

Evolution of the shell structure in medium-mass nuclei:

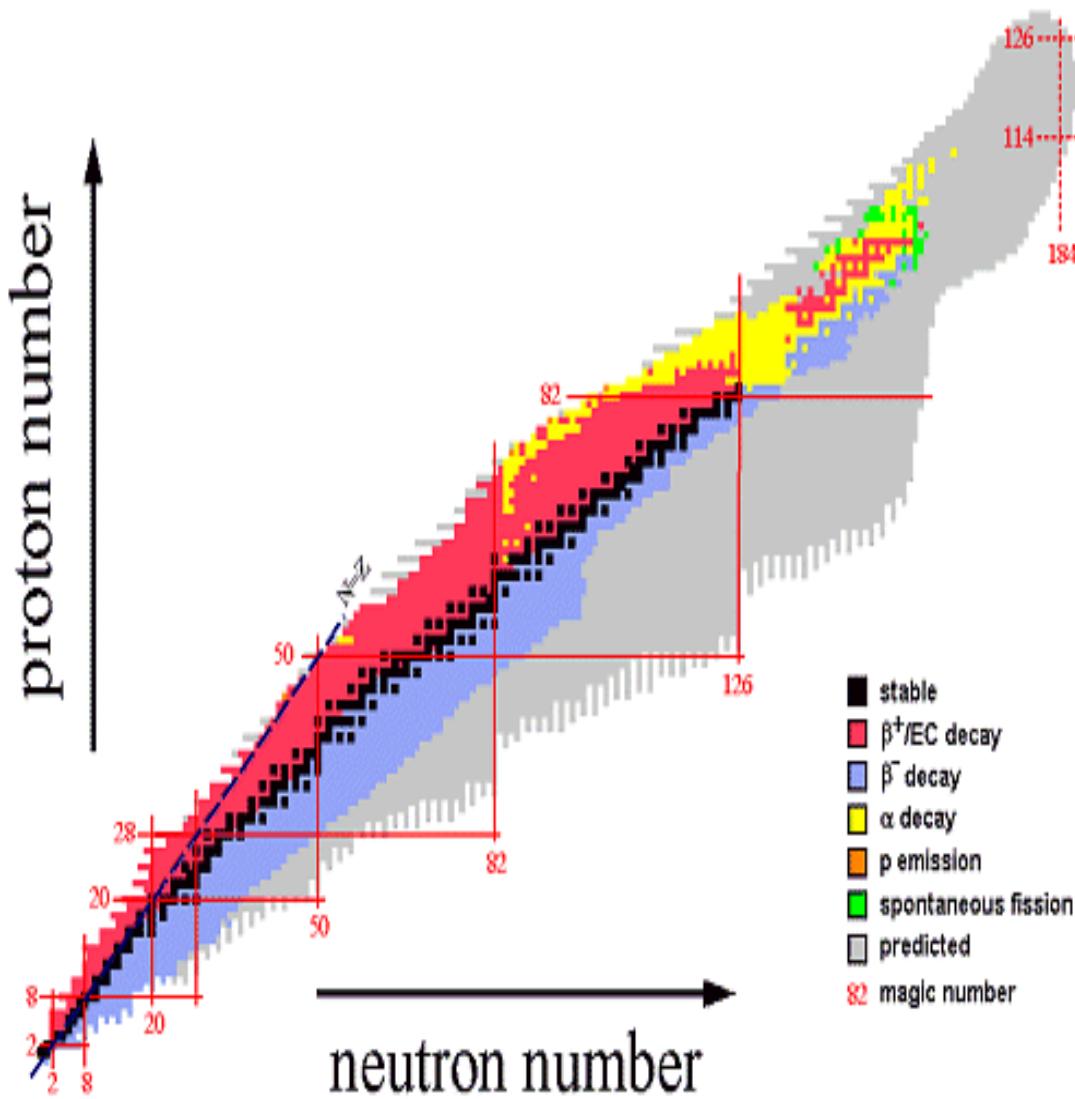
Search for the $2d_{5/2}$ neutron orbital in ^{69}Ni



