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On the
Freely Adjustable Parameters
of the
Standard Model

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A theory of scalar mesons

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Abstract

We discuss the effect of the instanton induced, six-fermion effective Lagrangian on the decays of the lightest scalar mesons in the diquark–antidiquark picture. This addition allows for a remarkably good description of light scalar meson decays. The same effective Lagrangian produces a mixing of the lightest scalars with the positive parity $a\bar{a}$ states. Comparing with previous work where the $a\bar{a}$ mesons are identified with the

Effective theories of spin $\frac{1}{2}$ baryons, scalar and pseudoscalar mesons, have an underlying theory: QCD, Which explains much about their masses, couplings and mixings ($\eta - \eta'$ mixing, $\omega - \phi$ mixing, diquark – teraquark mixing, etc.).

Ingredients: composite fields, *instanton effects*, ...

The Standard Model itself also has at least one scalar, spin $\frac{1}{2}$ fermions and vector particles.

All renormalizable couplings that comply with the symmetries of the model, are incalculable today.

Is there no underlying theory here?

What about quantum gravity ?

Strategy: look for inconsistencies in Standard Model particles coupled to gravity. *They are there* :

The textbook theory (Einstein-Hilbert action with indefinite series of renormalization counter terms) has a flaw much more serious than the undesired counter terms:

Black holes do not appear to behave quantum mechanically.

How do we describe creation and annihilation of black holes?

The theory is inconsistent *unless we impose constraints on the present theories* – not all interactions are allowed! Restrictions may well be very tight!

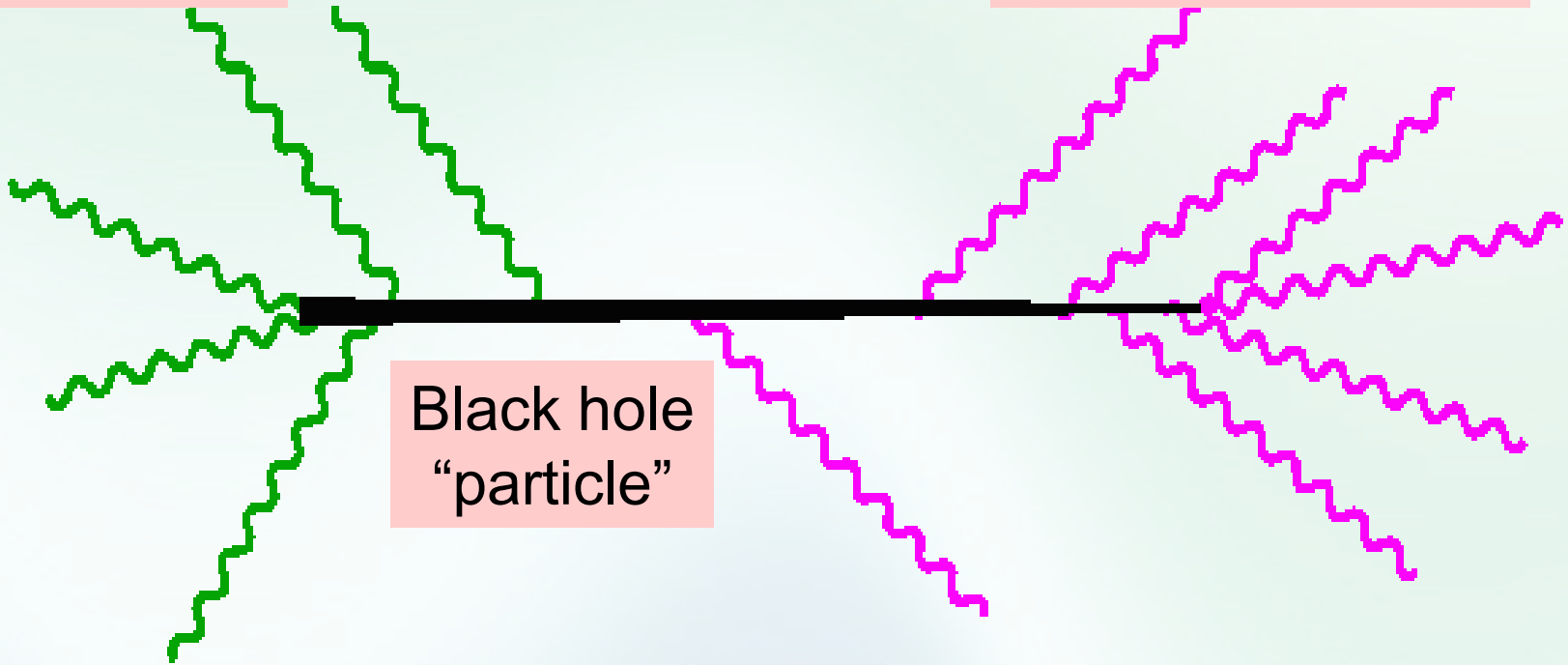
Are black holes just
“elementary particles”?

Are elementary particles
just “black holes”?

Imploding
matter

Hawking particles

Black hole
“particle”



Are black holes made of elementary particles
“elementary” or just “black holes”?

