Measurement of the τ Lepton Anomalous Magnetic Moment at B-Factories

Introduction, Status & Perspectives

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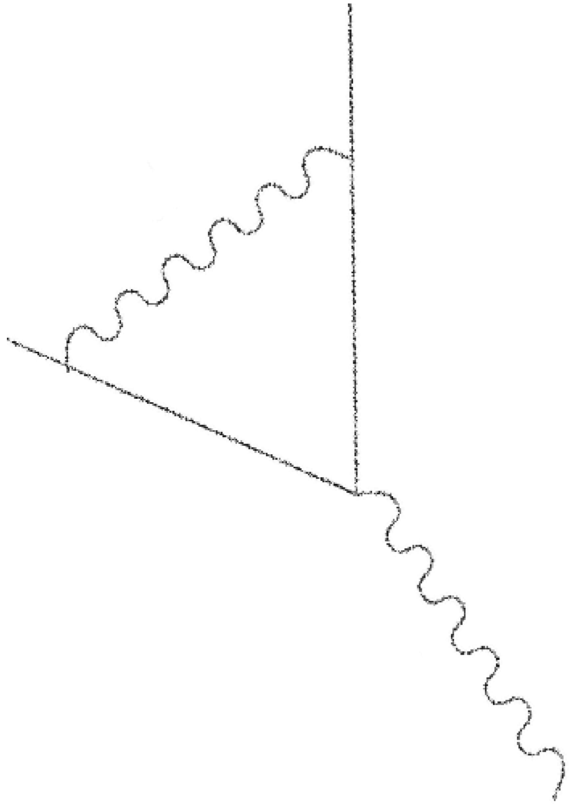
Outline

- *Measurement interest*
- *g-2 at B-factories*
- *Underlying physics*
- *Observables*
- *Ongoing work on BaBar & preliminary results*
  - *- Preselection*
  - *- Cross-section*
  - *- Asymmetries*
- *Polarized beams*
- *Outlook*



Interest of the Measurement

- Precision measurement of $a_{e,\mu,\tau} = (g_{e,\mu,\tau} - 2)/2 \rightarrow$ very sensitive test of SM
- Different contributions at loop level, dependence on fermion masses

- a_e and a_μ nowadays known with very high precision (see PDG)

$$a_e = (1159.6521810 \pm 0.0000007) \cdot 10^{-6}$$
$$a_\mu = (1165.92080 \pm 0.00054 \pm 0.00033) \cdot 10^{-6}$$

- Measurements of a_l for light leptons based on static interaction with an external field
  $\rightarrow$ impossible for the τ because of it's short lifetime $\tau = (290.6 \pm 1.0) \times 10^{-15}$
- The mesaurement from the DELPHI collaboration of $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ cross section at LEP2 (arXiv:hep-ex/0406010v1 (2004)) yelds

$$-0.052 < a_\tau < 0.013 \text{ at } 95\% \text{ CL}$$

- SM predicted value: $a_{SM,\tau} = 1177.21(5) \times 10^{-6}$, very precise, but not yet tested!
- If Δa_μ measured by BNL-E821 due to SUSY even bigger effects expected on a_τ

g-2 at B-factories

- It is not possible to measure interaction of τ with an external field
  $\rightarrow$ we have to infer about a_τ from cross section and partial widths
- B-factories are (nearly..) an ideal laboratory:
  $\rightarrow$ high luminosity
  $\rightarrow$ high production cross section: 0.919 nb at $\sqrt{s} = 10.54$ GeV/c
  $\rightarrow$ negligible contribution from higher order diagrams at $\sqrt{s} \sim m(\Upsilon(4s))$
  $\rightarrow$ electro-weak interference effects well known and under control
- Main issues related to CM boost and detector resolution
- In principle different final states could be used for the measurement
- If we use final states in which both τs decay in two bodies we can also use spin correlations (hepex/0709.2496v1 (2007)). (JHEP01 (2009) 62)
  $\rightarrow$ possibility to study more observables with high statistics
- In this study we focused on processes of type $\tau^\pm \longrightarrow h^\pm \nu$
  in particular, at present, only $h^\pm = \pi^\pm$ has been investigated

Underlying Physics

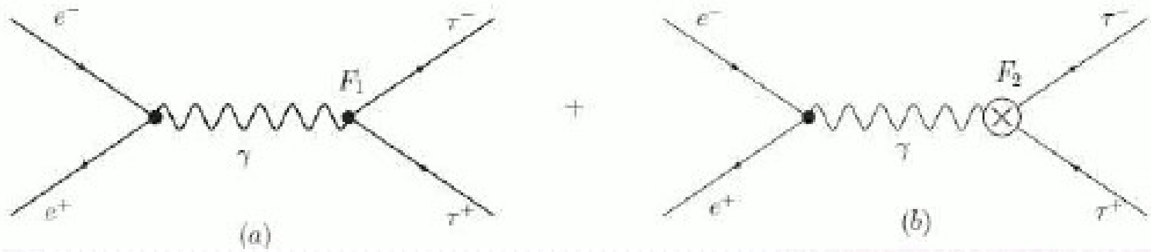
- Let's consider $e^+e^- \to \tau^+\tau^-$ in pure QED
- Most general Lorentz and gauge invariant coupling for the $\tau\tau\gamma$ vertex

$$< f(p_-)f(p_+)|J^\mu|0 >=$$
$$= e\bar{u}(p_-)[\gamma^\mu F_1 + \frac{1}{2m_f}(iF_2 + F_3\gamma^5)\sigma^{\mu\nu}q_\nu - (q^2\gamma^\mu - q^\mu\gamma^\nu q_\nu)\gamma^5 F_A]v(p_+)$$

- $F_1(s)$ and $F_2(s)$ are the electric and magnetic form factors which depend on energy, $F_1(0) = 1$, $F_2(0) = a_\tau$
- $F_2(0) = a_\tau$ depends also on lepton mass so $F_{2,e}(Y(4s))$ contribution can be neglected

$$F_2(s) = (2.65 - 2.45i) \cdot 10^{-4} \text{ at } \sqrt{s} \simeq 10.5$$

- $F_3 \to$ electric dipole form factor $\to$ contribution vanishes up to 3 loops in SM
- $F_A \to$ anapole form factor $\to$ suppressed by q^2/M_Z^2 w.r.t. leading QED corrections



