A quantum cellular automaton extension of quantum field theory

Giacomo Mauro D'Ariano
Università di Pavia

Open Problems in Quantum Mechanics Workshop
20-22 June 2012, Frascati

arXiv:1012.0535

INFORMATION-THEORETICAL PRINCIPLES

Principles for Quantum Theory


PRA **84** 012311 (2012)

P1. ***Causality***

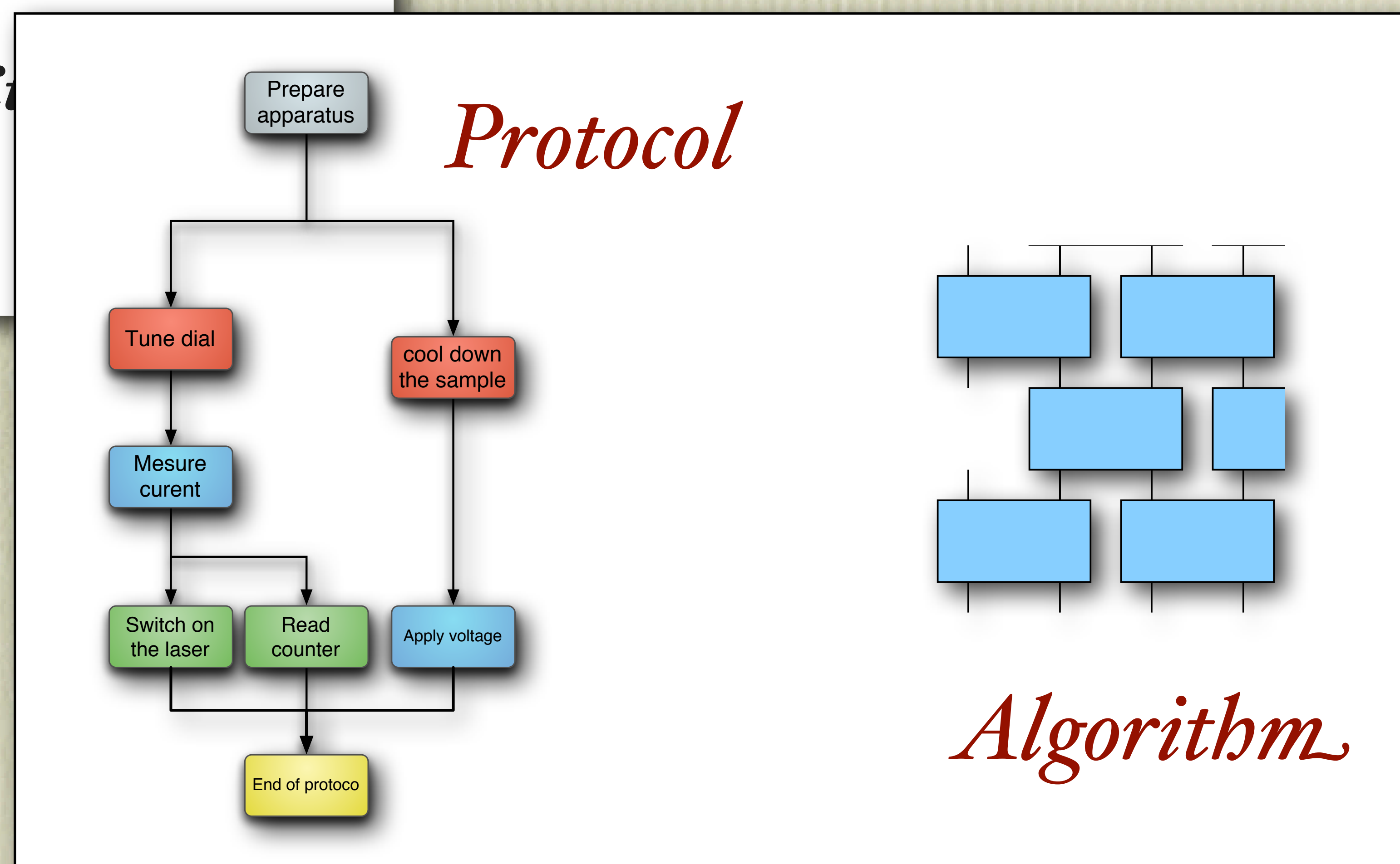
P2. ***Local Readability***

P3. ***Conservation of Information (Purification)***

P4. ***Indivisibility of Composition***

P5. ***Discriminability of Specific Ini***

P6. ***Lossless Compressibility***



INFORMATIO

Principles for Qua

P1. ***Causality***

P2. ***Local Readability***

P

P3. ***Conservation of Inform***

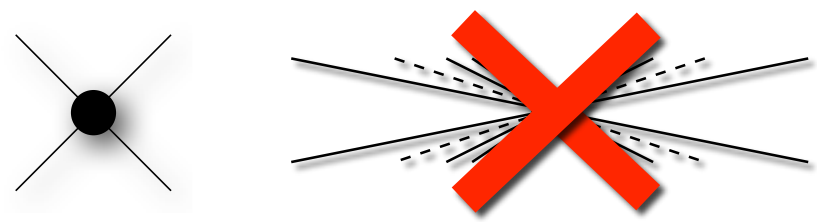
ic Init

Deutsch-Church-Turing principle

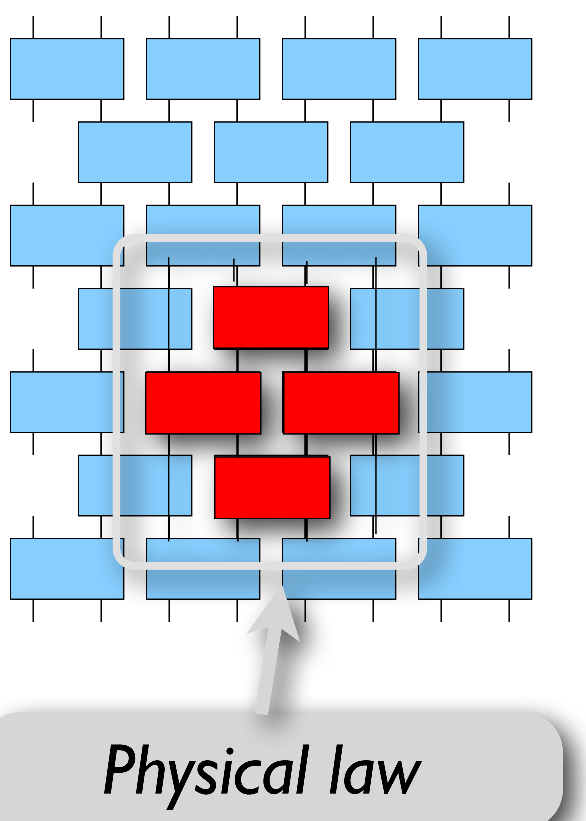
Every finite physical protocol must be perfectly simulated by a quantum computer made with a finite number of qubits and a finite number of gates

Quantum-information density is bounded

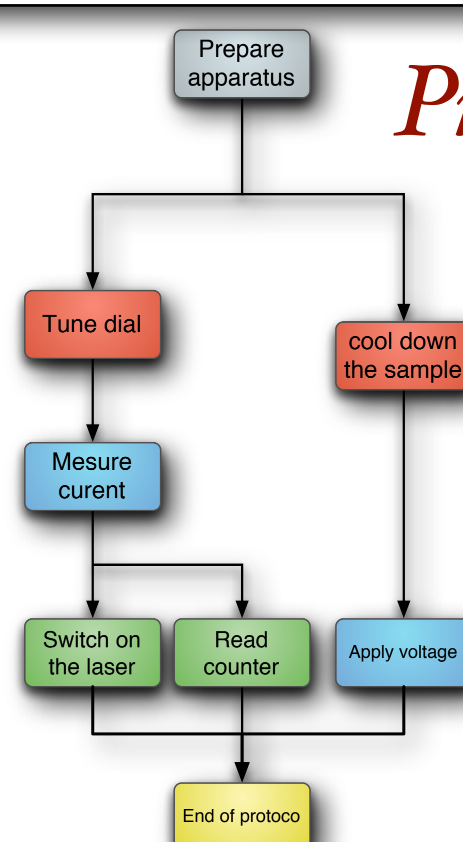
Locality of interactions



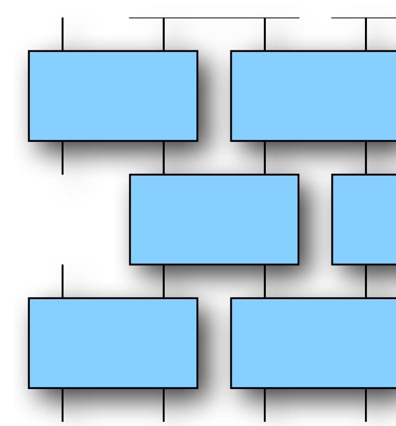
Topological Homogeneity



Protocol

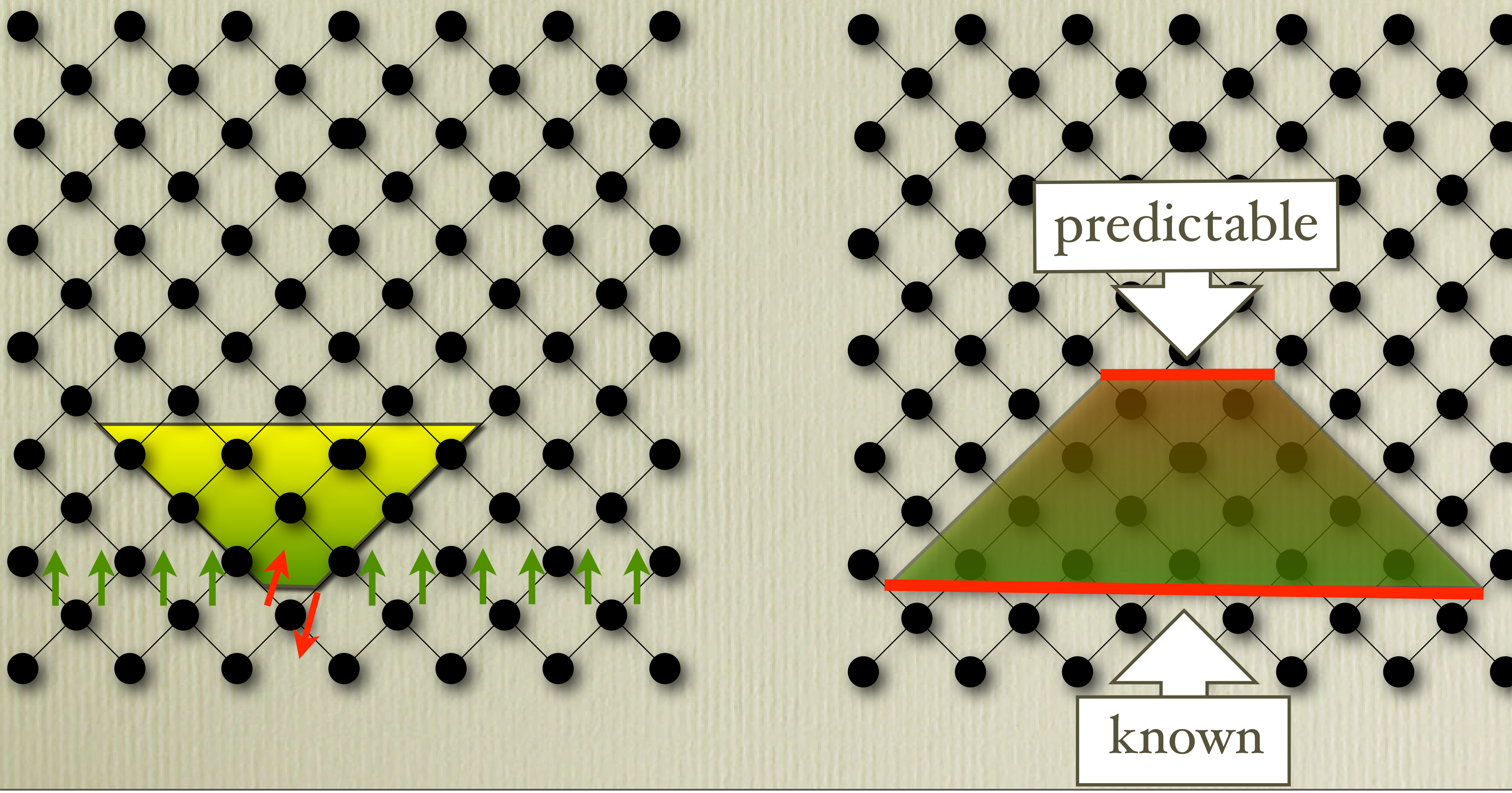


Algorithm

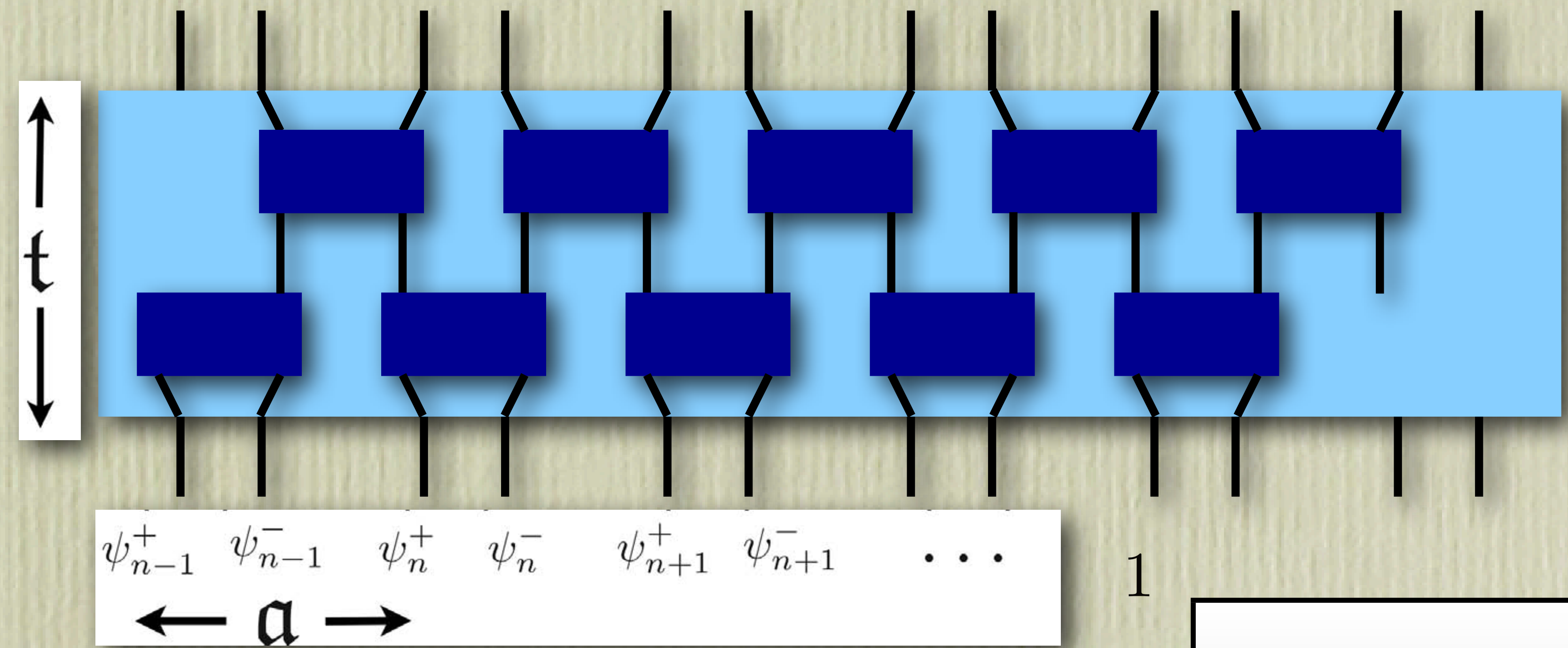


FINITE INFORMATION

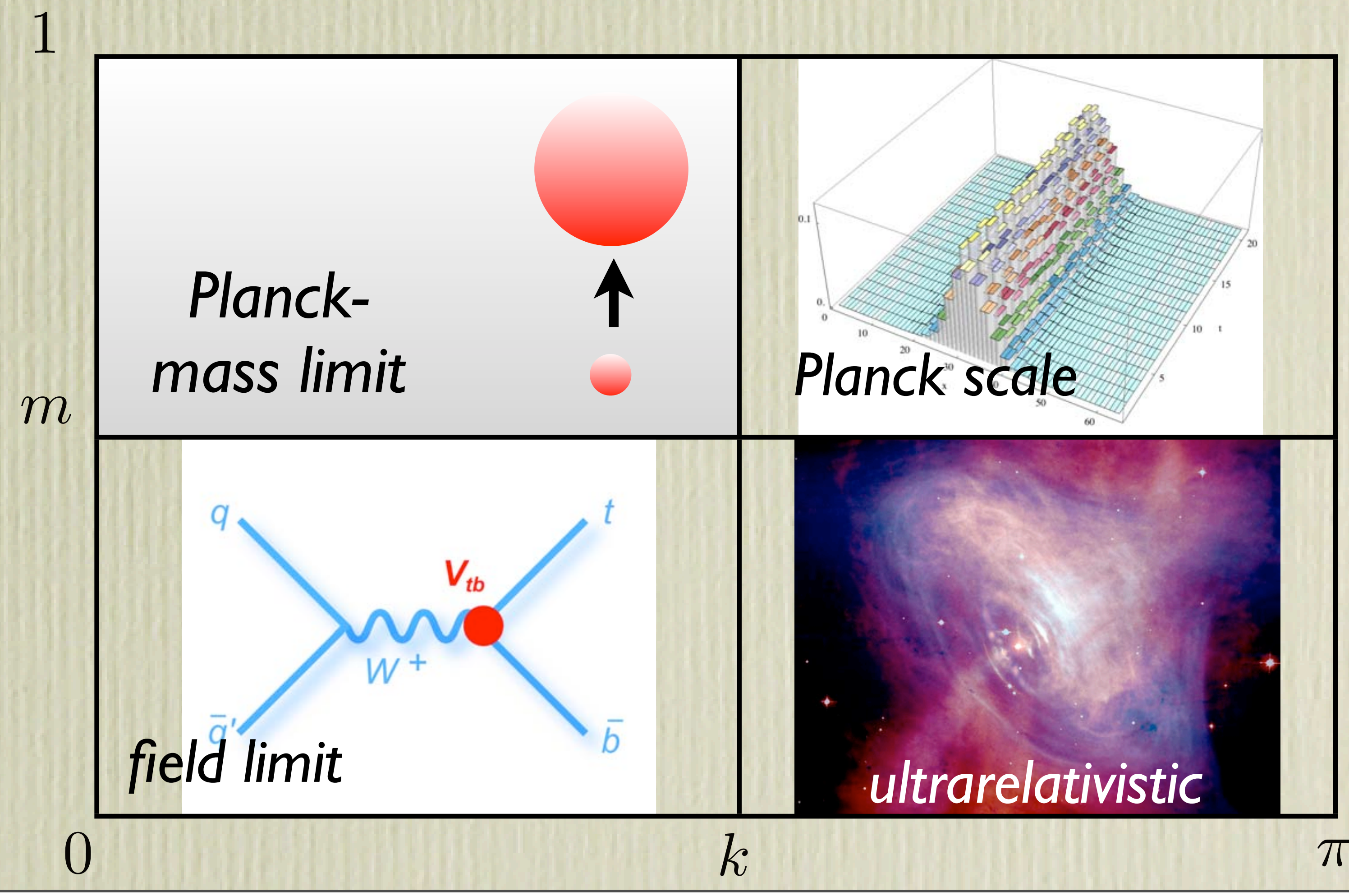
Localized states over a *locally quiescient state* (vacuum)