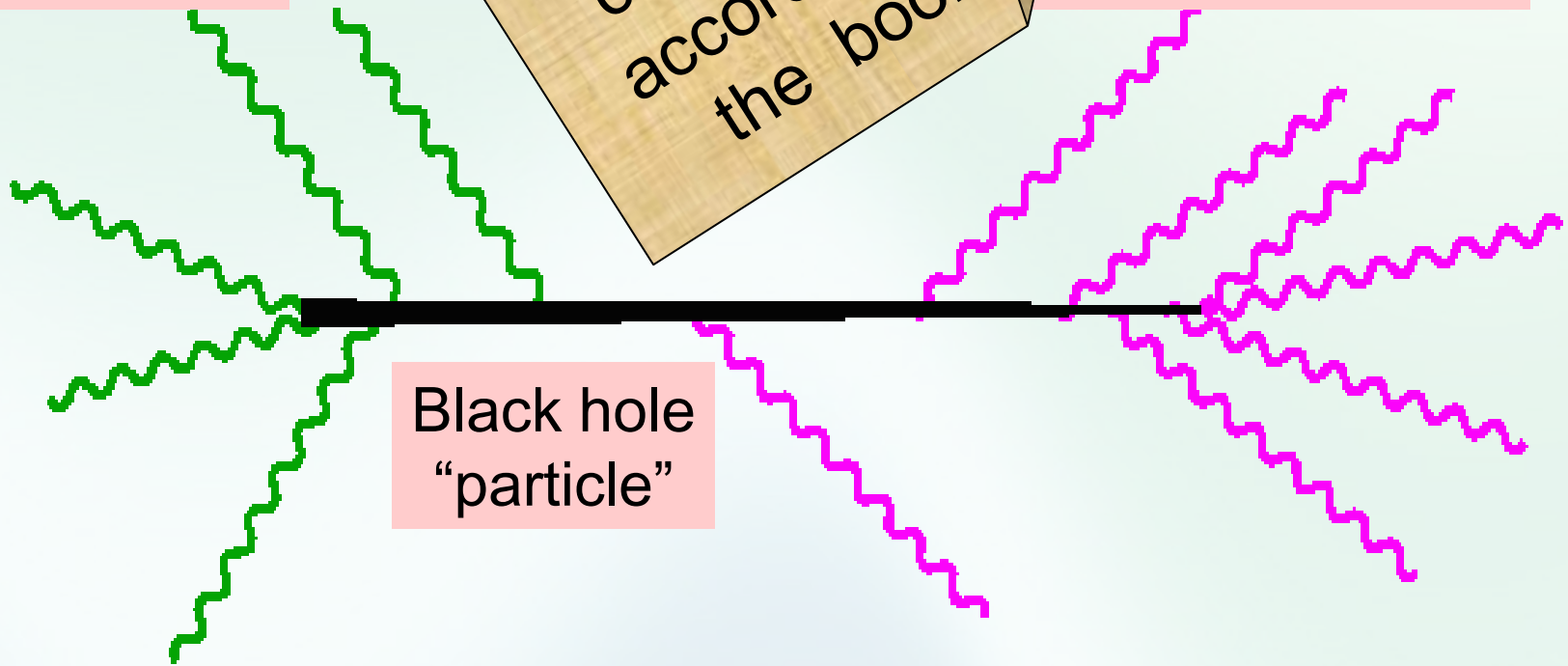
NO!!

