Multigluon correlations in JIMWLK

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Abstract

We describe recent progress in understanding two-particle correlations in the dilute-dense system, e.g. in forward dihadron production in deuteron-gold collisions. This requires computing the energy dependence of higher point Wilson line correlators from the JIMWLK renormalization group equation. We find that the large N_c approximation used so far in the phenomenological literature is not very accurate. On the other hand a Gaussian finite N_c approximation is surprisingly close to the full result.

Outline

Topics of interest:

1. *d*Au collisions at forward rapidity;
2. Ridge in pp/AA; length in η

Outline:

1. 4- and 6-point functions of Wilson lines in JIMWLK evolution
2. Unequal rapidity correlations in JIMWLK evolution

References:

1. A. Dumitru, J. Jalilian-Marian, T. Lappi, B. Schenke and R. Venugopalan, Phys.Lett. B706 (2011) 219
2. F. Gelis, T. Lappi and R. Venugopalan, Phys. Rev. D **79** (2009) 094017; T.L., B. Schenke, R. Venugopalan, in progress

2-particle correlation in forward pA

- Quark from p (large x) from pdf
- Radiate gluon
- Propagate eikonally in color field of target A $\Longrightarrow$ Wilson lines U

$$\frac{\mathrm{d}\sigma^{qA\to qgX}}{\mathrm{d}^3k_1\ \mathrm{d}^3k_2} \propto \int_{\mathbf{x}_T,\bar{\mathbf{x}}_T,\mathbf{y}_T,\bar{\mathbf{y}}_T} e^{-i\mathbf{k}_{T1}\cdot(\mathbf{x}_T-\bar{\mathbf{x}}_T)}\ e^{-i\mathbf{k}_{T2}\cdot(\mathbf{y}_T-\bar{\mathbf{y}}_T)}\ \mathcal{F}(\bar{\mathbf{x}}_T-\bar{\mathbf{y}}_T,\mathbf{x}_T-\mathbf{y}_T)$$

$$\Big\langle \hat{Q}(\mathbf{y}_T,\bar{\mathbf{y}}_T,\bar{\mathbf{x}}_T,\mathbf{x}_T)\ \hat{D}(\mathbf{x}_T,\bar{\mathbf{x}}_T) - \hat{D}(\mathbf{y}_T,\mathbf{x}_T)\hat{D}(\mathbf{x}_T,\bar{\mathbf{z}}_T) - \hat{D}(\mathbf{z}_T,\bar{\mathbf{x}}_T)\hat{D}(\bar{\mathbf{x}}_T,\bar{\mathbf{y}}_T)$$

$$+\frac{C_F}{N_c}\hat{D}(\mathbf{z}_T,\bar{\mathbf{z}}_T) + \frac{1}{{N_c}^2}\Big(\hat{D}(\mathbf{y}_T,\bar{\mathbf{z}}_T) + \hat{D}(\mathbf{z}_T,\bar{\mathbf{y}}_T) - \hat{D}(\mathbf{y}_T,\bar{\mathbf{y}}_T)\Big)\Big\rangle_{\text{target}}$$

$(\mathbf{z}_T = z\mathbf{x}_T + (1-z)\mathbf{y}_T,\ \bar{\mathbf{z}}_T = z\bar{\mathbf{x}}_T + (1-z)\bar{\mathbf{y}}_T.)$

Need target expectation values of Wilson line operators

$$\hat{D}(\mathbf{x}_T-\mathbf{y}_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T)$$

$$\hat{Q}(\mathbf{x}_T,\mathbf{y}_T,\mathbf{u}_T,\mathbf{v}_T) \equiv \frac{1}{N_c}\,\mathrm{Tr}\,U(\mathbf{x}_T)U^\dagger(\mathbf{y}_T)U(\mathbf{u}_T)U^\dagger(\mathbf{v}_T)$$

JIMWLK evolution

Need Wilson lines from probability distribution $W_y[U]$.

Energy/rapidity dependence of $W_y[U]$ from JIMWLK RGE:

$$\partial_y W_y[U(\mathbf{x}_T)] = \mathcal{H} W_y[U(\mathbf{x}_T)]$$

Here the JIMWLK Hamiltonian is:

$$\mathcal{H} \equiv \frac{1}{2} \int\limits_{\mathbf{x}_T \mathbf{y}_T \mathbf{z}_T} \frac{\delta}{\delta \widetilde{\mathcal{A}}_c^+(\mathbf{y}_T)} \mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) \cdot \mathbf{e}_T^{ca}(\mathbf{y}_T, \mathbf{z}_T) \frac{\delta}{\delta \widetilde{\mathcal{A}}_b^+(\mathbf{x}_T)},$$

$$\mathbf{e}_T^{ba}(\mathbf{x}_T, \mathbf{z}_T) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}_T - \mathbf{z}_T}{(\mathbf{x}_T - \mathbf{z}_T)^2} \left(1 - U^\dagger(\mathbf{x}_T) U(\mathbf{z}_T)\right)^{ba}$$

