

9 October 2024

**Davide Racco**

**ETH** zürich



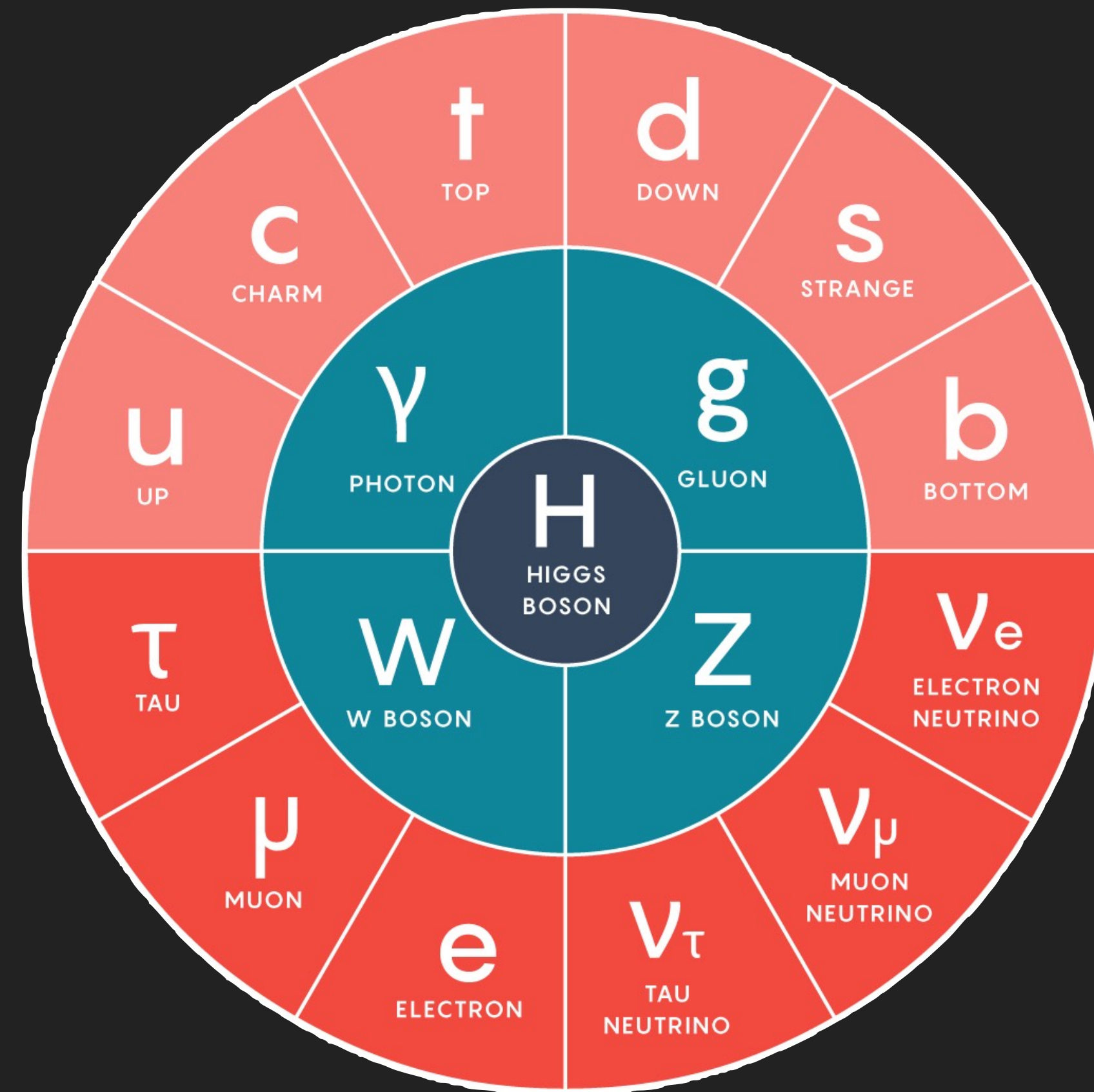
Universität  
Zürich<sup>UZH</sup>



Istituto Nazionale di Fisica Nucleare  
Laboratori Nazionali di Frascati

Production mechanisms  
for DM: from freeze-in to  
gravitational production

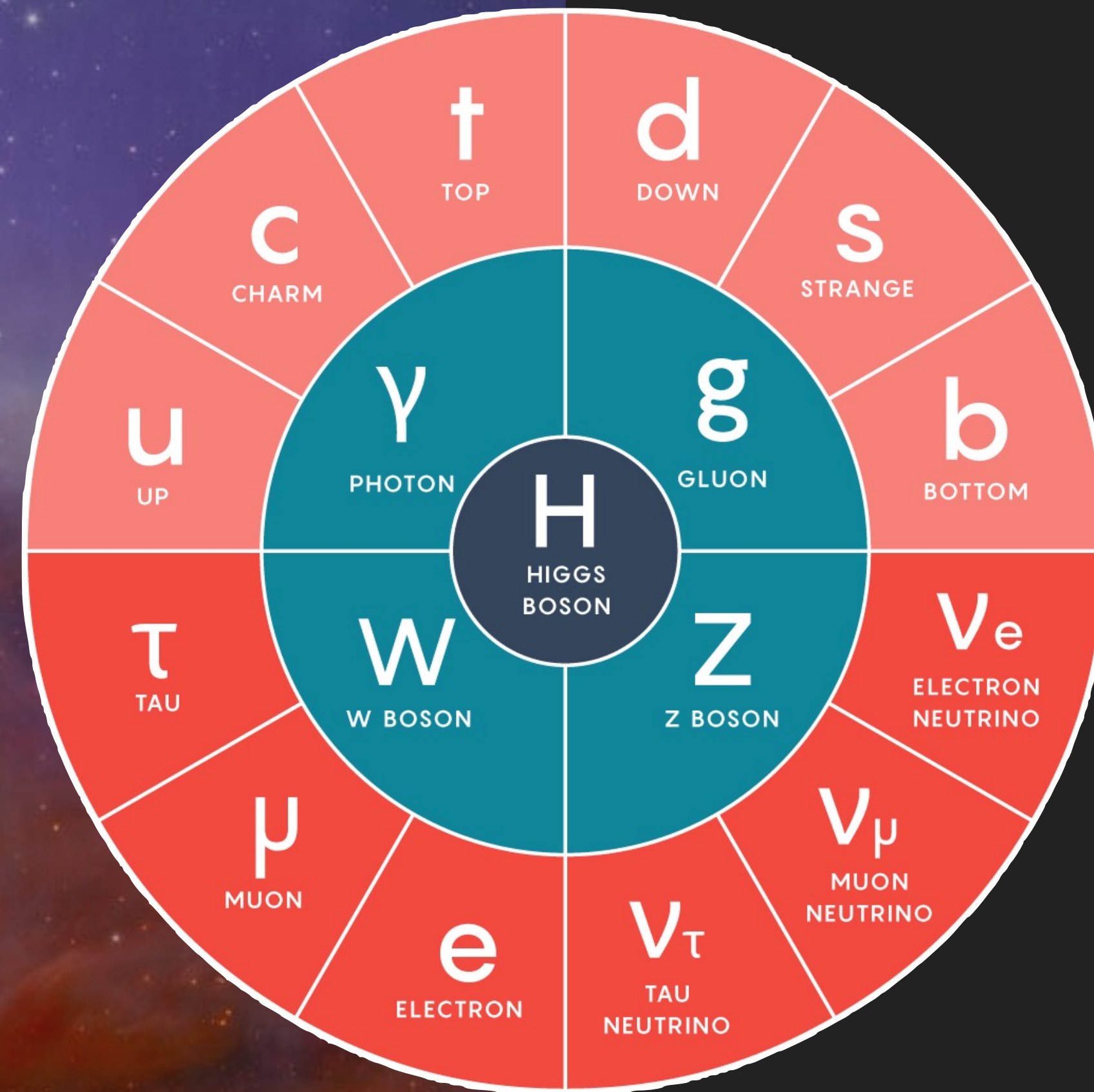






## *Universe history*

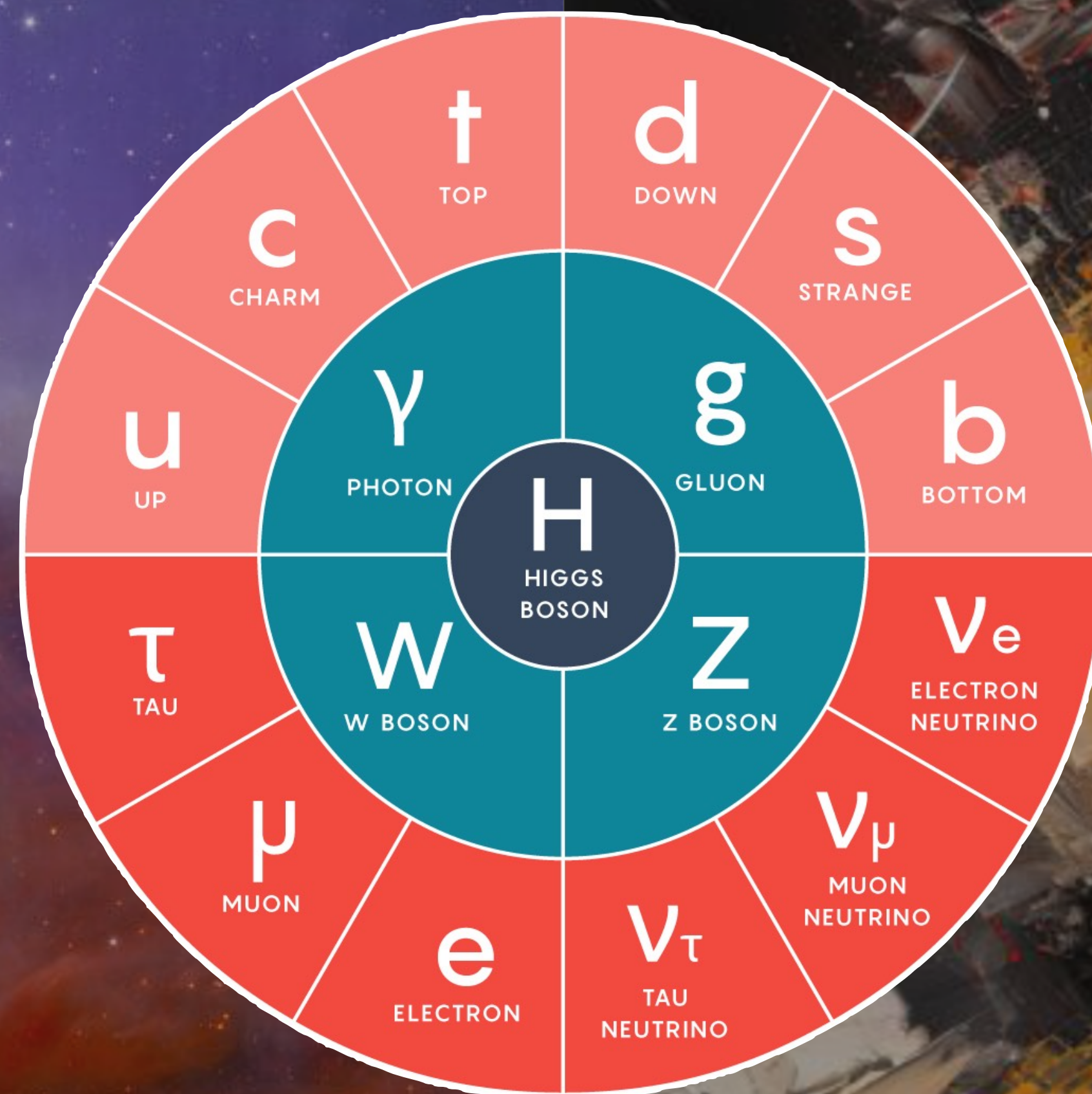
- ▶ Dark Matter
- ▶ Inflation
- ▶ Higgs instability
- ▶ Baryogenesis





## *Universe history*

- ▶ Dark Matter
- ▶ Inflation
- ▶ Higgs instability
- ▶ Baryogenesis

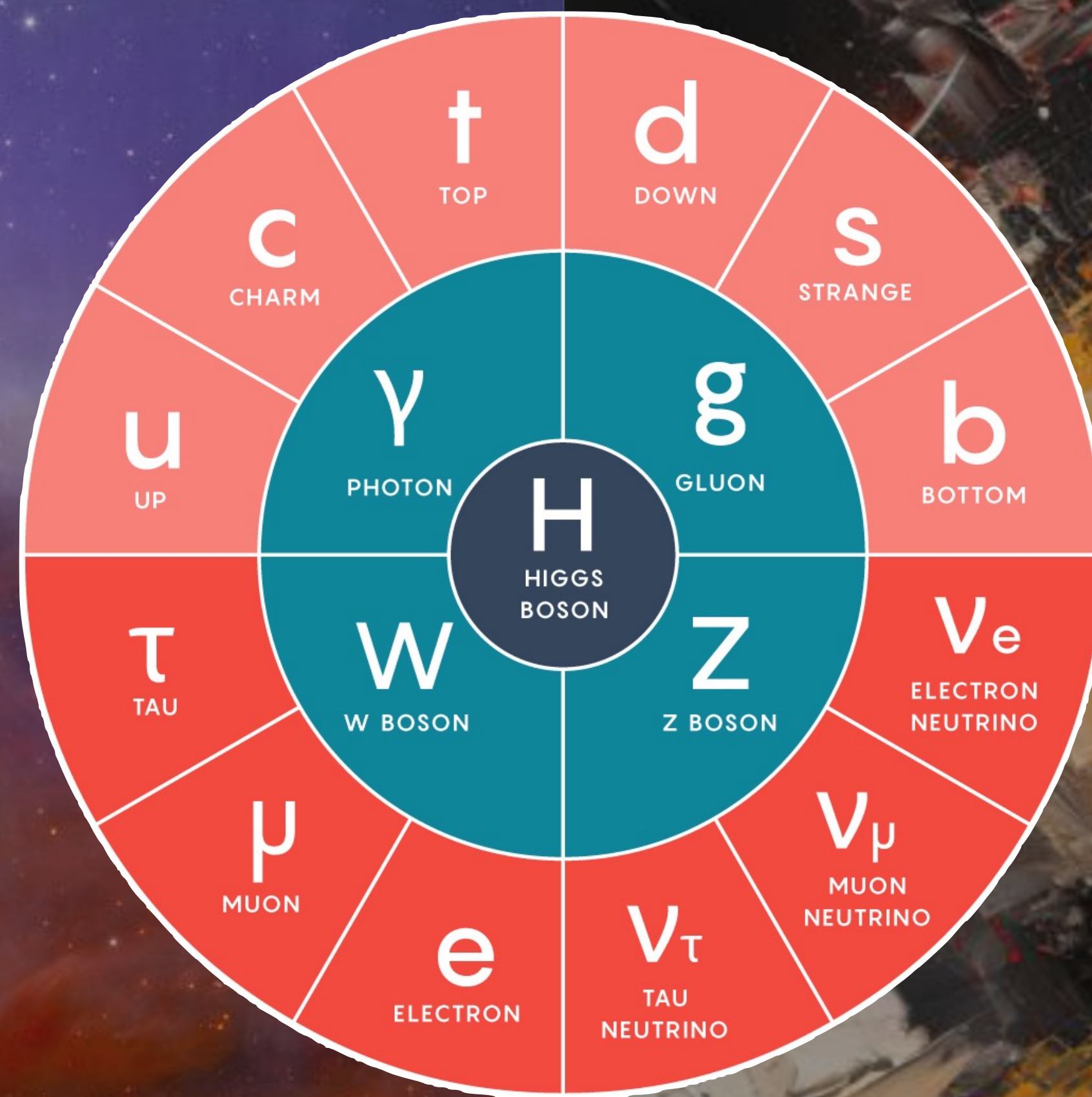


## *(B)SM structure*

- ▶  $\nu$  masses
- ▶ Flavour
- ▶ Unification



- Universe history*
- ▶ Dark Matter
  - ▶ Inflation
  - ▶ Higgs instability
  - ▶ Baryogenesis

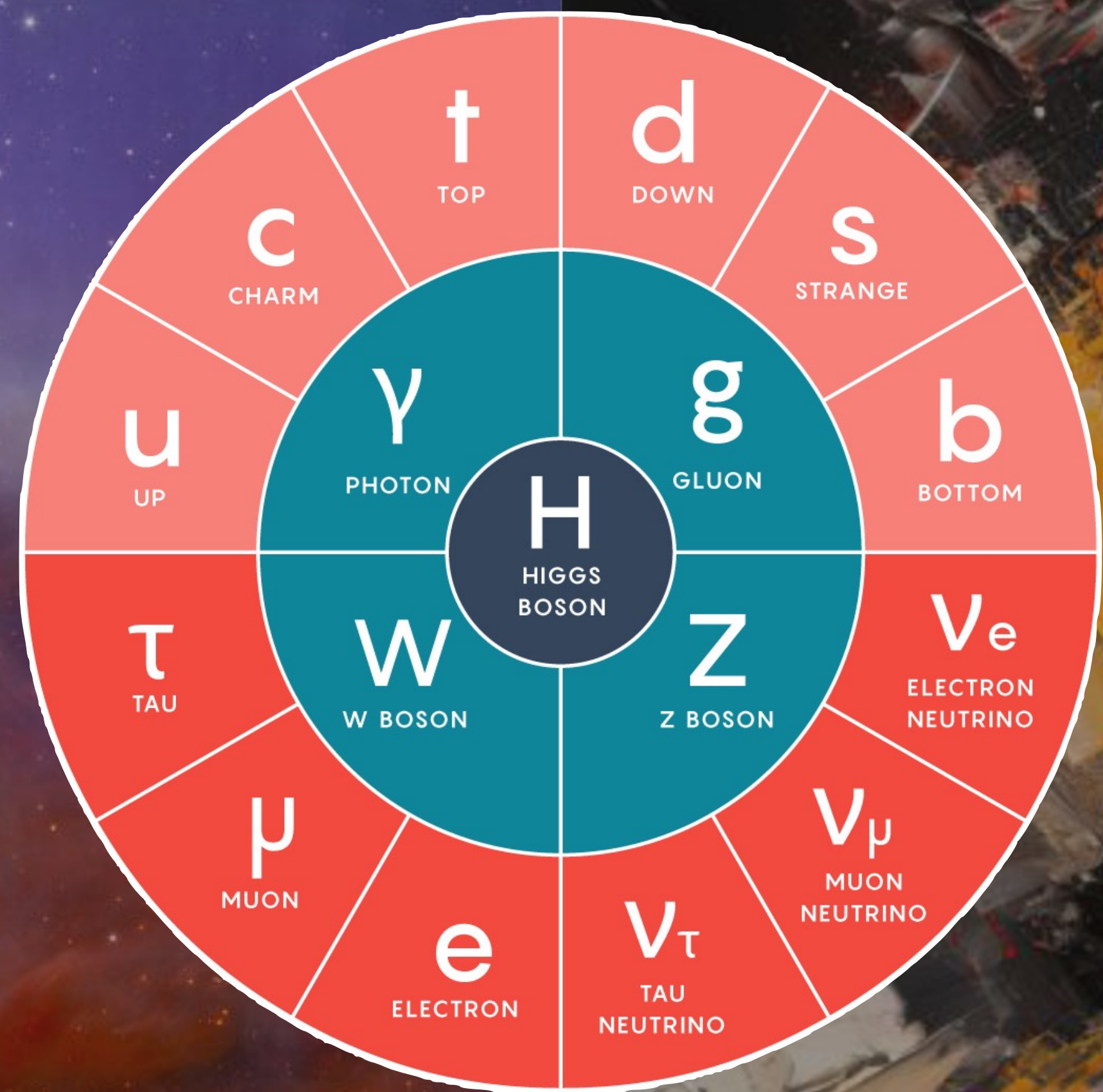


- SM tunings*
- ▶ Vacuum energy
  - ▶ Higgs mass
  - ▶ Strong CP

- (B)SM structure*
- ▶  $\nu$  masses
  - ▶ Flavour
  - ▶ Unification



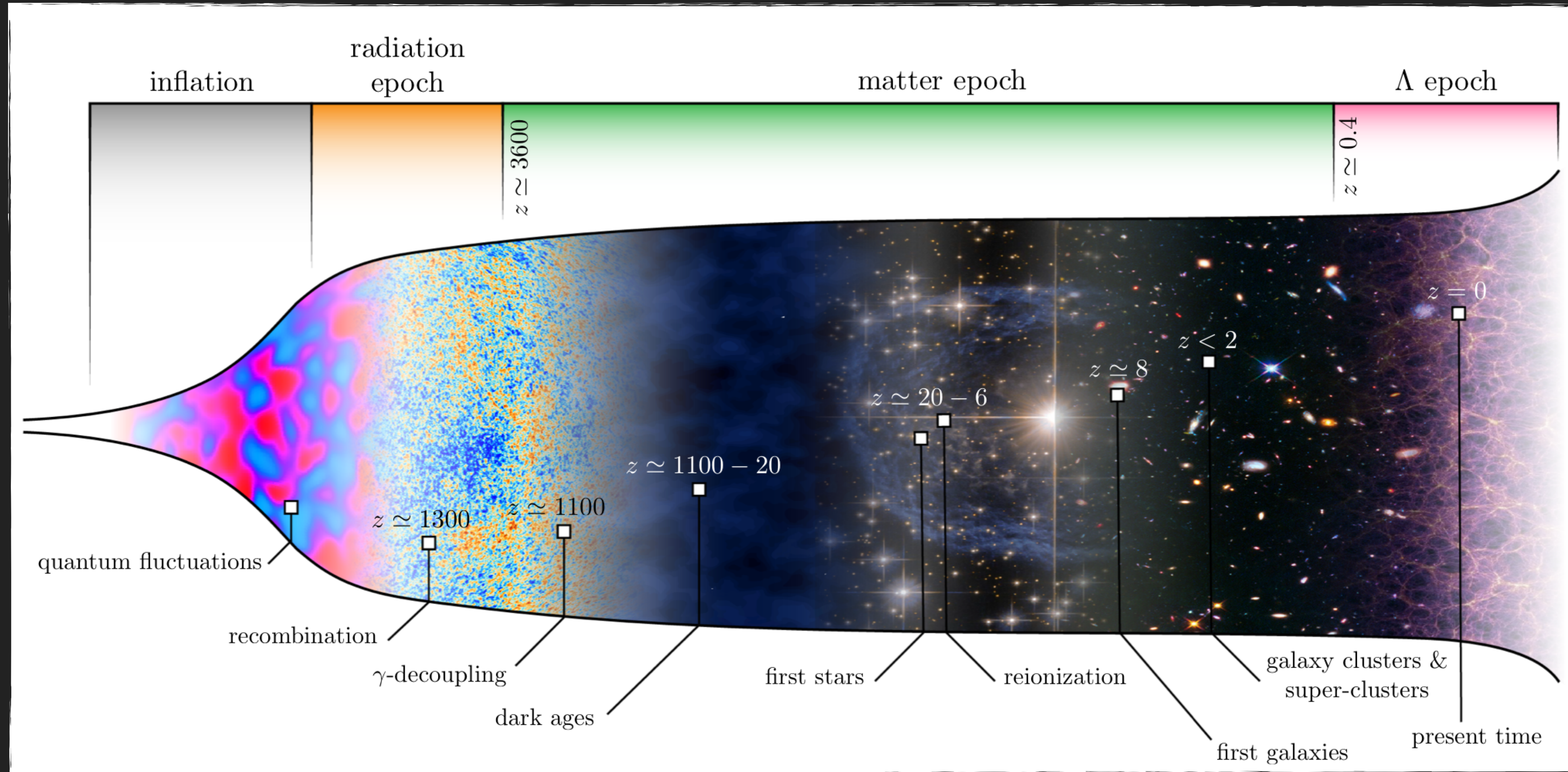
- Universe history*
- ▶ Dark Matter
  - ▶ Inflation
  - ▶ Higgs instability
  - ▶ Baryogenesis



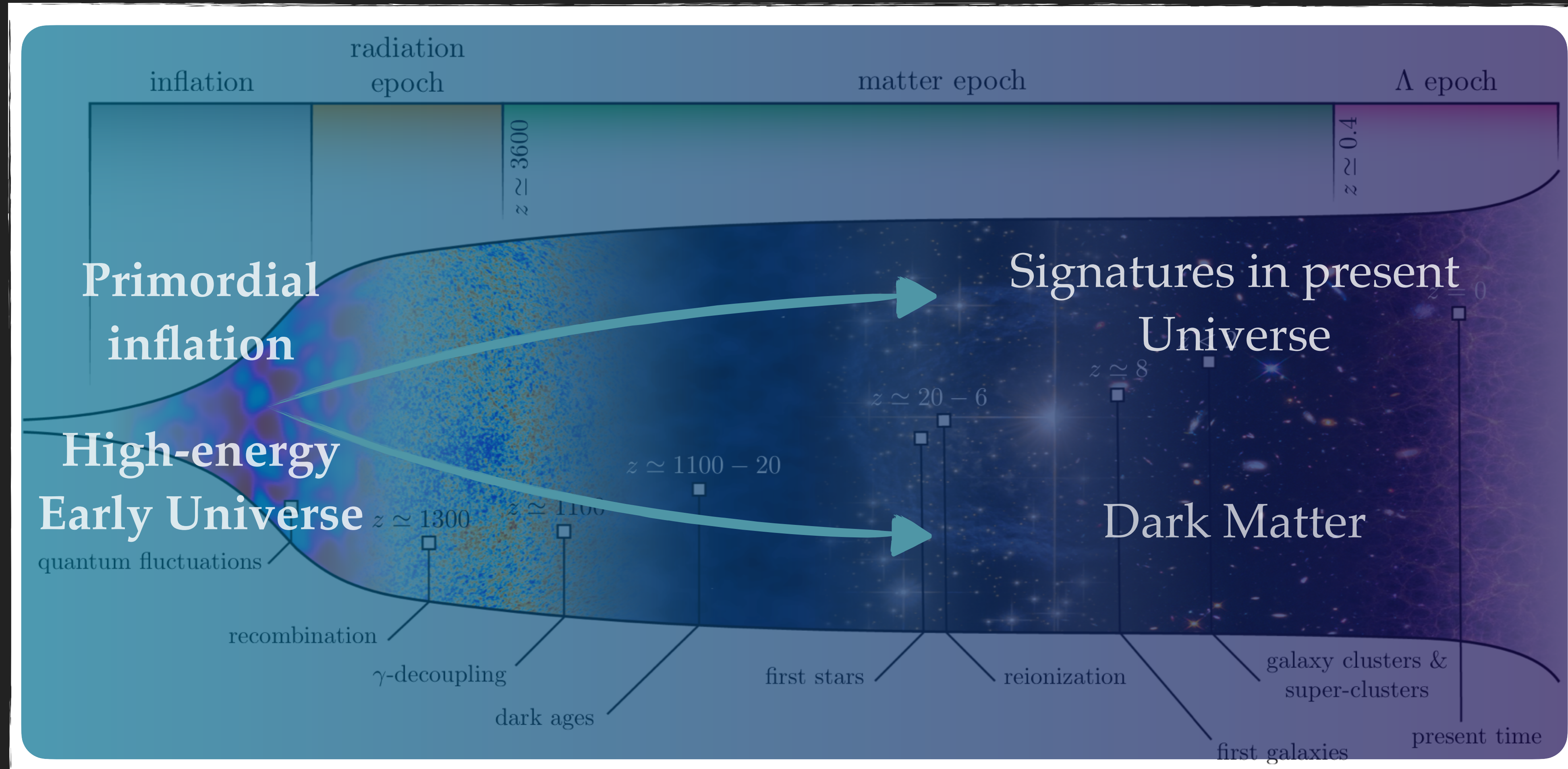
- SM tunings*
- ▶ Vacuum energy
  - ▶ Higgs mass
  - ▶ Strong CP

- (B)SM structure*
- ▶  $\nu$  masses
  - ▶ Flavour
  - ▶ Unification

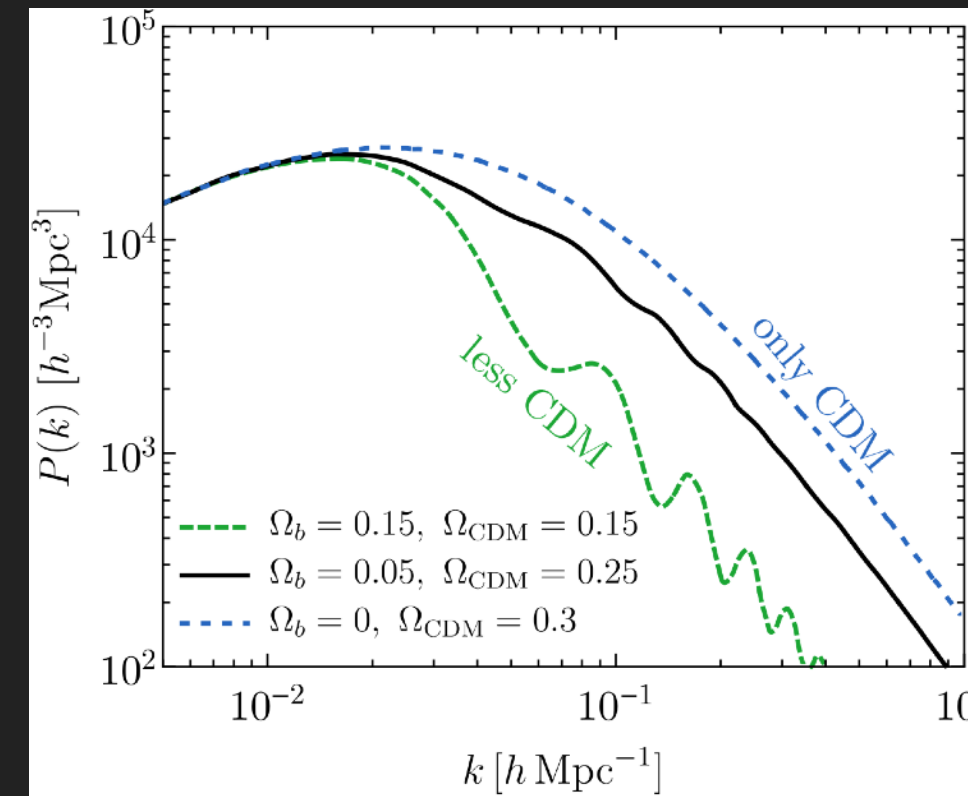
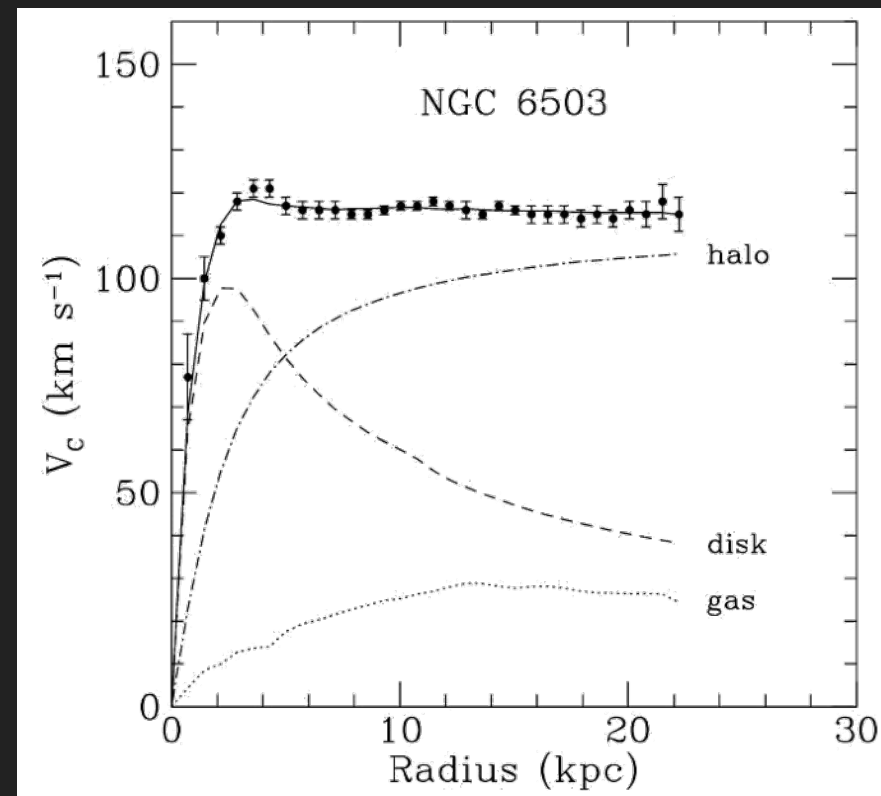






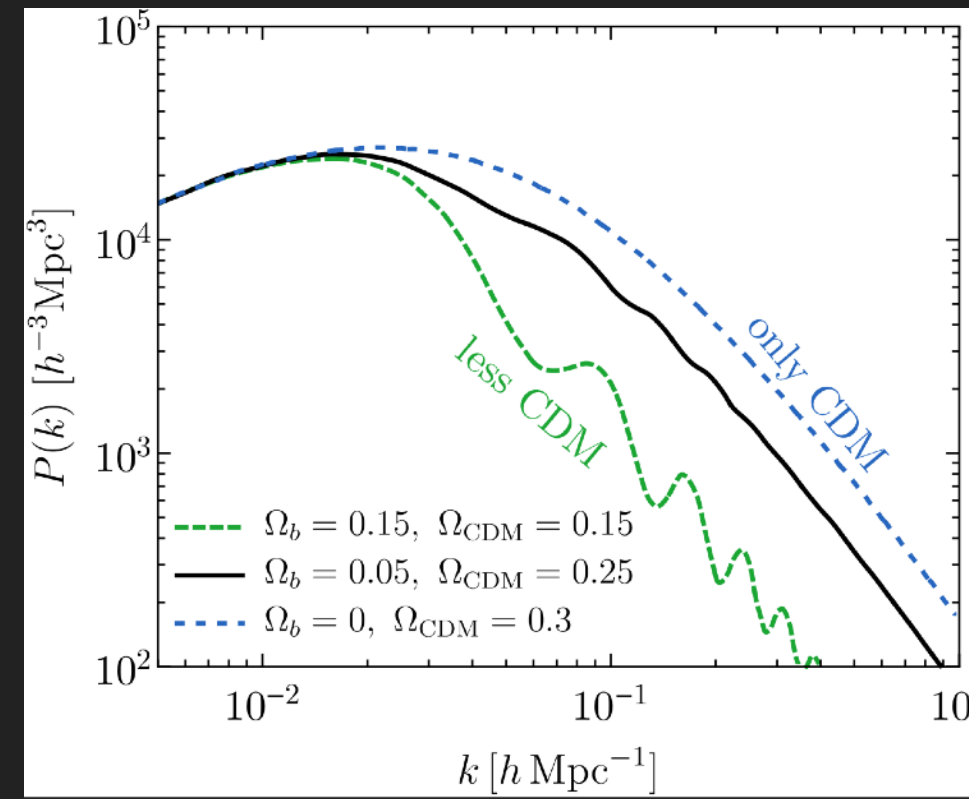
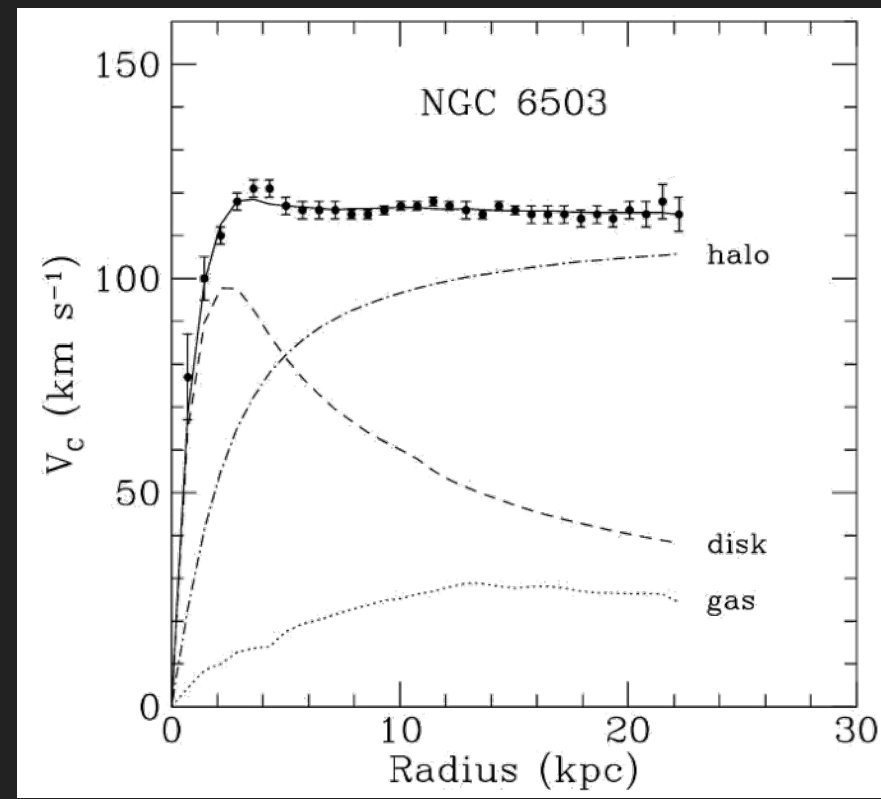






- ▶ All evidence from *gravitational* interactions
- ▶ Exp. searches look for other interactions with us





- ▶ All evidence from *gravitational* interactions
- ▶ Exp. searches look for other interactions with us



SM tunings, parameters

Universe History

QCD Axion  
Pre-infl. Post-infl.

$\nu_R$

WIMPs

Ultra-light DM

Freeze-in DM

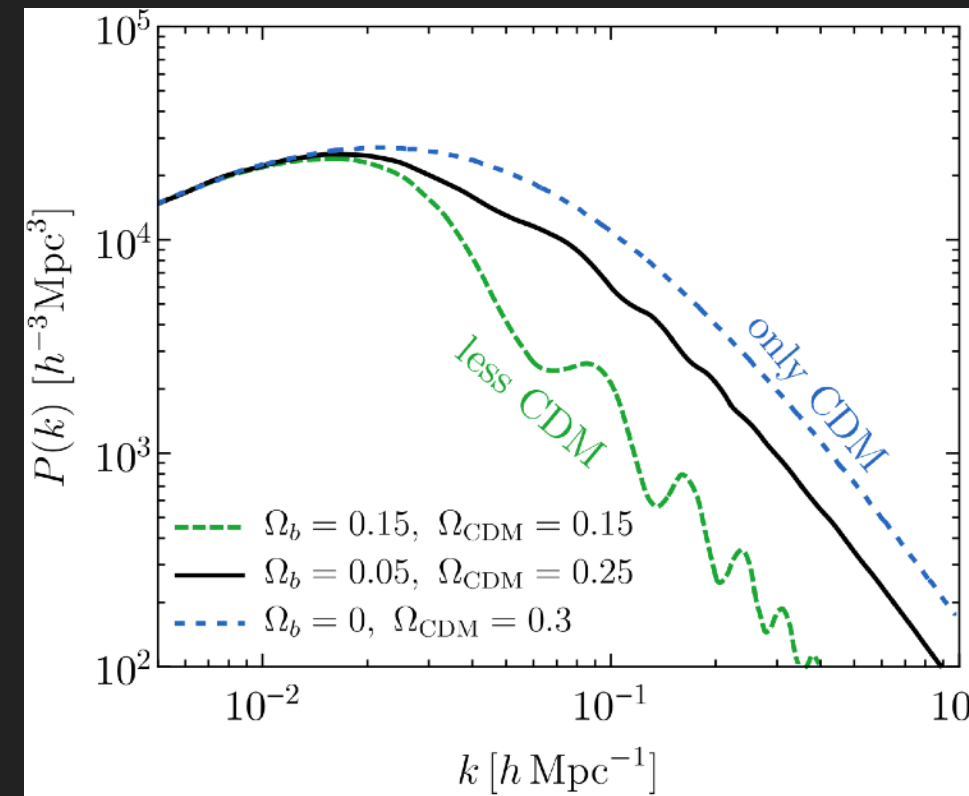
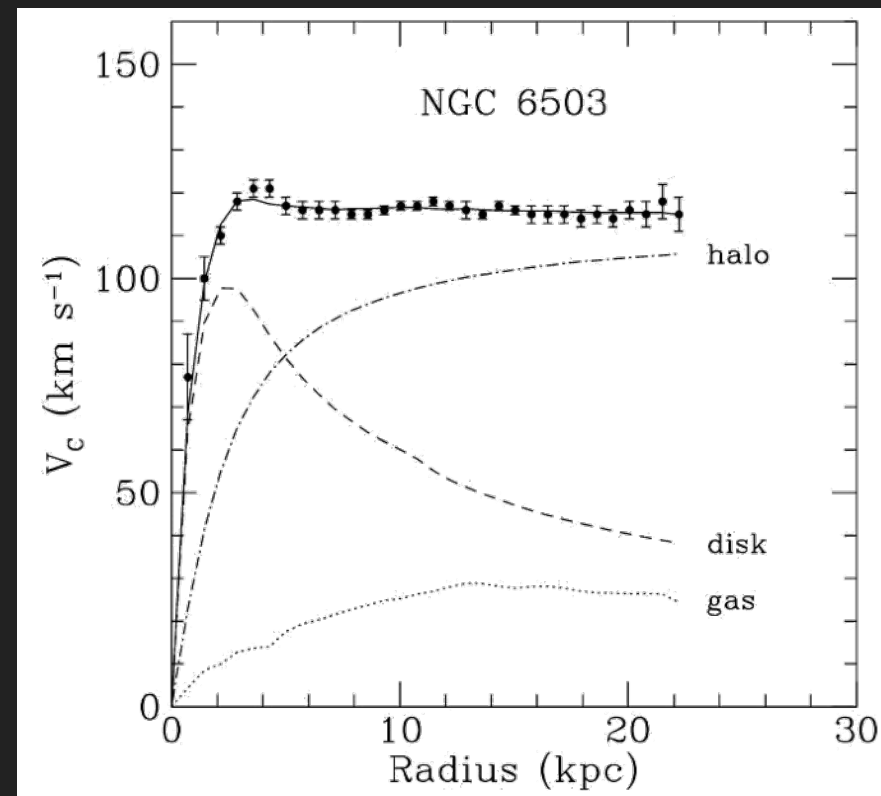
WIMPzillas

Asymmetric DM

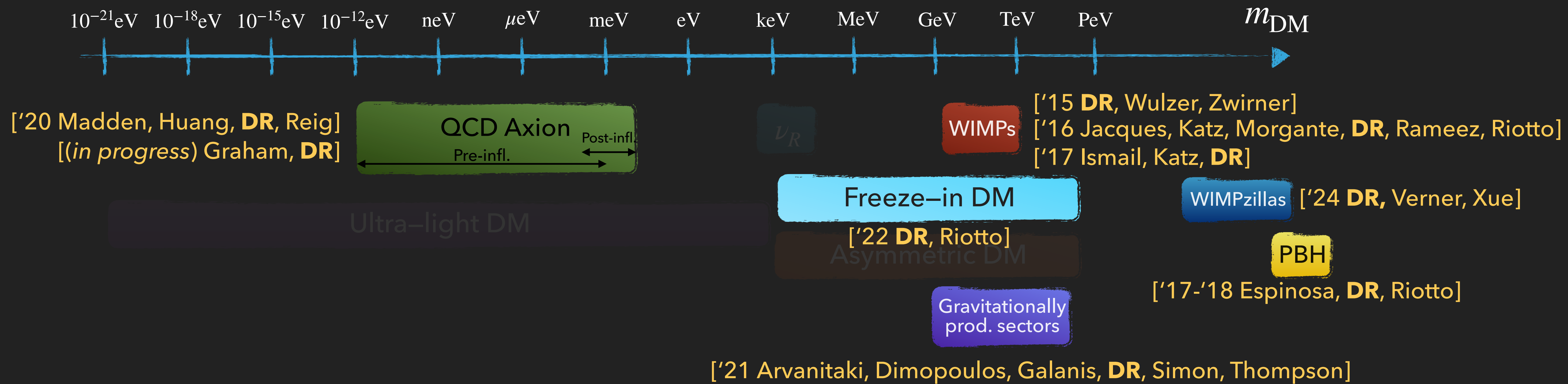
PBH

Gravitationally prod. sectors

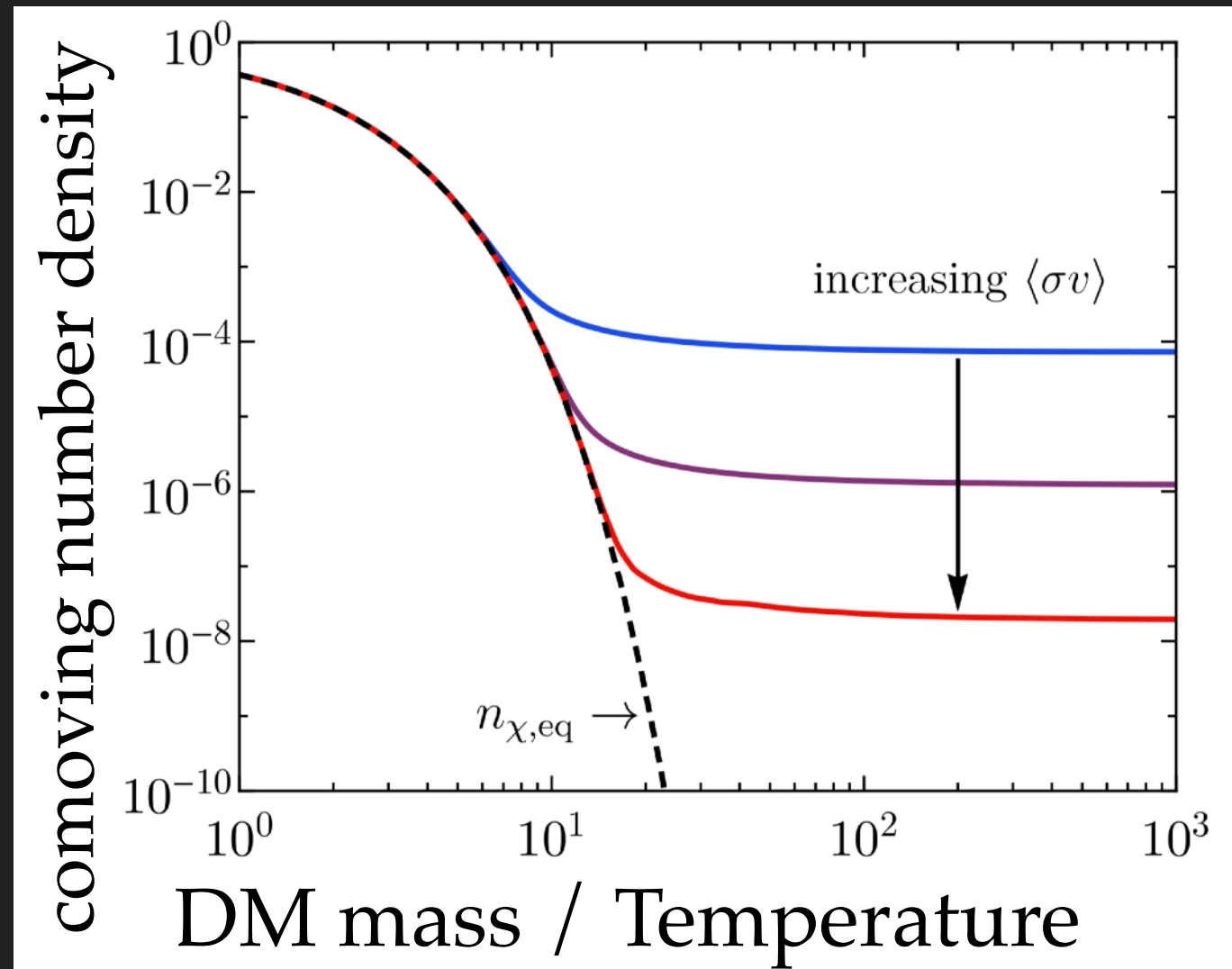




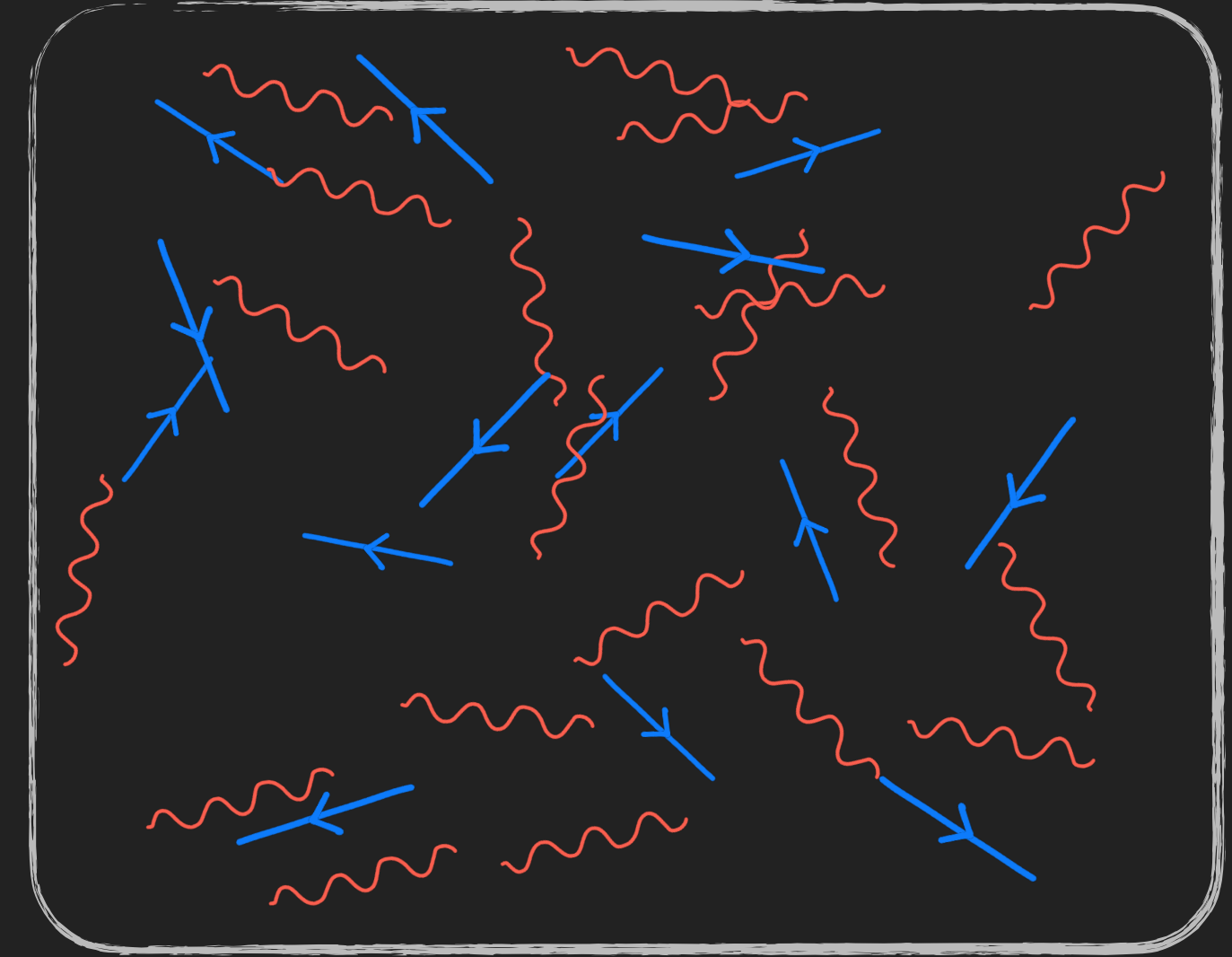
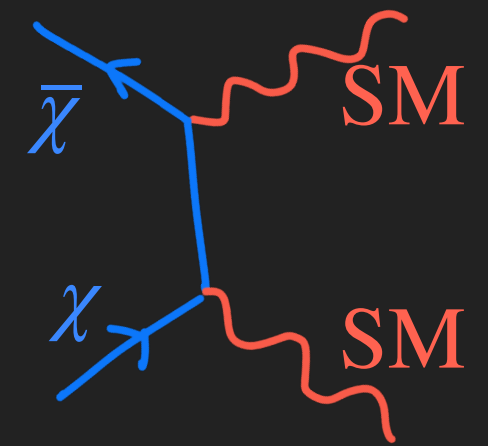
- ▶ All evidence from *gravitational* interactions
- ▶ Exp. searches look for other interactions with us