If you do the
calculations
according to
the book

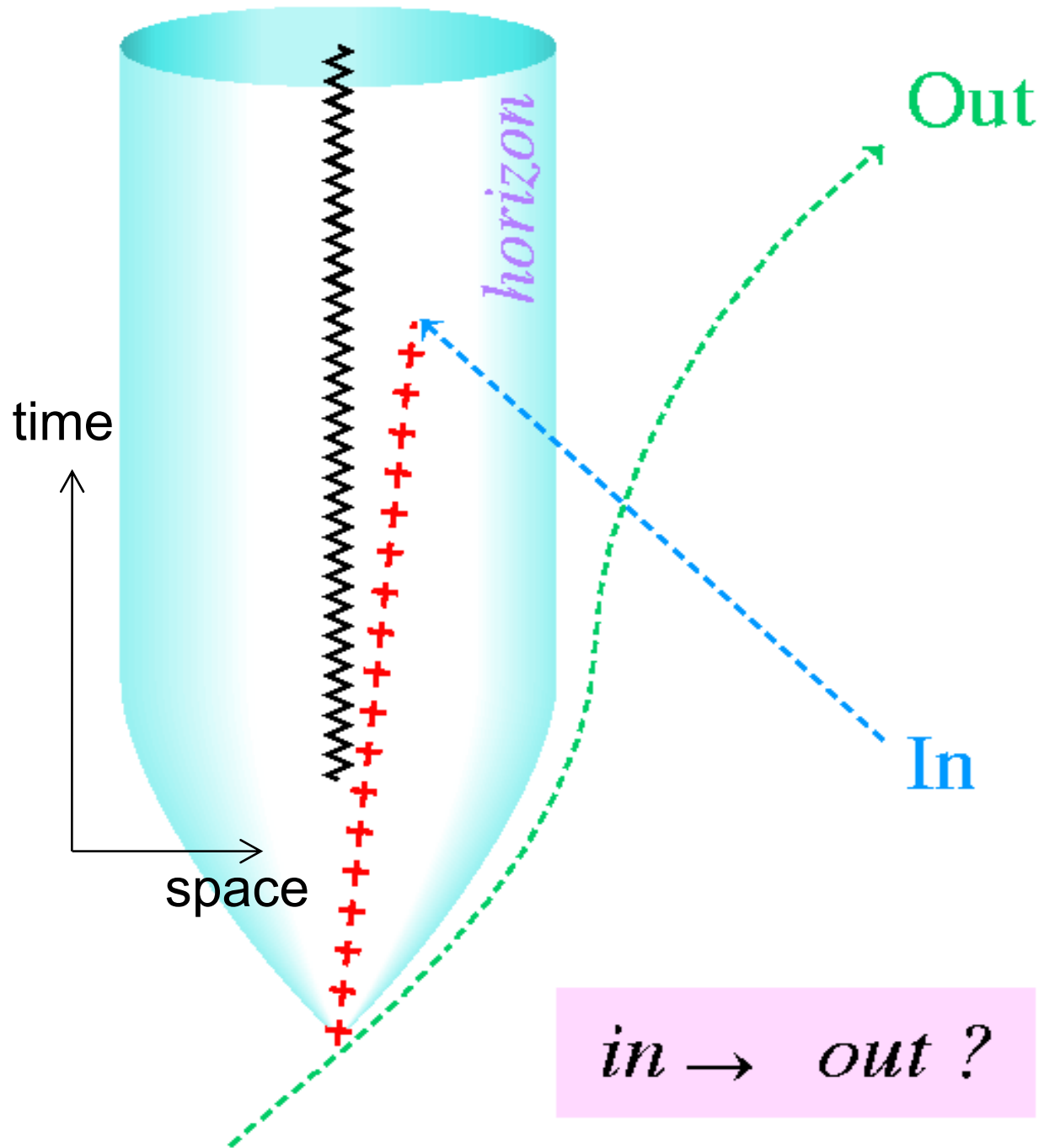
Imploding
matter

Outgoing particles

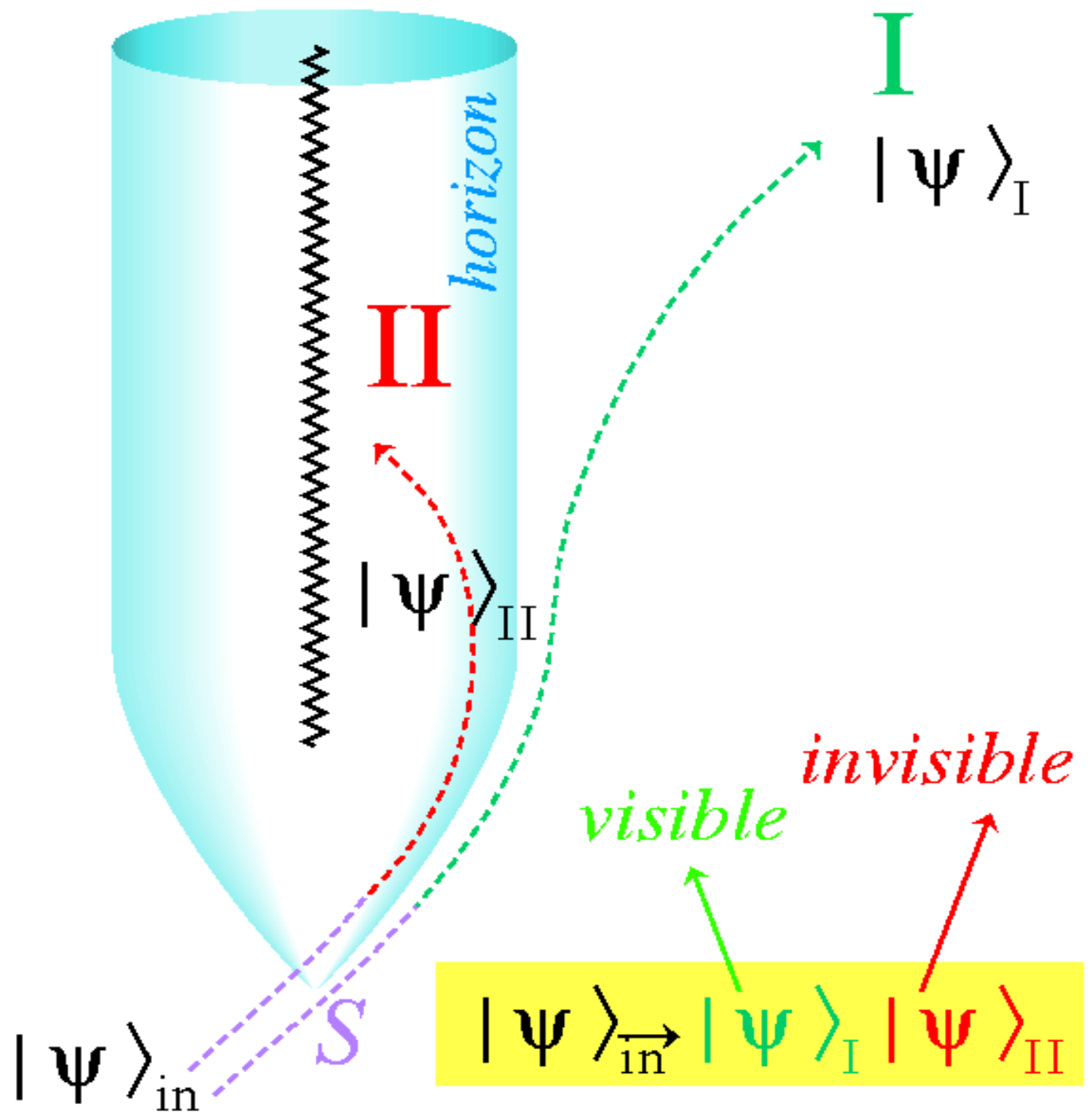
Black hole
“particle”



Violation
of
causality



The Information Problem



No Feynman diagrams for black holes ..

