CPT Symmetry and Lorentz Symmetry Tests

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All of physics as we know it exhibits Lorentz symmetry—invariance under rotations and boosts—and CPT symmetry.

These invariances have been tested in matter-antimatter comparisons, meson oscillations, atomic clocks, and astrophysical polarimetery.

There are numerous candidate quantum gravity theories with LV, but nobody knows whether these are the exception or the rule.

In the last ten years, there has been growing interest in the possibility that Lorentz and CPT symmetries may not be exact.

There are two broad reasons for this interest:

Reason One: Many theories that have been put forward as candidates to explain quantum gravity involve LV in some regime.

(For example, string theory, non-commutative geometry, loop quantum gravity...)

Reason Two: Lorentz symmetry is a basic building block of both quantum field theory and the General Theory of Relativity, which together describe all observed phenomena.

Anything this fundamental should be tested. Much of the story of modern theoretical physics is how important symmetries do not hold exactly.

There is no excellent beauty that hath not some strangeness in the proportion. — Francis Bacon

Ultimately, we don't know where Lorentz violation might come from. However, any theory with CPT violation must also be Lorentz-violating.

[Greenberg, PRL 89, 231602 (2002)]

So it would be good to have a systematic framework for studying any possible Lorentz and CPT violations. This framework is the standard model extension (SME), which uses the known tools of effective field theory to describe all possible forms of Lorentz violation involving standard model fields.

Standard Model Extension (SME)

Idea: Look for all operators that can contribute to Lorentz violation.
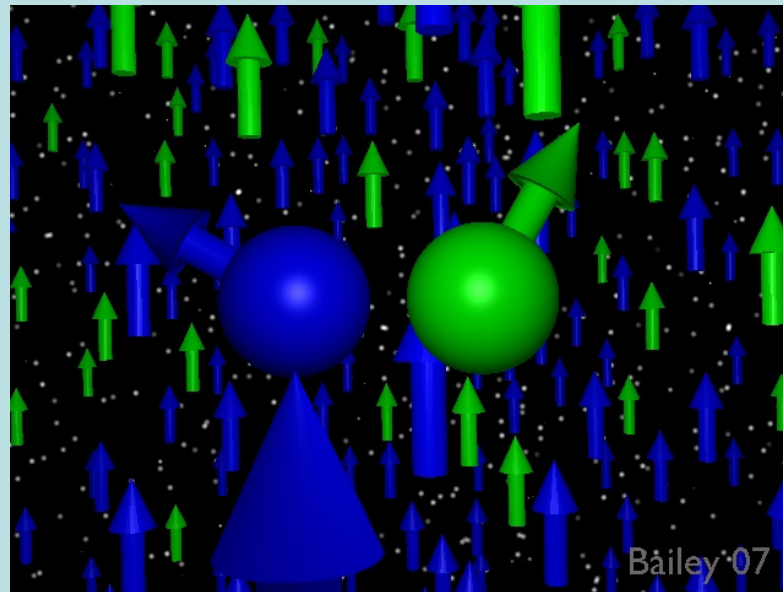
[Kostelecký and Colladay, PRD 58, 116002 (1998)]

Then one usually adds restrictions:

- locality

- superficial renormalizability

- $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance

- etc...

Many other formalisms turn out to be special cases of the SME.

Lorentz violating operators have objects built up from standard model fields, contracted with constant background tensors.

Bailey 07

The vacuum becomes a nontrivial medium, like an (anisotropic, birefringent) crystal.

Either spatial isotropy or Lorentz boost invariance might be broken. To test isotropy, one needs to compare experiments done in different directions.
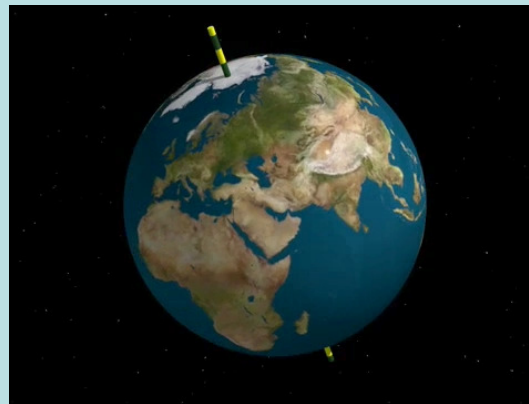
Earth-based laboratories will see slightly different local physics as the planet rotates and revolves.