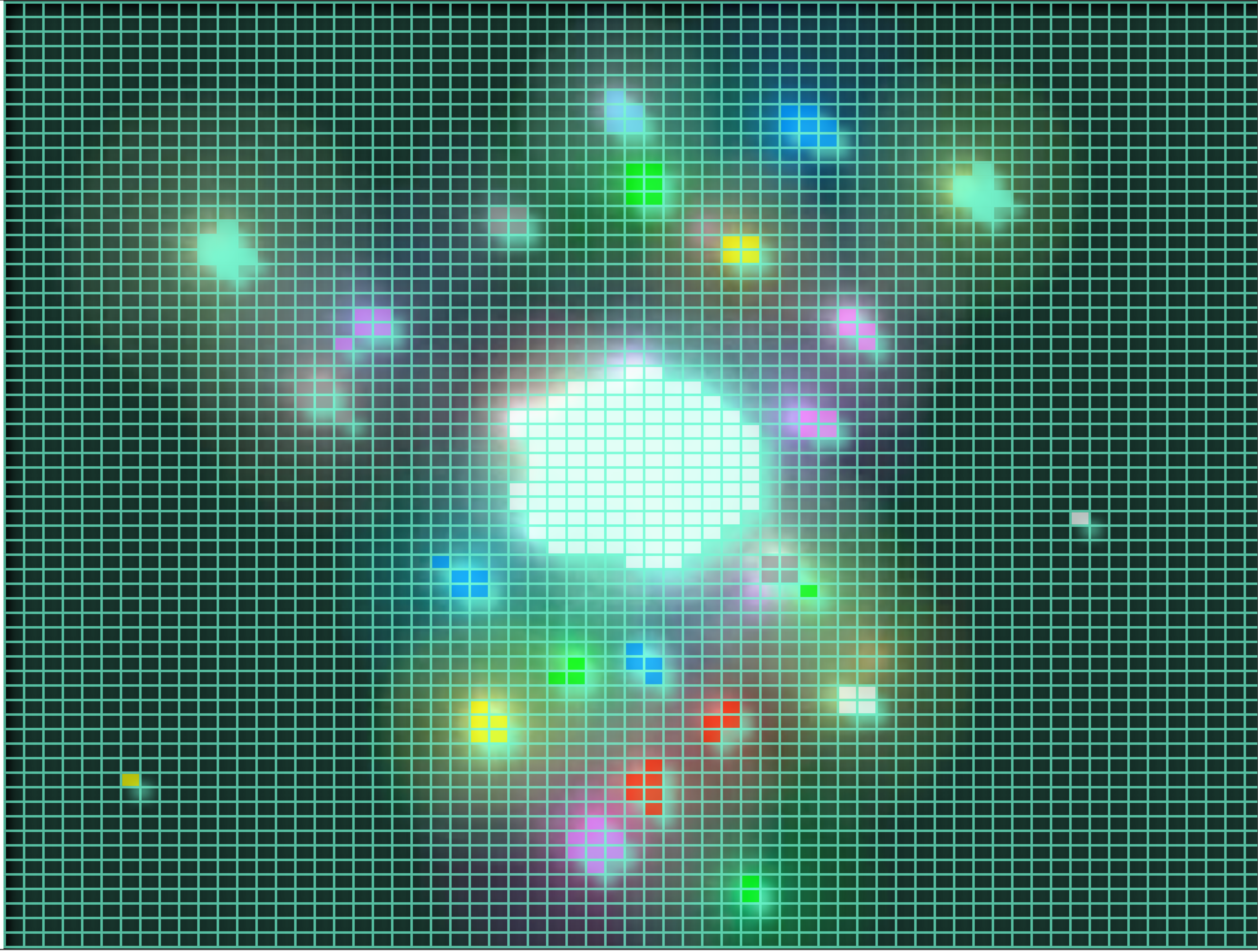


PROGRAM: QCA-EXTENSION OF QFT



Planck length





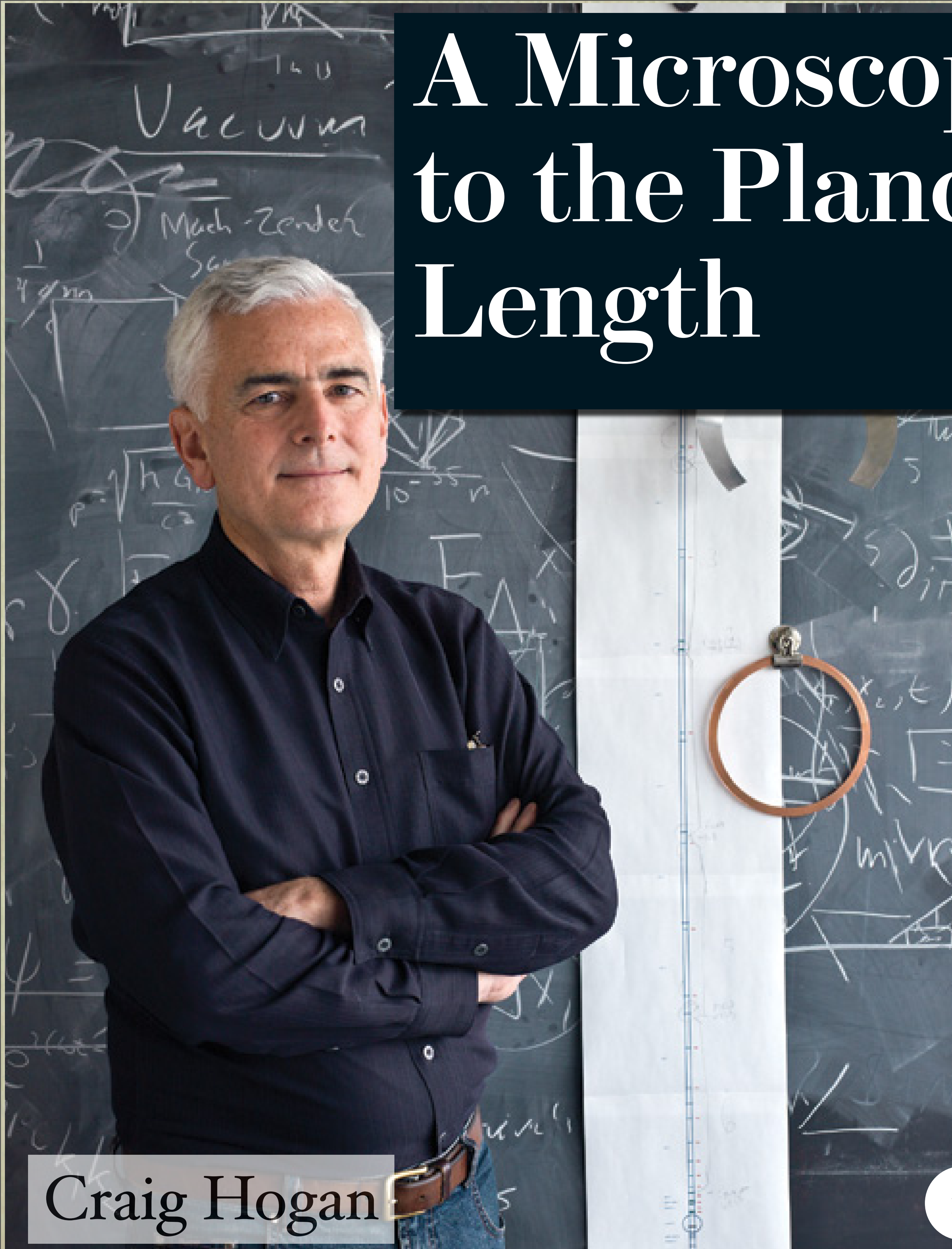
PHYSICS

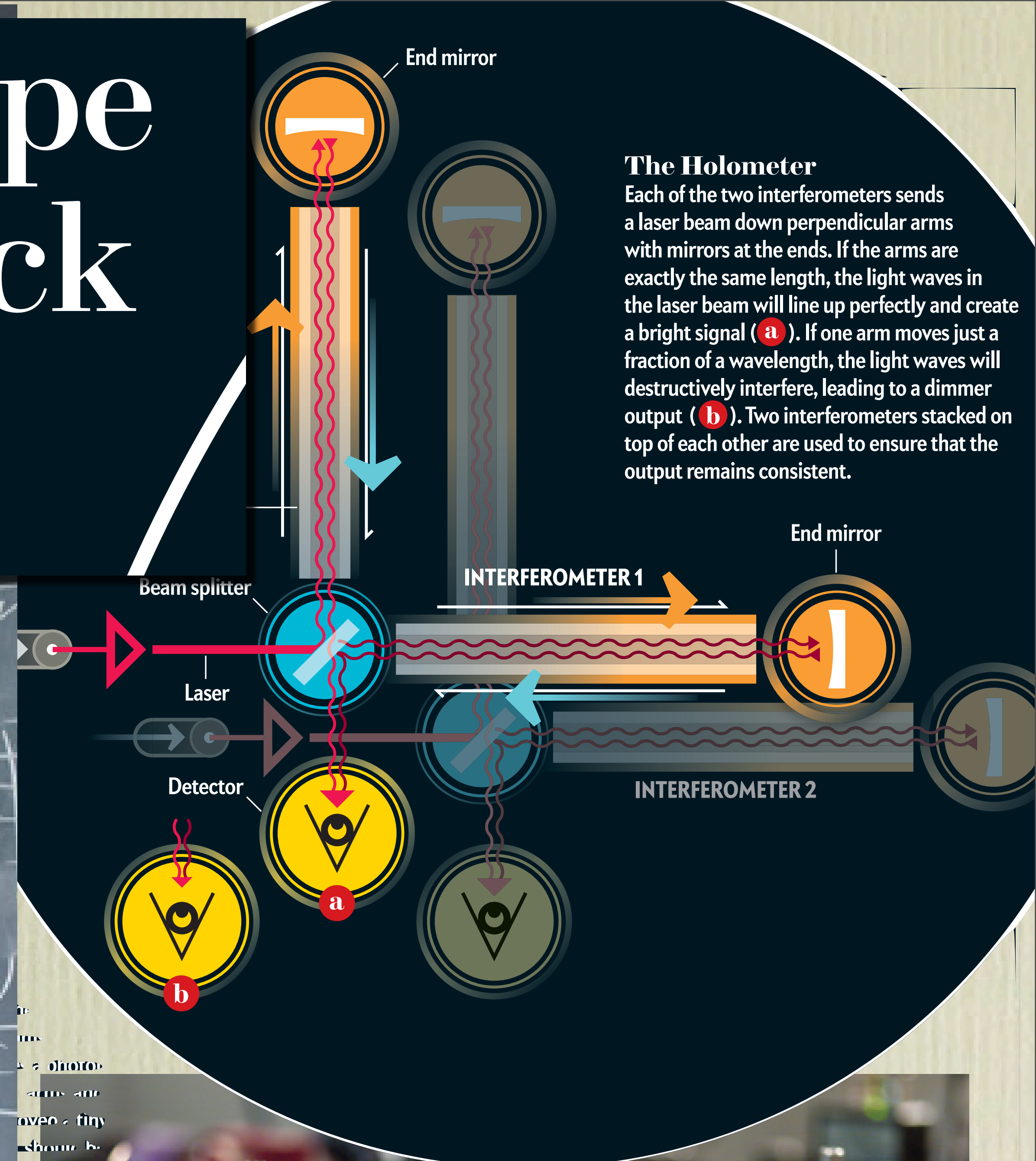
IS SPACE DIGITAL?

An experiment going up outside of Chicago will attempt to measure the intimate connections among information, matter and spacetime. If it works, it could rewrite the rules for 21st-century physics

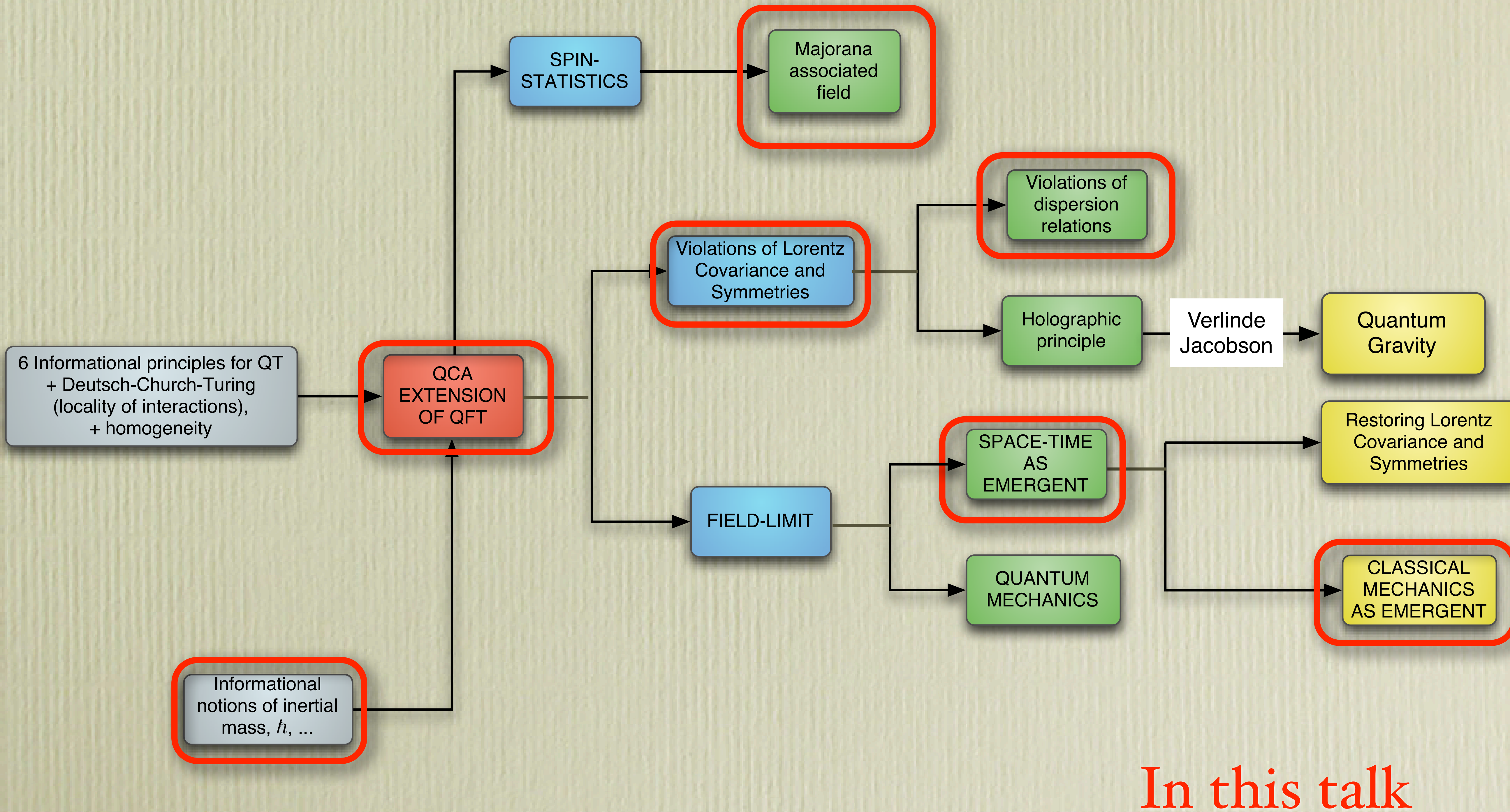
By Michael Moyer

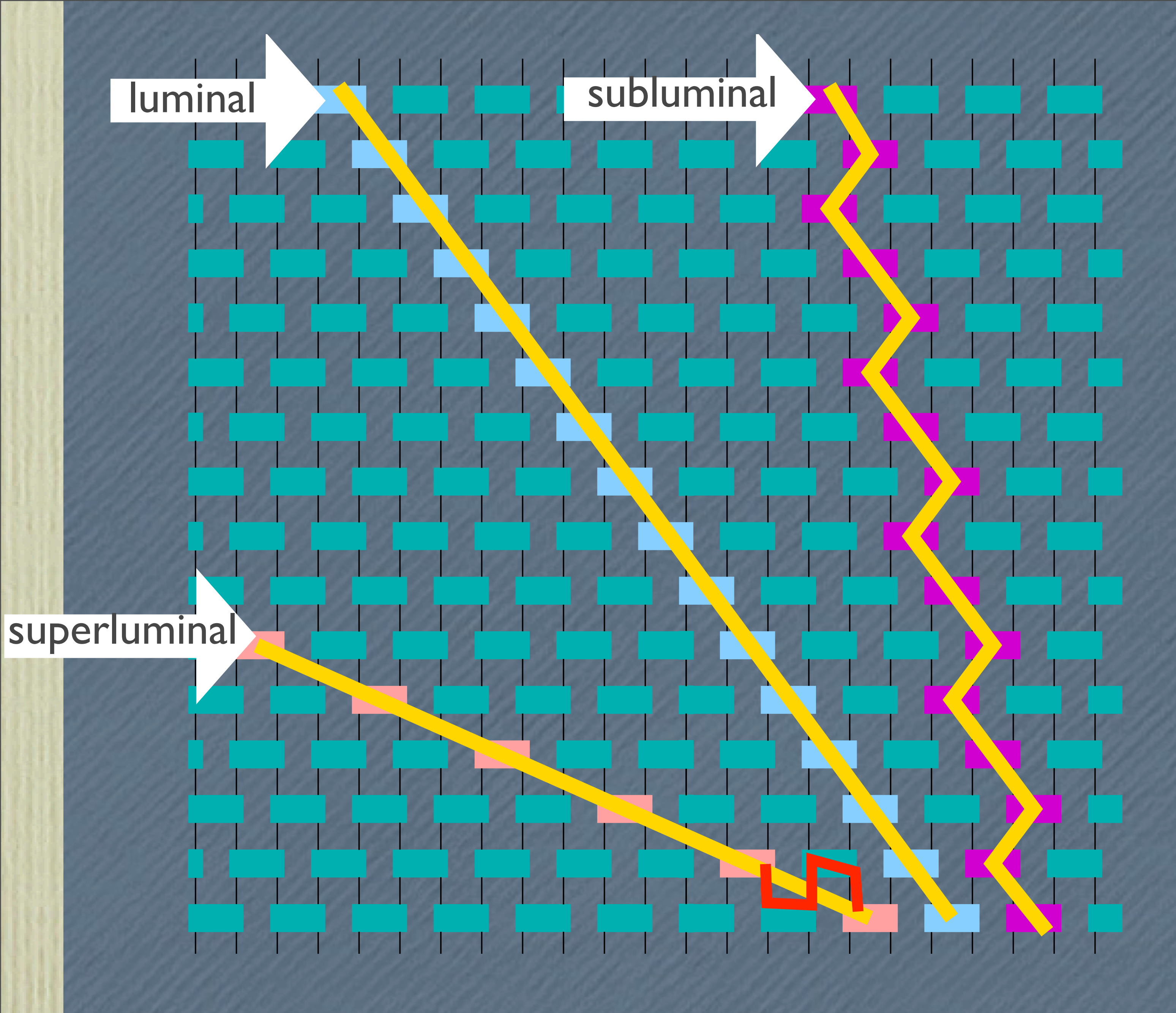
A Microscope to the Planck Length



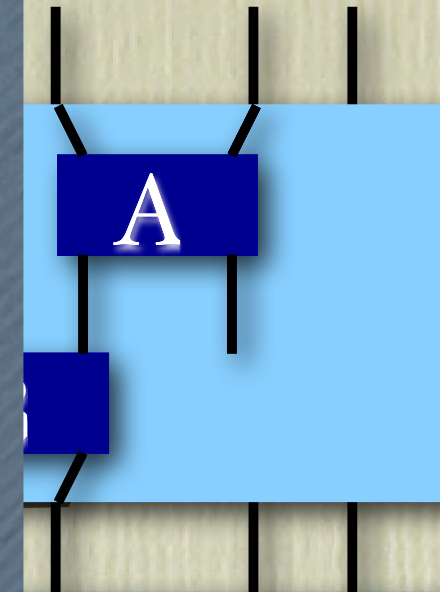


PROGRAM: QCA-EXTENSION OF QFT





AUTOMATON



$$\boldsymbol{\psi} := \begin{bmatrix} \cdots \\ \boldsymbol{\psi}_n \\ \boldsymbol{\psi}_{n+1} \\ \cdots \end{bmatrix}$$

