

Maiani-70

*Symposium in Honor of Luciano Maiani on the  
Occasion of his 70th Birthday*

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Dipartimento di Fisica, Universita' degli Studi  
"Sapienza"

# Chiral Symmetry in QCD and super-QCD and its implications

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# PART I. History

1974, infancy of QCD, doubts in everything, everything is  
**NEW** ✎ ✎ ✎ Epic, legendary times 😊

First Application was deep inelastic scattering;  
Neither theory nor data were good enough!

Second QCD application:  $\Delta I = 1/2$  Rule:

$$\frac{\Gamma(K_S \rightarrow \pi^+ \pi^-)}{\Gamma(K^+ \rightarrow \pi^+ \pi^0)} = 450 \qquad \text{Expected} \sim 9/4$$

Guido Altarelli, L. Maiani, Octet Enhancement of Nonleptonic Weak Interactions in Asymptotically Free Gauge Theories, Phys.Lett. B52 (1974) 351-354;

M.K. Gaillard, Benjamin W. Lee,  $\Delta I = 1/2$  Rule for Nonleptonic Decays in Asymptotically Free Field Theories, Phys.Rev.Lett. 33 (1974) 108.

$$\mathcal{O}_1 = \bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma^\mu u_L - \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L, \quad (8_f, \Delta I = 1/2)$$

$$\mathcal{O}_+ = \bar{s}_L \gamma_\mu d_L \bar{u}_L \gamma^\mu u_L + \bar{s}_L \gamma_\mu u_L \bar{u}_L \gamma^\mu d_L$$

includes  $\Delta I = 3/2$

J. Schwinger, 1964  $\rightarrow 9/4$

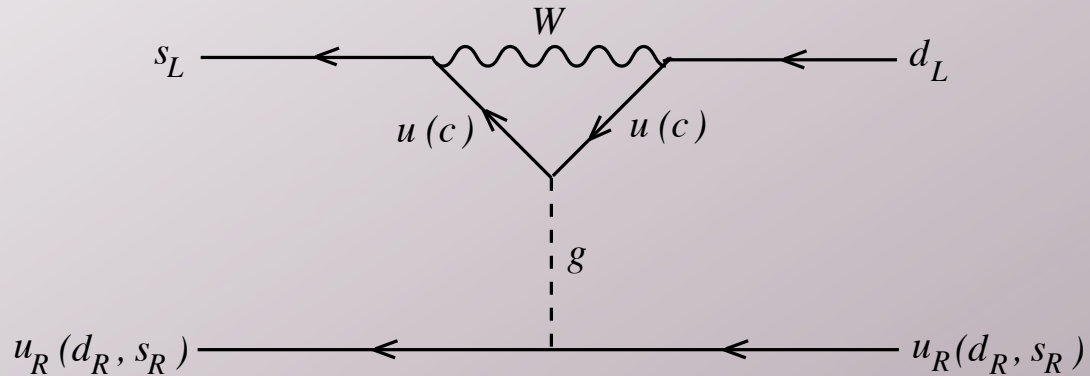
K. Wilson, 1969,  $\rightarrow (M_W)^K$  enhancement/suppression

AMGL, 1974  $\rightarrow \ln (M_W/\Lambda)$  enhancement/suppression

AMGL: Good news - 🍷 🍷 🍷 Inspirational,  $\mathcal{O}_1$  enhanced  $\mathcal{O}_+$  suppressed;

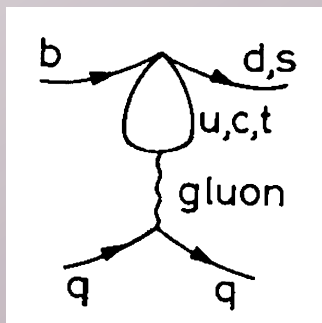
Bad news:  $9/4 \sim 10$  rather than 450 ☹️ ☹️ ☹️

# SVZ, penguins, 1974 (Heavy Quarks Enter the Game)



GIM cancellation: **believed**  $(m_c^2 - m_u^2)/\Lambda^2$

actual  $\ln(m_c^2/\Lambda^2)$     ✈ ✈ ✈



← J. Ellis et al.

