Theoretical progresses in off-equilibrium behaviour

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I met Luciano for the first time in 1969 when I was a student.

At that time Luciano and Nicola were attempting to resum leading divergences in weak interaction, i.e. $G\Lambda^2$ terms.

One of the aims was a self-consistent determination of the Cabibbo angle. However this led to unwanted terms in the effective weak interaction Hamiltonian, that were not there ($\Delta S = 2$). The argument involved quite complex considerations in field theory (very advanced for that time).

In Fall 1969 Luciano went to Harvard (by boat). Nicola received in winter a letter by Luciano (I remember he read to me and Massimo Testa).

The problem is solved. We have a consistent theory, but we have thrown the baby with the dirty water.

(Abbiamo buttato via il bambino con l'acqua sporca.)

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Is the 3104 MeV Vector Meson the φ_c or the W_0?

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(ricevuto il 20 Novembre 1974)

The exciting discovery [1,2] of a new neutral vector meson at a mass $M = 3104$ MeV [2], confirmed at Frascati [3], is a turning point in our understanding of fundamental interactions. Waiting for more definite experimental information, we shall base our discussion on the following values for the decay rates:

(1) $$\Gamma_e(=\Gamma_\mu) = 5 \text{ keV},$$

(2) $$\Gamma_{\text{total}} \simeq \Gamma_h = 50 \text{ keV},$$

where Γ_e and Γ_μ are the decay rates into e^+e^- and $\mu^+\mu^-$ pairs and Γ_h is the decay rate into hadrons.

The only expected narrow-width hadronic 1^--particle is the φ_c, *i.e.* a bound state of charmed quarks [4] ($\varphi_c \simeq p'\bar{p}'$).

This identification leads to serious difficulties in that the expected width for φ_c is in the $(1 \div 10)$ MeV range or more, *i.e.* at least a factor of twenty larger than eq. (2).

A more exciting alternative, strongly suggested by eqs. (1) and (2), is to identify this particle with the intermediate boson which mediates weak neutral currents (W_0).

My first paper with Luciano.

Je ne regrette rien!

The phenomenological analysis was correct.

We were not brave enough (as De Rujula and Glashow) to use QCD to compute the width.

At that time people were very shy in using field theory to do computations.

Deep inelastic scattering was related to the short distance Wilson expansion on the light cone (Brandt and Preparata) and it was considered safe (Symanzik analysis). It involved the study of the anomalous dimensions of twist 2 operators.

To use it in other cases was considered unsafe (e.g. $\mu^+\mu^-$ production in $p-p$ collisions).

It took long time before the whole stuff was reformulated in the parton language introducing effective (q^2) dependent distributions and factorizing divergences and to prove the relevant theorems.

The paper was soon forgotten, but 34 years later:

Dear Sra. Carlucci:

Thank you for your letter of inquiry about the paper published in Nuovo Cimento by Altarelli et al. just after the Ting-Richter discovery. As you are perhaps aware, the authors speculation that the then newly discovered boson might be the neutral weak intermediary is false. This is nothing for them to be ashamed of. It is the business of theorists to speculate, and we often find that our speculations are wrong. I have published more than a few papers that have turned out to have been wrong. So have most of my colleagues. Thats the name of the game! (...)

Scientists publish speculative results not because they are true, but because they may be true. If they refrained from publishing their speculations for fear that they may not always be true, there would be little progress in science. Even our greatest heroes, Galileo, Newton and Einstein, have published speculations that turned out to be quite false. I can supply citations, should you wish to question their scientific competence.

Sincerely

Sheldon Lee Glashow

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BOUNDS ON THE FERMIONS AND HIGGS BOSON MASSES IN GRAND UNIFIED THEORIES

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ABSTRACT

In the framework of grand unifying theories the requirement that no interaction becomes strong and no vacuum instability develops up to the unification energy, is shown to imply upper bounds to the fermion masses as well as upper and lower bounds to the Higgs boson mass. These bounds are studied in detail for the case of the unifying groups SU(5) or O(10).

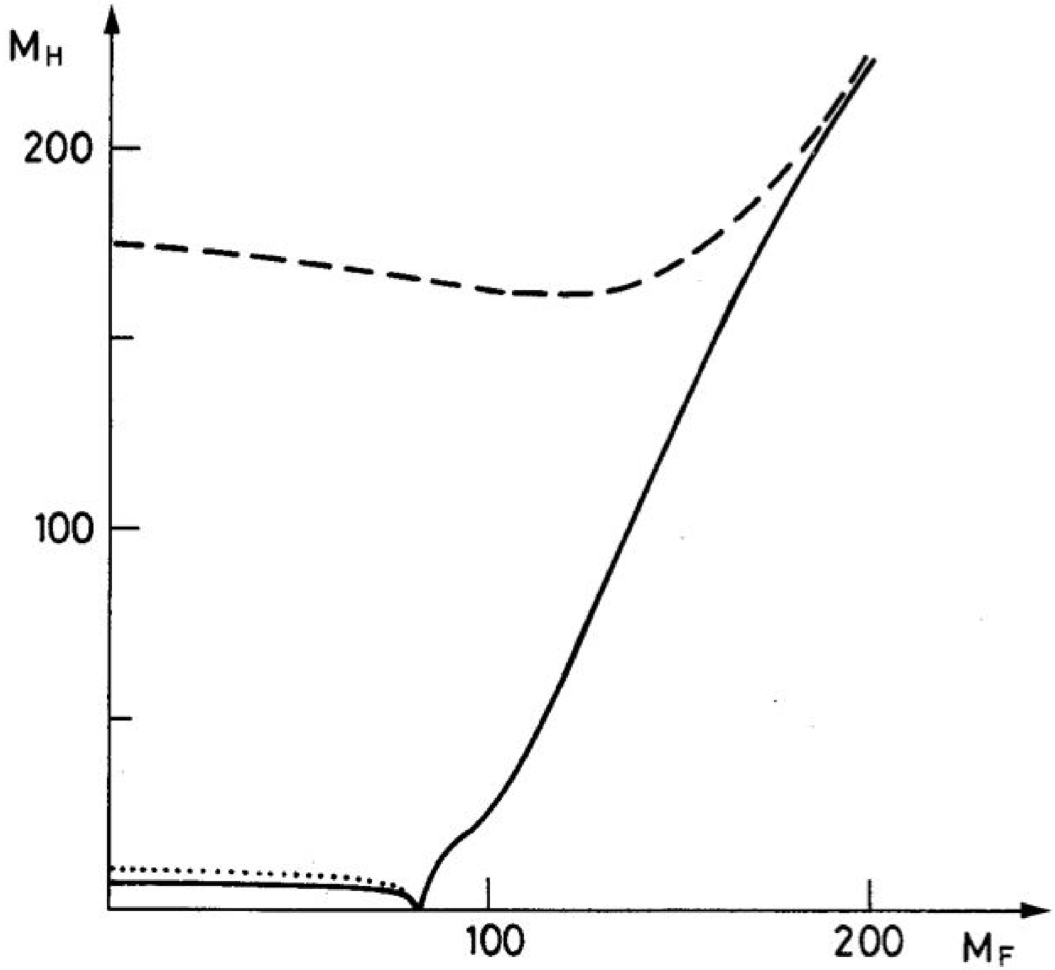


FIG. 1

The typical time scale in condensed matter physics is the picosecond.

Sometimes the time to approach equilibrium is much larger, seconds, years ....

Two scenarios:

- There is a process with large activation energy that is responsible of the very large activation time.
- All the elementary processes are fast and the slow behaviour is a collective effect.

Collective effects are well understood when they happen at a second order phase transition point (*critical slowing down*), where at equilibrium there are large scales excitations that involve a large number of atoms.