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Numerics using Langevin formulation

$$U_{y+\mathrm{d}y}(\mathbf{x}_T) = U_y(\mathbf{x}_T) e^{-i\,\mathrm{d}y\,\alpha(\mathbf{x}_T, y)}$$

$$\alpha^a(\mathbf{x}_T, y) = \sigma^a(\mathbf{x}_T, y) + \int_{\mathbf{z}_T} \mathbf{e}_T^{ab}(\mathbf{x}_T, \mathbf{z}_T) \eta_T{}^b(\mathbf{z}_T, y)$$

Approximations for 4-point function $\langle \hat{Q} \rangle$

Cf. talk by Triantafyllopoulos

Motivation for approximations

Getting the dipole $\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \rangle$ is easy from BK; an approximation using only $\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \rangle$ is much easier for practical work.

- In phenomenology of 2-particle correlations, (Marquet 2007, Tuchin 2009, Albacete & Marquet 2010) only used "naive large N_c" approximation:

$$\left\langle \hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) \right\rangle = \left\langle \frac{1}{N_c} \operatorname{Tr} U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) U(\mathbf{u}_T) U^\dagger(\mathbf{v}_T) \right\rangle$$
$$\underset{N_c \to \infty}{\approx} \frac{1}{2} \left\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \right\rangle \left\langle \hat{D}(\mathbf{u}_T, \mathbf{v}_T) \right\rangle + \left\langle \hat{D}(\mathbf{x}_T, \mathbf{v}_T) \right\rangle \left\langle \hat{D}(\mathbf{u}_T, \mathbf{y}_T) \right\rangle$$

- We also compare to "Gaussian" approximation, where $\left\langle \hat{Q}(\mathbf{x}_T, \mathbf{y}_T, \mathbf{u}_T, \mathbf{v}_T) \right\rangle$ is related to $\left\langle \hat{D}(\mathbf{x}_T, \mathbf{y}_T) \right\rangle$ assuming Gaussian correlators for Wilson lines.

  "GT" of Kuokkanen, Rummukainen, Weigert

Choose two coordinate configurations

Evaluating cross section nontrivial see talk by H. Mäntysaari

$\Longrightarrow$ Study $\hat{Q} = \frac{1}{N_c} \mathrm{Tr}\, U(\mathbf{x}_T) U^\dagger(\mathbf{y}_T) U(\mathbf{u}_T) U^\dagger(\mathbf{v}_T)$ for 2 special configs

Line $\mathbf{u}_T = \mathbf{x}_T; \quad \mathbf{v}_T = \mathbf{y}_T$ "Naive large N_c" $Q_|^{\mathrm{naive}}(r) = D(r)^2$

$$Q_|^{\mathrm{Gaussian}}(r) \approx \frac{N_c+1}{2}\Big(D(r)\Big)^{2\frac{N_c+2}{N_c+1}} - \frac{N_c-1}{2}\Big(D(r)\Big)^{2\frac{N_c-2}{N_c-1}}$$

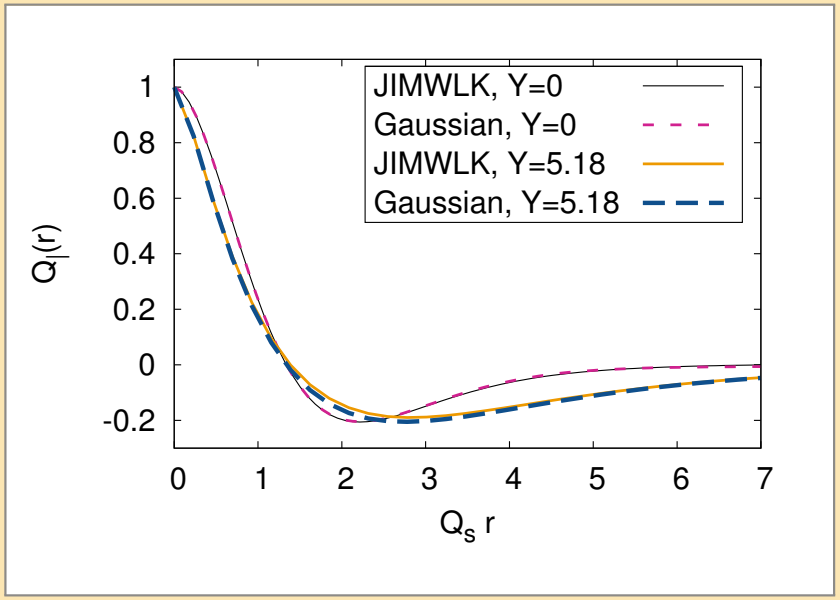
Square "Naive large N_c" $Q_\square^{\mathrm{naive}}(r) = D(r)^2$

$$Q_\square^{\mathrm{Gaussian}}(r) \approx (D(r))^2 \left[\frac{N_c+1}{2} \left(\frac{D(r)}{D(\sqrt{2}r)} \right)^{\frac{2}{N_c+1}} - \frac{N_c-1}{2} \left(\frac{D(\sqrt{2}r)}{D(r)} \right)^{\frac{2}{N_c-1}} \right].$$