$$\bar{s}_L \gamma_\mu t^a d_L \mathbf{D}_\mu \mathbf{G}^{\mu\nu a} \rightarrow$$

$$\mathcal{O}_5 = \bar{s}_L \gamma_\mu t^a d_L (\bar{u}_R \gamma^\mu t^a u_R + \bar{d}_R \gamma^\mu t^a d_R + \bar{s}_R \gamma^\mu t^a s_R), \quad (\mathbf{8}, \Delta I = 1/2)$$

One of two penguins,  
coefficient is rather small,  
**BUT**

LR chiral structure. Chiral enhancement:

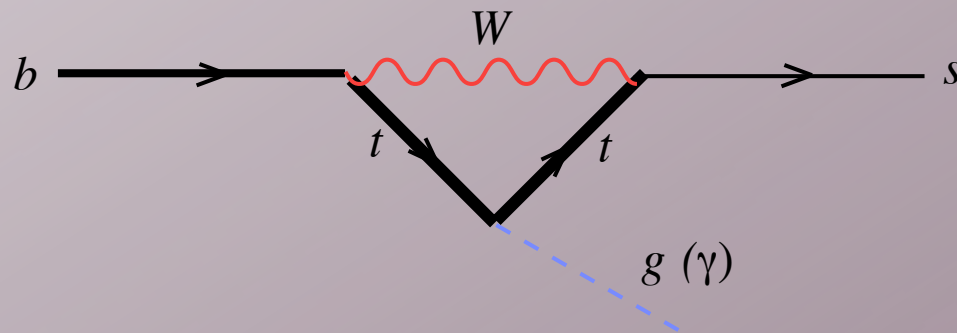
$$\frac{m_\pi^2}{m_u + m_d} \sim 2 \text{ GeV} \quad \text{versus } \Lambda \sim 200 \text{ MeV}; \text{ factor of 10 in ampl.}$$

Leutwyler, Gell-Mann, Weinberg:  $m_{u,d}$  quark mass terms very small

- 👉 Coleman-Witten, 1980: 'Proof' of  $\chi$ SB in large- $N$  QCD
- 👉 Leutwyler and many others: Development of  $\chi$ Pt 👉
- 👉 Matrix elements in lattice QCD (Martinelli and others)

Qualitatively and semi-quantitatively we are on the safe ground and in chartered waters, together with penguins.  
High precision theoretical predictions are still elusive because of the large-distance dynamics.

Penguins are much better off in heavier quark decays,  $bs \rightarrow \gamma$



# PART II. Present

Chiral symmetry restoration versus nonrestoration in highly excited states

## Outline

“Theory of QCD strings is hard to develop!  
For long strings (high excitations) it should be much easier.”