The characteristic times are much larger then the microscopic time, but they remains microscopic (e.g. they diverge at T_c as $|T - T_c|^{-1.4}$).

In glassy systems the characteristic times diverge much faster (e.g. exponentially, i.e. $\exp(A/(T-T_c))$, or as $|T-T_c|^{-10.}$) when we approach the transition temperature:

The systems are (on human scale) in an off-equilibrium situation: they are often called glasses.

Sometimes it may be convenient to speak of a collective barrier energy $E(T)$

$$\tau \propto \tau_0 \exp\left(\frac{E(T)}{kT}\right)$$

A divergence in $E(T)$ is a collective phenomenon that must be explained (universal behavior, universality classes, renomalization group...)

In off-equilibrium systems we do not have anymore a Boltzmann-Gibbs probability distribution.

We stay at quasi-equilibrium: only slow degrees of freedom are off-equilibrium.

- Can we define a temperature in off-equilibrium systems quasi-equilibrium?

  Two temperature scenarios: one for the slow an one for the fast modes.

- Can we find a substitute of the Boltzmann-Gibbs probability distribution?

  The replica approach can be used in some cases.

The study of *soluble* mean field models has been crucial to obtaining insight on what it could happens in these case.

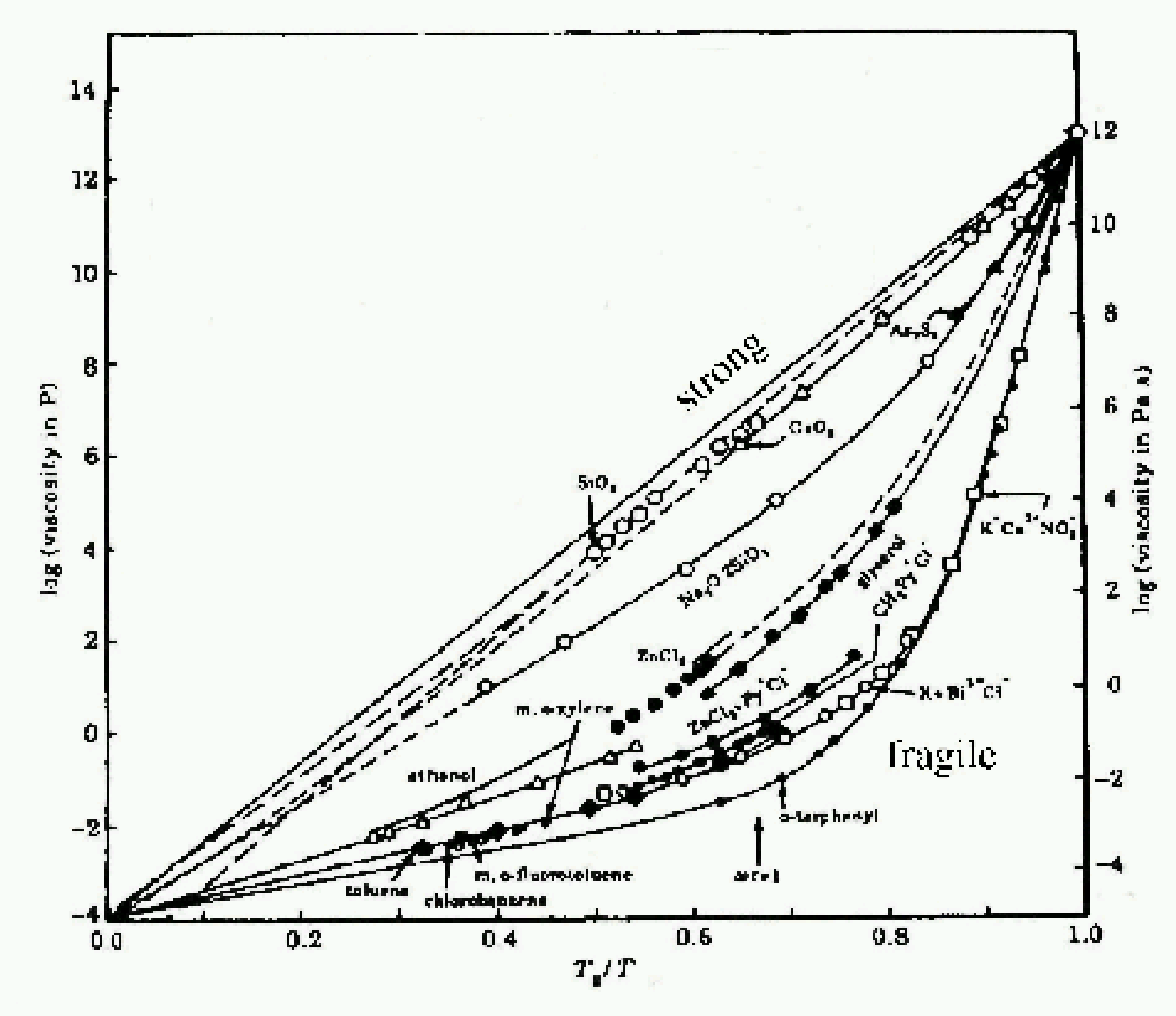
The dynamics (and statics) was much more complex of what we could guess before computations.

In this talk I will shortly discuss:

- A mini introduction to structural glasses and spin glasses.
- Generalized fluctuation dissipation relations and the definition of a scale dependent temperature.
- Some experimental and numeric results for aging in structural glasses and spin glasses.

Structural glasses: the viscosity increases of 18 order of magnitudo (Angell's Plot)

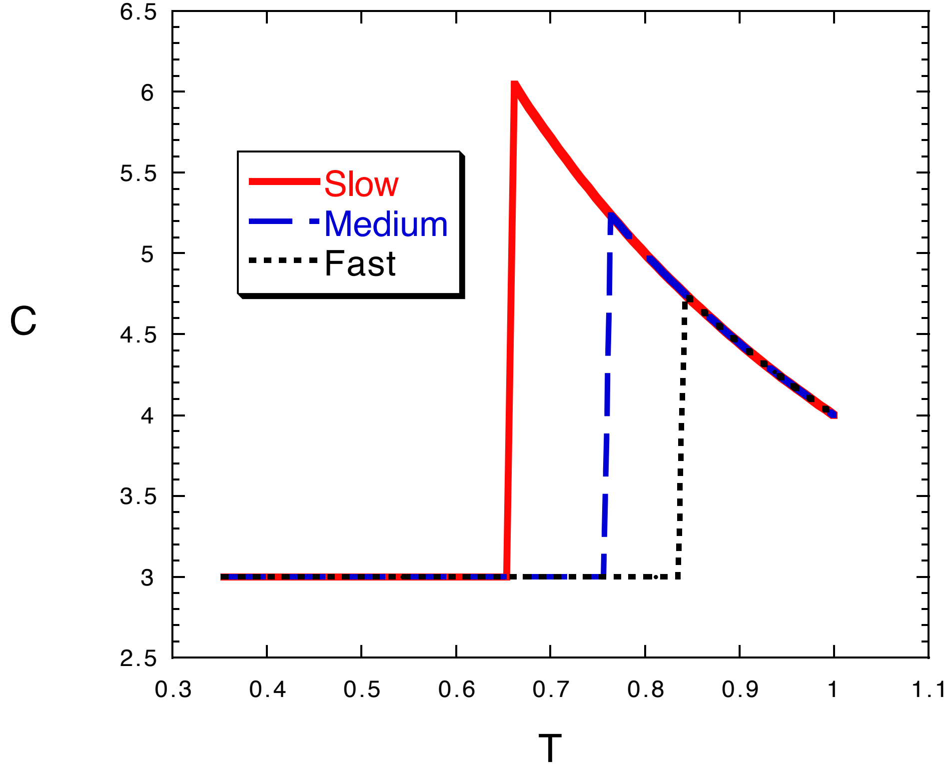
The characteristic times increase from picosecond to hours and more