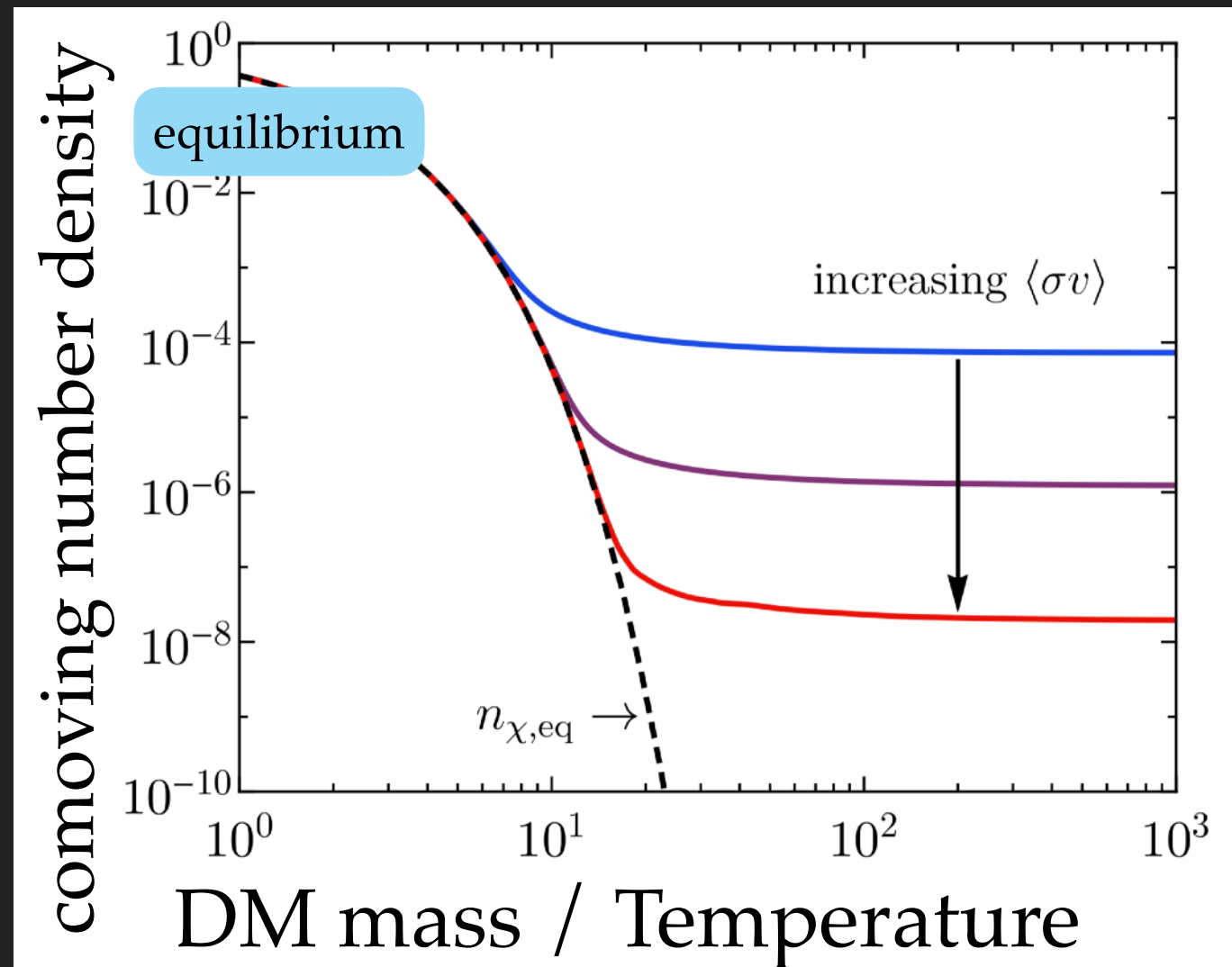




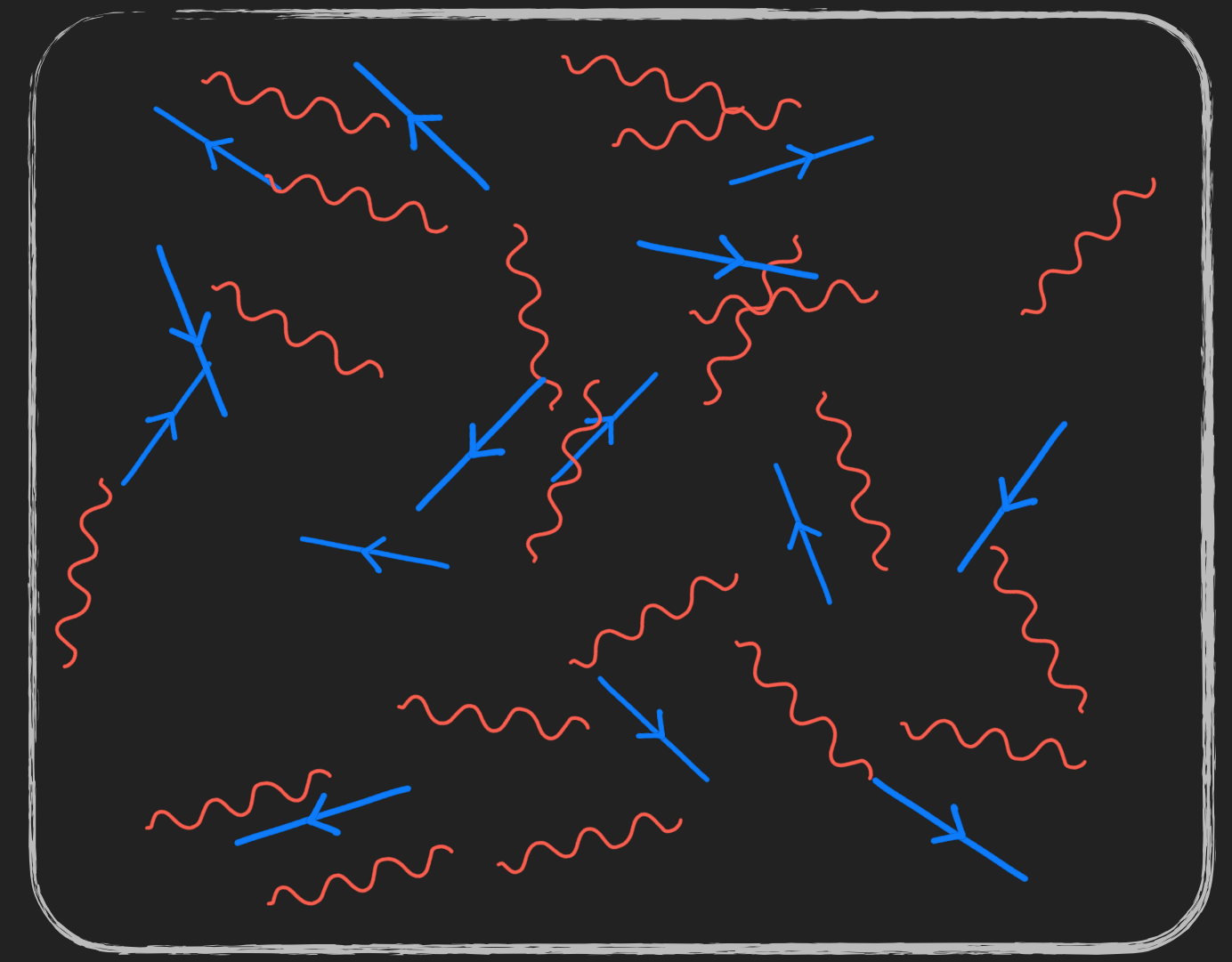
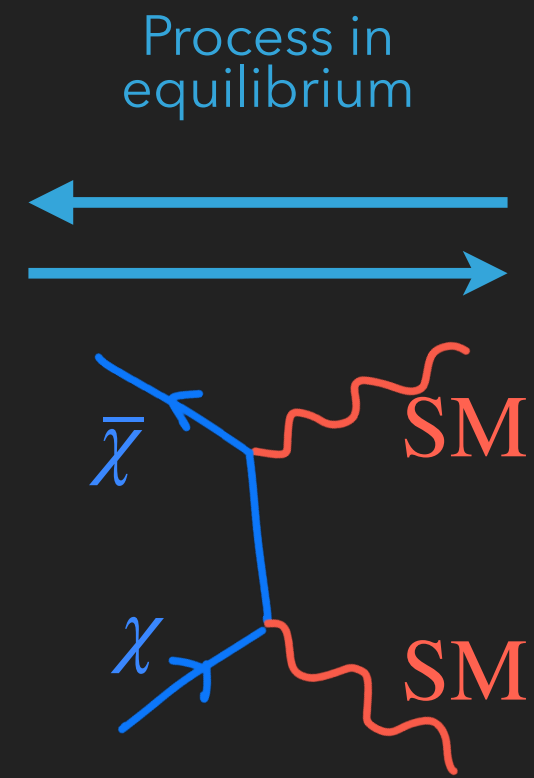
## Freeze-Out



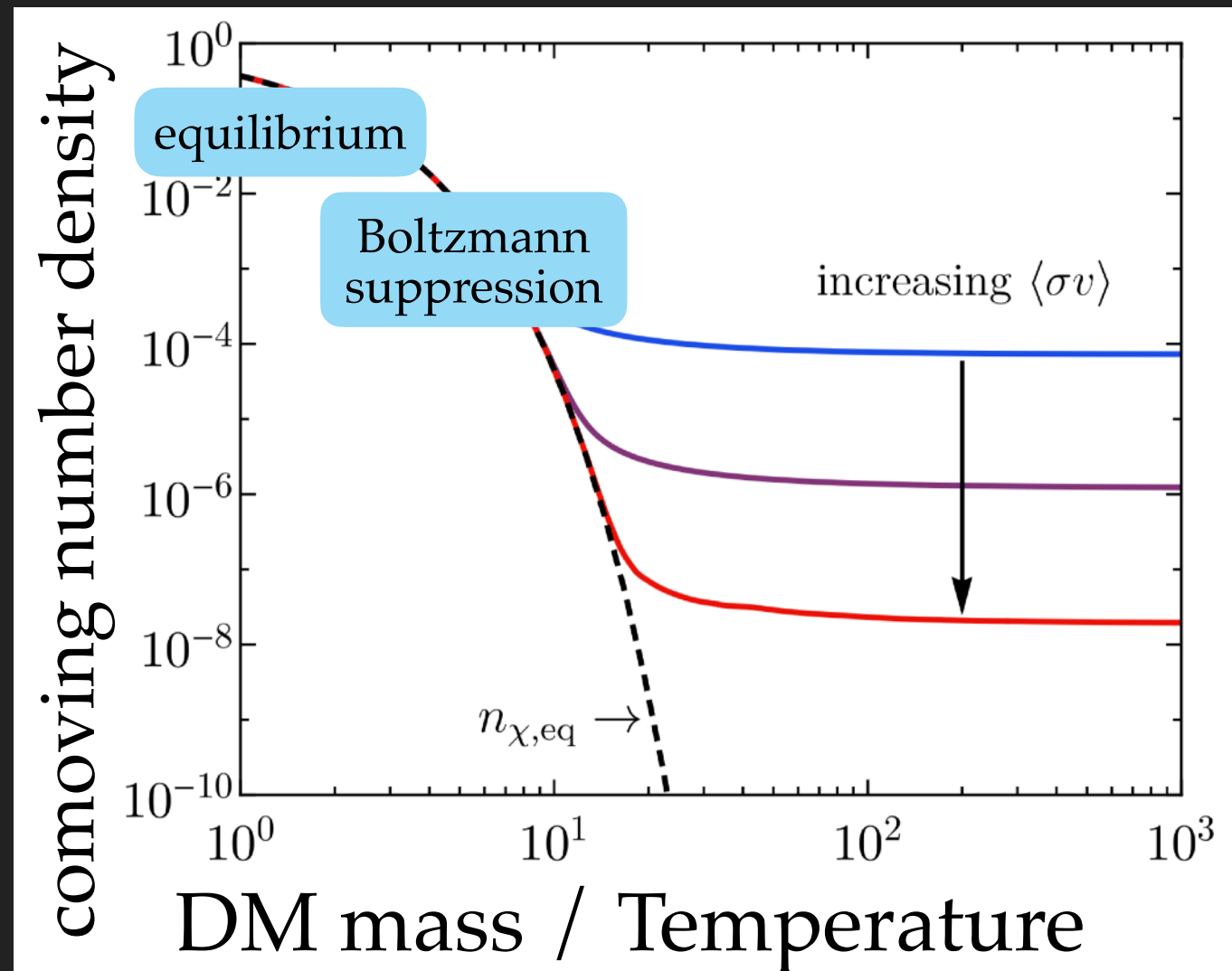




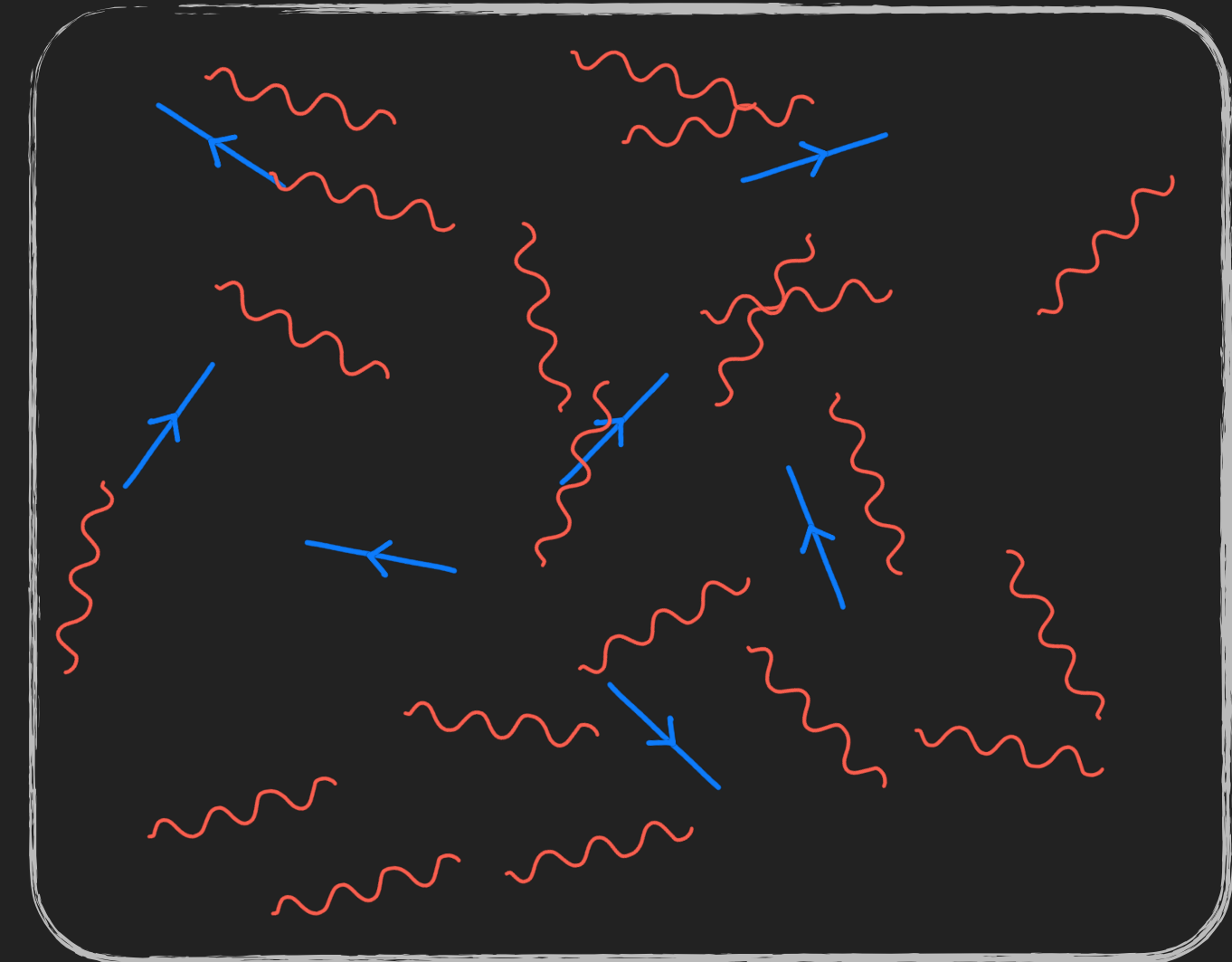
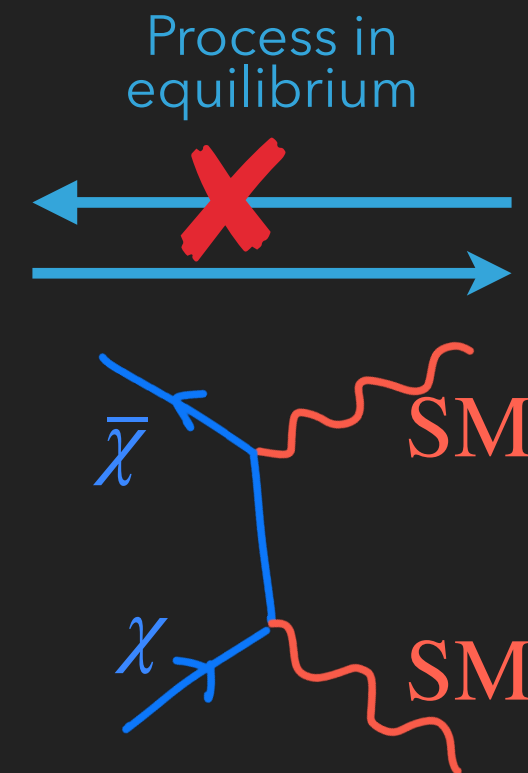
## Freeze-Out



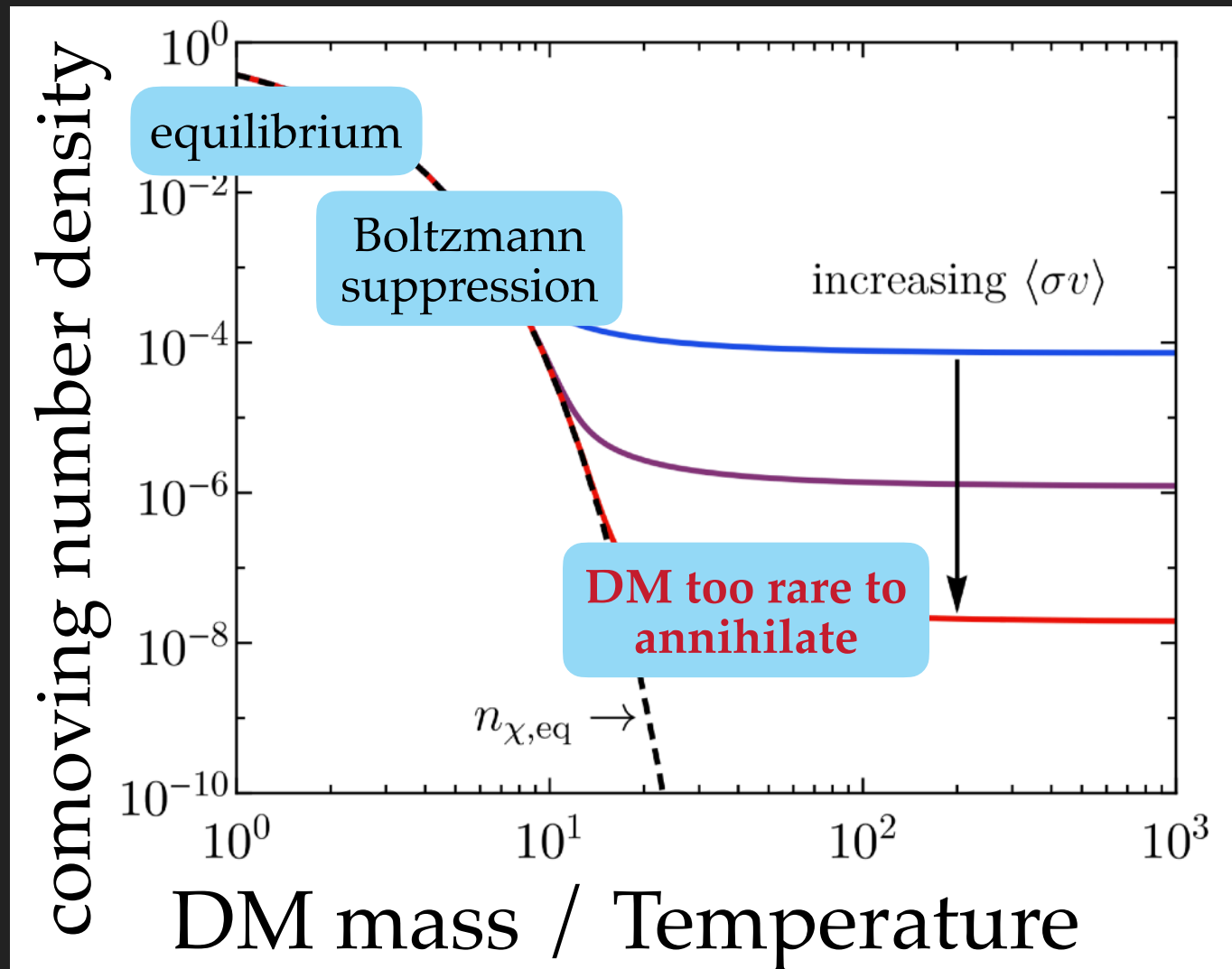




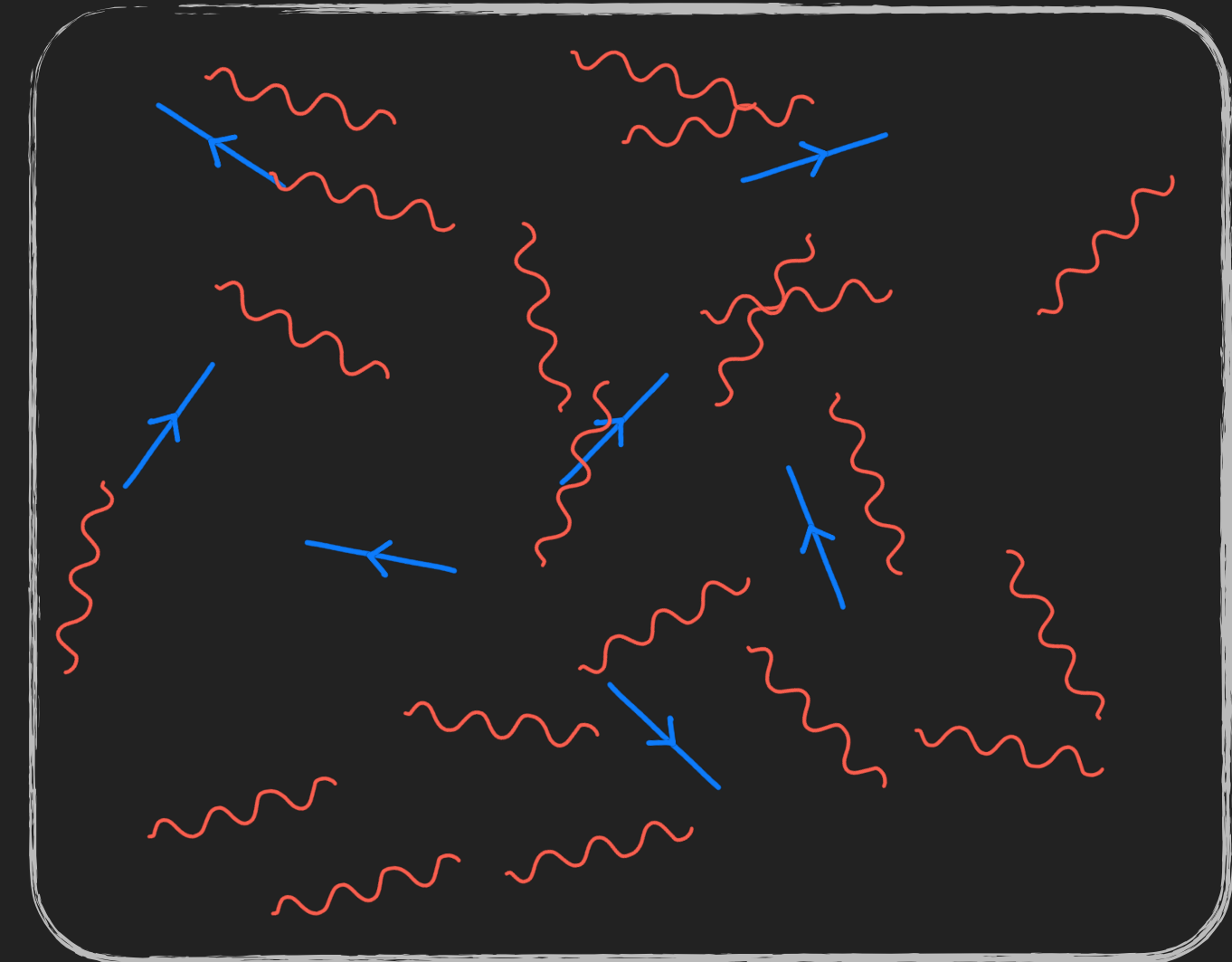
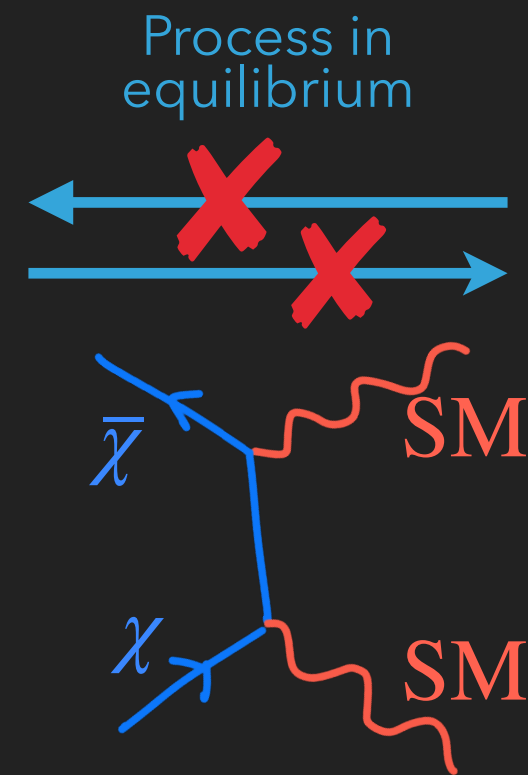
## Freeze-Out



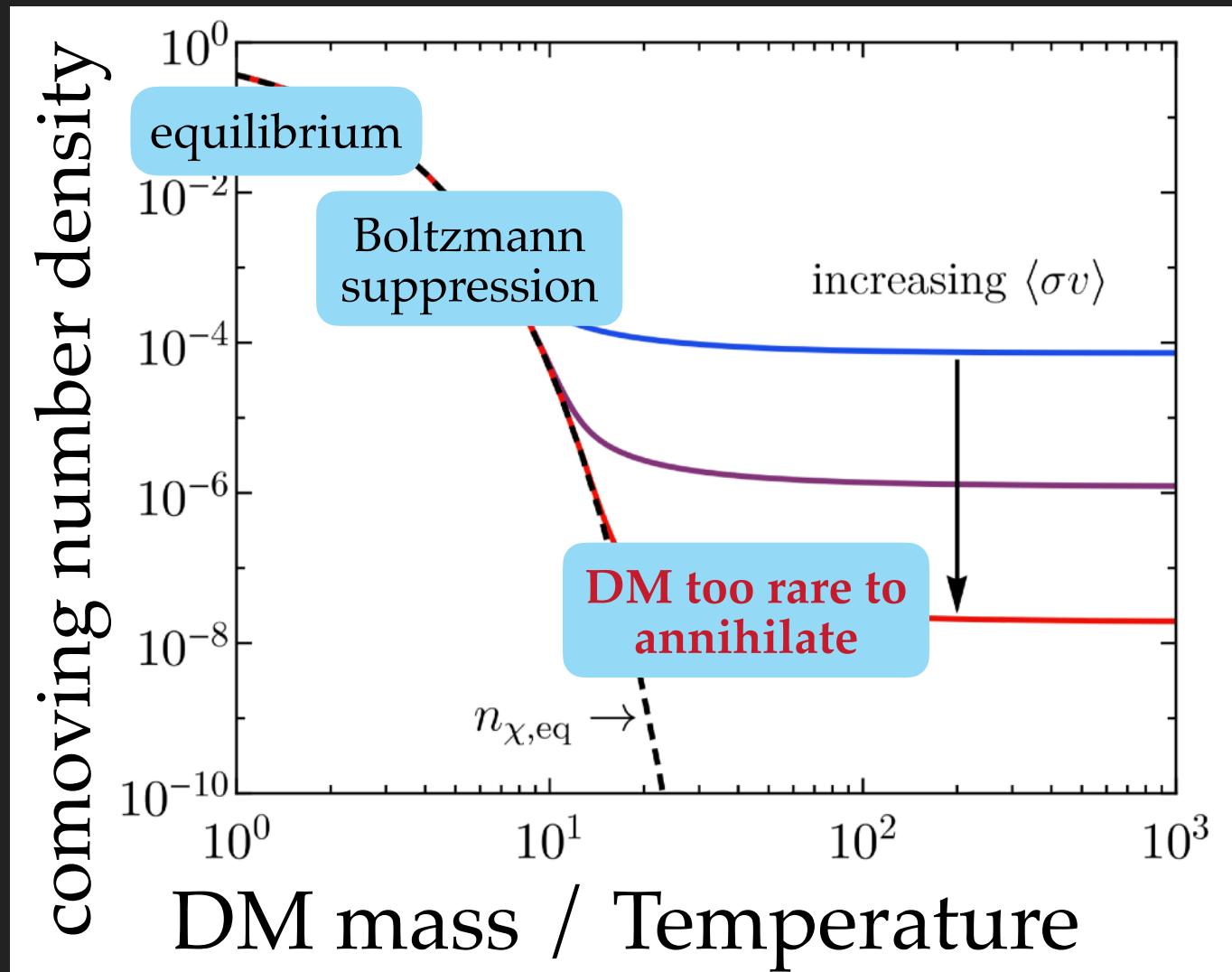




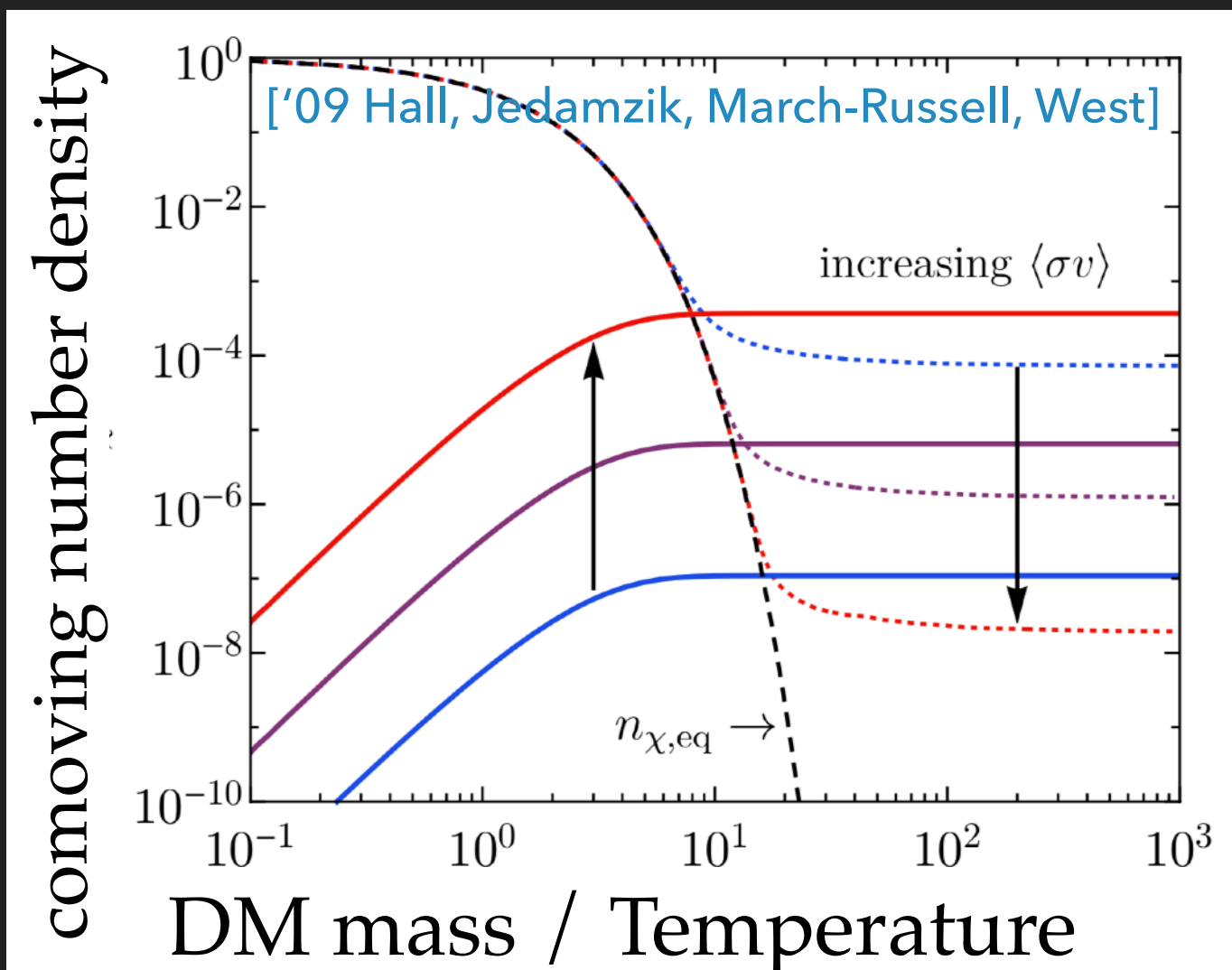
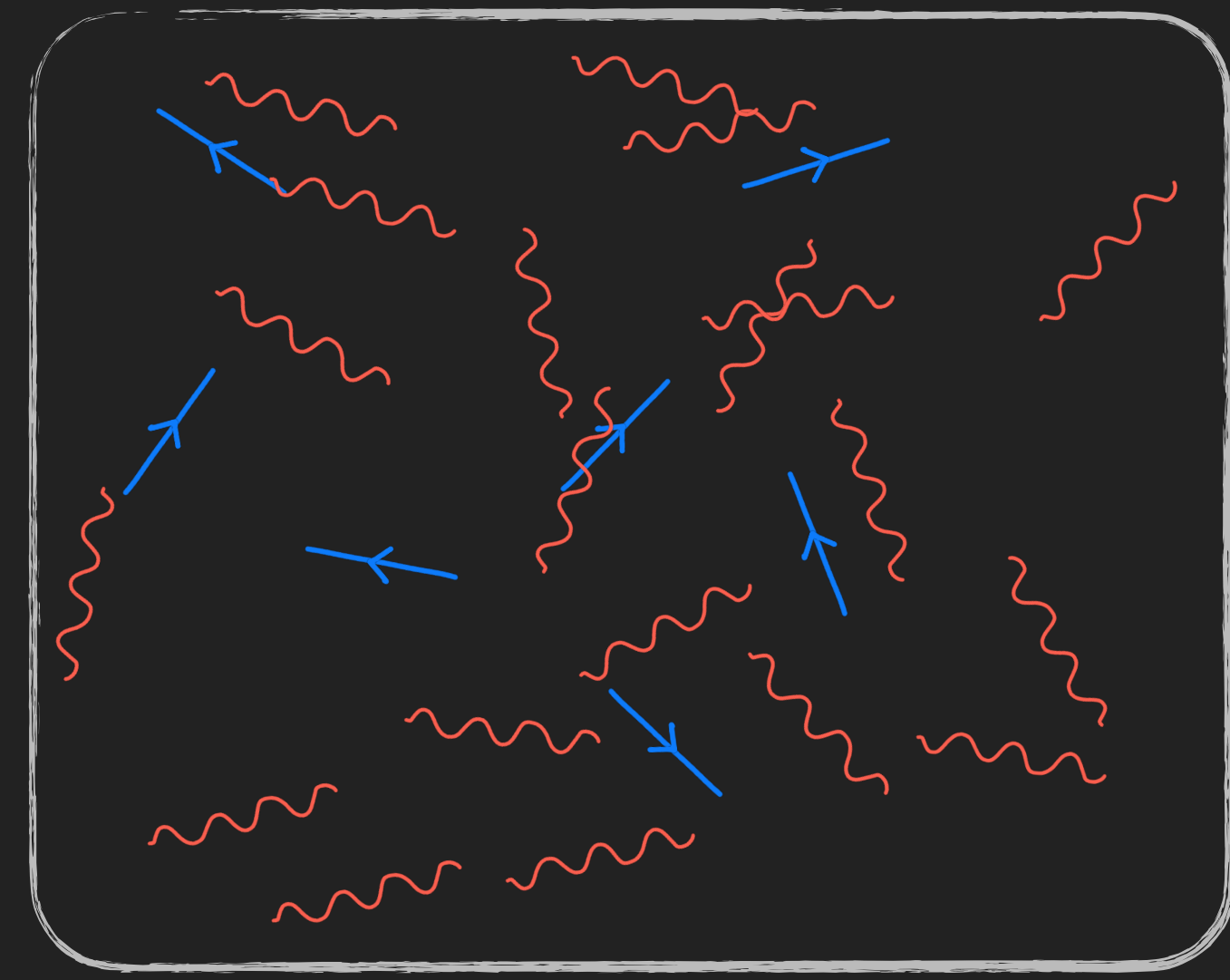
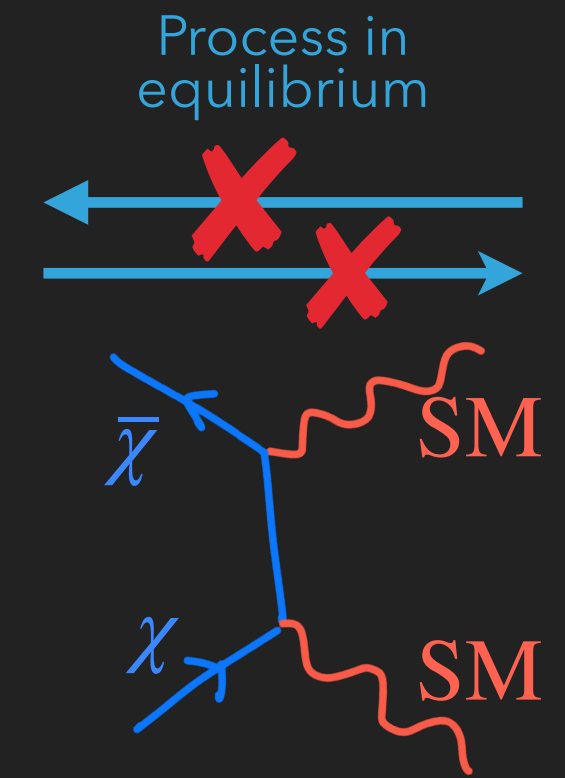
## Freeze-Out





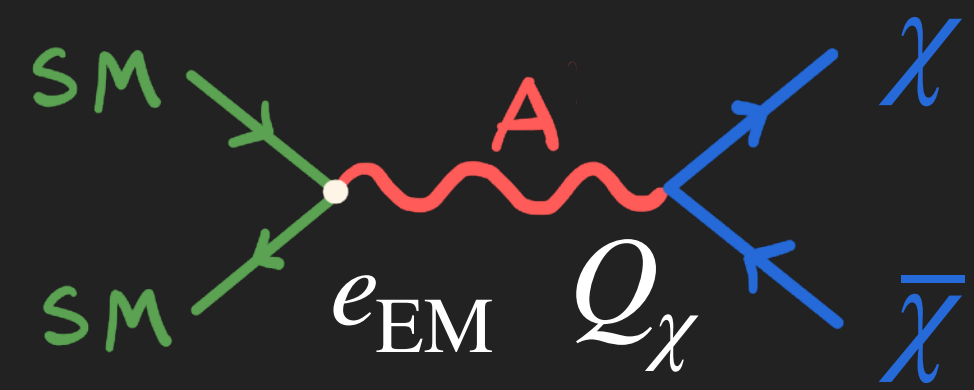


## Freeze-Out

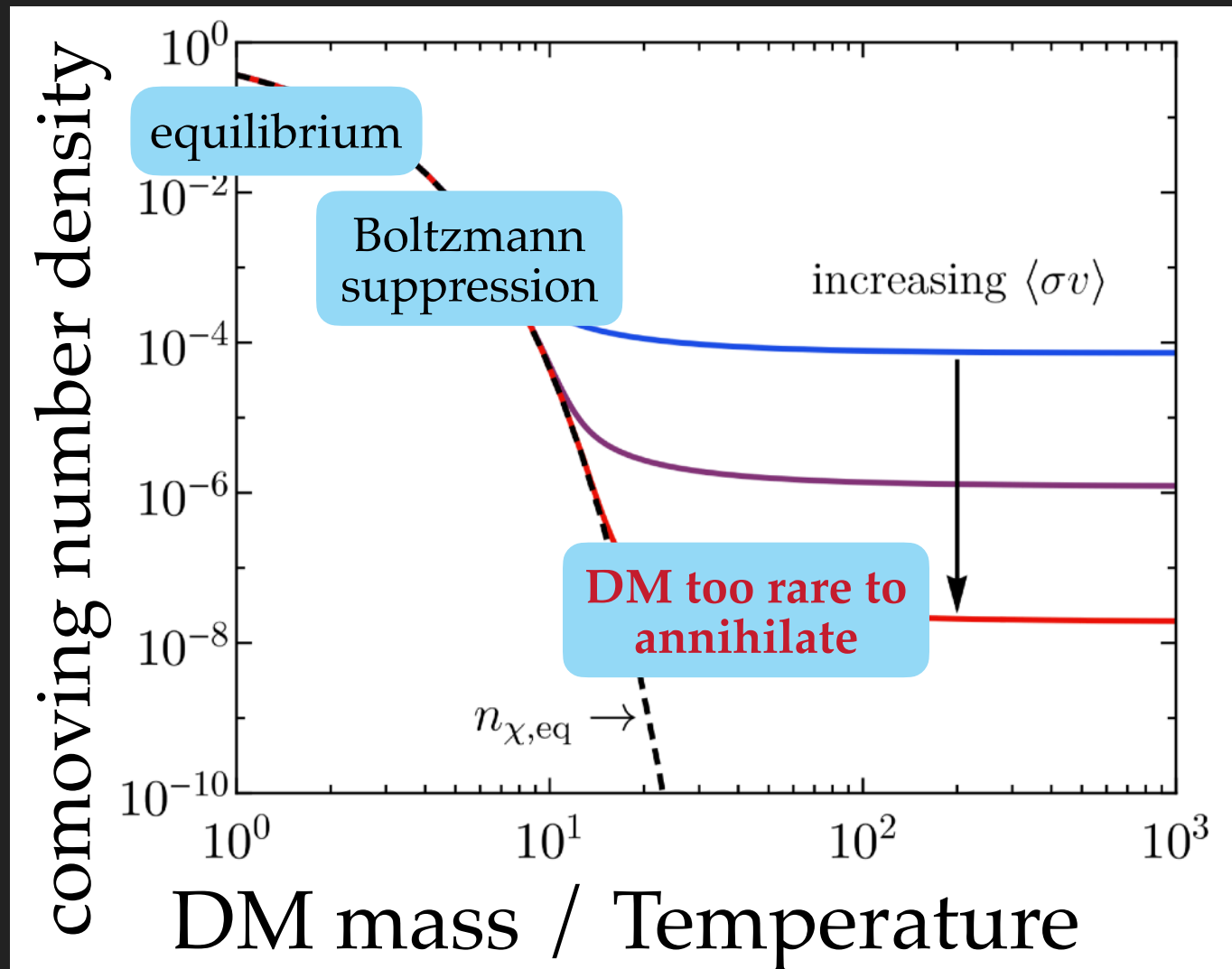


## Freeze-In

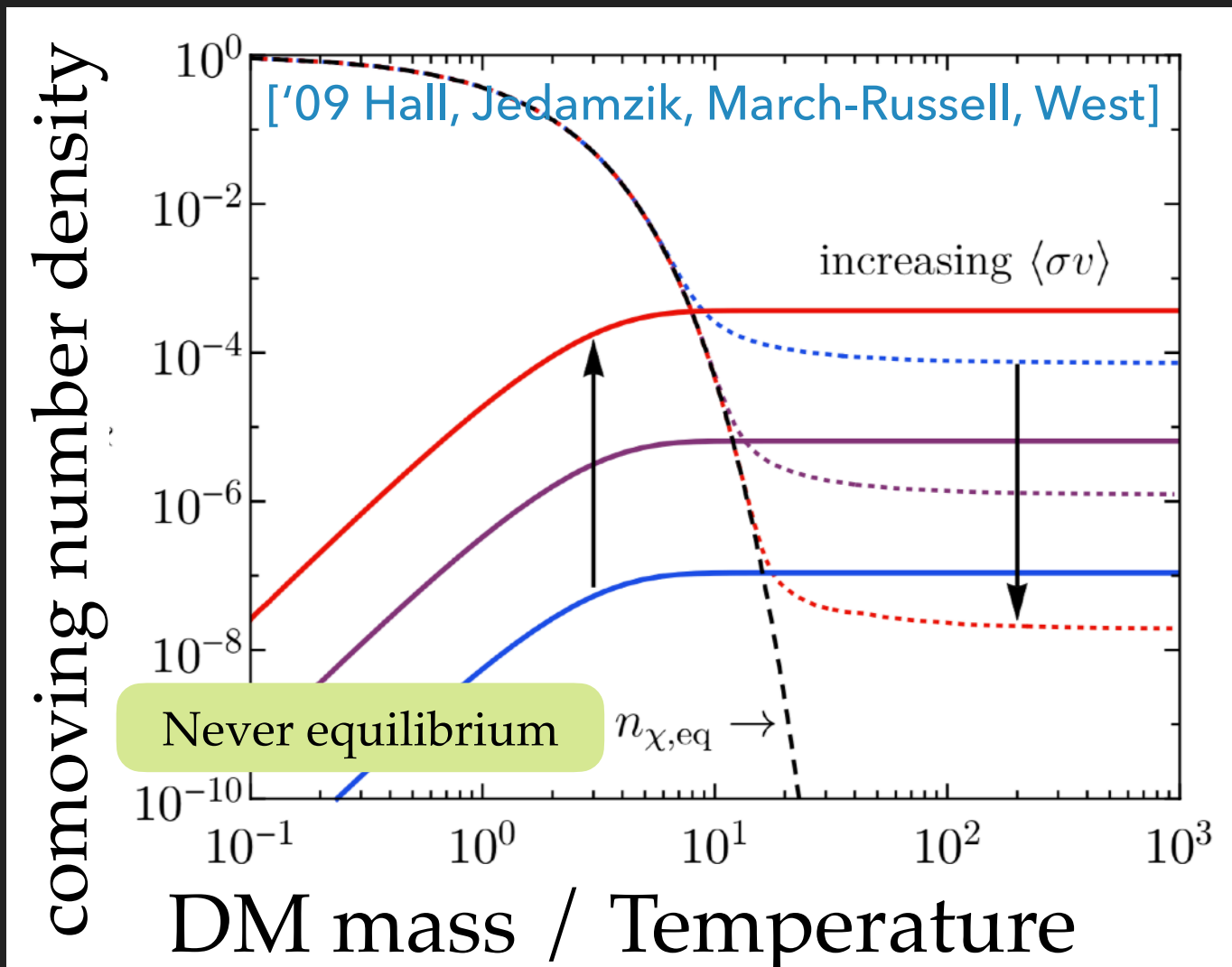
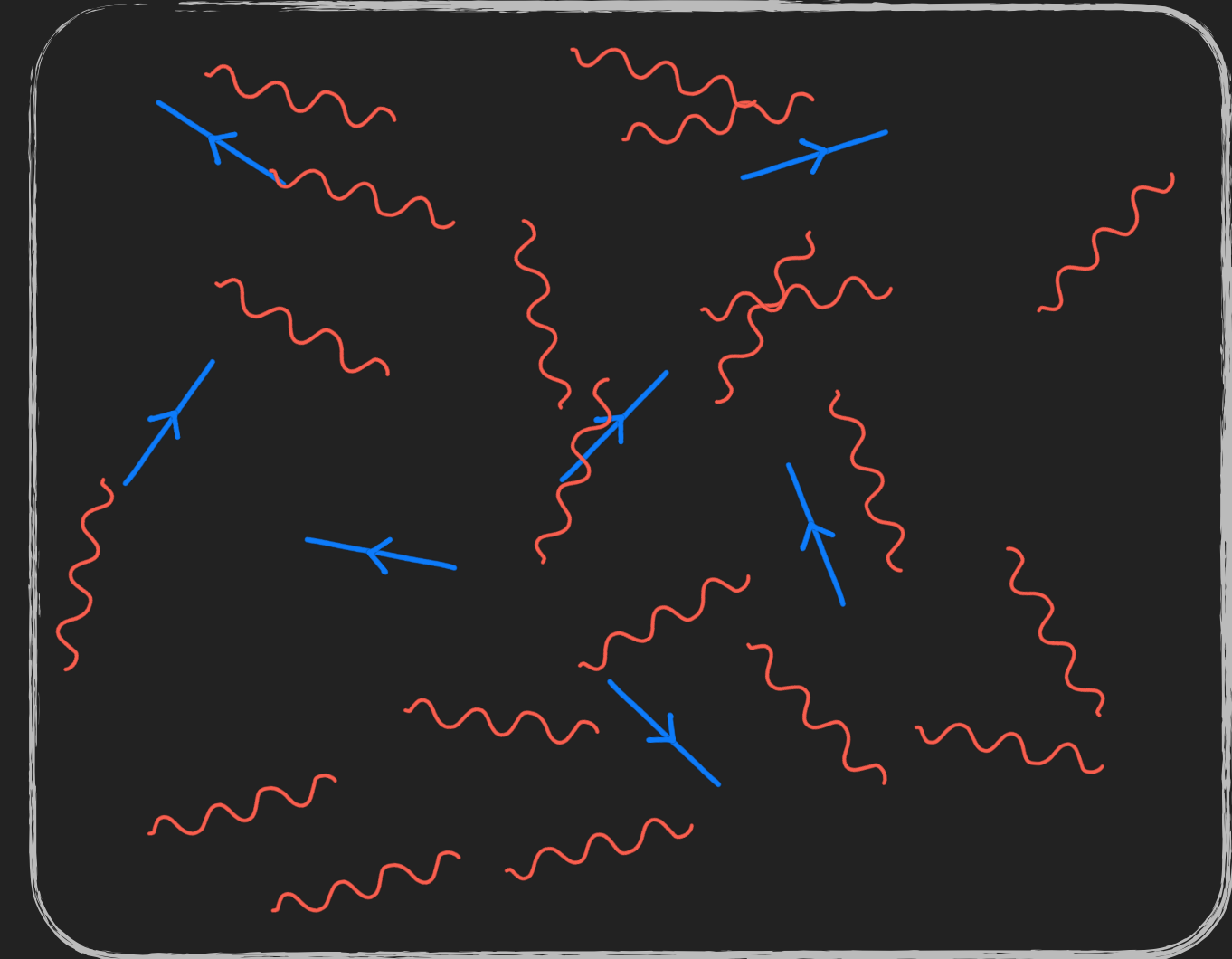
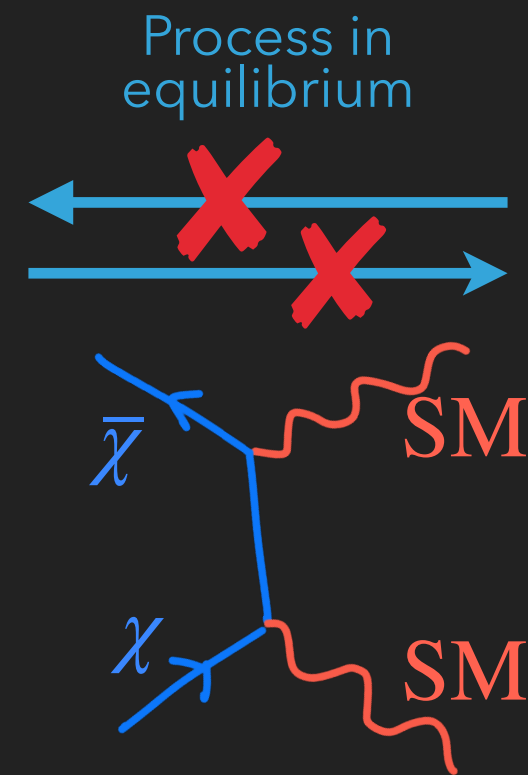
- ▶ Particle with small coupling