Black holes are fundamentally different from elementary particles ...

Black holes are heavier, while particles are lighter than M_{Planck}

But particles and black holes both carry gravitational fields, they both decay into other particles ...

From a physical point of view, why should they differ?

Shouldn't they blend at M_{Planck} ?

Holography:

All degrees of freedom of some section of the universe
reside on its BOUNDARY

How do we reconcile this with LOCALITY?



Unitarity,
Causality, ...

One avenue to be further explored :
A STRONG NEW GAUGE SYMMETRY IS NEEDED –
LOCAL CONFORMAL SYMMETRY ?

An other conjecture: *Quantum states are
NOT the most fundamental physical degrees
of freedom!*

What does superstring theory say ?

There are states made out of stacks of “branes”. These behave just as **black holes**, and have a density of states that appears to agree with the Hawking radiation formula.

But superstring theory cannot tell what happens to an observer who enters a large black hole ...

Is the horizon transparent or not?

The real question has not been answered!

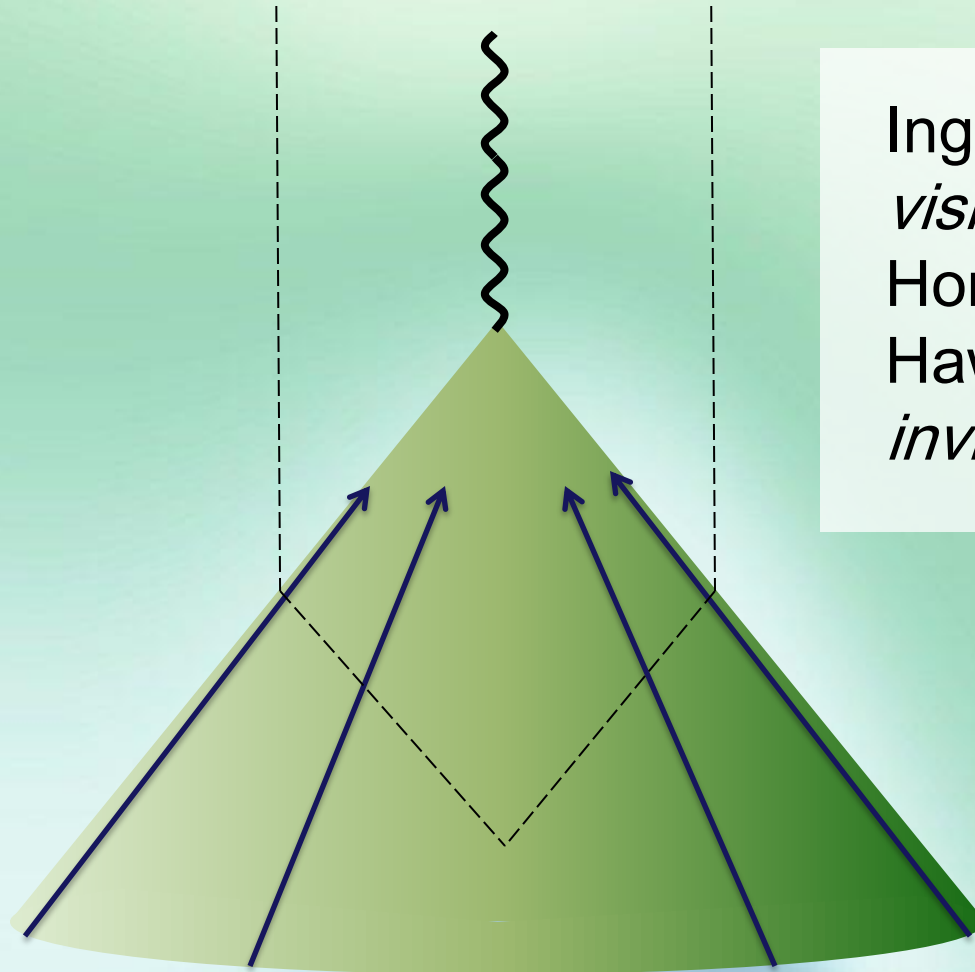
Black hole complementarity

An observer going in experiences the *original vacuum*,
Hence sees **no Hawking particles**, but does observe
objects behind horizon

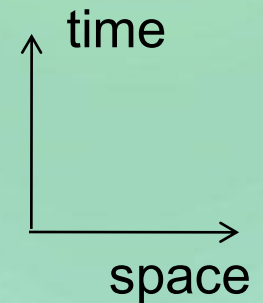
An observer staying outside sees
no objects behind horizon,
but does observe the **Hawking particles**.

They both look at the same “reality”, so there
should exist a *mapping* from one picture to the
other and back.