The Lagrange density for a Lorentz-violating free Fermion theory is:

$$\mathcal{L} = \overline{\psi}\left(i\Gamma^{\mu}\partial_{\mu} - M\right)\psi$$

$$M = m + \slashed{a} - \slashed{b}\gamma_5 + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu}$$

$$\Gamma^{\mu} = \gamma^{\mu} + c^{\nu\mu}\gamma_{\nu} - d^{\nu\mu}\gamma_{\nu}\gamma_5 + e^{\mu} + if^{\mu}\gamma_5 + \frac{1}{2}g^{\lambda\nu\mu}\sigma_{\lambda\nu}$$

a, b, e, f, and g also violate CPT.

A separate set of coefficients will exist for every elementary particle in the theory.

At high energies, where boost invariance violation can be studied, the c-type Lorentz violation mentioned earlier is the most important.

(Its effects grow as γ^2.)

Neglecting higher order corrections, the maximum fermion velocity in a direction $\hat{e}$ is:

$$v < 1 - c_{00} - c_{(0j)}\hat{e}_j - c_{jk}\hat{e}_j\hat{e}_k$$

This turns out to have readily measurable consequences.

The momentum and velocity don't generally point in the same direction. This can distort circular synchrotron orbits into ellipses

More generally, momentum eigenstates may not be eigenstates of velocity.

Measurement Type	System	Coefficients	log Sensitivity	Source
oscillations	K (averaged)	a (d, s)	−20	E773 Kostelecký
	K (sidereal)	a (d, s)	−21	KTeV
	D (averaged)	a (u, c)	−16	FOCUS
	D (sidereal)	a (u, c)	−16	FOCUS
	B (averaged)	a (d, b)	−16	BaBar, BELLE, DELPHI, OPAL
	neutrinos	a, b, c, d	−19 to −26	SuperK Kostelecký, Mewes
birefringence	photon	k_{AF} (CPT odd)	−43	Carroll, Field, Jackiw
		k_F (CPT even)	−32 to −37	Kostelecký, Mewes
resonant cavity	photon	k_F (CPT even)	−17	Muller et al.
anomaly frequency	e-/e+	b (e)	−23	Dehmelt et al.
	e- (sidereal)	$b, c,$ d (e)	−23	Mittleman et al.
	mu/anti-mu	b (mu)	−22	Bluhm, Kostelecký, Lane
cyclotron frequency	H-/anti-p	c (e, p)	−26	Gabrielse et al.
hyperfine structure	H (sidereal)	b, d (e, p)	−27	Walsworth et al.
	muonium (sid.)	b, d (mu)	−23	Hughes et al.
clock comparison	various	b, c, d (e, p, n)	−22 to −30	Kostelecký, Lane
	He-Xe	b, d (n)	−32	Bear et al. Cane et al.
torsion pend.	spin-polarized solid	b, d (e)	−29	Heckel et al. Hou et al.
gamma-ray astronomy	e- /photons	c, d (e)	−15 to −20	Altschul

CPT in Meson Oscillations

Meson oscillations are a wonderful venue for precision experiments, using interferometry to enhance sensitivity.

That's how CP violation was discovered, and it could be the same for CPT violation.

The most sensitive accelerator tests of Lorentz symmetry involve CPT tests with neutral mesons.

CPT-violating quantities, such as the $B^0 - \overline{B}^0$ mass difference are controlled by the phase

$$\delta_{CPT} \propto \gamma \frac{v_\mu \left(a_d^\mu - a_b^\mu \right)}{m_{B_H} - m_{B_L}}$$

[Kostelecký, PRL **80**, 1818 (1998)]

The dependence on the meson velocity has important consequences.

Experiments at higher energies are more sensitive, even when they apparently have the same sensitivity to the $B^0 - \overline{B}^0$ mass difference.

The rate of CPT violation also generally depends on the meson direction, and so will change as the Earth-based laboratory rotates.

CPT violation has been searched for in neutral K, D, and B meson systems, using both time-averaged and day-night asymmetry measurements.

The sensitivity of BaBar data on $B^0 - \overline{B}^0$ oscillations has produced bounds at the $10^{-14} - 10^{-15}$ GeV level. This used data from 232 million $B^0 - \overline{B}^0$ pairs, looking for CPT violation in direct semi-leptonic decays. This included a search for sidereal variations.