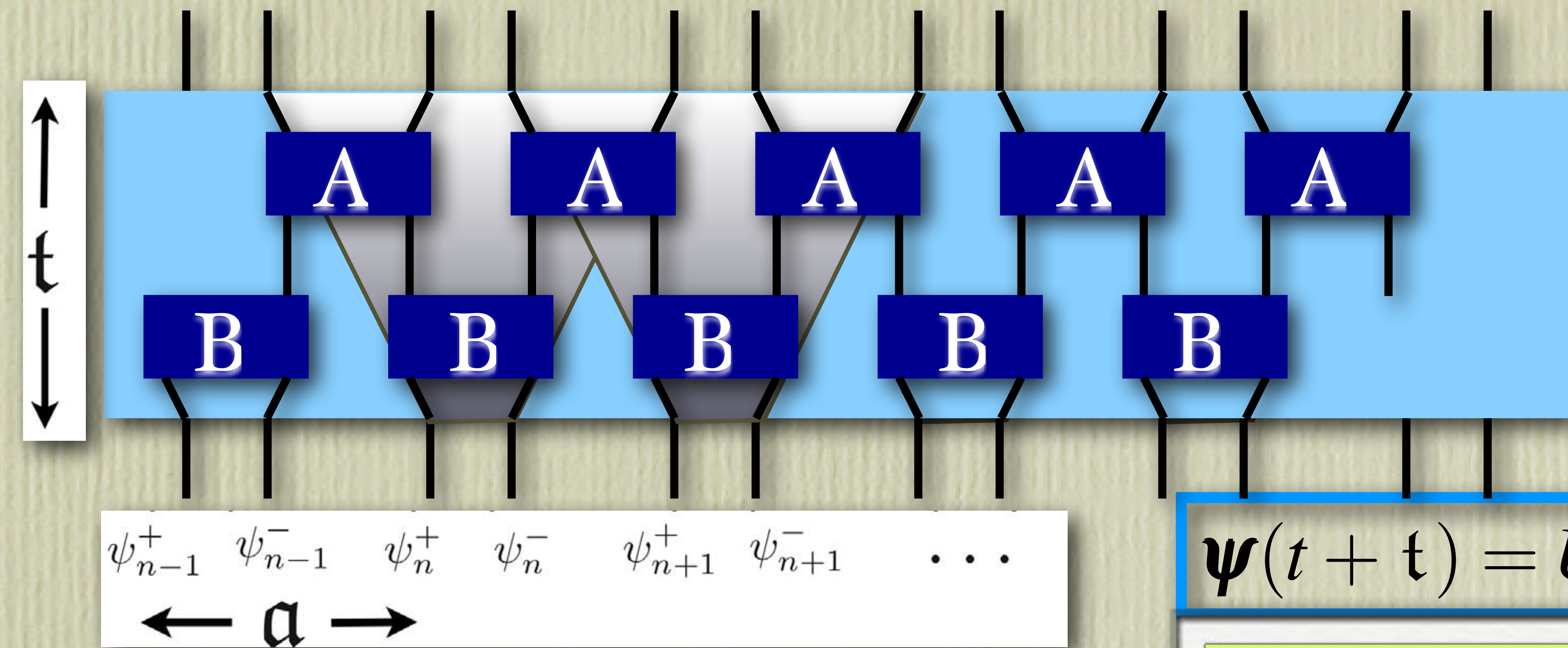
$$\boldsymbol{\psi}_n := \begin{bmatrix} \psi_n^+ \\ \psi_n^- \end{bmatrix}$$

$$\boldsymbol{\psi}(t+\mathrm{t}) = U^\dagger \boldsymbol{\psi}(t) U = \mathbf{U}\boldsymbol{\psi}(t)$$

$$\mathbf{U} = \begin{bmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\mathrm{c} \\ -i\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{bmatrix}$$

$$A_n = \exp\left[-i\theta\left(\sigma_{2n-1}^- \sigma_{2n}^+ + \sigma_{2n-1}^+ \sigma_{2n}^-\right)\right]$$
$$B_n = \exp\left[-i\tfrac{\pi}{2}\left(\sigma_{2n}^+ \sigma_{2n+1}^- + \sigma_{2n}^- \sigma_{2n+1}^+\right)\right]$$

THE DIRAC QU-AUTOMATON



$$\boldsymbol{\psi} := \begin{bmatrix} \dots \\ \boldsymbol{\psi}_n \\ \boldsymbol{\psi}_{n+1} \\ \dots \end{bmatrix}$$

$$\boldsymbol{\psi}_n := \begin{bmatrix} \psi^+_n \\ \psi^-_n \end{bmatrix}$$

$$\boldsymbol{\psi}(t+\mathfrak{t}) = U^\dagger \boldsymbol{\psi}(t) U = \mathbf{U}\boldsymbol{\psi}(t)$$

Planck length

$$\mathrm{c}^2 + \mathrm{s}^2 = 1$$

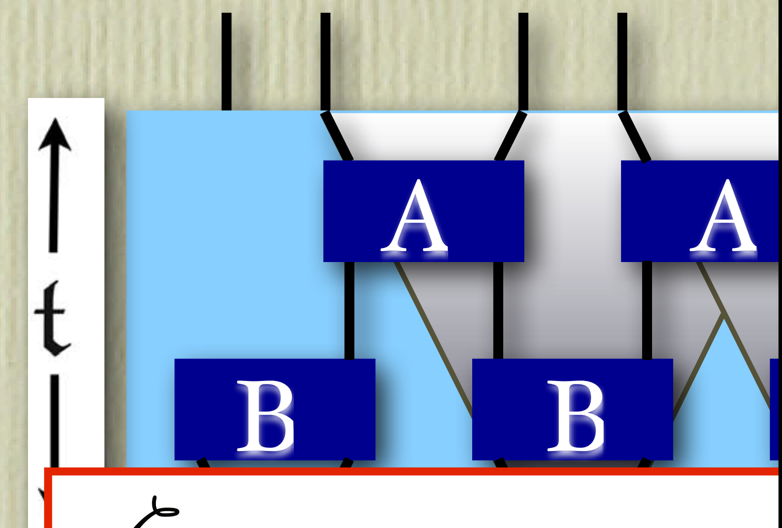
$$\mathbf{U} = \begin{bmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\mathrm{c} \\ -i\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{bmatrix}$$

$$A_n = \exp\left[-i\theta\left(\sigma^-_{2n-1}\sigma^+_{2n} + \sigma^+_{2n-1}\sigma^-_{2n}\right)\right]$$
$$B_n = \exp\left[-i\tfrac{\pi}{2}\left(\sigma^+_{2n}\sigma^-_{2n+1} + \sigma^-_{2n}\sigma^+_{2n+1}\right)\right]$$

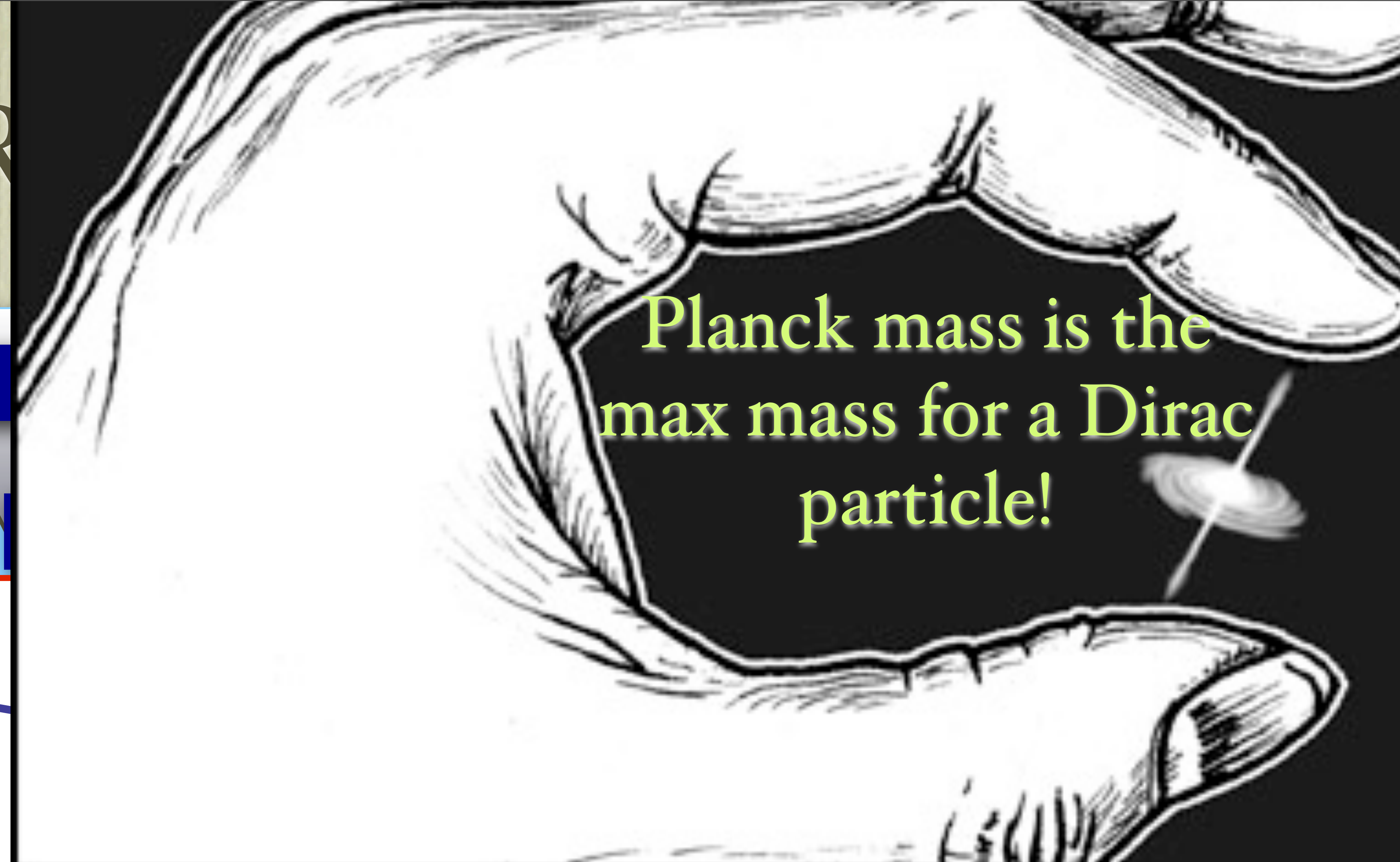
$$\mathrm{c} = \cos\theta = \omega\,\mathfrak{t} = \frac{\mathfrak{a}}{\lambda} = \frac{m}{\mathfrak{m}}, \qquad \mathrm{s} = \sin\theta = \zeta = \sqrt{1-\left(\frac{m}{\mathfrak{m}}\right)^2},$$

$$\mathfrak{m} := \frac{\hbar}{\mathfrak{a}\,\mathrm{c}}$$

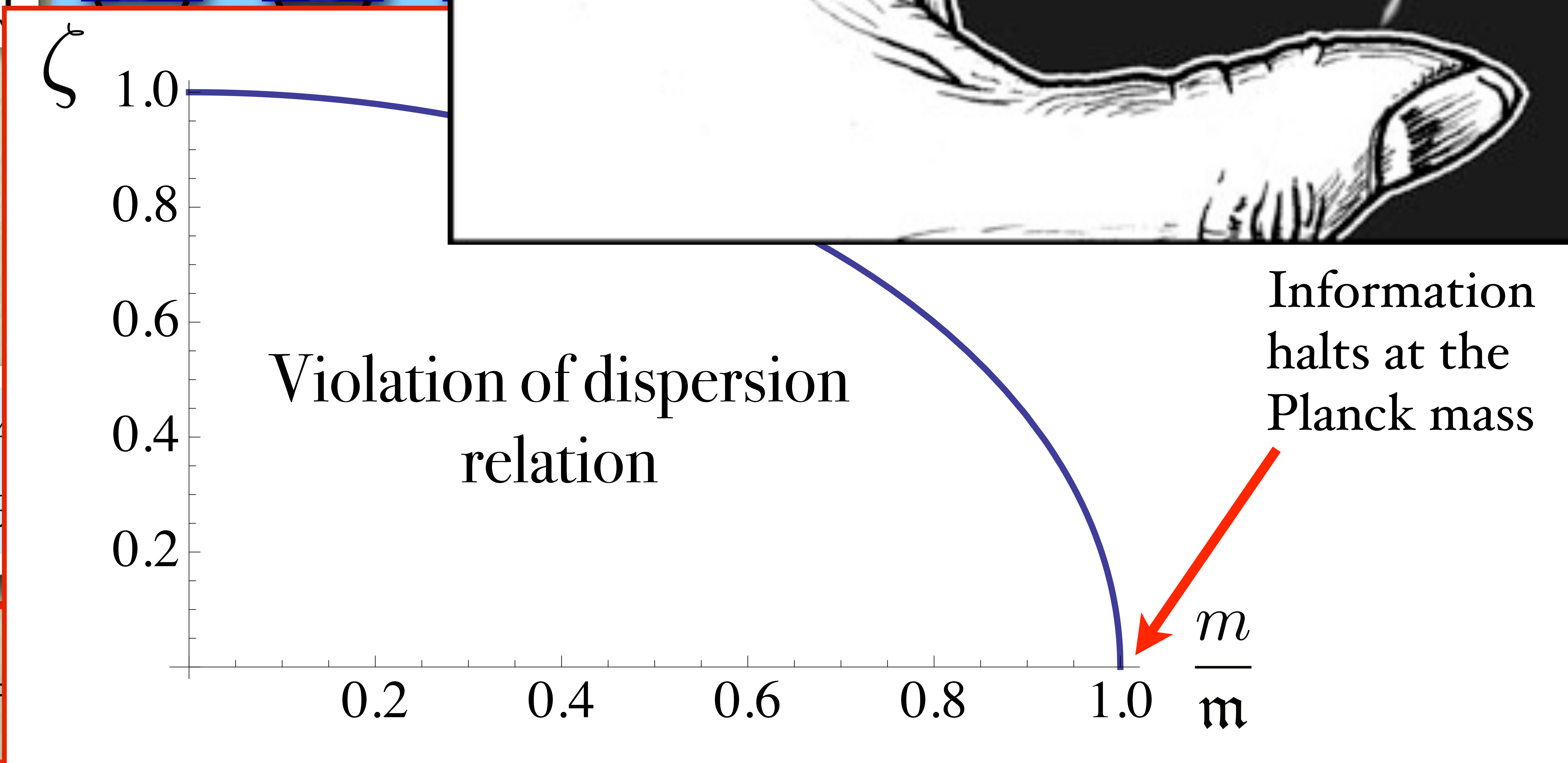
THE DIR



t
A
A
B
B

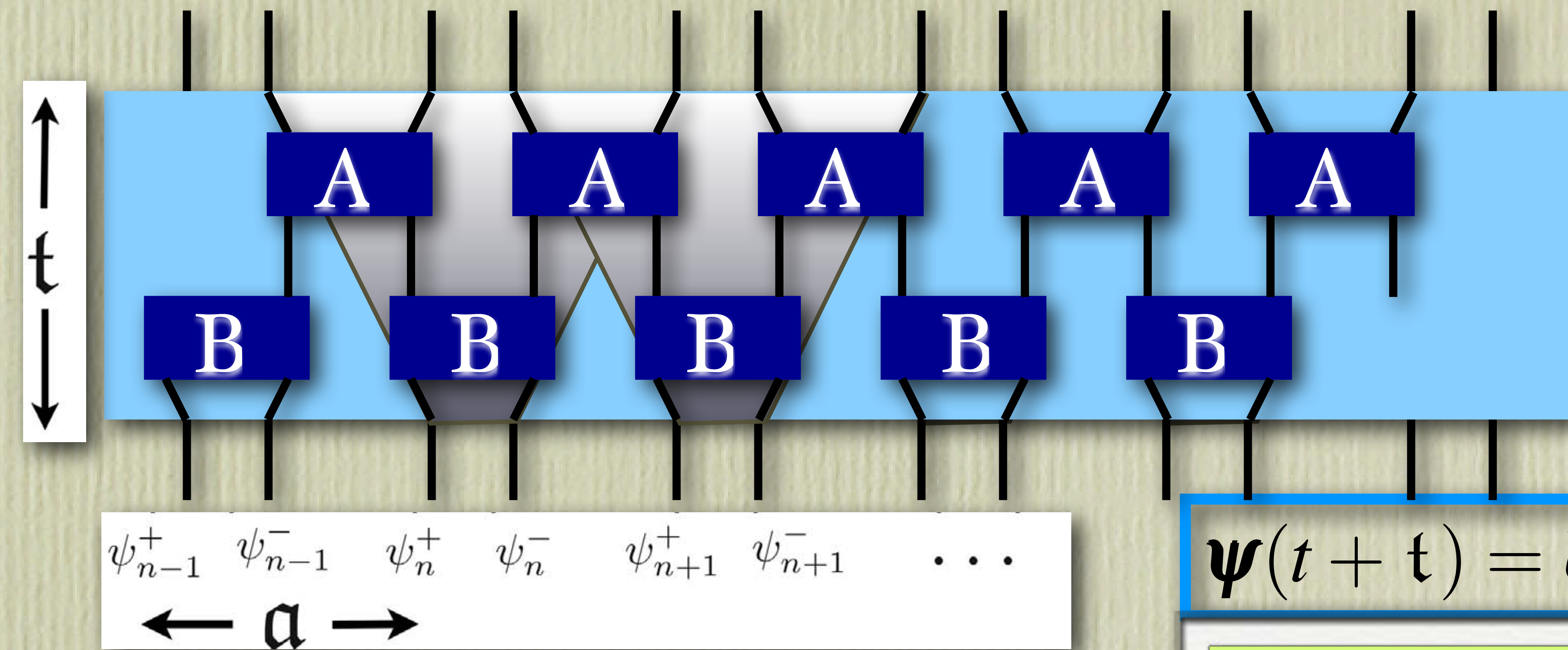


Planck mass is the max mass for a Dirac particle!