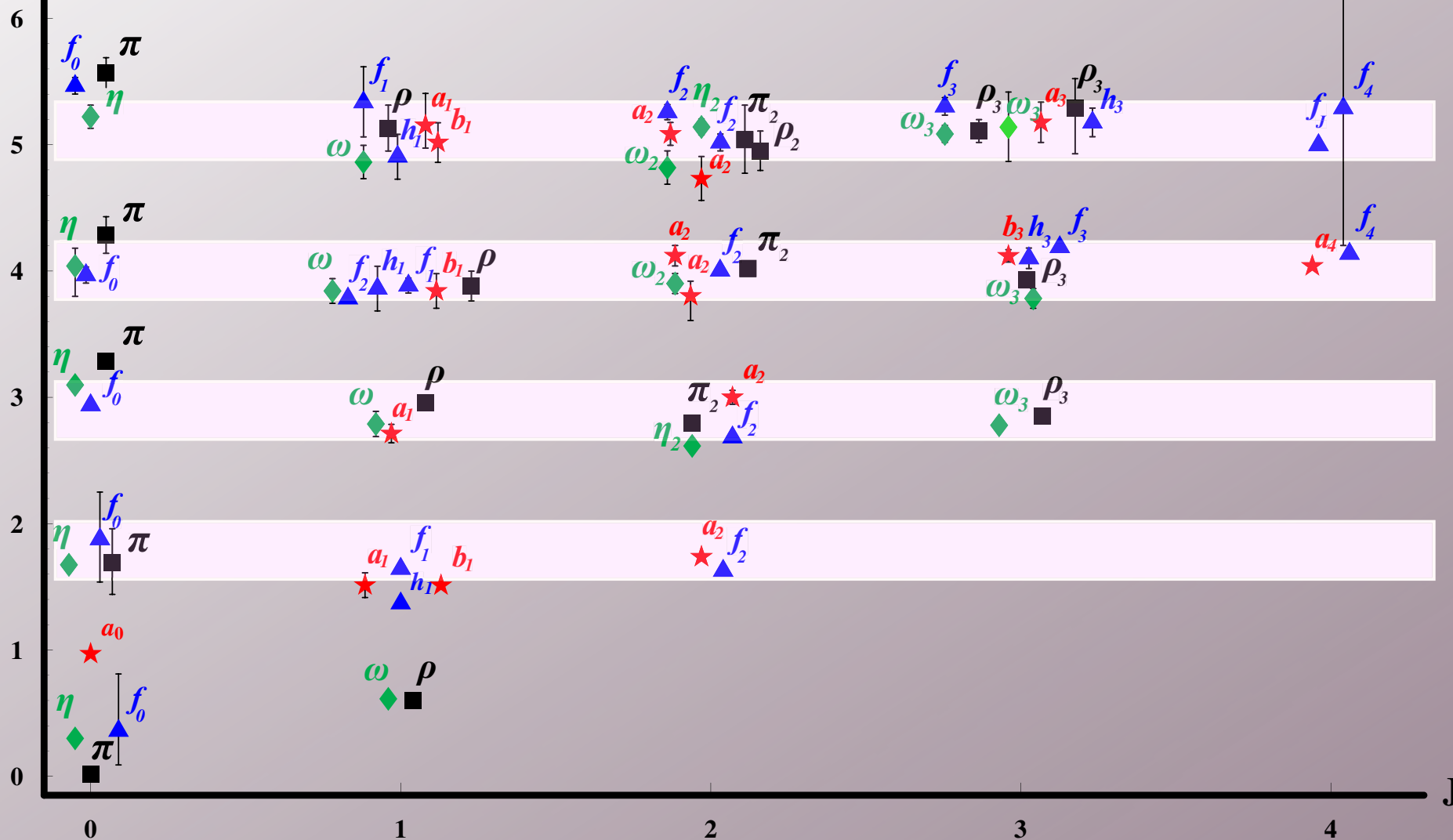
*Divine revelation/Common wisdom*

- \* Nonlinear versus linear realization
- \* Regge phenomenology
- \* Quasiclassical long string
- \* AdS/CFT and related approaches
- \* \* \* **Conclusions**

$M,^2 \text{ GeV}^2$

▲  $I = 0, P = +$     ★  $I = 1, P = +$

◆  $I = 0, P = -$     ■  $I = 1, P = -$





# Chiral Symmetry → Goldstone vs. Linear:

$$\text{SU}(N_f)_L \times \text{SU}(N_f)_R \rightarrow \text{SU}(N_f)_V$$

If  $|\pm\rangle$  are opposite parity states, the axial current  $a^\mu$

$$\begin{aligned} \langle + | a^\mu | - \rangle &= g(q^2) (p_+ + p_-)_\nu \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \\ &= g(q^2) \left[ (p_+ + p_-)_\mu - q_\mu \frac{M_+^2 - M_-^2}{q^2} \right] \end{aligned}$$

★ Linear realization:

$$M_+^2 = M_-^2, \quad g_A = g(0) = 1$$

★ Nonlinear realization:

$$g_{\pi+-} = f_\pi^{-1} g_A (M_+^2 - M_-^2)$$

← generalized Goldberger-Treiman relation

★ No constraints on  $g_A$ , the axial charge vanishes!

Glozman et al conjectured: At high energies the chiral condensate becomes less important. Pions decouple  $\rightarrow$  Asymptotically linear realization,

$$g_A \rightarrow 1, \quad \Delta M \equiv M_+ - M_- \rightarrow 0$$

inside the given representation  
while  $g_A \rightarrow 0$  for "outside" transitions

How fast?

\* If the distance  $\Delta M^2_{\text{chiral}} \ll \Delta M^2_{\text{radial}} \sim n^0$

A natural scaling law would be:

$$\Delta M^2_{\text{chiral}} \sim M^{-2} \sim (n^{-1}, J^{-1})$$

We will argue (from linearity of Regge trajectories, quasiclassical picture, etc.):

For highly excited states

$$\frac{M_+^2 - M_-^2}{a_1 \rho} = \Delta J_0 / \alpha' \sim \Lambda^2$$

and  $g_A \sim n^{-1/2}$  for all axial transition amplitudes.