Mohamad Moukaddam
UDS, IPHC Strasbourg, France

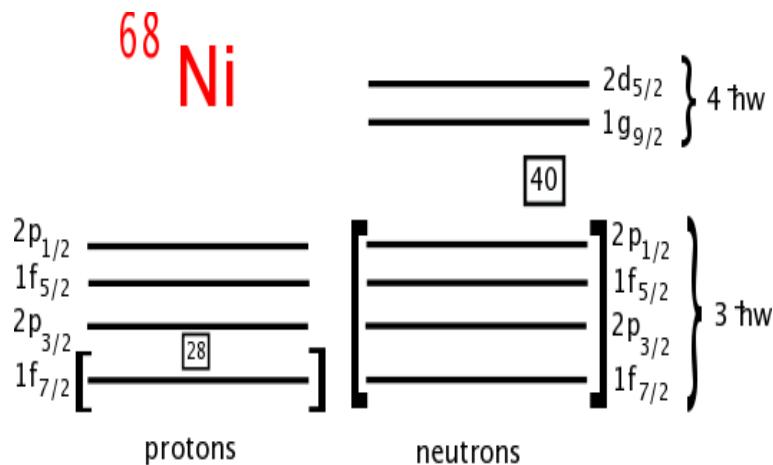
DREB 2012
26 - 29 March, Pisa , Italy

Introduction

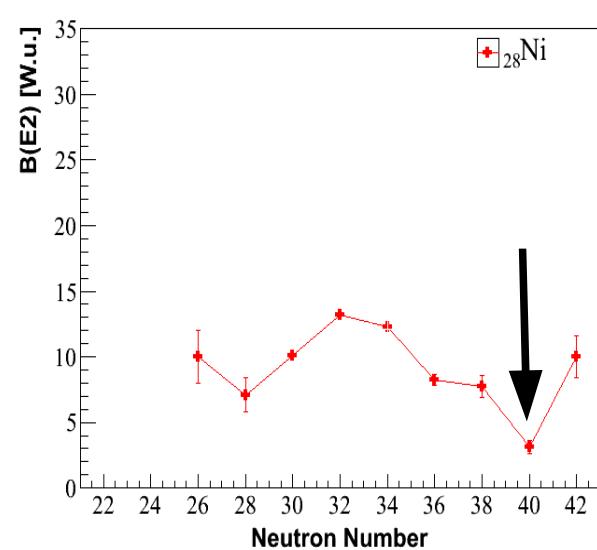
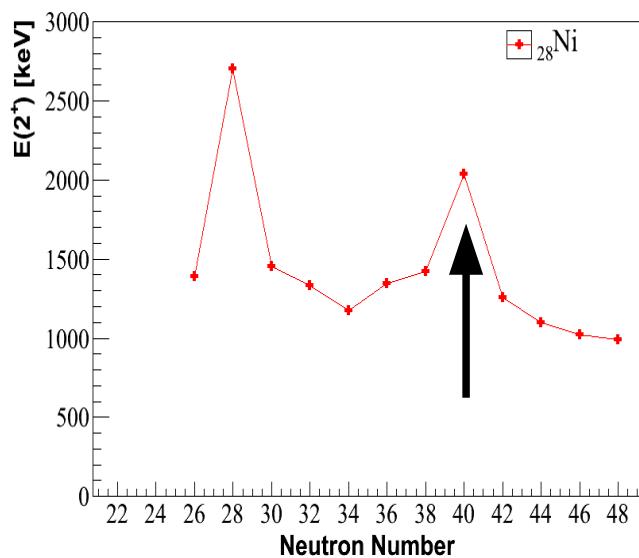


- **Light-mass region :**
 - HO shell closures : 2, 8, 20
- **heavy-mass region :**
 - Spin-orbit shell closures : 50, 82..
- **Structure evolution of neutron-rich nuclei at N=40**

N=40 Region : ^{68}Ni

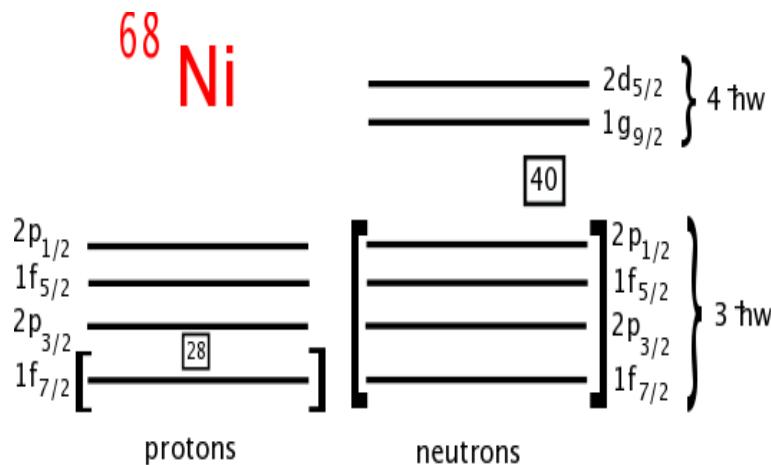


- Semi-magic nucleus ($Z=28$)
- $E(2^+) \sim 2 \text{ MeV}$ *Broda et al, PRL, 74, 868, 1995*
- $B(E2) \sim 3.2 \text{ W.u.}$ *Sorlin, PRL, 88, 092501, 2002*
- No trace for magicity according to S_{2n}
Rahaman, EPJ, 32, 87, 2007