Observables: Cross Section

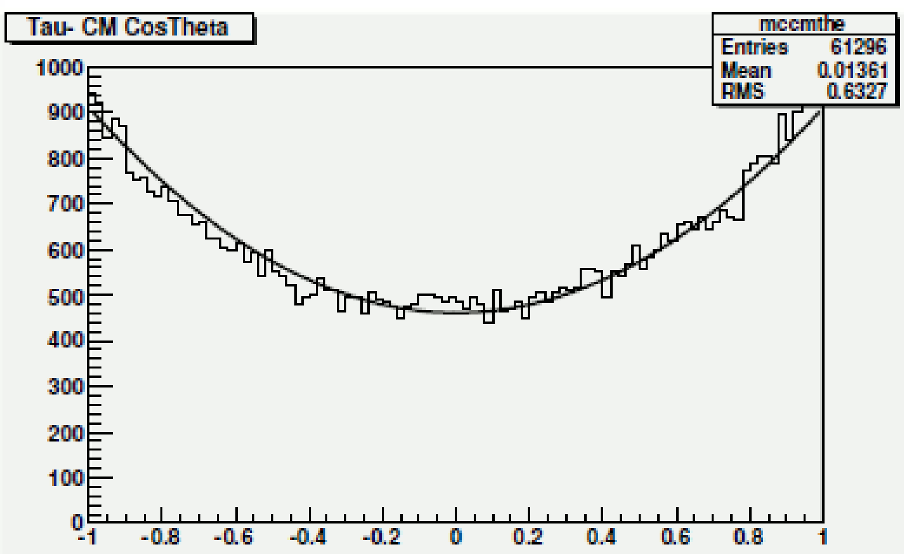
- From the previous form for the ττγ coupling, the differential cross-section for τ production becomes

$$\frac{d\sigma}{d\cos\theta_{\tau^-}} = \frac{\pi\alpha^2}{2s}\beta[(2-\beta^2\sin^2\theta_{\tau^-})|F_1(s)|^2 + 4\mathcal{R}e(F_2(s))]$$

- Choosing hv final states modifies the CS only by an overall factor BR1*BR2
- Theoretically most sensible observable to *Re(F₂)*
- Actual precision will depend mainly on reconstruction capabilities

from Bernabeu et al. (hepex/0709.2496v1 (2007))

EXPERIMENT ⇓	Cross Section $Re\{F_2\}$
Babar+Belle $2ab^{-1}$	4.6×10^{-6}
Super B/Flavor Factory (1 yr. running) $15ab^{-1}$	1.7×10^{-6} πρ only!
Super B/Flavor Factory (5 yrs. running) 75 ab^{-1}	7.5×10^{-7}

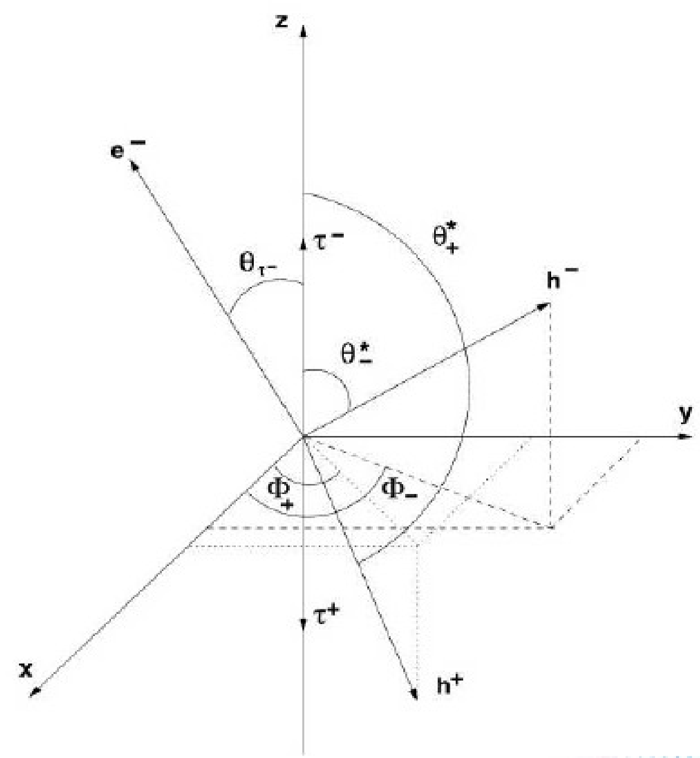


Observables: Asymmetries

- Taking the complete spin-dependent CS

$$\frac{d\sigma(s^+,s^-)}{d\cos\theta_{\tau^-}} = \frac{d\sigma^0(s^+,s^-)}{d\cos\theta_{\tau^-}} + \frac{d\sigma^S(s^+,s^-)}{d\cos\theta_{\tau^-}} + \frac{d\sigma^{SS}(s^+,s^-)}{d\cos\theta_{\tau^-}}$$

- Constant, linear e quadratic terms in tau spins → s^+ s^- are related to hadron flight directions in the tau rest frames



- With appropriate integration intervals one can define up to 5 asymmetry parameters, e.g.:

$$A_{TT} \equiv -\frac{\alpha_-\alpha_+}{\sigma}\left(\int_{-\pi/2}^{\pi/2} d\phi_- - \int_{\pi/2}^{3\pi/2} d\phi_-\right)\left(\int_{-\pi/2}^{\pi/2} d\phi_+ - \int_{\pi/2}^{3\pi/2} d\phi_+\right) d^2\sigma_{TT}$$

$$= -\frac{\pi\alpha^2\beta}{6\,s}\,\frac{\alpha_-\alpha_+}{\sigma}\left[(2-\beta^2)\,|F_1|^2 + 4\mathrm{Re}\,\{F_2\}\right]\,Br_+\,Br_-$$

$$d^2\sigma_{TT} = \frac{\pi\alpha^2\beta}{96\,s}\,[-(\alpha_-\alpha_+)]\,(\mathcal{X}\mathcal{X})\,(\cos\phi_-\,\cos\phi_+)\,d\phi_+\,d\phi_-\,Br_+\,Br_- \quad \mathcal{X}\mathcal{X} = (2-\beta^2)\,|F_1|^2 + 4\mathrm{Re}\,\{F_2\}$$

Dependence on linear combinations of F_1^2, $Re(F_2)$, $Im(F_2)$

- Sensivity to both $Re(F_2)$ and $Im(F_2)$!

		Babar+Belle	Super B/Flavour Factory	
			(1 yr. running)	(5 yrs. running)
	Correlations	$2ab^{-1}$	$15ab^{-1}$	$75ab^{-1}$
$Re\{F_2\}$	$TT--LT$	7.6×10^{-5}	2.8×10^{-5}	1.2×10^{-5}
	$LL--LT$	5.2×10^{-5}	1.9×10^{-5}	8.5×10^{-6}
	$NN--LT$	5.1×10^{-5}	1.8×10^{-5}	8.3×10^{-6}
	Global	2.9×10^{-5}	1.1×10^{-5}	4.7×10^{-6}
$Im\{F_2\}$ (from ref. [10])	Normal single-τ Asymm.	2.1×10^{-5}	7.8×10^{-6}	3.5×10^{-6}

from Bernabeu et al.