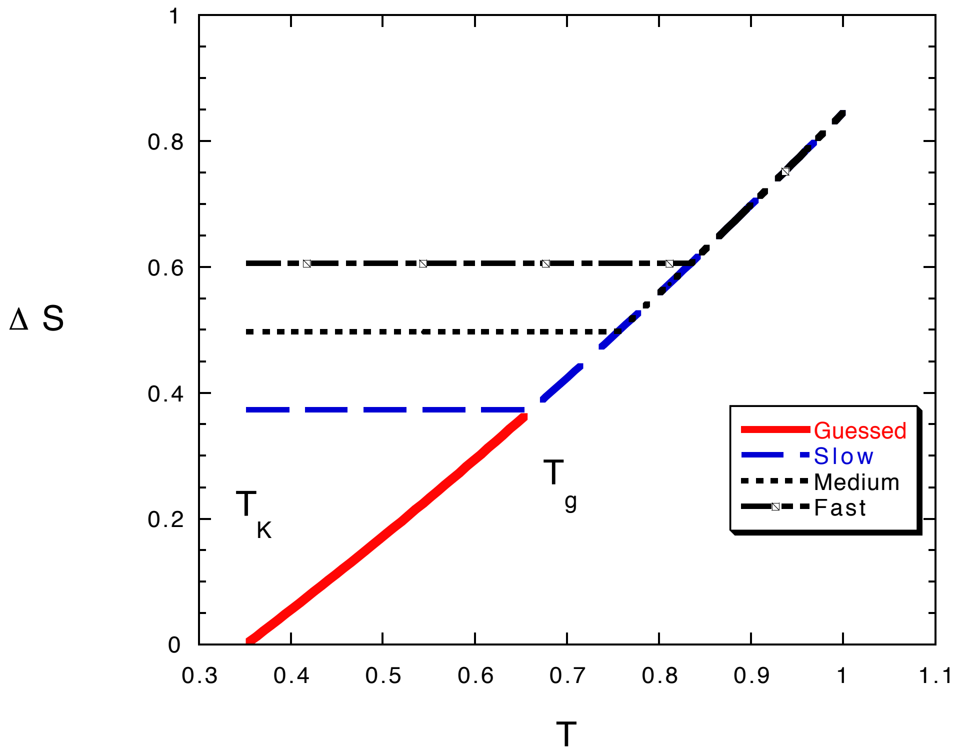
Strong glasses: $\eta \propto \exp\left(\frac{B}{T}\right)$

Fragile glasses: $\eta \propto \exp\left(\frac{B}{T - T_K}\right)$

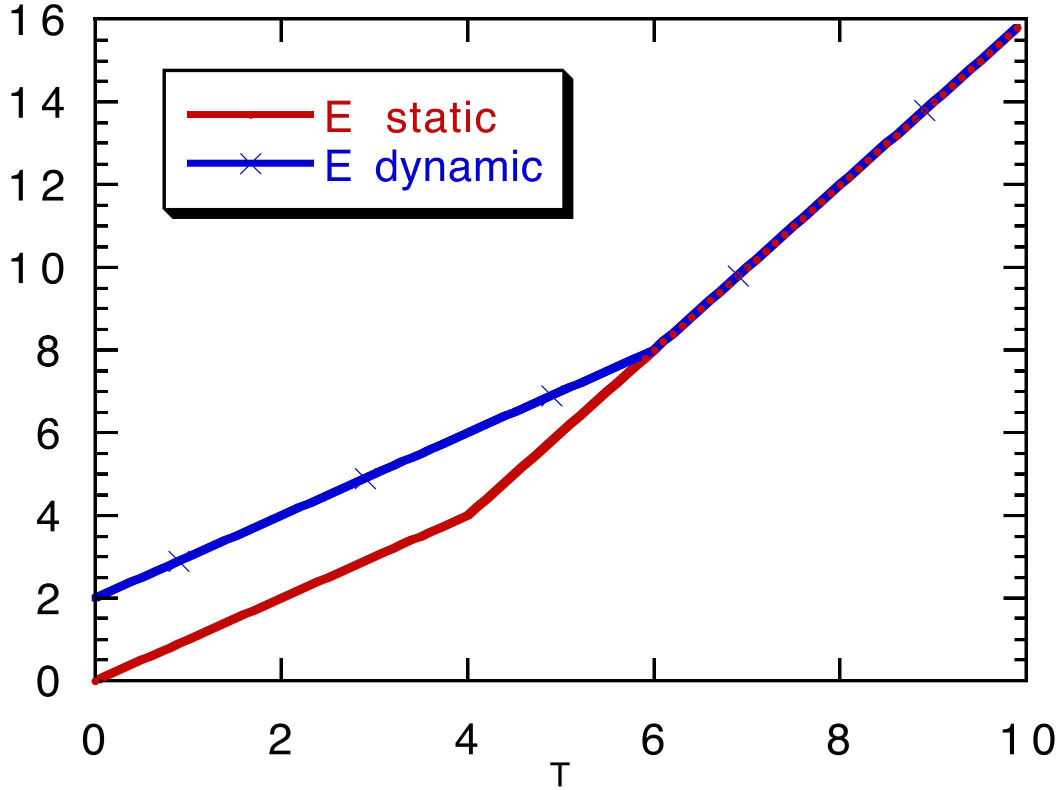
Time dependent specific heat



Time dependent entropy



Schematic view of the internal energy as function of the temperature:



The position of the dynamic line slightly depends (on a logarithmic scale) on the cooling speed.

The simplest Ising spin glasses has the following Hamiltonian:

$$H_J = \sum_{i,k=1,N} J_{i,k}\sigma_i\sigma_k$$

$\sigma = \pm 1$, i and k are *neighbours*. The J are random (they take both sign).

Some bonds are ferromagnetic, some are antiferromagnetic and it is impossible to find a configuration such

$$J_{i,k}\sigma_i\sigma_k < 0 \qquad \forall i,k$$

Finding the ground state is a difficult task (in all senses). Minimal descent brings you in a local minimum but there are many local minima. There is an exponentially large number of minima).

From an analytic point of view finding the ground state is an NP-complete problem; i.e. in the worst case any algorithm takes an exponentially large CPU time.

A physical system will take an incredible very large time to reach equilibrium.

There are many variations (Ising, Heisenberg, anisotropy, space dimensions...)

Experimental study of spin glasses by Mydosh in the 70's. Experimental evidence of a transition

Deep theoretical studies of spin glass theory started with the Edwards Anderson paper and the Sherrington Kirkpatrick model.

The Sherrington Kirkpatrick model is conceptually simple, it defines the mean field theory of spin glasses.

Unfortunately the theory of spin glasses is difficult. Already it took a long time before understanding mean-field theory (there are still open problems).

We can compute analytically nearly all the statics properties. The dynamics is more complex. Some quantities are under control, others not.

We need to compute dynamic in terms of static (equilibrium) quantities.