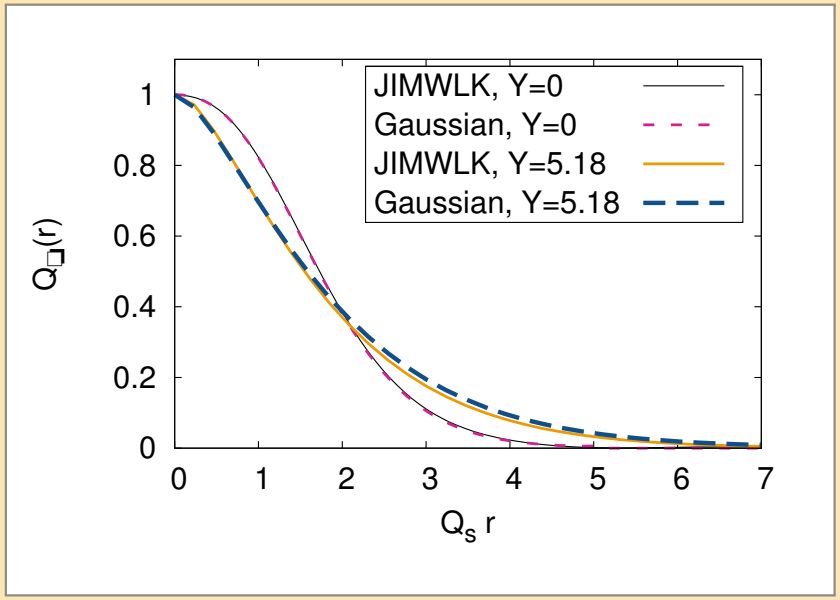
Gaussian is good

Initial condition $y = 0$ satisfies Gaussian approximation by construction.

But JIMWLK stays very close at later rapidities.