The CPT violation bounds are limited by statistics. SuperB tests could give an order of magnitude improvement.

Searching for sidereal variations is a very powerful way to eliminate systematic errors, since nothing should be oscillating with that period (as opposed to the solar day period).

The analysis will need to rely on Monte Carlo to find fraction of events that are actually related to $B^0 - \overline{B}^0$ pairs and their expected dilepton production rates (in the absence of CPT violation). This is potentially a significant source of systematic error.

However, we expect the largest source of systematic error to be in particle tracking. CPT violation appears as a difference in dilepton event rates, depending on whether the B^0 or $\overline{B}^0$ decays first.

Correctly ordering the decays requires good directional information at short times in the SVT. Problems with this may be the largest (~20%) source of systematic error.

Particle Tracking Tests

Changes to the relativistic energy-momentum relation affect the synchrotron orbits of particles in a magnetic field.

$\dot{\vec{p}} = \vec{v} \times \vec{B}$ still holds, but $\vec{v}(\vec{p})$ may be rather complicated.

$\vec{v}(\vec{p})$
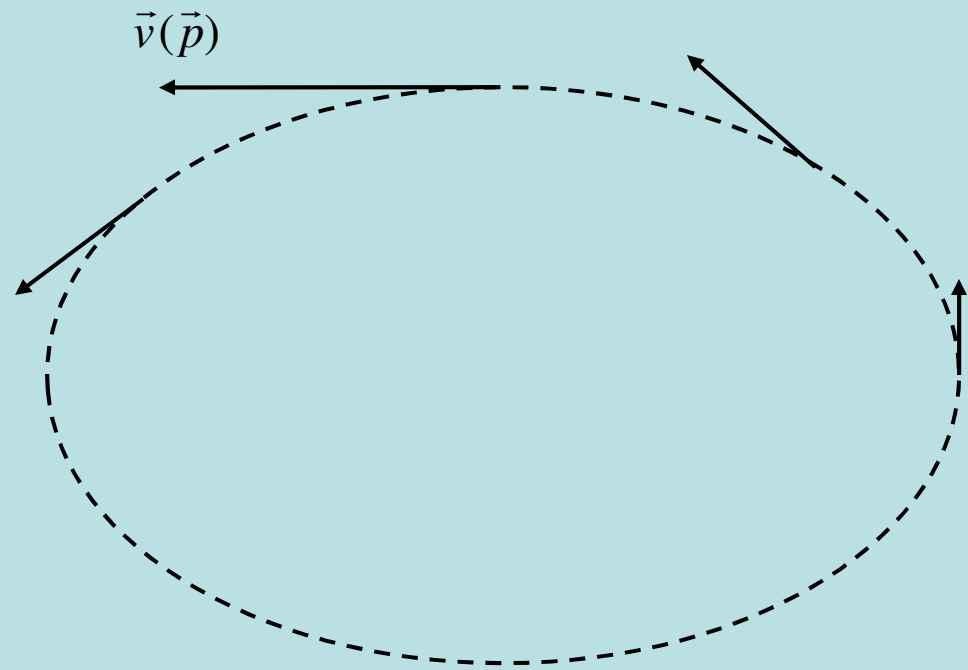
Just measuring orbital eccentricities at SuperB is not practical. Systematic effects will dominate.

However, we can look at sidereal variations in the observed ellipticity, as the detector orientation rotates with the Earth. The eccentricity $\mathscr{E}$ oscillates, with Fourier components up to four times the Earth's sidereal rotation frequency.

[BDA, PRD 84, 076006 (2011)]

The Fourier observables are complicated:

$$\mathcal{A}_\Omega = \beta\gamma(\alpha^2 + \beta^2 - 4)c_- c_{XZ} + \alpha\gamma(\alpha^2 + \beta^2 - 4)c_- c_{YZ}$$
$$-2\alpha\gamma(\alpha^2 + \beta^2 - 4)c_{XY}c_{XZ} + 2\beta\gamma(\alpha^2 + \beta^2 - 4)c_{XY}c_{YZ}$$
$$[+2\beta\gamma(\alpha^2 + \beta^2)c_Q c_{XZ} - 2\alpha\gamma(\alpha^2 + \beta^2)c_Q c_{YZ}]$$