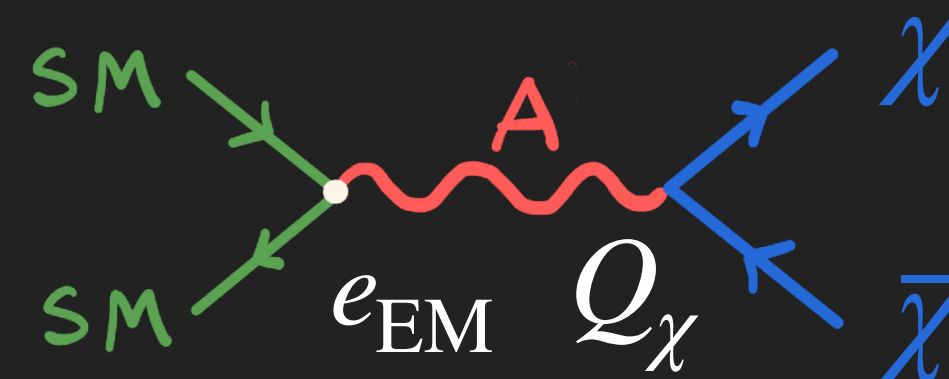


## Freeze-Out

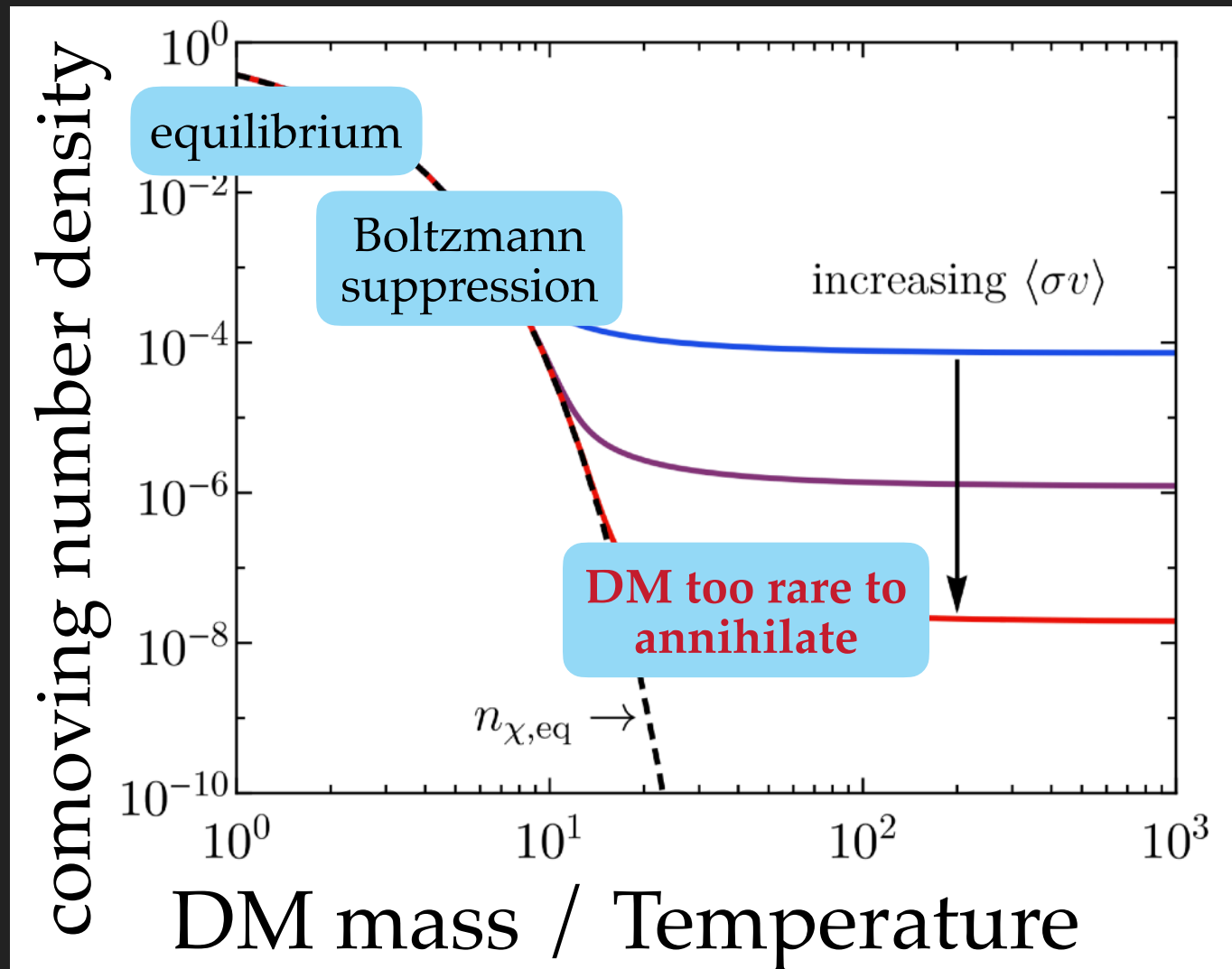


## Freeze-In

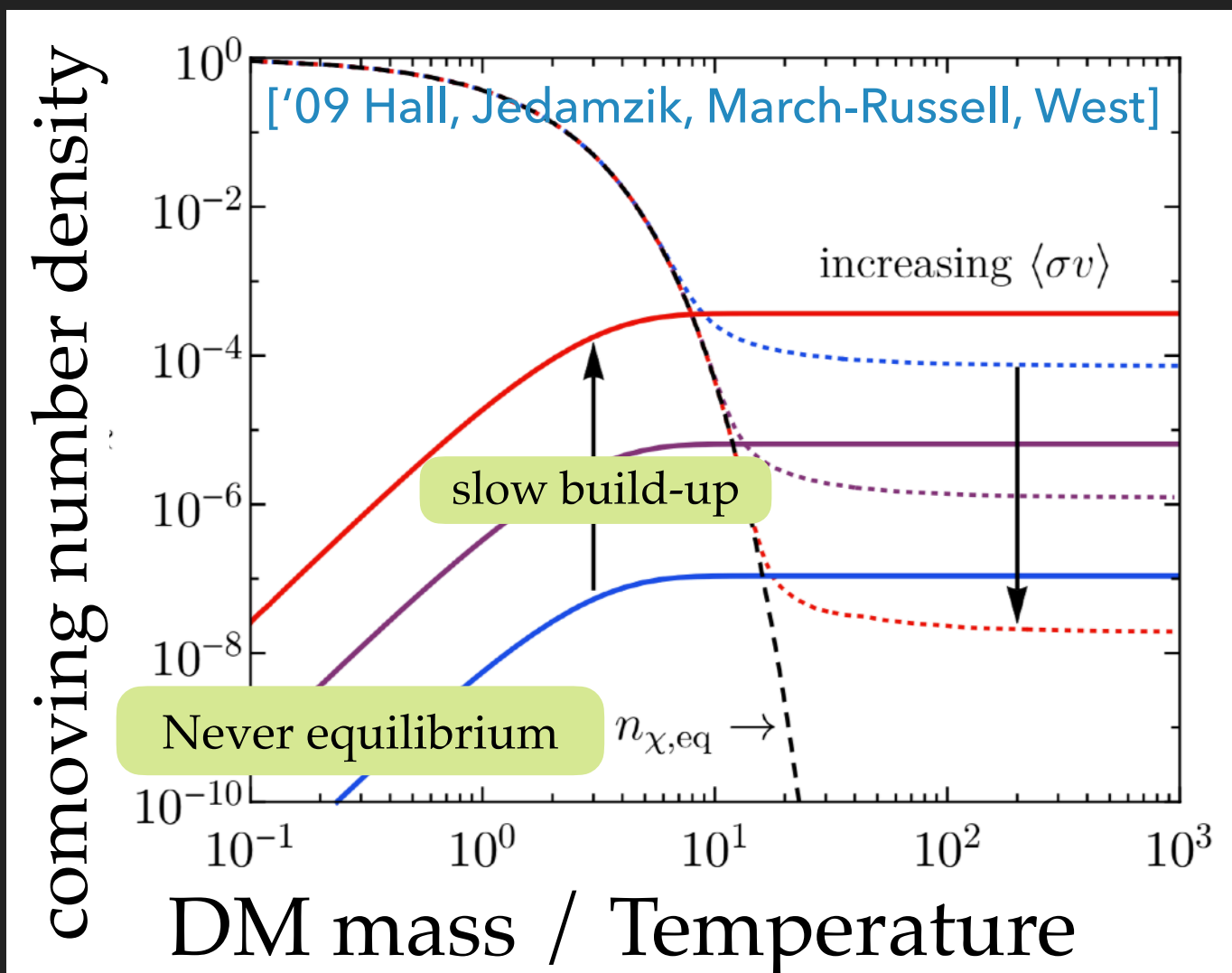
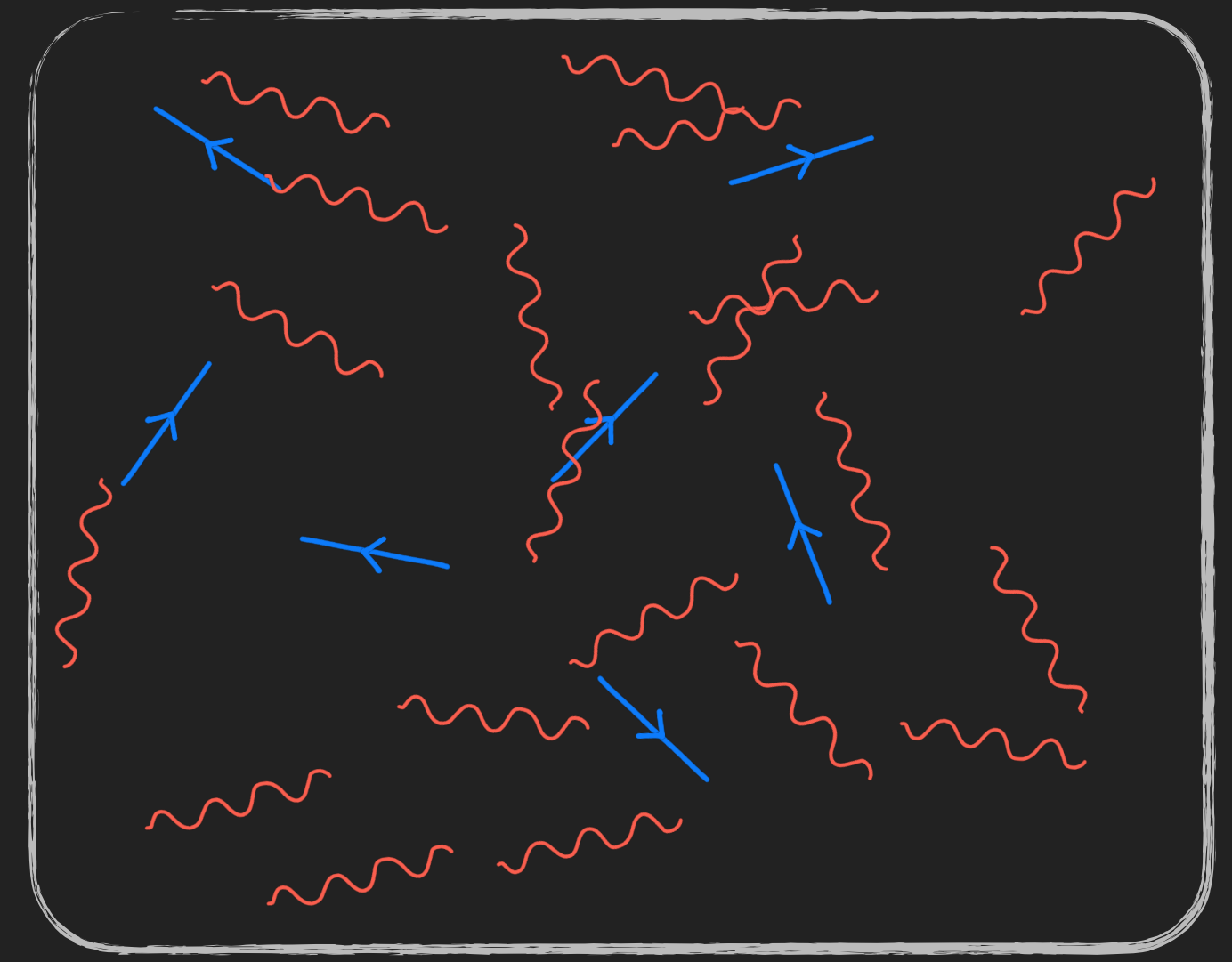
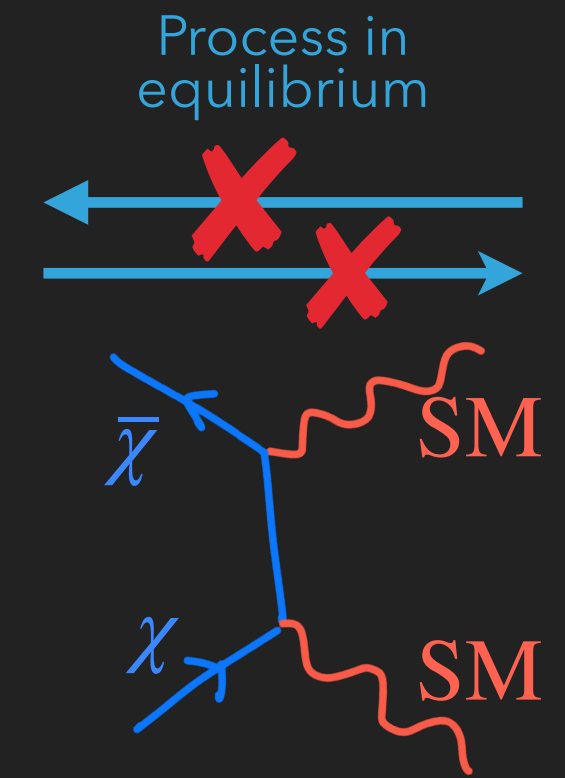
- ▶ Particle with small coupling





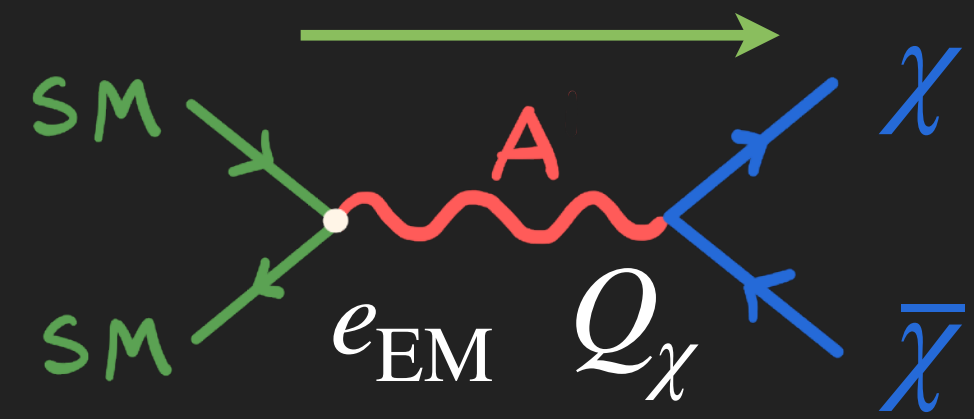


## Freeze-Out

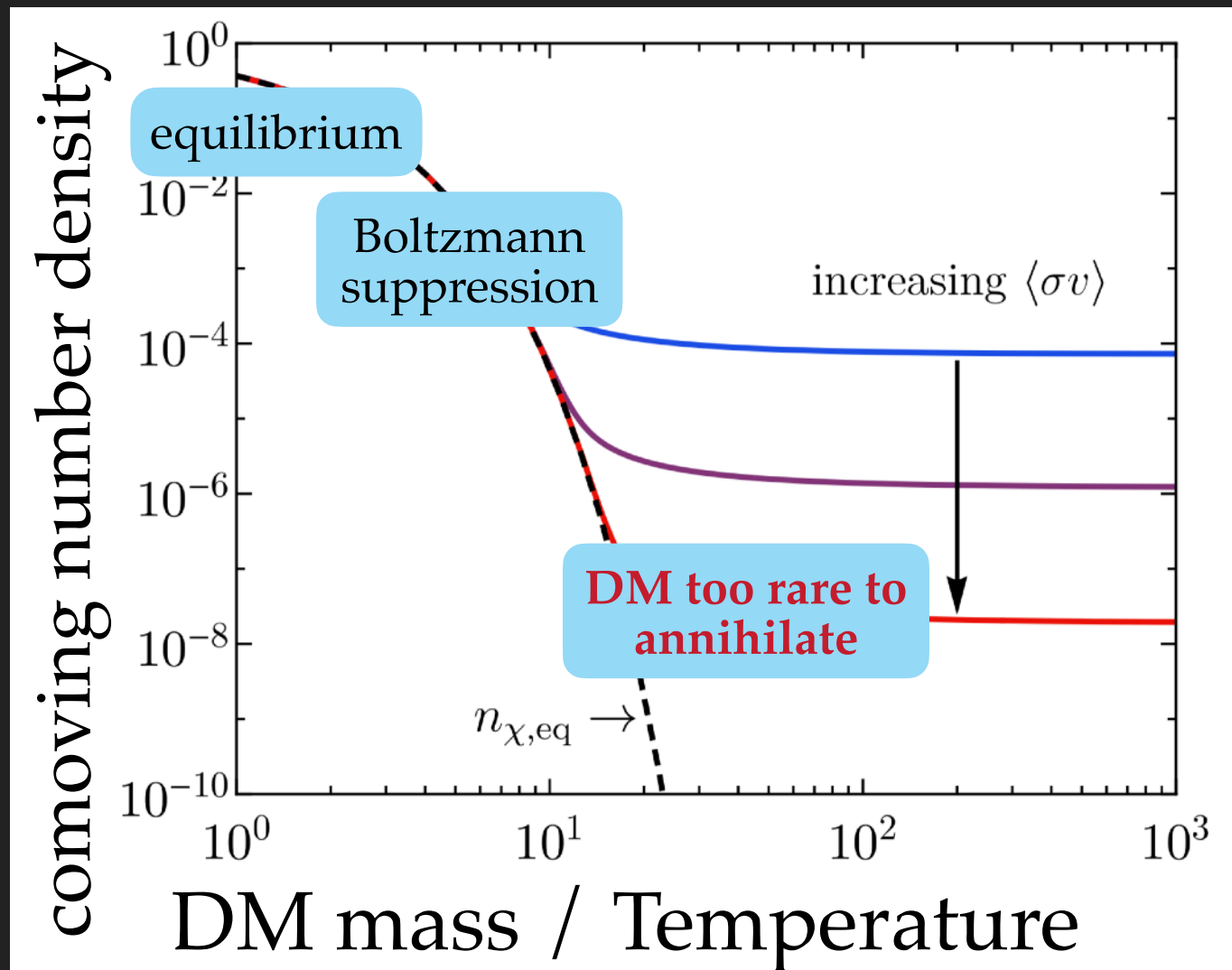


## Freeze-In

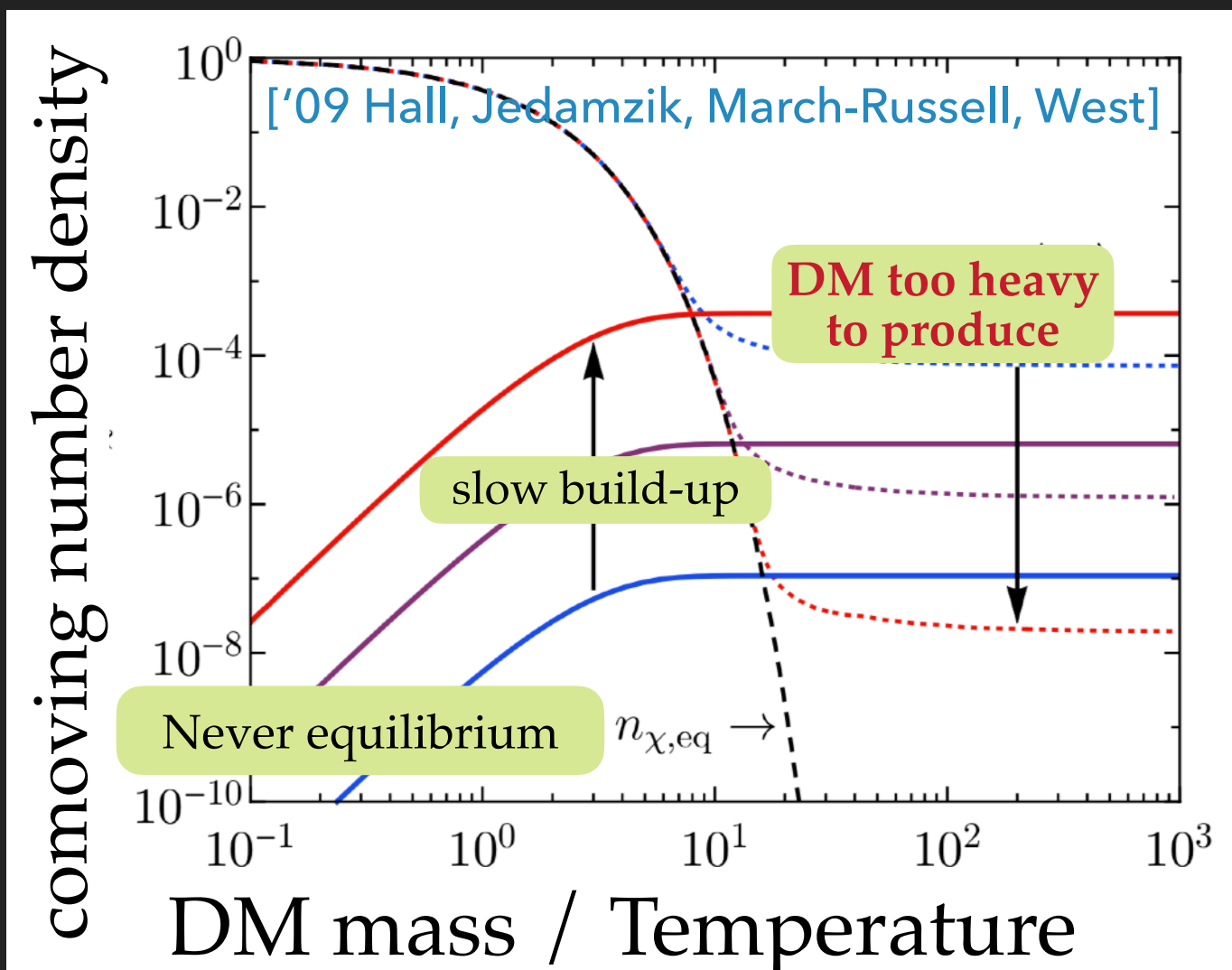
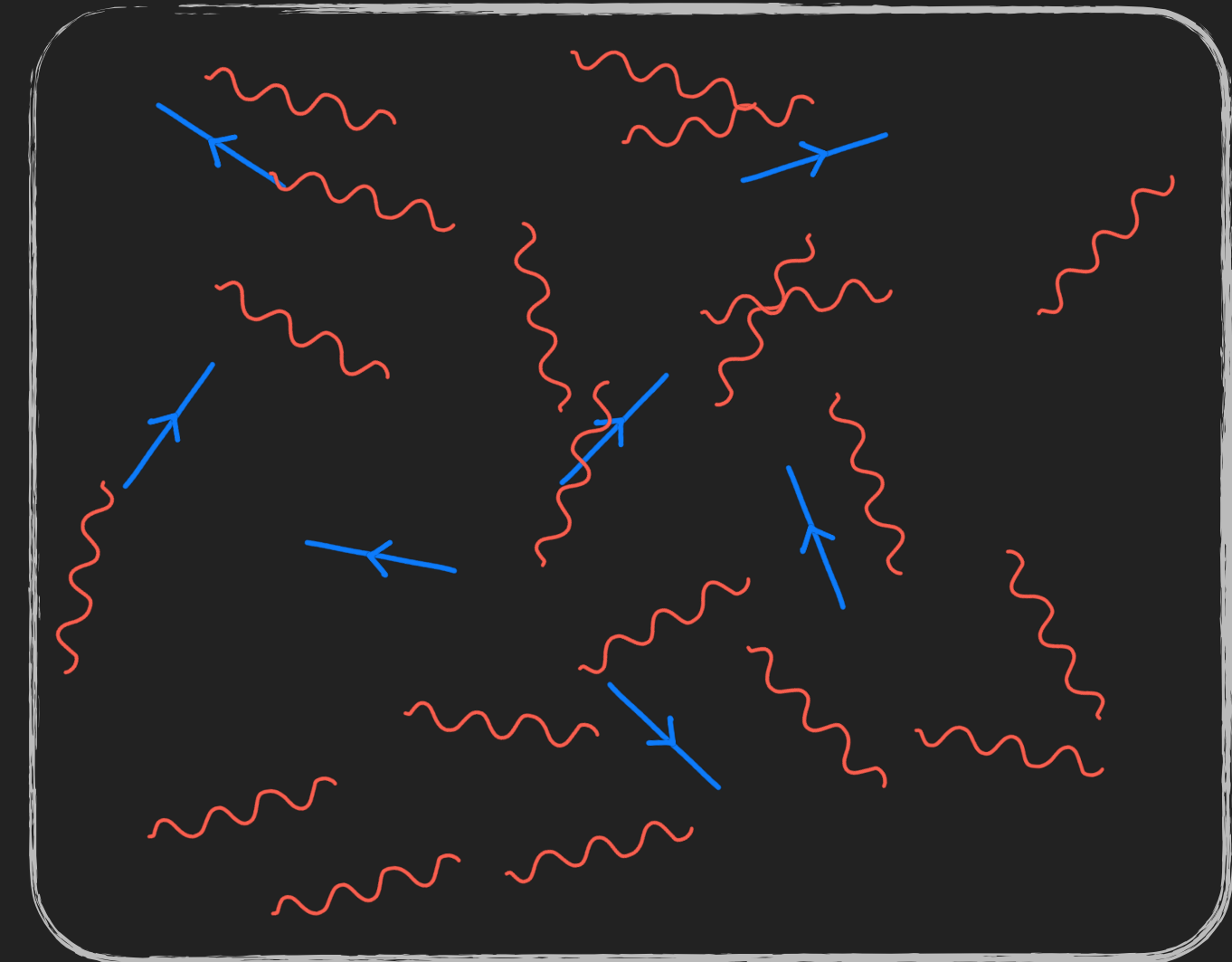
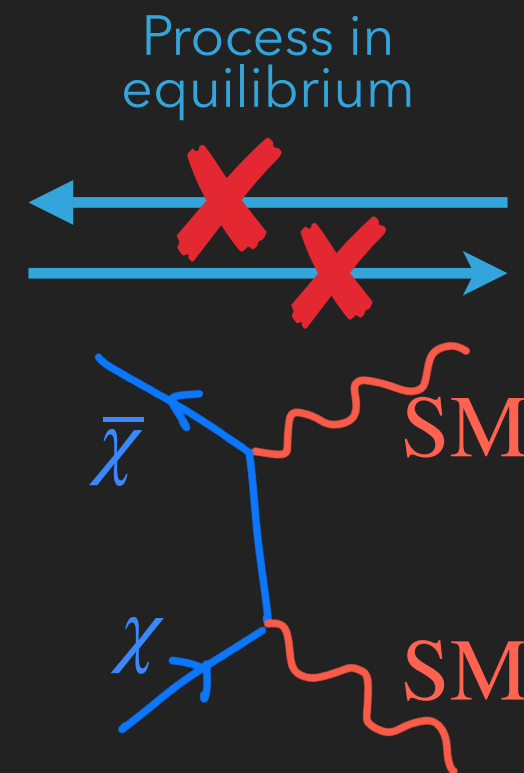
- ▶ Particle with small coupling





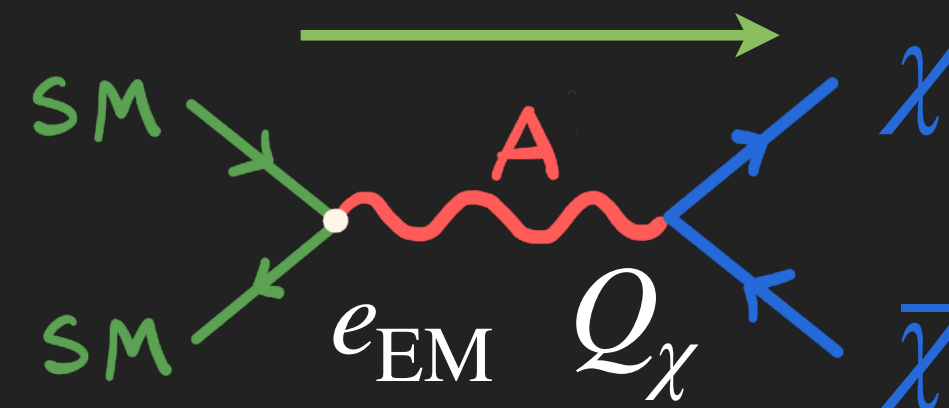


## Freeze-Out

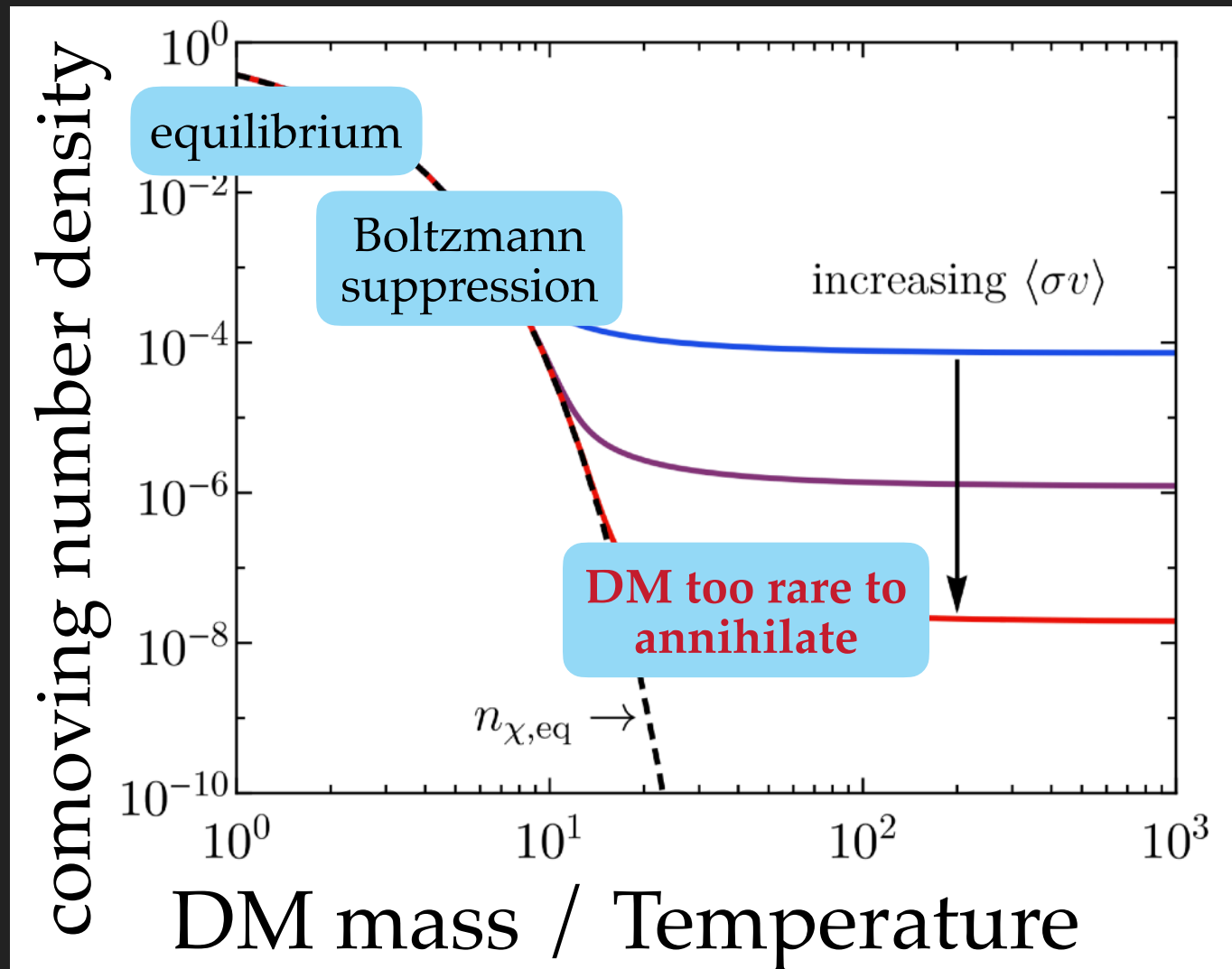


## Freeze-In

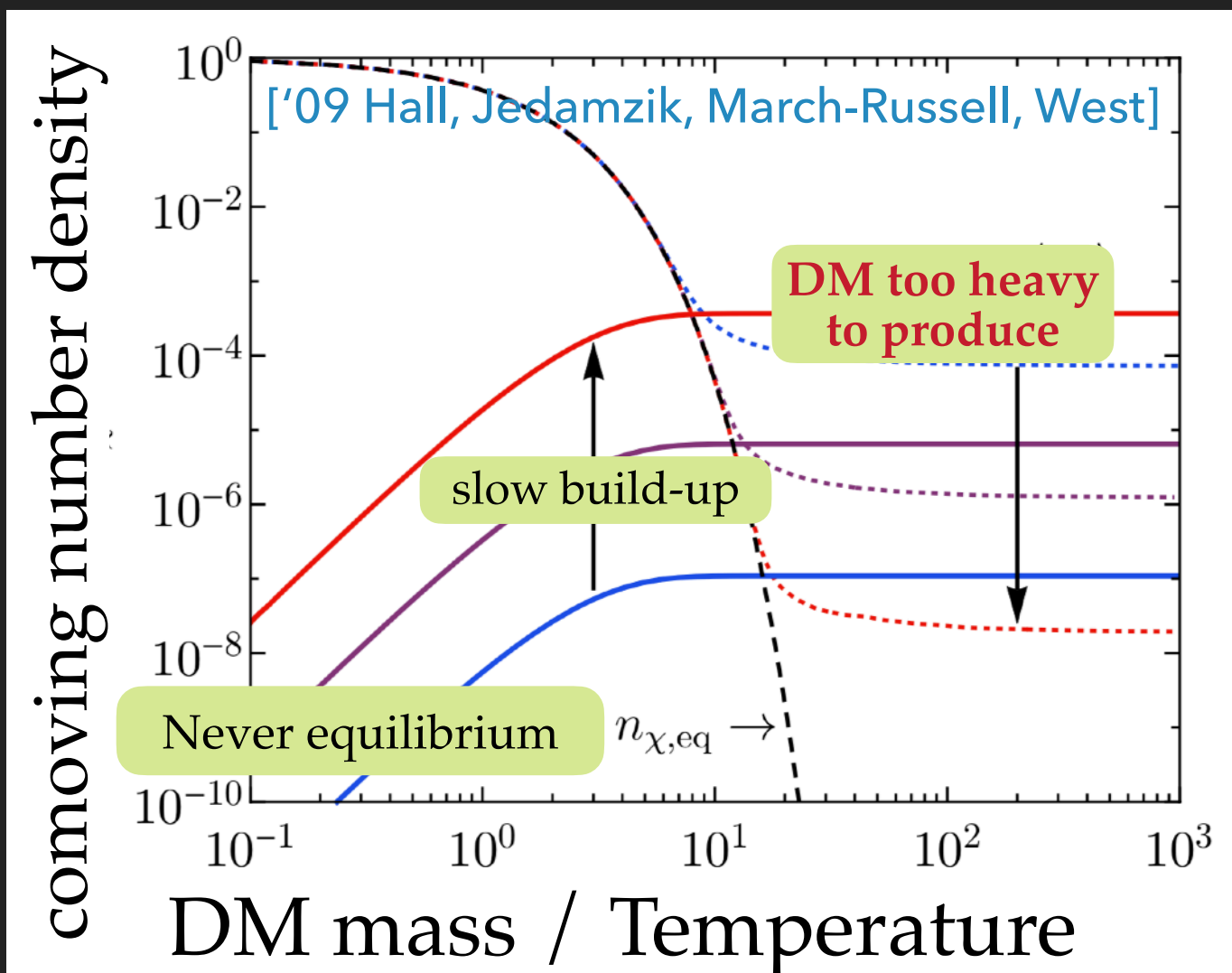
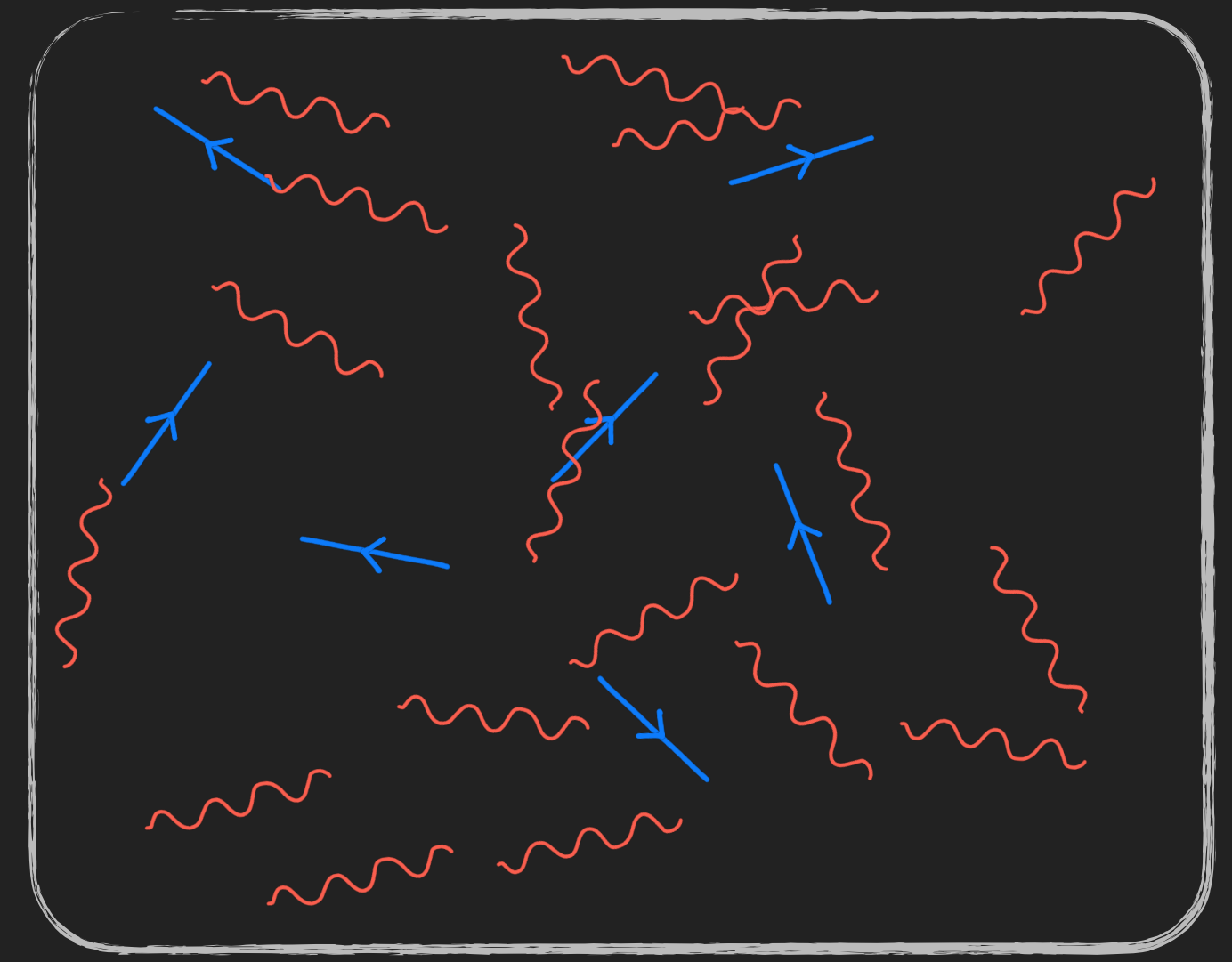
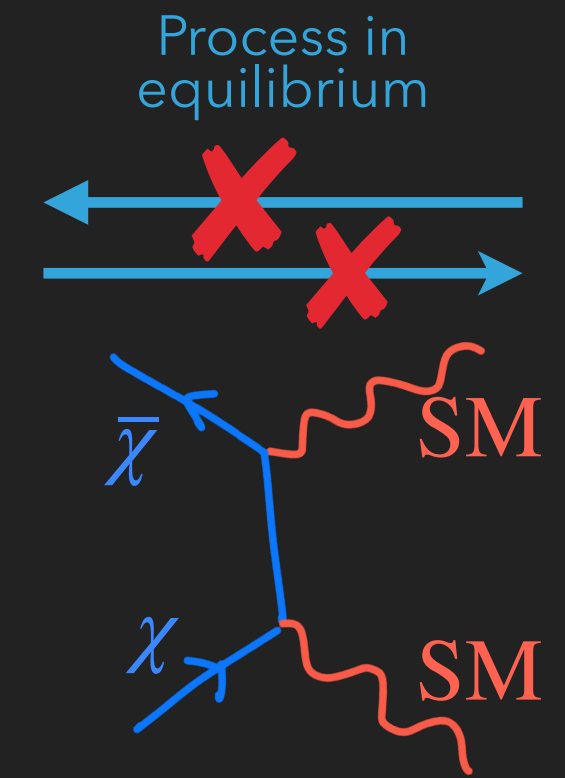
- ▶ Particle with small coupling





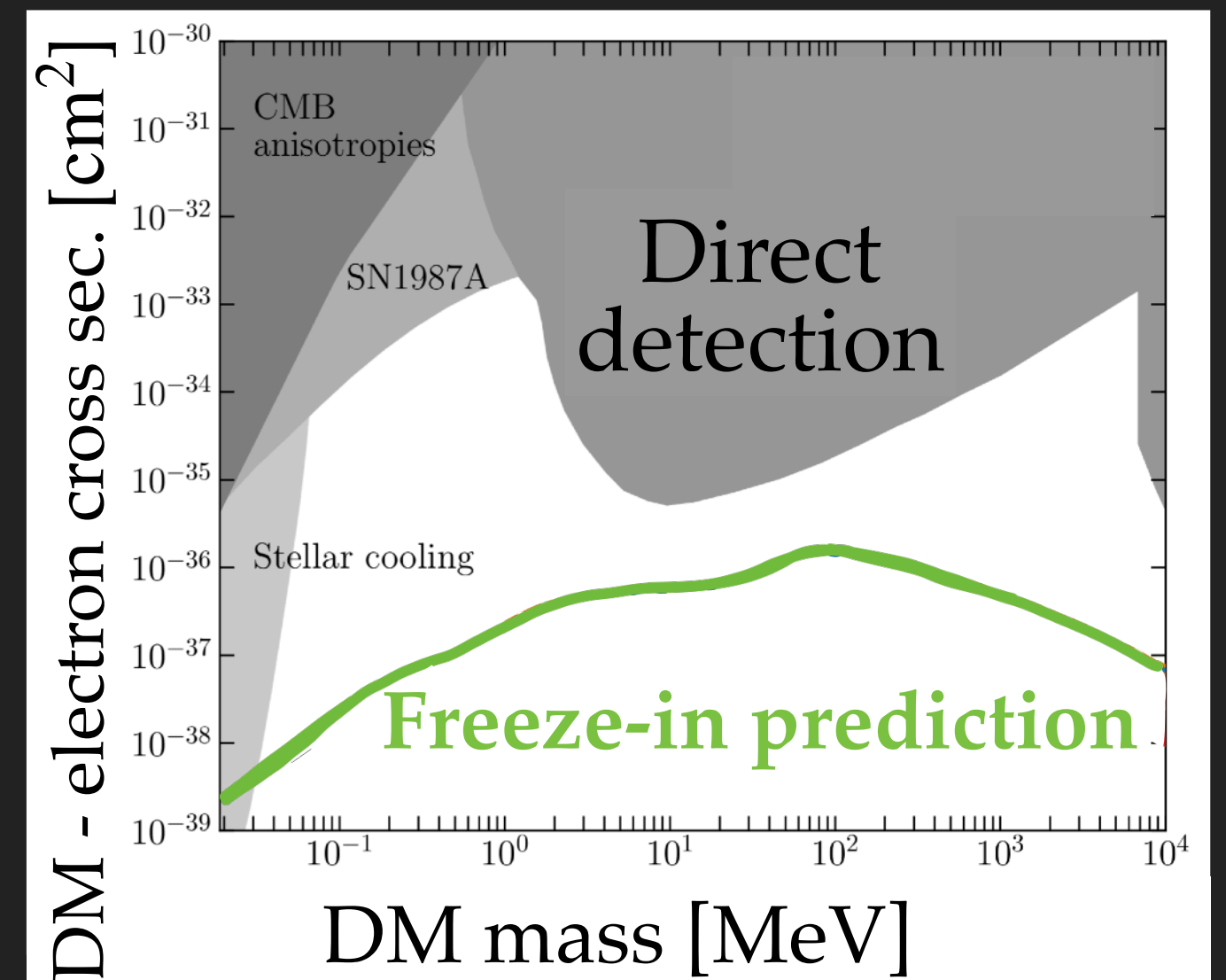
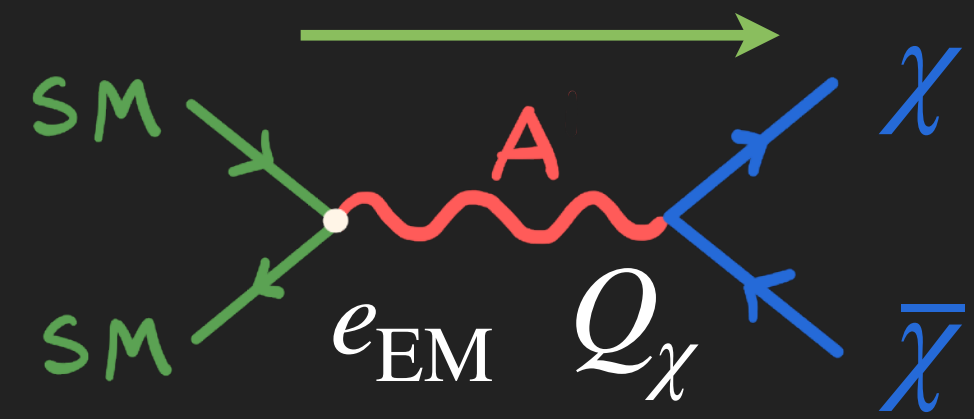


## Freeze-Out



## Freeze-In

► Particle with small coupling

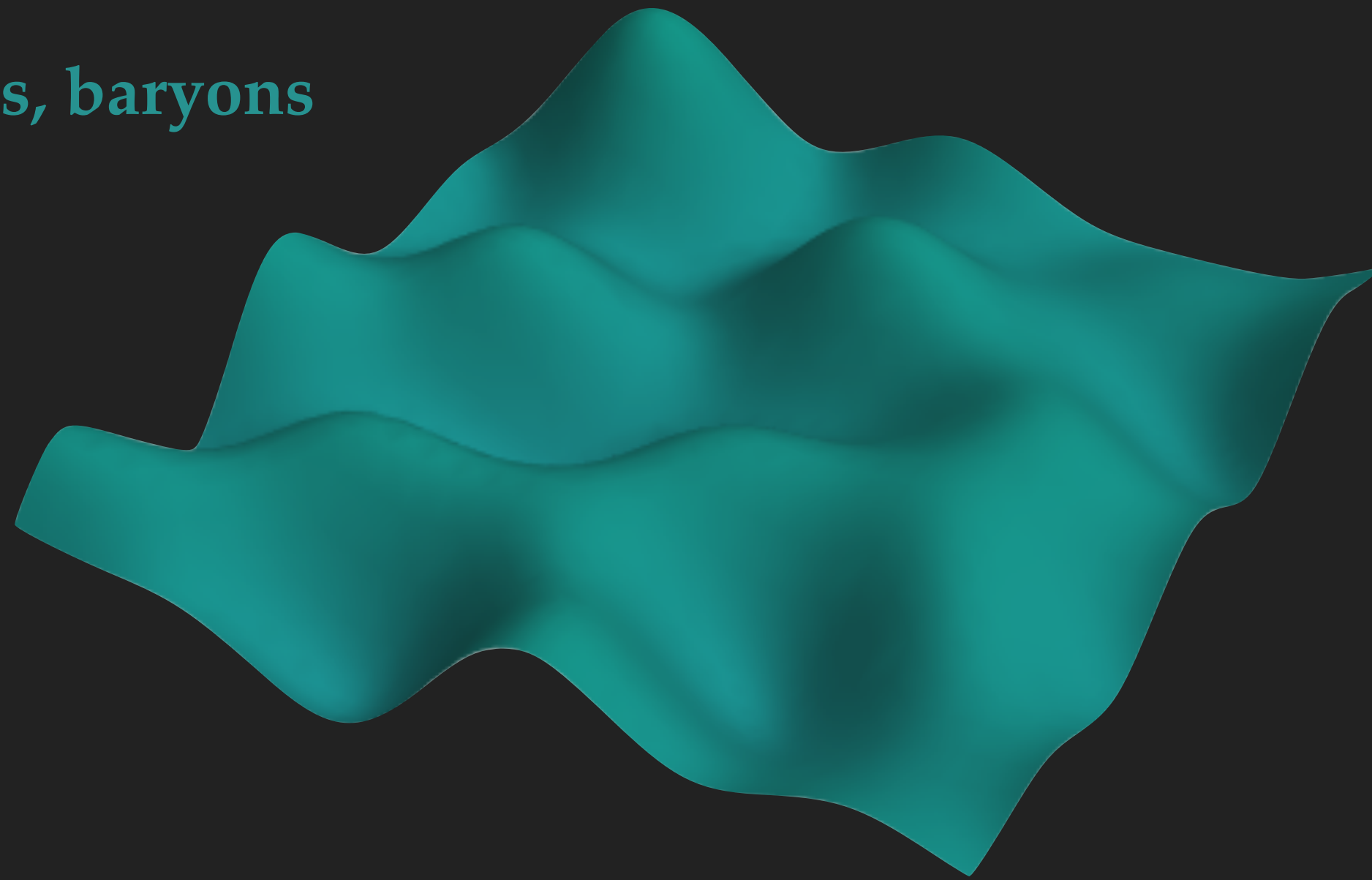




- ▶ DM and SM never in equilibrium. Do they share same perturbations?

[‘22 Bellomo, Berghaus, Boddy]

Photons, baryons



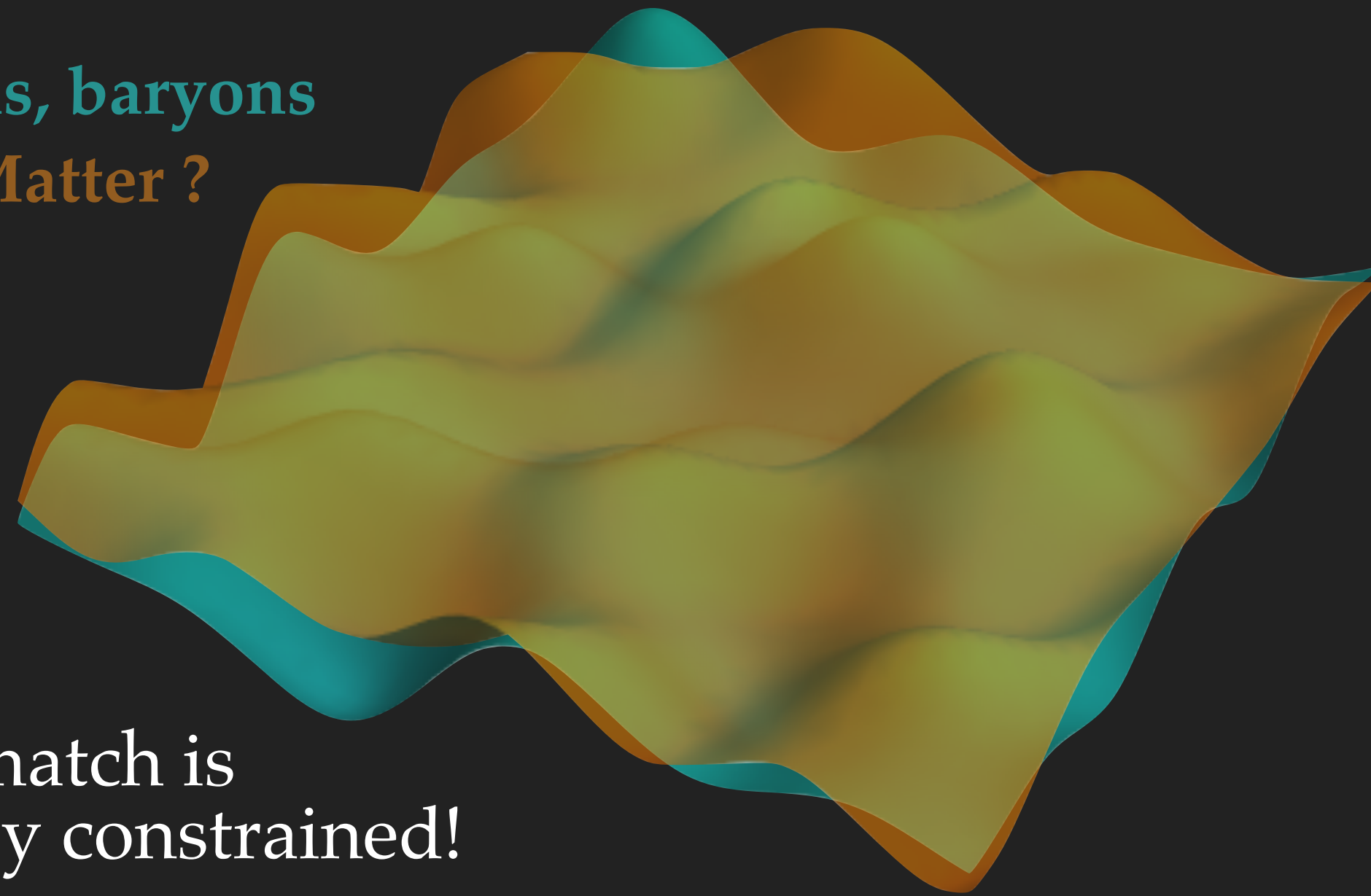
$$\frac{\text{perturbation}}{\text{average}} \rightarrow \frac{\delta\rho(t, \mathbf{x})}{\rho(t)}$$



- ▶ DM and SM never in equilibrium. Do they share same perturbations?

[’22 Bellomo, Berghaus, Boddy]

Photons, baryons  
Dark Matter ?



A mismatch is strongly constrained!

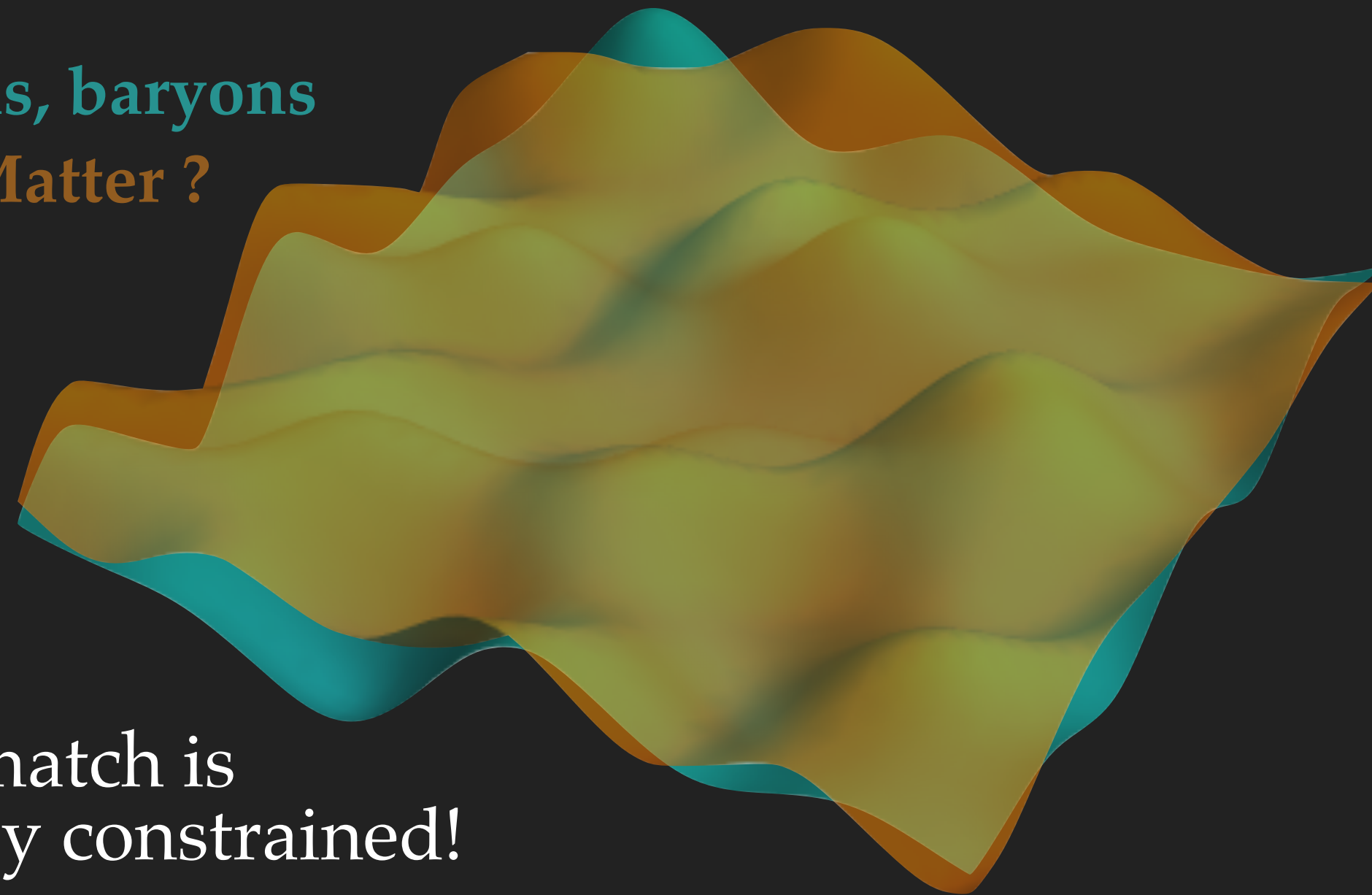
$$\frac{\text{perturbation}}{\text{average}} \rightarrow \frac{\delta\rho(t, \mathbf{x})}{\rho(t)}$$



- ▶ DM and SM never in equilibrium. Do they share same perturbations?