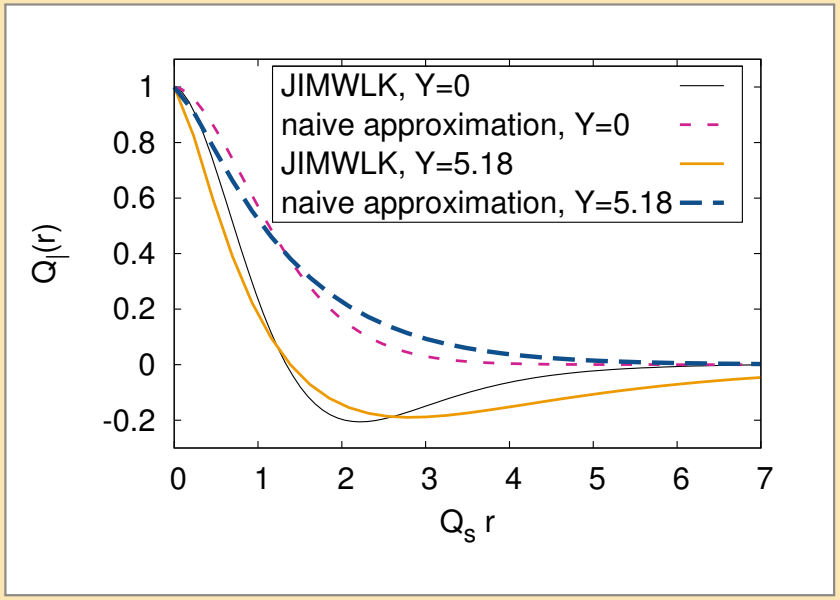


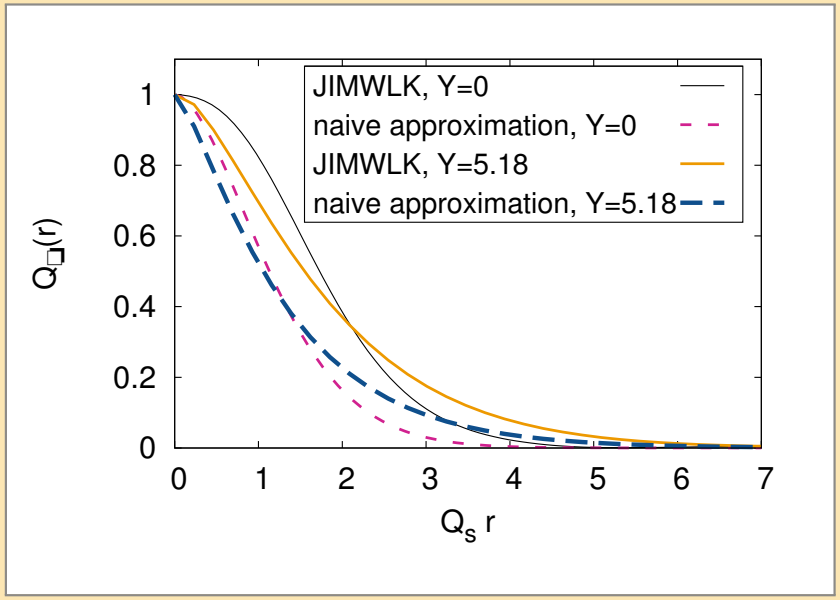
Line



Square

Naive large N_c is not good





Line

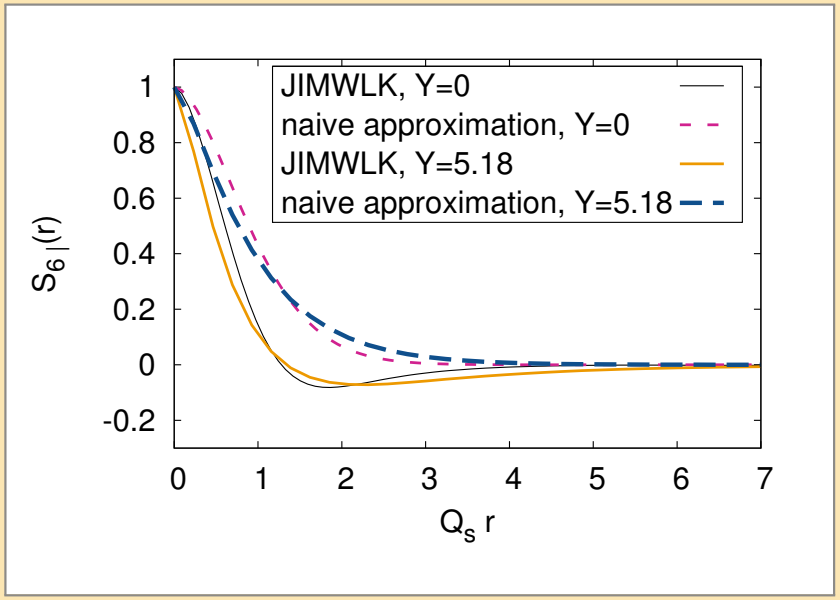
Square

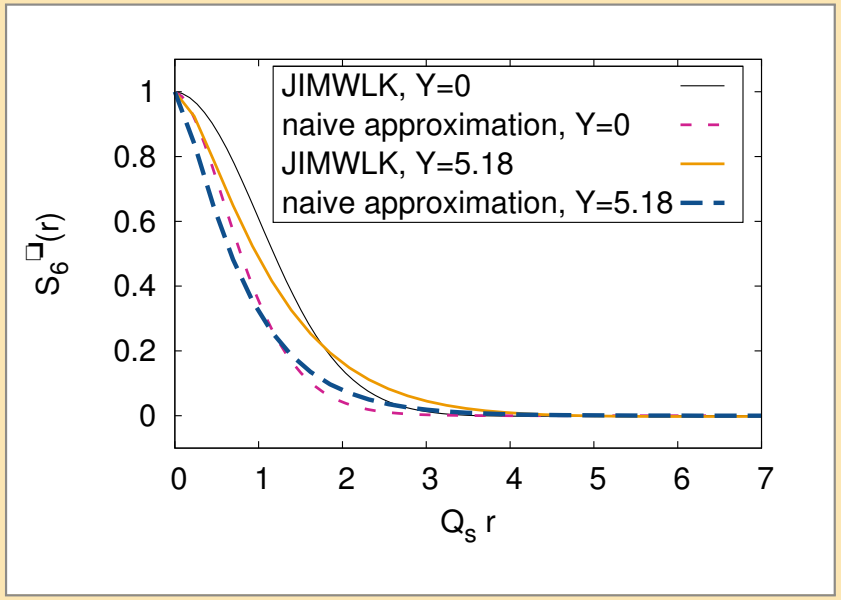
Even characteristic length/momentum scale differs by factor ~ 2.

6pt function

- Actually cross section has not $\langle \hat{Q} \rangle$, but $\langle \hat{Q}\hat{D} \rangle$.