ζ
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
$\frac{m}{\mathfrak{m}}$
Violation of dispersion relation
Information halts at the Planck mass

THE DIRAC QU-AUTOMATON



Planck length

$$\boldsymbol{\psi} := \begin{bmatrix} \dots \\ \boldsymbol{\psi}_n \\ \boldsymbol{\psi}_{n+1} \\ \dots \end{bmatrix}$$

$$\boldsymbol{\psi}_n := \begin{bmatrix} \psi^+_n \\ \psi^-_n \end{bmatrix}$$

$$\boldsymbol{\psi}(t+\mathfrak{t}) = U^\dagger \boldsymbol{\psi}(t) U = \mathbf{U}\boldsymbol{\psi}(t)$$

$$\mathrm{c}^2 + \mathrm{s}^2 = 1$$

$$\mathbf{U} = \begin{bmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\mathrm{c} \\ -i\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{bmatrix}$$

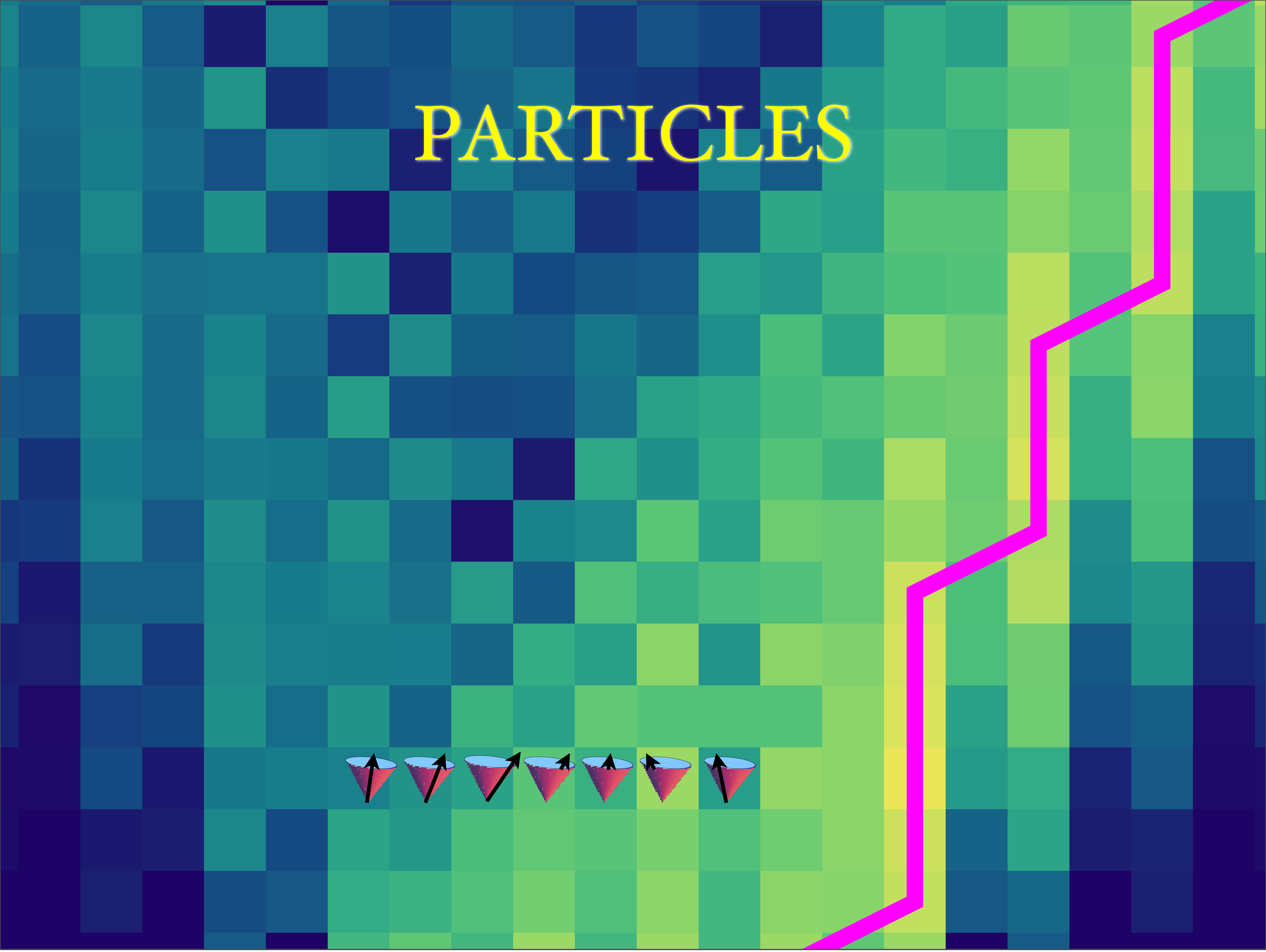
$$A_n = \exp\left[-i\theta\left(\sigma^-_{2n-1}\sigma^+_{2n} + \sigma^+_{2n-1}\sigma^-_{2n}\right)\right]$$

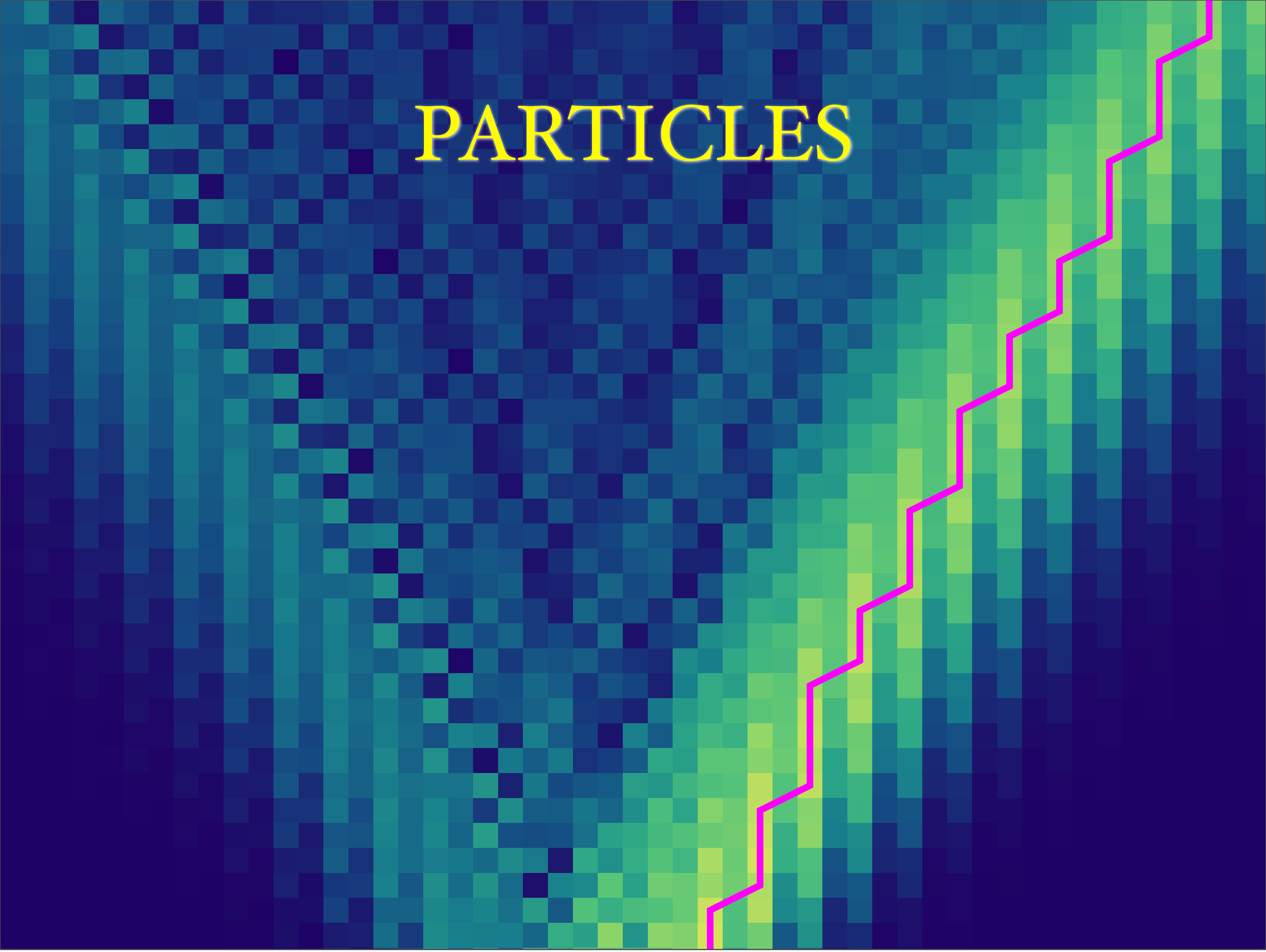
$$B_n = \exp\left[-i\tfrac{\pi}{2}\left(\sigma^+_{2n}\sigma^-_{2n+1} + \sigma^-_{2n}\sigma^+_{2} \dots\right)\right]$$

$$\mathrm{c} = \cos\theta = \omega\mathfrak{t} = \frac{\mathfrak{a}}{\lambda} = \frac{m}{\mathfrak{m}}, \qquad \mathrm{s} = \dots$$

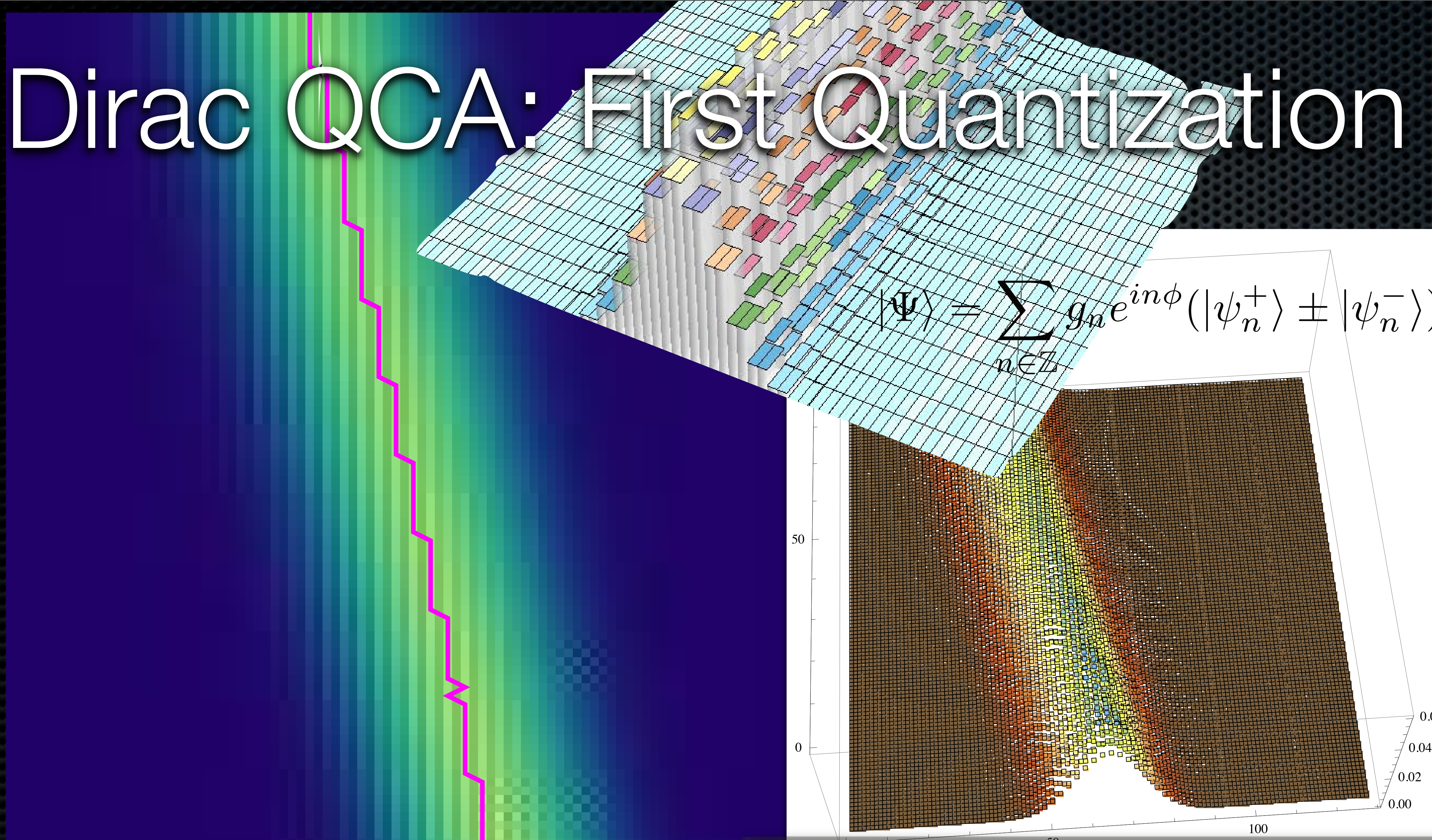
$$\hbar := \mathfrak{mac}$$

PARTICLES

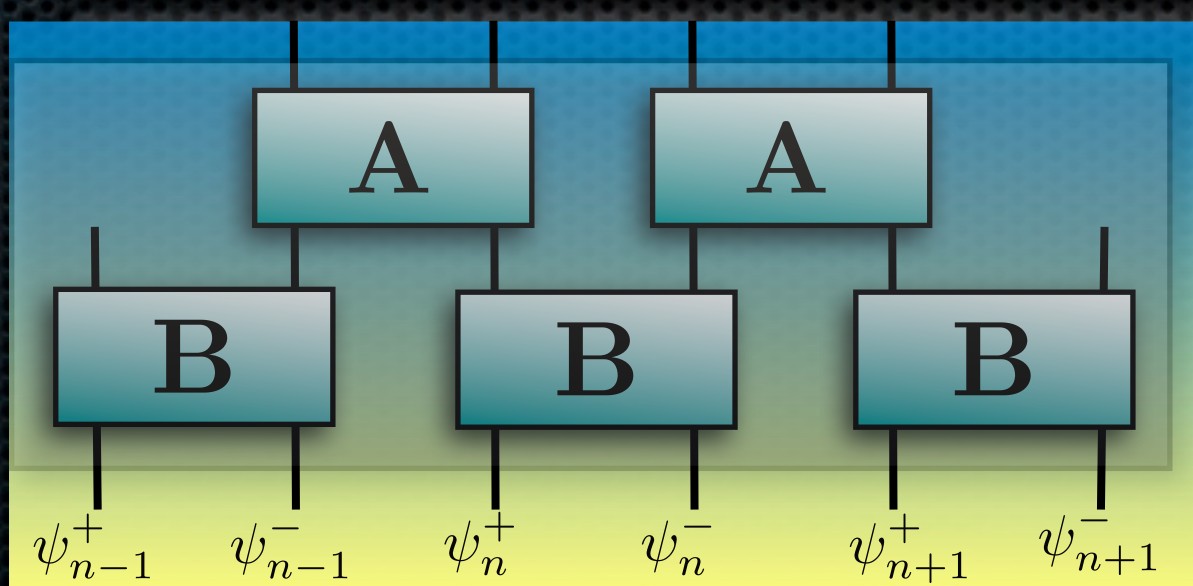



PARTICLES

Dirac QCA: First Quantization

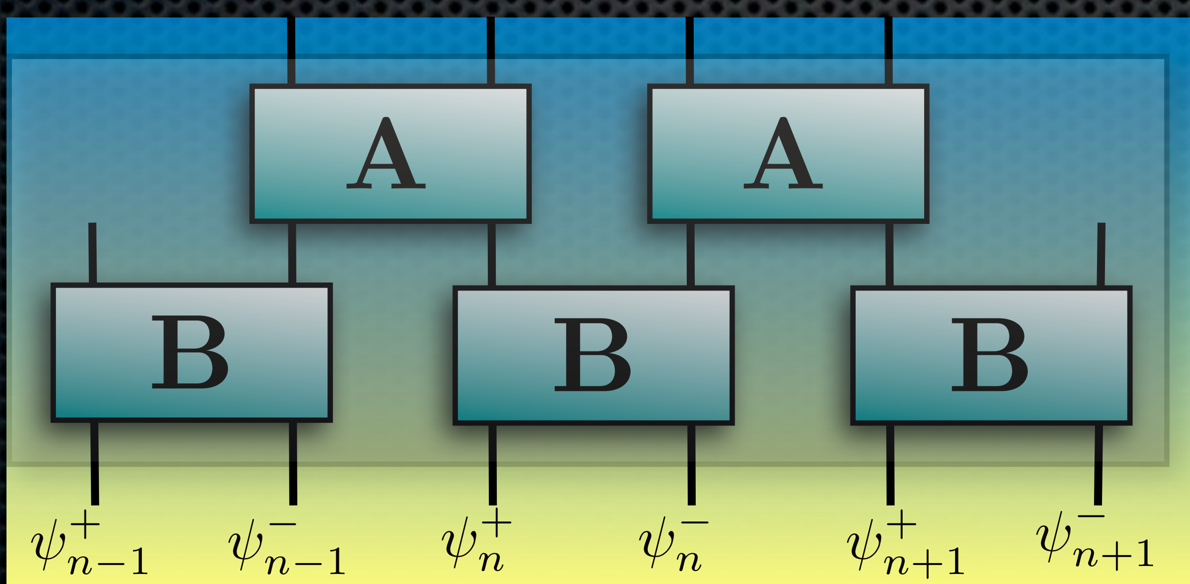


$$|\Psi\rangle = \sum_{n\in\mathbb{Z}} g_n e^{in\phi}\left(|\psi_n^+\rangle \pm |\psi_n^-\rangle\right)$$



$$\mathbf{U} = \begin{bmatrix} s\,\widehat{\partial}_- & -ic \\ -ic & s\,\widehat{\partial}_+ \end{bmatrix}$$

Dirac QCA: First Quantization



$$\mathbf{U} = \begin{bmatrix} s\,\widehat{\partial}_- & -ic \\ -ic & s\,\widehat{\partial}_+ \end{bmatrix}$$