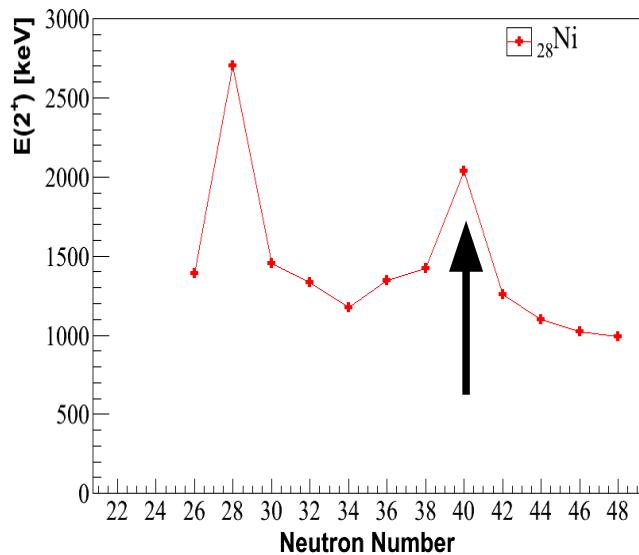


Magic signature

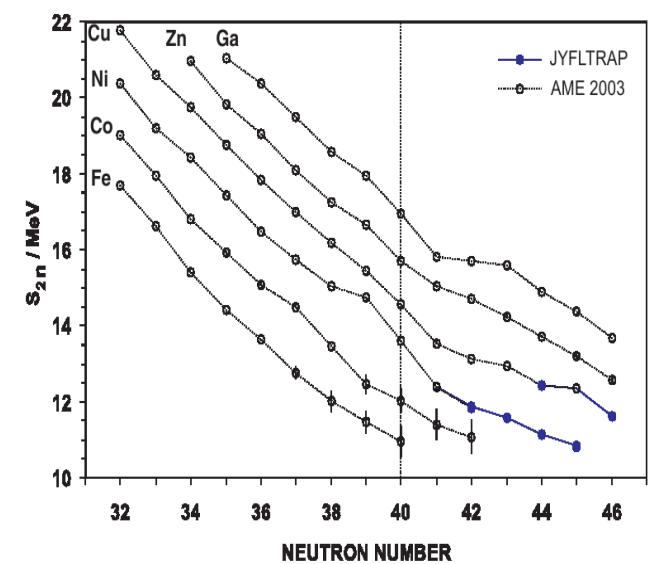
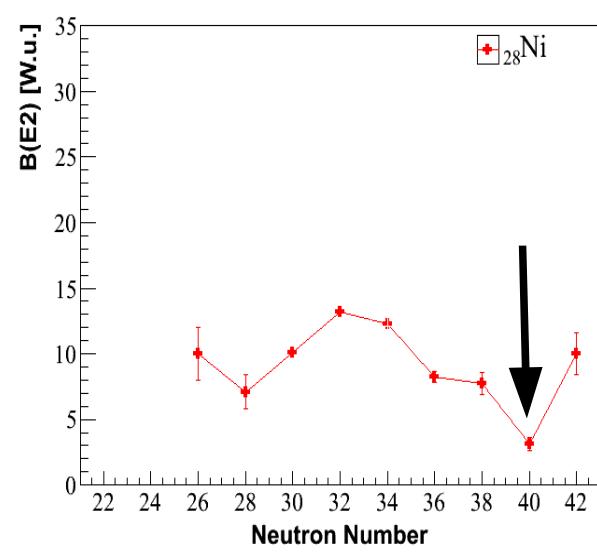
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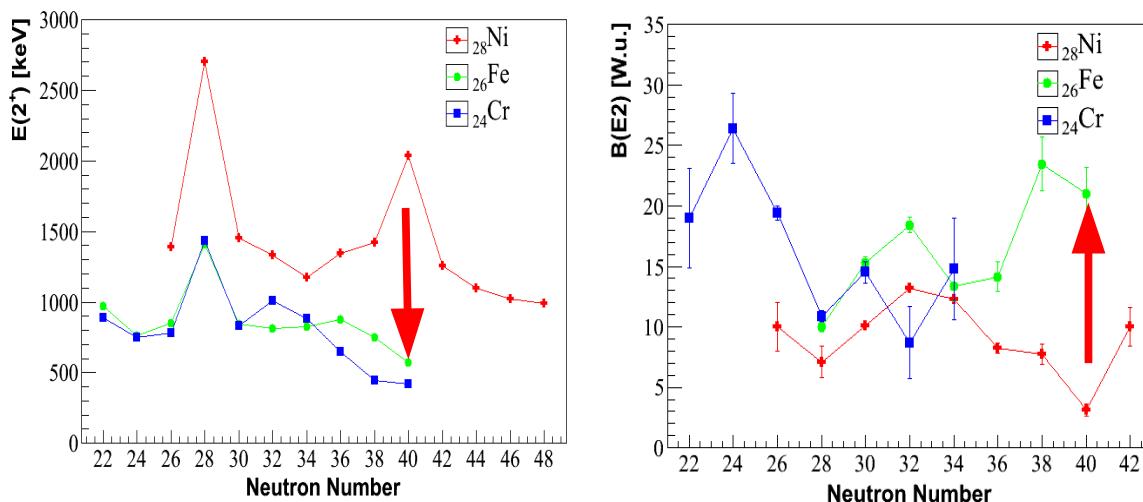
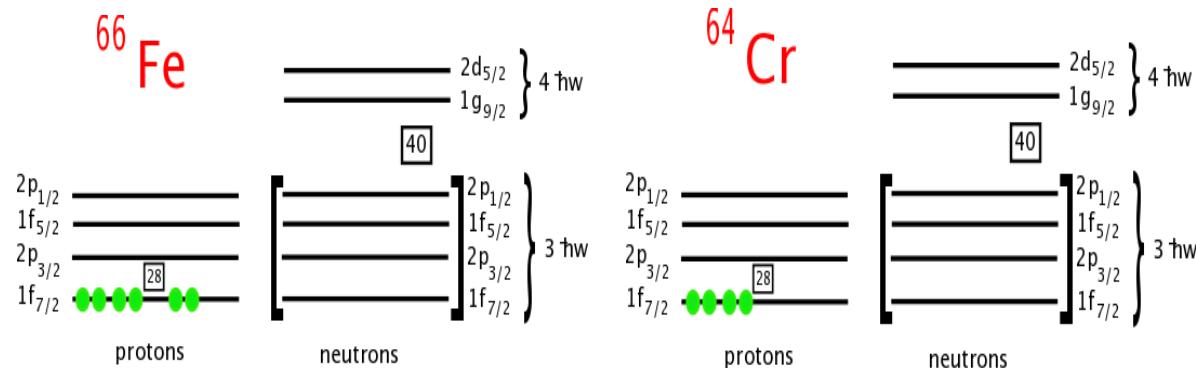


Magic signature



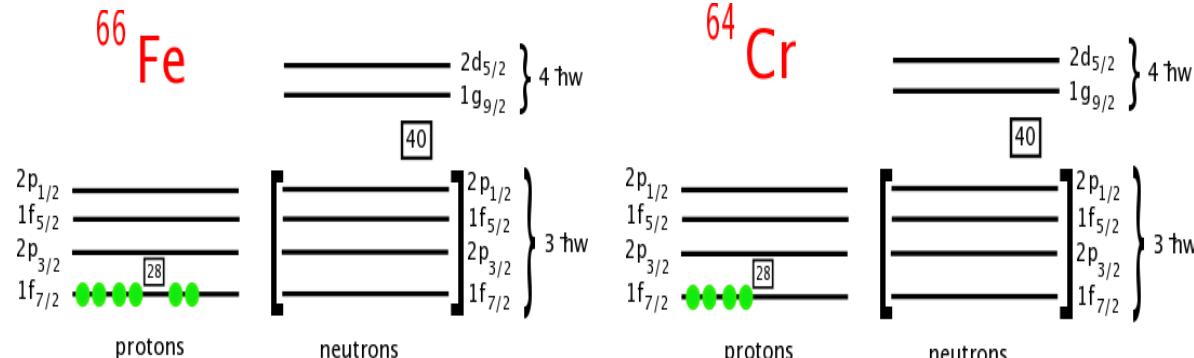
No trace for magicity!

