Time-dependent and eikonal approximations to analyse breakup of halo nuclei

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Breakup reaction

Breakup used to study exotic nuclear structures
e.g. halo nuclei: halo dissociates from core
by interaction with target

Coulomb breakup used to infer radiative-capture rates of astrophysical interest [^{7}Be(p,γ)^{8}B,^{14}C(n,γ)^{15}C,...]

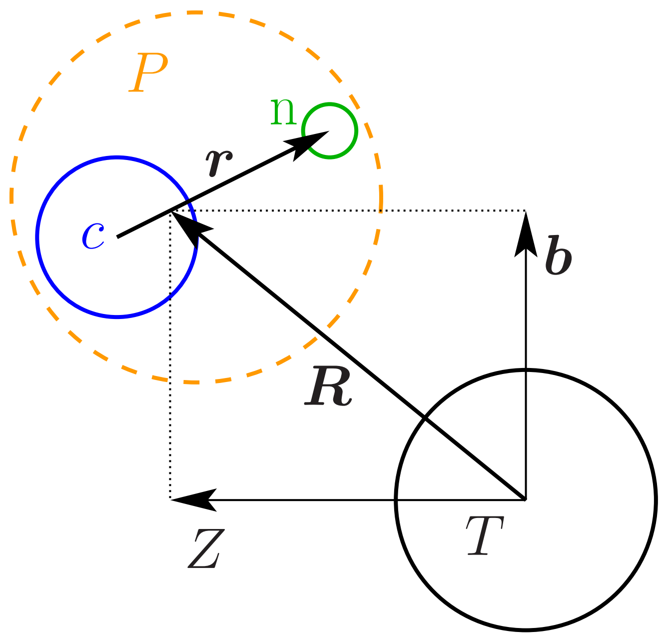
Outline

- Description of breakup models: CDCC, time-dependent (TD), eikonal (DEA)
- Comparison.
  When and why chose a particular model?

Framework

Projectile (P) modelled as a two-body system:
core (c)+loosely bound neutron (n) described by

$$H_0 = T_r + V_{cn}(\boldsymbol{r})$$



V_{cn} adjusted to reproduce
bound state Φ_0
and resonances

Target T seen as
structureless particle

P-T interaction simulated by optical potentials
$\Rightarrow$ breakup reduces to three-body scattering problem:

$$[T_R + H_0 + V_{cT} + V_{nT}]\,\Psi(\boldsymbol{R},\boldsymbol{r}) = E_T\Psi(\boldsymbol{R},\boldsymbol{r})$$

with initial condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\Phi_0(\boldsymbol{r})$

CDCC

Solve the three-body scattering problem:

$$\left[T_R + H_0 + V_{cT} + V_{\mathrm{n}T}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

by expanding Ψ on eigenstates of H_0

$\Psi(\boldsymbol{r},\boldsymbol{R}) = \sum_i \chi_i(\boldsymbol{R})\Phi_i(\boldsymbol{r})$ with $H_0\Phi_i = \epsilon_i\Phi_i$

Leads to set of coupled-channel equations (hence CC)

$$\left[T_R + \epsilon_i + V_{ii}\right]\chi_i + \sum_{j\neq i} V_{ij}\chi_j = E_T\chi_i,$$

with $V_{ij} = \langle\Phi_i|V_{cT} + V_{\mathrm{n}T}|\Phi_j\rangle$

The continuum has to be discretised (hence CD)

[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]

Fully quantal approximation

No approx. on P-T motion, no restriction on energy

But expensive computationally (at high energies)

Time-dependent model

P-T motion described by classical trajectory $\boldsymbol{R}(t)$

[Esbensen, Bertsch and Bertulani, NPA 581, 107 (1995)]

[Typel and Wolter, Z. Naturforsch. A54, 63 (1999)]

P structure described quantum-mechanically by H_0

Time-dependent potentials simulate P-T interaction

Leads to the resolution of time-dependent Schrödinger equation (TD)

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},\boldsymbol{b},t) = [H_0 + V_{cT}(t) + V_{nT}(t)]\Psi(\boldsymbol{r},\boldsymbol{b},t)$$