Planckian Particles

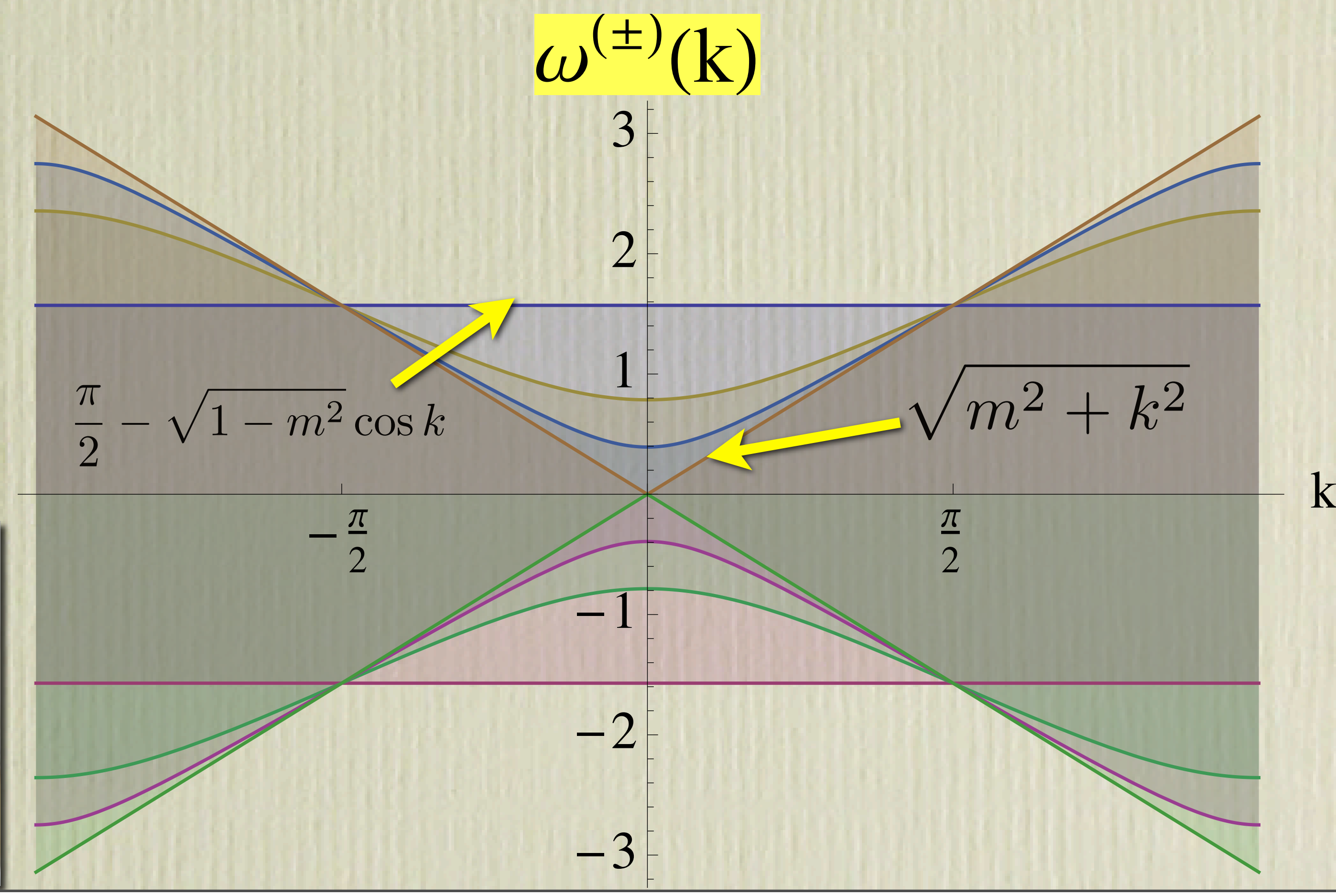
(Foldy-Wouthuysen) $$\mathbf{U} = \begin{pmatrix} \mathrm{s}\,\widehat{\partial}_- & -i\,\mathrm{c} \\ -i\,\mathrm{c} & \mathrm{s}\,\widehat{\partial}_+ \end{pmatrix} = \begin{pmatrix} \mathrm{s}\,e^{ik} & -i\,\mathrm{c} \\ -i\,\mathrm{c} & \mathrm{s}\,e^{-ik} \end{pmatrix}$$

Eigenvectors in *k*-space

$$\frac{1}{2N^{(\pm)}(k)} \begin{pmatrix} i\,\mathrm{c} \\ \mathrm{s}\,e^{ik} - e^{\pm i\omega(k)} \end{pmatrix} \qquad N^{(\pm)}(k) = \sqrt{1 - \mathrm{s}\cos[\mathrm{k} \pm \omega(\mathrm{k})]}$$

Dispersion relation

$$\boxed{\omega(k) = \cos^{-1}(\mathrm{s}\cos\mathrm{k})}$$



Alessandro Bisio

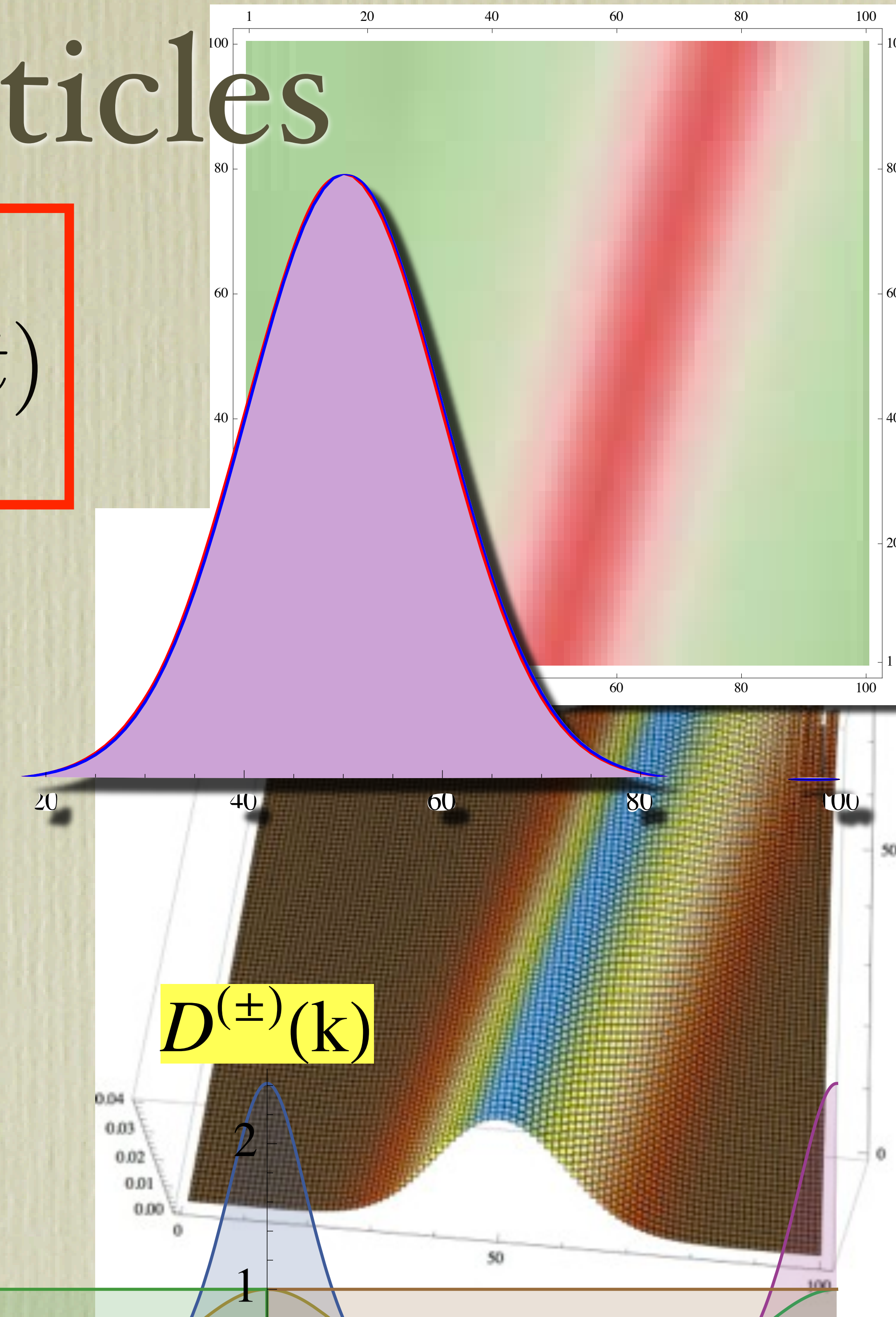
Alessandro Tosini

Planckian Particles

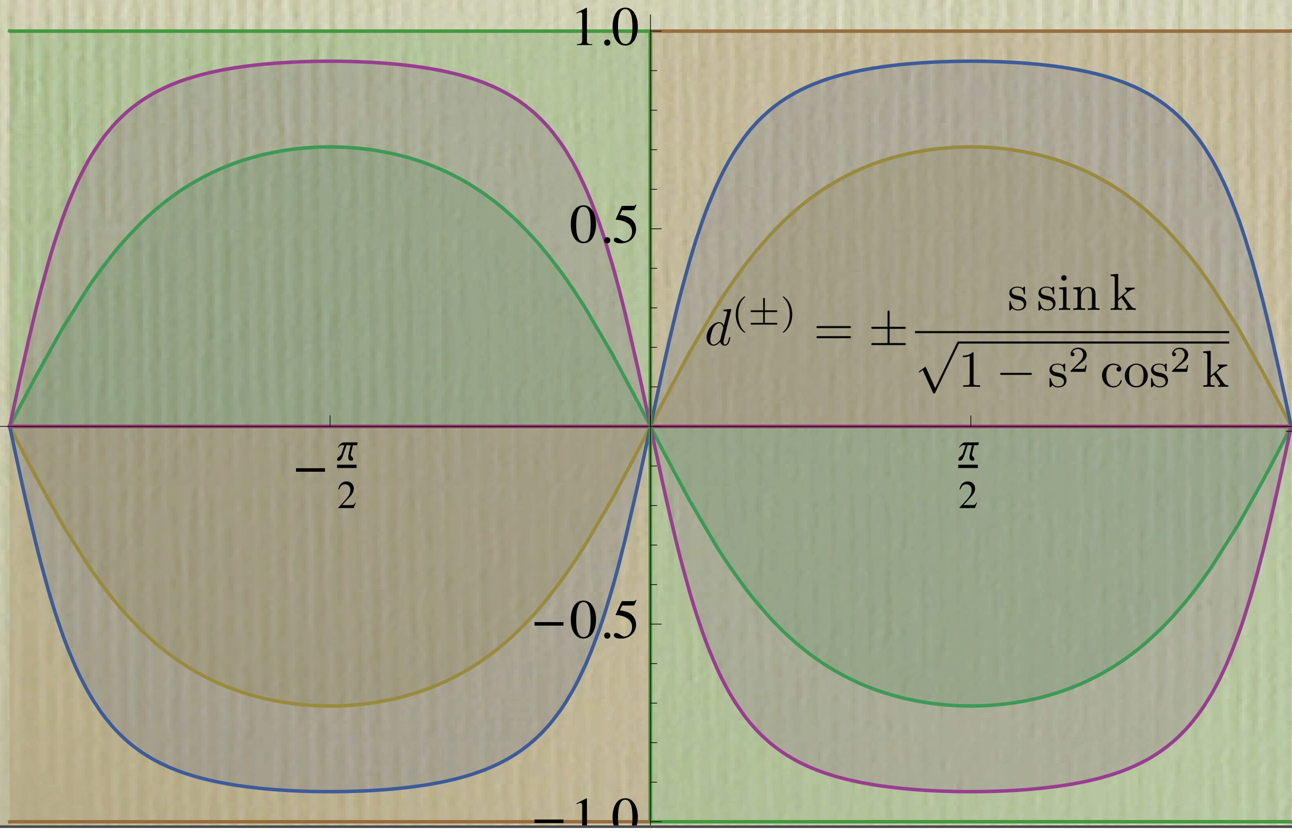
$$\boxed{i\partial_t A(x,t) = \left[-id\partial_x - \frac{D}{2}\partial_x^2\right] A(x,t)}$$

$$A(x,t) = \frac{1}{\sqrt[4]{2\pi\Delta^2(t)}} \exp\left[-\frac{(x-x(t))^2}{4\Delta^2(t)}\right]$$

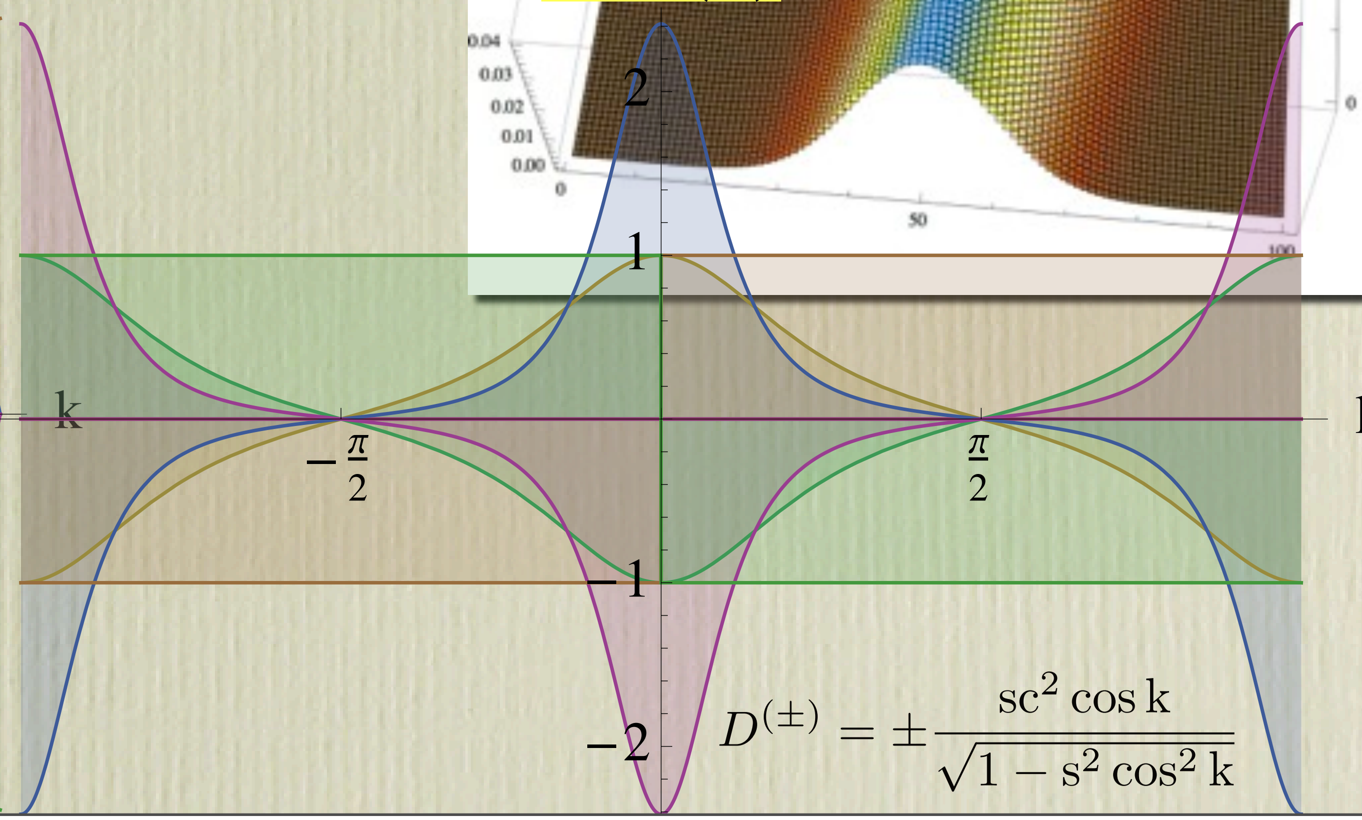
$$x(t) = d \qquad \Delta(t) = \Delta\sqrt{1 + i\frac{D}{2\Delta^2}t}$$



$d^{(\pm)}(\mathrm{k})$

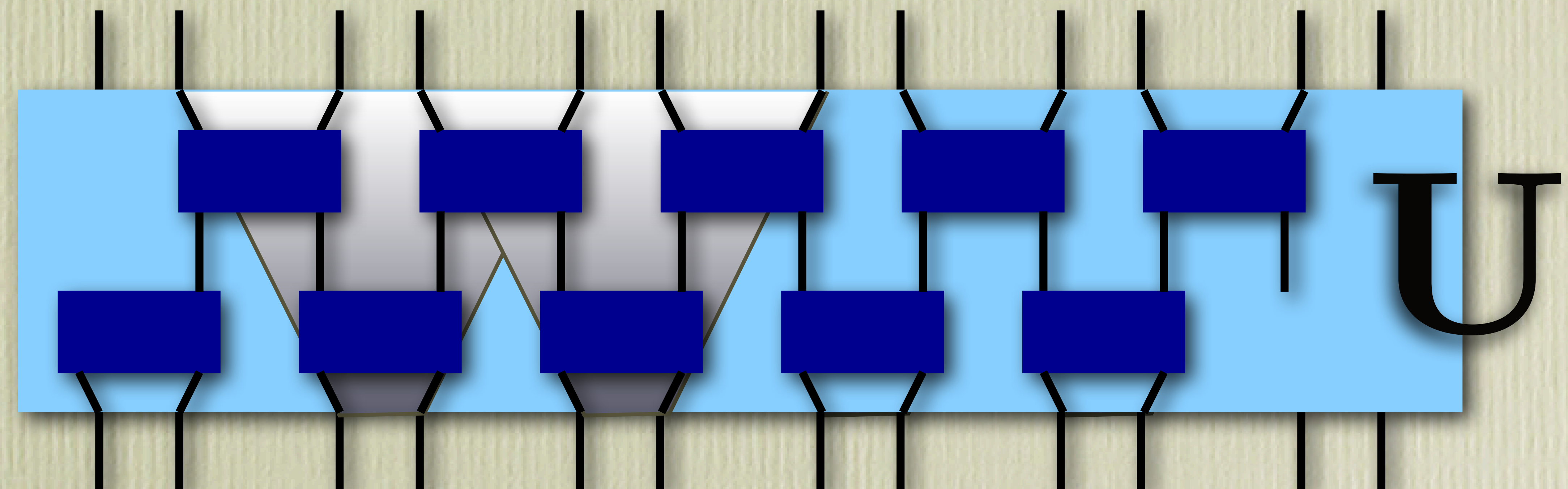


$D^{(\pm)}(\mathrm{k})$



MECHANICS EMERGING FROM COMPUTATION

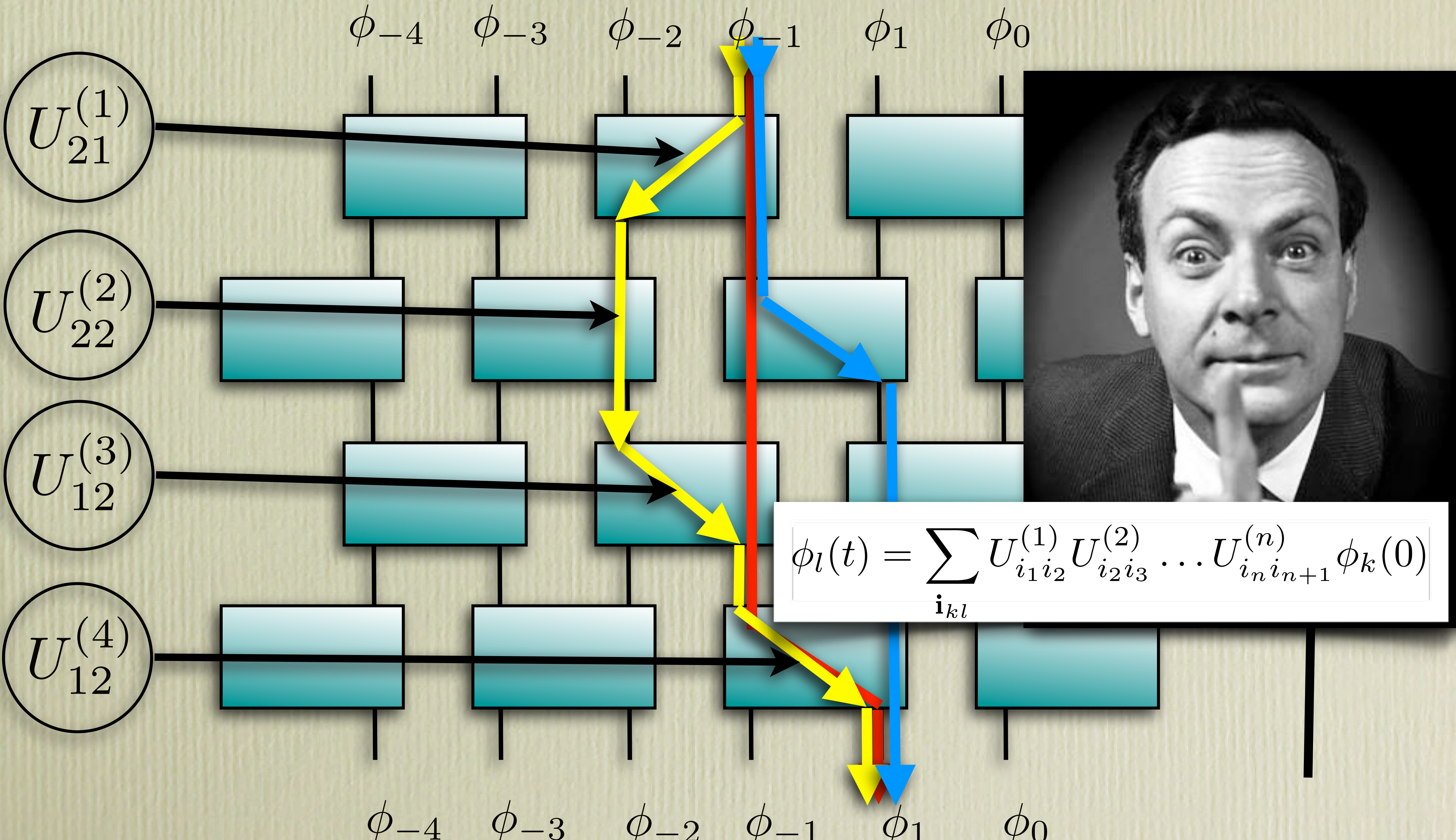
$$i\hbar\widehat{\partial}_t \boldsymbol{\psi} = [\boldsymbol{\psi}, H]$$



$$H = \frac{i\hbar}{2\mathfrak{t}} \boldsymbol{\psi}^\dagger (\mathbf{U} - \mathbf{U}^\dagger) \boldsymbol{\psi}$$

MECHANICS EMERGING FROM COMPUTATION

- **PATH-INTEGRAL**



$$\phi_l(t) = \sum_{\mathbf{i}_{kl}} U^{(1)}_{i_1 i_2} U^{(2)}_{i_2 i_3} \ldots U^{(n)}_{i_n i_{n+1}} \phi_k(0)$$

Are we able to simulate our theory (even with a quantum computer)?