As expected, the "naive large N_c" approximation for the 6pt function is as bad as for the 4pt function.





Line

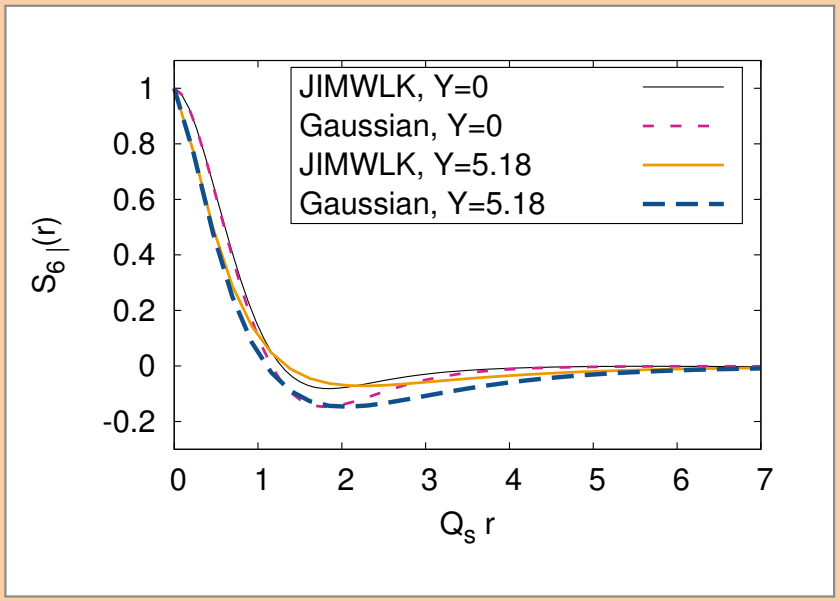
Square

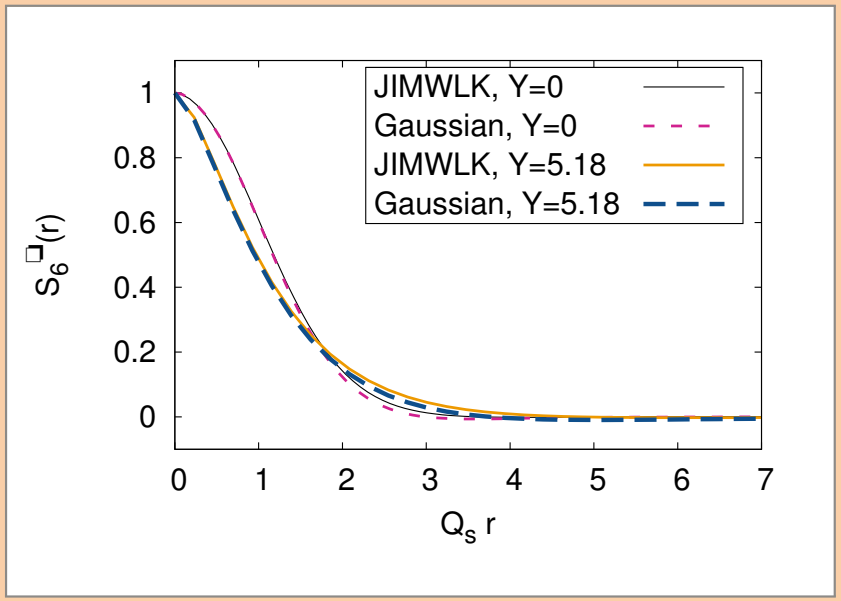
6pt function: “factorized” Gaussian approximation

- Actually cross section has not $\langle \hat{Q} \rangle$, but $\langle \hat{Q}\hat{D} \rangle$.
- Gaussian (MV) for this not known when this work was done, cf. Triantafyllopoulos
- But: we know Kuokkanen et al that $\langle \hat{D}\hat{D} \rangle \approx \langle \hat{D} \rangle \langle \hat{D} \rangle$ works pretty well
- Compare JIMWLK with “factorized” ∼Gaussian approx. $\langle \hat{Q}\hat{D} \rangle \approx \langle \hat{Q} \rangle \langle \hat{D} \rangle$