[‘22 Bellomo, Berghaus, Boddy]

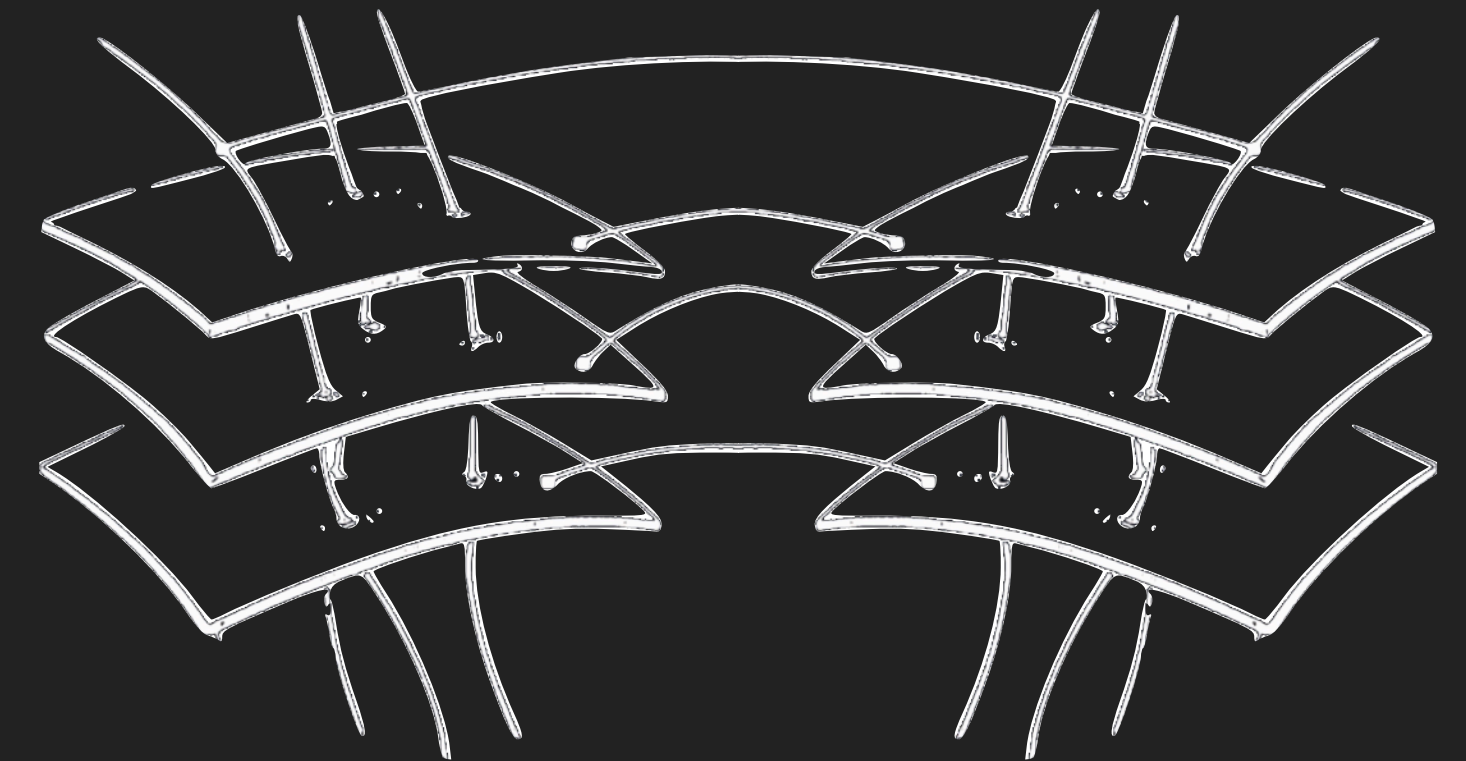
Photons, baryons  
Dark Matter ?



A mismatch is strongly constrained!

$$\frac{\text{perturbation}}{\text{average}} \rightarrow \frac{\delta\rho(t, \mathbf{x})}{\rho(t)}$$

- ▶ Diffeo. invariance: choose time foliation, or *clock*

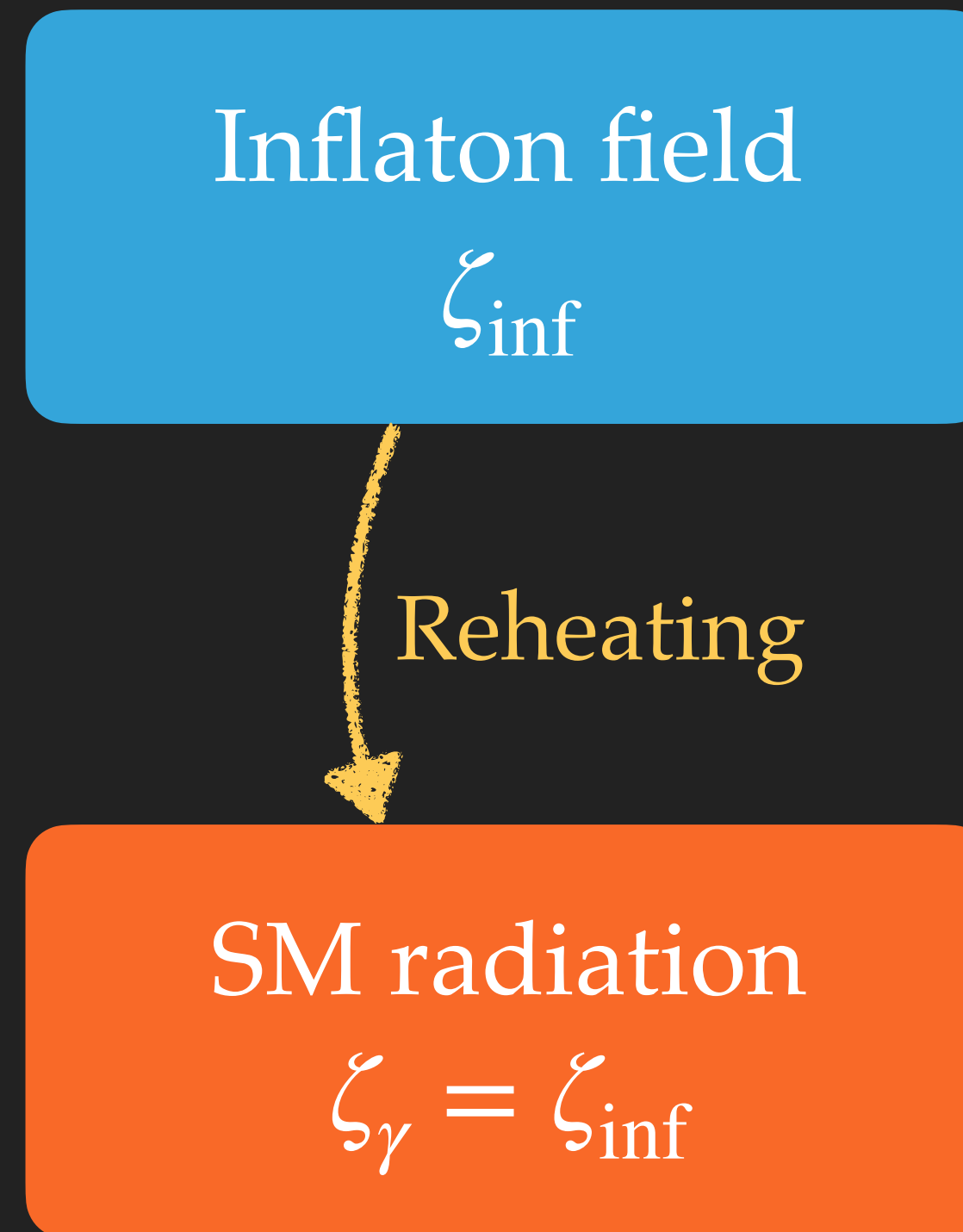


- ▶ different gauge:  $\delta\rho(t, \mathbf{x}) \rightarrow \delta\rho(t, \mathbf{x}) + \dot{\rho}(t)\delta t$
- ▶ Gauge invariant curvature perturbation:

$$\zeta = -\psi - H \frac{\delta\rho_{\text{tot}}(t, \mathbf{x})}{\dot{\rho}_{\text{tot}}(t)}$$

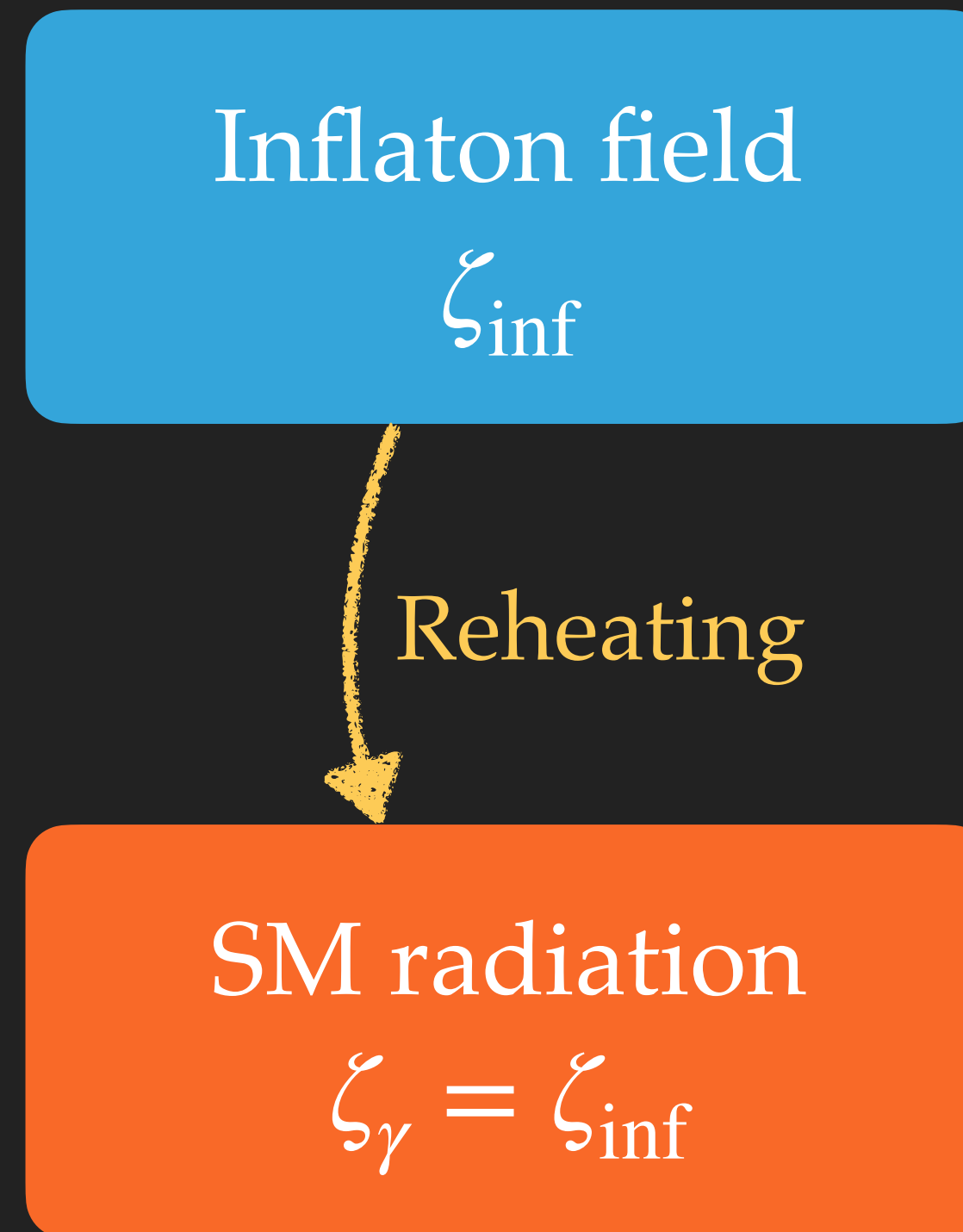


- ▶ Thermal bath:  
 $\zeta_\gamma(t, \mathbf{x}) \leftrightarrow \delta T(t, \mathbf{x})$
- ▶ **Adiabatic perturbations:**  
any fluid component  
following these pert.





- ▶ Thermal bath:  
 $\zeta_\gamma(t, \mathbf{x}) \leftrightarrow \delta T(t, \mathbf{x})$
- ▶ Adiabatic perturbations:  
 any fluid component following these pert.



Different source of  $\zeta$ ?

$$\zeta_\alpha = -\psi - H \frac{\delta\rho_\alpha(t, \mathbf{x})}{\dot{\rho}_\alpha(t)}$$

$$\mathcal{S}_{\alpha,\beta} = 3(\zeta_\alpha - \zeta_\beta)$$

- ▶ Isocurvature perturbations: a fluid component with  $\neq$  curv. pert.



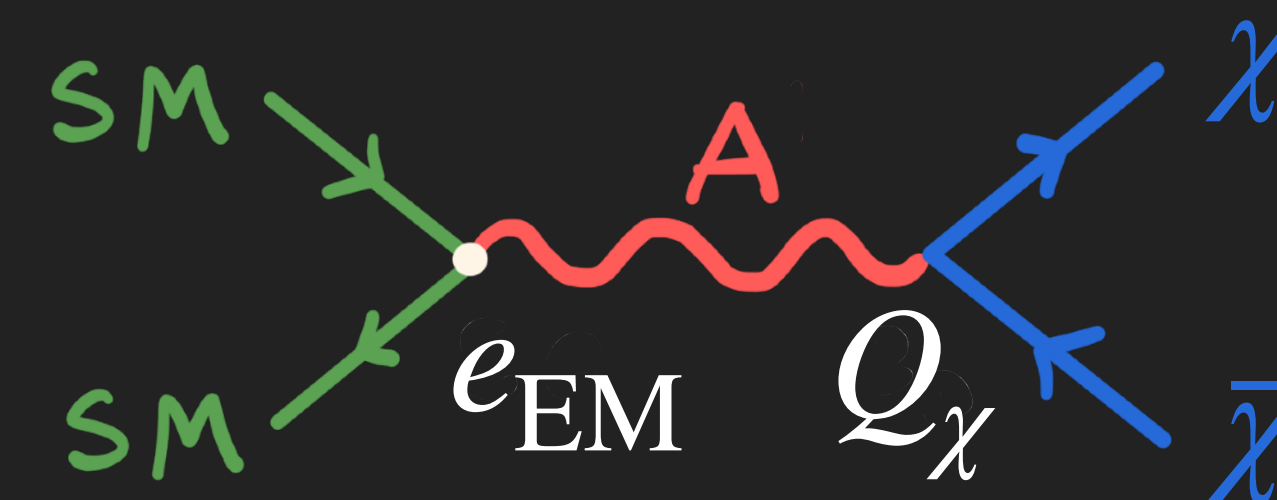
- ▶ Freeze-in DM is never in thermal equilibrium
- ▶ It originates from SM though:

$$\dot{\rho}_{\text{DM}}(t, \mathbf{x}) = -3H (\rho_{\text{DM}}(t, \mathbf{x}) + P_{\text{DM}}(t, \mathbf{x})) + \Gamma(t, \mathbf{x})$$

must be included in  $\dot{\rho} \rightarrow \zeta$

$$\Gamma = \langle T_{\text{SM}} \sigma_{\text{ann}} v_{\text{rel}} \rangle n_{\text{SM}}^2$$

Millicharge DM:  $\Gamma = \left( \frac{9\alpha_{\text{EM}} Q_\chi^2 \zeta(3)^2}{2\pi^4} \right) T_{\text{SM}}^5(t, \mathbf{x})$





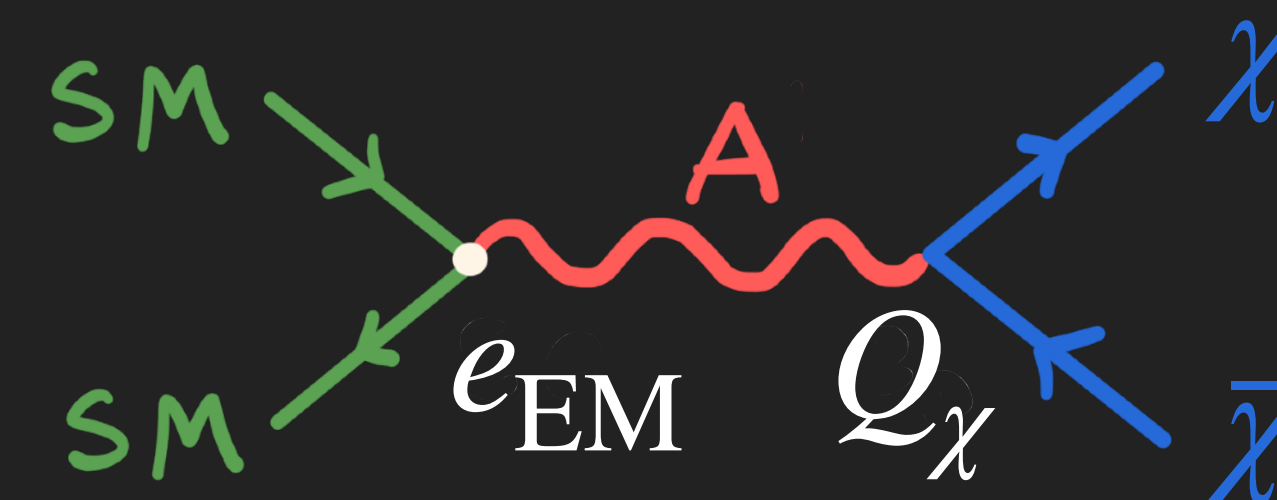
- Freeze-in DM is never in thermal equilibrium
- It originates from SM though:

$$\dot{\rho}_{\text{DM}}(t, \mathbf{x}) = -3H(\rho_{\text{DM}}(t, \mathbf{x}) + P_{\text{DM}}(t, \mathbf{x})) + \Gamma(t, \mathbf{x})$$

must be included in  $\dot{\rho} \rightarrow \zeta$

$$\Gamma = \langle T_{\text{SM}} \sigma_{\text{ann}} v_{\text{rel}} \rangle n_{\text{SM}}^2$$

Millicharge DM:  $\Gamma = \left( \frac{9\alpha_{\text{EM}} Q_\chi^2 \zeta(3)^2}{2\pi^4} \right) T_{\text{SM}}^5(t, \mathbf{x})$



[‘04 Weinberg]

## SINGLE-CLOCK ARGUMENT

$$\rho_{\text{DM}}(t, \mathbf{x}) \leftrightarrow T_{\text{SM}}(t, \mathbf{x})$$

$$\frac{\delta \rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} = \frac{\delta T}{\dot{T}} = \frac{\delta \rho_\gamma}{\dot{\rho}_\gamma}$$

NB: regardless of thermalisation!



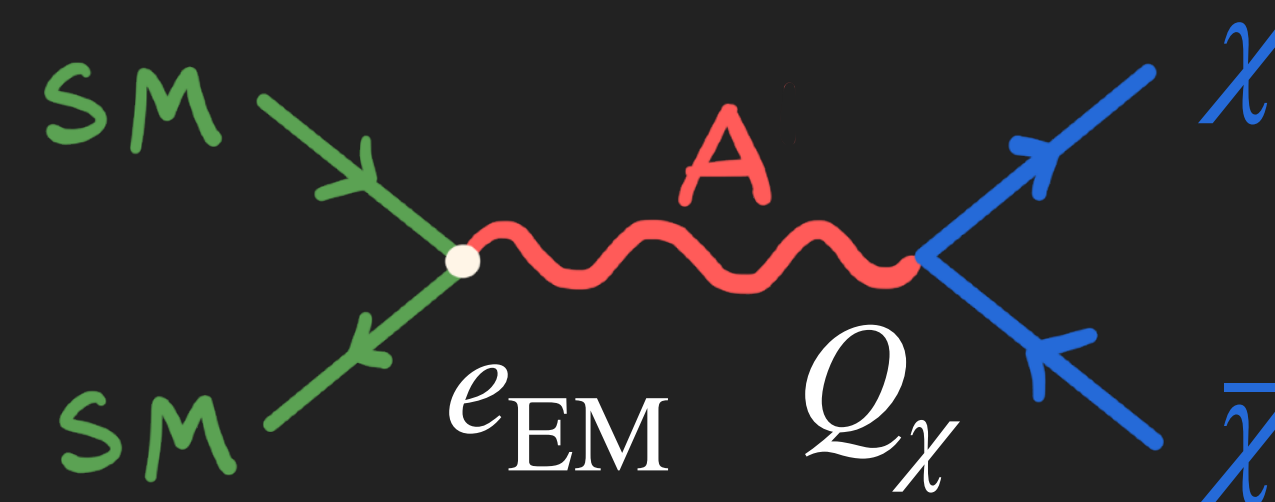
- ▶ Time evolution for energy density:

$$\dot{\rho}_{\text{DM}} = -3H (\rho_{\text{DM}} + P_{\text{DM}}) + Q_{\text{DM}},$$

$$\dot{\rho}_{\gamma} = -3H (\rho_{\gamma} + P_{\gamma}) + Q_{\gamma}.$$

$$Q_{\text{DM}} = \Gamma(\rho_{\gamma}),$$

$$Q_{\gamma} = -\Gamma(\rho_{\gamma}).$$



- ▶ Derive evolution equation for curvature perturbations on *large scales*
- ▶ Freeze-in relevant around  $T_{\text{SM}} \sim m_{\chi} \gtrsim \text{MeV}$ , way before recombination ( $\sim \text{eV}$ ), and then shuts off



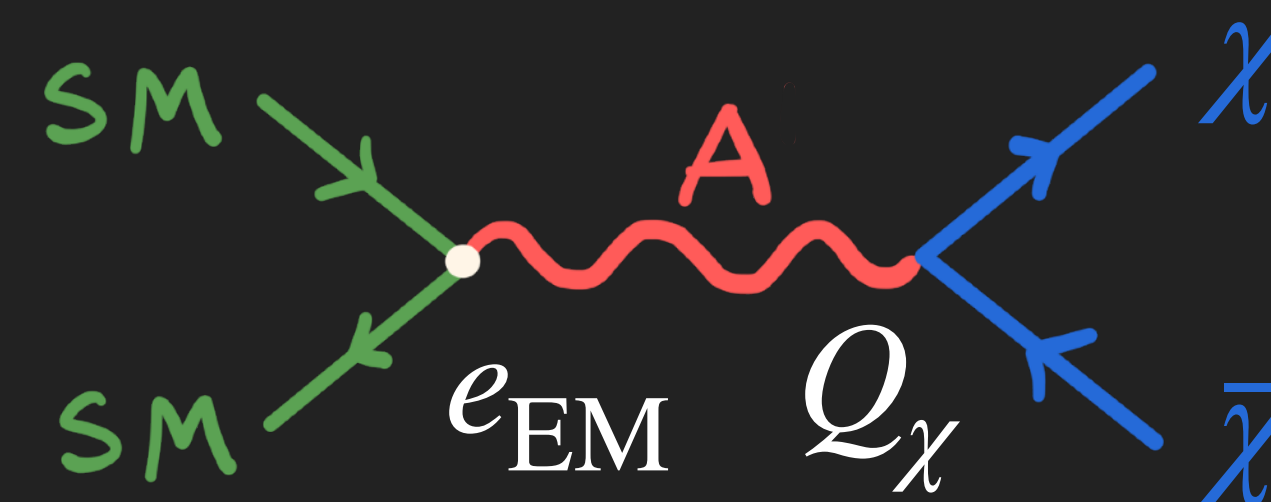
- ▶ Time evolution for energy density:

$$\dot{\rho}_{\text{DM}} = -3H (\rho_{\text{DM}} + P_{\text{DM}}) + Q_{\text{DM}},$$

$$\dot{\rho}_{\gamma} = -3H (\rho_{\gamma} + P_{\gamma}) + Q_{\gamma}.$$

$$Q_{\text{DM}} = \Gamma(\rho_{\gamma}),$$

$$Q_{\gamma} = -\Gamma(\rho_{\gamma}).$$



- ▶ Derive evolution equation for curvature perturbations on *large scales*
- ▶ Freeze-in relevant around  $T_{\text{SM}} \sim m_{\chi} \gtrsim \text{MeV}$ , way before recombination ( $\sim \text{eV}$ ), and then shuts off

$$P_{\text{DM}} = P_{\text{DM}}(\rho_{\text{DM}}) \rightsquigarrow P_{\text{DM}}(T_{\text{SM}}) \quad \text{DM pressure}$$

$$Q_{\text{DM}} = Q_{\text{DM}}(\rho_{\text{SM}}) \rightsquigarrow Q_{\text{DM}}(T_{\text{SM}}) \quad \text{Energy transfer rate}$$

- ▶ Single-clock argument:  $T_{\text{SM}}$  only source of perturbations here



- ▶ Final result:

$$\Gamma = \langle T_{\text{SM}} \sigma_{\text{ann}} v_{\text{rel}} \rangle n_{\text{SM}}^2$$

$$\dot{\mathcal{S}}_{\text{DM},\gamma} = \frac{3\dot{\rho}}{\dot{\rho}_{\text{DM}}^2} \left( \frac{\dot{\rho}_{\gamma}^2 - \dot{\rho}_{\text{DM}}^2}{\dot{\rho}_{\gamma}} \frac{\Gamma}{2\rho} - \dot{\Gamma} \right) (\zeta - \zeta_{\gamma}) \propto \mathcal{S}_{\text{DM},\gamma}$$

- ▶ Isocurvature can be only sourced by itself, **and only if  $\Gamma \neq 0$**
- ▶ It is exactly zero on large scales, so it remains zero:

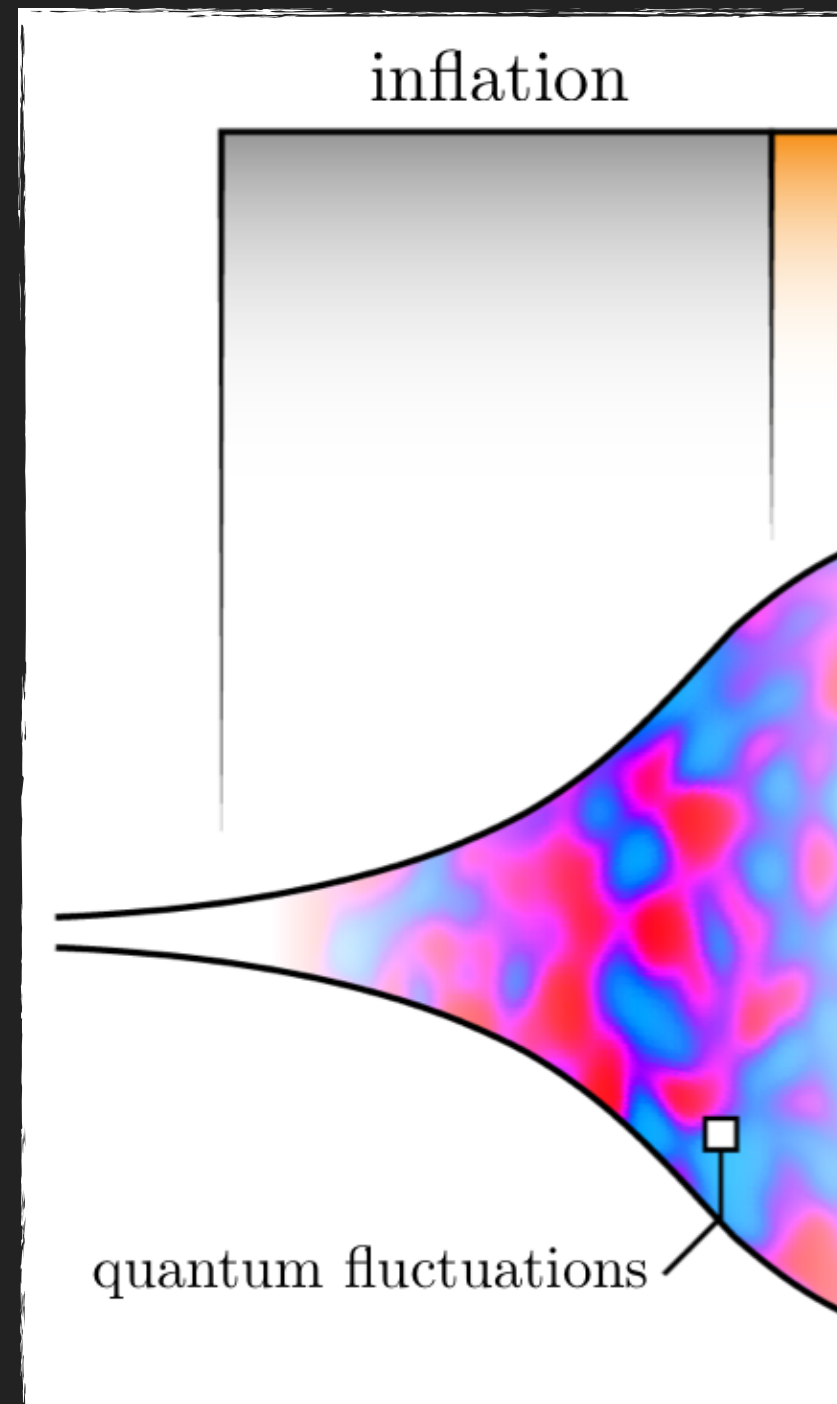
$$\zeta(\mathbf{x}, t \ll t_{\text{F-IN}}) = \zeta_{\gamma}(\mathbf{x}, t \ll t_{\text{F-IN}}) \quad \mathcal{S}_{\text{DM},\gamma}(\mathbf{x}, t \gg t_{\text{F-IN}}) = 0$$

- ▶ Large gap of scales ( $> 10^6$ ) between horizon and freeze-in and CMB scales

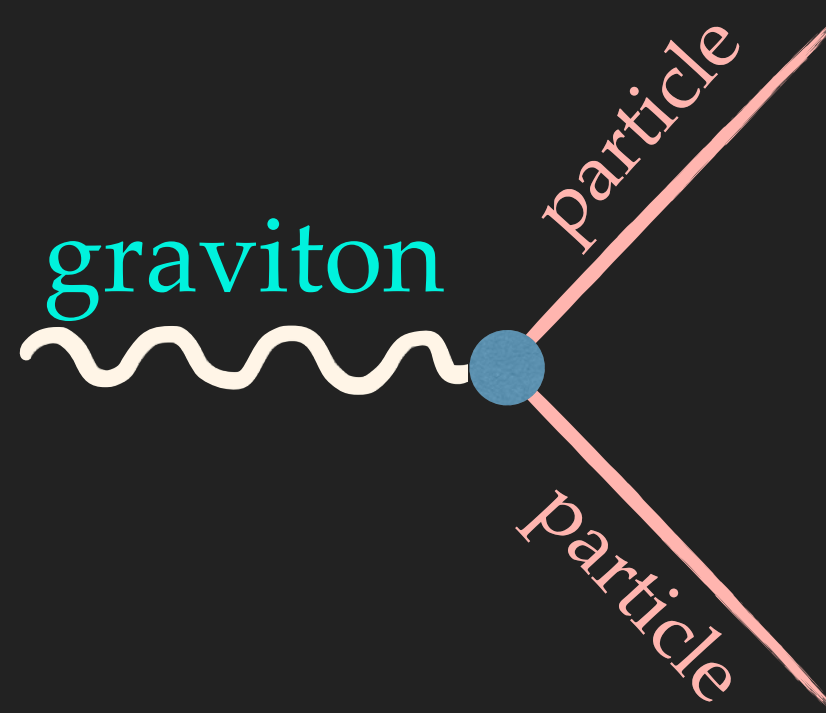


# GRAVITATIONAL PRODUCTION (1)

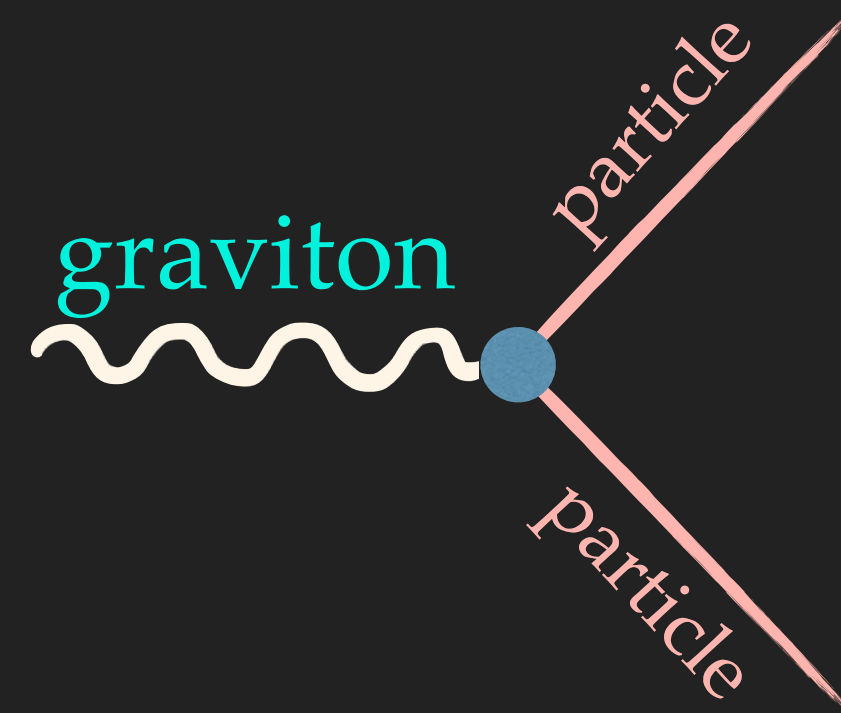
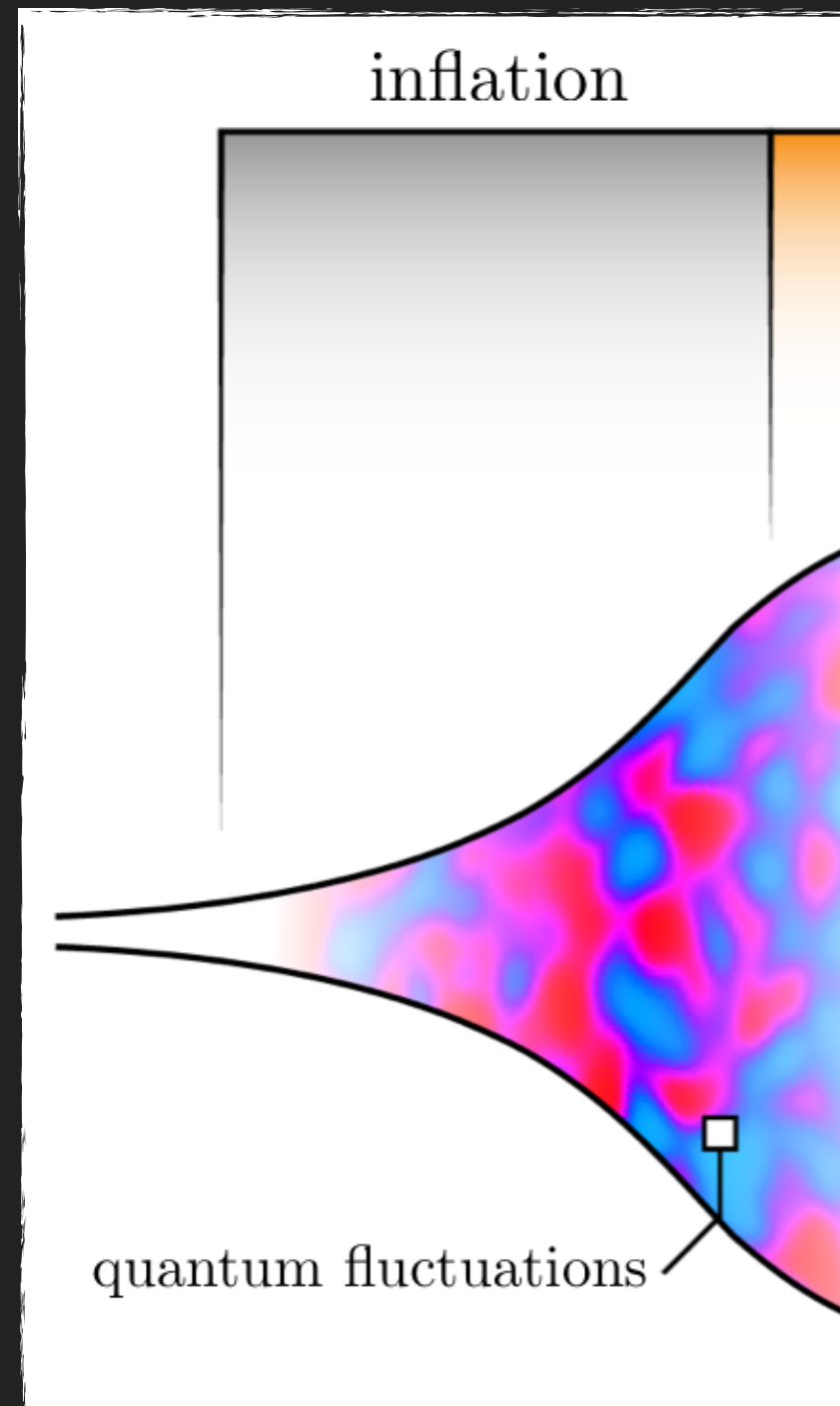
[ '39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; ... ]<sup>11</sup>



Oscillator with  $\omega(t)$  → level crossing  
time-dependent  $\omega_k(t)$  in expanding Universe → from initial vacuum to non-vacuum later







Oscillator with  $\omega(t)$   $\longrightarrow$  level crossing  
 time-dependent  $\omega_k(t)$  in expanding Universe  $\longrightarrow$  from initial vacuum to non-vacuum later

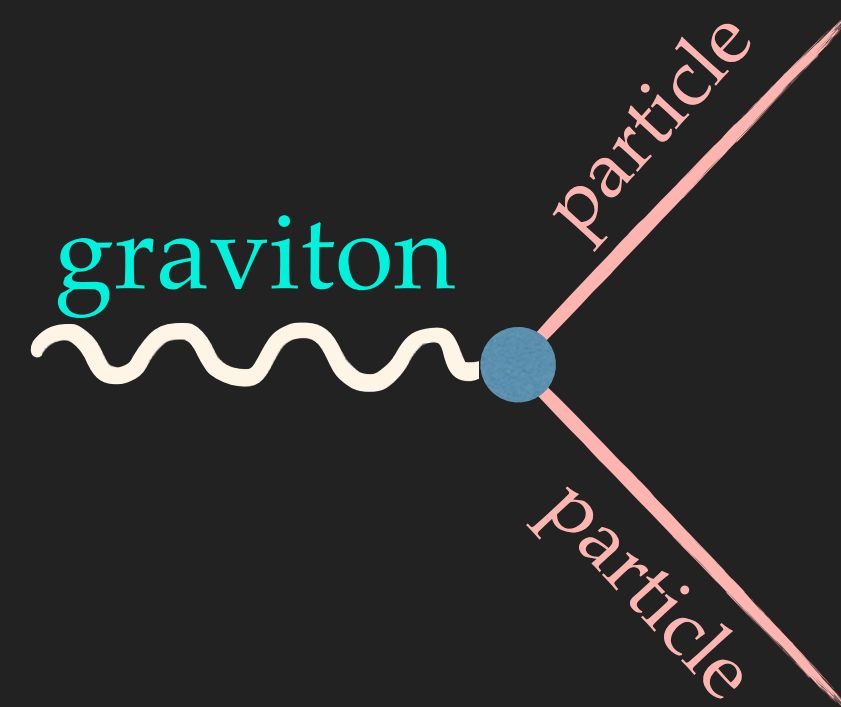
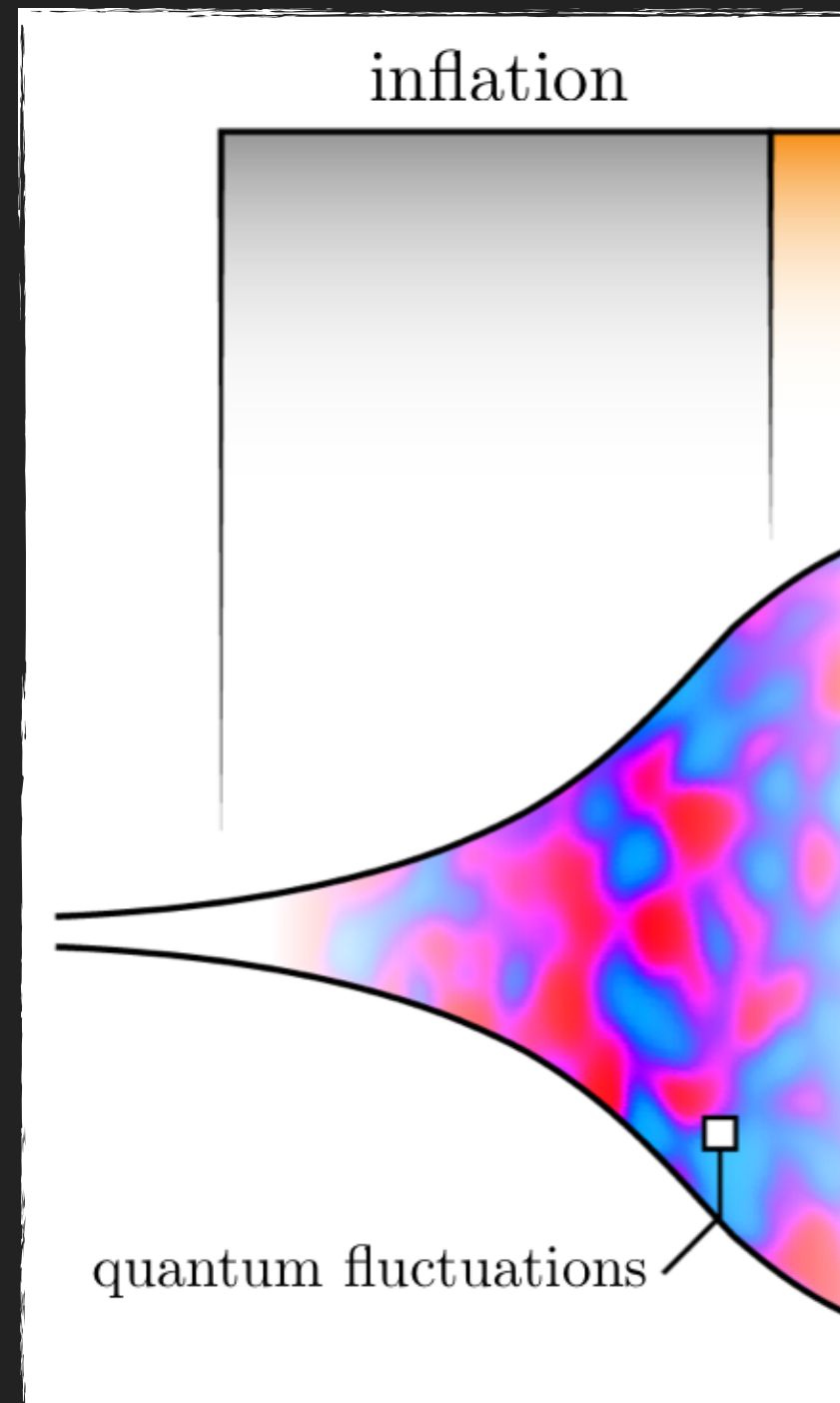
$$\ddot{u}_k(t) + \omega_k^2 u_k(t) = 0$$

$$u_k(t) = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t}$$

$$\phi \sim \int \left( a_k u_k + a_k^\dagger u_k^* \right)$$

$$a_k |0\rangle = 0 \quad \forall k$$





Oscillator with  $\omega(t)$   $\longrightarrow$  level crossing  
 time-dependent  $\omega_k(t)$  in expanding Universe  $\longrightarrow$  from initial vacuum to non-vacuum later

$$\ddot{u}_k(t) + \omega_k^2 u_k(t) = 0$$

$$u_k(t) = \frac{1}{\sqrt{\omega_k}} e^{-i\omega_k t}$$

$$\phi \sim \int \left( a_k u_k + a_k^\dagger u_k^* \right)$$

$$a_k |0\rangle = 0 \quad \forall k$$

$$\ddot{u}_k(t) + \omega_k^2(t) u_k(t) = 0$$

$$u_k(t) \approx \frac{1}{\sqrt{\omega_k(t)}} e^{-i \int^t \omega_k(t') dt'}$$

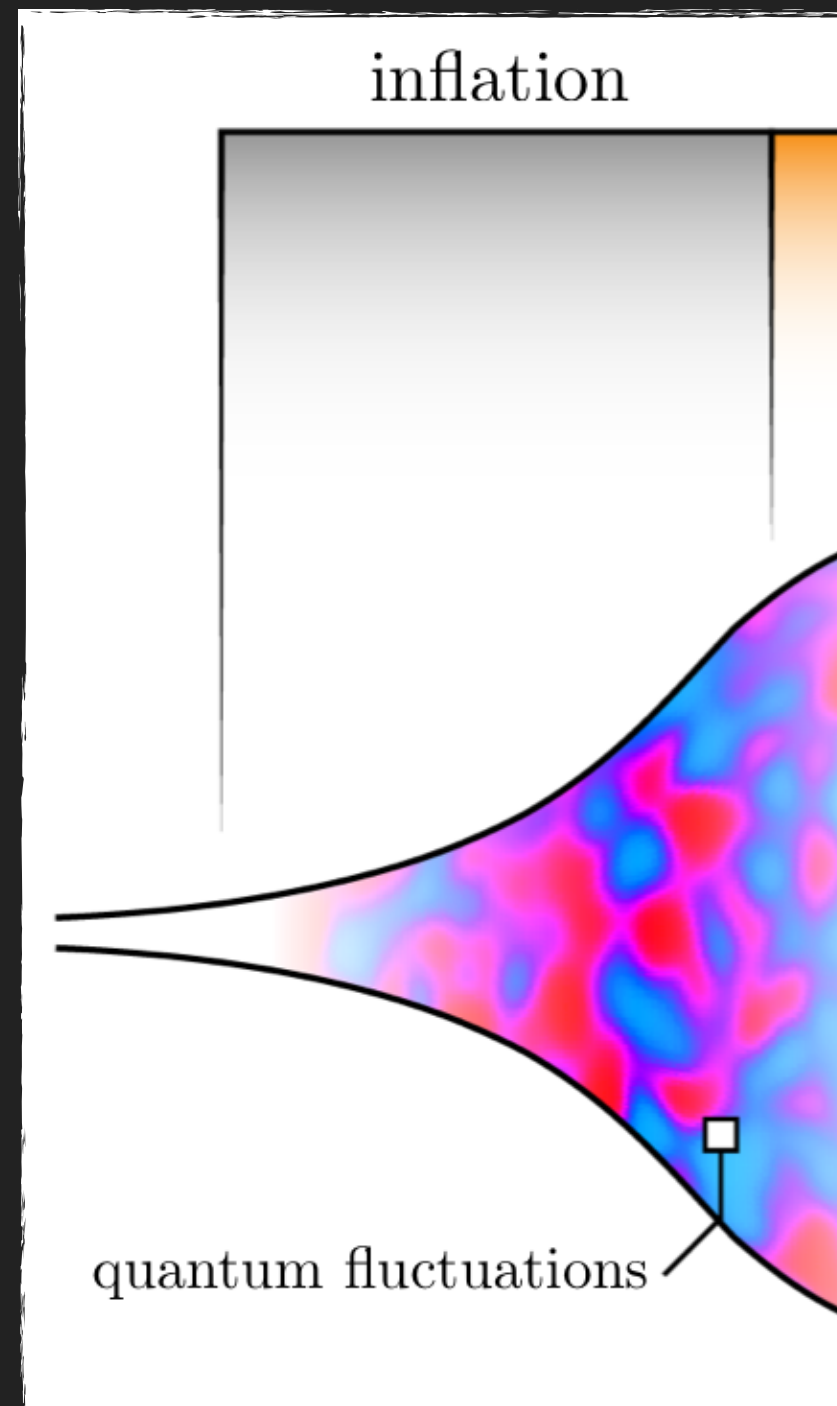
$$\phi \sim \int \left( a_k^{(\text{out})} u_k^{(\text{out})} + a_k^{\dagger(\text{out})} u_k^{*(\text{out})} \right)$$

$$a_k^{(\text{in})} \neq a_k^{(\text{out})} \implies |0^{(\text{in})}\rangle \neq |0^{(\text{out})}\rangle$$

Particle production!

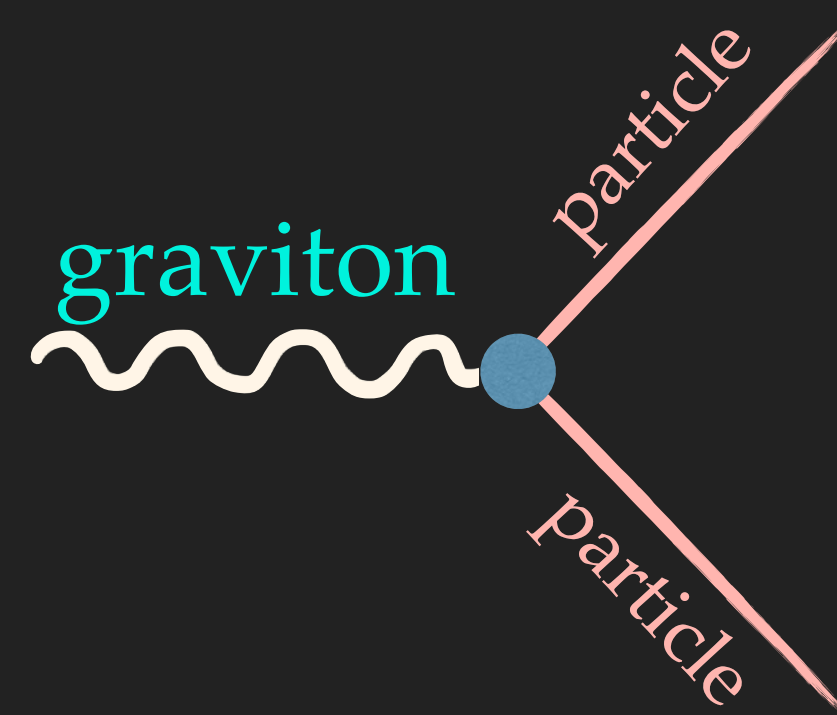
$\omega_k(t)$  : mass term, ...