Solved for each $\boldsymbol{b}$ with initial condition $\Psi \underset{t\to-\infty}{\longrightarrow} \Phi_0$

Many programs have been written to solve TD

Lacks quantum interferences between trajectories

Dynamical Eikonal Approximation

Three-body scattering problem:

$$[T_R + H_0 + V_{cT} + V_{nT}]\,\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0$

Eikonal approximation: factorise $\Psi = e^{iKZ}\widehat{\Psi}$

$$T_R\Psi = e^{iKZ}[T_R + vP_Z + \frac{\mu_{PT}}{2}v^2]\widehat{\Psi}$$

Neglecting T_R vs P_Z and using $E_T = \frac{1}{2}\mu_{PT}v^2 + \epsilon_0$

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - \epsilon_0 + V_{cT} + V_{nT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi} \underset{Z\to-\infty}{\longrightarrow} \Phi_0(\boldsymbol{r})$

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

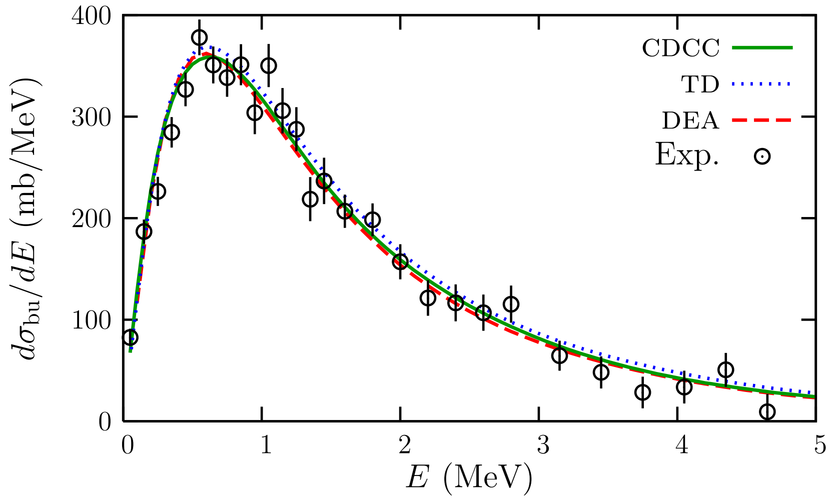
Same equation as TD with straight line trajectories

^{15}C + Pb @ $68A$MeV

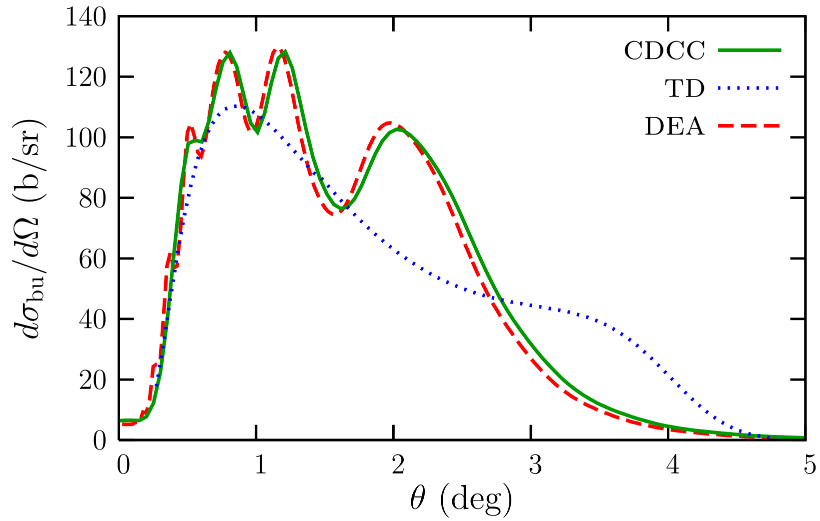
Comparison of CDCC, TD, and DEA

[PC, Esbensen, and Nunes, accepted in PRC]