N=40 Region : Deformation in Fe et Cr

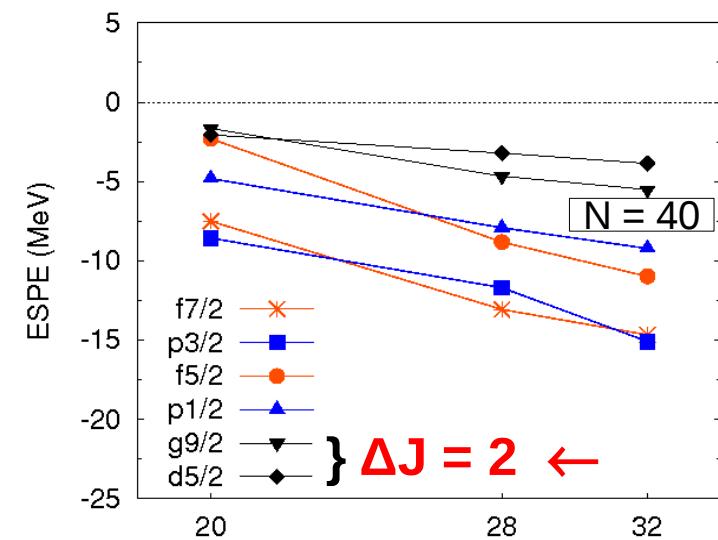


- Fe et Cr : deformation increases when approaching N = 40
- N = 40 HO shell-closure is quickly washed out with removal of proton pairs

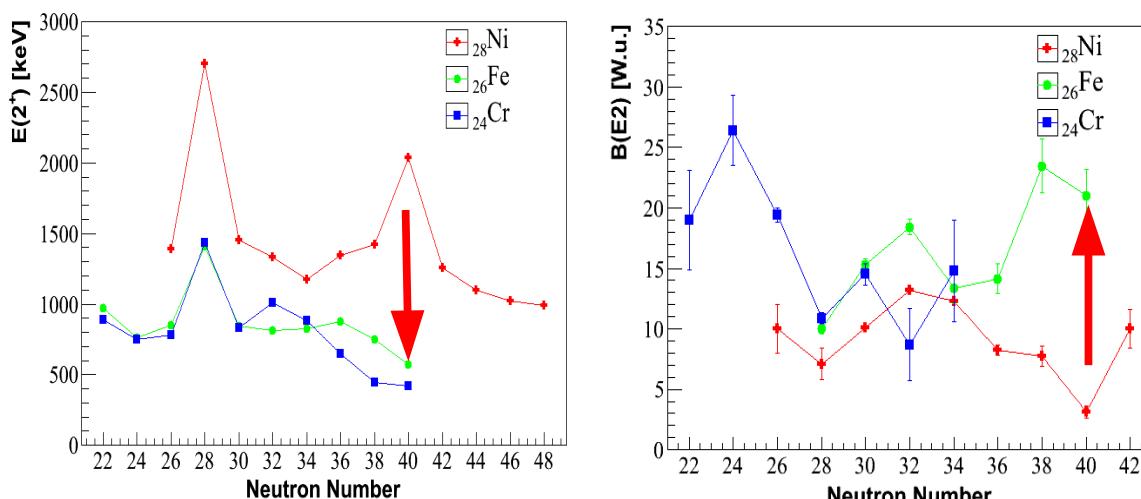
N=40 Region : Deformation in Fe et Cr



2d_{5/2} plays a major role in the deformation mechanism at N = 40 *Caurier et al. EPJ, A, 15, 2002, 145*