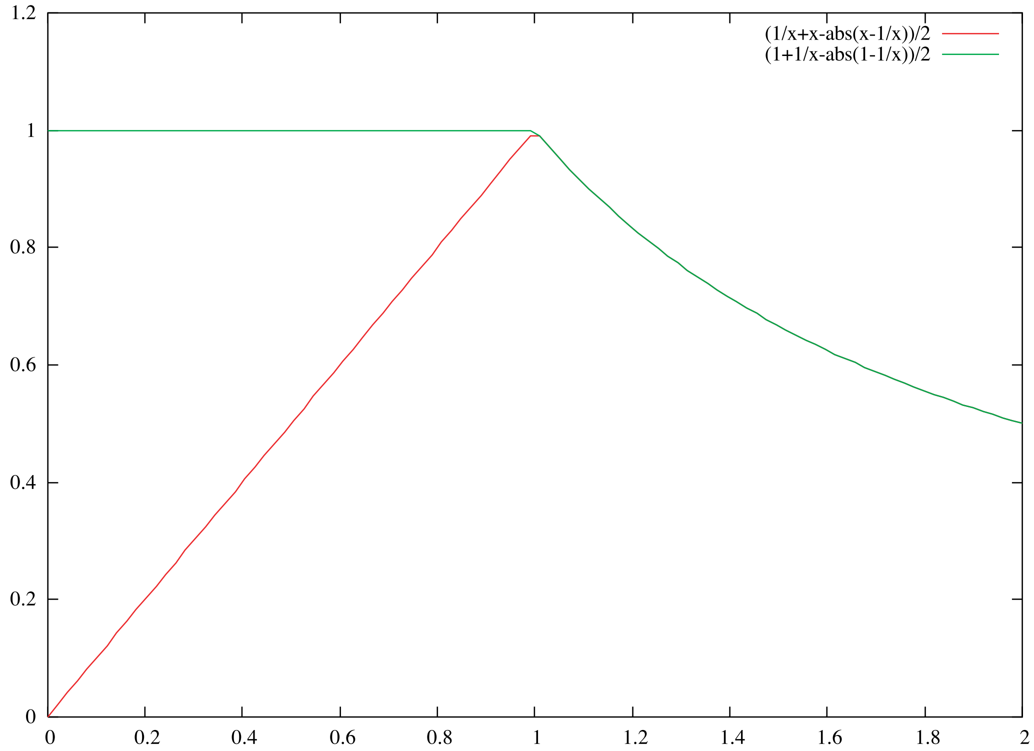
A first comparison with experiments

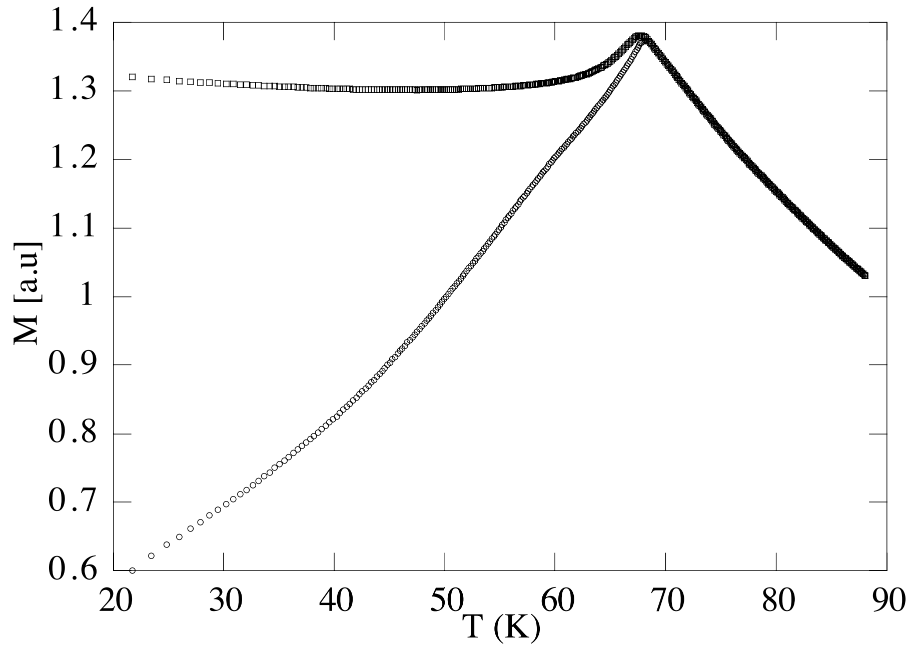
Spin glasses (amorphous magnets) are realized in nature in many ways, e.g. in alloys: $Fe_5\ Au_{95}$. Here the interaction between two iron spins (RKKY) is proportional to $\sin(2k_F r)/r^3$

In spin glasses we can define two physically relevant susceptibilities.

- χ_{LR}, i.e. the response within a state, that is observable when one changes the magnetic field at fixed temperature and one does not wait too much.
- χ_{eq}, the true equilibrium susceptibility, that is very near to χ_{FC}, i.e. the field cooled susceptibility, where one cools the system in presence of a field.

The difference of the two susceptibilities is the hallmark of replica symmetry breaking.