[ '39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; ... ]



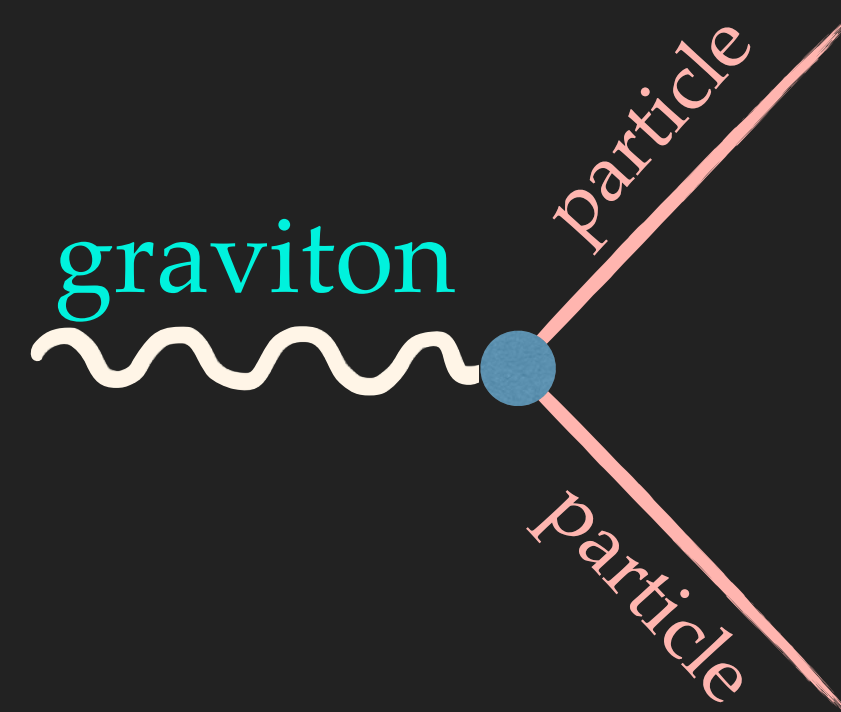
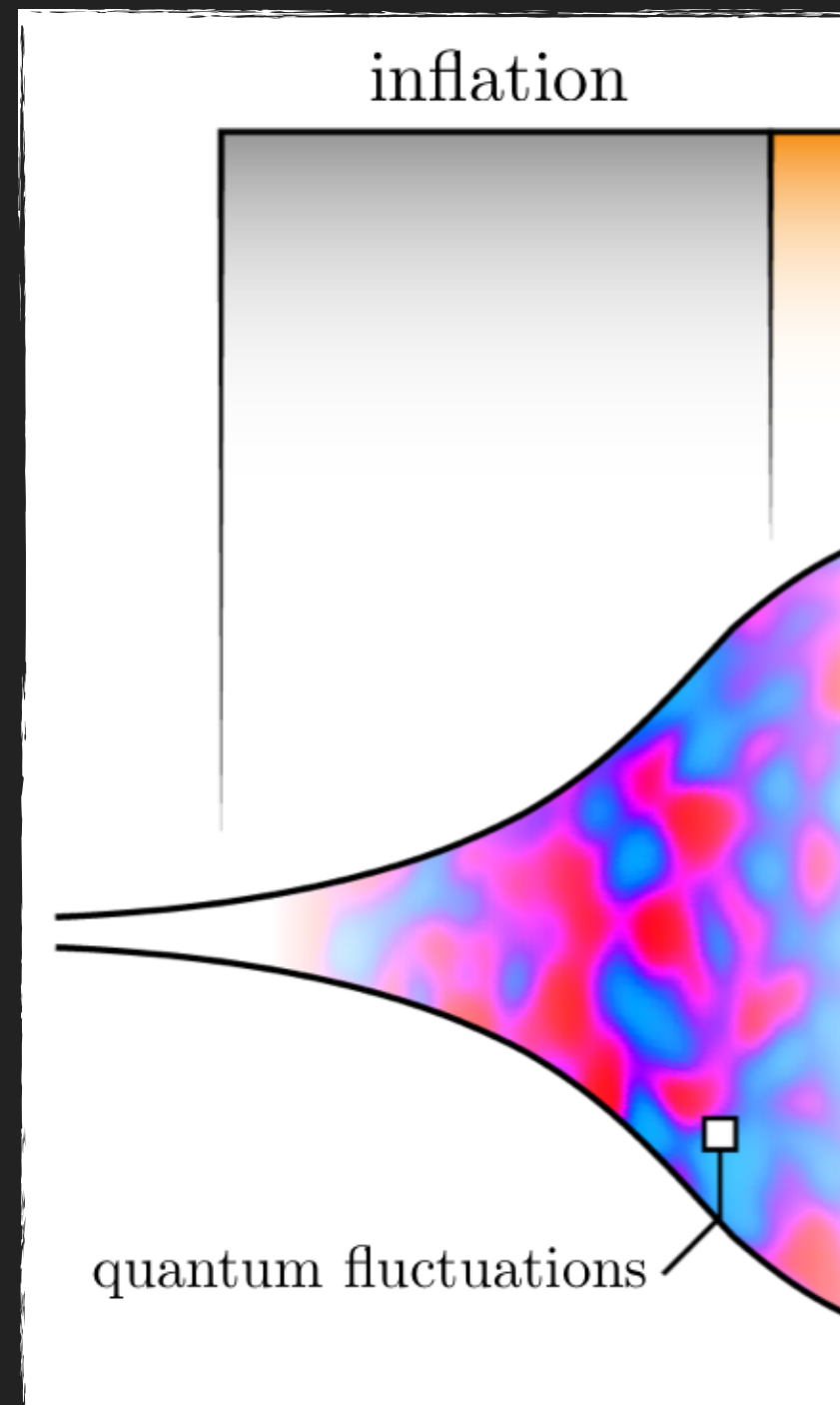
Time-varying bkg  $\rightarrow \omega_k(t) \rightarrow$  particle production

Violation of scale invariance  $\rightarrow$  time-dependent equations





[ '39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; ... ]



Time-varying bkg  $\rightarrow \omega_k(t) \rightarrow$  particle production

Violation of scale invariance  $\rightarrow$  time-dependent equations

Scale transformation:  $g_{\mu\nu} \rightarrow \lambda^2 g_{\mu\nu}, \phi \rightarrow \lambda^\# \phi$

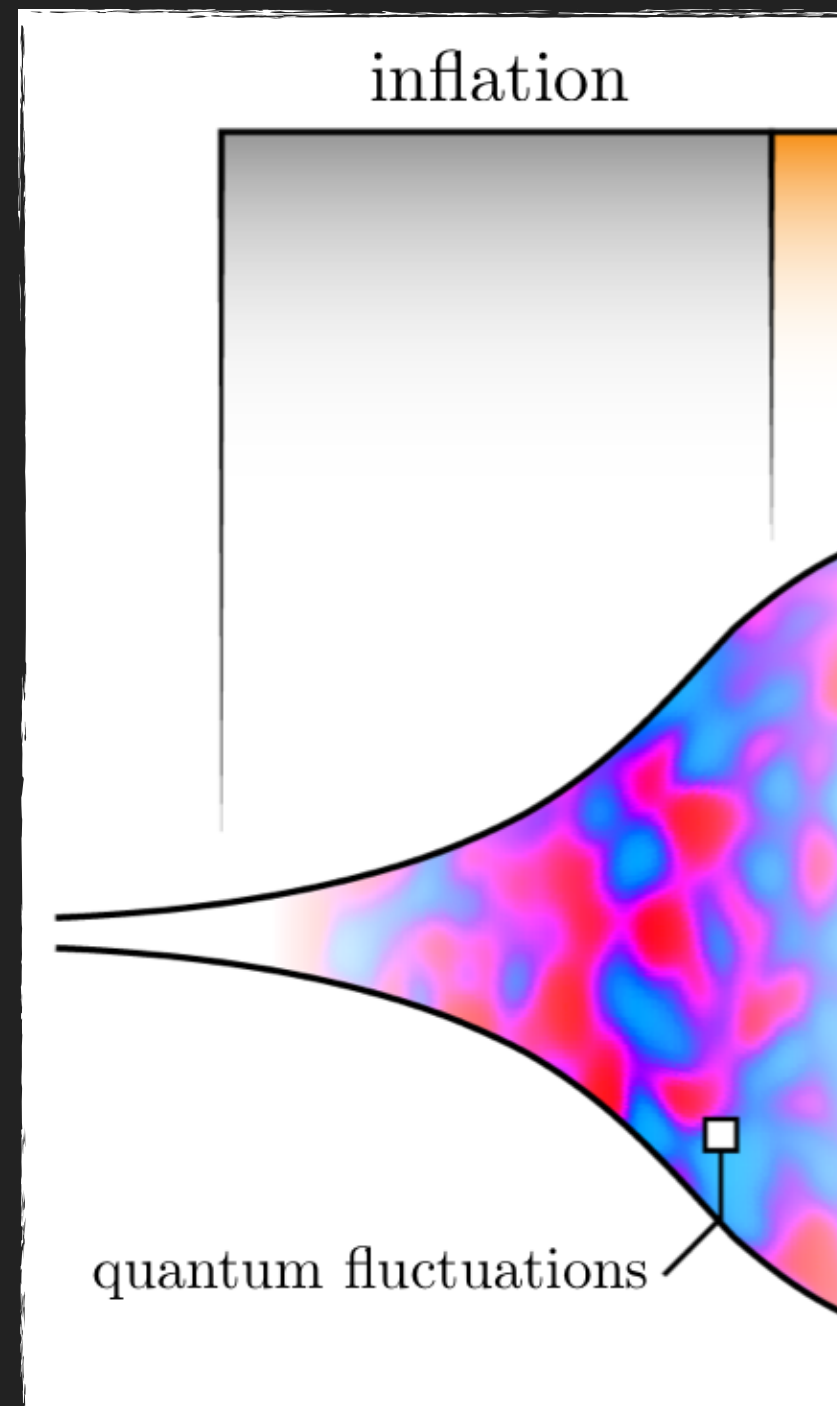
Irreducible time dependence in equations of motion

Scale invariance:  $T^\mu_\mu = 0$ , broken e.g. by mass

Coupling matter-gravity:  
 $\mathcal{L} = h_{\mu\nu} T^{\mu\nu}$



[ '39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; ... ]

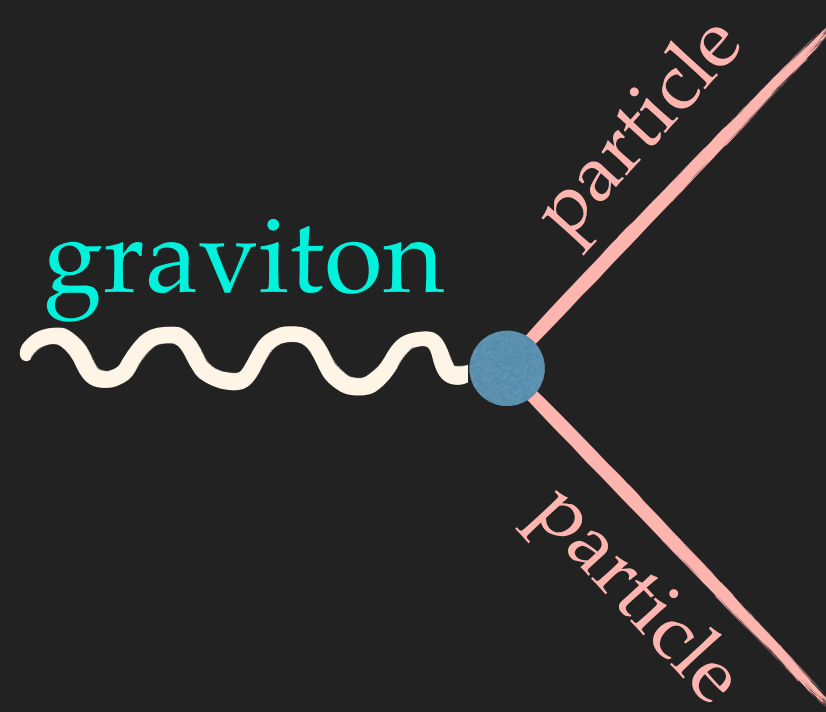


Time-varying bkg  $\rightarrow \omega_k(t) \rightarrow$  particle production

Violation of scale invariance  $\rightarrow$  time-dependent equations

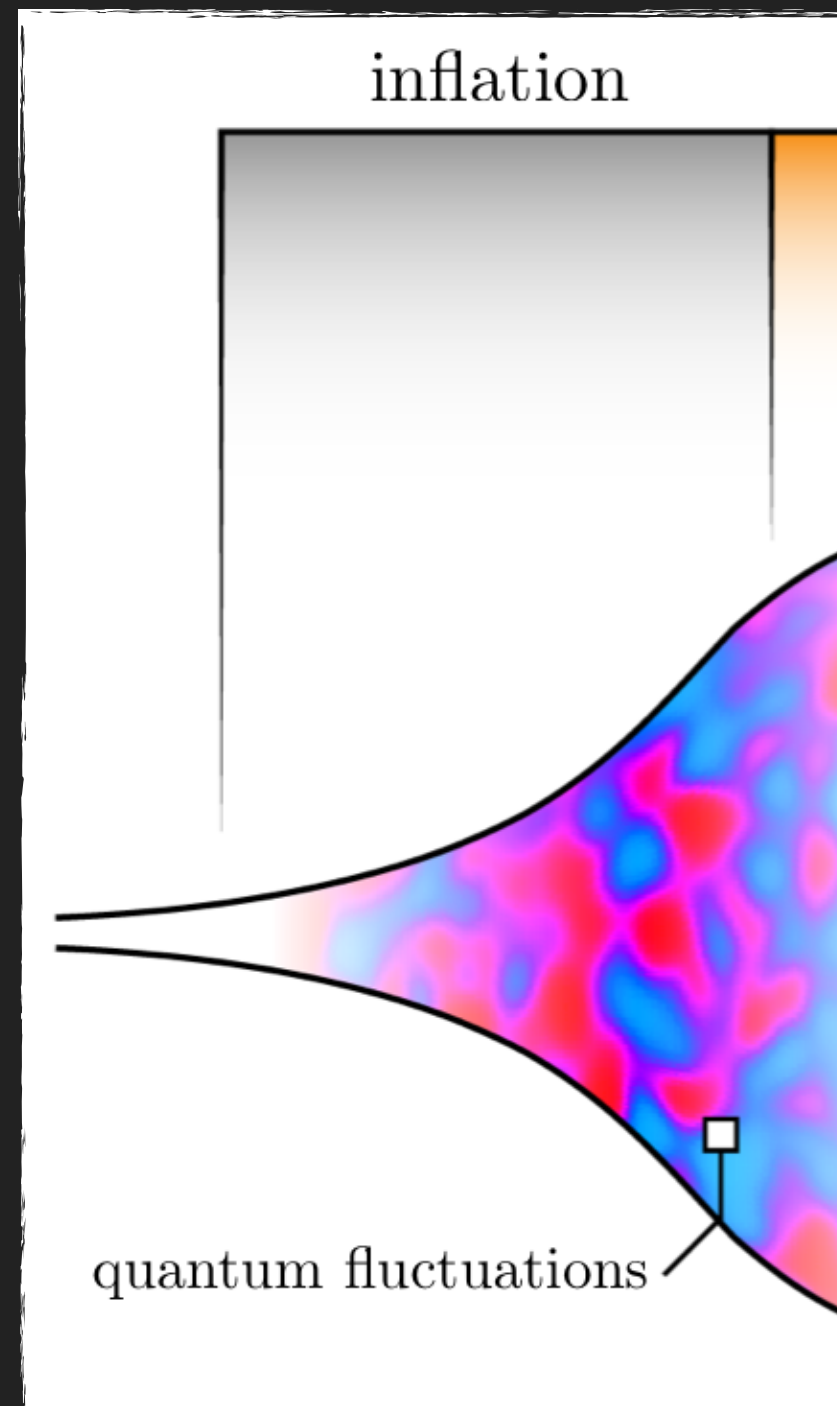
Inflationary de-Sitter  $\approx$  "bath" at "temperature"  $T_{\text{dS}} \sim \frac{H_I}{2\pi}$

$$\rho_{k,\text{exit}}^{(\text{scalar})} \sim H_I^4$$





[ '39 Schrödinger; '69 Parker; '77 Gibbons, Hawking; '79 Birrell, Davies; '87 Ford; ... ]

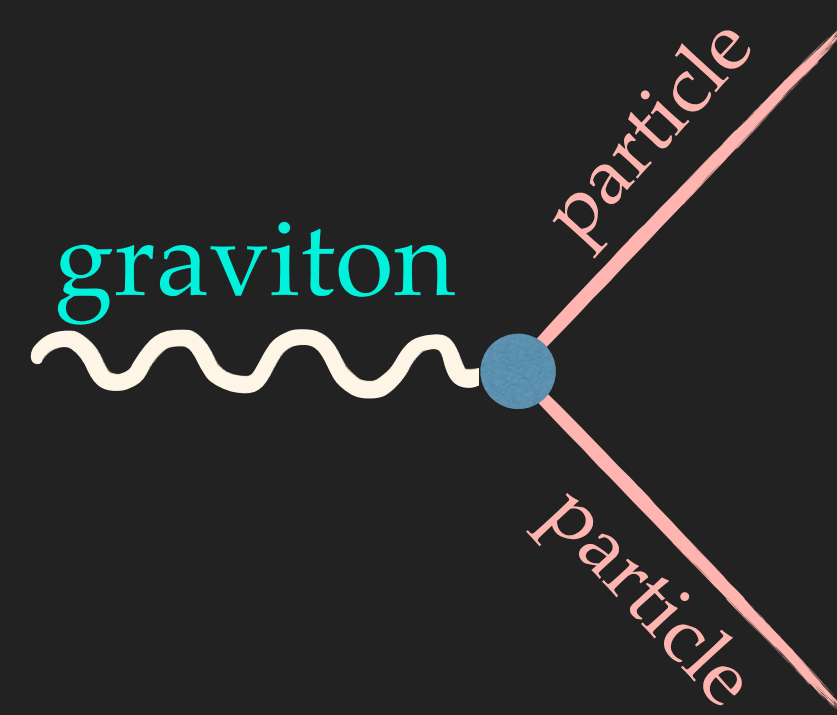


Time-varying bkg  $\rightarrow \omega_k(t) \rightarrow$  particle production

Violation of scale invariance  $\rightarrow$  time-dependent equations

Inflationary de-Sitter  $\approx$  "bath" at "temperature"  $T_{\text{dS}} \sim \frac{H_I}{2\pi}$

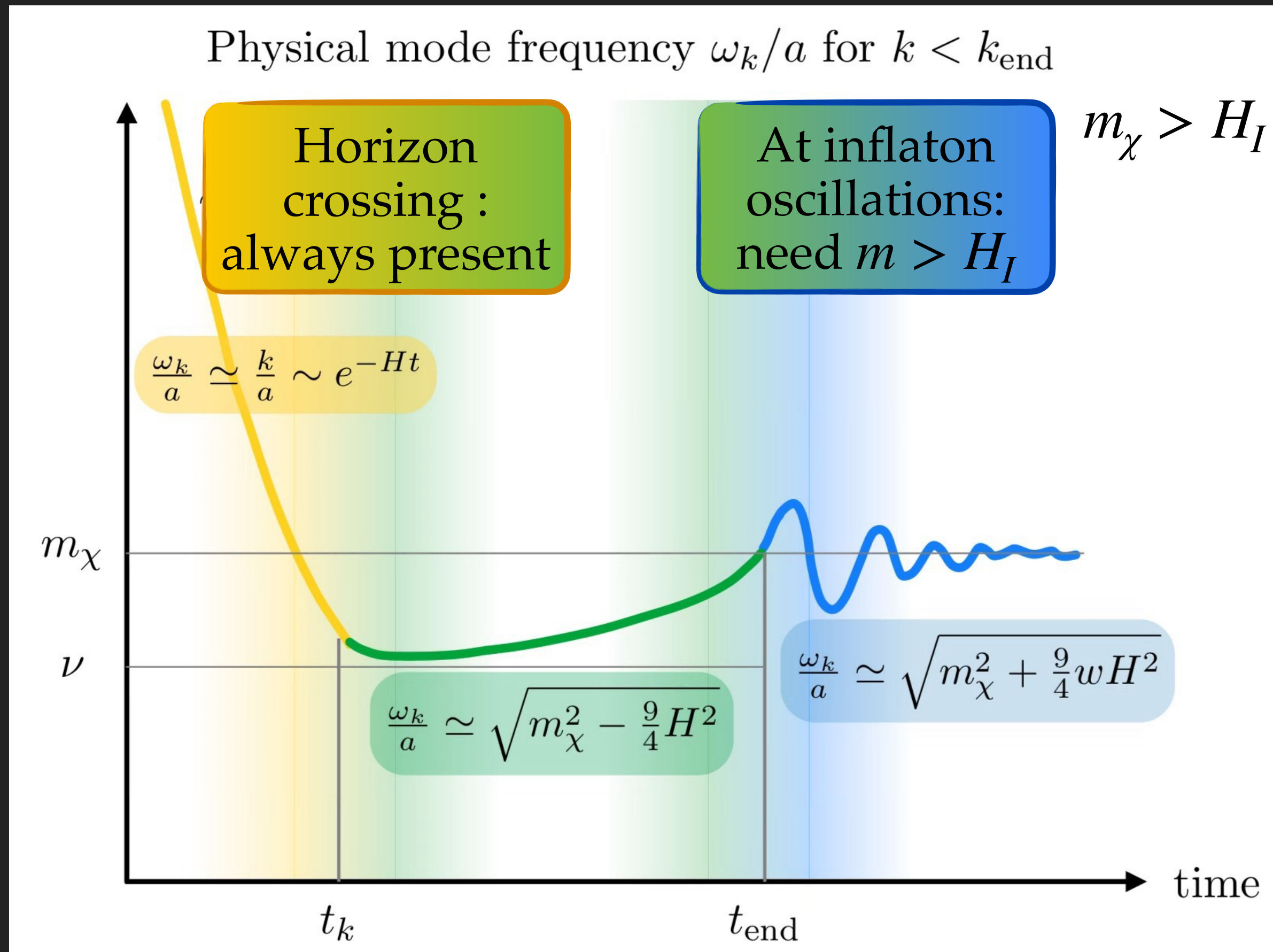
$$\rho_{k,\text{exit}}^{(\text{scalar})} \sim H_I^4$$



► Useful, but only analogy, not a strict equivalence

$$\frac{\omega_k(t)}{a(t)} = \sqrt{\frac{k^2}{a(t)^2} + m^2 + \frac{9}{4}w(t)H(t)^2}$$

[’24 DR, Verner, Xue]



[’98 Chung, Kolb, Riotto; ’99 Kofman, Linde, Starobinsky; ’18 Chung, Kolb, Long; ’19 Li, Nakama, Sou, Wang, Zhou; ’21 Ling, Long; ’23 Brandenberger, Kamali, Ramos; ...]

► Time-dependent  $\omega_k(t)$ :

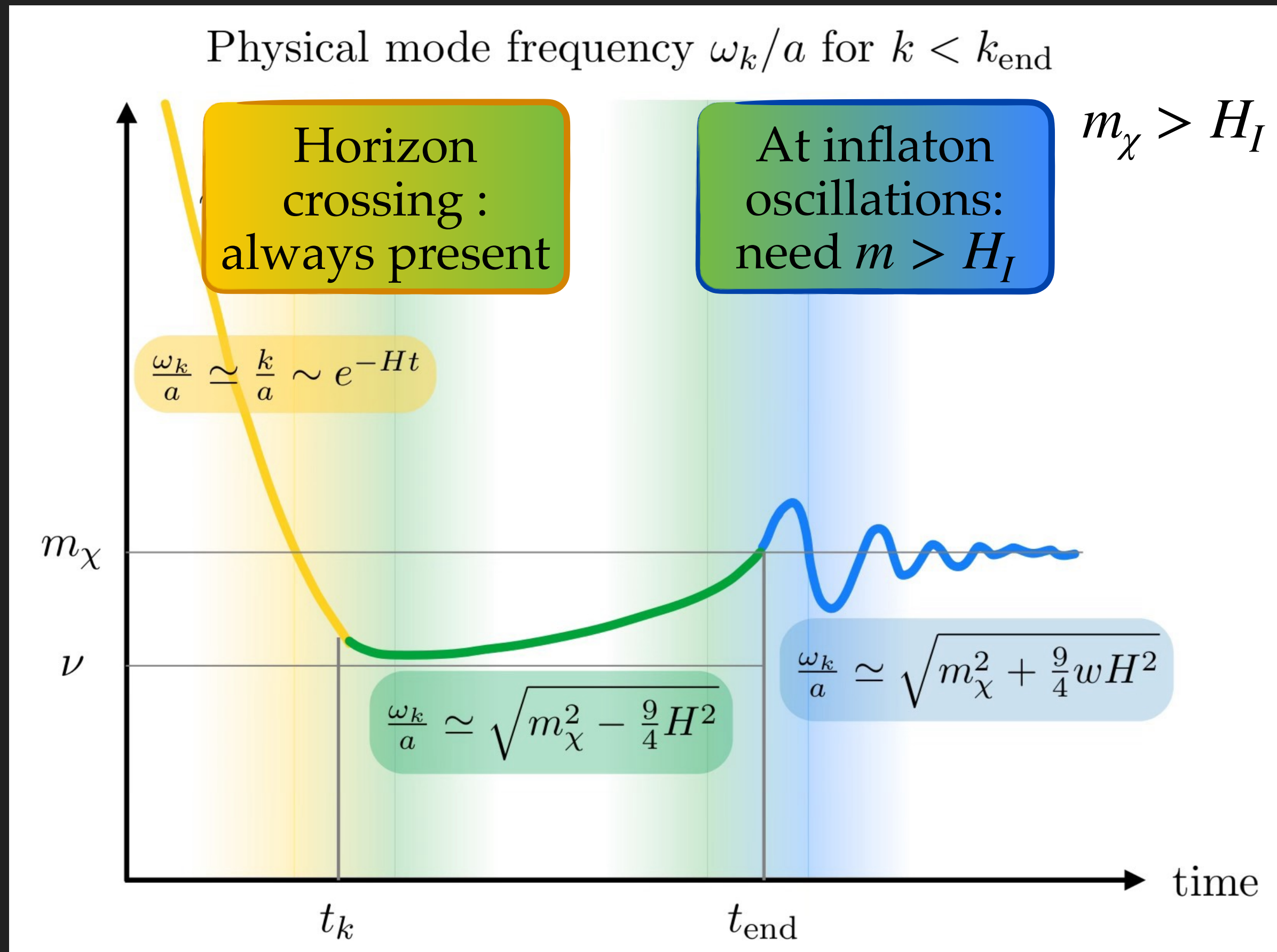
$$n_{k,(\text{late times})} \sim k^3 |\beta_k|^2 \quad |0_k\rangle^{(\text{in})} \neq |0_k\rangle^{(\text{out})}$$

$$\beta_k = \int_{t_i}^t dt' \frac{1}{2} \frac{\dot{\omega}_k}{\omega_k} \exp\left(-2i \int_{t_i}^{t'} \frac{\omega_k}{a} dt''\right)$$



$$\frac{\omega_k(t)}{a(t)} = \sqrt{\frac{k^2}{a(t)^2} + m^2 + \frac{9}{4}w(t)H(t)^2}$$

['24 DR, Verner, Xue]



['98 Chung, Kolb, Riotto; '99 Kofman, Linde, Starobinsky; '18 Chung, Kolb, Long; '19 Li, Nakama, Sou, Wang, Zhou; '21 Ling, Long; '23 Brandenberger, Kamali, Ramos; ...]

► Time-dependent  $\omega_k(t)$ :

$$n_{k,(\text{late times})} \sim k^3 |\beta_k|^2 \quad |0_k\rangle^{(\text{in})} \neq |0_k\rangle^{(\text{out})}$$

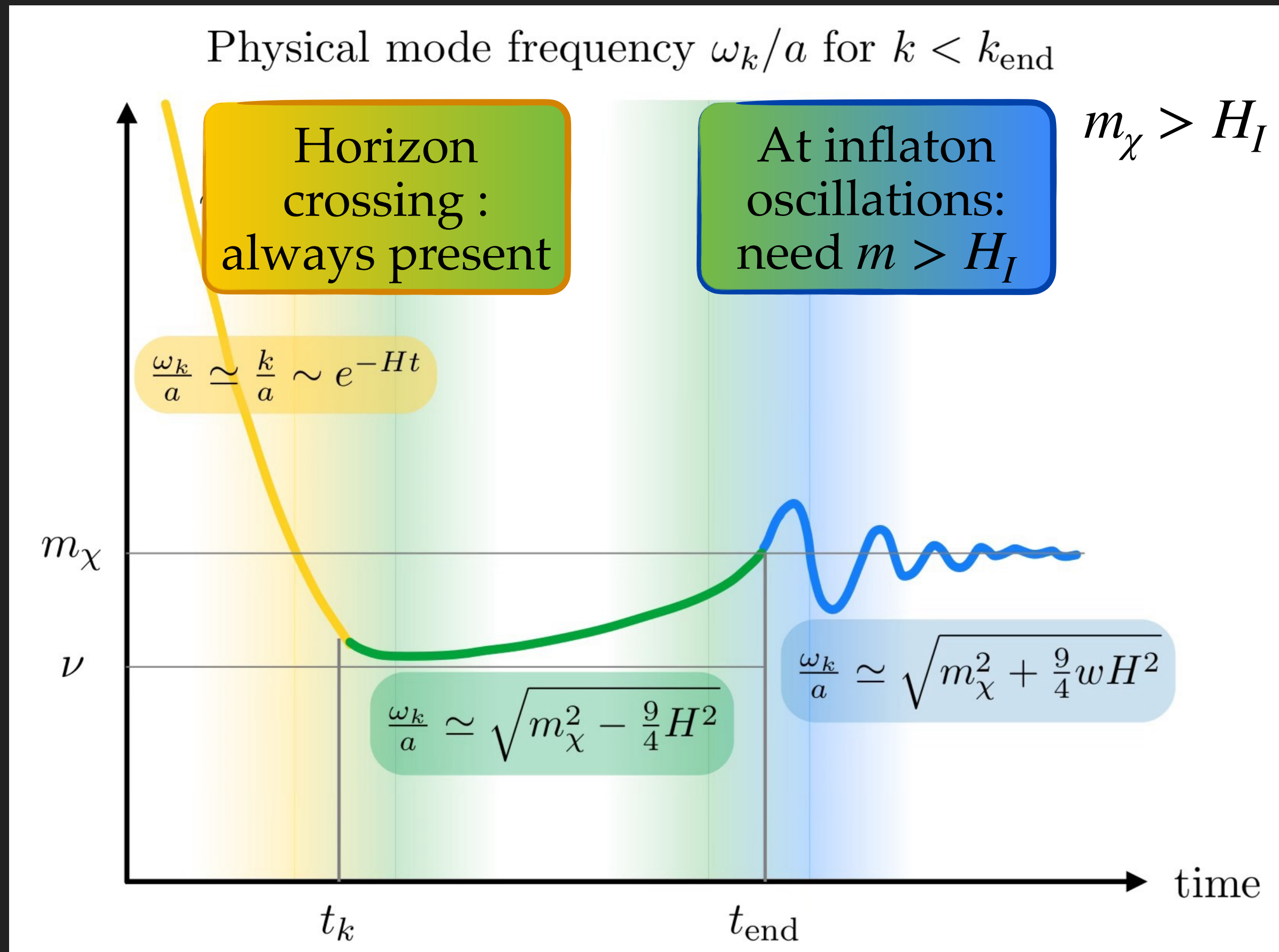
$$\beta_k = \int_{t_i}^t dt' \frac{1}{2} \frac{\dot{\omega}_k}{\omega_k} \exp\left(-2i \int_{t_i}^{t'} \frac{\omega_k}{a} dt''\right)$$

► Non-adiabaticity  $\dot{\omega}_k/\omega_k^2$  : larger at

- Hubble crossing (*model independent*)
- heavy  $m > H_I$  : after end of inflation (*depends on preheating*)

$$\frac{\omega_k(t)}{a(t)} = \sqrt{\frac{k^2}{a(t)^2} + m^2 + \frac{9}{4}w(t)H(t)^2}$$

['24 DR, Verner, Xue]



['98 Chung, Kolb, Riotto; '99 Kofman, Linde, Starobinsky; '18 Chung, Kolb, Long; '19 Li, Nakama, Sou, Wang, Zhou; '21 Ling, Long; '23 Brandenberger, Kamali, Ramos; ...]

▶ Time-dependent  $\omega_k(t)$ :

$$n_{k,(\text{late times})} \sim k^3 |\beta_k|^2 \quad |0_k\rangle^{(\text{in})} \neq |0_k\rangle^{(\text{out})}$$

$$\beta_k = \int_{t_i}^t dt' \frac{1}{2} \frac{\dot{\omega}_k}{\omega_k} \exp\left(-2i \int_{t_i}^{t'} \frac{\omega_k}{a} dt''\right)$$

▶ Non-adiabaticity  $\dot{\omega}_k/\omega_k^2$  : larger at

- ▶ Hubble crossing (*model independent*)
- ▶ heavy  $m > H_I$  : after end of inflation (*depends on preheating*)

▶ Phase  $\exp(i \int \omega dt)$

- ▶  $e^{2ik\eta}$  at early times
- ▶ heavy  $m > H_I$  : rapid phase  $\rightarrow$  saddle appr.  $\rightarrow n_k \sim \exp(-\pi m/H_I)$



**Gravitational production**

## Gravitational production



Single  
Species

Massive vector  $A'$

[<sup>'15</sup> Graham, Mardon, Rajendran]

Fermion  $\psi$

[<sup>'99</sup> Kuzmin, Tkachev]

[<sup>'11</sup> Chung, Everett, Yoo, Zhou]



## Gravitational production



Single Species

Massive vector  $A'$

[ '15 Graham, Mardon, Rajendran ]

Fermion  $\psi$

[ '99 Kuzmin, Tkachev ]

[ '11 Chung, Everett, Yoo, Zhou ]

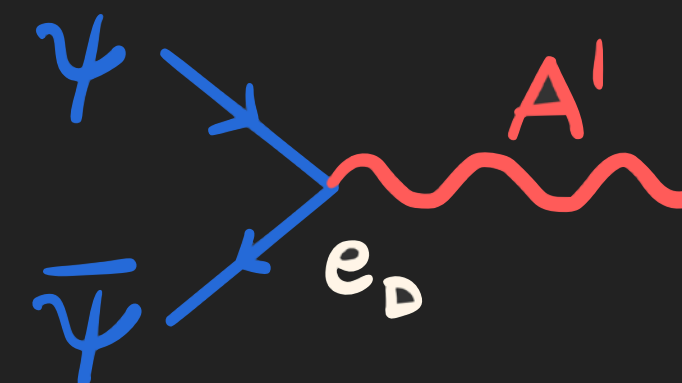
Dark Sector

[ '21 Arvanitaki, Dimopoulos, Galanis, DR, Simon, Thompson ]

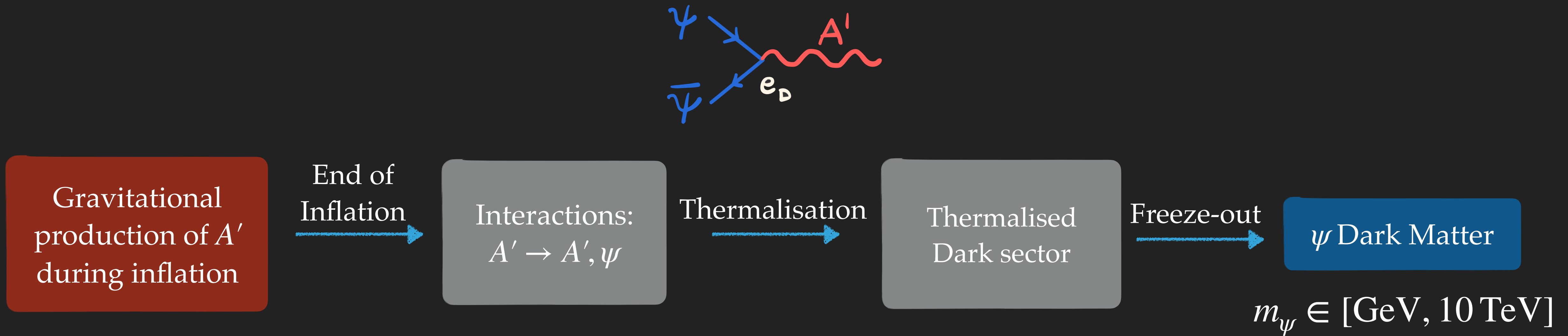
Massive Dark QED

$\psi$  dark matter

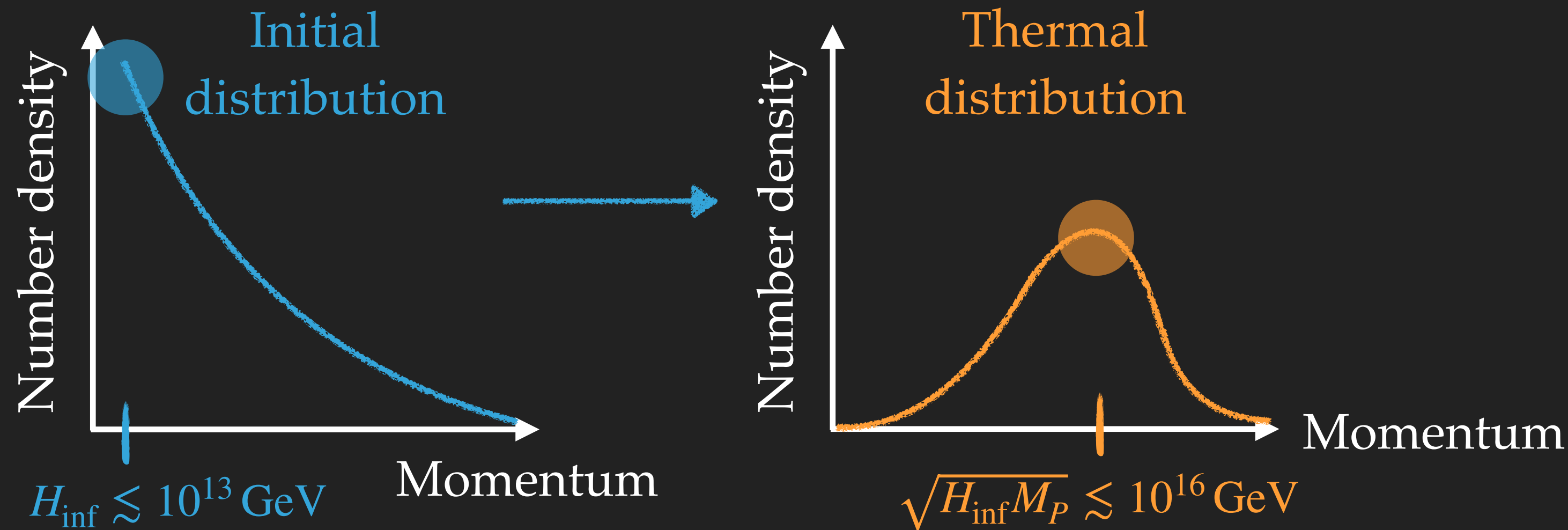
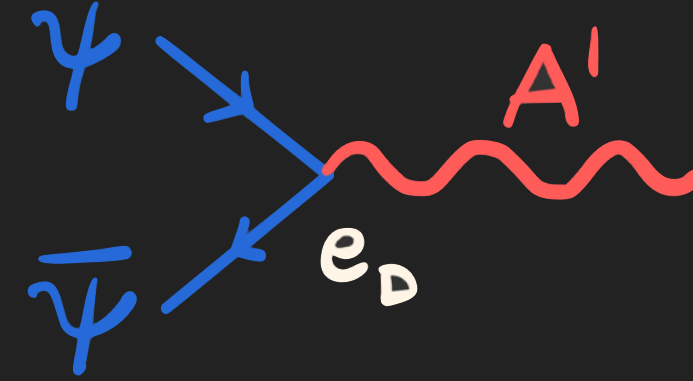
$A'$  grav. prod.



- Complexity and thermalisation in the dark sector change the Dark Matter target

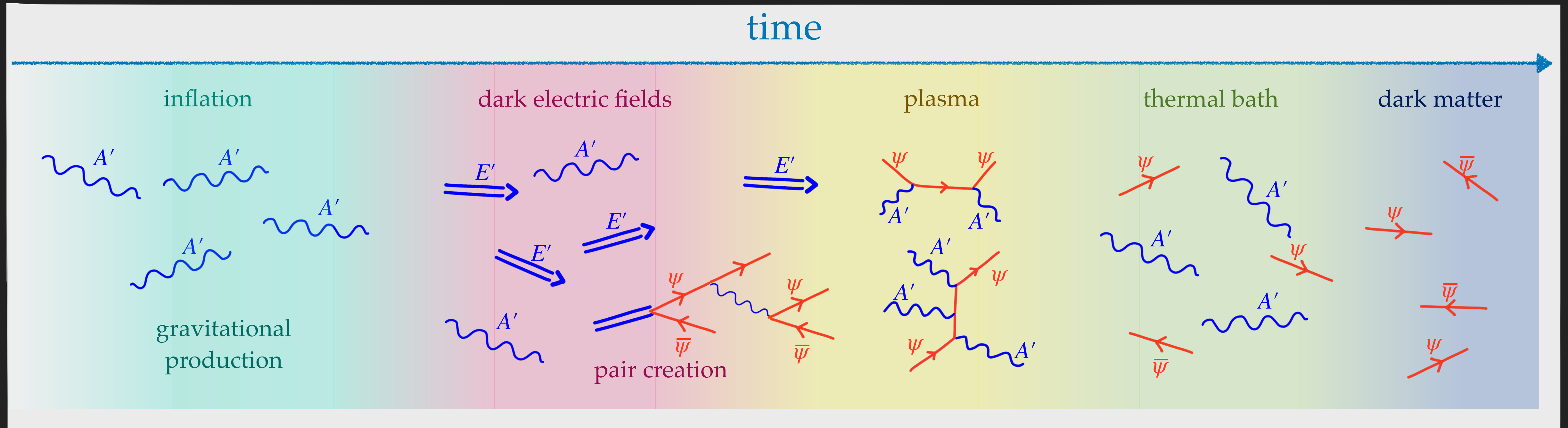




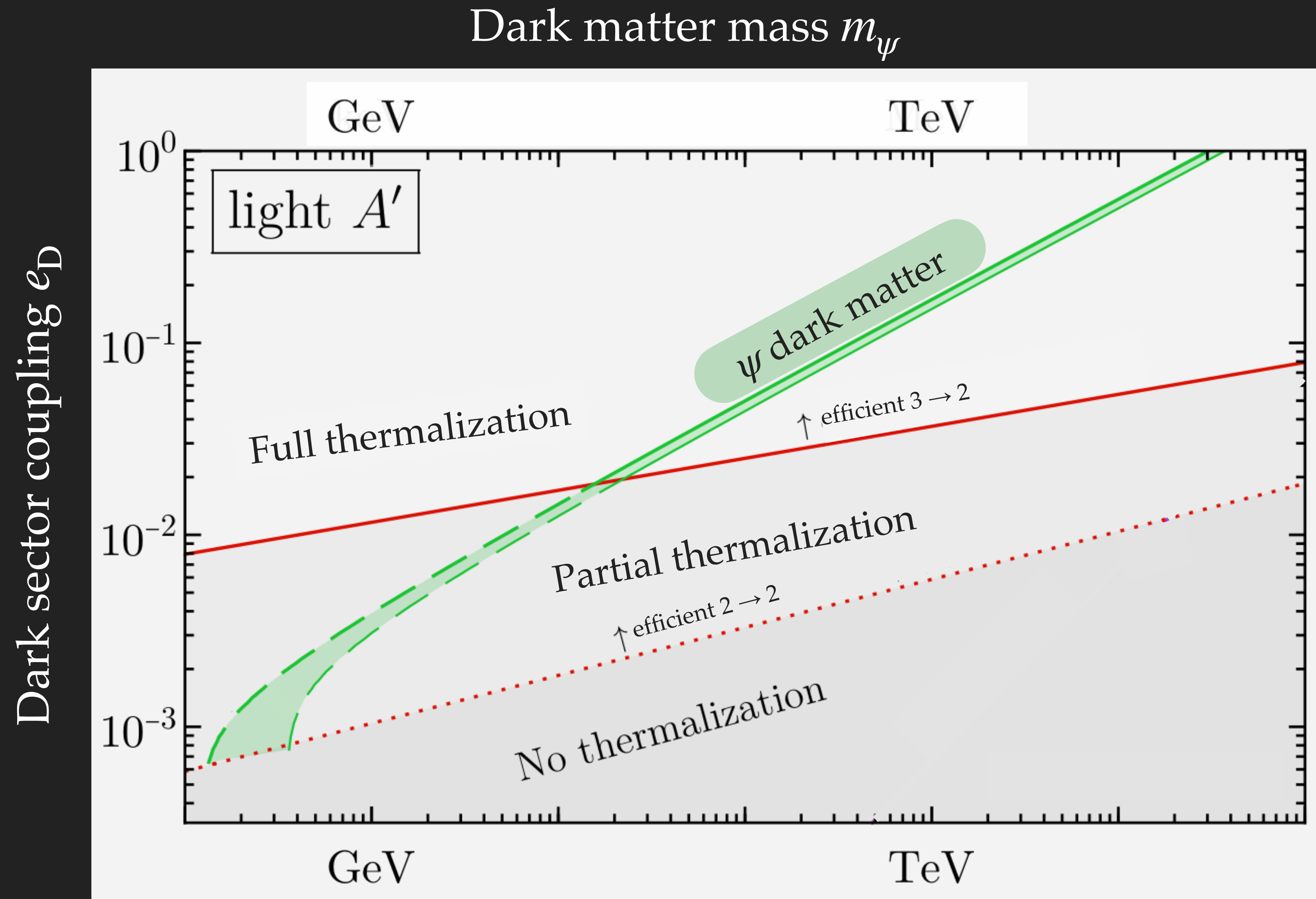


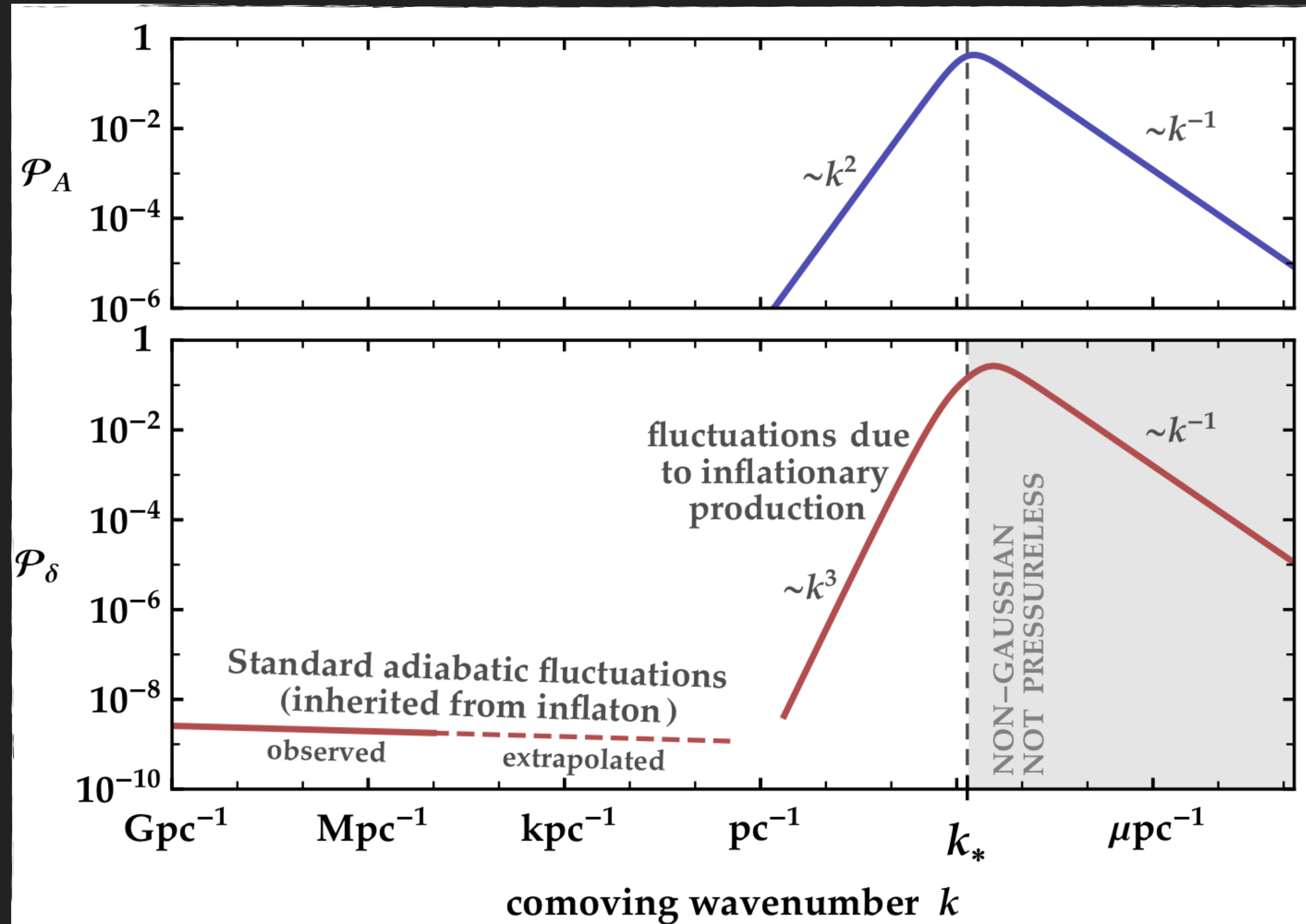
Universality:

$$T_{\text{DM}} \sim \sqrt{\frac{H_{\text{inf}}}{M_P}} T_{\text{SM}}$$



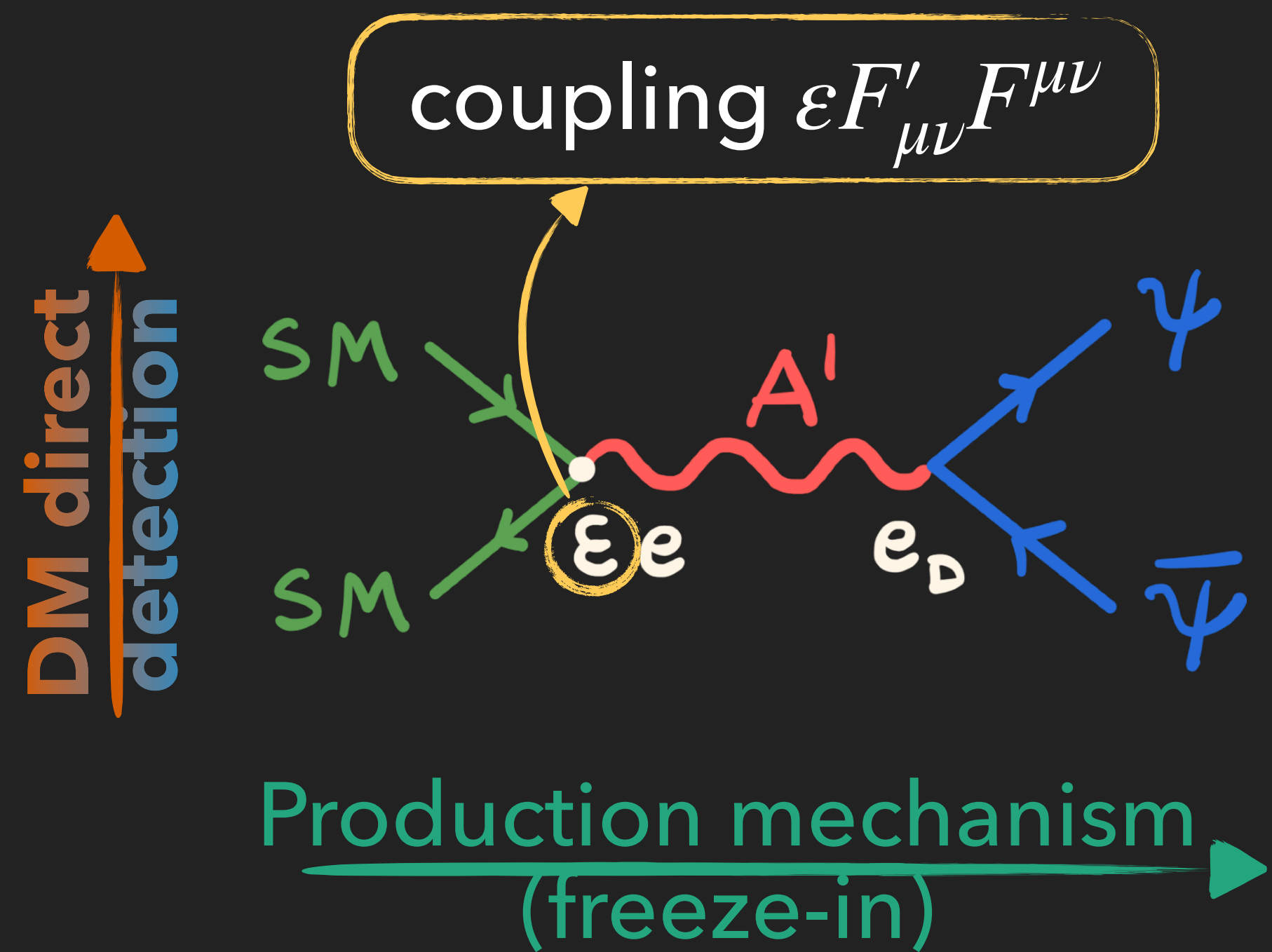


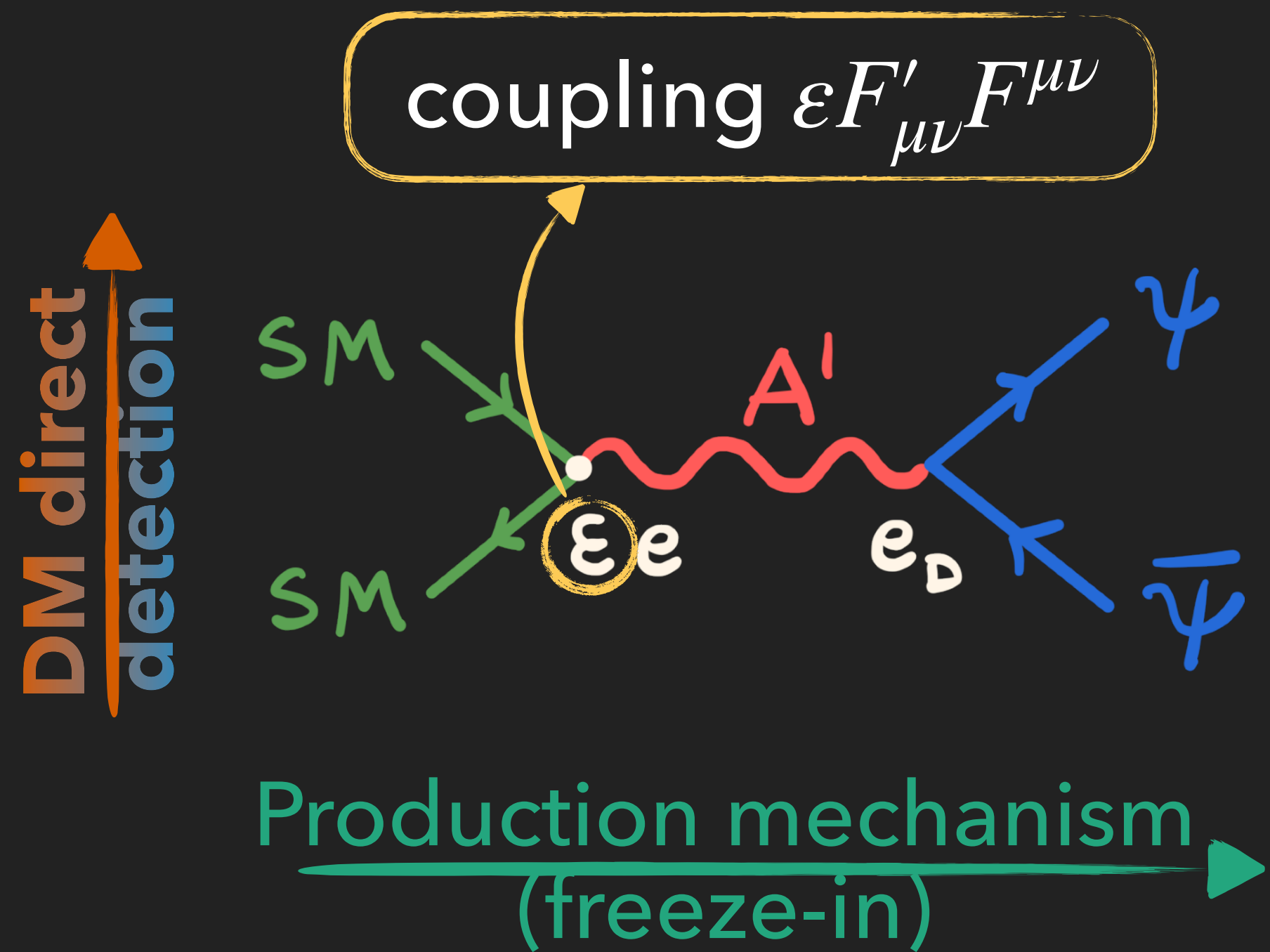




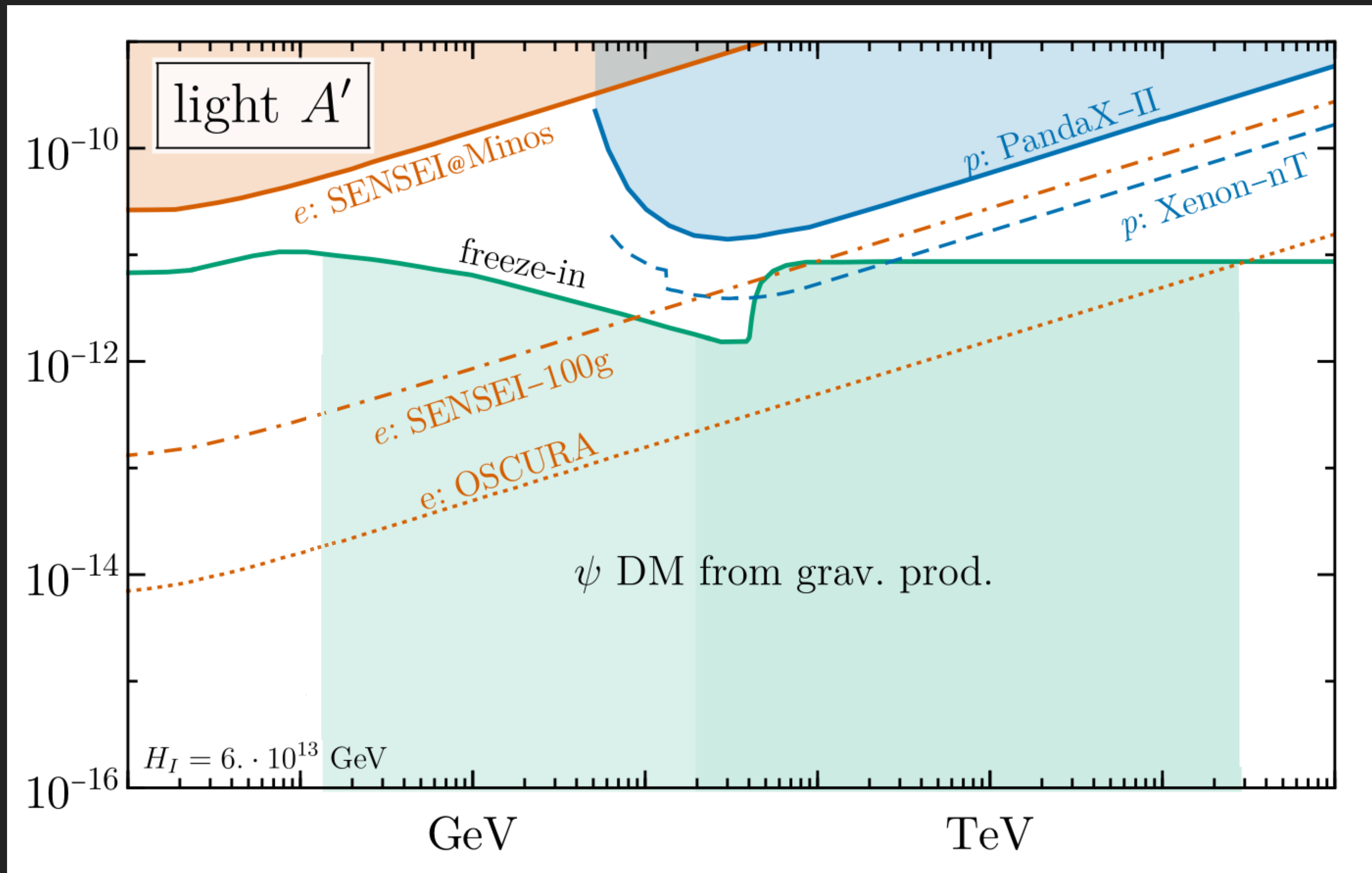
$$k_*^{-1} \sim 10^{10} \text{ km} \cdot \sqrt{\frac{10^{-5} \text{ eV}}{m_{A'}}} \sim 0.3 \text{ mpc} \cdot \sqrt{\frac{10^{-5} \text{ eV}}{m_{A'}}$$