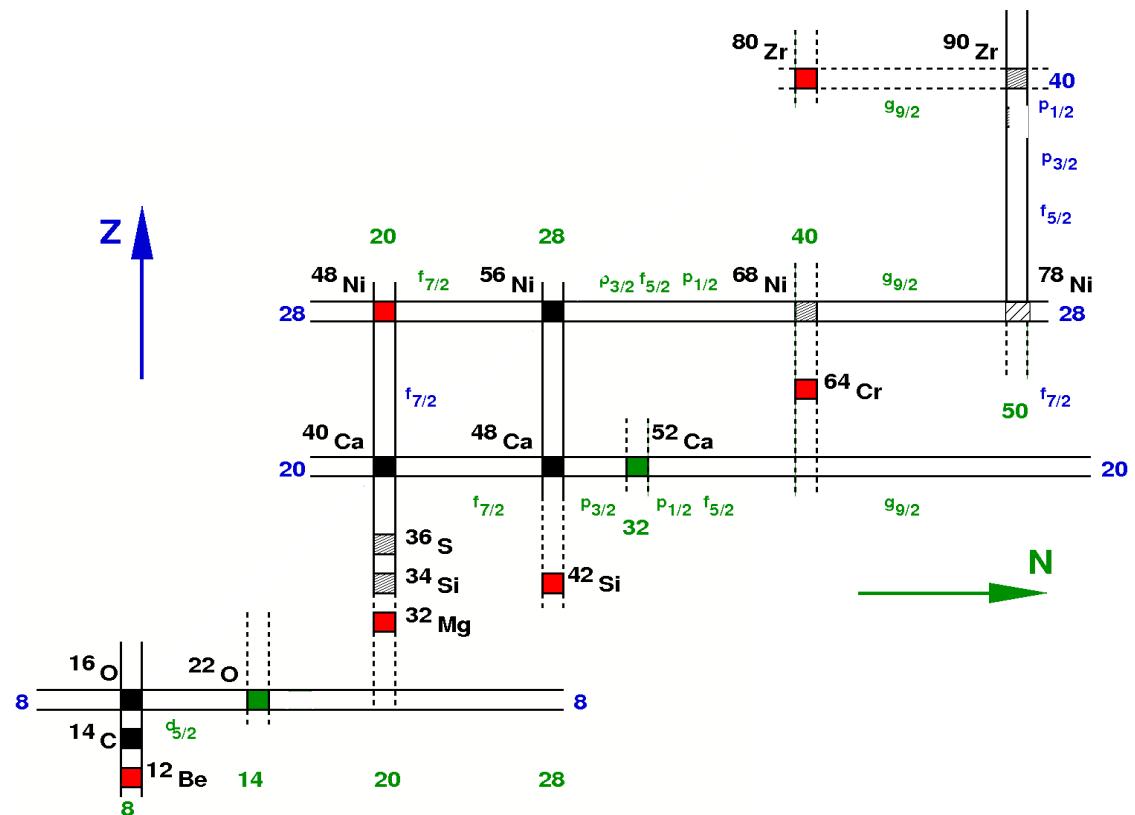


Lenzi et al., PRC, 82, 054301, 2010



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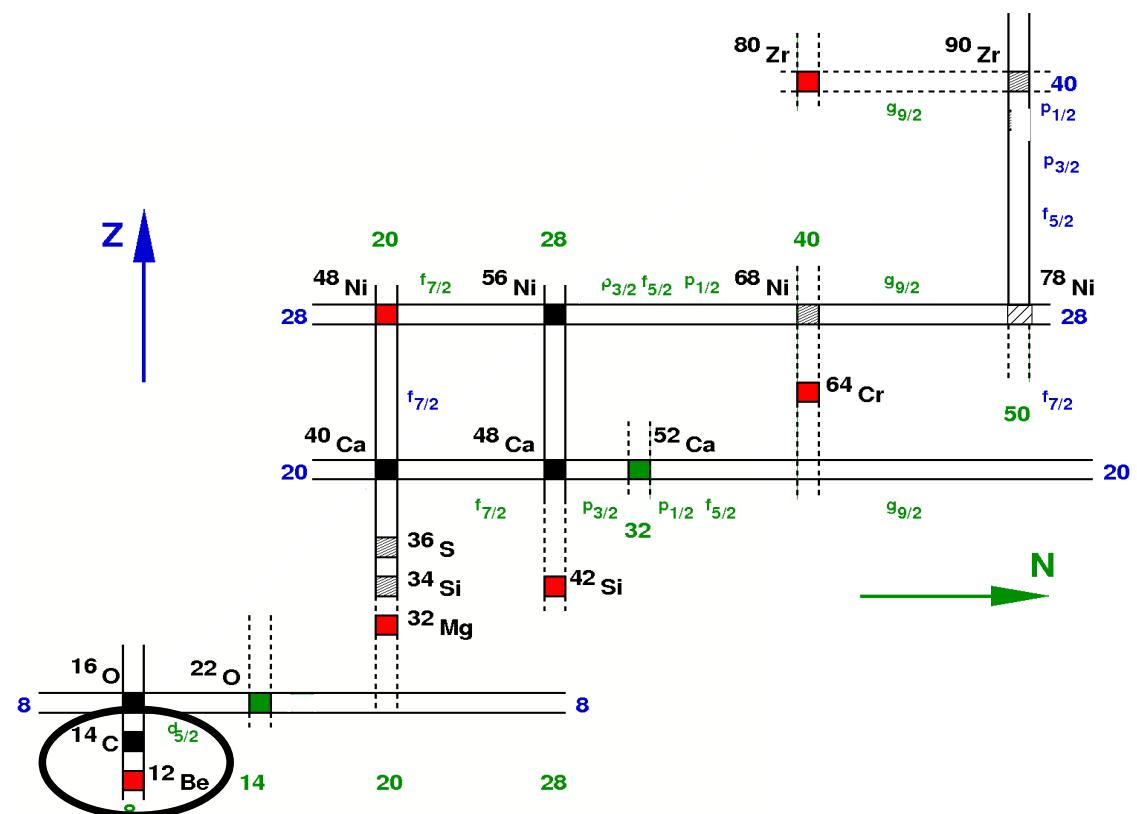
Island of inversion and quadrupole deformations



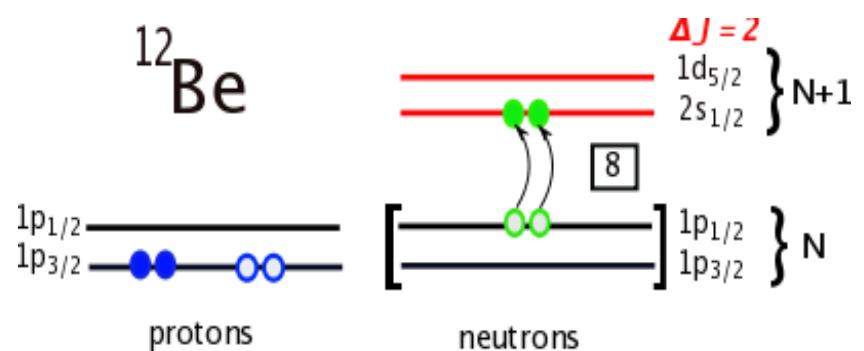
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Caurier et al. EPJ, A, 15, 2002, 145