Reconstruction of τ flight direction

- In CM frame τ lies on a cone with opening angle around hadron flight direction

$$\cos\theta_{\pm} = \frac{2E_{\tau^\pm}E_{h^\pm} - m_\tau^2 - m_{h^\pm}^2}{|p_{\tau^\pm}||p_{h^\pm}|}$$

- Imposing same primary vertex

$$\begin{aligned} |\vec{P}_{\tau-}||\vec{p}_{h-}|\cos\theta_- &= \vec{P}_{\tau-}\cdot\vec{p}_{h-} \\ |\vec{P}_{\tau-}||\vec{p}_{h+}|\cos\theta_+ &= -\vec{P}_{\tau-}\cdot\vec{p}_{h+} \end{aligned}$$

- Using precedent equations we can solve for τ momenta up to a twofold ambiguity
- Possible the solve the ambiguity exactly in symmetric machines using impact parameters (Phys.Lett.B 313 458.)
- Not clear, in general, if it can be efficiently done in B-factories: at present theoretical gain completely lost to efficiency and systematics

STRATEGY $\longrightarrow$

- retain both solutions and study how the distributions of observables change

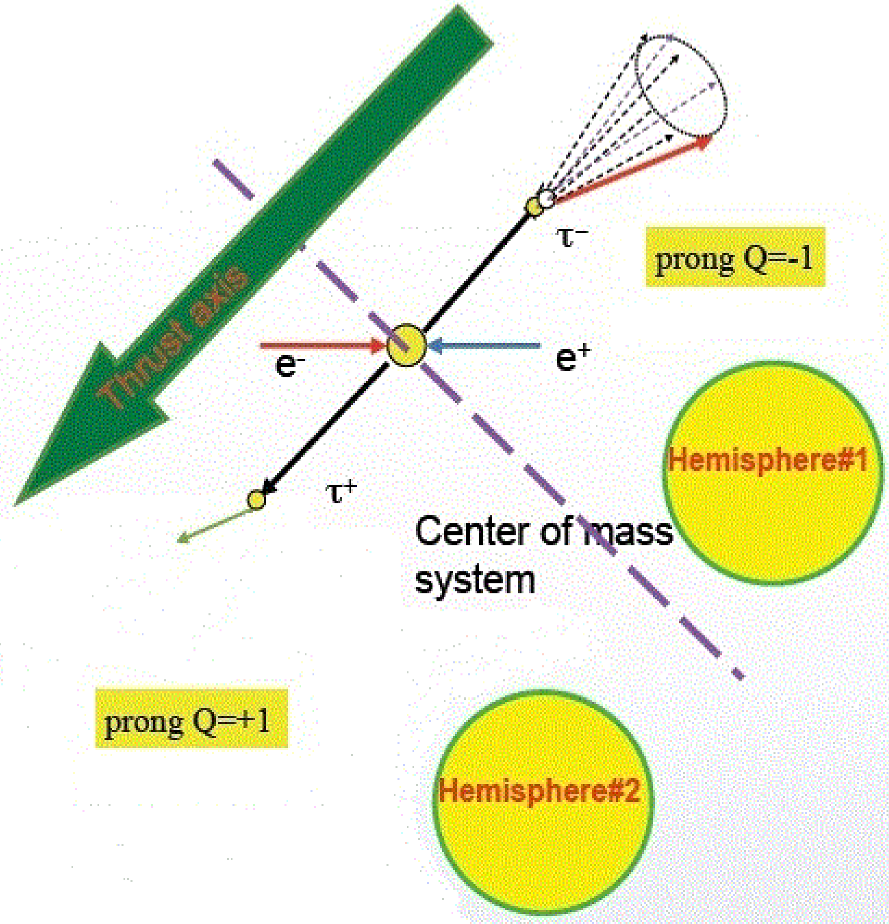
Progress: event pre-selection

- **Work started for analysis on BaBar data**
- **Standard BaBar skims for 1-1 events used in preselection**

- Preliminary cuts on reconstructed tracks:
  - $0.41 < \theta_{LAB} < 2.46$ EMC acceptance
  - Transverse momentum of $p_T > 0.1$ GeV/c
  - Maximum total momentum $p_{max} < 10$GeV/c
- Trigger requirments (or of the following):
  - at least 1 track with $p_T > 600$ MeV/c, doca $d_{xy} < 1$ cm (transverse) $d_z < 7$ cm (longitudinal)
  - at least 2 tracks with $p_T > 250$ MeV/c, with $d_{xy} < 1.5$ cm and $d_z < 10$ cm
  - at least two clusters (EMC) with $E > 350$MeV, total invariant mass $M_{inv} > 1.5$GeV/c^2
- Tau background filter:
  - Two tracks and no net charge, $p_1 + p_2 < 9$GeV/c, $E_1 + E_2 < 5$GeV
  - $E_{CM} - p_1 - p_2 > 0$ and $(p_1 + p_2)/(E_{CM} - p_1 - p_2) > 0.07$ accounts for neutrino presence
- Tau candidate selection: the thrust axis defines the topology.

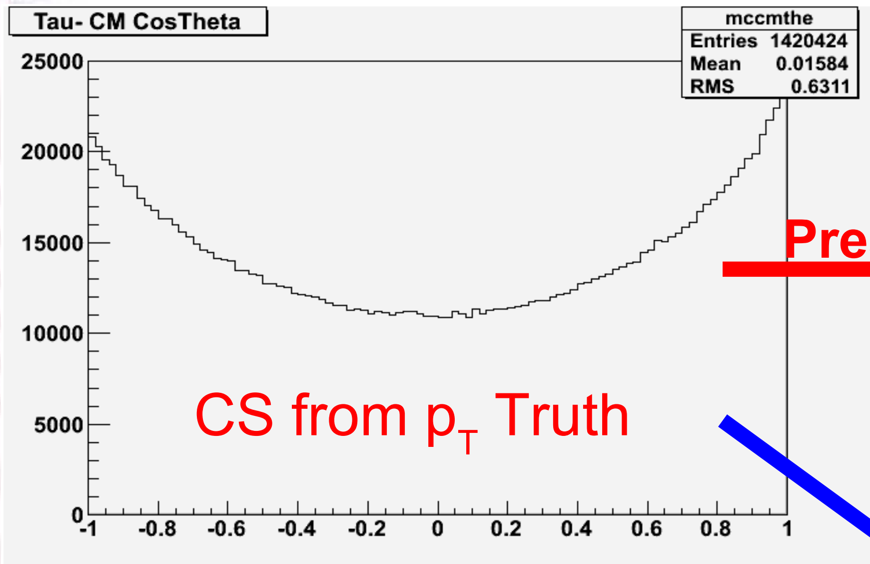
$$T = \frac{\Sigma_i |\hat{T} \cdot \vec{p}_i|}{\Sigma_i |\vec{p}_i|}$$

The event is divided in 2 non overlapping emispheres.