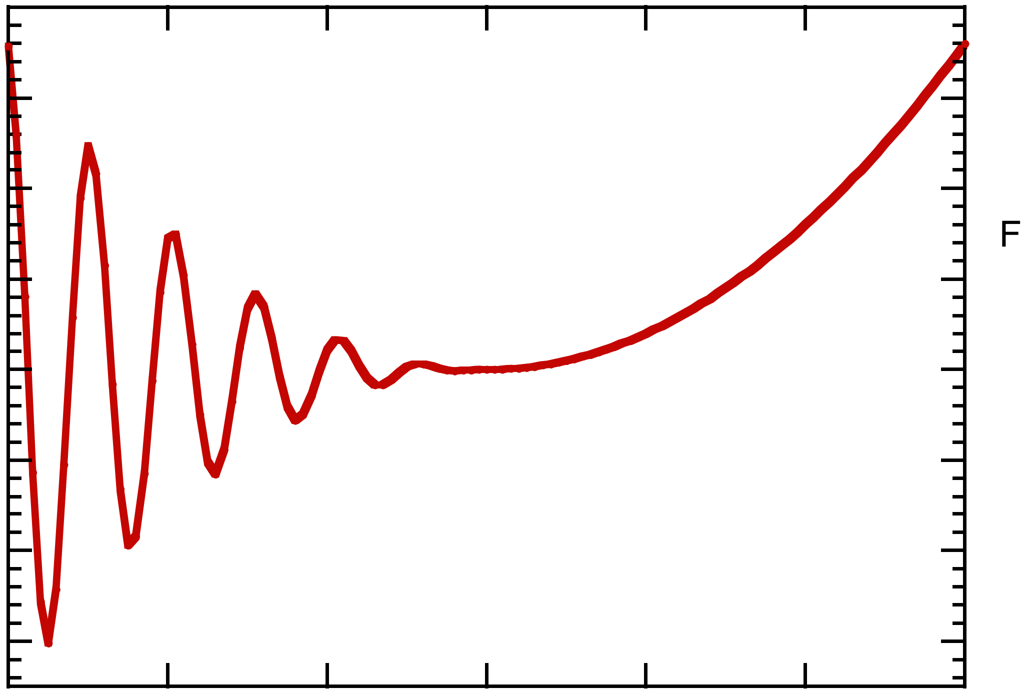


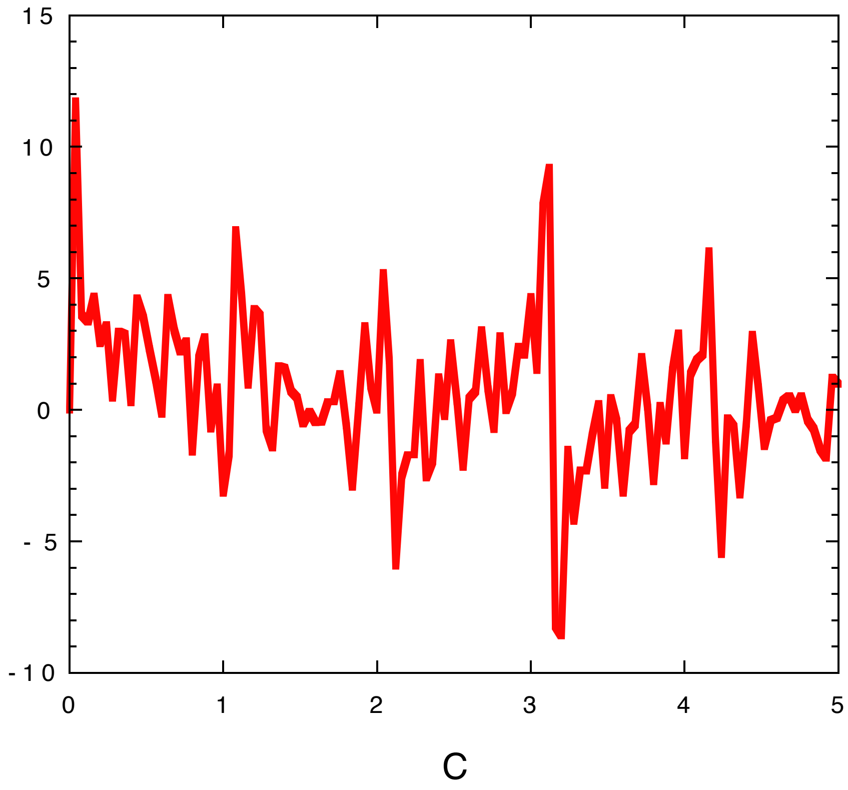


At the left we show the analytic results for the SK model, at the right we have experimental data (C. Djurberg, K. Jonasson and P. Nordblad) on metallic spin glasses.

The similarities among the two graphs are striking.

Artistic views of the free energy landscape:





In the transition region.

Deep in the many valleys region.

Lessons from mean field theory

Mean field theory predicts the existence of an exponentially large number of valleys of in the free energy landscape.

In the simpler case the number of valleys as function of the free energy of the valley F is

$$\mathcal{N}(F) \propto \exp(\beta_V(F-F_0)) \qquad (\beta_V < \beta)$$

This formula recalls $\mathcal{N}(E) \propto \exp(\beta(E-E_0))$ that is equivalent to $\frac{dS}{dE} = \beta$.

Two temperatures (Cugliandolo Kurchan): one for the configurations inside the valley and one for the valleys. How to grip experimentally β_V?

- Short times: the standard temperature measured with fast thermometers.
- Long times: β_V is measured with slow thermometers that are affected by the jumping between valleys.

Fluctuation dissipation theorem **at equilibrium**.

For a magnetic system, where $M(t)$ is the total magnetization, we define the response function $r(t)$:

$$r(t) = \frac{\delta M(t)}{\delta h(0)} ,$$

where $h(0)$ is a field that is added at time zero.

If we add a field h at time 0 and we keep it for al positive times, we can define a relaxation function:

$$R(t) = \frac{\partial M(t)}{\partial h} = \int_0^t dt' r(t') \qquad r(t) = \frac{dR}{dt} .$$

Fluctuation dissipation theorem implies

$$r(t) = -\frac{1}{T}\frac{dC}{dt} .$$

We eliminate the time t in a parametric way: we define $R(C)$. FDT implies that

$$\frac{dR}{dC} = -\frac{1}{T} .$$

Fluctuation dissipation relations can be proved by assuming that the energy of a weakly coupled harmonic oscillator is kT, independently of the frequency of harmonic oscillator.

We take an harmonic oscillator and we couple to the system by adding a term in the Hamiltonian $\epsilon x(t) M(t)$.

$$\frac{d^2x}{dt^2} - \omega^2 x = \epsilon M(t) \quad \langle M(t) M(t') \rangle = C(t-t') \quad \langle M(t) x(t') \rangle = \epsilon r(t - r')$$

Zeroth law of thermodynamics: two bodies in thermal contact reach the same temperature.

The harmonic oscillator has an energy T for all ω iff

$$r(t) = -\frac{1}{T}\frac{dC}{dt}$$

One can do the computation just at order ϵ^2.

Off-equilibrium fluctuation dissipation relations (Cugliandolo Kurchan)

We start from an high temperature system:

we cool the system at the final temperature at time $-t_w$.

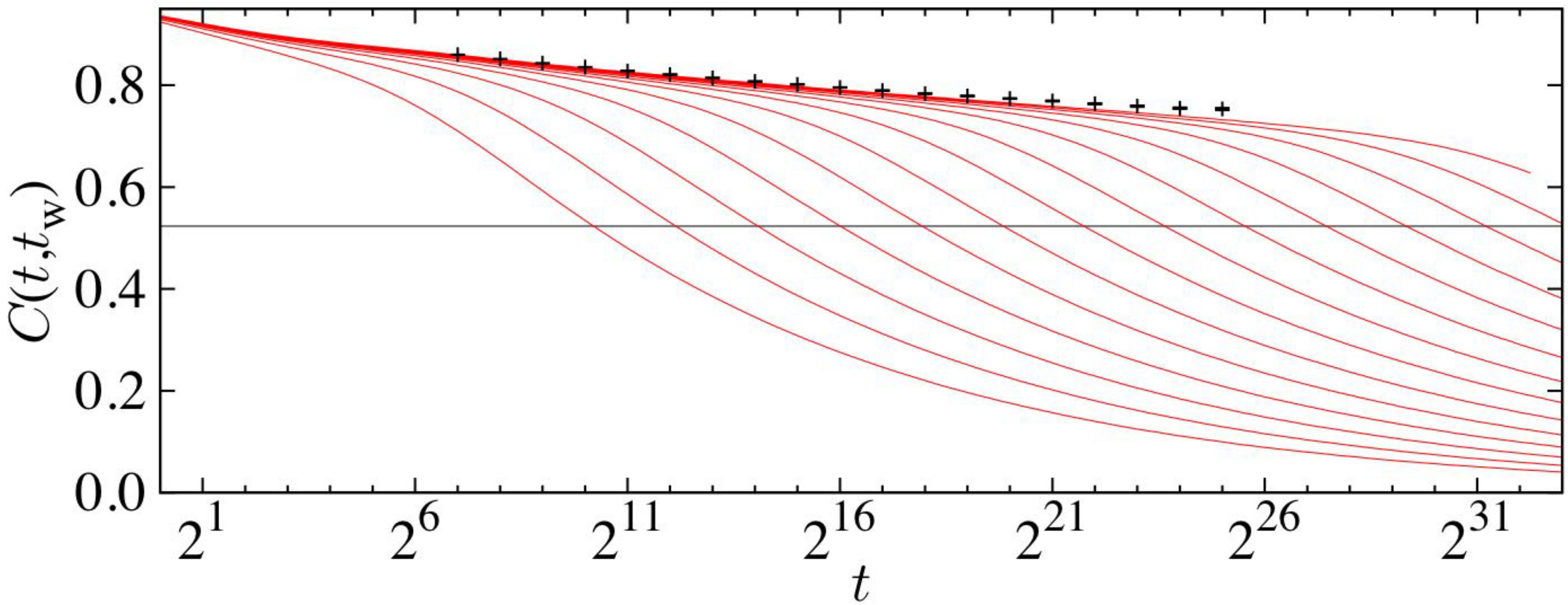
$$C(t_w, t) = \langle M(t) M(0) \rangle \ .$$

We are in the aging regime

$$\text{if} \;\; t << t_w \;\; C(t_w, t) = C(t) \, , \qquad\qquad \text{if} \;\; t/t_w = O(1) \;\; C(t_w, t) \approx \mathcal{C}(t/t_w) \ .$$

In the same way we define the relaxation function $R(t_w, t)$.

In the limit $t_w \to \infty$ we recover the equilibrium correlation and response functions.