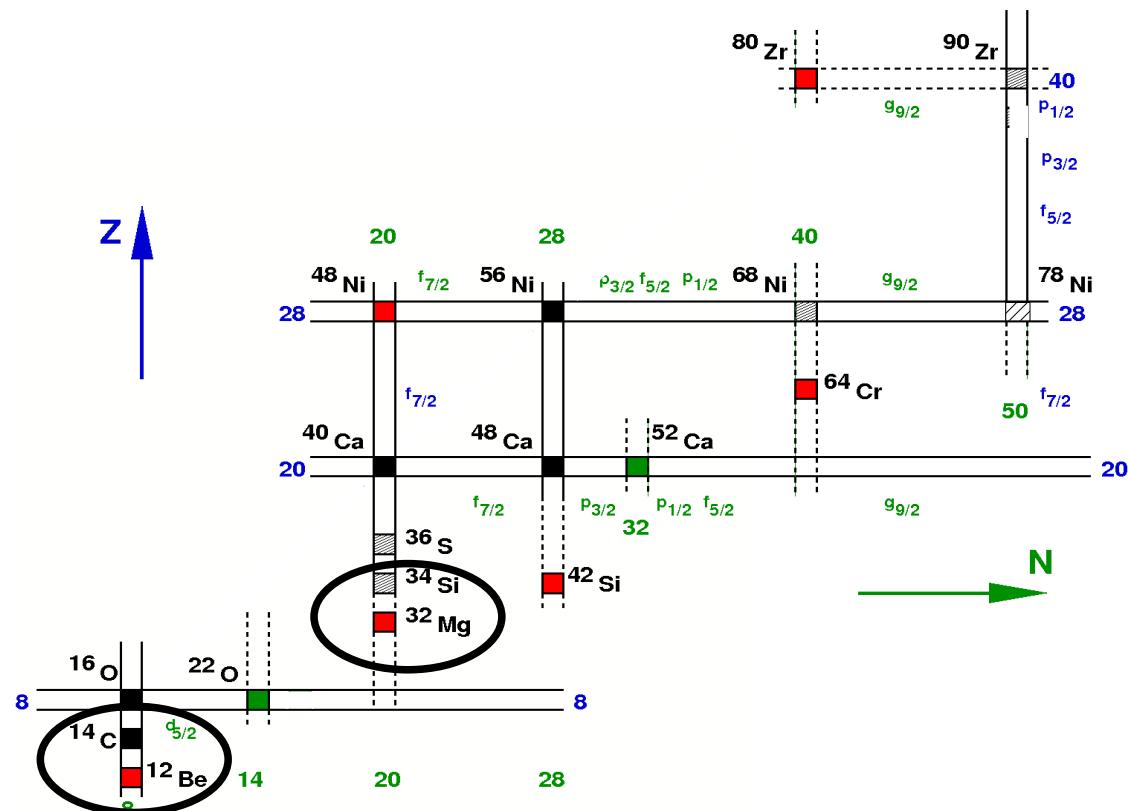
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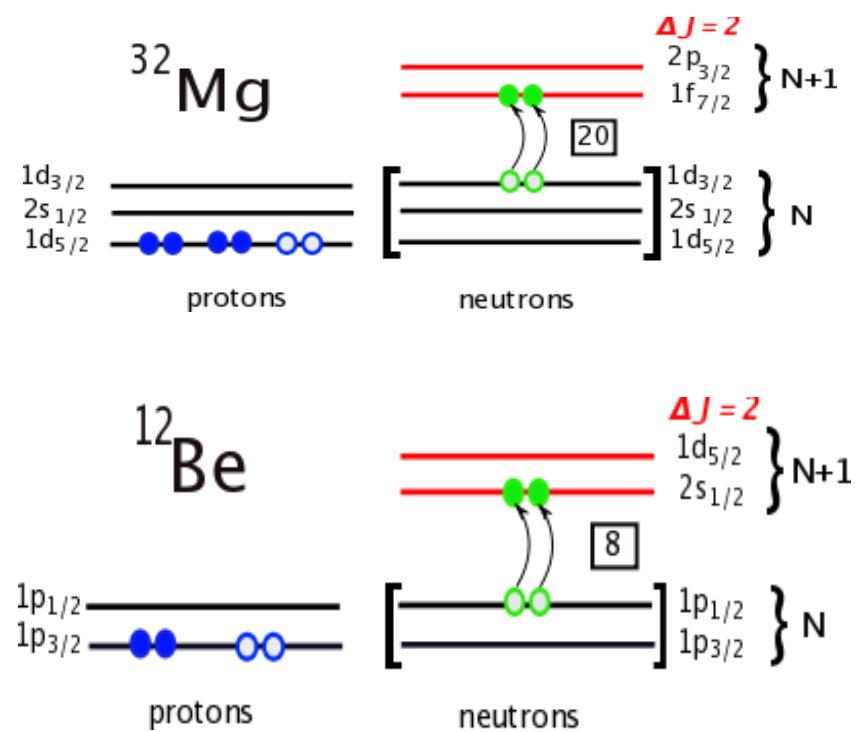
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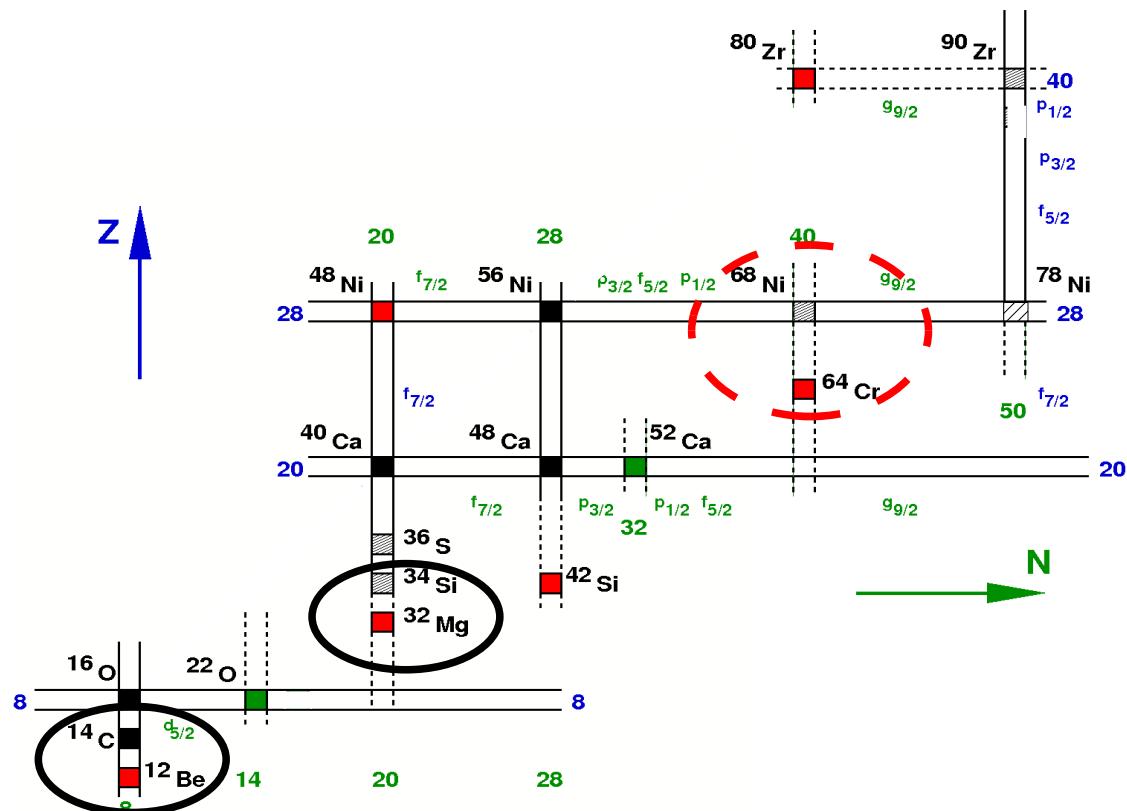