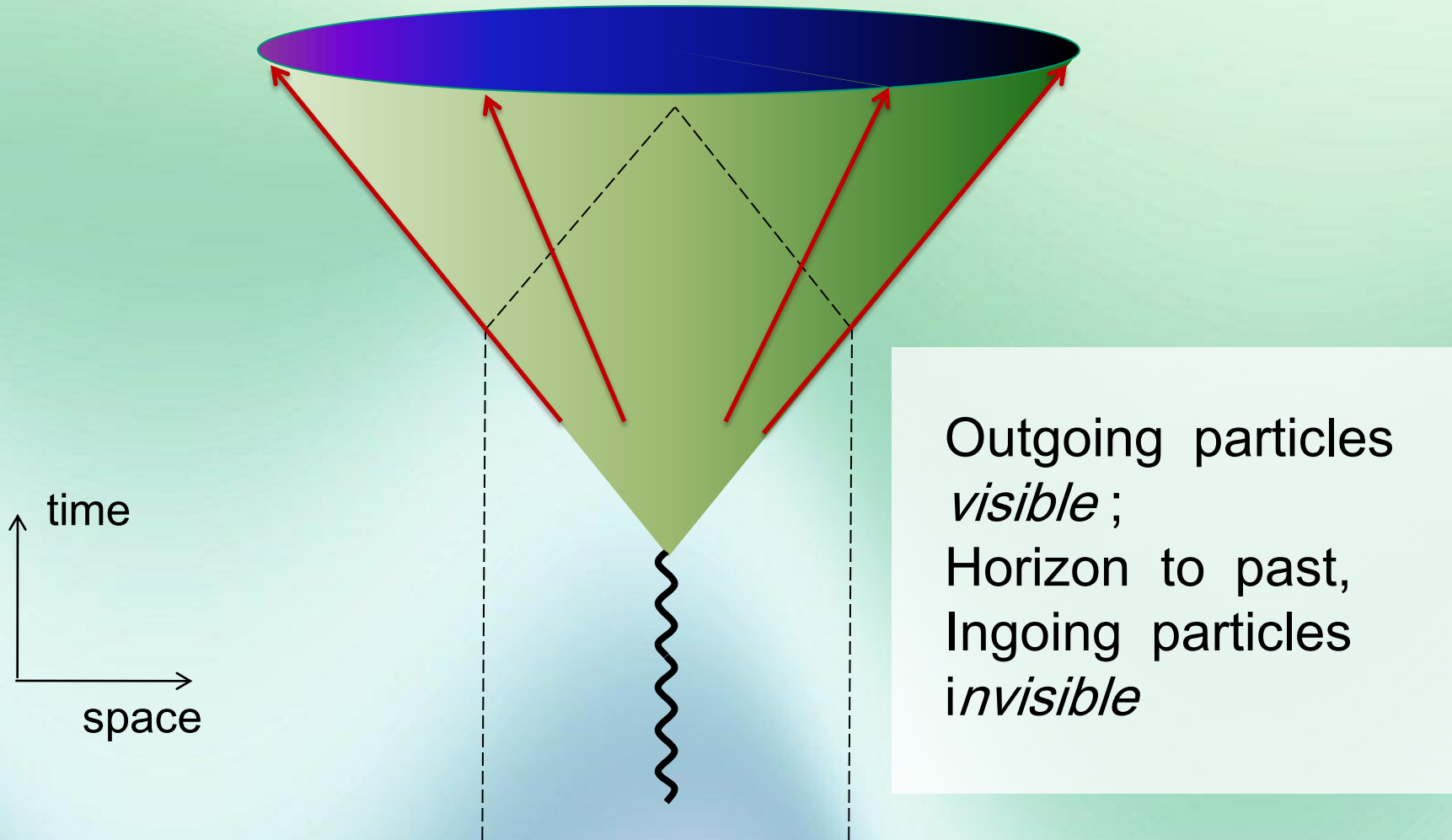
Extreme version of complementarity



Ingoing particles
visible ;
Horizon to future,
Hawking particles
invisible



Extreme version of complementarity



THE LOCAL CONFORMAL GROUP IN CANONICAL QUANTUM GRAVITY

$$g_{\mu\nu} = \omega^2 \hat{g}_{\mu\nu} , \quad \det(\hat{g}_{\mu\nu}) = -1 ,$$

$$\omega = (-\det(g_{\mu\nu}))^{1/8}$$

Exact local conformal invariance emerges *formally* in canonical quantum gravity:

$$\int Dg_{\mu\nu} e^{i(S^{EH} + S^M)} = \int D\hat{g}_{\mu\nu} \int D\omega e^{i(S^{EH}(\hat{g},\omega) + S^M(\hat{g},\omega))};$$

$$\int D\omega e^{i(S^{EH}(\hat{g},\omega) + S^M(\hat{g},\omega))} = e^{iS^{eff}(\hat{g})};$$

$S^{eff}(\hat{g}_{\mu\nu})$ does not depend on ω , therefore is *locally conformally invariant!*

Would this make $S^{eff}(\hat{g}_{\mu\nu})$ “renormalizable” !!?

Unfortunately **no** !?

The conformal anomaly

The conformal transformation is:

$$\hat{g}_{\mu\nu} \rightarrow \lambda^2(x) \hat{g}_{\mu\nu}$$

$$\tilde{\omega} \rightarrow \lambda^{-1}(x) \tilde{\omega} \quad (\text{this field may be integrated over } \textit{first!})$$

$$\phi \rightarrow \lambda^{-1}(x) \phi$$

Note that not the original $g_{\mu\nu}$ but $\hat{g}_{\mu\nu}$ serves as “background metric”.