Aging in Monte Carlo simulations of a 3-d spin glass (Janus collaboration).

We plot the correlation function for different t_w: from 2^6 to 2^{32}.

10^{11} time steps for systems with half million spins.

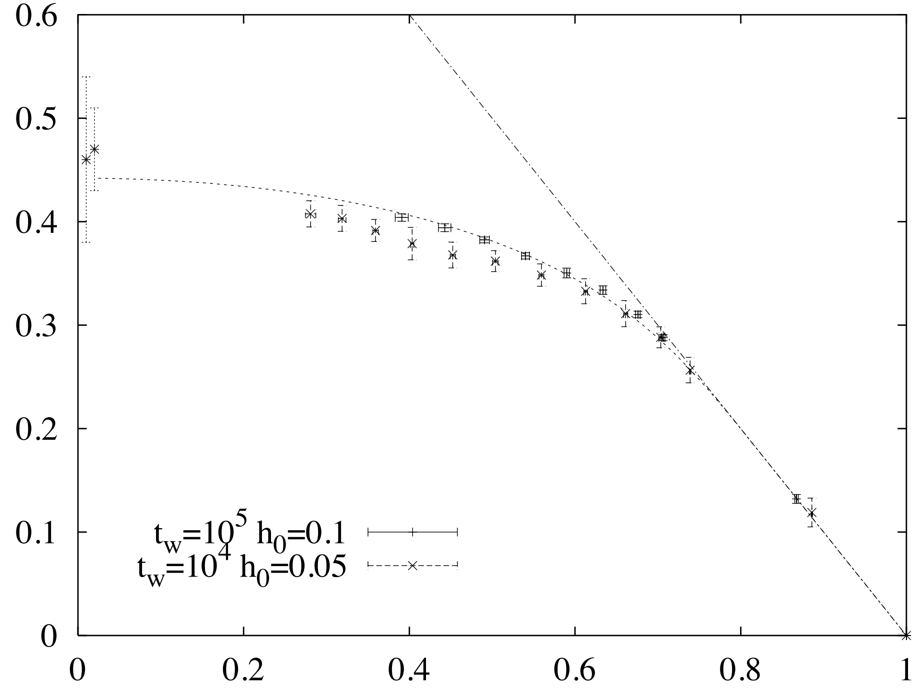
We have defined the off-equilibrium relaxation ($R(t_w, t)$) and correlation ($C(t_w, t)$) functions.

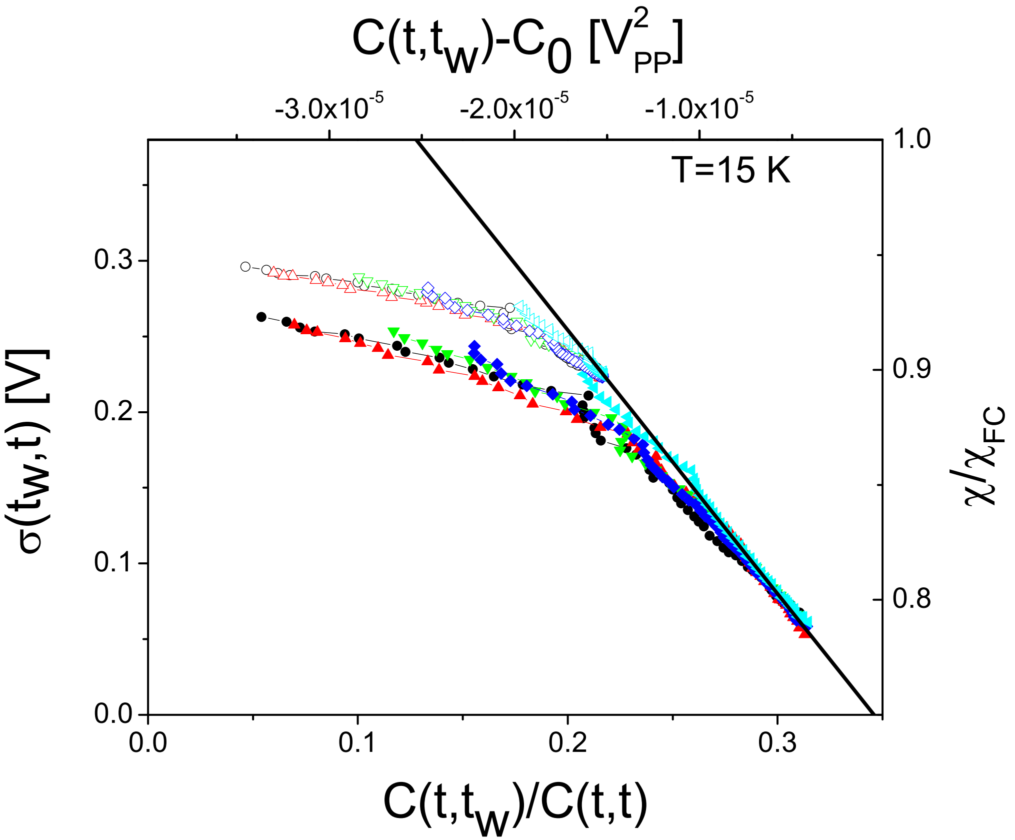
For large values of t_w we eliminate the time (t) in a parametric way: we find