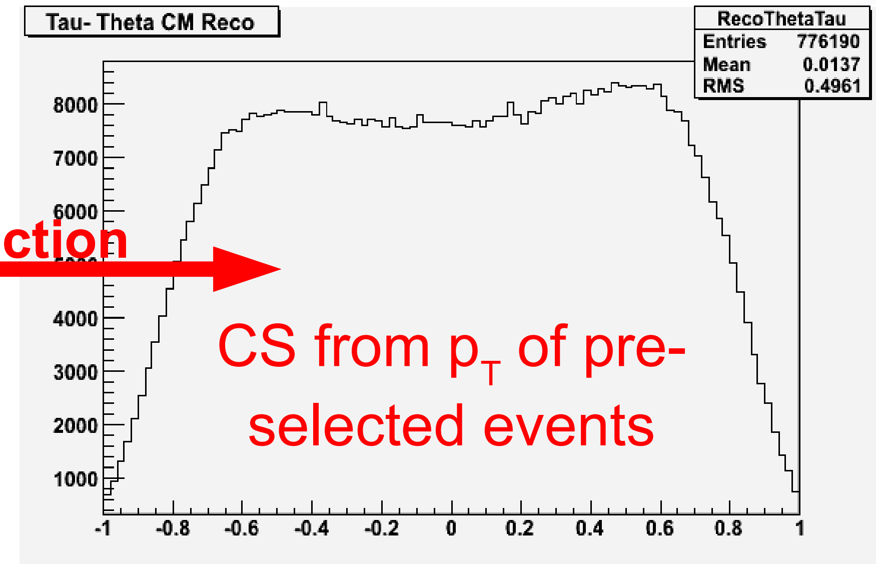


Overall efficiency on signal ~50%

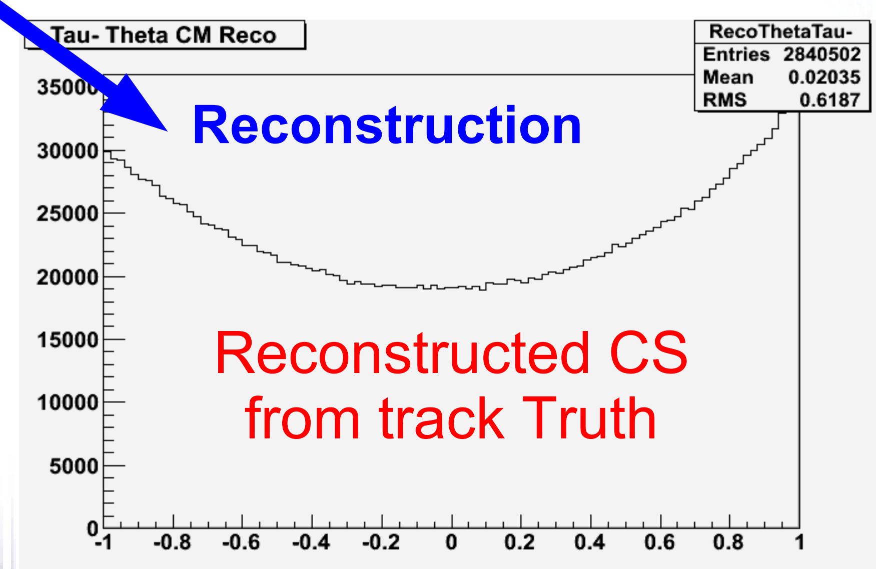
Progress: Cross section



Preselection



Reconstruction



- **Most information lost in preselection**
- **Dramatic effect of detector acceptance**
- **Magnetic form factor enters only as on offset, angular dependence is in $|F_1|^2$**

Differential CS not suited to extract $Re(F_2)$!

Progress: asymmetries (1)

- We are interested in azimuthal and polar distributions of hadrons in tau rest frames
- Reconstruction and selection cause distortions
- Individual distributions of hadrons have to be flat independently of g-2 (all effect in primary vertex, none in decay)

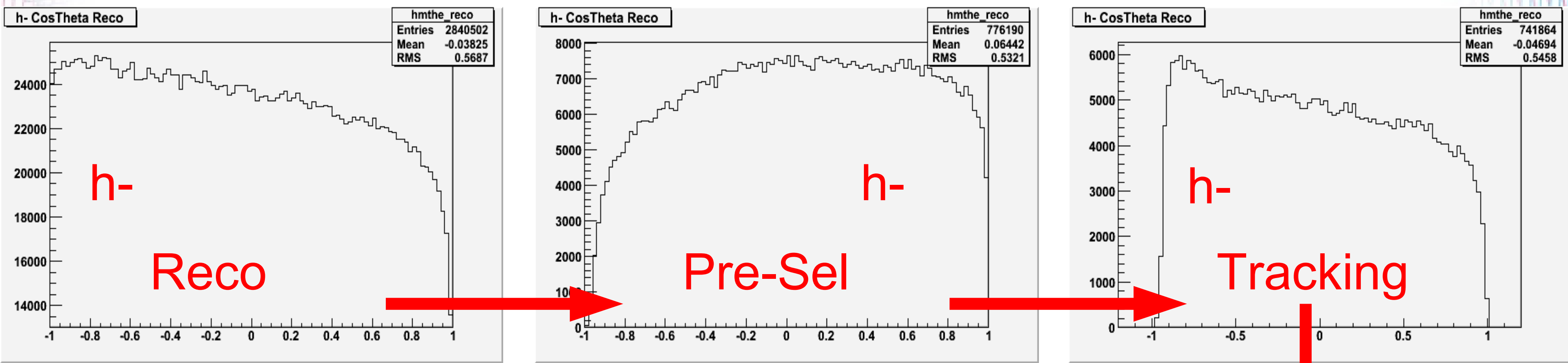
Idea is to find reweighting factors for individual angular distributions and THEN look for correlations

Crucial Points:

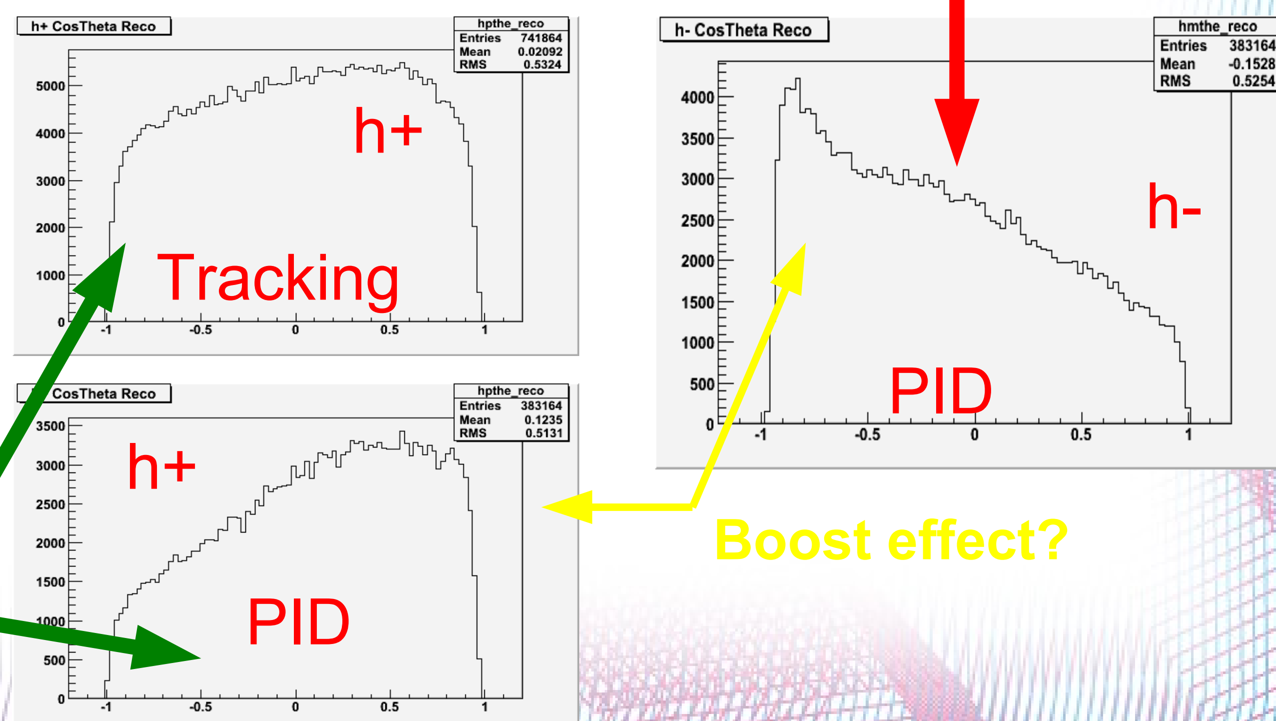
- Understand how good MC matches data
- Careful study of correlations between angular variables

Progress: asymmetries (2)

Polar Distribution

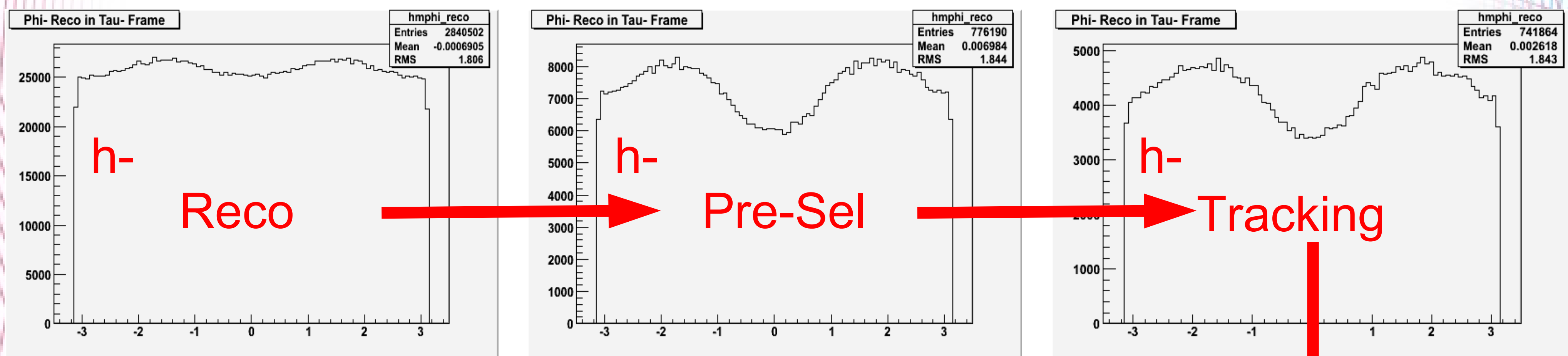


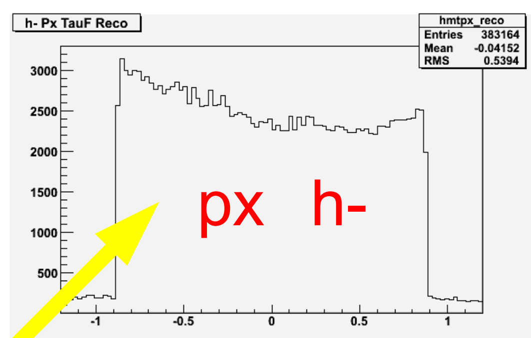
- **Remarks:**
- **Polar distributions is asymmetricaly distorted at every sel. step**
- **Reconstruction and pre-selection effects charge symmetric due to boost**
- **Tracking & PID effects not charge symmetric**

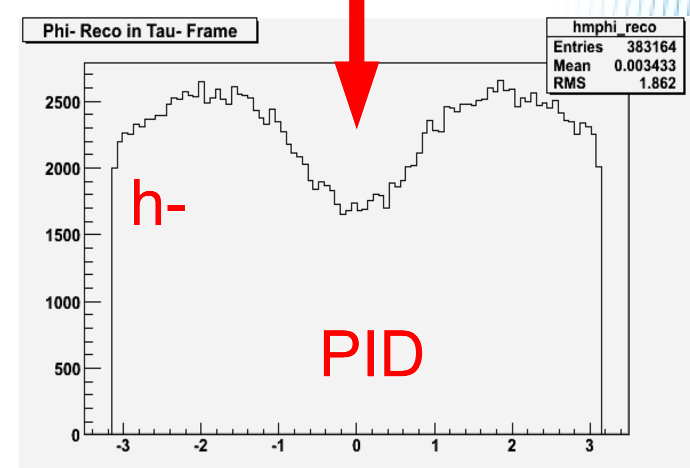


Progress: asymmetries (3)

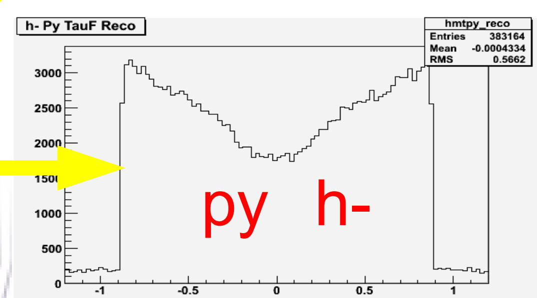
Azimuthal Distribution







- **Remarks:**
- **Azimuthal distributions remains symmetric**
- **No charge dependent effects**
- **Dist. shape originates from different behaviour of px and py due to frame choice**