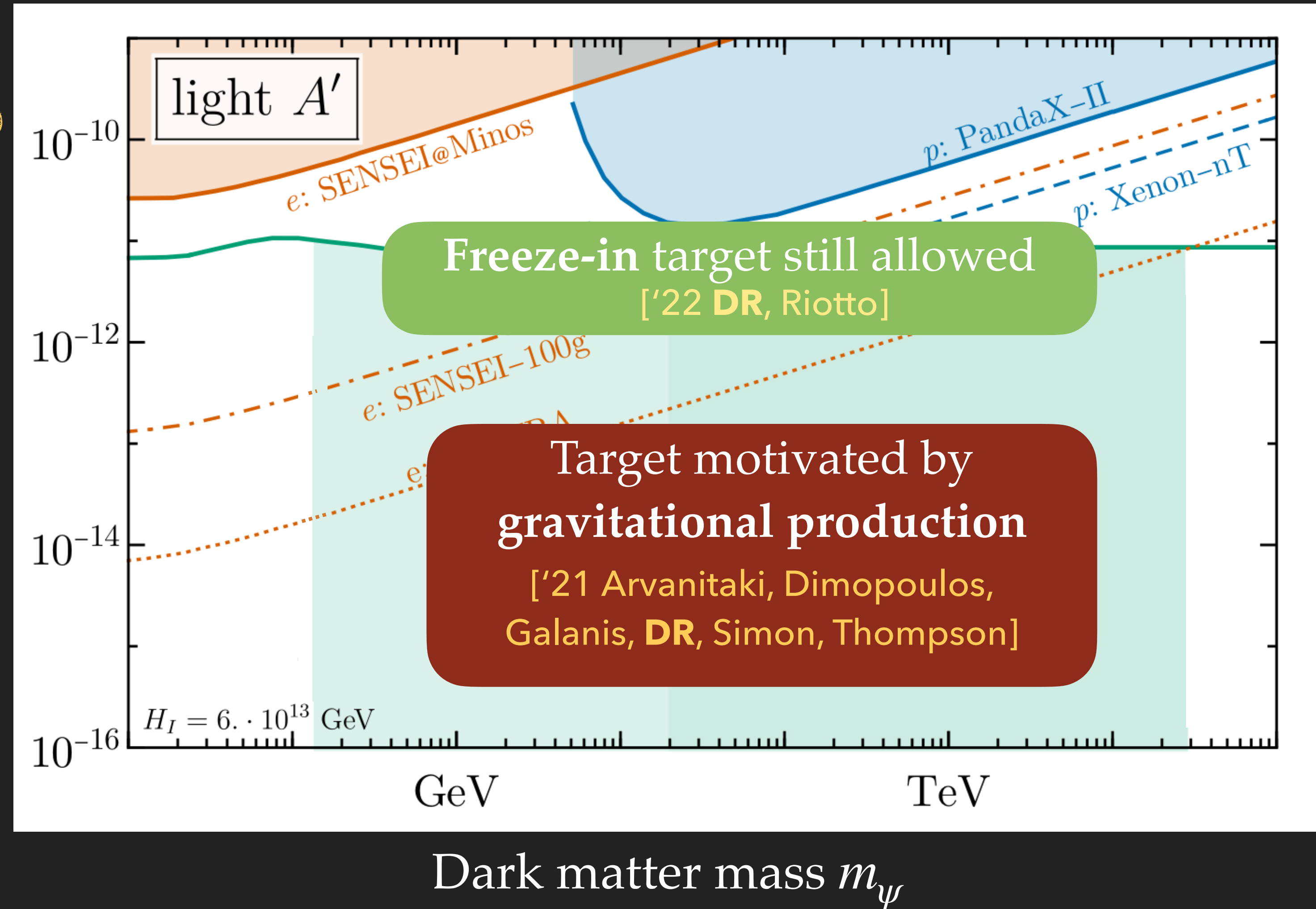
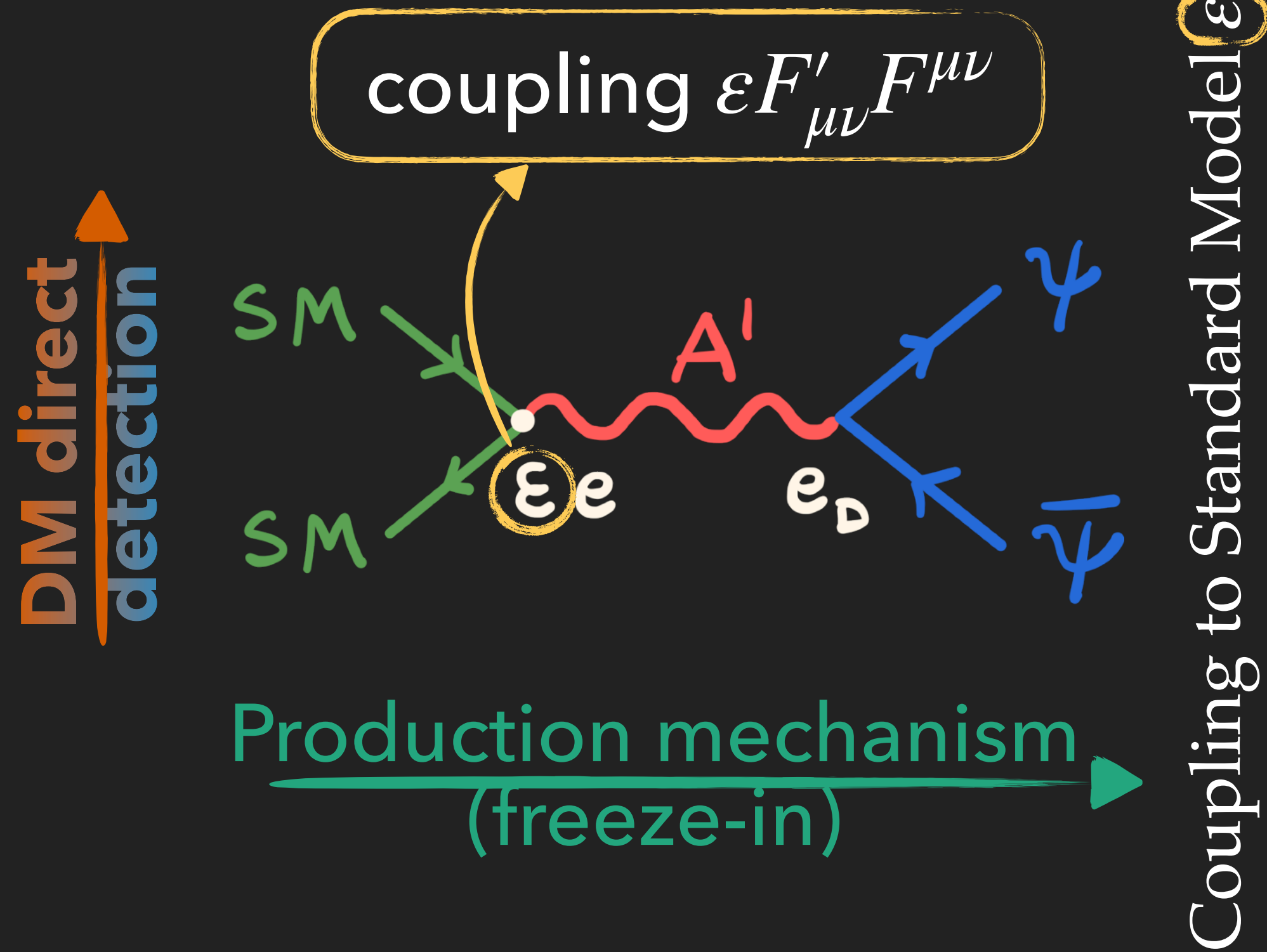


Coupling to Standard Model  $\epsilon_D$



Dark matter mass  $m_\psi$









**Thank you for your attention!**



BACKUP SLIDES

- ▶ Choose spatial coordinates to reabsorb  $\zeta_L(\mathbf{x})$  on long scales:

perturbations  $\zeta_L$   
on large scales



homogeneous  
on large scales

$$ds^2 = - dt^2 + a(t)^2 e^{2\zeta(\mathbf{x})} d\mathbf{x}^2$$

$$\mathbf{x}' = e^{\zeta_L(\mathbf{x})} \mathbf{x}$$

- ▶ Super-horizon fluctuations seeded by inflation:

$$T(\mathbf{x}', t) = T_{\text{bkg}}(t) e^{\zeta_L(\mathbf{x})/5}$$

- ▶ Compute any quantity in this frame:

$$\Gamma(T(\mathbf{x}', t))$$





- ▶ SM and DM are coupled by the energy transfer:

$$T^{\mu\nu} = T_{\text{DM}}^{\mu\nu} + T_{\gamma}^{\mu\nu}$$

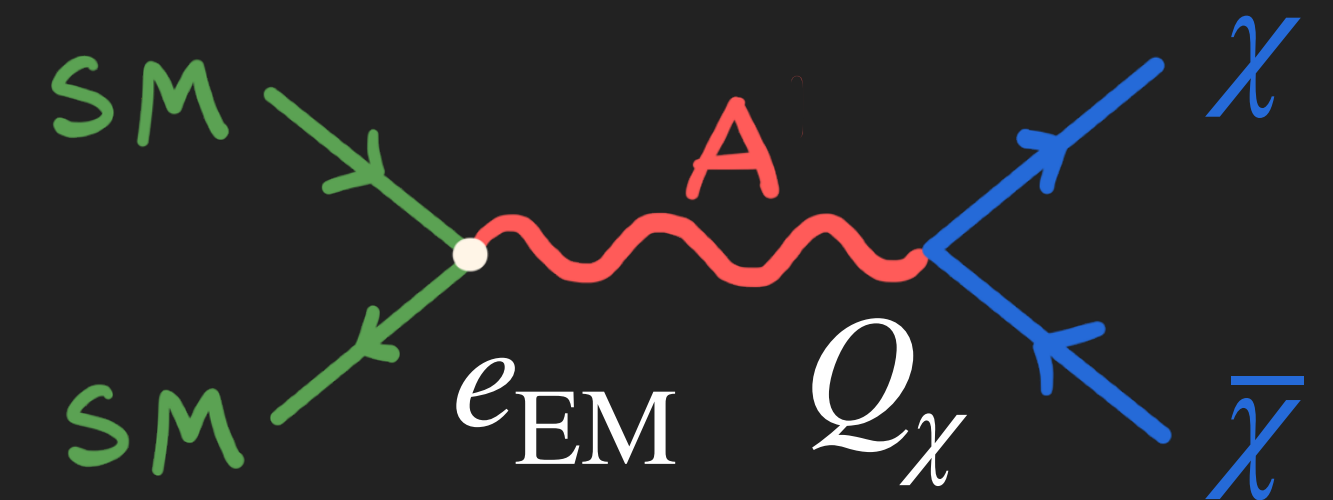
$$\begin{aligned} \nabla_{\mu} T_{\text{DM}}^{\mu\nu} &= Q_{\text{DM}}^{\nu}, \\ \nabla_{\mu} T_{\gamma}^{\mu\nu} &= Q_{\gamma}^{\nu}, \end{aligned}$$

$$Q_{\text{DM}}^{\nu} + Q_{\gamma}^{\nu} = 0.$$

- ▶ Time evolution for energy density:

$$\begin{aligned} \dot{\rho}_{\text{DM}} &= -3H (\rho_{\text{DM}} + P_{\text{DM}}) + Q_{\text{DM}}, \\ \dot{\rho}_{\gamma} &= -3H (\rho_{\gamma} + P_{\gamma}) + Q_{\gamma}. \end{aligned}$$

$$\begin{aligned} Q_{\text{DM}} &= \Gamma(\rho_{\gamma}), \\ Q_{\gamma} &= -\Gamma(\rho_{\gamma}). \end{aligned}$$



- ▶ Transfer rate depends **only** on  $T_{\text{SM}}(t, \mathbf{x})$
- ▶ Freeze-in relevant around  $T_{\text{SM}} \sim m_{\chi} \gtrsim \text{MeV}$ , way before recombination ( $\sim \text{eV}$ ), and then shuts off

- Perturbations to densities, transfer rate, metric:

$$ds^2 = -(1 + 2\varphi)dt^2 + 2aB_{,i}dtdx^i + a^2 [(1 - 2\psi)\delta_{ij} + 2E_{,ij}] dx^i dx^j,$$

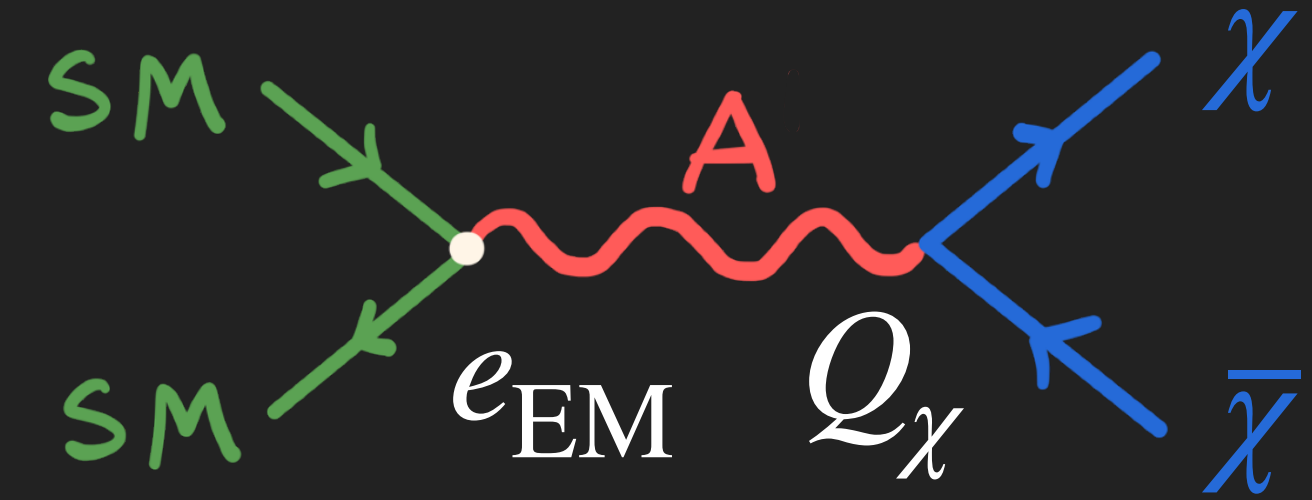
$$-Q_{\text{DM}}(1 + \varphi) - \delta Q_{\text{DM}} \quad \text{and} \quad -Q_{\gamma}(1 + \varphi) - \delta Q_{\gamma},$$

$$\begin{aligned} \dot{\delta\rho}_{\text{DM}} + 3H(\delta\rho_{\text{DM}} + \delta P_{\text{DM}}) - (\rho_{\text{DM}} + P_{\text{DM}}) 3\dot{\psi} &= Q_{\text{DM}}\varphi + \delta Q_{\text{DM}}, \\ \dot{\delta\rho}_{\gamma} + 3H(\delta\rho_{\gamma} + \delta P_{\gamma}) - (\rho_{\gamma} + P_{\gamma}) 3\dot{\psi} &= Q_{\gamma}\varphi + \delta Q_{\gamma}. \end{aligned}$$

$$\begin{aligned} \zeta_{\text{DM}} &= -\psi - H \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}}, \\ \zeta_{\gamma} &= -\psi - H \frac{\delta\rho_{\gamma}}{\dot{\rho}_{\gamma}}. \end{aligned}$$

$$\mathcal{S}_{\text{DM}\gamma} = 3(\zeta_{\text{DM}} - \zeta_{\gamma}) = -3H \left( \frac{\delta\rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} - \frac{\delta\rho_{\gamma}}{\dot{\rho}_{\gamma}} \right).$$

includes  $\Gamma$  in  $\dot{\rho}$



- Large-scale limit

[‘03 Malik, Wands, Ungarelli]



► Gauge-invariant result:

$$\dot{\zeta}_{\text{DM}} = \frac{3H^2}{\dot{\rho}_{\text{DM}}} \delta P_{\text{intr,DM}} - \frac{H}{\dot{\rho}_{\text{DM}}} (\delta Q_{\text{intr,DM}} + \delta Q_{\text{rel,DM}})$$

$$\dot{\zeta}_{\gamma} = \frac{3H^2}{\dot{\rho}_{\gamma}} \delta P_{\text{intr},\gamma} - \frac{H}{\dot{\rho}_{\gamma}} (\delta Q_{\text{intr},\gamma} + \delta Q_{\text{rel},\gamma})$$

$$\delta P_{\text{intr,DM}} = \delta P_{\text{DM}} - \frac{\dot{P}_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \delta \rho_{\text{DM}}$$

$$\delta P_{\text{intr},\gamma} = \delta P_{\gamma} - \frac{\dot{P}_{\gamma}}{\dot{\rho}_{\gamma}} \delta \rho_{\gamma}$$

$$\delta Q_{\text{intr,DM}} = \delta Q_{\text{DM}} - \frac{\dot{Q}_{\text{DM}}}{\dot{\rho}_{\text{DM}}} \delta \rho_{\text{DM}}$$

$$\delta Q_{\text{intr},\gamma} = \delta Q_{\gamma} - \frac{\dot{Q}_{\gamma}}{\dot{\rho}_{\gamma}} \delta \rho_{\gamma}$$

$$\delta Q_{\text{rel,DM}} = \frac{Q_{\text{DM}} \dot{\rho}}{2\rho} \left( \frac{\delta \rho_{\text{DM}}}{\dot{\rho}_{\text{DM}}} - \frac{\delta \rho}{\dot{\rho}} \right) = -\frac{Q_{\text{DM}}}{6H\rho} \dot{\rho}_{\gamma} \mathcal{S}_{\text{DM},\gamma}$$

$$\delta Q_{\text{rel},\gamma} = \frac{Q_{\gamma} \dot{\rho}}{2\rho} \left( \frac{\delta \rho_{\gamma}}{\dot{\rho}_{\gamma}} - \frac{\delta \rho}{\dot{\rho}} \right) = \frac{Q_{\gamma}}{6H\rho} \dot{\rho}_{\text{DM}} \mathcal{S}_{\text{DM},\gamma}$$

► Intrinsic non-adiabatic pressure perturbation

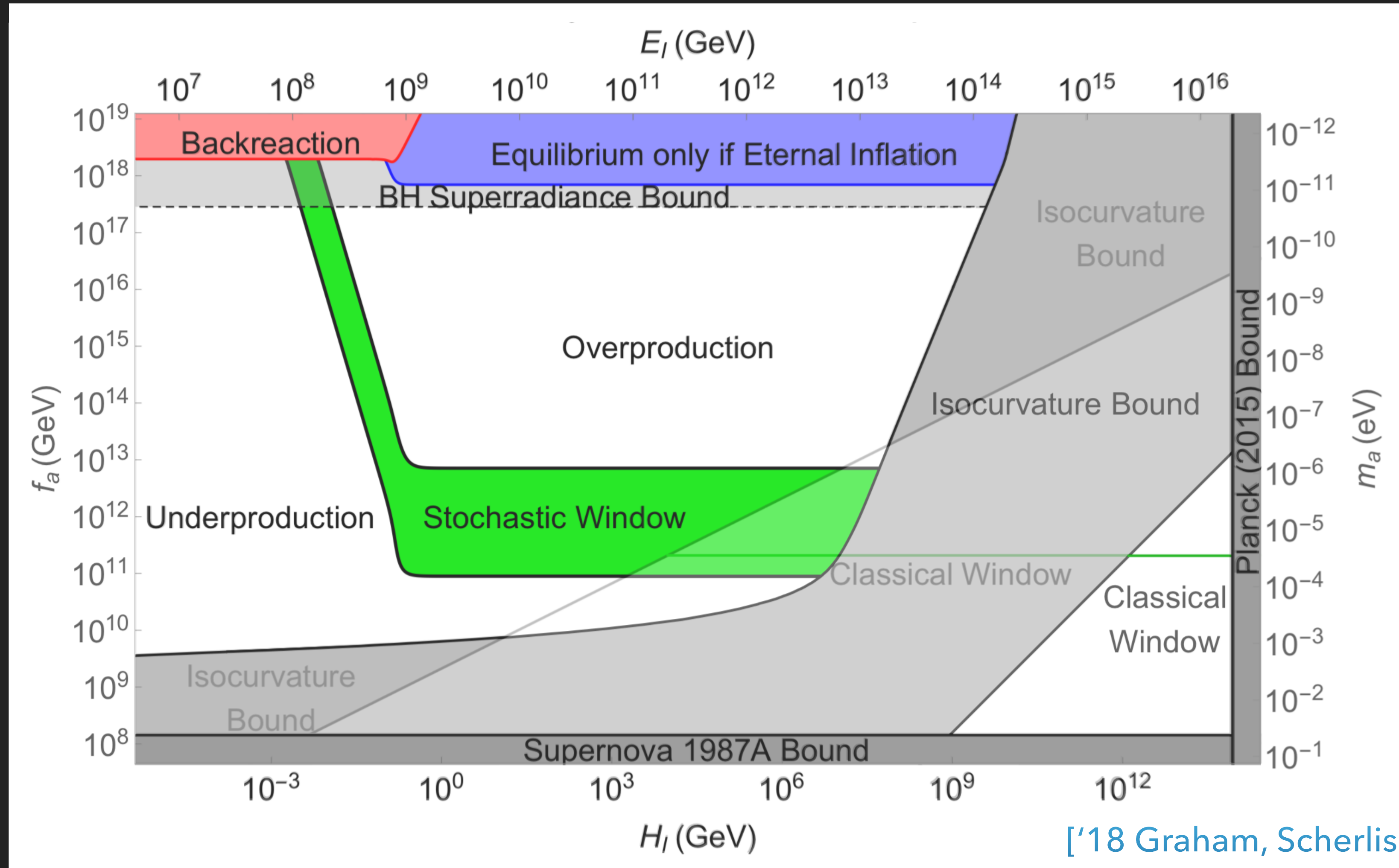
► Vanish when  $P_{\text{DM}}$  is only a function of  $\rho_{\text{DM}}$

► Intrinsic non-adiabatic energy transfer

► If  $\Gamma$  is only a function of  $T_{\text{SM}}$ , it's  $\propto \mathcal{S}_{\text{DM},\gamma}$

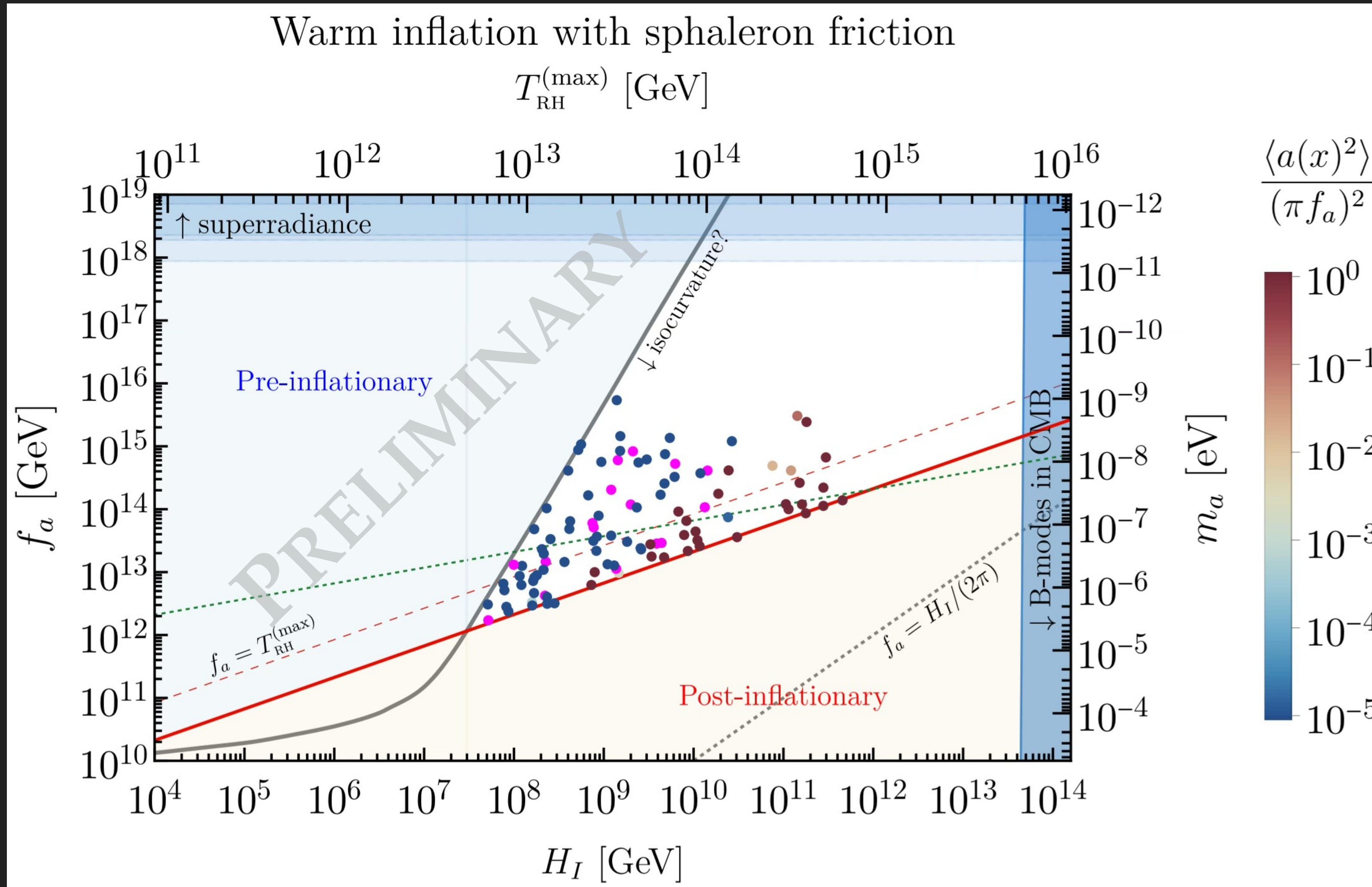
► Non-adiabatic perturbed energy transfer

► It is  $\propto \mathcal{S}_{\text{DM},\gamma}$

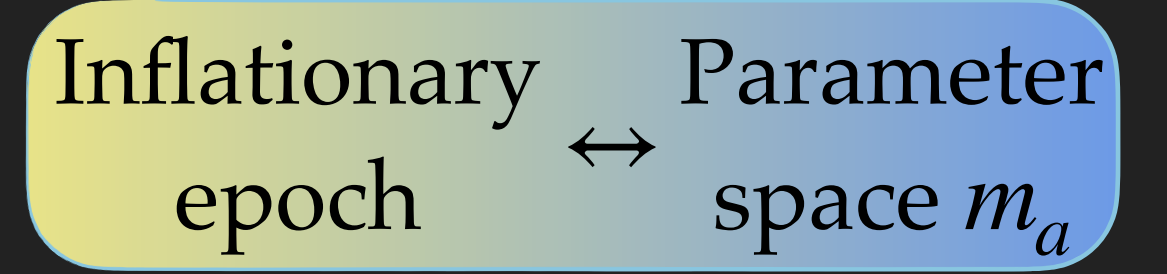




- ▶ Couplings like  $\xi R |\Phi|^2$  for saxion field:  $f_{a,\text{inf}} \nearrow$ ,  $\delta\theta_{\text{misalignment}} \sim \frac{H_I}{f_{a,\text{inf}}} \searrow$ ,  $\delta_{\text{iso}} \searrow$



[in progress, Graham, DR]



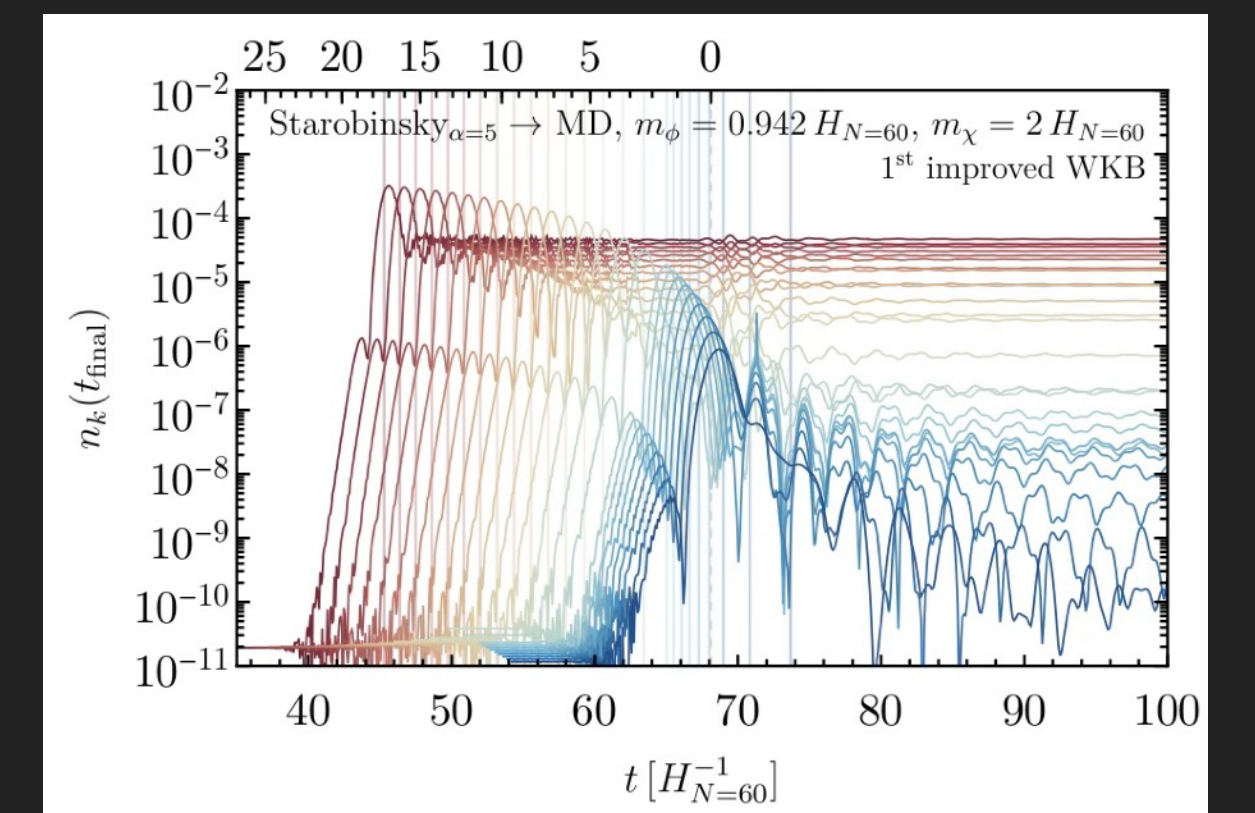
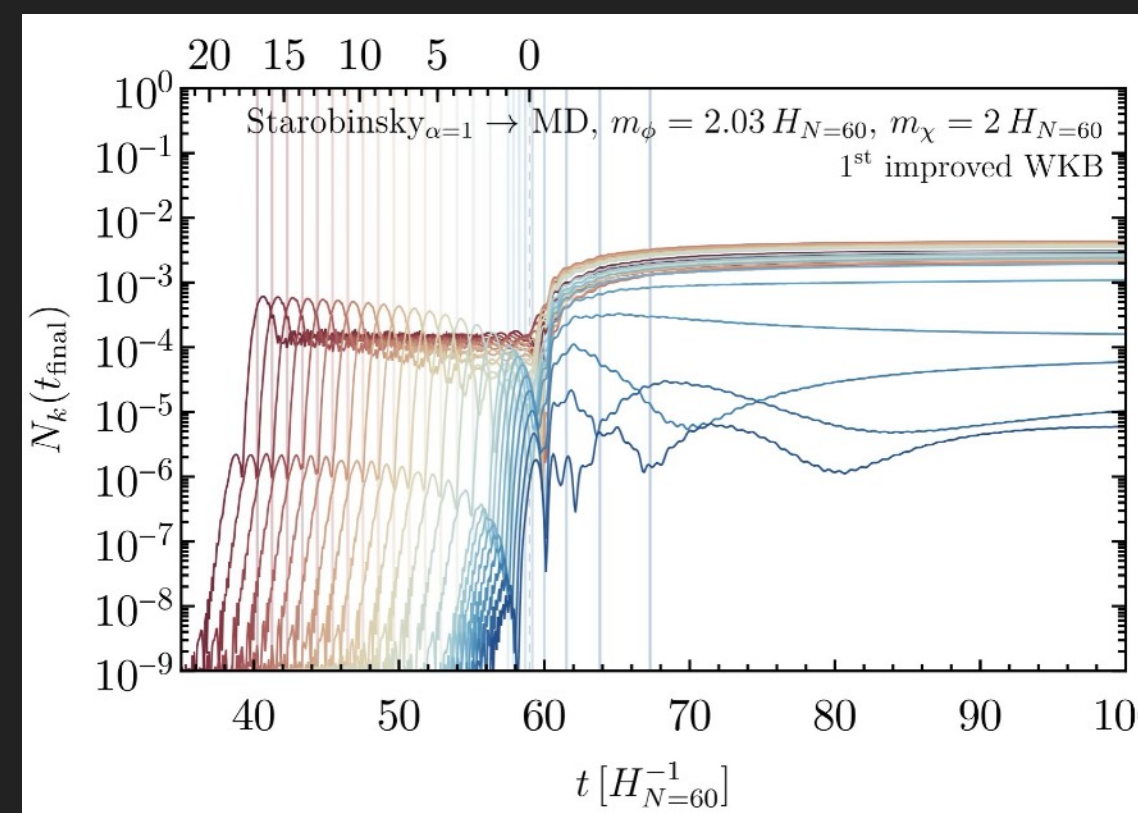
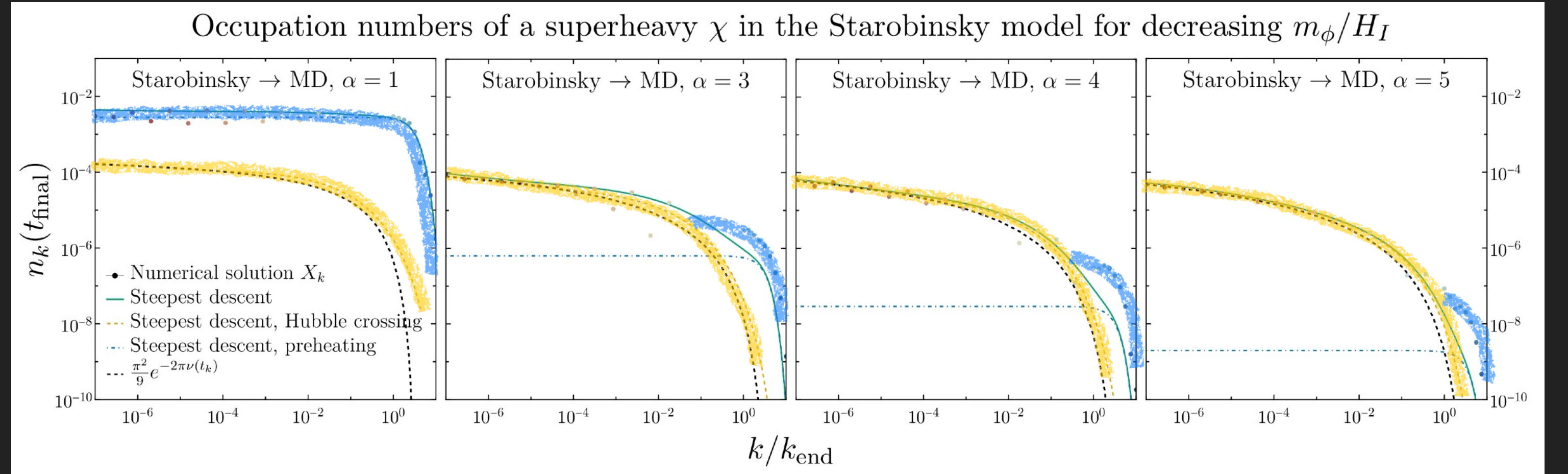
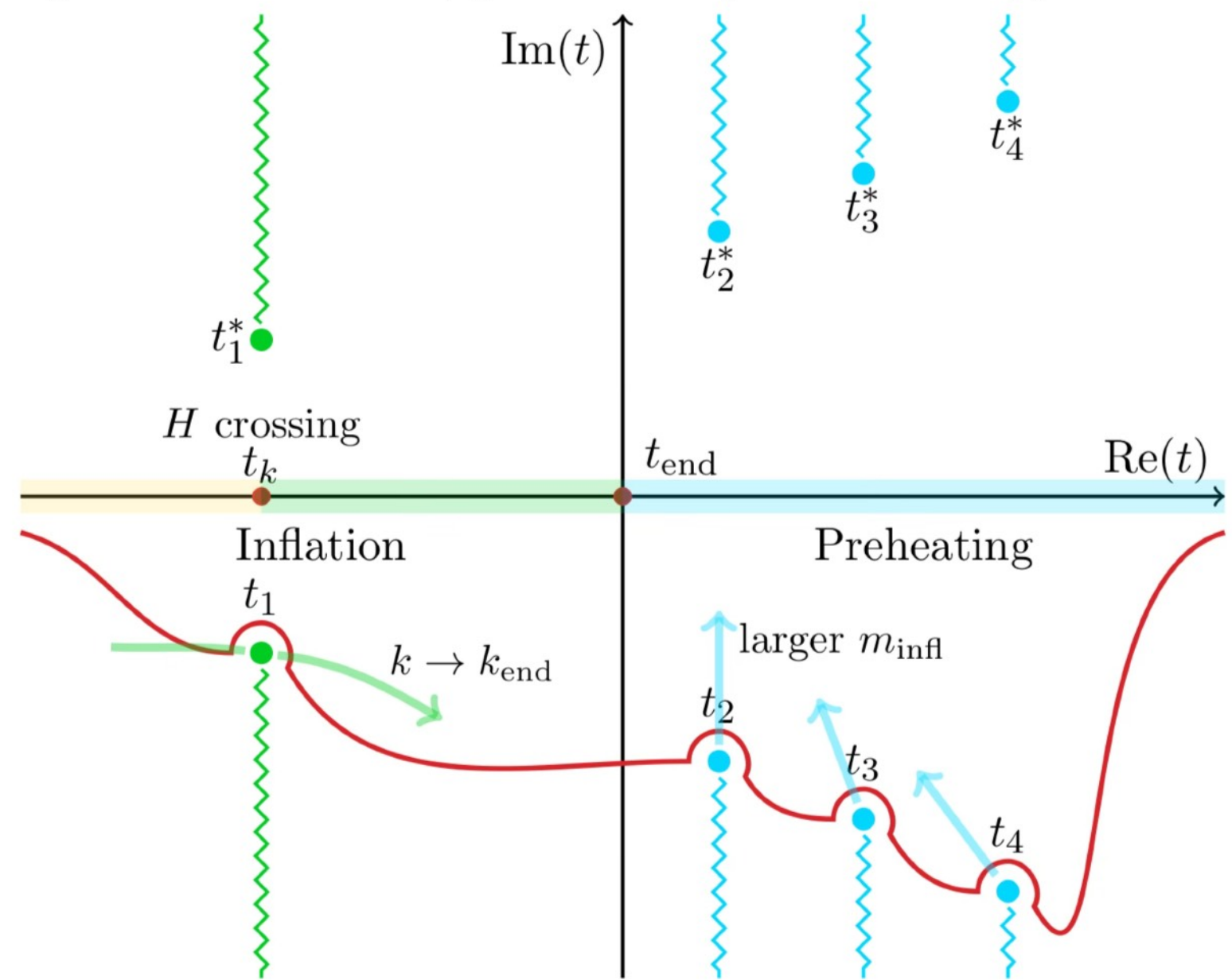


► Zeros of  $\omega_k(t)$  in complex plane:

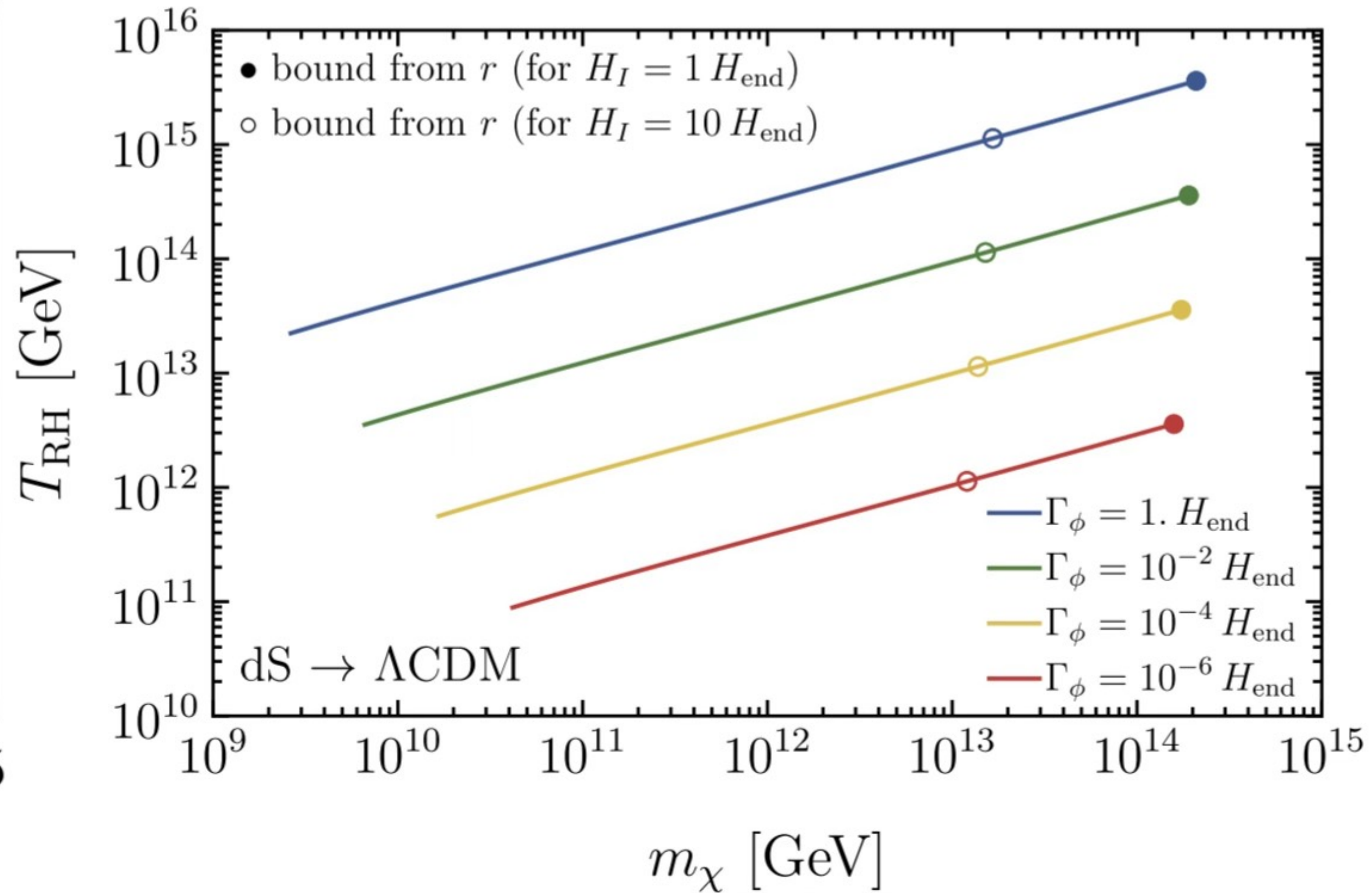
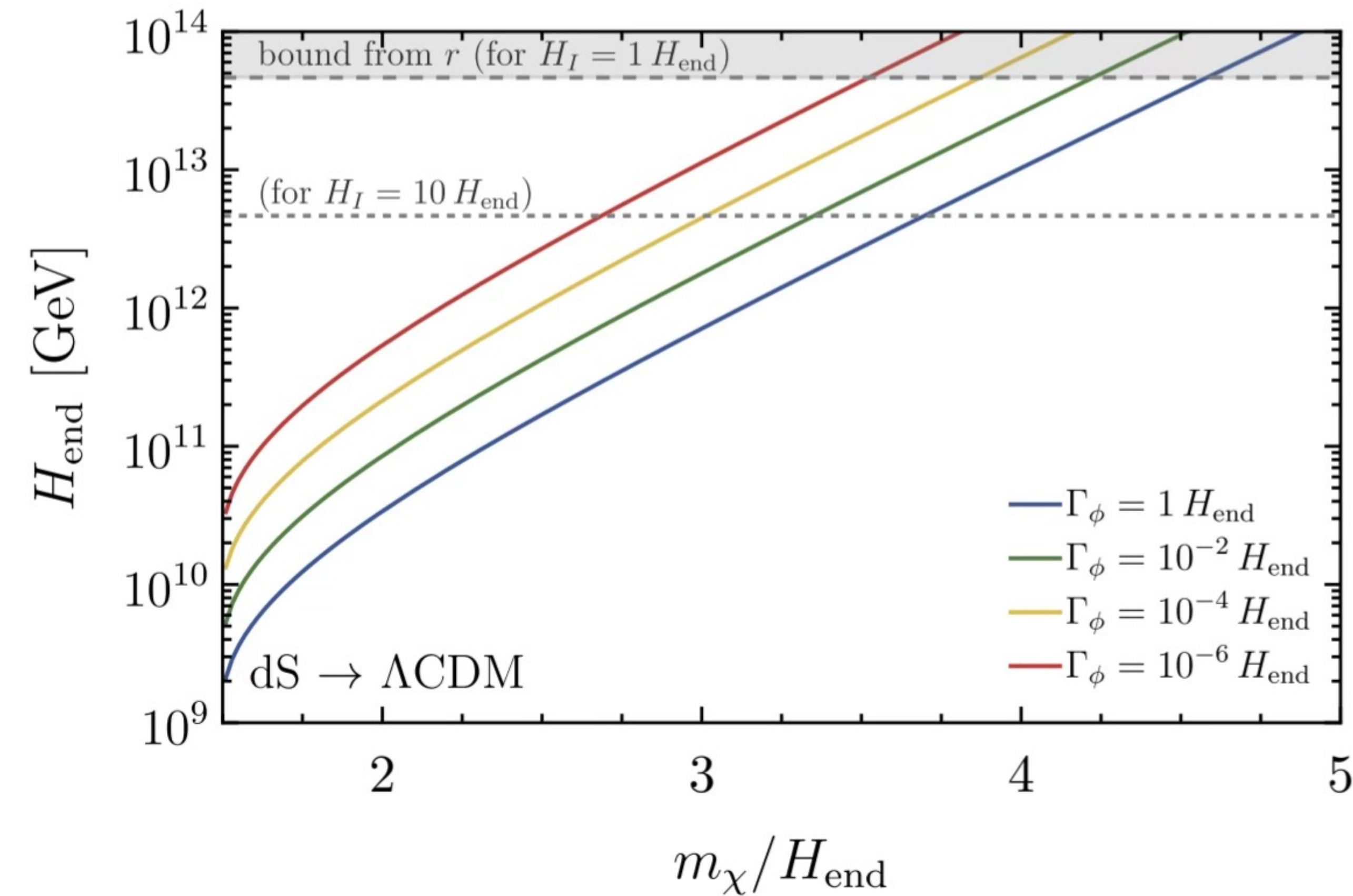
At inflaton oscillations:  
need  $m_\phi > H_I$