6pt: “factorized ” ∼Gaussian works pretty well

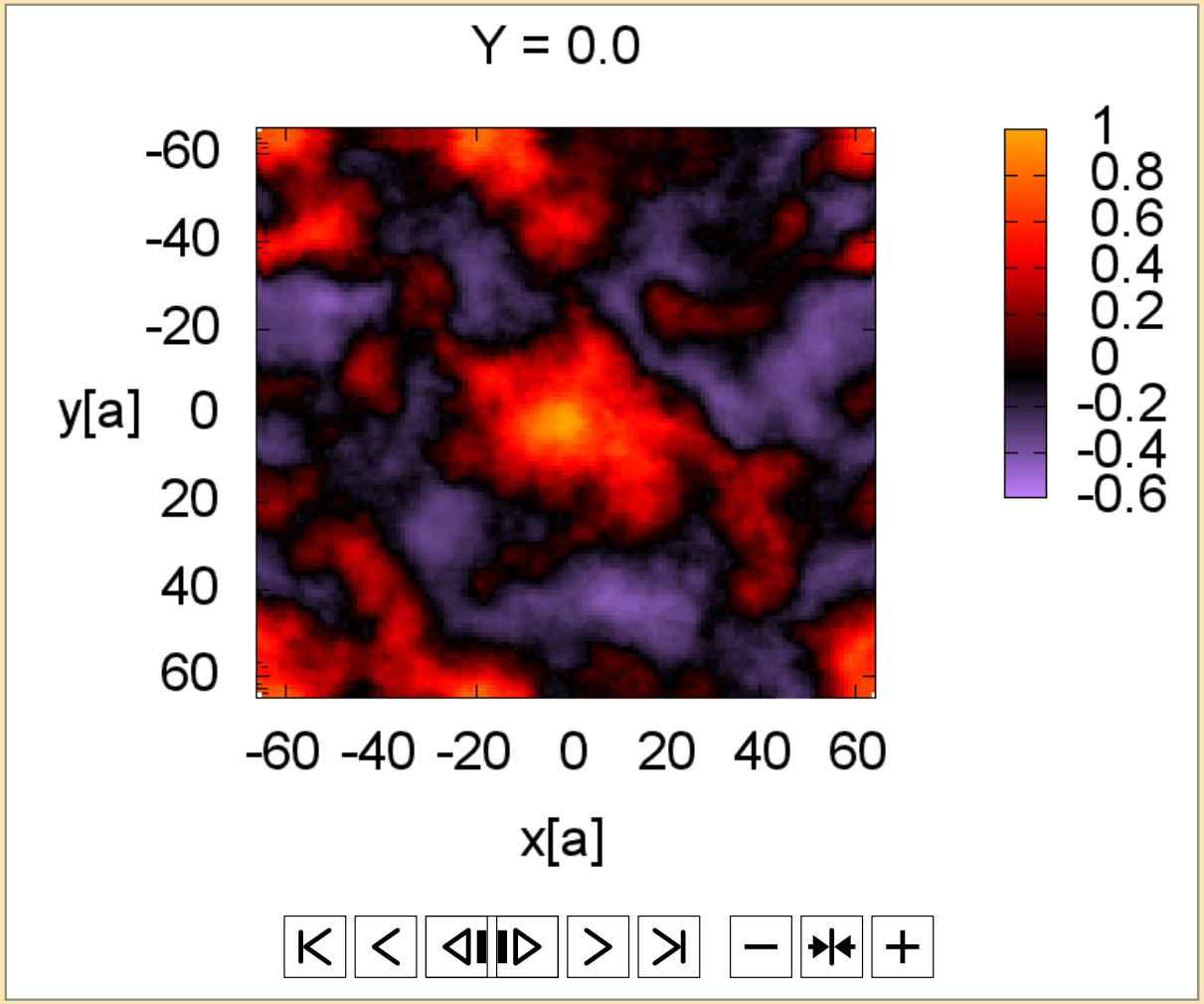
At least for small r, which counts for $p_T \gtrsim Q_s$





Line Square

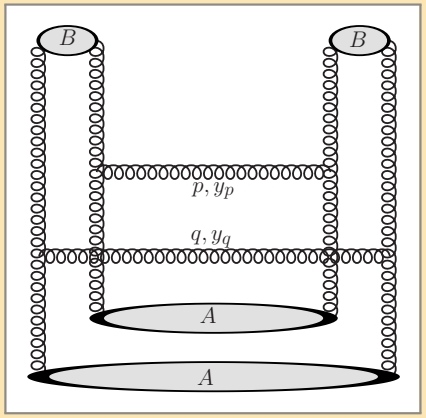
Visualization of correlations in JIMWLK



Correlation $\frac{1}{N_c}\mathrm{Re}\,\mathrm{Tr}\,U^\dagger(0,0)U(x,y)$ between origin $(0,0)$ and (x,y)
$\Longrightarrow$ correlation length decreases for increasing energy.