Polarized Beams

- Both $|F_1|^2$ and $\mathrm{Re}(F_2)$ have the same properties under C, P and T
- To extract them we need observables were they enter with different weights
- Using polarized beams one can define observables depending on the decay products of a single tau, linear in F_2, with different behaviour under P
- We can consider longitudinal and transverse polarization states of outgoing tau
- This defines 2 new asymmetry parameters A_N and A_L

$$A_L^{\pm} = \frac{\sigma_{FB}^{\pm}(+)|_{\rm Pol} - \sigma_{FB}^{\pm}(-)|_{\rm Pol}}{\sigma} = \mp \alpha_{\perp} \frac{3}{4(3-\beta^2)} \left[|F_1|^2 + 2\,\mathrm{Re}\{F_2\} \right]$$

$$\sigma_{FB}^{\pm}(+)|_{\rm Pol} \equiv \int_0^1 d(\cos\theta_{\pm}^{*}) \left. \frac{d\sigma_{FB}^{S}}{d(\cos\theta_{\pm}^{*})} \right|_{\mathrm{Pol}(e^-)} = \mp\alpha_{\pm}\, Br(\tau^+ \to h^+ \bar{\nu}_\tau) \times Br(\tau^- \to h^- \nu_\tau)\, \frac{\pi\alpha^2}{4s}\beta \left[|F_1|^2 + 2\,\mathrm{Re}\{F_2\} \right]$$

$$\sigma_{FB}^{\pm}(-)|_{\rm Pol} \equiv \int_{-1}^0 d(\cos\theta_{\pm}^{*}) \left. \frac{d\sigma_{FB}^{S}}{d(\cos\theta_{\pm}^{*})} \right|_{Pol(e^-)} = -\sigma_{FB}^{\pm}(+)|_{\rm Pol}\,.$$

πρ only! EXPERIMENT ⇓	Transverse and Longitudinal Asymmetry combined* $\mathrm{Re}\{F_2\}$
Super B/Flavor Factory (1 yr. running) $15ab^{-1}$	3.7×10^{-6}
Super B/Flavor Factory (5 yrs. running) 75 ab^{-1} ($15ab^{-1}$)	1.7×10^{-6}

Gain a factor ~3 on $\mathrm{Re}(F_2)$ wrt unpol beams

Preliminary Conclusions & Work Plan

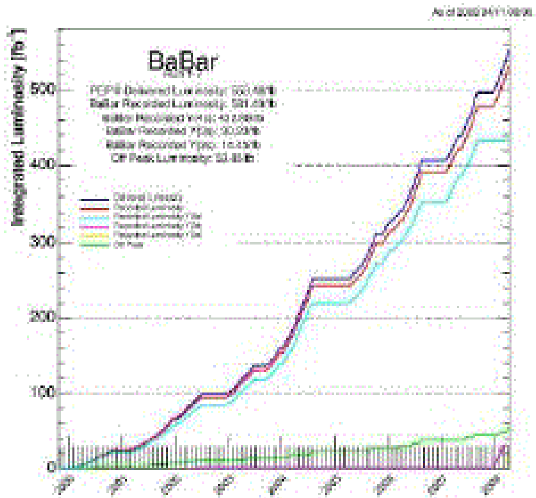
- The measurement of g-2 seems feasible using asymmetries
- Overall efficiency on signal at this point around 30%
- Asymmetries based on symmetric integration over polar variables A_{NN} A_{TT} better suited due to reconstruction and selection effects
- Actual sensivity will depend mainly on systematics

Open questions & Roadmap:

- Study of backgrounds and definition of selection cuts
- Definition and test of reweighting technique
- Preliminary check of sensivity
- Inclusion all hadronic two-body decay channels
- Study possibility to use also leptonic decay channels
- Begin production of events with beam polarization and studies on polarization dependent observables
- Try to solve the ambiguity problem using IP as constraint

Backup Slides

BaBar Data Sample



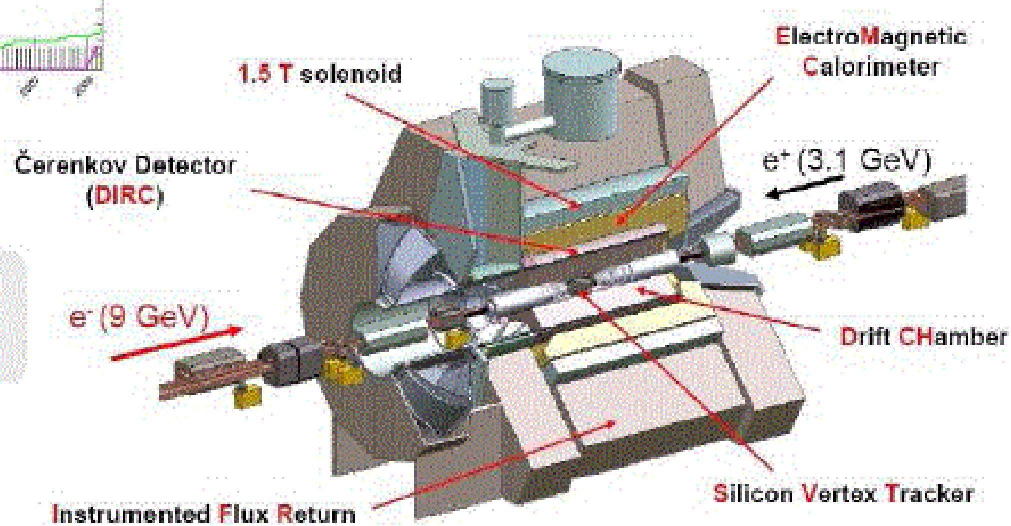
$e^+e^- \to \Upsilon(4S)$:

Run1 to Run6 $\to L_{on+off} = 474\ fb^{-1}$

$e^+e^- \to \Upsilon(3S), \Upsilon(2S)$:

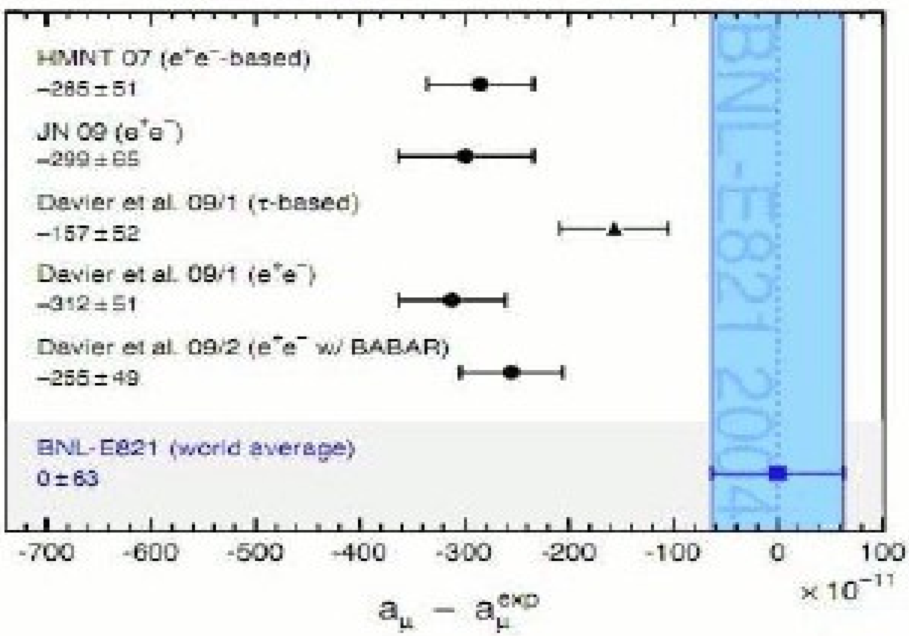
Run7 $\to L_{on+off} = 47\ fb^{-1}$

PEP-II is also a charm-Factory: *BABAR* recorded around 690M of $e^+e^- \to c\bar{c}$ events



Tau g-2 & BNL821

VALUE	CL%	DOCUMENT ID		TECN	COMMENT
>-0.052 and <0.013	**OUR LIMIT**				
>-0.052 and <0.013	95	[1] ABDALLAH	04K	DLPH	$e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2
* * * We do not use the following data for averages, fits, limits, *etc.* * * *					
<0.107	95	[2] ACHARD	04G	L3	$e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2
>-0.007 and <0.005	95	[3] GONZALEZ-SP...	00	RVUE	$e^+e^- \to \tau^+\tau^-$ and $W \to \tau\nu_\tau$
>-0.052 and <0.058	95	[4] ACCIARRI	98E	L3	1991 – 1995 LEP runs
>-0.068 and <0.065	95	[5] ACKERSTAFF	98N	OPAL	1990 – 1995 LEP runs
>-0.004 and <0.006	95	[6] ESCRIBANO	97	RVUE	$Z \to \tau^+\tau^-$ at LEP
<0.01	95	[7] ESCRIBANO	93	RVUE	$Z \to \tau^+\tau^-$ at LEP
<0.12	90	GRIFOLS	91	RVUE	$Z \to \tau\tau\gamma$ at LEP
<0.023	95	[8] SILVERMAN	83	RVUE	$e^+e^- \to \tau^+\tau^-$ at PETRA



The complete cross section

$$\frac{d\sigma(s^+,s^-)}{d\cos\theta_{\tau^-}} = \frac{d\sigma^0(s^+,s^-)}{d\cos\theta_{\tau^-}} + \frac{d\sigma^S(s^+,s^-)}{d\cos\theta_{\tau^-}} + \frac{d\sigma^{SS}(s^+,s^-)}{d\cos\theta_{\tau^-}}$$

$$\frac{d\sigma^0(s^+,s^-)}{d\cos\theta_{\tau^-}} = \frac{\pi\alpha^2}{8s}\beta[(2-\beta^2\sin^2\theta_{\tau^-})|F_1(s)|^2 + 4\mathcal{R}e(F_1(s)F_2^*(s))]$$

$$\frac{d\sigma^S(s^+,s^-)}{d\cos\theta_{\tau^-}} = \frac{\pi\alpha^2}{4s}\gamma\beta^3(s^- + s^+)_y(\cos\theta_{\tau^-}\sin\theta_{\tau^-})\mathcal{I}m(F_2(s))$$

$$\frac{d\sigma^{SS}(s^+,s^-)}{d\cos\theta_{\tau^-}} = \frac{\pi\alpha^2}{8s}\beta(s_+^x s_-^x C_{xx} + s_+^y s_-^y Cyy + s_+^z s_-^z C_{zz} + (s_+^x s_-^z + s_+^z s_-^x)C_{xz}^+)$$

$$\begin{aligned}
C_{xx} &= ((2-\beta^2)|F_1|^2 + 4\mathcal{R}e(F_2))\sin^2\theta_{\tau^-} \\
C_{yy} &= -\beta^2|F_1|^2\sin^2\theta_{\tau^-} \\
C_{zz} &= |F_1|^2(2\cos^2\theta_{\tau^-} + \beta^2\sin^2\theta_{\tau^-} + 4\mathcal{R}e(F_2)\cos^2\theta_{\tau^-}) \\
C_{xz}^+ &= \frac{1}{\gamma}(|F_1|^2 + \gamma^2(2-\beta^2)\mathcal{R}e(F_2))\sin 2\theta_{\tau^-}
\end{aligned}$$

Asymmetries: both τs measured, symmetric polar integration, Re(F_2)

$$A_{TT} = -\frac{\alpha_-\alpha_+}{\sigma}\left(\int_{-\pi/2}^{\pi/2} d\phi_- - \int_{\pi/2}^{3/2\pi} d\phi_-\right)\left(\int_{-\pi/2}^{\pi/2} d\phi_+ - \int_{\pi/2}^{3/2\pi} d\phi_+\right) d^2\sigma_{TT}$$

$$= -\frac{\pi\alpha^2\beta}{6s}\frac{\alpha_-\alpha_+}{\sigma}((2-\beta^2)|F_1|^2 + 4\mathcal{R}e(F_2))\mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

$$d^2\sigma_{TT} = -\frac{\pi\alpha^2\beta}{96s}(\alpha_-\alpha_+)(\cos\phi_- \cos\phi_+)\mathcal{X}\mathcal{X} d\phi_- d\phi_+ \mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

$$A_{NN} = -\frac{\alpha_-\alpha_+}{\sigma}\left(\int_0^{\pi} d\phi_- - \int_{\pi}^{2\pi} d\phi_-\right)\left(\int_0^{\pi} d\phi_+ - \int_{\pi}^{2\pi} d\phi_+\right) d^2\sigma_{NN}$$

$$= \frac{\pi\alpha^2\beta}{6s}\frac{\alpha_-\alpha_+}{\sigma}\beta^2|F_1|^2\mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

$$d^2\sigma_{NN} = -\frac{\pi\alpha^2\beta}{96s}(\alpha_-\alpha_+)(\sin\phi_- \sin\phi_+)\mathcal{Y}\mathcal{Y} d\phi_- d\phi_+ \times \mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