Horizon crossing :  
always present

Integration contour for  $\beta_k$  in the steepest descent approximation





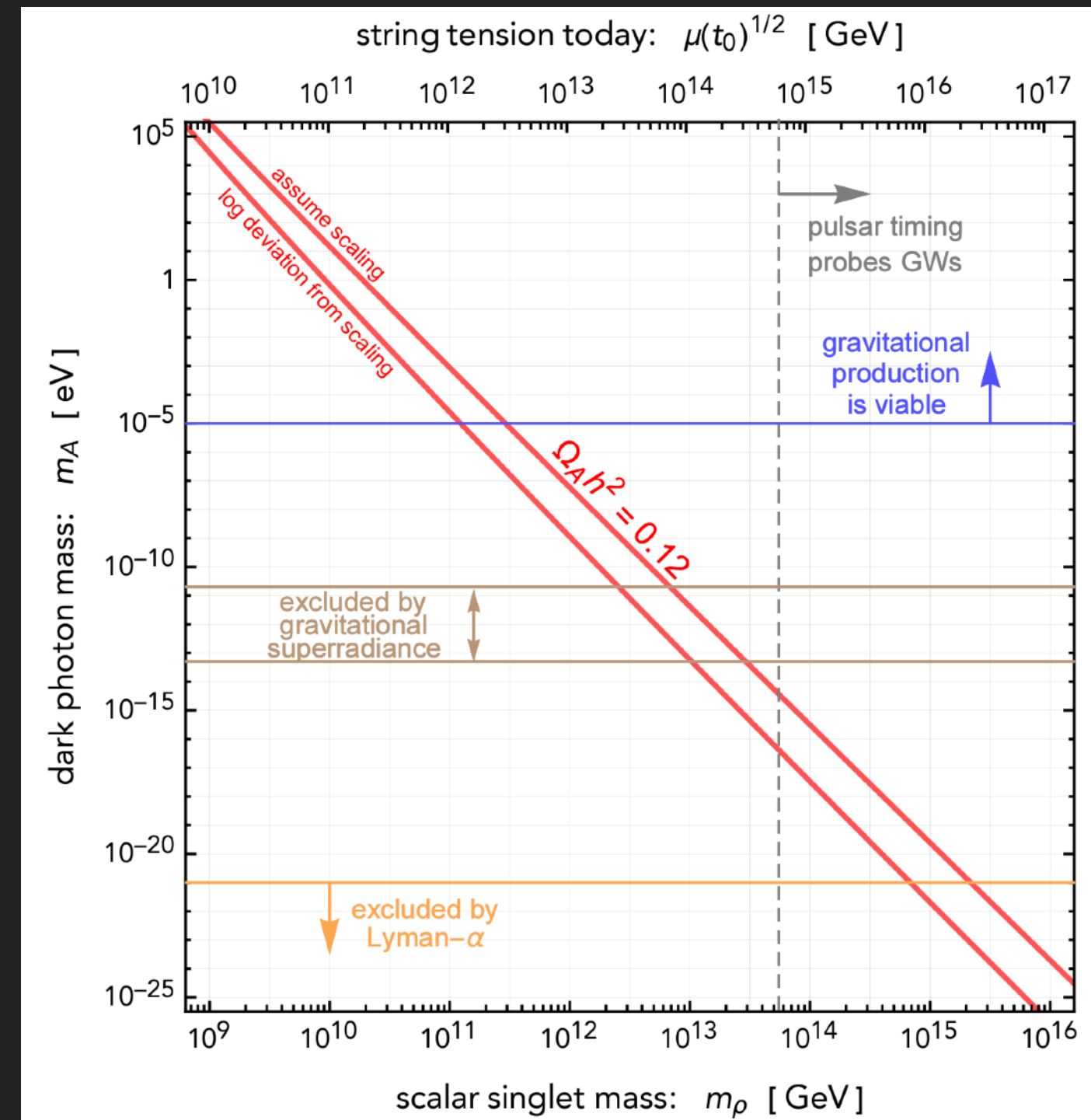


▶ We consider pure Stückelberg mass: Proca theory, perfectly valid QFT

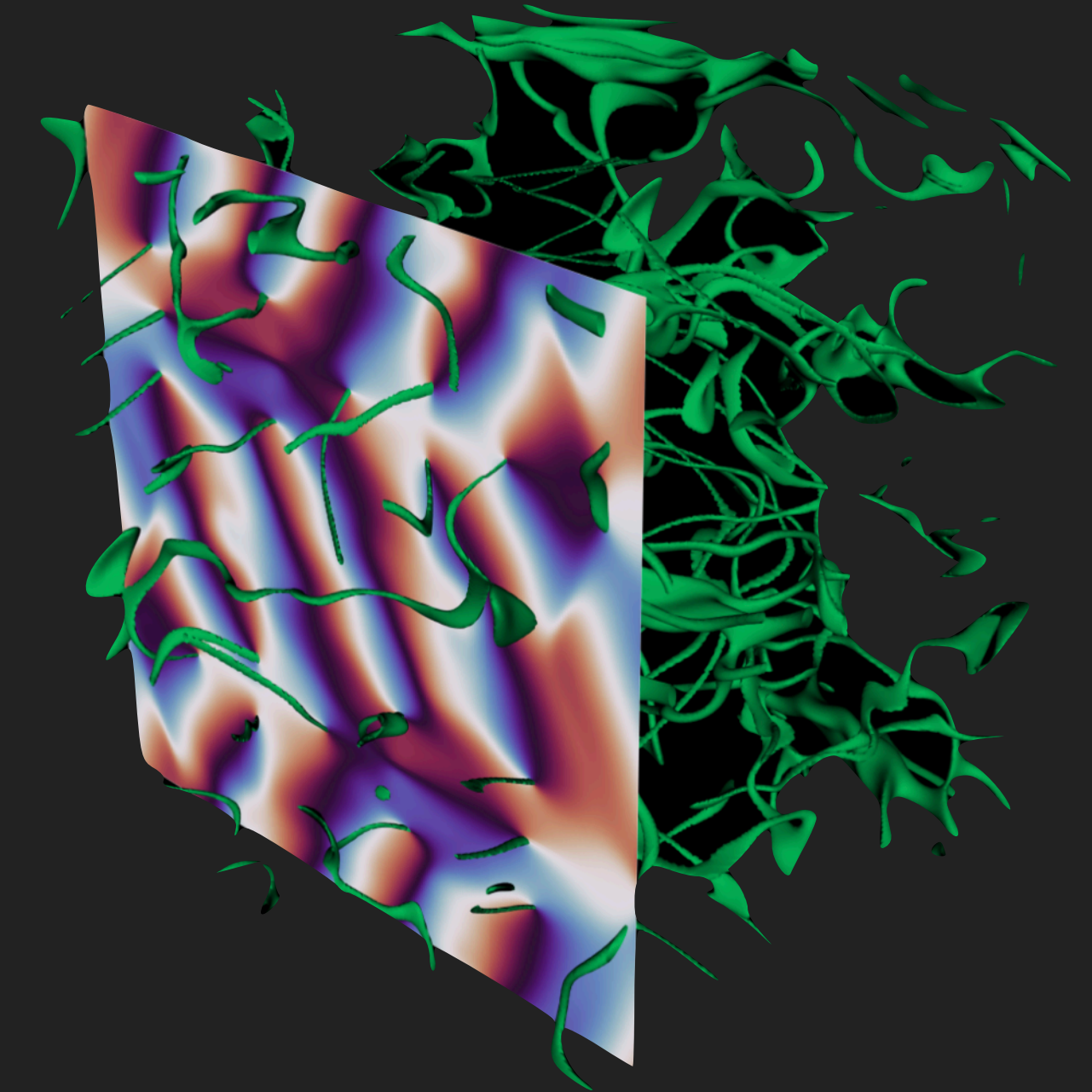
▶ If assume Higgs mechanism, string network produced if

$$v < H_I \rightarrow \langle g \rangle \sim \frac{m_A}{H_I}$$

['19 Long, Wang]



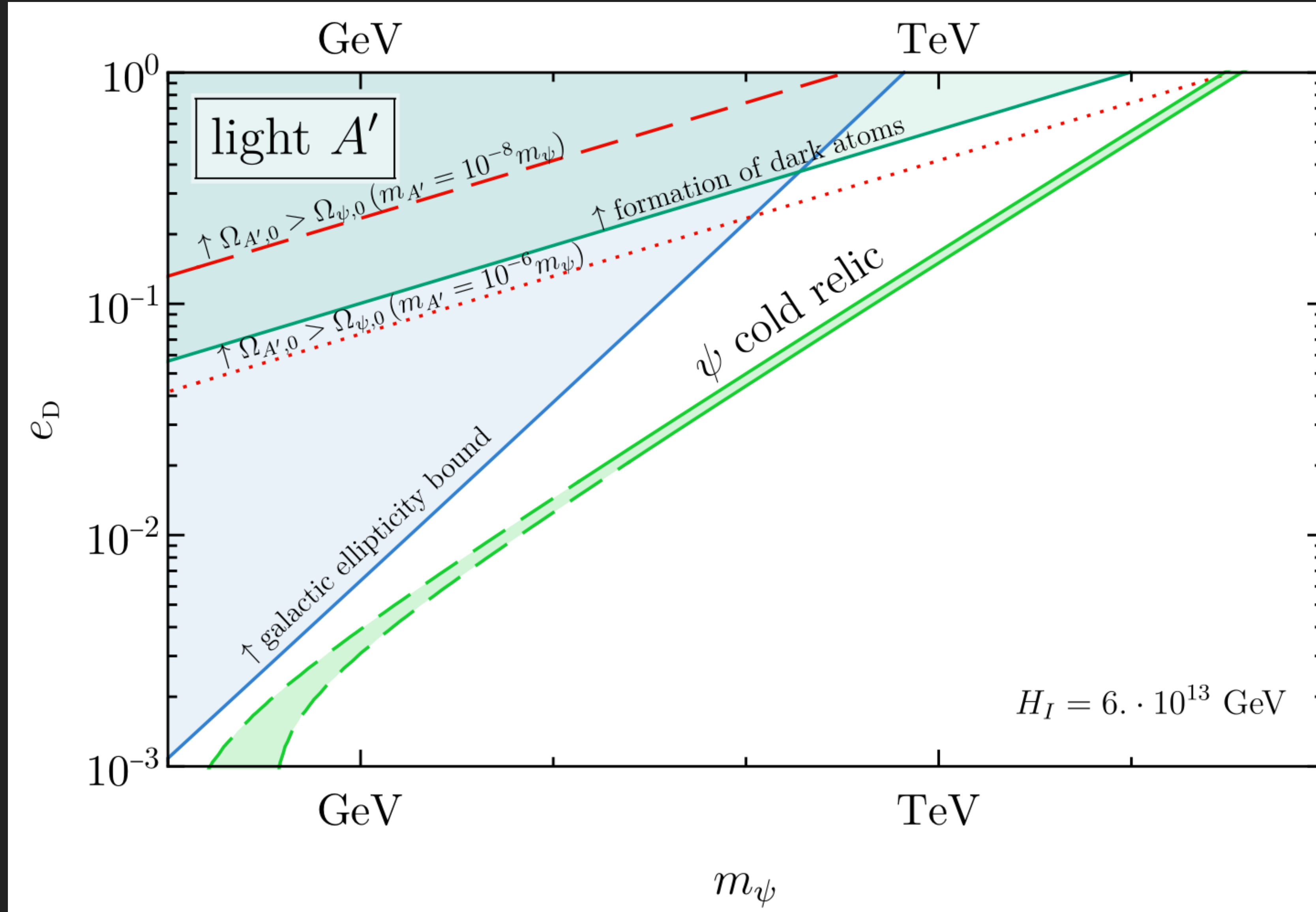
['22 Redi, Tesi]  
['22 East, Huang]

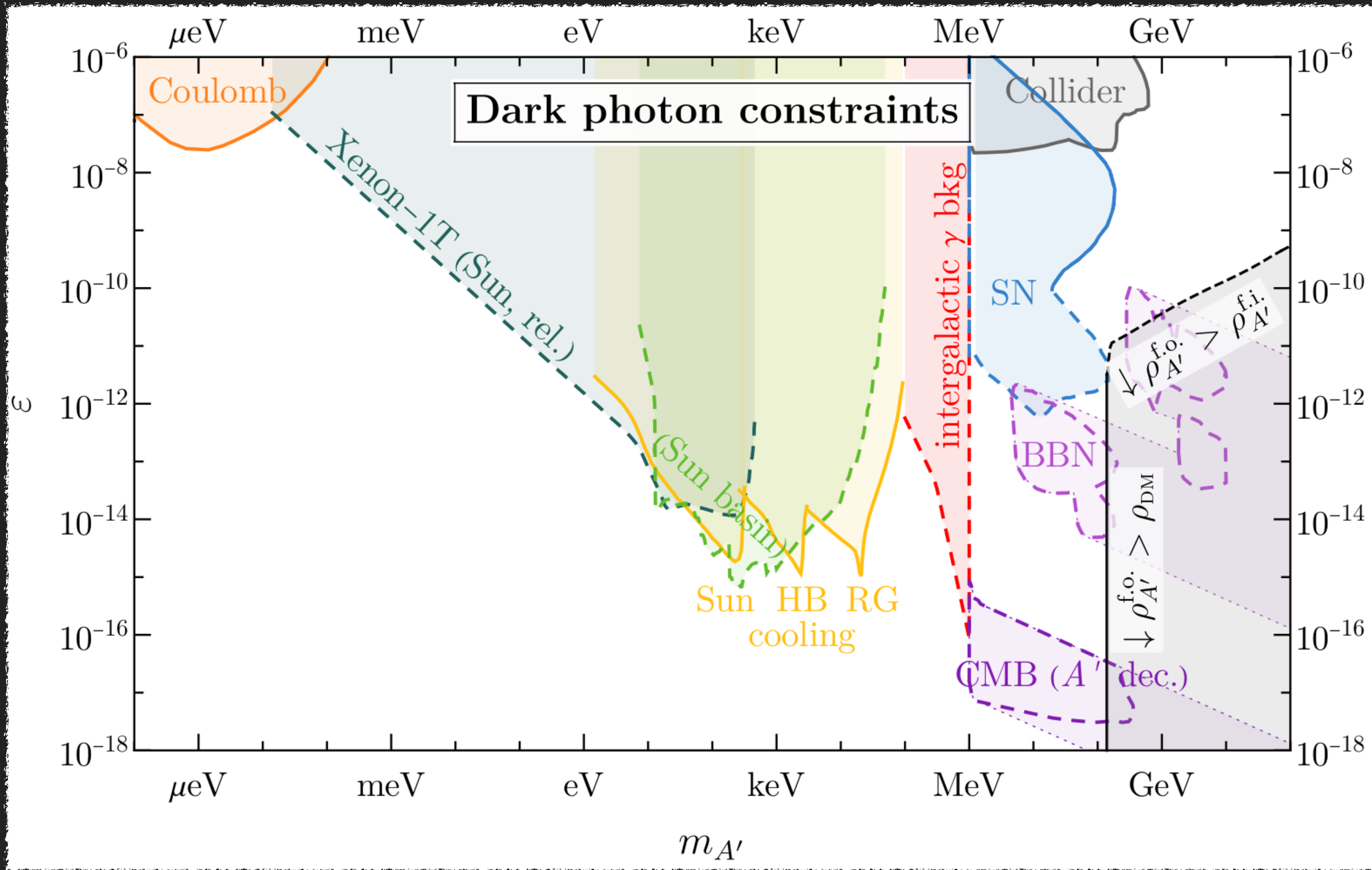


▶ For dark QED, even assuming dark Higgs:  $\psi$  can be DM for  $e_D \gg g$ , or  $A'$  is DM with  $e_D \gtrsim g$  ( $H_I \lesssim \mathcal{O}(10)$  GeV,  $m_{A'} \sim \mathcal{O}(1)$  GeV)

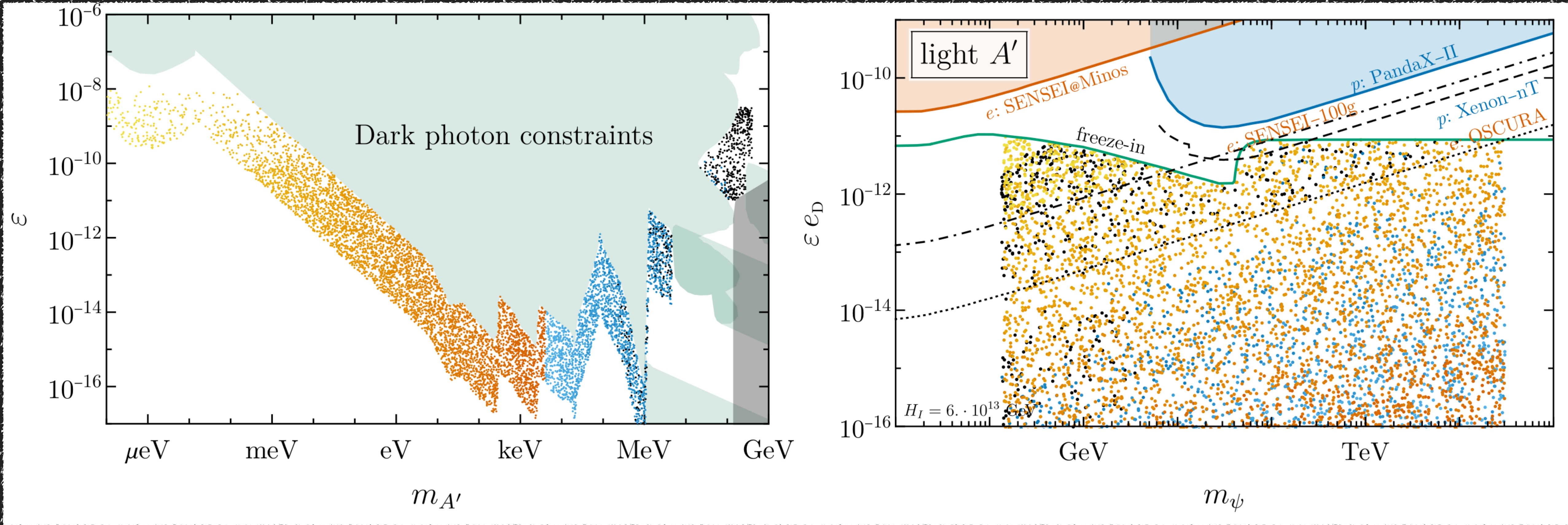
▶ This constraint is milder in dark QED than pure  $A'$ : we can afford much larger  $m_{A'}$





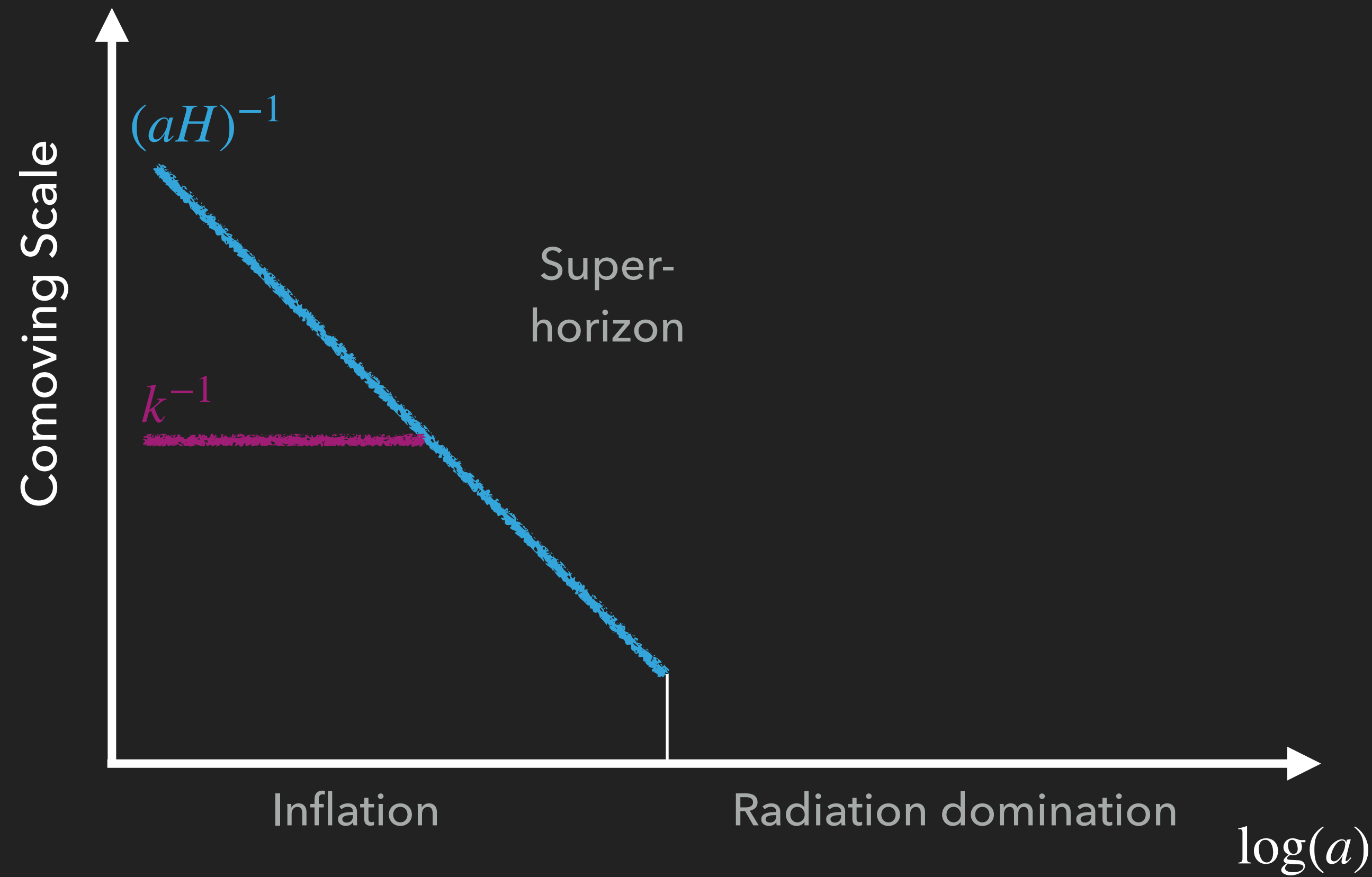




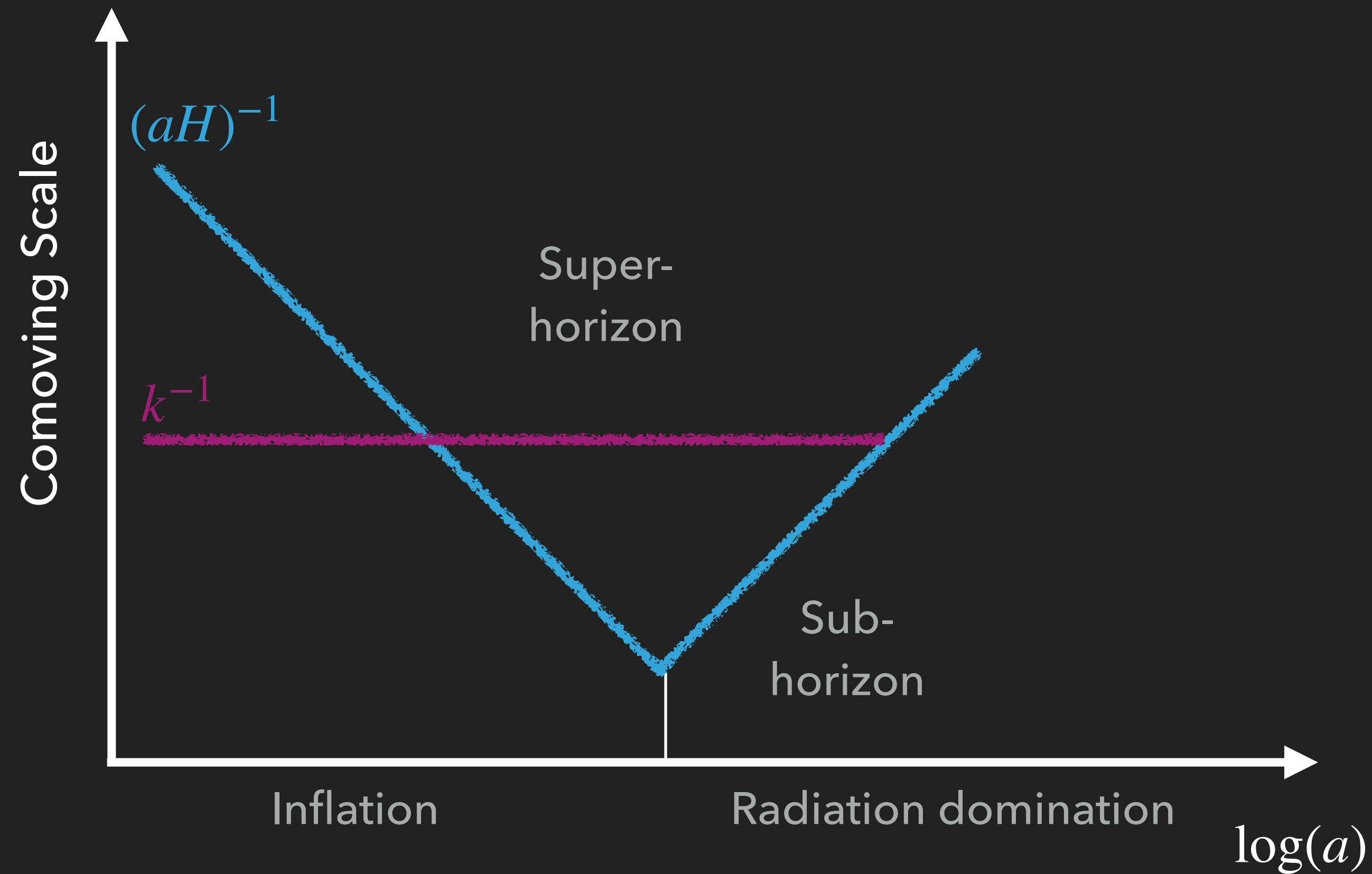






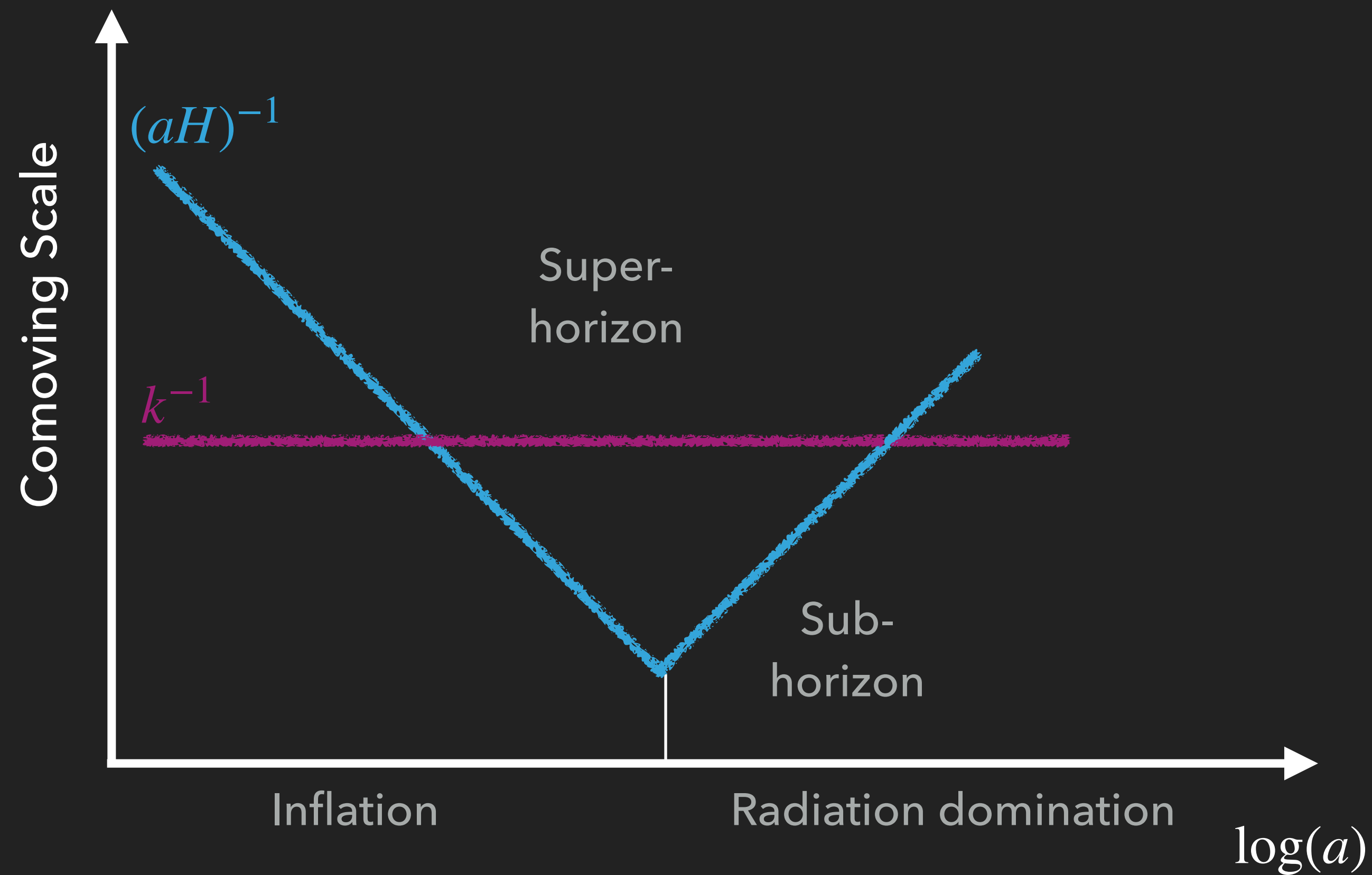


►  $\rho_{k,\text{exit}} \sim H_I^4$ , the mode  $A'_{L,k}$  freezes

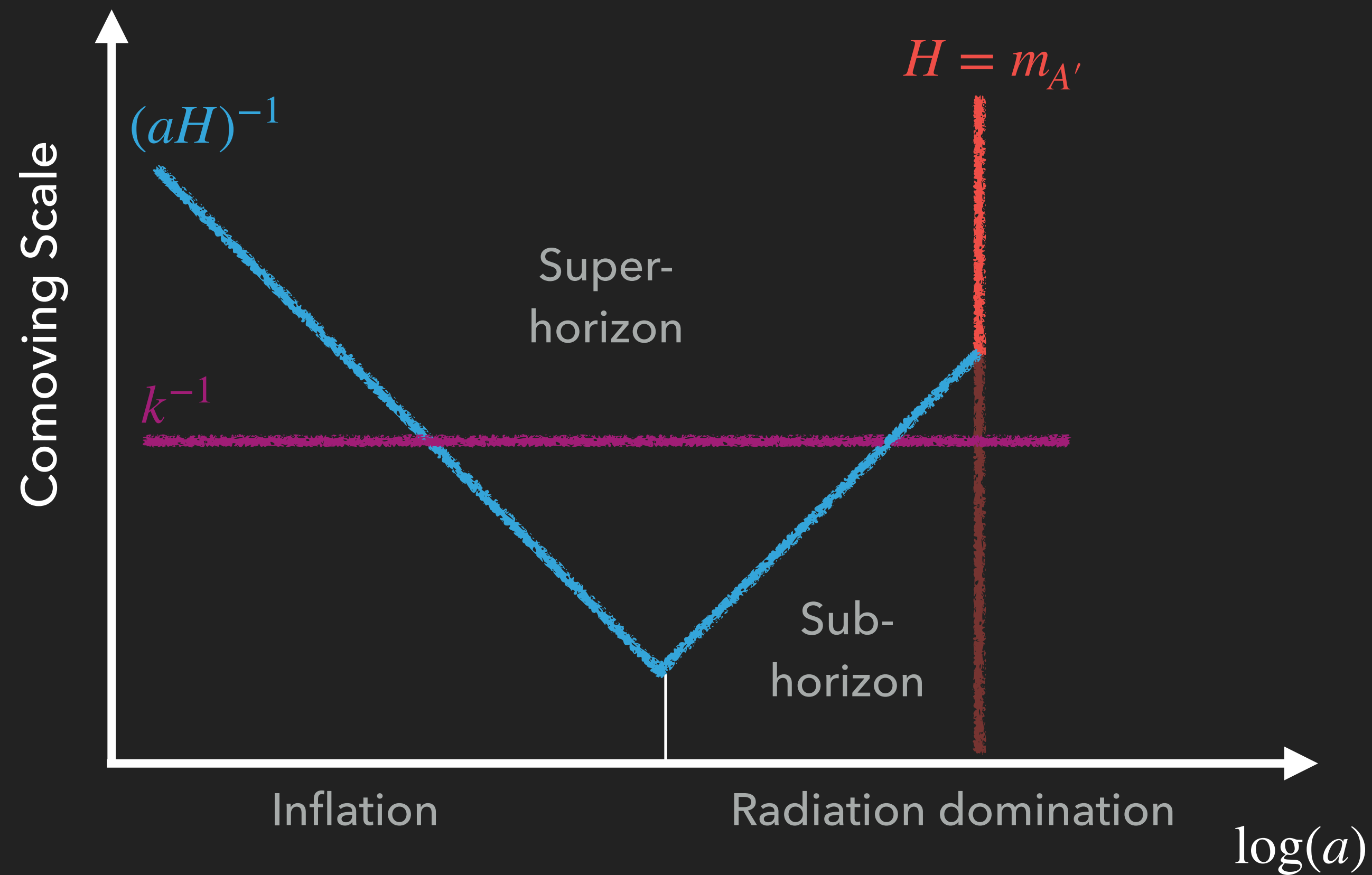


- ▶  $\rho_{k,\text{exit}} \sim H_I^4$ , the mode  $A'_{L,k}$  freezes
- ▶ Super-horizon:  $\rho_k \sim m_{A'}^2 A'^2 \sim a^{-2}$



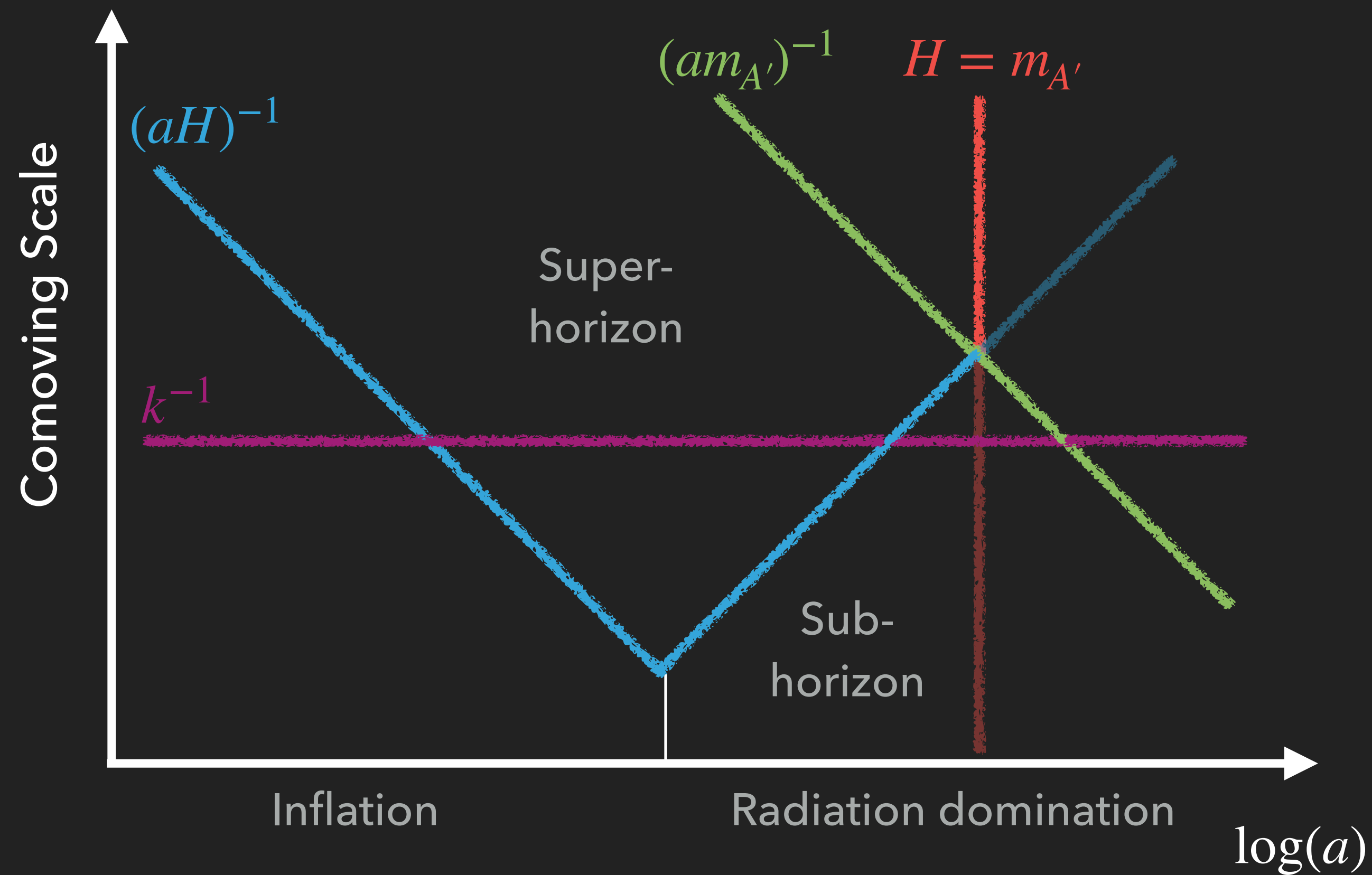


- ▶  $\rho_{k,\text{exit}} \sim H_I^4$ , the mode  $A'_{L,k}$  freezes
- ▶ Super-horizon:  $\rho_k \sim m_{A'}^2 A'^2 \sim a^{-2}$
- ▶ Hor. entry: oscillation,  $\rho_k \sim a^{-4}$

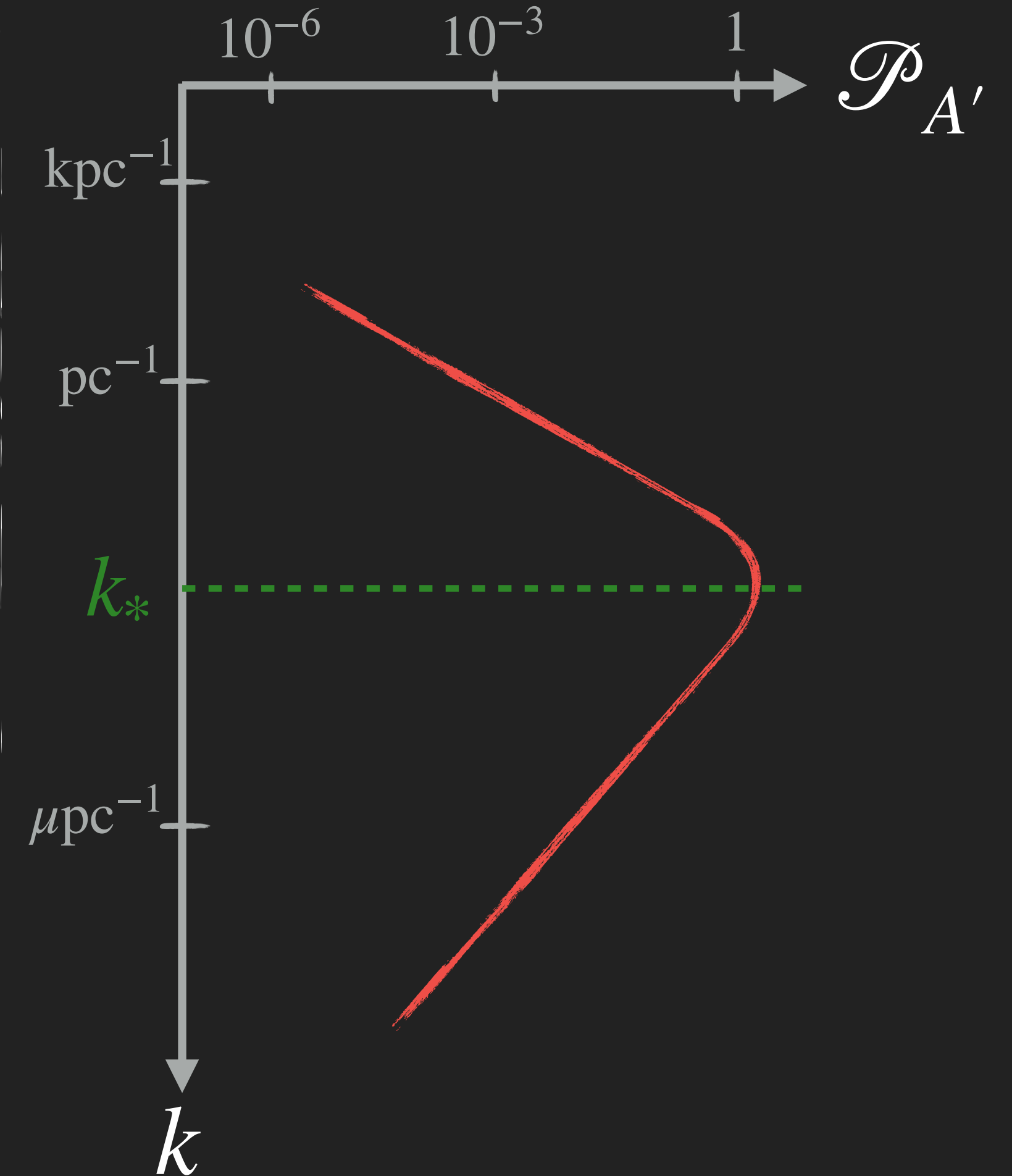
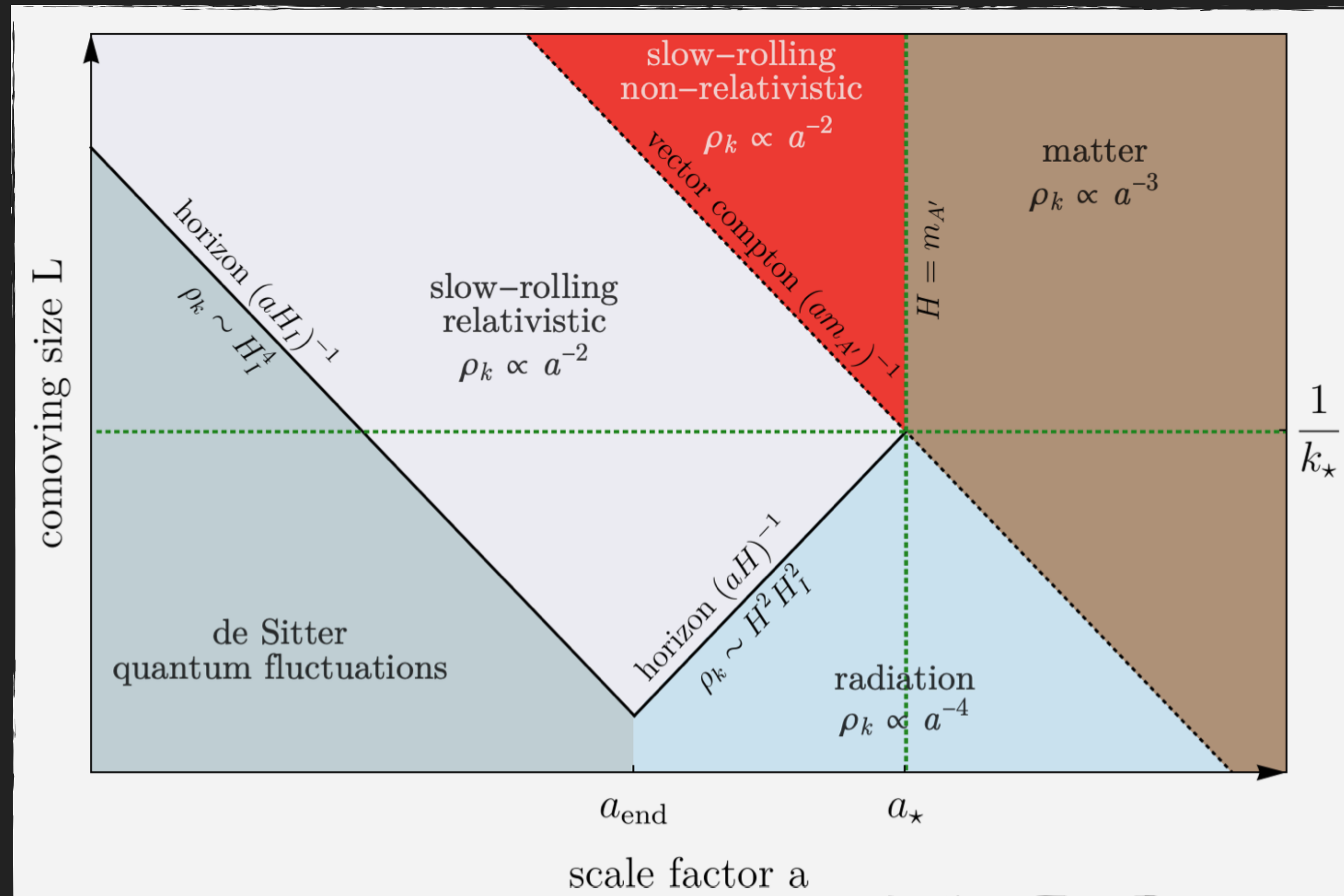


- ▶  $\rho_{k,\text{exit}} \sim H_I^4$ , the mode  $A'_{L,k}$  freezes
- ▶ Super-horizon:  $\rho_k \sim m_{A'}^2 A'^2 \sim a^{-2}$
- ▶ Hor. entry: oscillation,  $\rho_k \sim a^{-4}$
- ▶ Time  $H = m_{A'}$ : all modes oscillate





- ▶  $\rho_{k,\text{exit}} \sim H_I^4$ , the mode  $A'_{L,k}$  freezes
- ▶ Super-horizon:  $\rho_k \sim m_{A'}^2 A'^2 \sim a^{-2}$
- ▶ Hor. entry: oscillation,  $\rho_k \sim a^{-4}$
- ▶ Time  $H = m_{A'}$ : all modes oscillate
- ▶ Mode non-relativistic  $\rho_k \sim a^{-3}$

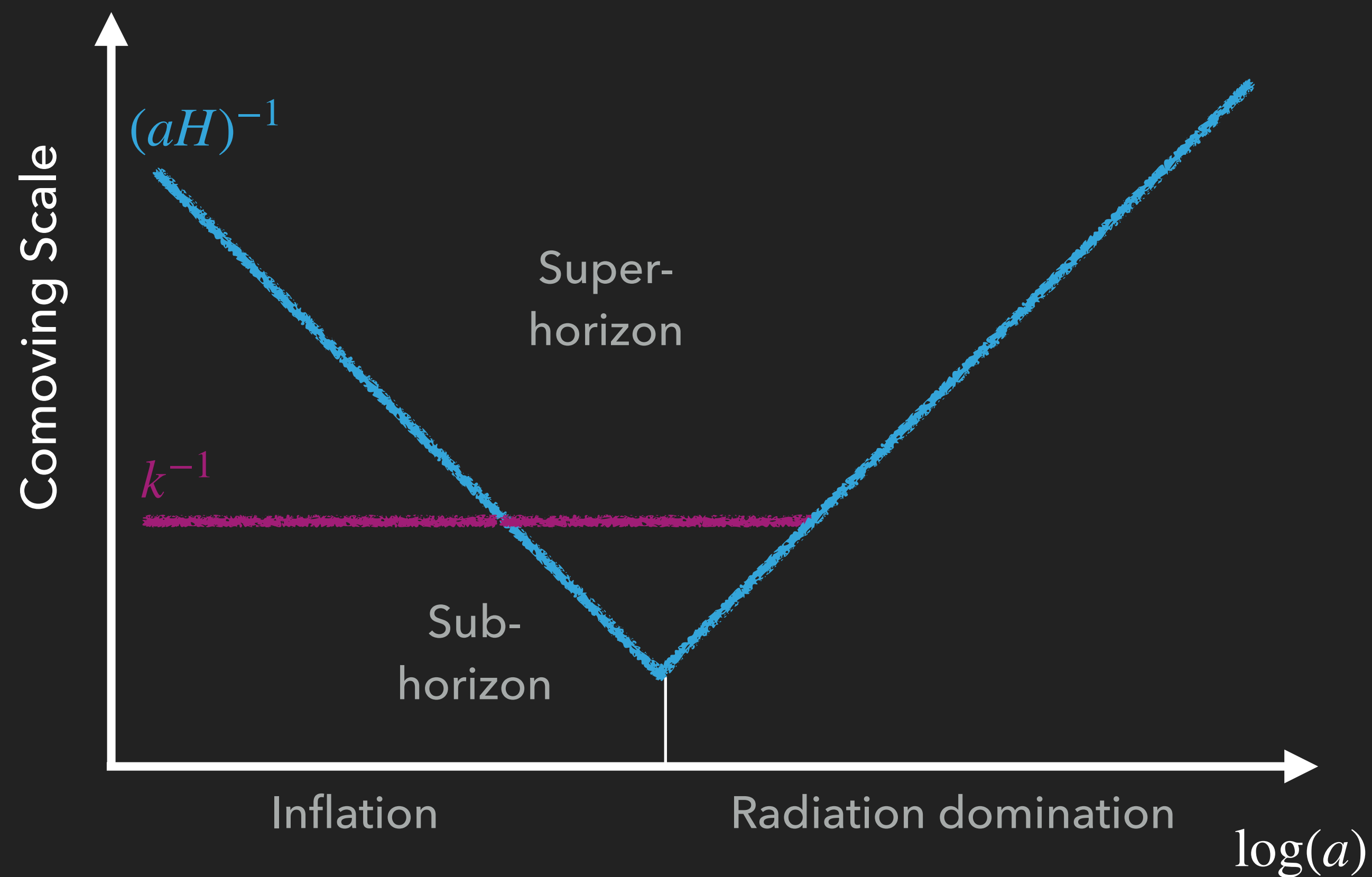


$$\frac{\Omega_{A'}}{\Omega_{\text{DM}}} \sim \sqrt{\frac{m_{A'}}{5 \cdot 10^{-5} \text{ eV}}} \left( \frac{H_I}{6 \cdot 10^{13} \text{ GeV}} \right)^2$$

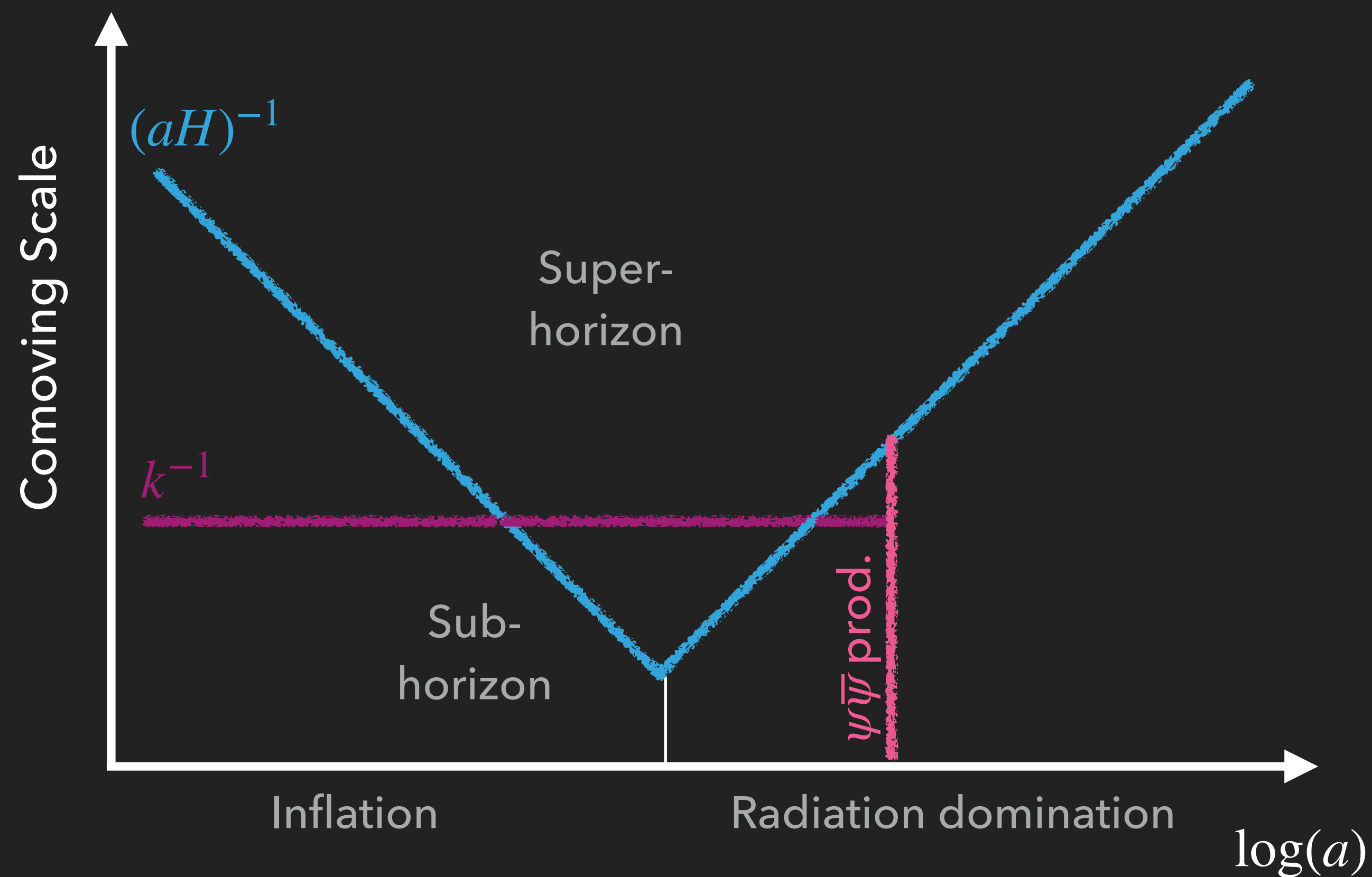




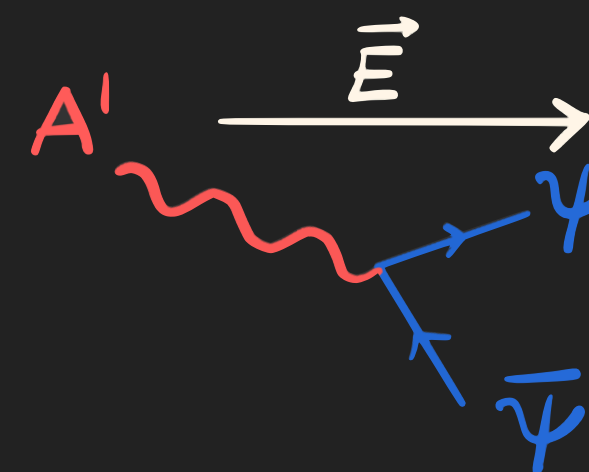
►  $A'_{L,k}$  produced during inflation