Unequal rapidity: ridge in AA or pp

- Equation derived in K. Dusling, F. Gelis, T.L. and R. Venugopalan, *Nucl. Phys.* **A836** (2010) 159
- Used e.g. in A. Dumitru, K. Dusling, F. Gelis, J. Jalilian-Marian, T. L. and R. Venugopalan, Phys. Lett. B **697** (2011) 21 ; K. Dusling and R. Venugopalan, arXiv:1201.2658 [hep-ph].



$$C(\mathbf{p},\mathbf{q}) \sim \int_{\mathbf{k}_T} \Big\{ \Phi_B^2(\mathbf{k}_T)\Phi_A(\mathbf{p}_T-\mathbf{k}_T) \times \Big[\Phi_A(\mathbf{q}_T+\mathbf{k}_T)+\Phi_A(\mathbf{q}_T-\mathbf{k}_T)\Big] + (\mathbf{k}_T \leftrightarrow -\mathbf{k}_T) + (A \leftrightarrow B) \Big\}$$

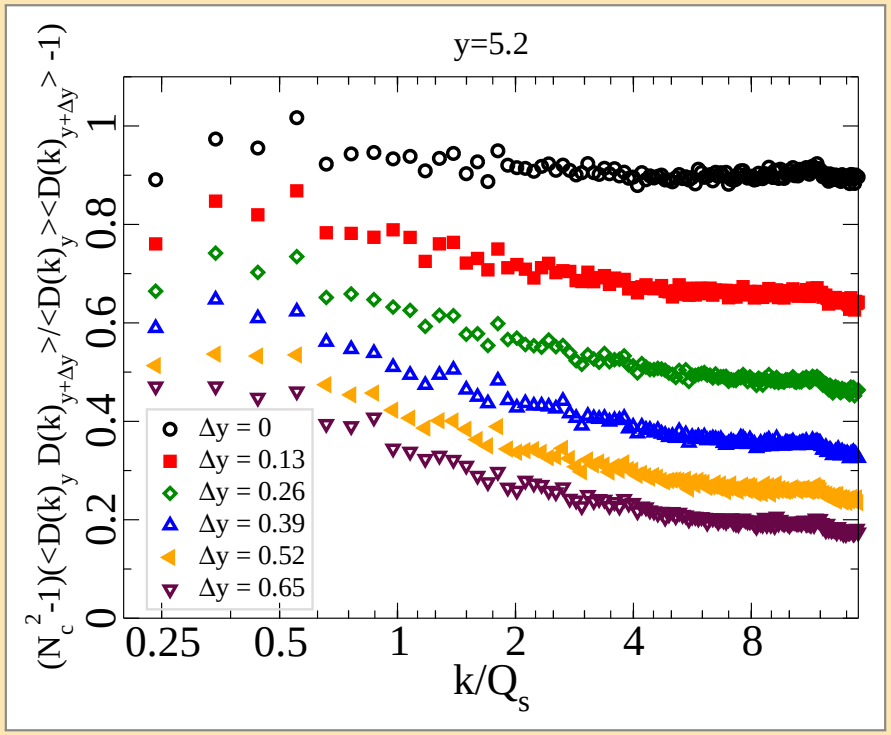
Based on leading α_s, (i.e. no decorrelation in rapidity) approximation

$$\left\langle \hat{D}(\mathbf{k}_T)_{y_p} \hat{D}(\mathbf{k}_T)_{y_q} \right\rangle - \left\langle \hat{D}(\mathbf{k}_T)_{y_p} \right\rangle \left\langle \hat{D}(\mathbf{k}_T)_{y_q} \right\rangle \sim \frac{1}{{N_c}^2-1} \left\langle \hat{D}(\mathbf{k}_T)_{y_{\min}} \right\rangle^2$$

Is this valid? Real evolution causes decorrelation; how fast?

Decorrelation in rapidity: preliminary results

$$(N_c^2 - 1)\left[\frac{\left\langle \hat{D}(\mathbf{k}_T)_y \hat{D}(\mathbf{k}_T)_{y+\Delta y}\right\rangle}{\left\langle \hat{D}(\mathbf{k}_T)_y\right\rangle \left\langle \hat{D}(\mathbf{k}_T)_{y+\Delta y}\right\rangle} - 1\right]$$