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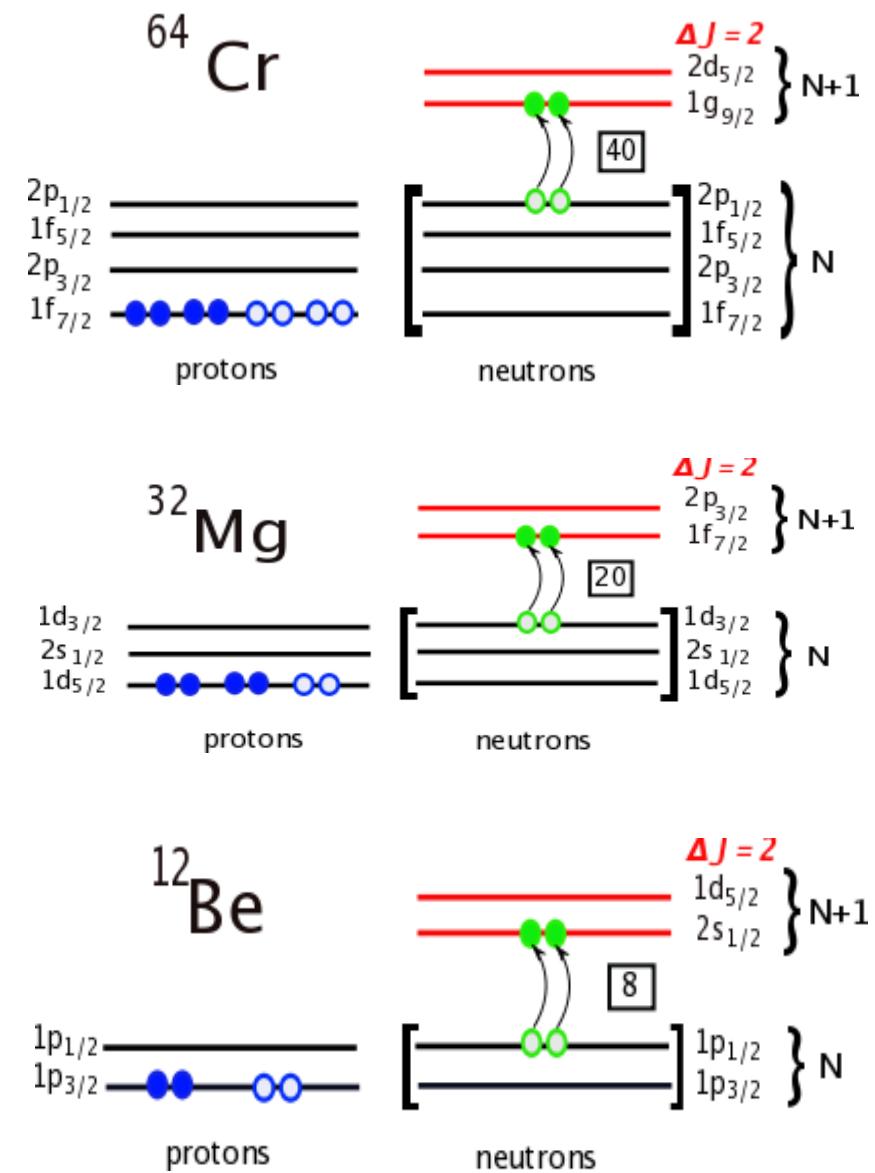
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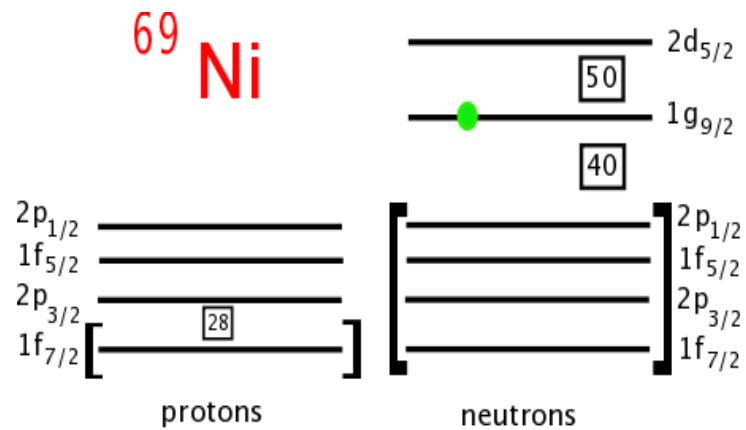
Island of inversion and quadrupole deformations