Local conformal transformation: $g_{\mu\nu}(x) \rightarrow \lambda(x)g_{\mu\nu}(x)$
This modifies the curvature of space-time –
hence also the *energy-momentum tensor*.

$$T_{\mu\nu}(x) \rightarrow T_{\mu\nu}(x) - \left(\frac{1}{8\pi G}\right)(D_\mu \partial_\nu \lambda - g_{\mu\nu} D^2 \lambda) + \dots$$

Clearly, the vacuum is not invariant under all local conformal transformations. Therefore this symmetry is **spontaneously broken** (à la Higgs)

In a theory with exact local conformal invariance, one must *fix the gauge*, which means that **one component** of $T_{\mu\nu}(x)$ can be fixed – at *all* space-time points x .

How can exact *local conformal gauge symmetry* restore locality, unitarity and causality for a black hole?

An observer near the future event horizon of a black hole observes particles and other material going into the black hole; nothing comes out: $T_{--}(\vec{x}, t) = 0$.
An observer witnessing the final explosion sees the *time reverse* of that: only outgoing matter. Nothing goes in: $T_{++}(\vec{x}, t) = 0$.

Use *radial light cone coordinates*

A *local conformal transformation* changes time- and distance scales at each space-time point \vec{x}, t . This affects *space-time curvature*, and therefore includes a modification of the energy-momentum tensor $T_{\mu\nu}(\vec{x}, t)$ by one number at each point. The in-out scattering matrix is now obtained by *gauge-fixing*: $T_{--}(\vec{x}, t) = 0$ for the *in* states, $T_{++}(\vec{x}, t) = 0$ for the *out* states.

It is possible to see how local conformal invariance can produce black hole complementarity for a **slowly decaying black hole**. The infalling observer sees the Schwarzschild metric at (and beyond) the horizon, so also at what would be $t > \infty$ for the distant observer,

$$ds^2 = -dt^2 \left(1 - \frac{2M}{r}\right) + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

A distant observer can describe a shrinking black hole as

$$ds^2 = \lambda(t^-) \left(-dt^2 \left(1 - \frac{2M}{r}\right) + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2 \right)$$

Which means that

$$r \rightarrow \lambda(t^-) r \quad , \quad M \rightarrow \lambda(t^-) M \quad ; \quad t^- \equiv t - f(r)$$

$$\hat{g}_{\mu\nu} \rightarrow \lambda^2(x) \hat{g}_{\mu\nu}$$

$$\tilde{\omega} \rightarrow \lambda^{-1}(x) \tilde{\omega}$$

$$\phi \rightarrow \lambda^{-1}(x) \phi$$

Gauge fixing:

Trivial gauge I: $\det(\hat{g}_{\mu\nu}) = -1$

Trivial gauge II: $\omega(x) = 1$

Black hole gauge: $T_{--}^{\text{matter}} = 0$

White hole gauge: $T_{++}^{\text{matter}} = 0$

Now $g_{\mu\nu} = \lambda^2(x) \hat{g}_{\mu\nu}$ is invariant!

So why does $T_{\mu\nu}^{\text{matter}}$ depend on λ ?? Answer:

$T_{\mu\nu}^{\text{matter}}$ is the energy momentum tensor obtained by varying $\mathcal{L}^{\text{matter}}(x)$ w.r.t. $\hat{g}_{\mu\nu}$, not $g_{\mu\nu}$!!

Radially symmetric metric for black / white hole:
start with Kruskal coordinates,

$$ds^2 = 4M^2 e^{\mu(x,y)} \left(\frac{4}{\rho(x,y)} dx dy + \rho^2(x,y) d\Omega^2 \right)$$

$$xy = (\rho(x,y) - 1) e^{\rho(x,y)} ; \quad \rho = \frac{r}{2M} , \quad \lambda = e^{\frac{\mu(x,y)}{2}}$$

Einstein tensor:

$$G_{xx} = \left(1 - \frac{1}{\rho^2} \right) \frac{\mu_x}{x} - \frac{1}{2} \mu_x^2 + \mu_{xx}$$

$$G_{yy} = \left(1 - \frac{1}{\rho^2} \right) \frac{\mu_y}{y} - \frac{1}{2} \mu_y^2 + \mu_{yy}$$

$$G_{xy} = 2 \frac{1-\rho}{\rho^2} \left(\frac{\mu_x}{y} + \frac{\mu_y}{x} \right) - \mu_x \mu_y - \mu_{xy}$$

$$G_{\theta\theta} = -\rho^3 e^{\rho} \left(\mu_{xy} + \frac{\mu_x \mu_y}{4} \right) - \frac{1}{2} \rho (x \mu_x + y \mu_y)$$

Black hole gauge:

$$\frac{\partial \mu}{\partial y} \equiv \mu_y = 0$$

White hole gauge:

$$\frac{\partial \mu}{\partial x} \equiv \mu_x = 0$$

Einstein tensor is then given by

$$G_{xx} = \left(1 - \frac{1}{\rho^2}\right) \frac{\mu_x}{x} - \frac{1}{2} \mu_x^2 + \mu_{xx}$$

$$G_{yy} = 0$$

$$G_{xy} = 2 \frac{1-\rho}{\rho^2} \frac{\mu_x}{y}$$

$$G_{\theta\theta} = -\frac{1}{2} \rho x \mu_x$$

$$G_{xx} = 0$$

$$G_{yy} = \left(1 - \frac{1}{\rho^2}\right) \frac{\mu_y}{y} - \frac{1}{2} \mu_y^2 + \mu_{yy}$$

$$G_{xy} = 2 \frac{1-\rho}{\rho^2} \frac{\mu_y}{x}$$

$$G_{\theta\theta} = -\frac{1}{2} \rho y \mu_y$$

For this, it is essential that local conformal symmetry be an exact symmetry; in the usual theories, it is explicitly broken by anomalies.