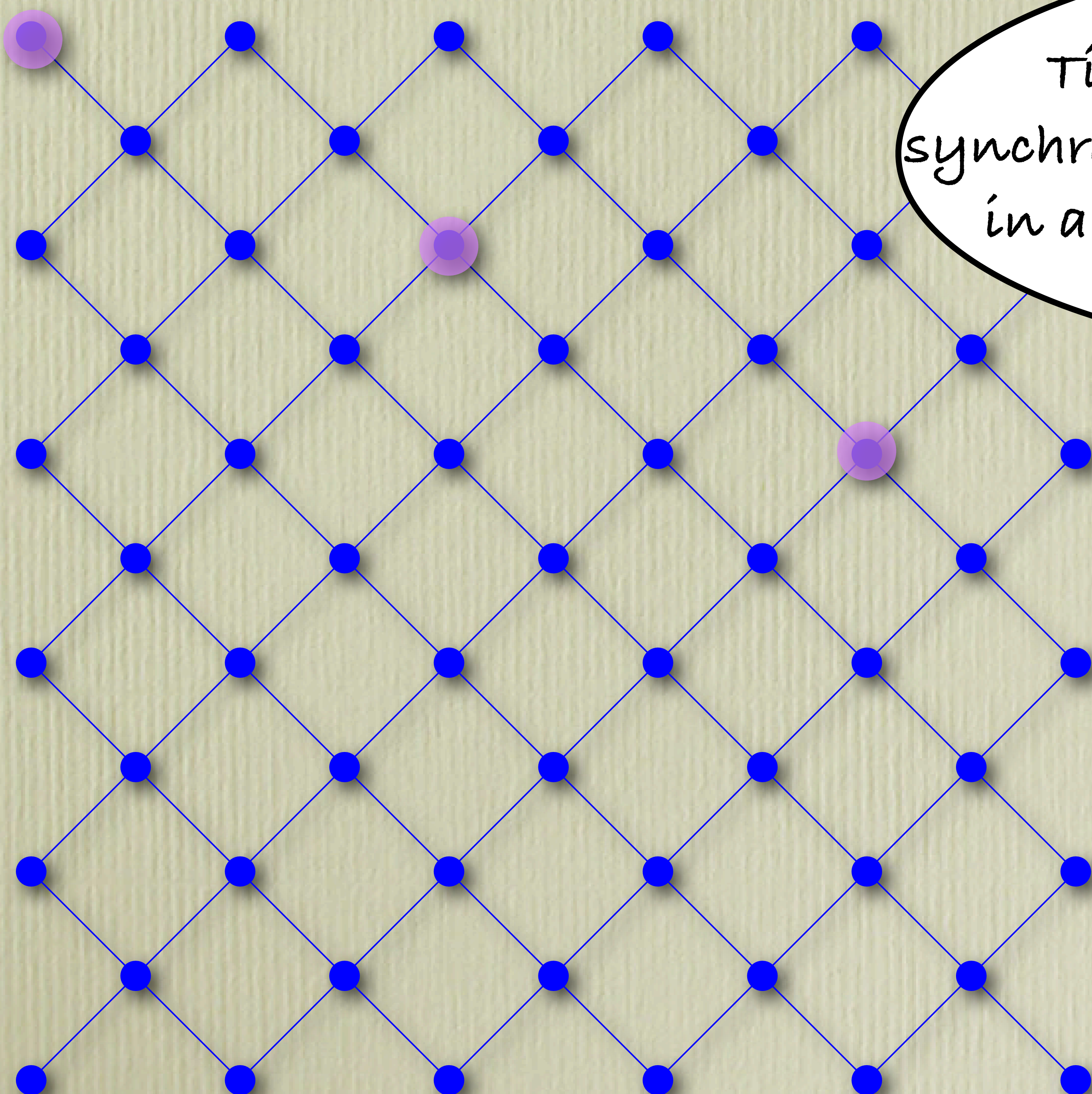
Simulating Physics with Computers
Richard P. Feynman

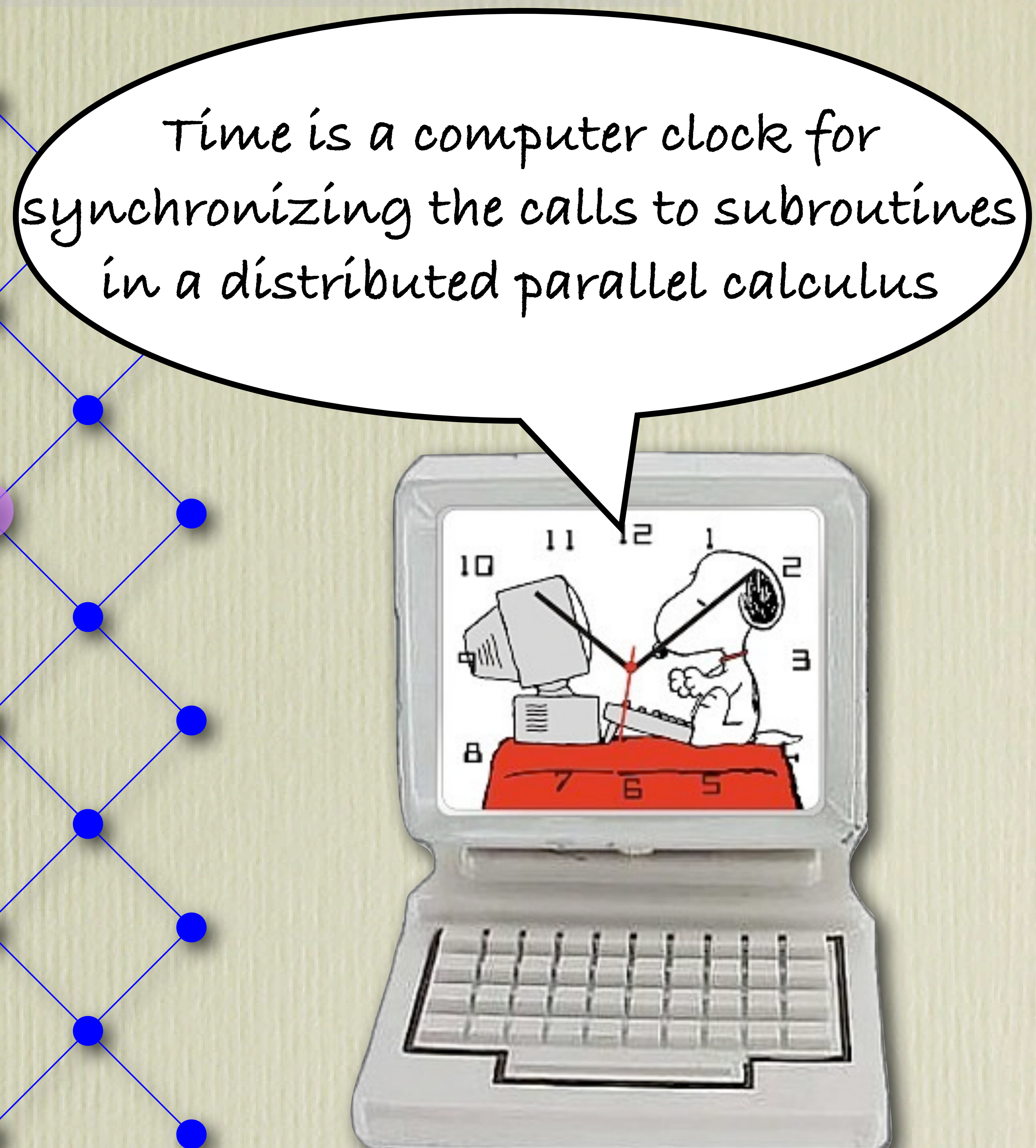
The question is, if we wrote a Hamiltonian which involved only these operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? I know, almost certainly, that we could do that for any quantum mechanical system which involves Bose particles. I'm not sure whether Fermi particles could be described by such a system. So I leave that open. Well, that's an example of what I meant by a general quantum mechanical simulator. I'm not sure that it's sufficient, because I'm not sure that it takes care of Fermi particles.

Int. J. of Th. Phys. **21 467** (1982)