**The Nambu-Goldstone mode persists!**

Two-dimensional 't Hooft model presents a (primitive) example of this type: no asymptotic restoration.

# Regge Phenomenology

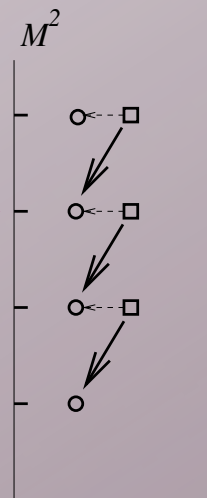
- ☀ Linear equidistant quark-meson trajectories (except  $M^2, J=0, 0$ );
- ☀ No parity degeneracy on the leading trajectory



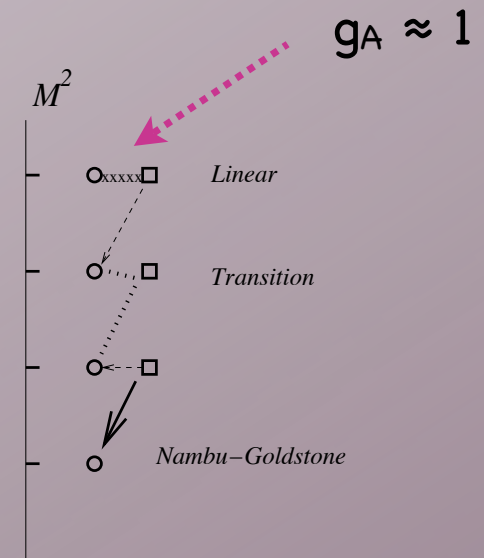
If the trajectories do not start converging "later" → asymptotic linear realization is not restored ...

Contrived scenario  
(not realized in the  
quasiclassical  
picture) →

All  $g_A$ 's different,  $\neq 1$

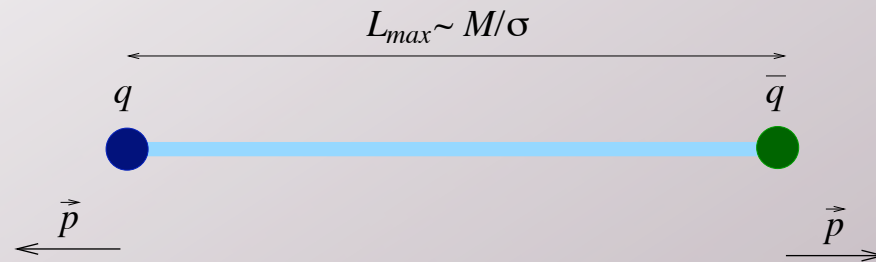


a) Nambu-Goldstone mode



b)

# Quasiclassical (long) string



The mass  $M_n$

$$M_n = 2p + \sigma r$$

Quantization

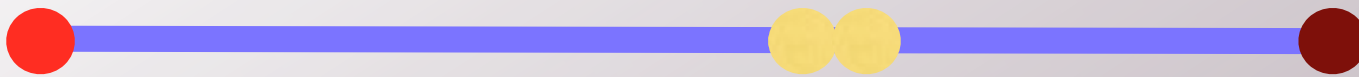
$$\int_0^{\ell_*} p(r) dr = \pi n \quad p(r) = (M_n - \sigma r)/2 \quad \ell_* = \frac{M_n}{\sigma}$$

gives

$$M_n^2 = 4\pi\sigma n \sim \Lambda^2 n$$

When  $L \neq 0$

$$n \rightarrow n_r + L$$



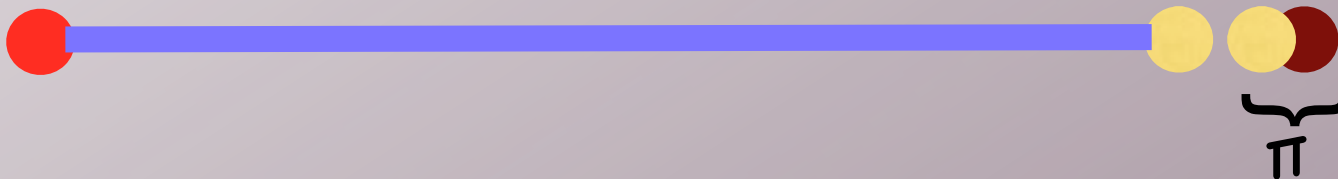
$$L \sim M$$

$$\Gamma_{\text{tot}} \sim (1/N) M \sim (1/N) \Lambda n^{1/2} \quad (\text{CNN, 1979})$$