The requirement that these anomalies cancel out may imply new restrictions on “Standard Model Interactions” !? These then can only be calculated if we know all interactions up to the Planck scale, which of course we don't. But perhaps new clues can be obtained as to what the unknown interactions may be ...

$$S^{EH} + S^M = \int d^4x \sqrt{-\hat{g}} \left(\frac{1}{16\pi G_N} \left(\omega^2 \hat{R} + 6 \hat{g}^{\mu\nu} \partial_\mu \omega \partial_\nu \omega \right) + \mathcal{L}^{\text{matt}}(\hat{g}, \omega) \right)$$

$$\mathcal{L}^{\text{matt}}(\hat{g}, \omega) = -\frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{R} \phi^2 - \frac{1}{2} m^2 \omega^2 \phi^2$$

$$\omega \rightarrow \sqrt{\frac{16\pi G_N}{12}} \tilde{\omega}$$

$$\mathcal{L} = +\frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \tilde{\omega} \partial_\nu \tilde{\omega} + \frac{1}{12} \hat{R} \tilde{\omega}^2$$

$$-\frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} \hat{R} \phi^2 - \frac{2}{3} \pi (G_N m^2) \tilde{\omega}^2 \phi^2$$

Can one *cancel* the infinity in the ω integral? **No!**

If $R_{\mu\nu} \neq 0$ there are 2 kinds of conformal anomalies.

- 1) The scaling anomaly in a flat background;
- 2) The conformal anomaly when

$$R_{\mu\nu}^{(\text{background})} \neq 0$$

The equations

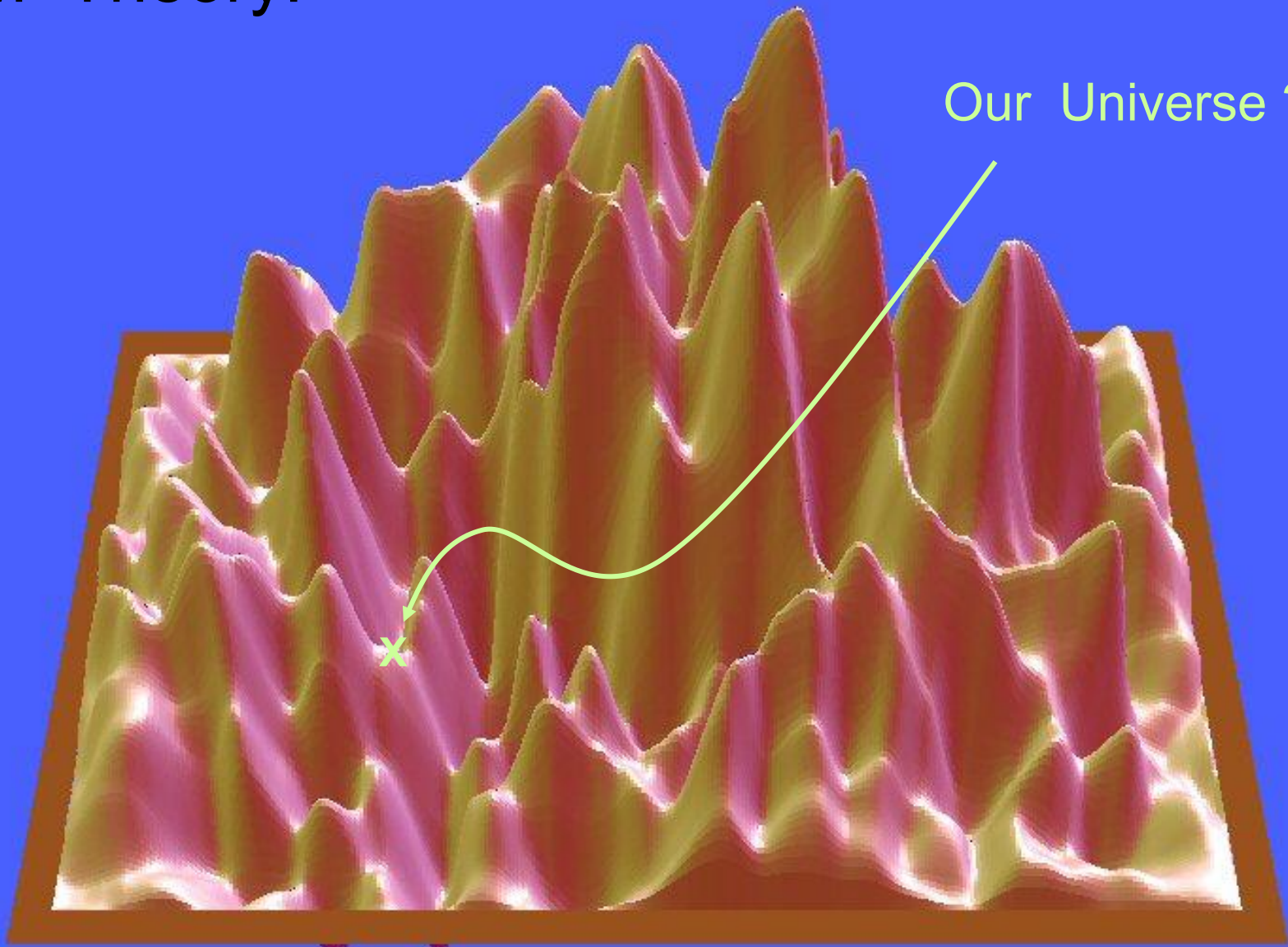
$$\frac{\mu d}{d\mu}(\tilde{\Lambda}, \mathbf{g}, \lambda, \mathbf{y}, \mathbf{y}^5, \tilde{\kappa}g_3, \tilde{\kappa}m_s, \tilde{\kappa}m_d, \dots) =$$
$$\vec{\beta}(\tilde{\Lambda}, \mathbf{g}, \lambda, \mathbf{y}, \mathbf{y}^5, \tilde{\kappa}g_3, \tilde{\kappa}m_s, \tilde{\kappa}m_d, \dots) = 0$$