$$\frac{dR}{dC} = -\frac{1}{T(C)}$$

Two temperatures scenario

- For $C > C_{plateaux}$ $T(C) = T$
- For $C < C_{plateaux}$ $T(C) = T_V$

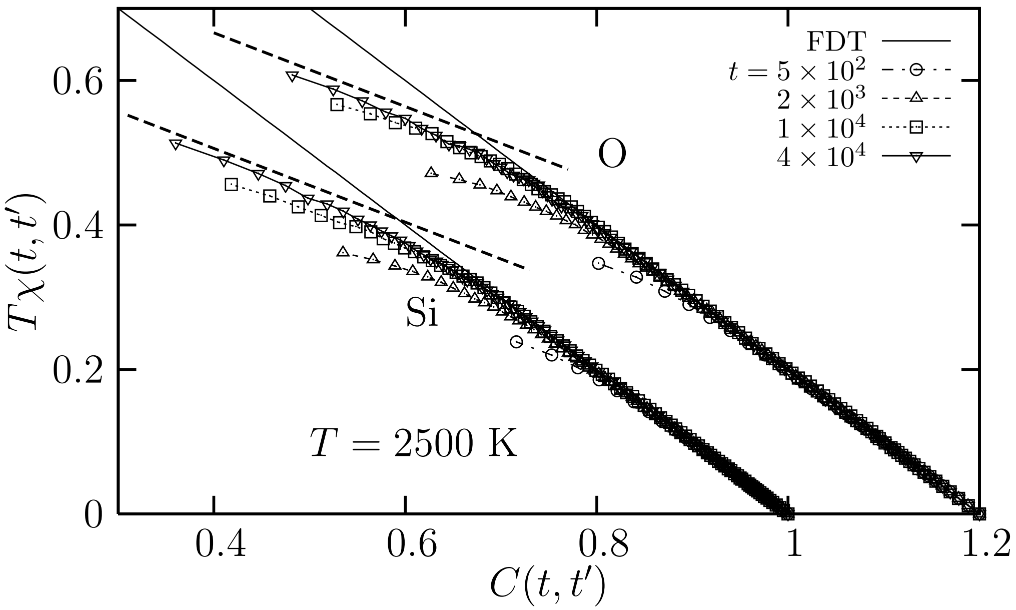


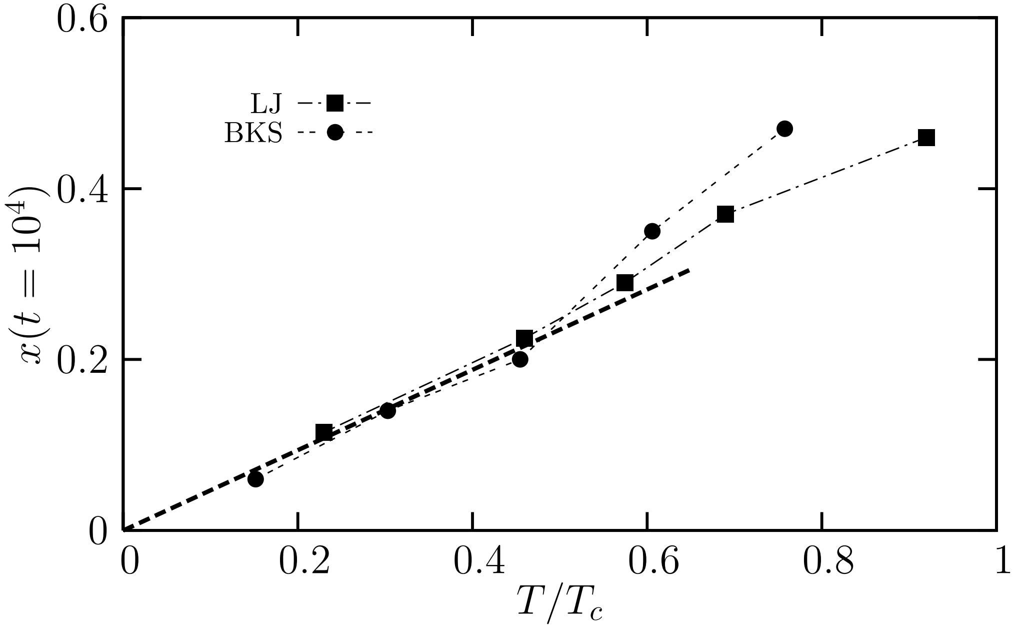


Simulations (spin glasses) Experiments (spin glasses)

and absolute theoretical prediction (no fitting parameters). Relaxation versus correlation

Numerical simulations in glasses (Silica)





. In the figure above $x(T) \equiv \frac{T_V}{T}$

One finds that T_V is temperature-independent at small temperature as theoretical expected and it is of the order of the critical temperature.

The free energy landscape does not change too much with the temperature.

Lessons from mean field: the mosaic space time picture.

Equilibrium: in a region of size L there are quite different valleys that have similar free energies. The relative order of these free energies strongly depends on the boundary conditions.

What we expect if we start the dynamics from an high temperature (random) configuration?

The system will go inside one of these valley if we observe on a scale less $\xi(t)$ and it will be look completely disordered on scale larger that $\xi(t)$. This is well in agreement with phenomenological theories of the glass transition that are forty years old.

We expect that the system at large times has dynamic heterogeneities. The dynamics length $\xi(t)$ is the scale of the correlated movement that happens at time t (cooperatively moving regions): i.e. a whole region of size $\xi(t)$ moves from one valley to an other valley.

I will discuss the these dynamics properties for spin glasses (in the case of structural glasses things are slightly different and less understood).

Dynamics for spin glasses in the low temperature phase

Let us consider some experiments that we can do only in simulations. We consider two (copies) clones of the same system and we study the dynamics starting from different random configurations. The two configurations are $\sigma(t)$ and $\tau(t)$.

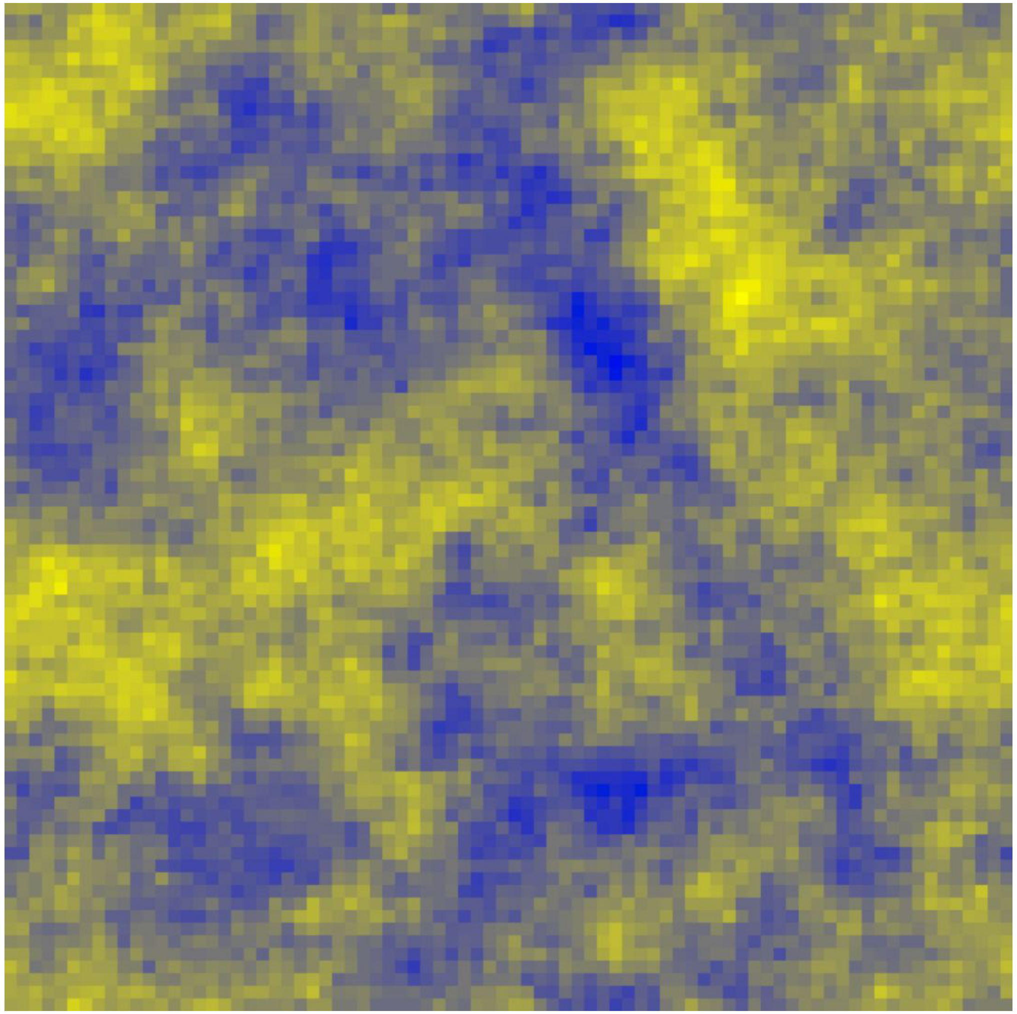
A key quantity is the overlap among two replicas at the same time:

$$q(x,t) = \sigma(x,t)\tau(x,t)$$

At time t the overlap will be correlated at distances less than $\xi(t)$. This effect can be measured by looking to the correlation functions of the overlap

$$C_4(x,t) = \langle q(x,t)q(0,t)\rangle \, ,$$

Very large scale simulations of spin glasses are possible.