$$\mathcal{B}_\Omega = \alpha\gamma(\alpha^2 + \beta^2 - 4)c_- c_{XZ} - \beta\gamma(\alpha^2 + \beta^2 - 4)c_- c_{YZ}$$
$$+2\beta\gamma(\alpha^2 + \beta^2 - 4)c_{XY}c_{XZ} + 2\alpha\gamma(\alpha^2 + \beta^2 - 4)c_{XY}c_{YZ}$$
$$[+2\alpha\gamma(\alpha^2 + \beta^2)c_Q c_{XZ} + 2\beta\gamma(\alpha^2 + \beta^2)c_Q c_{YZ}]$$

$$\mathcal{A}_{2\Omega} = +2(\alpha^4 - \beta^4)c_{XZ}^2 + 2(\beta^4 - \alpha^4)c_{YZ}^2 + 8\alpha\beta(\alpha^2 + \beta^2)c_{XZ}c_{YZ}$$
$$[-\frac{1}{2}(\alpha^2 - \beta^2)(\alpha^2 + \beta^2 - 2)c_- c_Q - 2\alpha\beta(\alpha^2 + \beta^2 - 2)c_Q c_{XY}]$$

$$\mathcal{B}_{2\Omega} = -4\alpha\beta(\alpha^2 + \beta^2)c_{XZ}^2 + 4\alpha\beta(\alpha^2 + \beta^2)c_{YZ}^2 + 4(\alpha^4 - \beta^4)c_{XZ}c_{YZ}$$
$$[+\alpha\beta(\alpha^2 + \beta^2 - 2)c_- c_Q - (\alpha^2 - \beta^2)(\alpha^2 + \beta^2 - 2)c_Q c_{XY}]$$

$$\mathcal{A}_{3\Omega} = \beta\gamma(\beta^2 - 3\alpha^2)c_- c_{XZ} + \alpha\gamma(\alpha^2 - 3\beta^2)c_- c_{YZ}$$
$$+2\alpha\gamma(\alpha^2 - 3\beta^2)c_{XY}c_{XZ} + 2\beta\gamma(3\alpha^2 - \beta^2)c_{XY}c_{YZ}$$

$$\mathcal{B}_{3\Omega} = \alpha\gamma(3\beta^2 - \alpha^2)c_- c_{XZ} + \beta\gamma(\beta^2 - 3\alpha^2)c_- c_{YZ}$$
$$+2\beta\gamma(\beta^2 - 3\alpha^2)c_{XY}c_{XZ} + 2\alpha\gamma(\alpha^2 - 3\beta^2)c_{XY}c_{YZ}$$

$$\mathcal{A}_{4\Omega} = \frac{1}{8}(\alpha^4 - 6\alpha^2\beta^2 + \beta^4)c_-^2 + 2\alpha\beta(\alpha^2 - \beta^2)c_- c_{XY} - \frac{1}{2}(\alpha^4 - 6\alpha^2\beta^2 + \beta^4)c_{XY}^2$$

$$\mathcal{B}_{4\Omega} = \frac{1}{2}\alpha\beta(\beta^2 - \alpha^2)c_-^2 + \frac{1}{2}(\alpha^4 - 6\alpha^2\beta^2 + \beta^4)c_- c_{XY} + 2\alpha\beta(\alpha^2 - \beta^2)c_{XY}^2.$$

$(\alpha,\ \beta,$ and $\ \gamma$ are lab frame angles.)

The biggest systematic limitation may be understanding the magnetic field in the detector.

The precision with which B is known and the accuracy of track dimensions determine the precision of the constraints on Lorentz violation.

Data from older machines will be used to estimate what kind of precision is needed.

For many unstable particles (such as $B^{\pm}$), the **only** constraints on LV come from the absence of the decay $\gamma \rightarrow B^{+} + B^{-}$ for TeV-scale cosmic ray photons.

Those bounds are at the 10^{-6} level for $B^{\pm}$, and they are strictly **one-sided**.

Filling in the other side of the bound at the $10^{-4} - 10^{-5}$ level would be very interesting, but may require 10^{-3} precision for B.

Conclusion

Tests of special relativity are still interesting and relevant.

Many Lorentz-violating coefficients are strongly constrained, but LV remains a strong candidate to appear in a fundamental theory.

SuperB provides an excellent opportunity to test some very poorly constrained forms of LV.

That's all, folks!