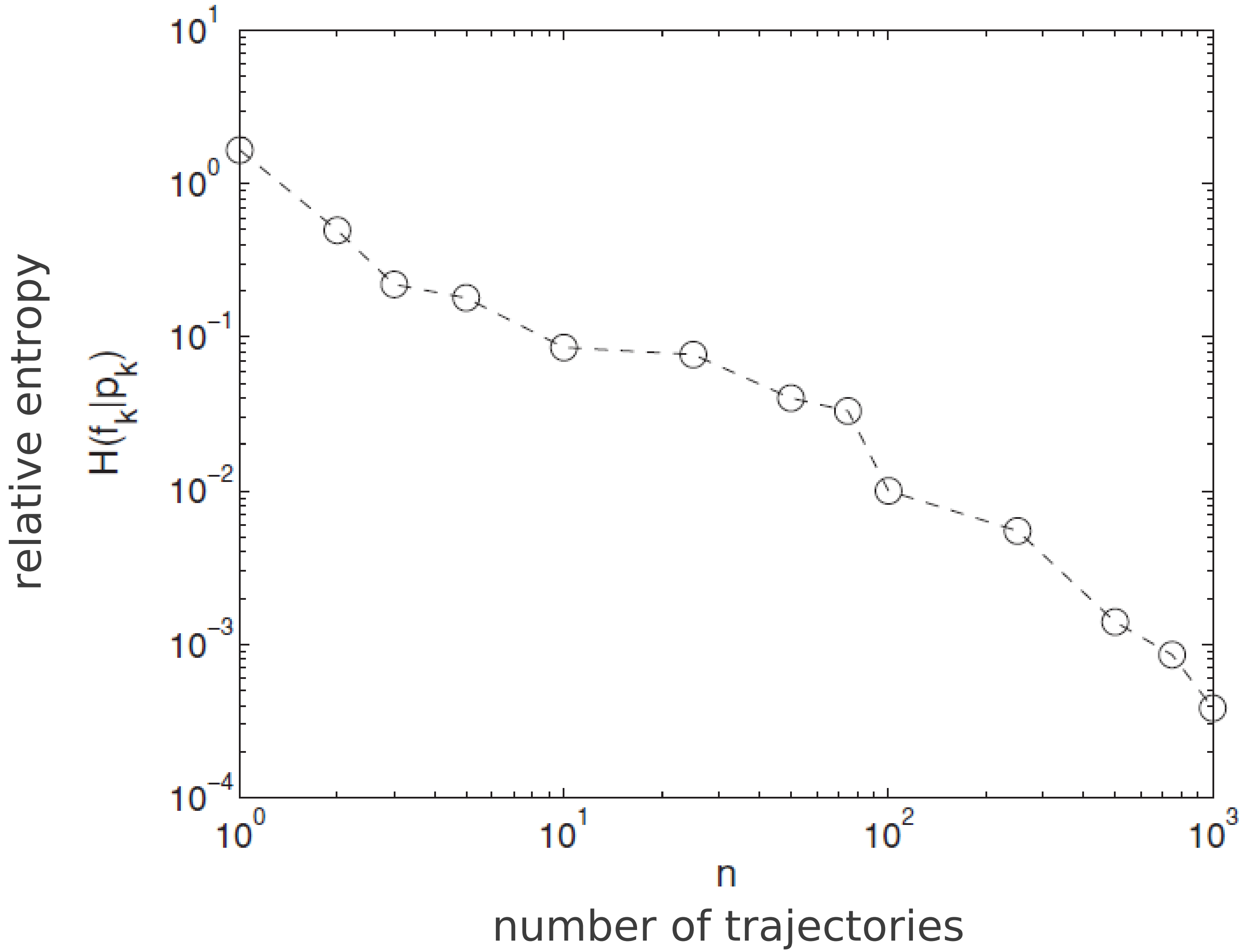
stochastic process analytically tractable



$$\mathrm{Prob}(c_1(\infty)=1)=\mathrm{Prob}(\text{ odd jumps})=1-\frac{1}{2}\exp\left(-2\int_0^\infty \mathrm{d}t \int \mathrm{d}q\ r_t^{(q)}\right)=|c_1(0)|^2$$ ✓

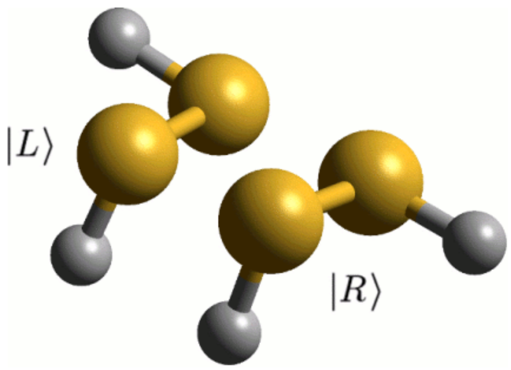
The statistical weights, $N>2$

numerical analysis confirms $\mathrm{Prob}(c_j(\infty)=1) \;=\; |c_j(0)|^2$

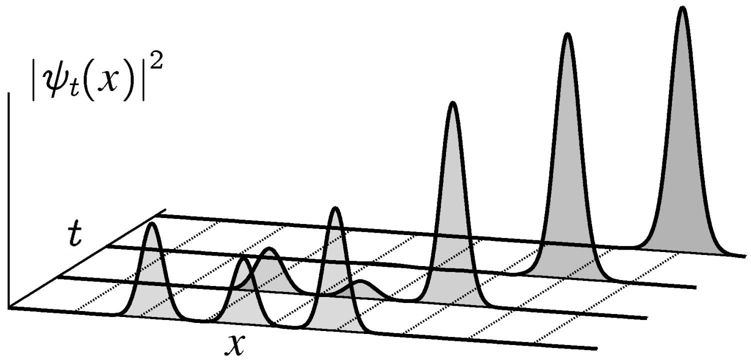


Summary

- Hund's paradox
  - – super-selecting chiral molecular configuration states –



- Pointer states of motion
  - – the pointer basis induced by collisional decoherence –



acknowledgments

theory

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experiments

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