(PSZ '05)

The # of channels  $\sim n$ ; each has  $\Gamma \sim (1/N) \Lambda n^{-1/2}$

$$A \rightarrow V\pi \text{ amplitude} \sim (\epsilon_i \epsilon_f) |\vec{p}_\pi| / N^{1/2} \sim (\epsilon_i \epsilon_f) \Delta M^2 g_A / f_\pi$$



$$\text{At maximum } g_A \sim \Lambda/M \sim n^{-1/2}$$

A part of the spectral degeneracy comes from spin independence.  
(Chromo)magnetic charges are screened in the vacuum!

An upper bound on spin-spin

$$\sim \Lambda^{-2} L^{-3} \sim \Lambda n^{-3/2}; \quad \Delta M^2 \sim 1/n$$

\* String degeneracy:

$$M = F(L+n_r)[1+O(1/n, 1/L)]$$

↑

$$(L+n_r)^{1/2}$$

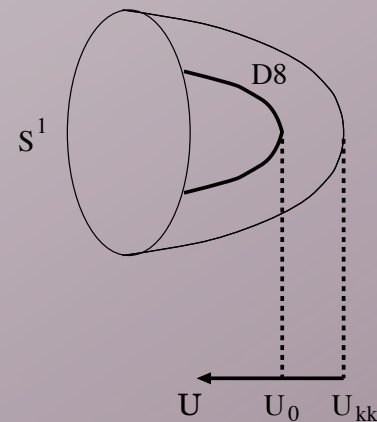
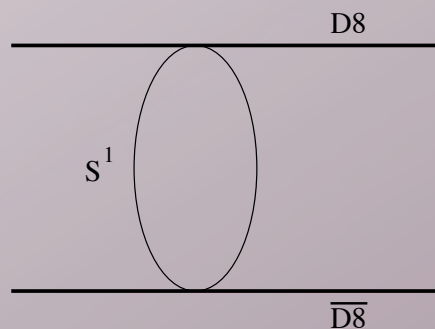
$$\Delta M^2 \sim 1/n$$

# AdS/CFT & AdS/QCD

- (a) Sakai & Sugimoto (top-down)
- (b) Karch, Katz, Son & Stephanov (bottom-up)
- (c) Casero, Kiritsis, Paredes (mixed)

Holographic description of hadrons with the fifth coordinate  $z$  is used.

$N_f$  test D8 -  $\overline{D8}$  brane pairs in the background of  $N_c$  D4 branes compactified on SUSY breaking  $S^1$ .





$$S^{\text{SS}} = \kappa \int d^4x dz \text{Tr} \left[ K^{-1/3} \frac{1}{2} F_{\mu\nu}^2 + K F_{\mu z}^2 \right]$$

$$K = 1 + z^2, \quad \kappa = \frac{\lambda N_c}{108\pi^3}.$$

The massless pion is built in and does not decouple from high excitations.  
Nambu-Goldstone mode

$$S^{\text{KKSS}} = \int d^4x dz e^{\Phi(z)} \sqrt{g} \left[ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right]$$

$$\Phi = z^2 \quad g_5^2 = 12\pi^2/N_c$$

QM eigenvalues of the holographic coordinate  $z$  give  $M^2$  in 4D. Characteristic  $z \sim n^{1/2}$ .

“ $\rho$ - $a_1$ ” splittings in KKSS are due to the  $X$  field (containing pion and its scalar partner).

KKSS:  $X \rightarrow \text{const.} \Rightarrow M^2_+ - M^2_- \sim 1/n$  BUT trajectories  
NONlinear

CKP:  $X \rightarrow z, \Rightarrow M^2_+ - M^2_- \sim \text{const.}$

Trajectories are linear & equidistant  
BUT  $\Rightarrow$  Nambu-Goldstone mode

# Conclusions

\* Linearity of the Regge trajectories & quasiclassical picture of long strings imply persistence of the Nambu-Goldstone mode of the chiral symmetry realization for high excitations.



Happy Birthday, Luchano! Many  
happy returns of the day!