SPACE-TIME AT PLANCK SCALE

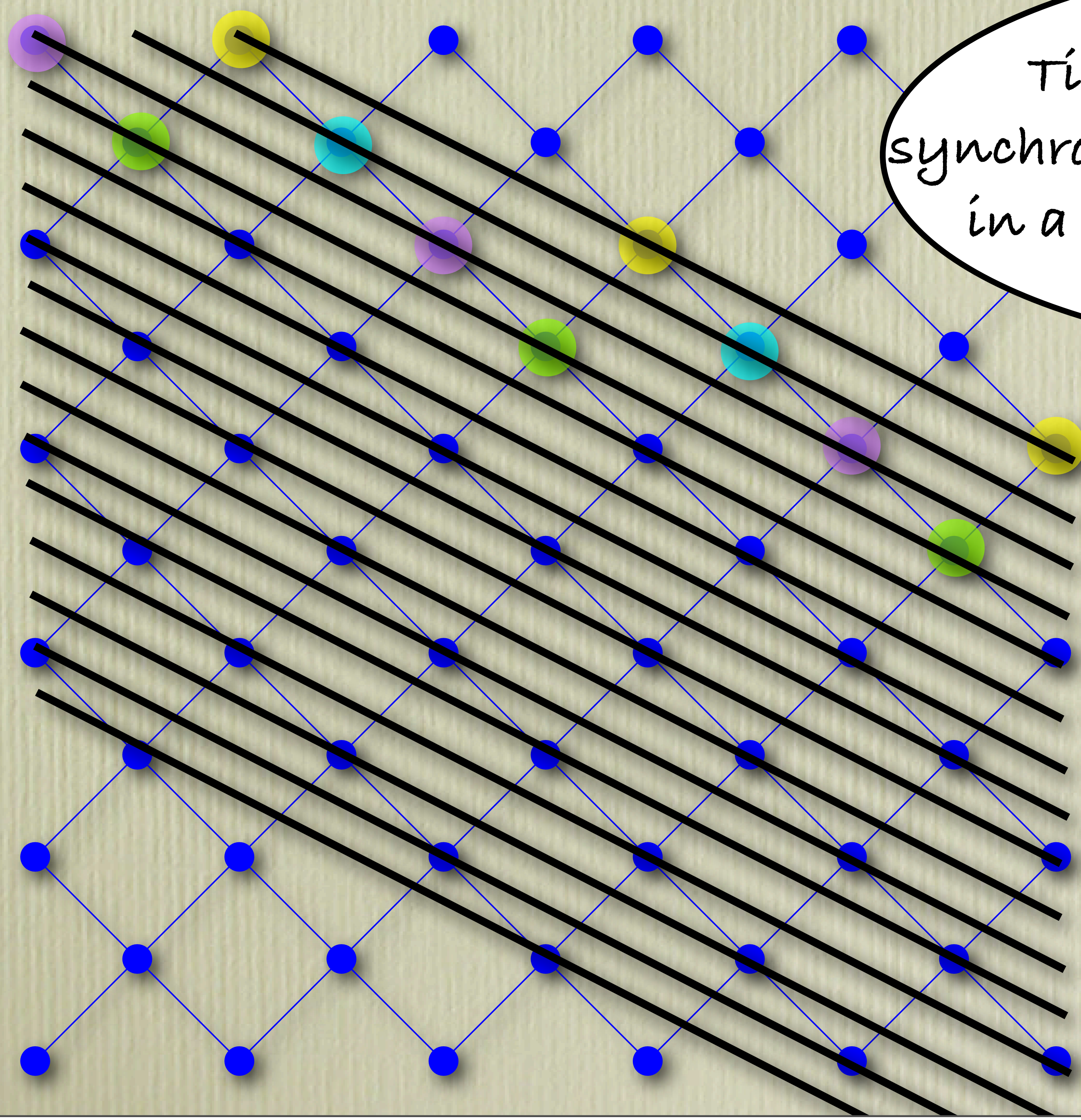
FOLIATION: TIME AS A COMPUTER CLOCK

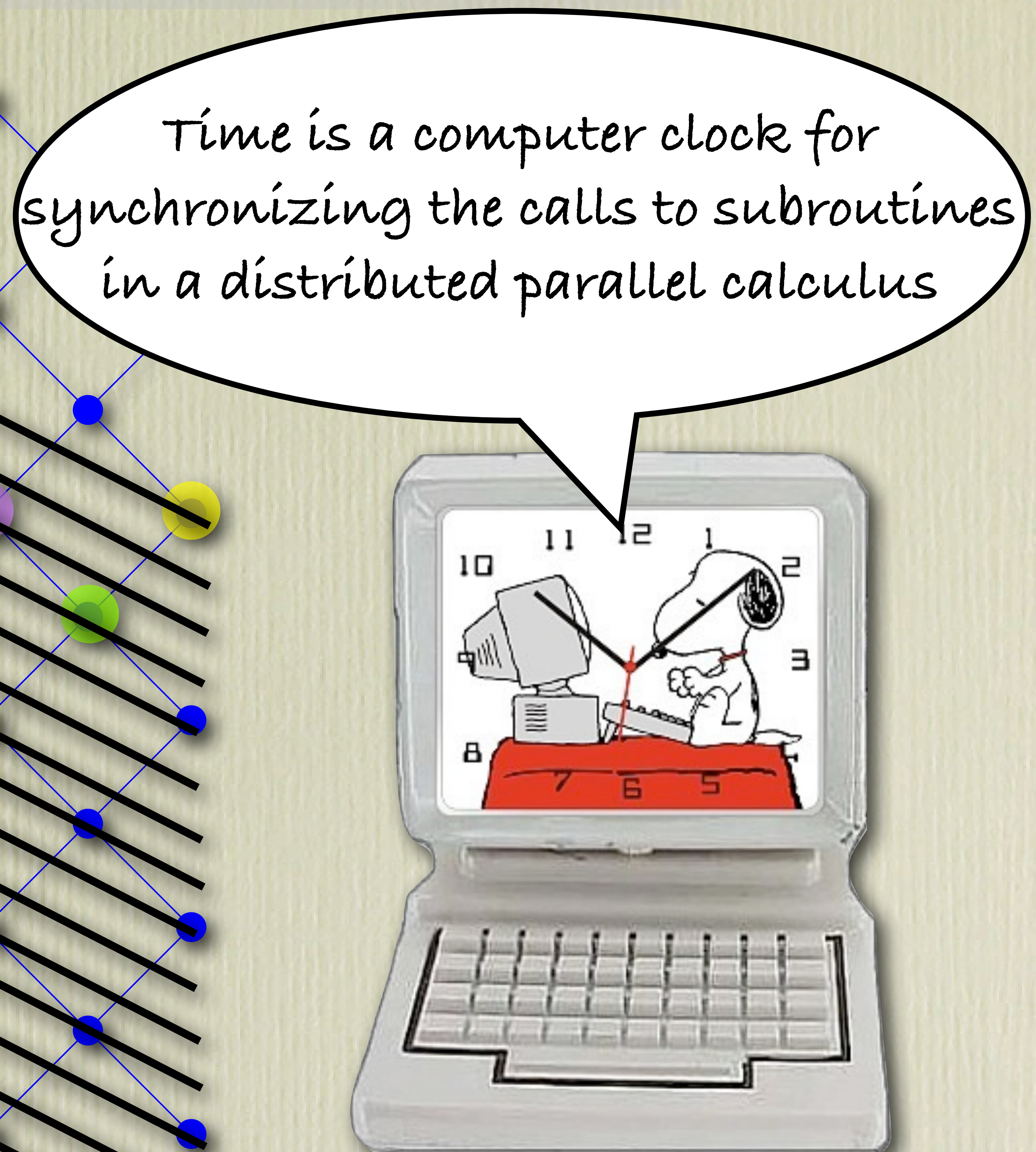




SPACE-TIME AT PLANCK SCALE

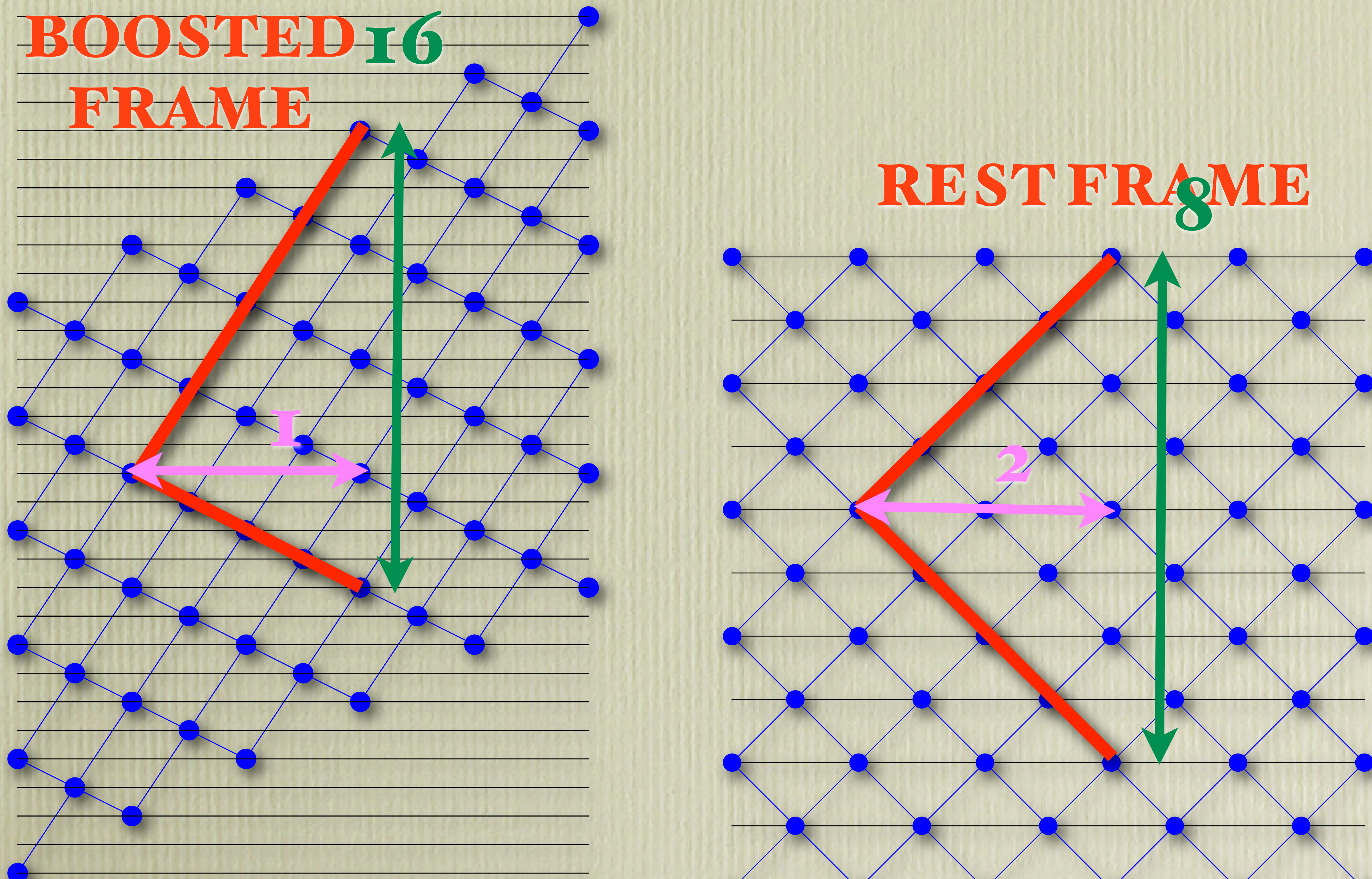
THE COMPUTATIONAL TOMONAGA-SCHWINGER





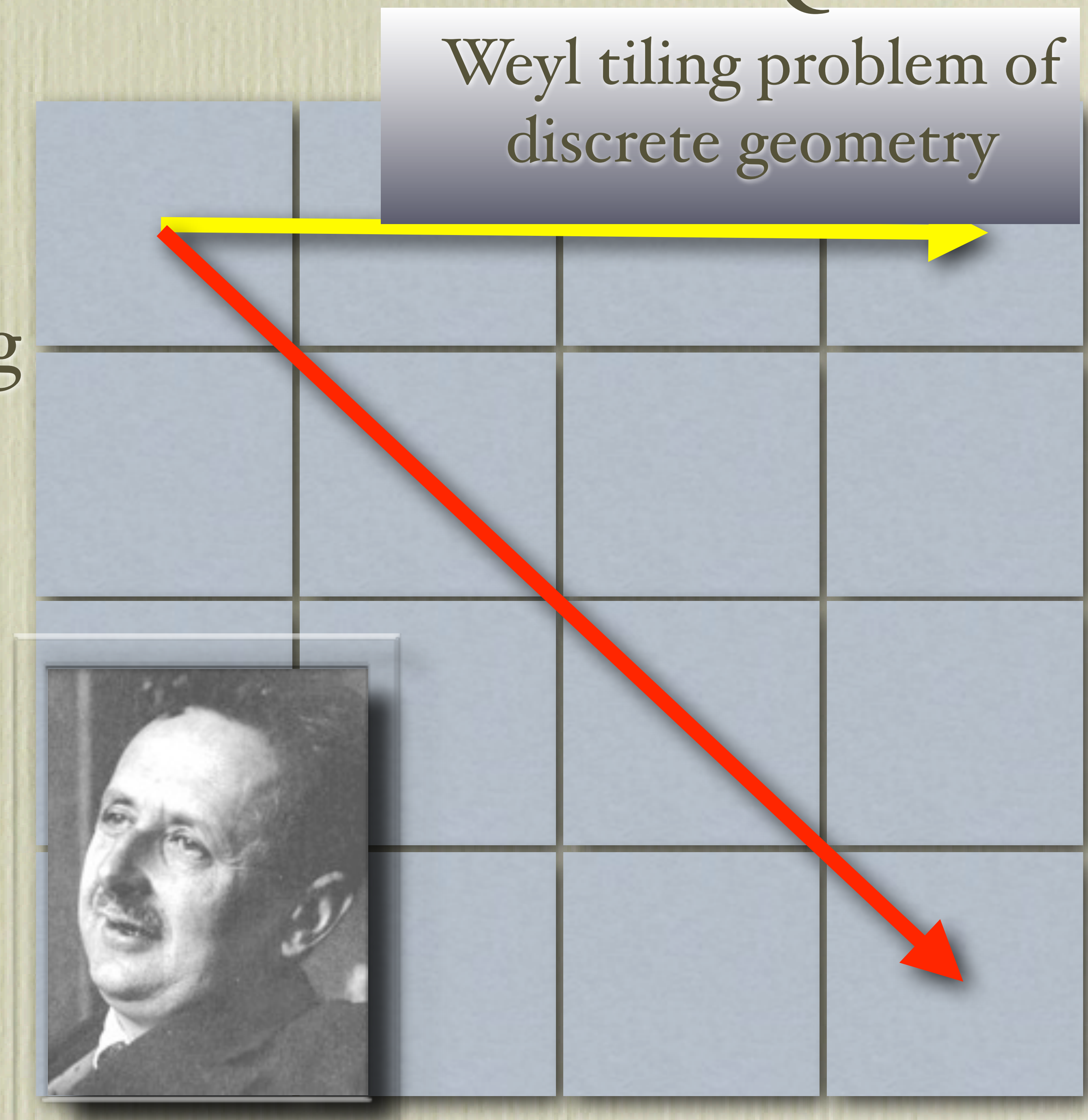
SPACE-TIME AT PLANCK SCALE

TIME-DILATION AND SPACE-CONTRACTION



Why information is quantum?

Should we consider a network-axiom for QT?



- “Direction” of information imprinted in the state using minimal informational resources.

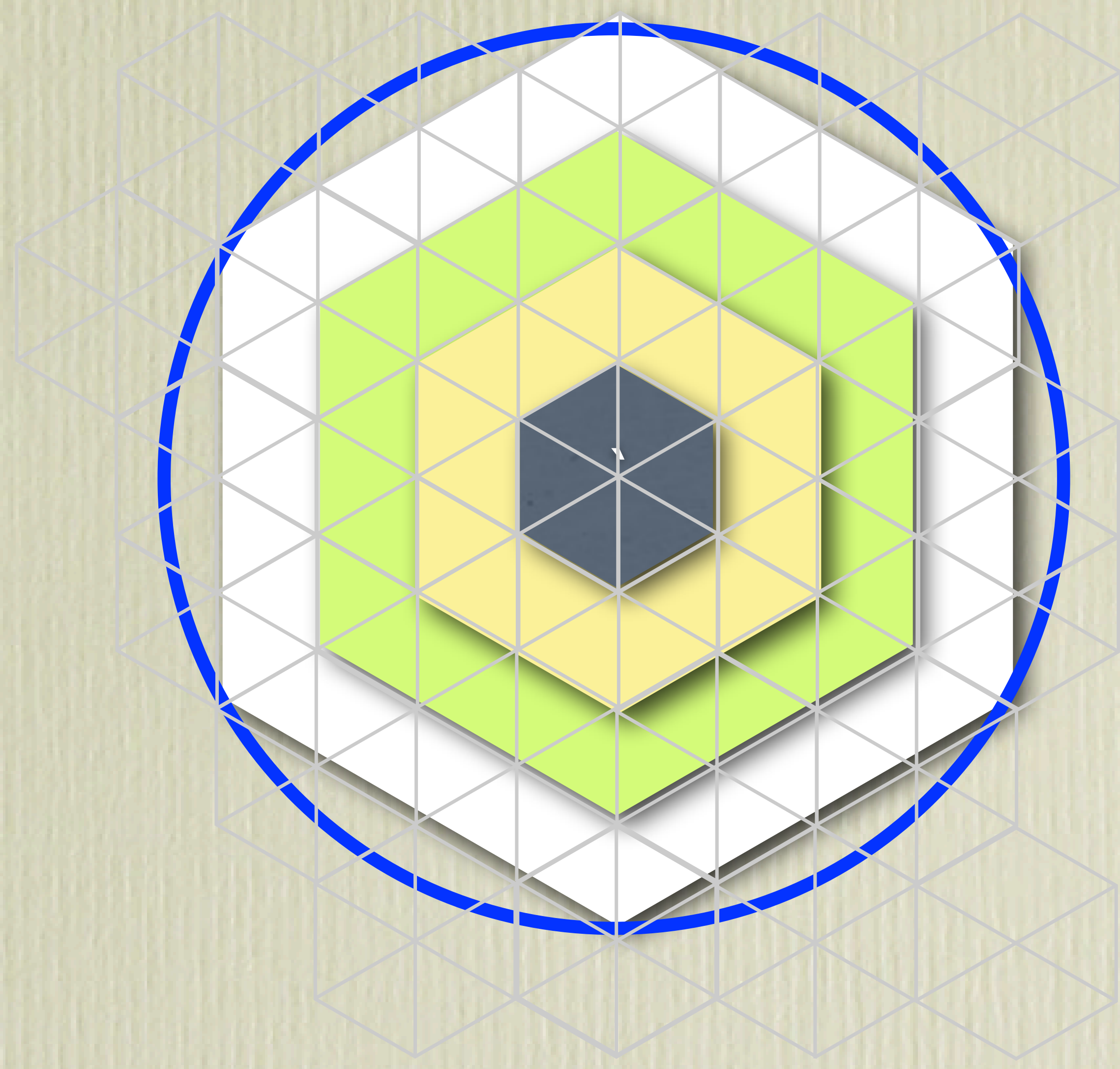
- A *Quantum-Digital World*: restoration of the isotropy of information flow.

Why information is quantum?

Should we consider a network-axiom for QT?

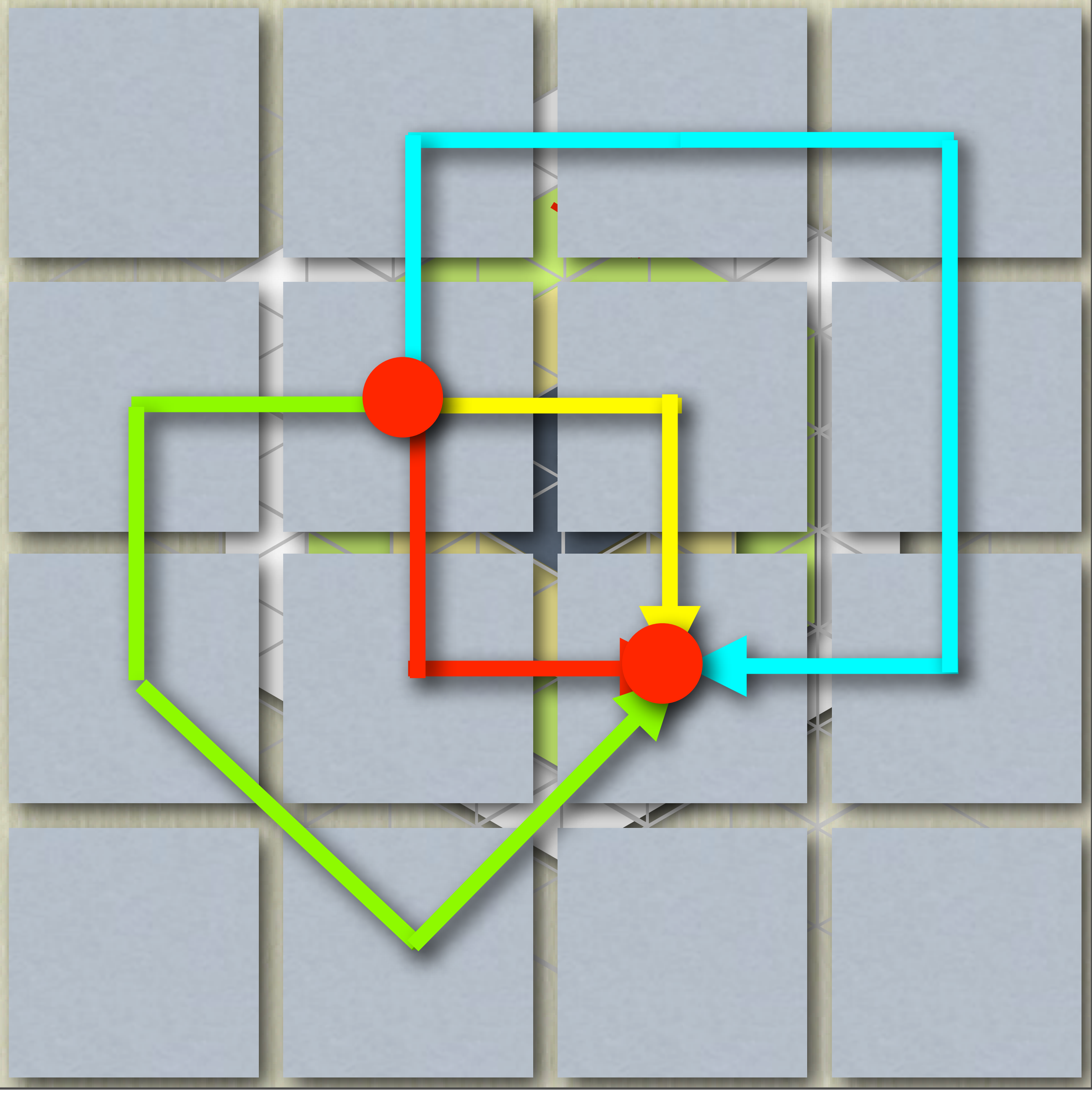
- "Direction" of information imprinted in the state using minimal informational resources.

- A *Quantum-Digital World*: restoration of the isotropy of information flow.

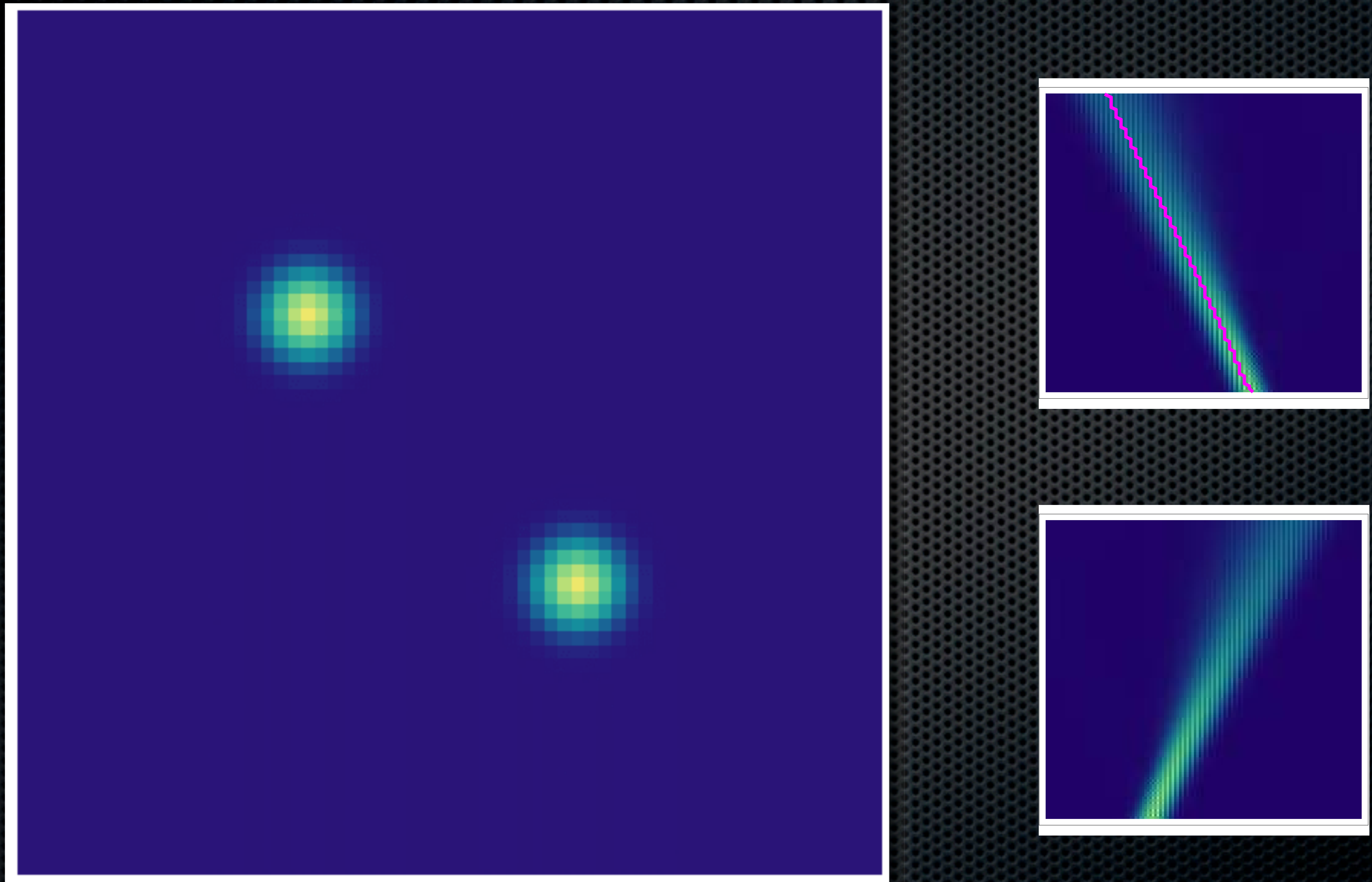


Why information is quantum?