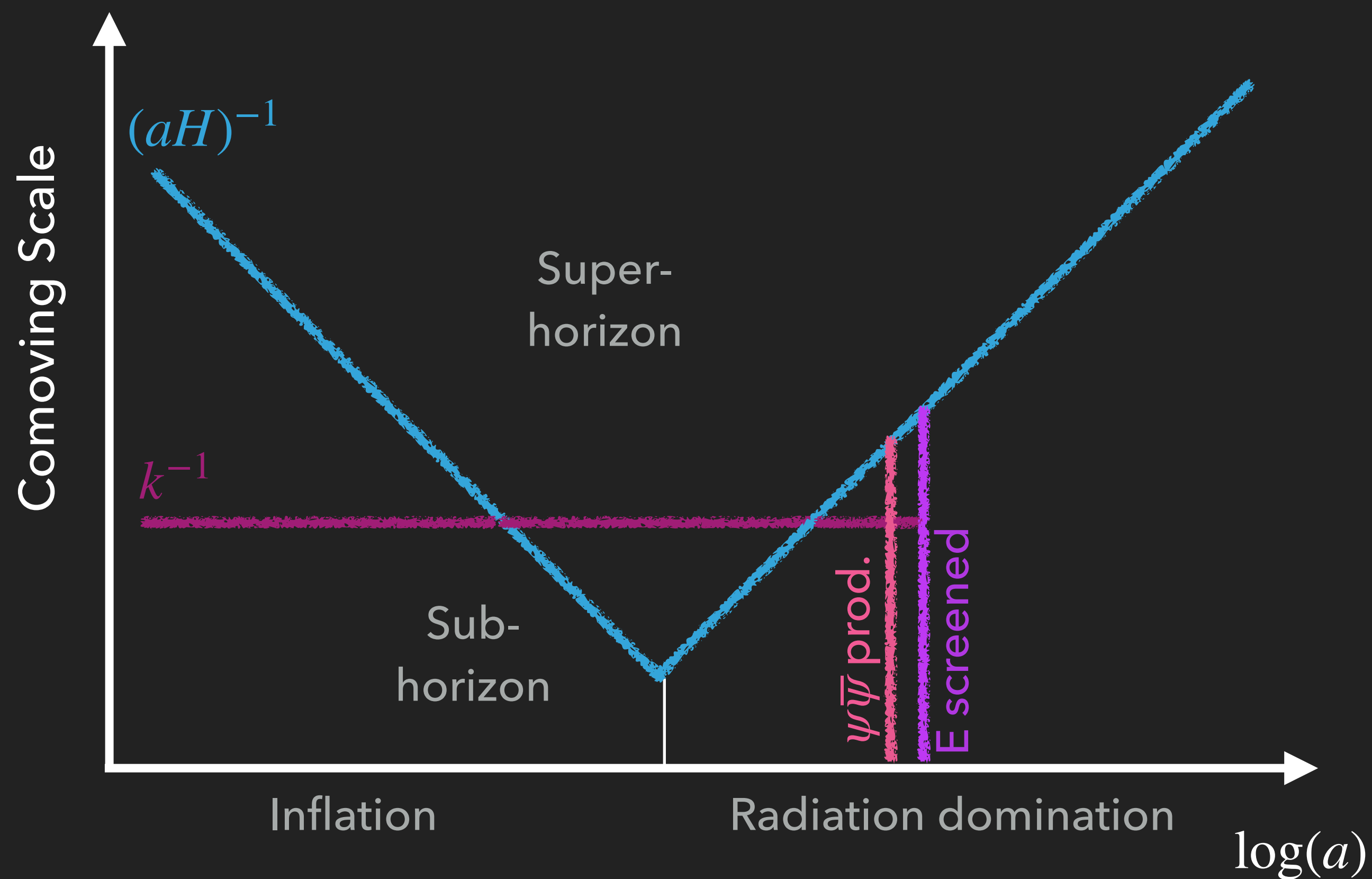




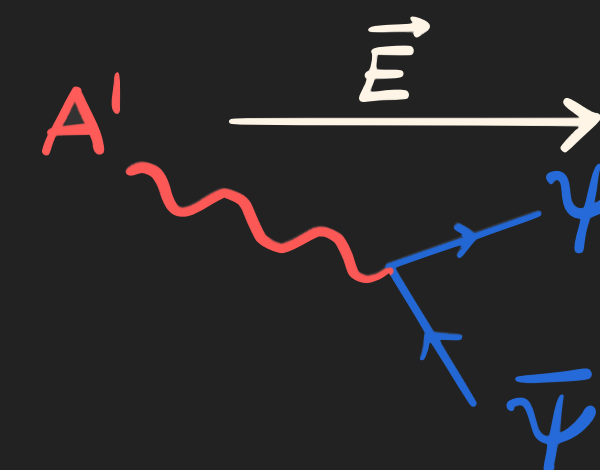


- ▶  $A'_{L,k}$  produced during inflation
- ▶ Hor. crossing: strong electric fields



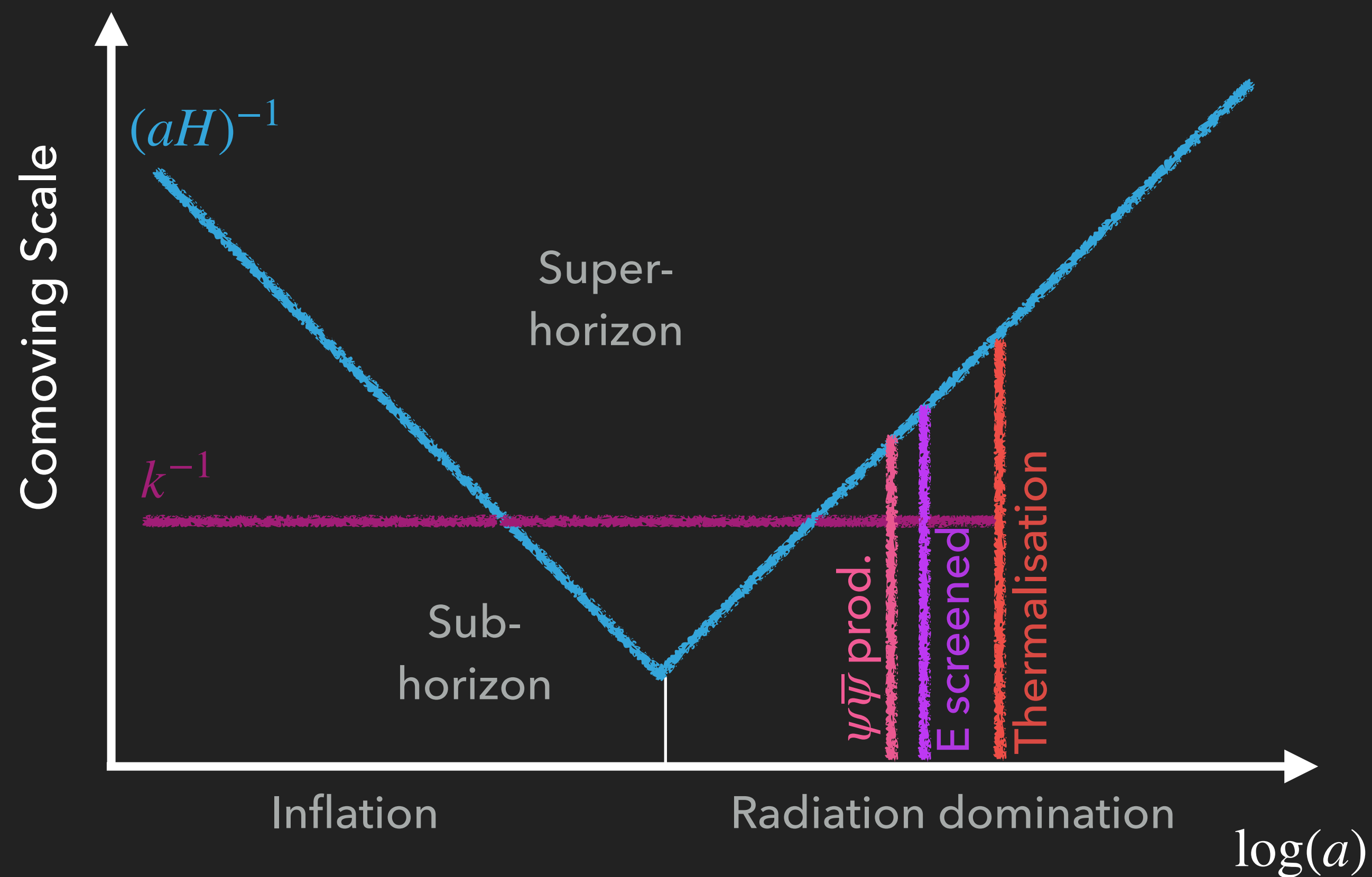


- ▶  $A'_{L,k}$  produced during inflation
- ▶ Hor. crossing: strong electric fields

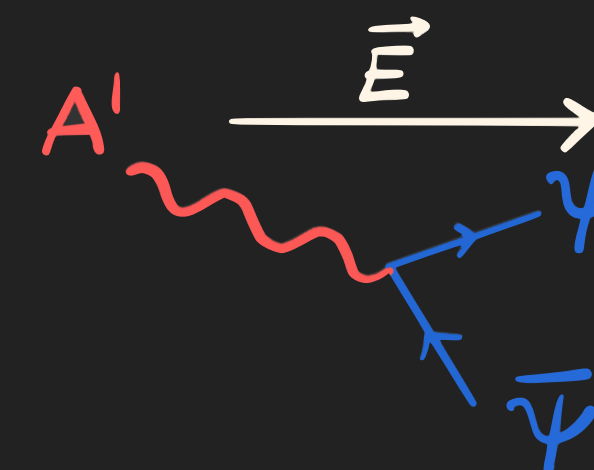


- ▶  $\psi$ 's partially screen the electric field

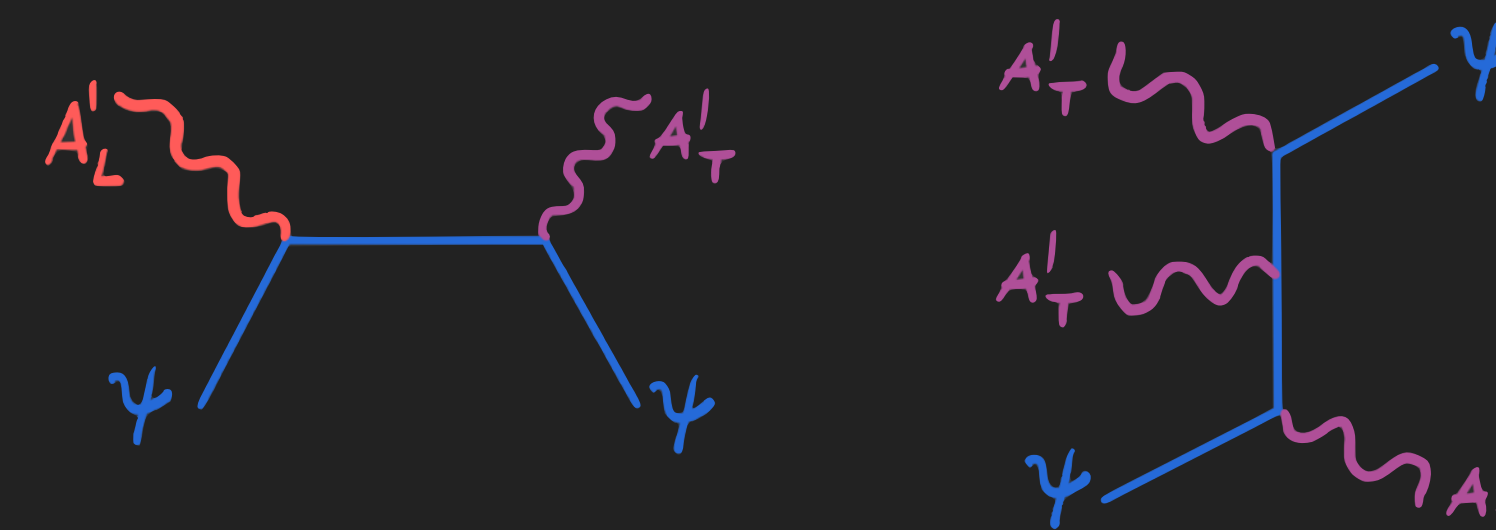


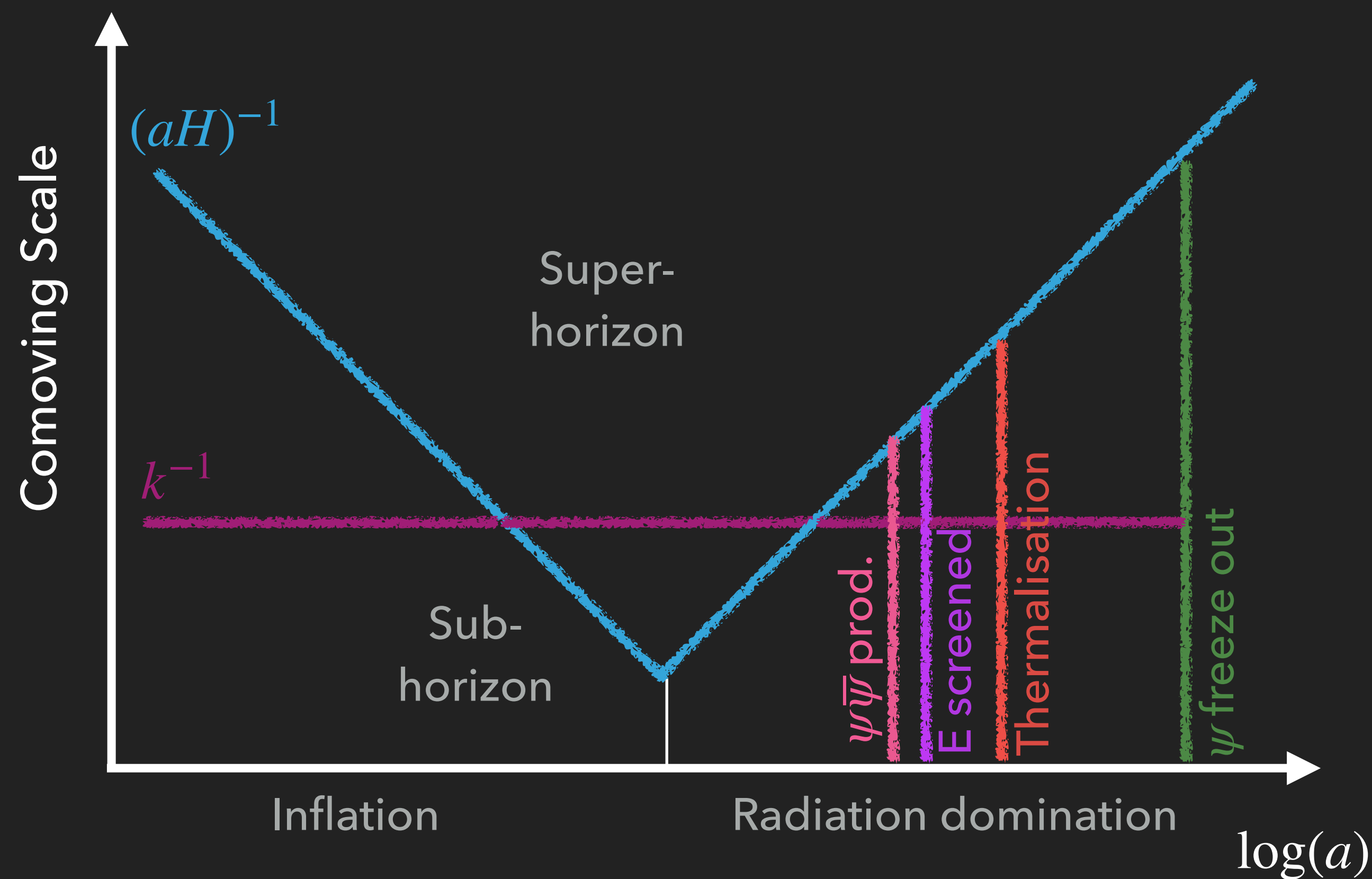


- ▶  $A'_{L,k}$  produced during inflation
- ▶ Hor. crossing: strong electric fields

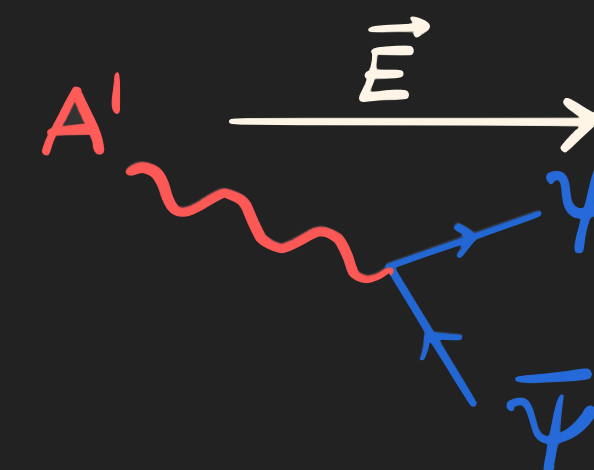


- ▶  $\psi$ 's partially screen the electric field
- ▶ Dark sector thermalises

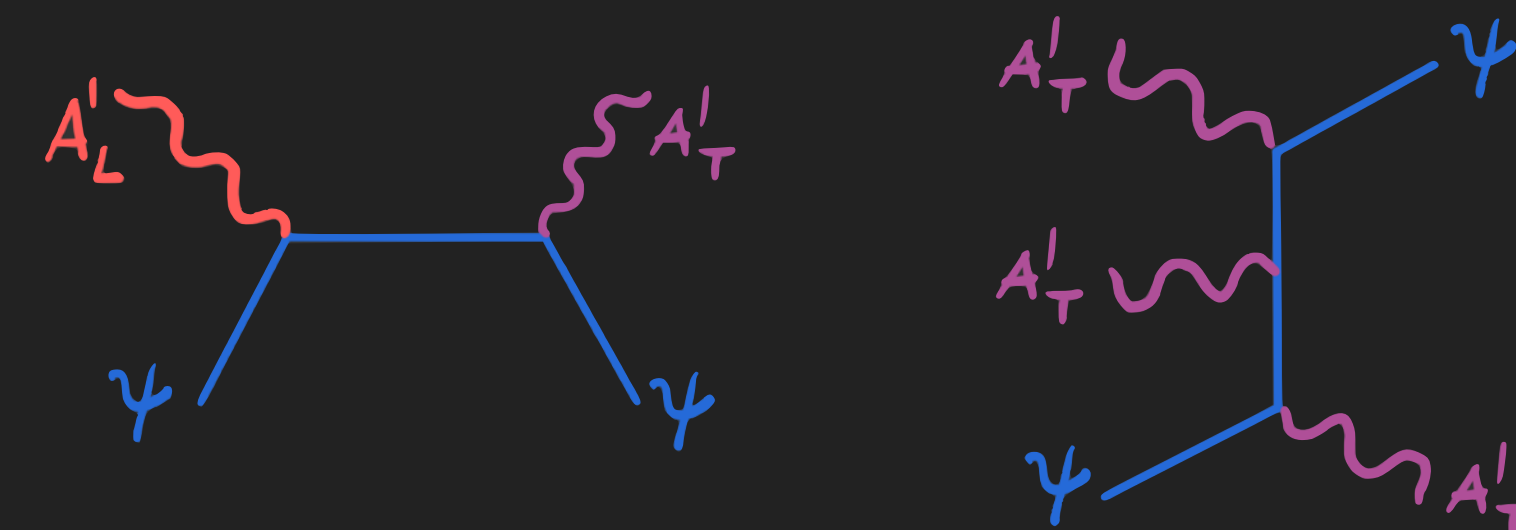




- ▶  $A'_{L,k}$  produced during inflation
- ▶ Hor. crossing: strong electric fields

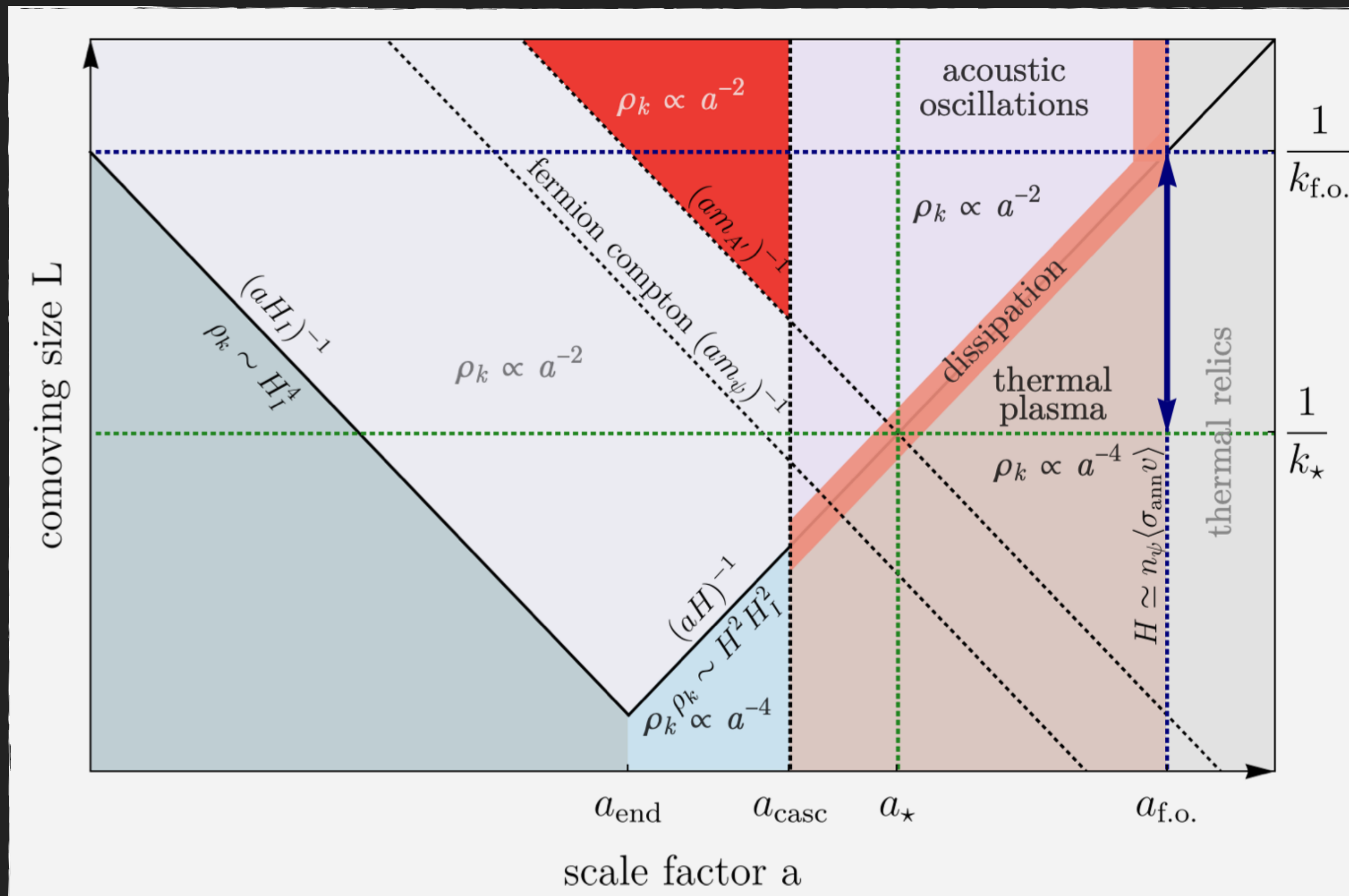


- ▶  $\psi$ 's partially screen the electric field
- ▶ Dark sector thermalises

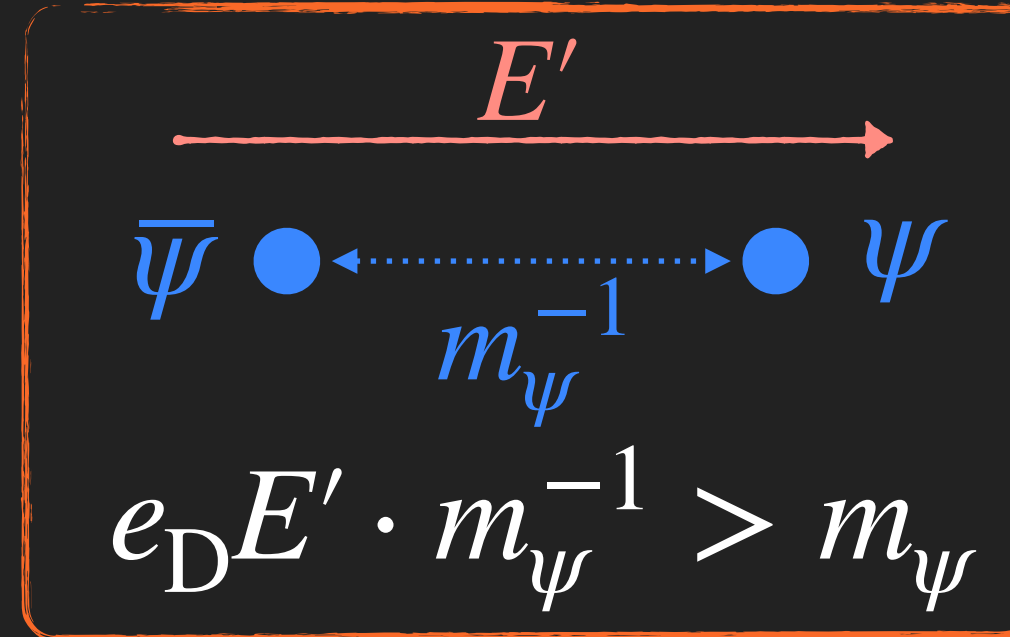


- ▶  $\psi$ 's freeze out  $\longrightarrow$  Dark Matter





Schwinger rate



Cascade rate

$$\mathcal{W}_{\text{Schwinger}} = (e_D E')^2 \exp\left(-\frac{\pi m_\psi^2}{e_D E'}\right)$$

$$\chi \approx \frac{e_D E' \omega_{A'}}{m_\psi^3}$$

$$E' \sim \partial_t A'_L \sim H \frac{m_{A'}^2}{H^2} \frac{H H_I}{m_{A'}} \sim m_{A'} H_I$$

$$\mathcal{W}_{\text{casc}} = \frac{dN_{\psi\bar{\psi}}}{dt dV} \sim n_{A'} \frac{m_{A'}^2}{\omega_{A'}^2} \cdot \begin{cases} e_D^3 \frac{E'}{m_\psi} e^{-8/(3\chi)} & \chi \lesssim 1 \\ e_D^{8/3} \frac{E'^{2/3}}{\omega_{A'}^{1/3}} & \chi \gg 1 \end{cases}$$

Longitudinal suppression



Maxwell eq.

$$(\omega^2 - k^2 - m_{A'}^2)\vec{A} = -\vec{J}$$

$$\vec{J} = e_D n_\psi \vec{u}$$

Lorentz eq.

$$m_\psi \partial_t \vec{u} = -e_D \vec{E} - 2m_\psi \nu \vec{u}$$

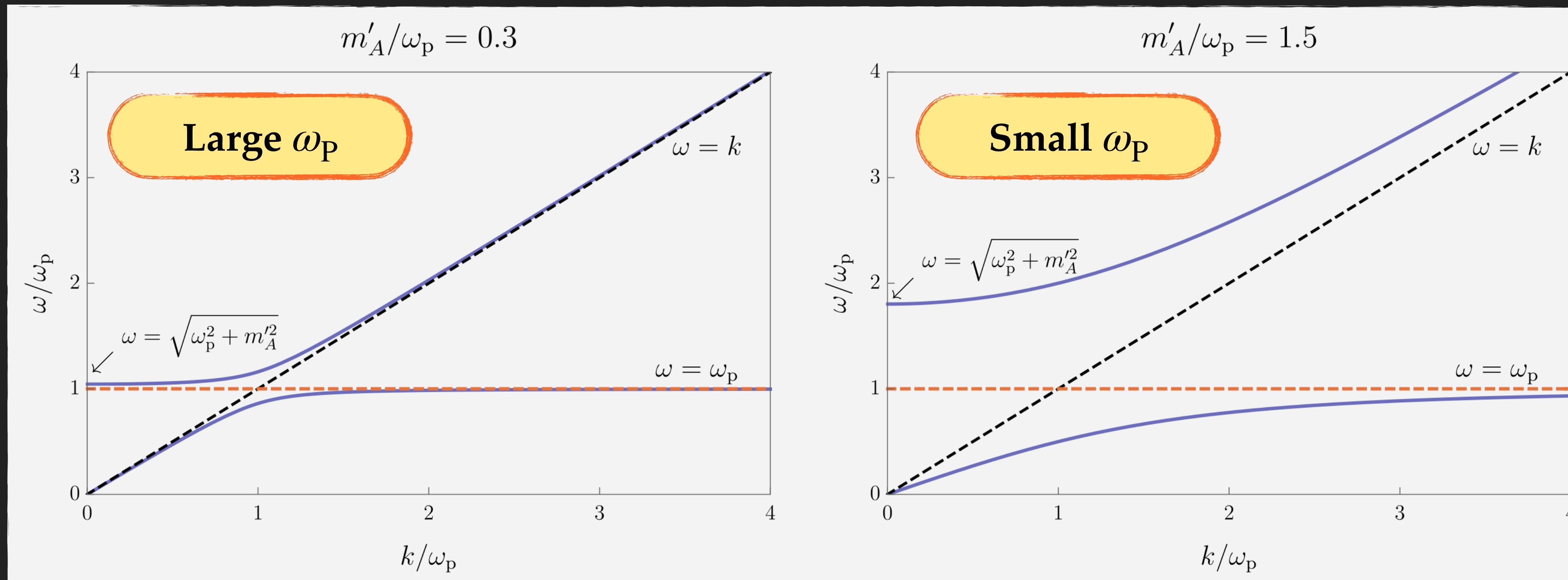
$\nu$  : collision rate of  $\psi$

Dispersion relation for  $A'_L$

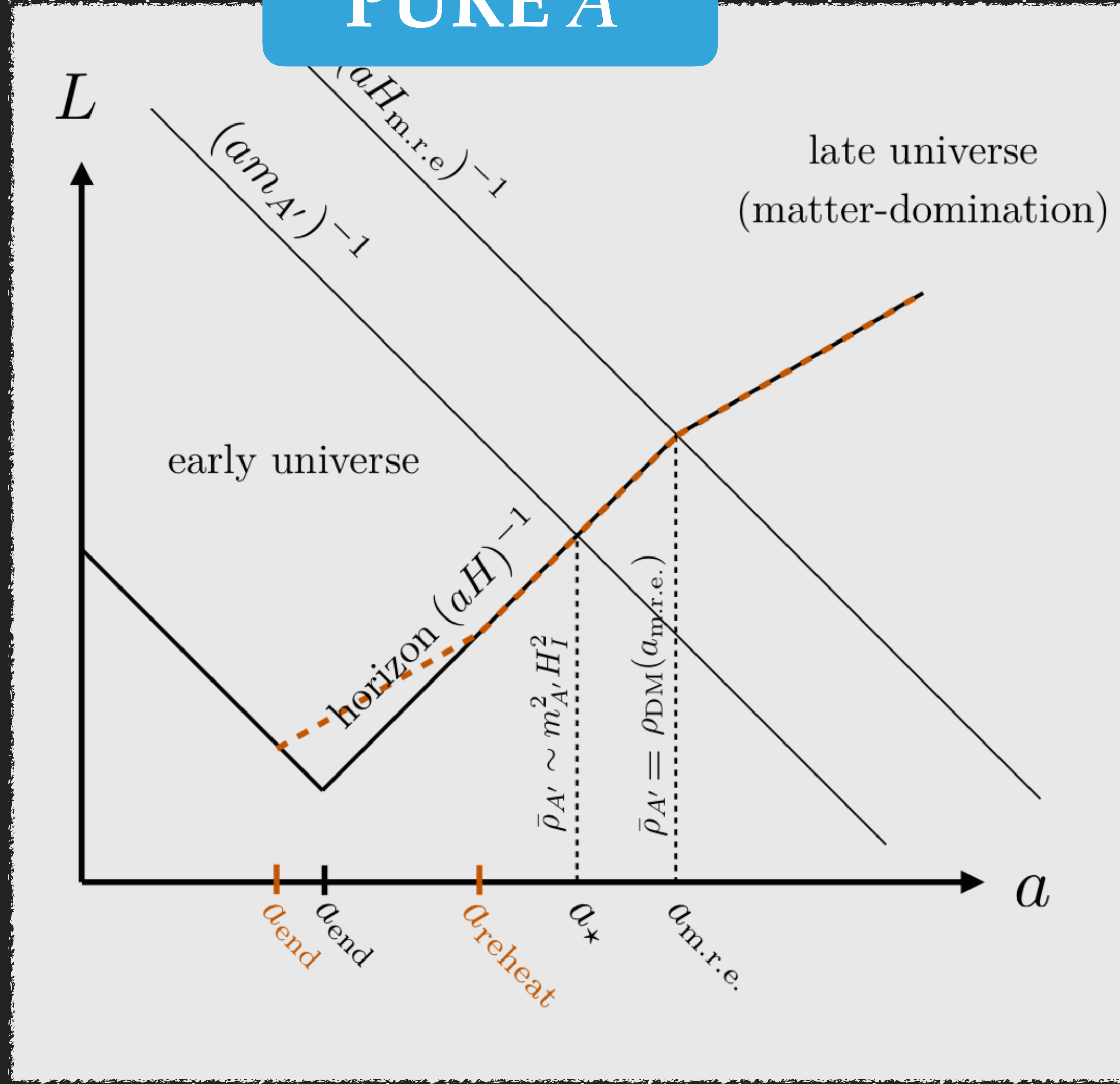
$$\omega^2 - k^2 - m_{A'}^2 = \omega_P^2 \frac{\omega}{\omega + 2i\nu} \left( 1 - \frac{k^2}{\omega^2} \right)$$

Plasma mass

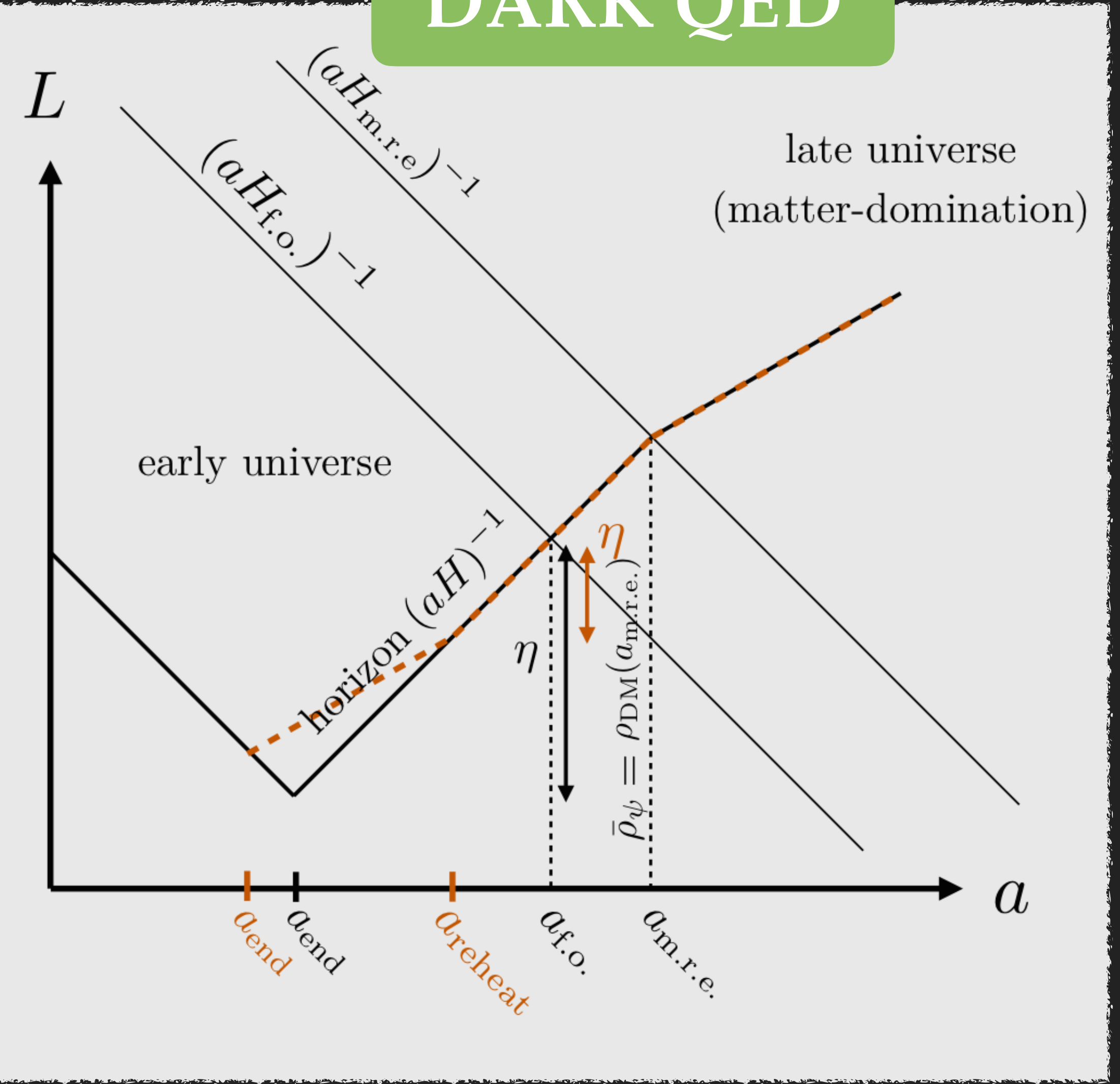
$$\omega_P^2 \equiv \frac{e_D^2 n_\psi}{E_\psi}$$



PURE A'



DARK QED

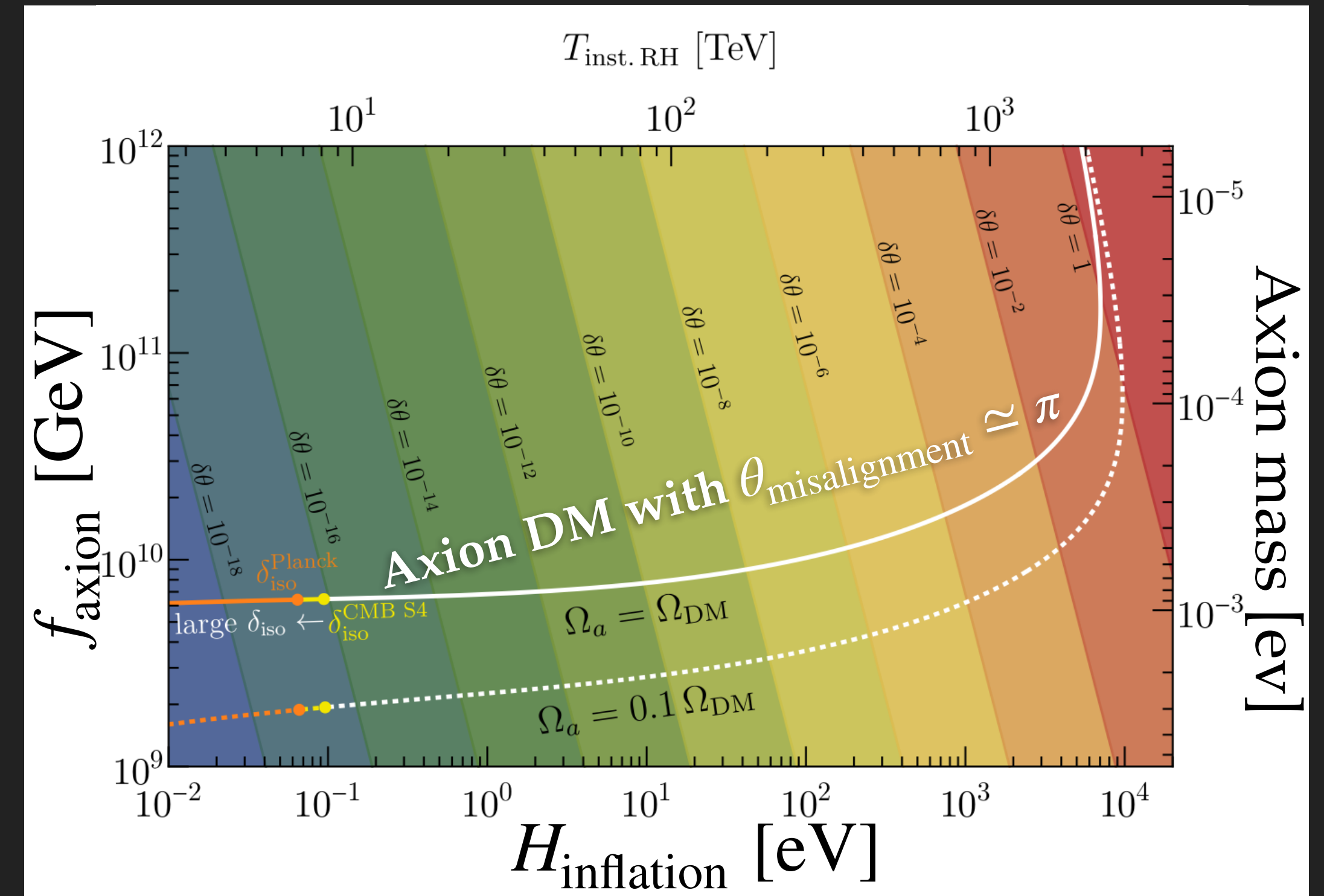




- ▶ Pre-inflationary scenario: depends on

$$\begin{cases} m_{\text{axion}} & \rightarrow \text{experimental target} \\ \theta_{\text{misalignment}} & \rightarrow \text{astro / cosmo implications?} \end{cases}$$

- ▶ Pre-inflationary scenario: depends on
    - $m_{\text{axion}} \rightarrow$  experimental target
    - $\theta_{\text{misalignment}} \rightarrow$  astro / cosmo implications?
  - ▶  $\theta_{\text{misalignment}} \simeq \pi \Rightarrow$  dense substructures
  - ▶ Can be realised in a minimal model
- ['19 Arvanitaki+]
- ['20 Huang, Madden, DR, Reig]

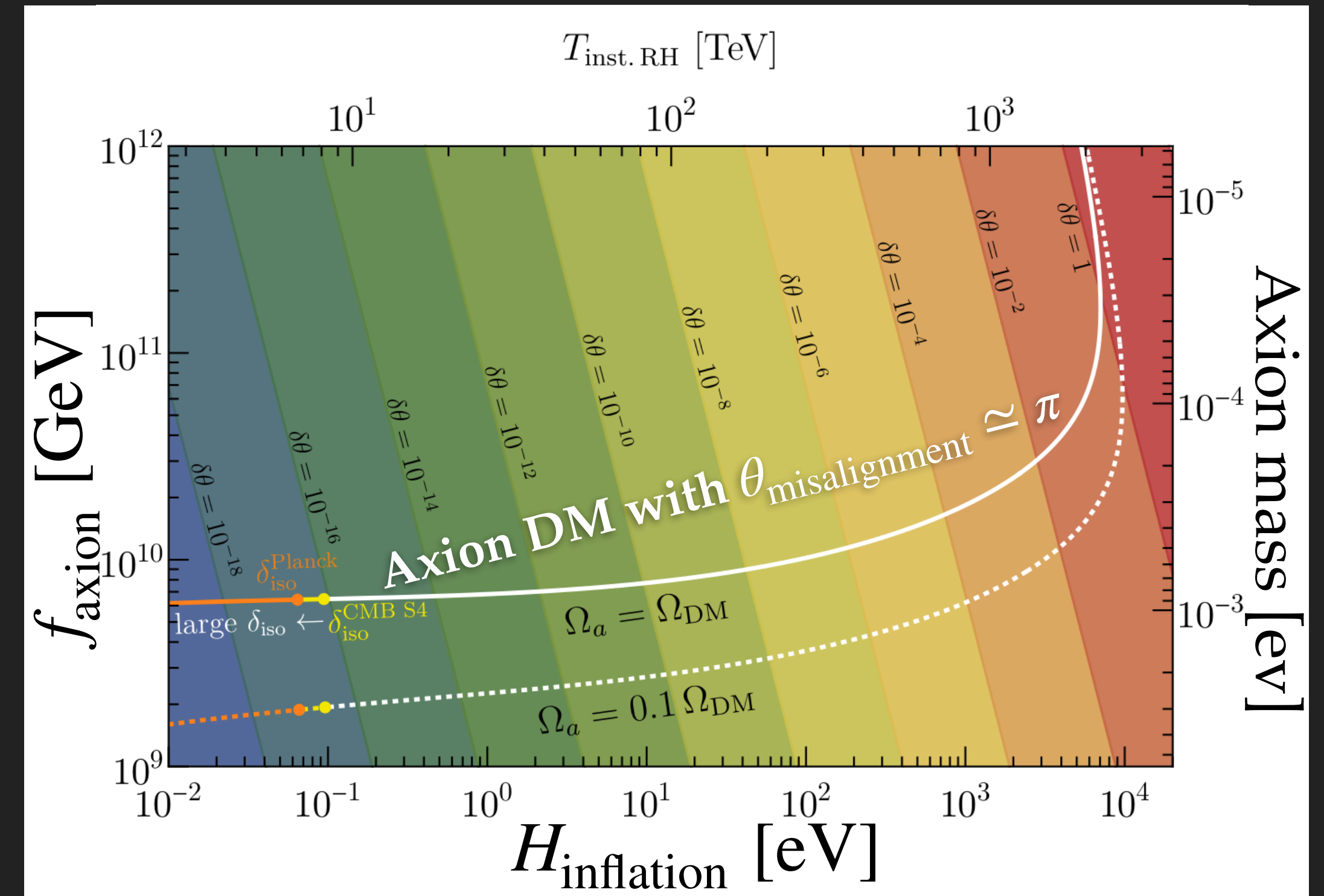




- ▶ Pre-inflationary scenario: depends on
  - $m_{\text{axion}} \rightarrow$  experimental target
  - $\theta_{\text{misalignment}} \rightarrow$  astro / cosmo implications?

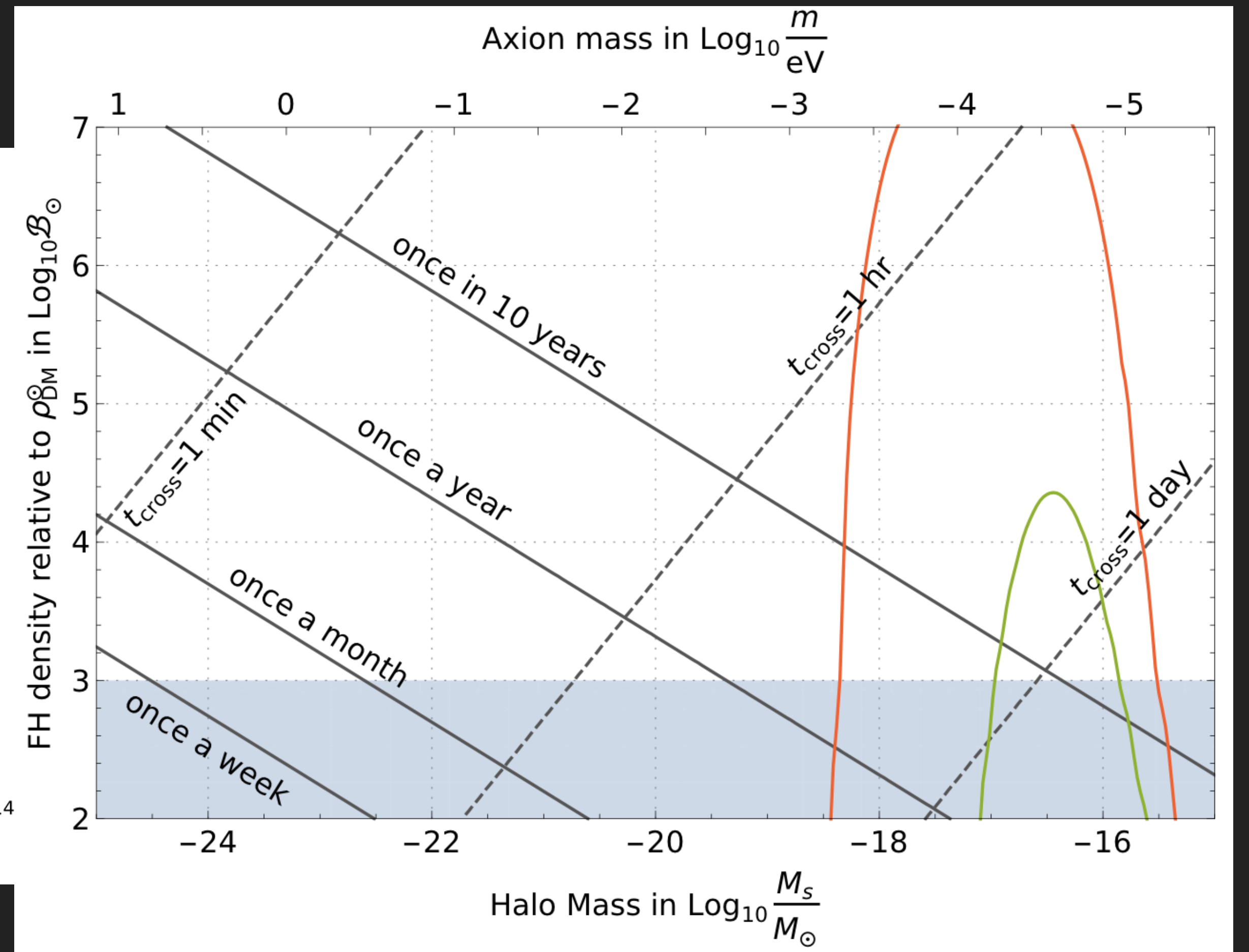
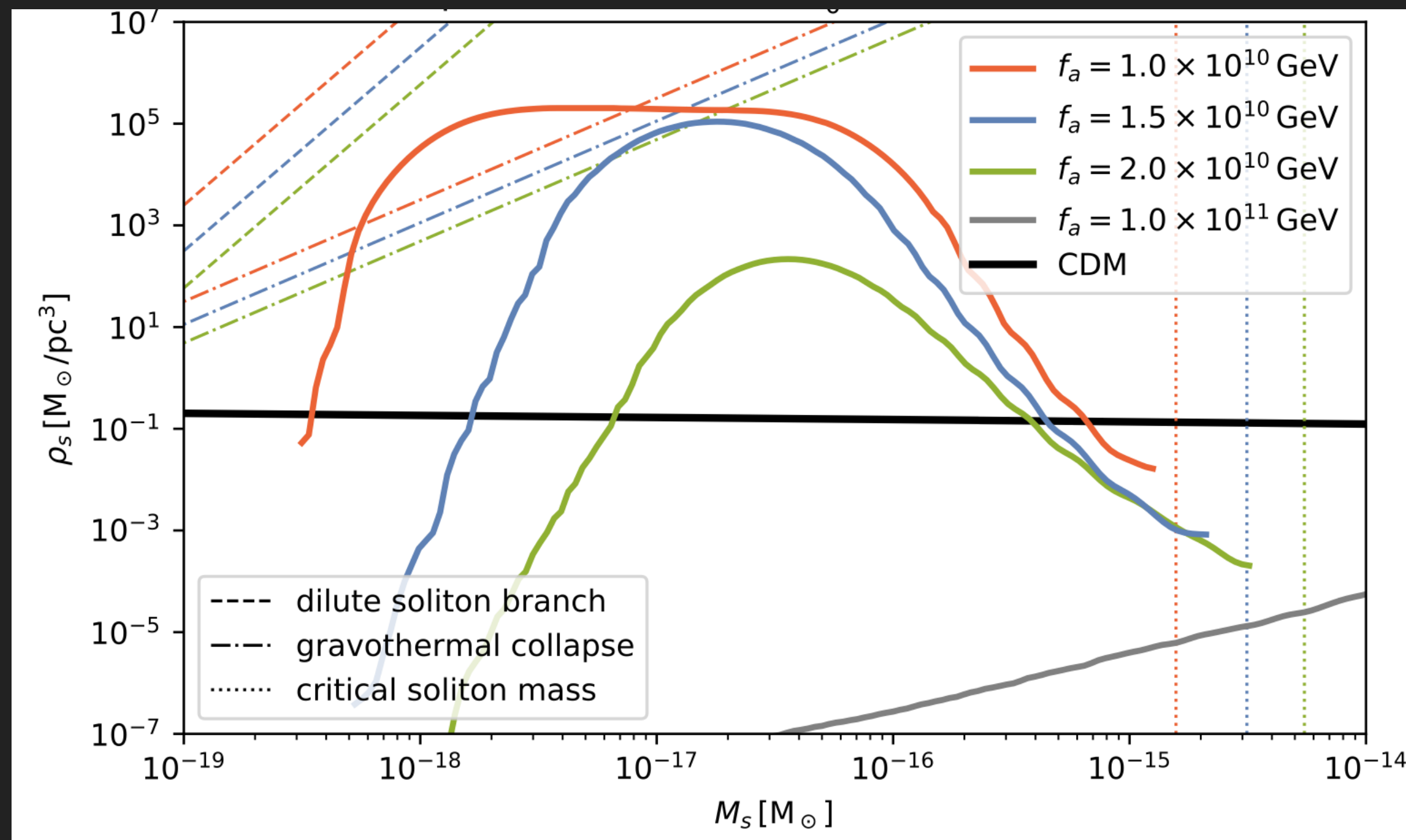
- ▶  $\theta_{\text{misalignment}} \simeq \pi \Rightarrow$  dense substructures
- ▶ Can be realised in a minimal model

[‘19 Arvanitaki+]  
[‘20 Huang, Madden, DR, Reig]



- ▶ Large  $H_{\text{inflation}} \Rightarrow$  fluctuations in  $\theta_{\text{misalignment}} \Rightarrow$  excluded by isocurvature
- ▶ How general is this conclusion?

[(work in progress) Graham, DR]





- Large initial misalignment: affect QCD axion mass, and clump DM substructures

$$\theta = \theta_{\text{SM}} + \frac{a}{f_{\text{axion}}} + \arg[M_{\text{quarks}}]$$

$y \phi \bar{q}q$

Colored fermion  
 $\lesssim 10 - 100 \text{ TeV}$

Scalar: flip sign

$m_{\text{axion DM}} \sim \text{meV}$