Have only isolated solutions ! All coupling parameters, including μ , are completely fixed by these equations, which can be worked out.

The only choices we have are discrete parameters: the group structures and symmetry patterns.

a LANDSCAPE of “Standard Models”

M Theory: The Landscape



Our Universe ?

The anthropic principle



Lessons in conformal invariance

$R^\mu_{\nu\alpha\beta}$ Riemann curvature

$R_{\nu\beta} = R^\sigma_{\nu\sigma\beta}$ Ricci curvature

$R = g^{\mu\nu} R_{\mu\nu}$ Ricci scalar

$W_{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} - \frac{1}{2}(g_{\mu\alpha} R_{\nu\beta} \pm \dots) + \frac{1}{6}R(g_{\mu\alpha}g_{\nu\beta} - \dots)$

$W^\sigma_{\nu\sigma\beta} = 0$ Weyl curvature

Under the transformation $g_{\mu\nu} \rightarrow \omega^2(x)g_{\mu\nu}$
the Riemann and Ricci curvature transform non-trivially,
but the Weyl curvature is conformal:

$$W_{\mu\nu\alpha\beta} \rightarrow \omega^2(x) W_{\mu\nu\alpha\beta}$$
$$\int d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} \rightarrow \int d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} \omega^{4-8+4}$$

$$W_{\mu\nu\alpha\beta} \rightarrow \omega^2(x) W_{\mu\nu\alpha\beta}$$

$$\int d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} \rightarrow \int d^4x \sqrt{-g} W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

$$W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 2R_{\mu\nu} R^{\mu\nu} + \frac{1}{3} R^2$$

Furthermore

$$\sqrt{-g} d^4x \left(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \right)$$

Is a pure derivative, hence the integral is a *topological invariant*. Use this to rewrite

$$\int \sqrt{-g} d^4x \left(W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta} \right) =$$

$$2 \int d^4x \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right)$$

Under a conformal transformation

$$g_{\mu\nu} \rightarrow \omega(x)^2 g_{\mu\nu} ; \phi \rightarrow \omega(x)^{-1} \phi$$

$$\int \sqrt{-g} d^4 x \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{12} R \phi^2 \right) \text{ is invariant}$$

$$\text{and } \int \sqrt{-g} d^4 x \left(R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) \text{ is invariant}$$

In our effective action (after integrating out ω), the overall coefficient is *infinite*. Hence, it has to be *renormalized*. A non-local, finite effective action will then result. The (massive) scalar matter field will require a quartic renormalized interaction – fine !
But how to renormalize the **Weyl action** ???

$$\mathcal{L}^{eff,div} = \frac{\sqrt{-\hat{g}}}{8\pi^2(4-n)} \left(\frac{1}{120} \left(\hat{R}_{\mu\nu} \hat{R}^{\mu\nu} - \frac{1}{3} \hat{R}^2 \right) + \right. \\ \left. + \frac{4}{9} \pi^2 (G_N m^2)^2 \phi^4 \right)$$

is conformally invariant in 4 dimensions !

But

$$\left(\frac{1}{4-n} \right) \rightarrow \frac{1}{2} \log(\Lambda^2 / k^2)$$

At $n \neq 4$ this “local term” is *not* conformally invariant.

Can one *cancel* that infinity ? **No!**

Matter fields (spin 0, $\frac{1}{2}$, or 1) all contribute to the same anomaly, **with the same sign !**

Coefficients:

$$\text{grav. } \omega \text{ field: } + \frac{1}{120}$$

$$\mathcal{N}_0 \text{ Scalars: } + \frac{1}{120} \mathcal{N}_0$$

$$\mathcal{N}_{1/2} \text{ Dirac Spinors: } + \frac{1}{20} \mathcal{N}_{1/2}$$

$$\mathcal{N}_1 \text{ Vector fields: } + \frac{1}{10} \mathcal{N}_1$$



Happy birthday Luciano !

THE END