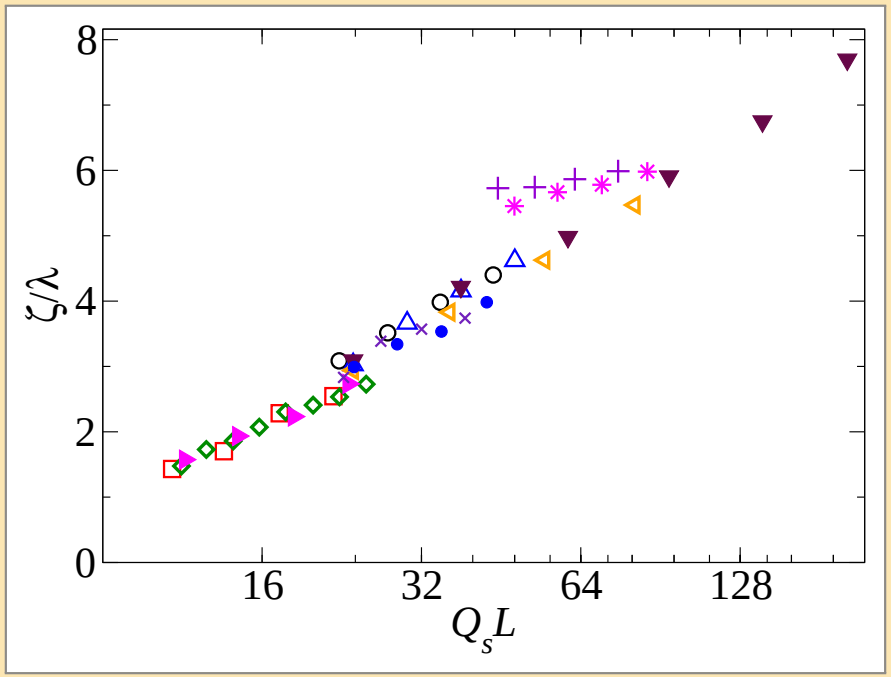


Decorrelation in rapidity: preliminary results

Weighted with same power of $\mathbf{k}_T$ as in double inclusive spectum, fit

$$\frac{\int \mathrm{d}^2\mathbf{k}_T \mathbf{k}_T{}^4 \left[\left\langle \hat{D}(\mathbf{k}_T)_y \hat{D}(\mathbf{k}_T)_{y+\Delta y} \right\rangle - \left\langle \hat{D}(\mathbf{k}_T)_y \right\rangle \left\langle \hat{D}(\mathbf{k}_T)_{y+\Delta y} \right\rangle \right]}{\int \mathrm{d}^2\mathbf{k}_T \mathbf{k}_T{}^4 \hat{D}(\mathbf{k}_T)_y} \sim \exp\{-\zeta y\}$$

Compare decorrelation speed ζ to evolution speed $Q_s^2 \sim e^{-\lambda y}$:



Different symbols: different lattice size, rapidity step, initial Q_s, coupling freeze ...

$\Longrightarrow$ Fast, and seems to be IR-unsafe (depends on system size L)

Conclusion

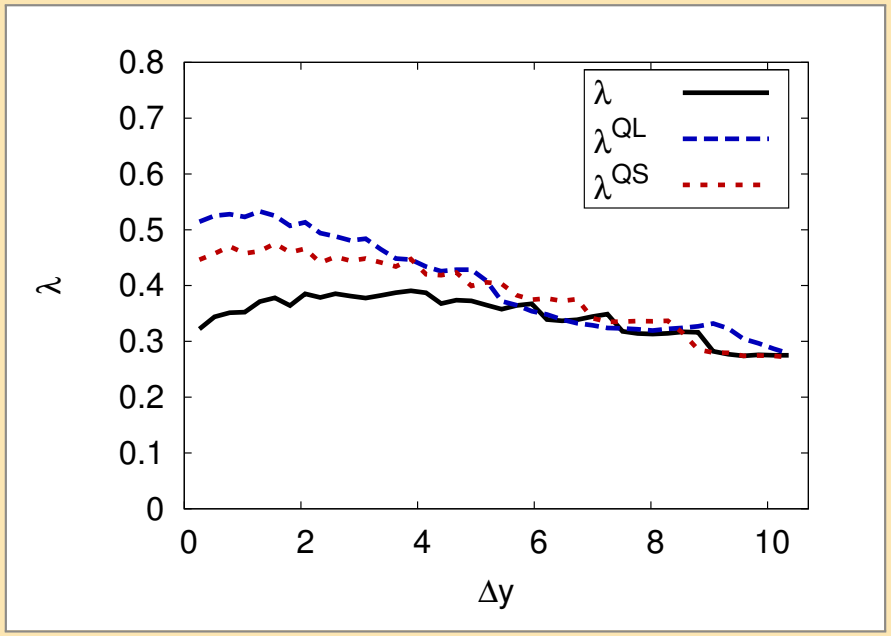
- Studied renormalization group evolution of multiparticle correlators in CGC/JIMWLK formalism.
- Higher point functions of Wilson lines in JIMWLK:
  - Significant deviation from approximations used in phenomenology.
  - Gaussian approximation works well.
- Unequal rapidity correlations:
  - IR safety?
  - Consequences for ridge?

Evolution speeds

Sefine characteristic momentum scale Q_s for each correlator.
Evolution speed is

$$\lambda \equiv \frac{\mathrm{d} \ln Q_s^2}{\mathrm{d}y}$$

Result: the higher point functions evolve “faster”



(This is transient effect specific to MV initial condition; goes away for high rapidity.)