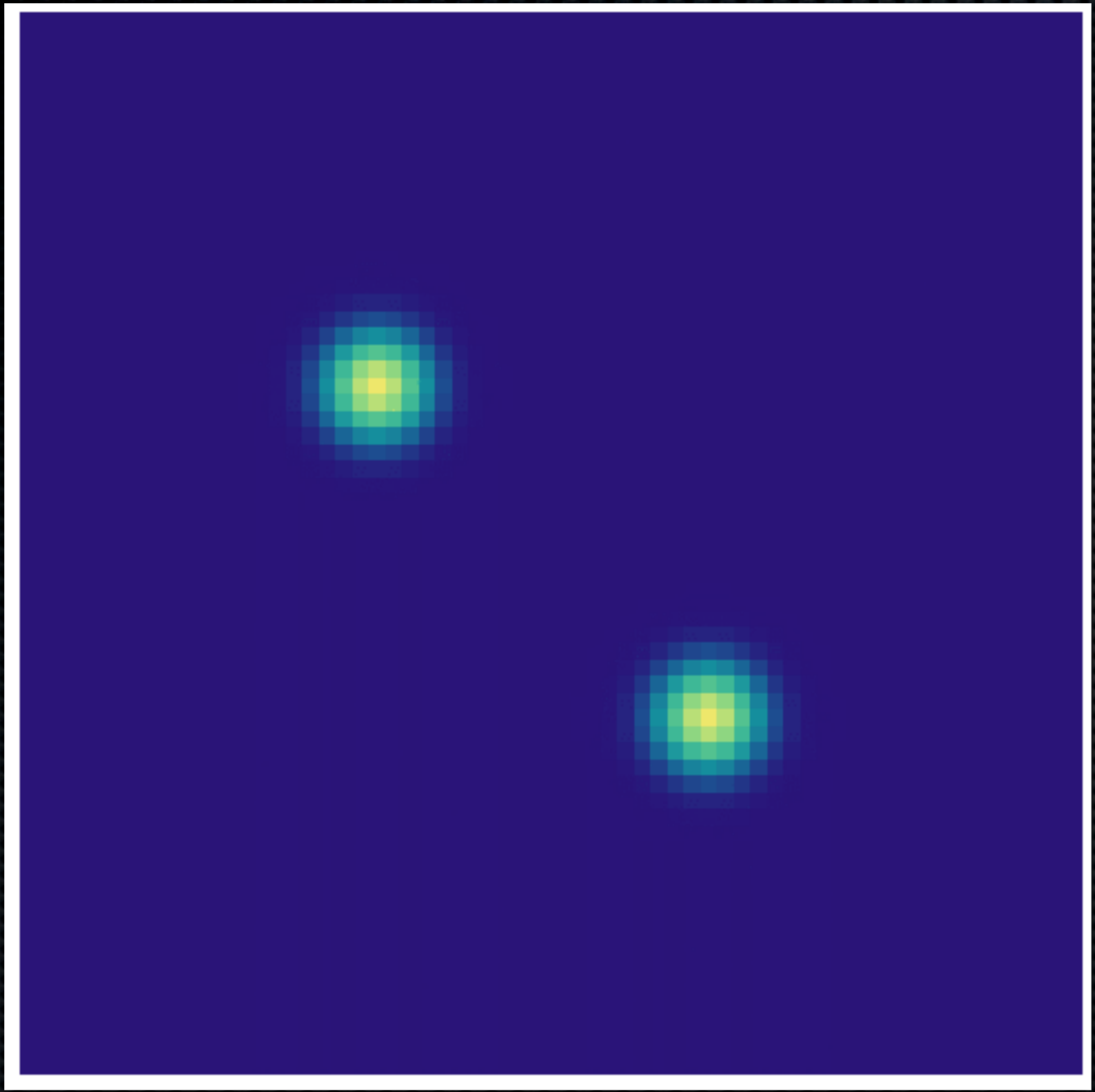
Should we consider a network-axiom for QT?

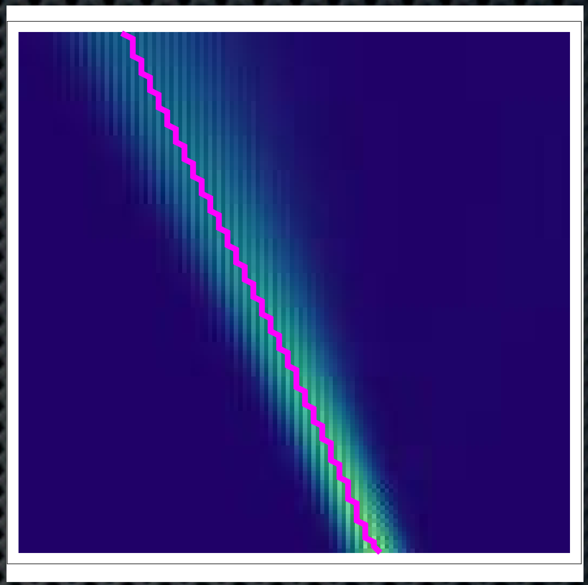


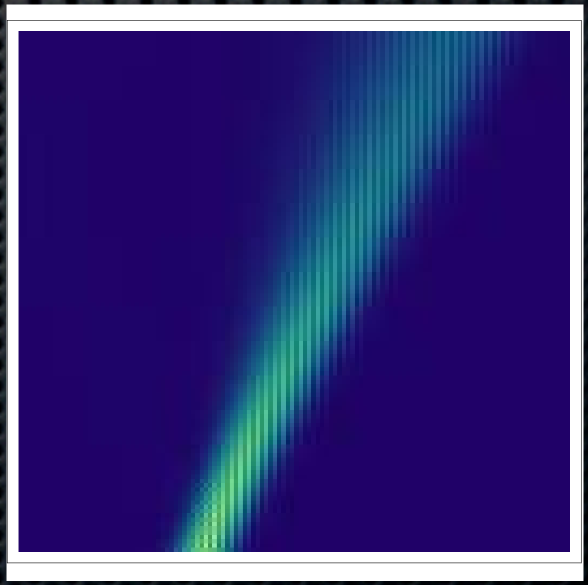
- "Direction" of information imprinted in the state using minimal informational resources.
- A *Quantum-Digital World*: restoration of the isotropy of information flow.



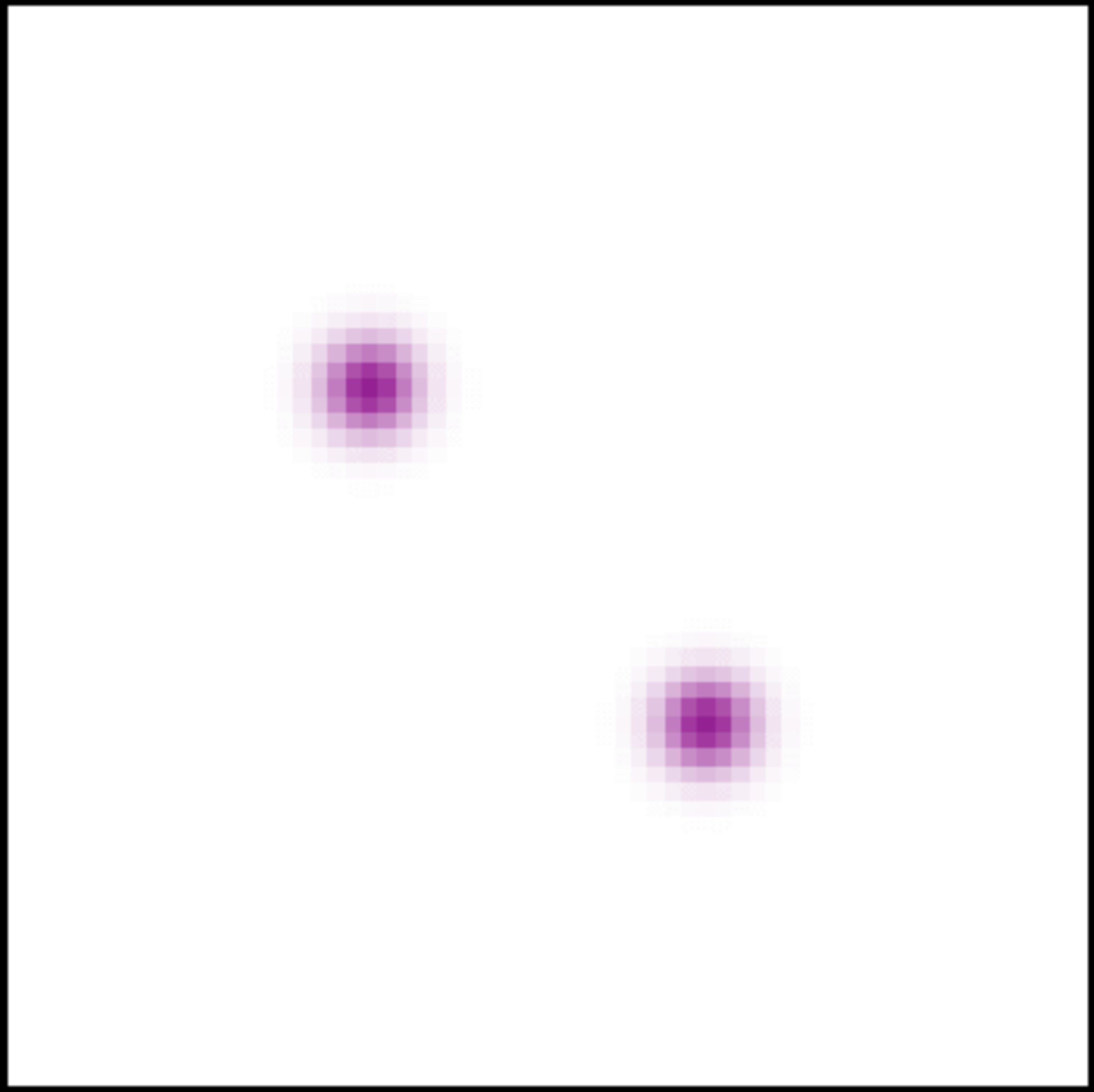
First Quantization: two-particle states

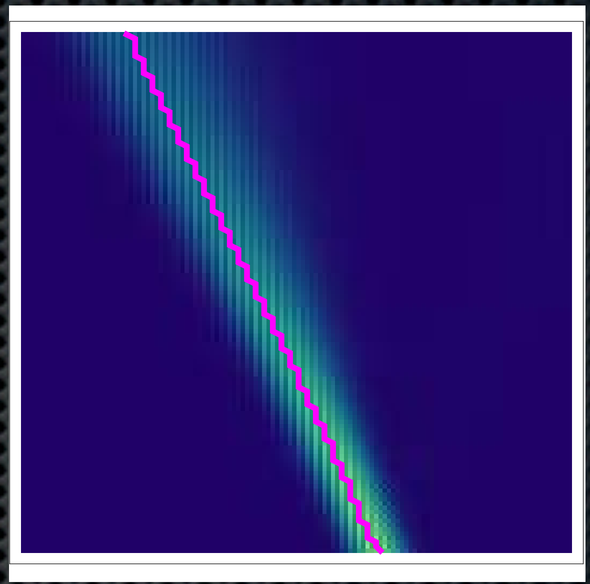


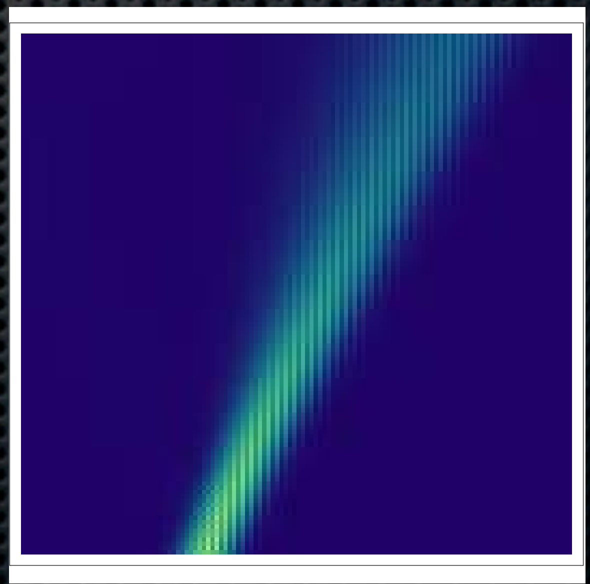




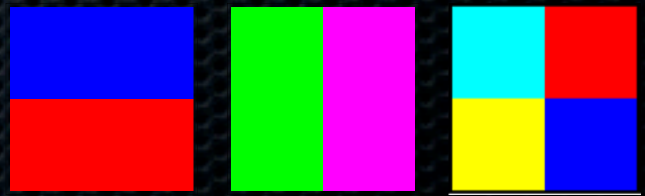
First Quantization: particle-antiparticle



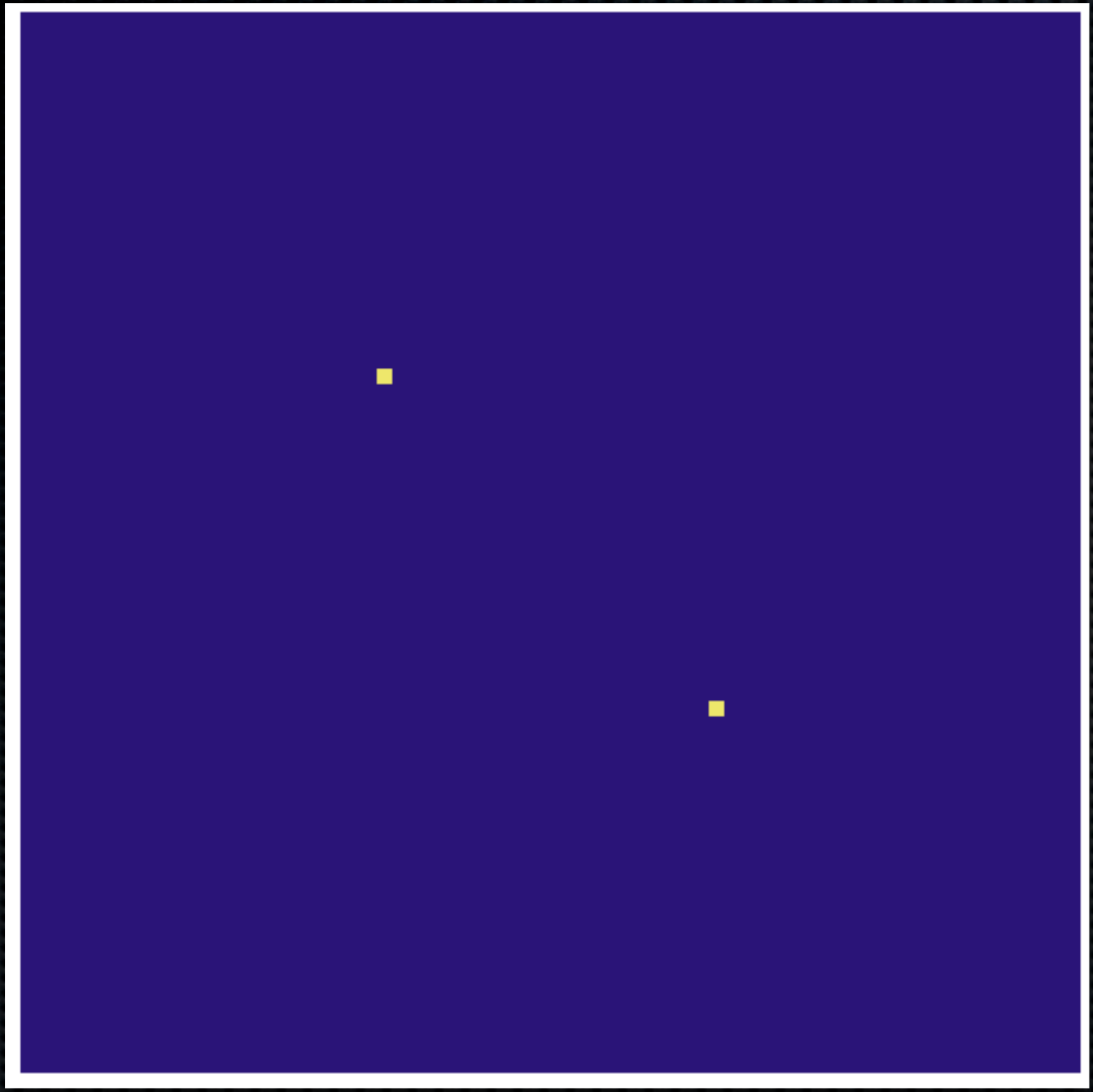


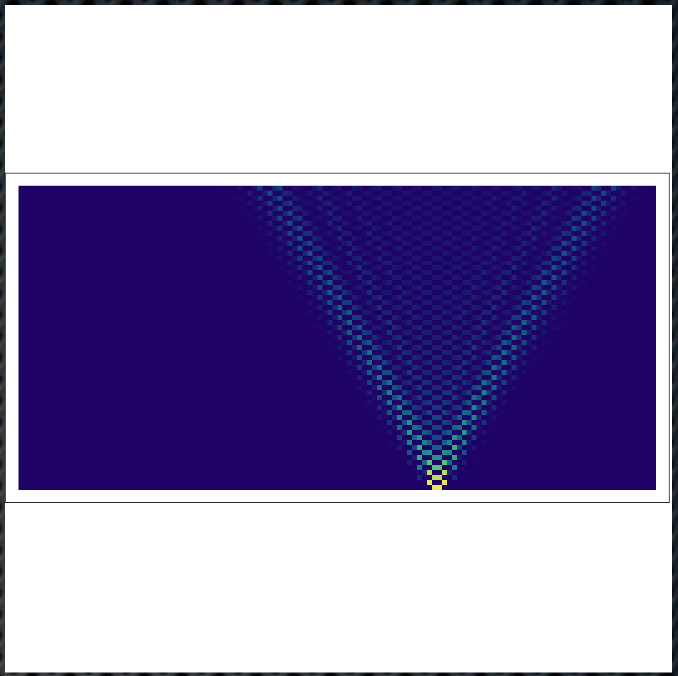


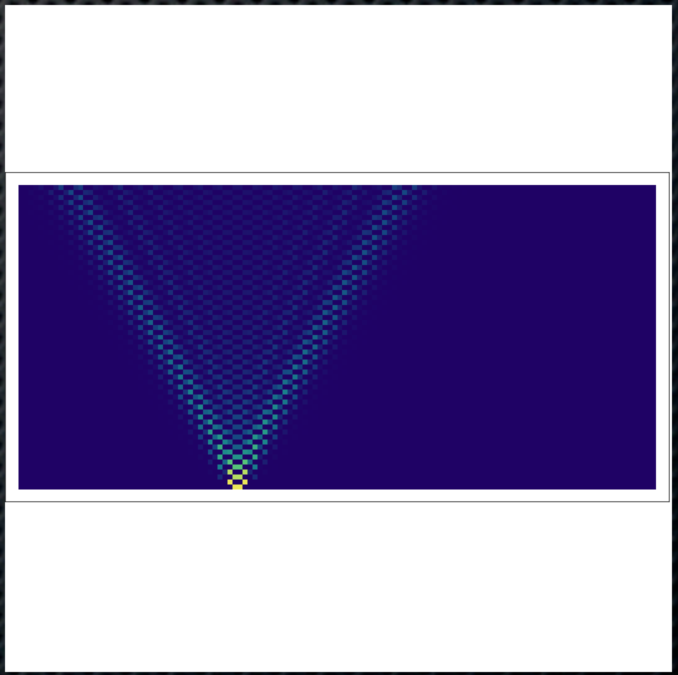
First Quantization: particle-antiparticle



singlet triplet







First Quantization: two-particle states

THANK YOU!