$d\sigma_{\rm bu}/dE$



$d\sigma_{\rm bu}/d\Omega$



All models agree

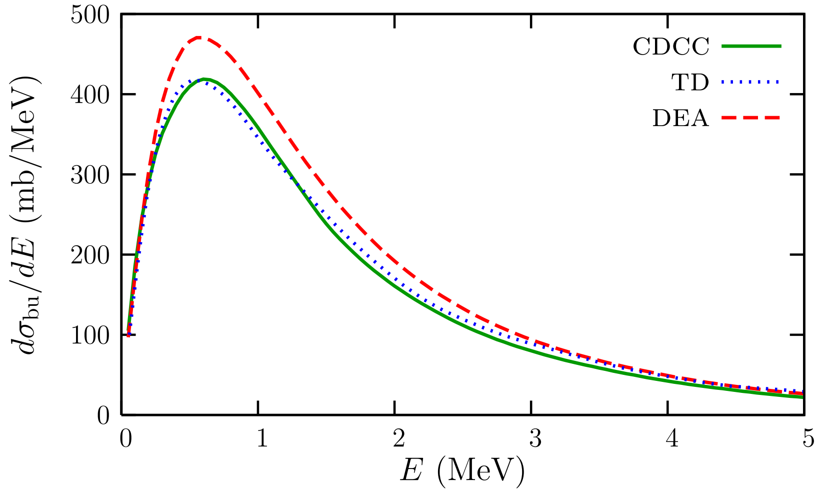
Data: [Nakamura *et al.* PRC 79, 035805 (2009)]

DEA agrees with CDCC

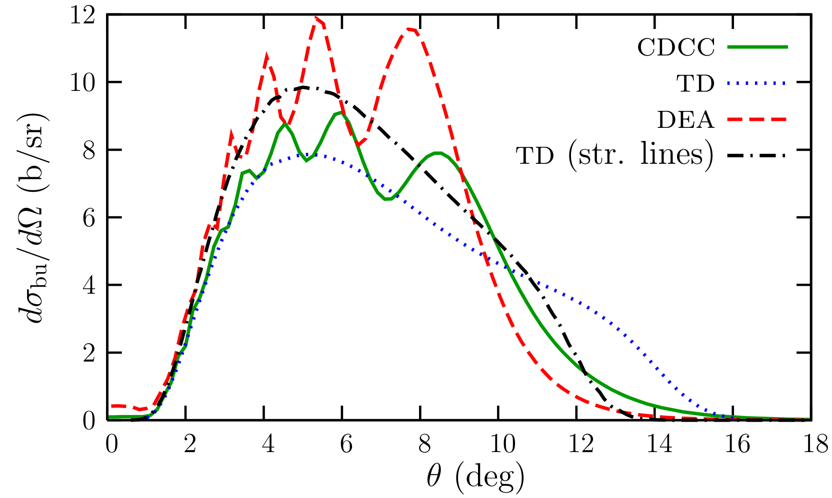
TD reproduces trend but lacks oscillations

^{15}C + Pb @ $20A$MeV

$d\sigma_{\rm bu}/dE$



$d\sigma_{\rm bu}/d\Omega$



TD $\equiv$ CDCC
DEA too high

TD gives trend of CDCC
(lacks oscillations)
DEA peaks too early

DEA$\neq$CDCC due to Coulomb deflection
(TD straight lines)

Comparison

	CDCC	TD	DEA	Eikonal
Coulomb bu	ok	ok	ok	diverges
nuclear bu	ok	no	ok	ok
Energy	all	all(?)	high	high
Observables	all	$\int d\theta$	all	all
CPU time	long	short	short	very short
(for this test)	208 h	14 h	48 h	1 h
RAM	64 GB	desktop	400 MB	desktop
Other reactions				
stripping	maybe	no	maybe	ok
transfer	maybe	no	no	no

Conclusion and outlook

Good understanding of reaction process
Next step: improving projectile description

CDCC is by no means the only one reaction model
TD and DEA (and eikonal) are reliable
(within their range of validity) AND are faster

⇒ use them to improve description of the projectile:

- core excitation
- two-neutron haloes (cf. E. C. Pinilla Beltran's poster)
- microscopic description

Range of validity can be extended

- describe stripping with CDCC (cf. K. Minomo's talk)
- low-E Coulomb breakup within DEA

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