Asymmetries: both τs measured, asymmetric polar integration, Re(F_2)

$$A_{LL} = -\frac{\alpha_-\alpha_+}{\sigma}\left(\int_{-1}^{0} d\cos\theta^*_- - \int_{0}^{1} d\cos\theta^*_-\right)\left(\int_{-1}^{0} d\cos\theta^*_+ - \int_{0}^{1} d\cos\theta^*_+\right)d^2\sigma_{LL}$$

$$= -\frac{\pi\alpha^2\beta}{6s}\frac{\alpha_-\alpha_+}{\sigma}((1+\beta^2)|F_1|^2 + 2\mathcal{R}e(F_2))\mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

$$d^2\sigma_{LL} = -\frac{\pi\alpha^2\beta}{6s}(\alpha_-\alpha_+)(\cos\theta^*_-\cos\theta^*_+)\mathcal{Z}\mathcal{Z}d\theta^*_-d\theta^*_+ \times \mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

$$A_{LT} = \left(\int_{-1}^{0} d\cos\theta^*_+ - \int_{0}^{1} d\cos\theta^*_+\right)\sigma_{LT}$$

$$= \frac{\pi\alpha^2\beta}{6s}\frac{\alpha_-\alpha_+}{\sigma}\left[\frac{1}{\gamma}|F_1|^2 + \gamma(2-\beta^2\mathcal{R}e(F_2))\right]\mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau)$$

$$d\sigma_{LT} = -\alpha_-\alpha_+\left(\int_{-\pi/2}^{\pi/2} d\phi_- - \int_{\pi/2}^{3/2\pi} d\phi_-\right)\left(\int_{0}^{2\pi} d\phi_+\right)\left(\int_{-1}^{1} d\cos\theta^*\right)d^4\sigma^{SS}$$

$$= -\frac{\pi\alpha^2\beta}{6s}\frac{\alpha_-\alpha_+}{\sigma}\mathcal{Z}\mathcal{X}\cos\theta^*_+ d\cos\theta^*_+\mathrm{BR}(\tau^+ \to h^+\bar{\nu}_\tau)\mathrm{BR}(\tau^- \to h^-\nu_\tau).$$

Asymmetries: single τ measured Im(F_2)

$$A_N^{\pm} = \frac{\sigma_L^{\pm} - \sigma_R^{\pm}}{\sigma} = \pm\alpha_{\pm}\frac{1}{2(3-\beta^2)}\beta^2\gamma\mathcal{I}m(F_2(s))$$

$$\sigma_L^{\pm} = \int_{\pi}^{2\pi} d\phi_{\pm}\frac{d\sigma_{FB}}{d\phi_{\pm}}, \ \sigma_R^{\pm} = \int_0^{\pi} d\phi_{\pm}\frac{d\sigma_{FB}}{d\phi_{\pm}} = -\sigma_L^{\pm}$$

$$\sigma_{FB}(\vec{s}_+, \vec{s}_-) = 2\pi\left(\int_0^1 d\cos\theta_{\tau^-}\frac{d\sigma}{d\Omega_{\tau^-}} - \int_{-1}^0 d\cos\theta_{\tau^-}\frac{d\sigma}{d\Omega_{\tau^-}}\right)$$

Asymmetries: polarization

A_T

$$A_T^{\pm} = \frac{\sigma_R^{\pm}|_{\rm Pol} - \sigma_L^{\pm}|_{\rm Pol}}{\sigma}$$

$$\sigma_L^{\pm}|_{\rm Pol} \equiv \int_{\pi/2}^{3\pi/2} d\phi_{\pm} \left[\left.\frac{d\sigma^S}{d\phi_{\pm}}\right|_{{\rm Pol}(e^-)} \right] = \pm Br(\tau^+ \to h^+ \bar{\nu}_\tau) Br(\tau^- \to h^- \nu_\tau) \times \alpha_{\pm} \frac{(\pi\alpha)^2\beta}{8s} \frac{1}{\gamma} \left[|F_1|^2 + (2-\beta^2)\gamma^2 {\rm Re}\left\{F_2\right\} \right],$$

$$\sigma_R^{\pm}|_{\rm Pol} \equiv \int_{-\pi/2}^{\pi/2} d\phi_{\pm} \left[\left.\frac{d\sigma^S}{d\phi_{\pm}}\right|_{{\rm Pol}(e^-)} \right] = -\sigma_L^{\pm}|_{\rm Pol}\,.$$

$$\left.\frac{d\sigma^S}{d\phi_{\pm}}\right|_{{\rm Pol}(e^-)} = \mp \frac{\pi^2\alpha^2\beta}{16s} Br(\tau^+ \to h^+ \bar{\nu}_\tau) Br(\tau^- \to h^- \nu_\tau) \times \frac{1}{\gamma} \left[|F_1|^2 + (2-\beta^2)\gamma^2 {\rm Re}\left\{F_2\right\} \right] \left[(\alpha_{\pm}) \cos\phi_{\pm} \right]$$

A_L

$$A_L^{\pm} = \frac{\sigma_{FB}^{\pm}(+)|_{\rm Pol} - \sigma_{FB}^{\pm}(-)|_{\rm Pol}}{\sigma} = \mp \alpha_{\pm} \frac{3}{4(3-\beta^2)} \left[|F_1|^2 + 2\,{\rm Re}\left\{F_2\right\} \right]$$

$$\sigma_{FB}^{\pm}(+)|_{\rm Pol} \equiv \int_0^1 d(\cos\theta_{\pm}^*) \left.\frac{d\sigma_{FB}^S}{d(\cos\theta_{\pm}^*)}\right|_{{\rm Pol}(e^-)} = \mp\alpha_{\pm} Br(\tau^+ \to h^+ \bar{\nu}_\tau) \times Br(\tau^- \to h^- \nu_\tau) \frac{\pi\alpha^2}{4s}\beta \left[|F_1|^2 + 2\,{\rm Re}\left\{F_2\right\} \right]$$

$$\sigma_{FB}^{\pm}(-)|_{\rm Pol} \equiv \int_{-1}^0 d(\cos\theta_{\pm}^*) \left.\frac{d\sigma_{FB}^S}{d(\cos\theta_{\pm}^*)}\right|_{Pol(e^-)} = -\sigma_{FB}^{\pm}(+)|_{\rm Pol}\,.$$

$$\left.\frac{d\sigma_{FB}^S}{d(\cos\theta_{\pm}^*)}\right|_{{\rm Pol}(e^-)} = \mp \frac{\pi\alpha^2\beta}{2s} Br(\tau^+ \to h^+ \bar{\nu}_\tau) Br(\tau^- \to h^- \nu_\tau) \times \left[|F_1|^2 + 2\,{\rm Re}\left\{F_2\right\} \right] \left[(\alpha_{\pm}) \cos\theta_{\pm}^* \right].$$