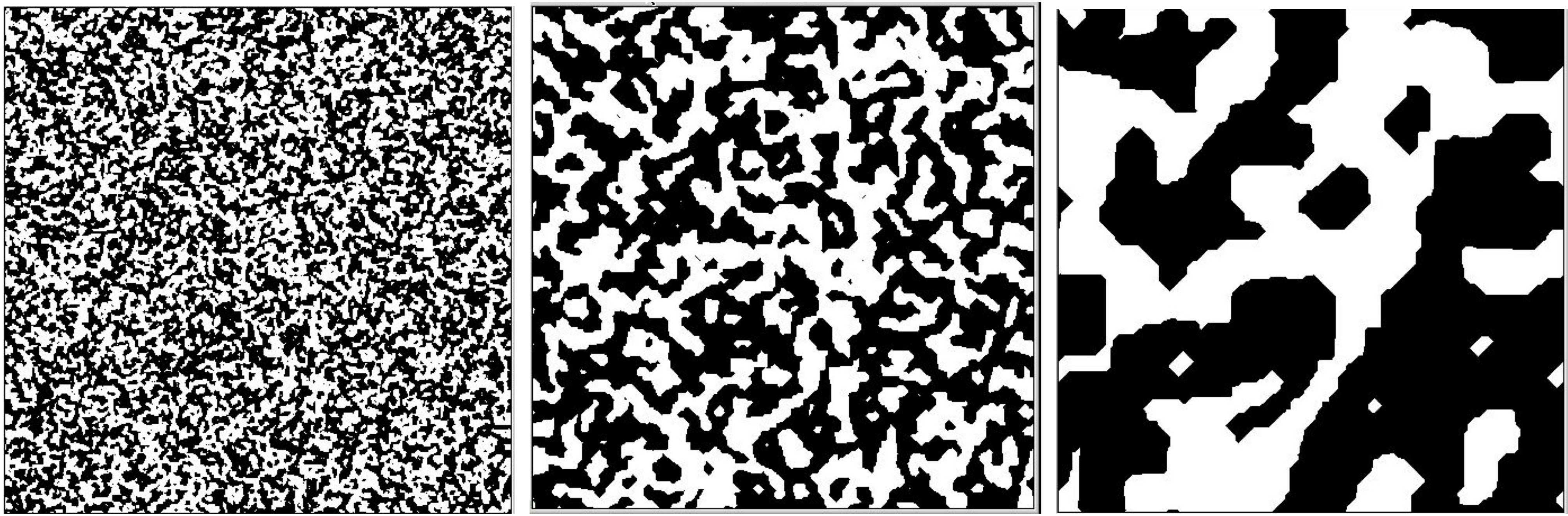


The instantaneous distribution of the overlap in a 80^3 cube after about 10^{11} Monte Carlo sweeps.

A large correlation length can be seen at bare eyes.

This effect is very similar to coarsening in Ising model .

We start from a configuration with random magnetization below the critical temperature. The correlation length increases with time.



Simulations are done on 512^2 system at $T = 0$.

Dynamic heterogeneities: cooperatively rearranging regions.

We have two configurations σ, one at time zero ($\sigma(0)$), one at time t ($\sigma(t)$).

The global correlations is

$$C(t) = \frac{\sum_x \sigma(x,0)\sigma(x,t)}{N} \equiv q(\sigma(t),\sigma(0))\,; \quad q(\sigma,\tau) \equiv \frac{\sum_x \sigma(x)\tau(x)}{N}\ .$$

The local correlation is given by $q_D(x) \equiv \sigma(x,0)\sigma(x,t)$.

Dynamical heterogeneities correspond to the presence of correlations in $q_D(x)$ at large distances:

$$C_{2,2}(x,t) = \langle q_D(x) q_D(0) \rangle$$

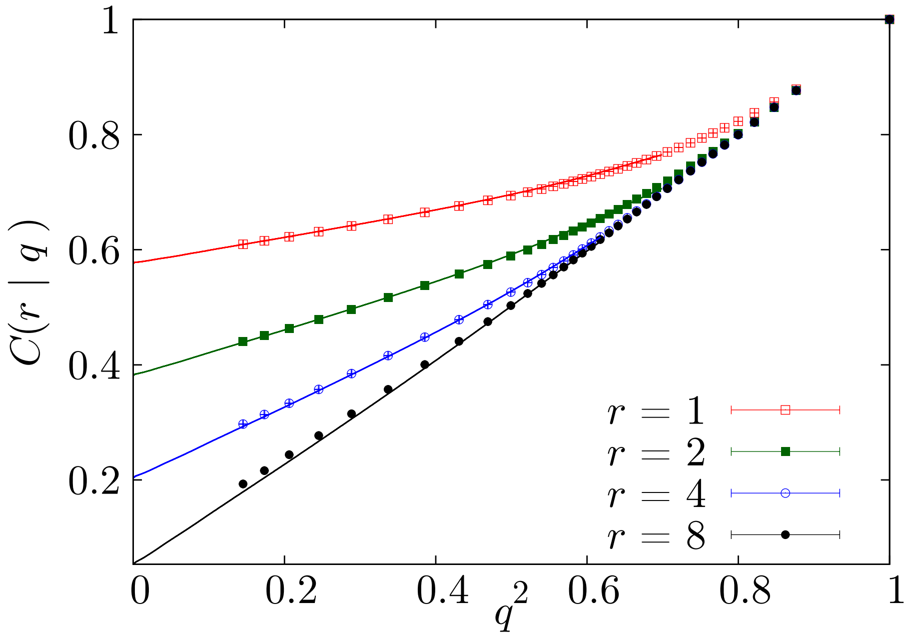
Eliminating the time we get $C_{2,2}(x|C)$.

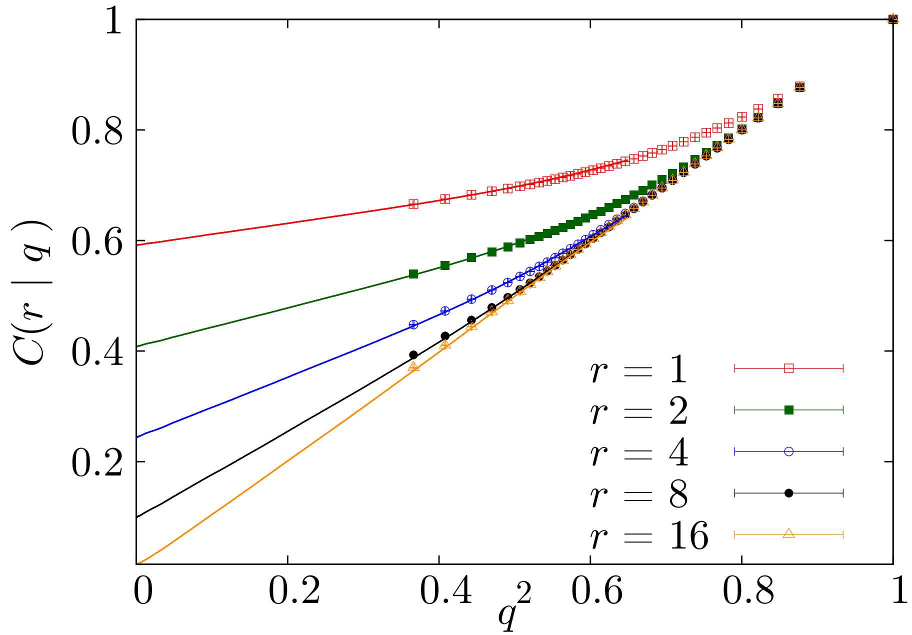
$C_{2,2}$: two spins at on time, two spins at another time in the same replica.

C_4: 4 spins at the same time in two replicas.

At small x there is a weak dependence on t_w;

at $x\xi(t_t)$ there is a strong dependence on t_w.





Data at $t_w = 10^8$ Data at $t_w = 10^{10}$

$q \equiv C$. Full lines are not fit! Full lines are absolute theoretical predictions

Open problems.

We must:

- Better understand the adiabatic approximation.
- Extend the results and the tests to structural glasses.
- Go beyond the adiabatic approximation and to compute the time dependence
- Do renormalization group computations.
- Perform careful experimental study of fluctuation-dissipation relations and make comparisons with theoretical predictions.