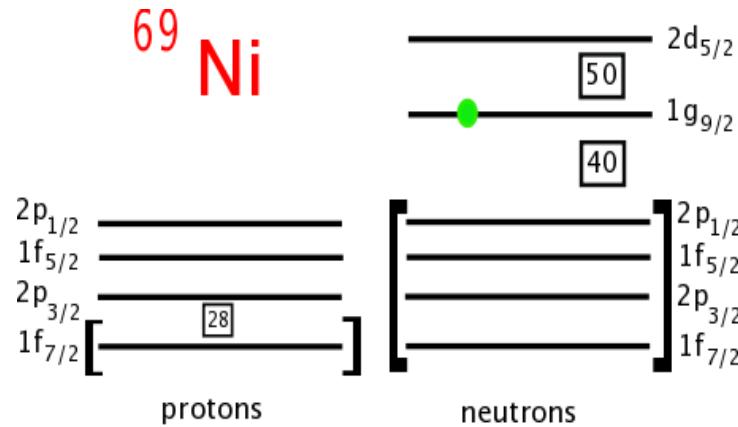
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^{69}Ni : search for the $2\text{d}_{5/2}$ neutron orbital



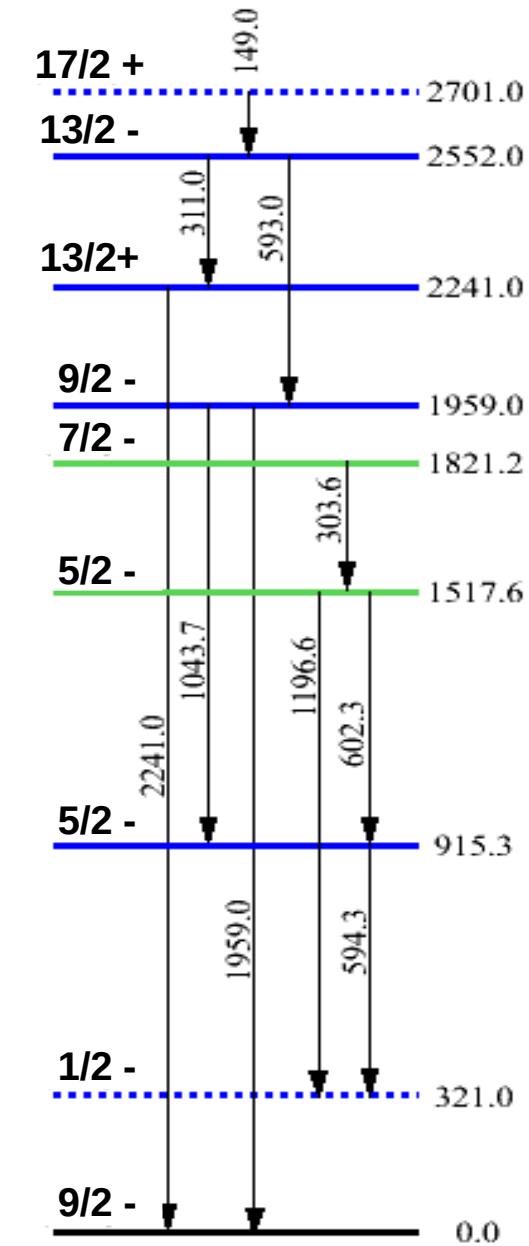
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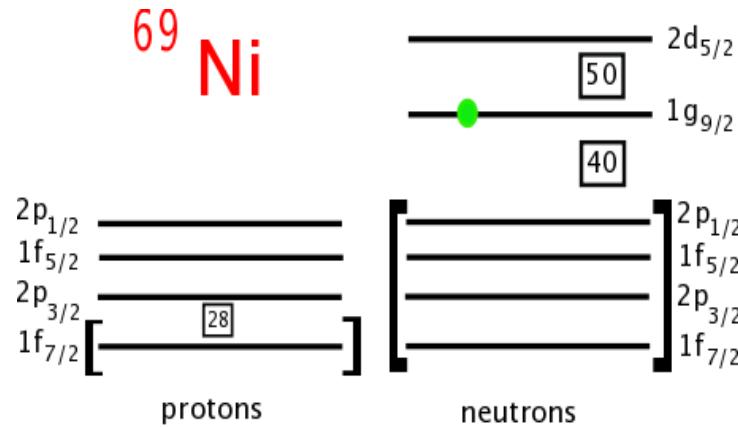
Previous experiments

- Isomer-state decay
(Grzywacz et al., PRL, 81, 766, 1998)
- β -decay
(Mueller et al., PRL, 83, 3613, 1999)

$2\text{d}_{5/2}$ ($5/2+$) neutron orbital not observed



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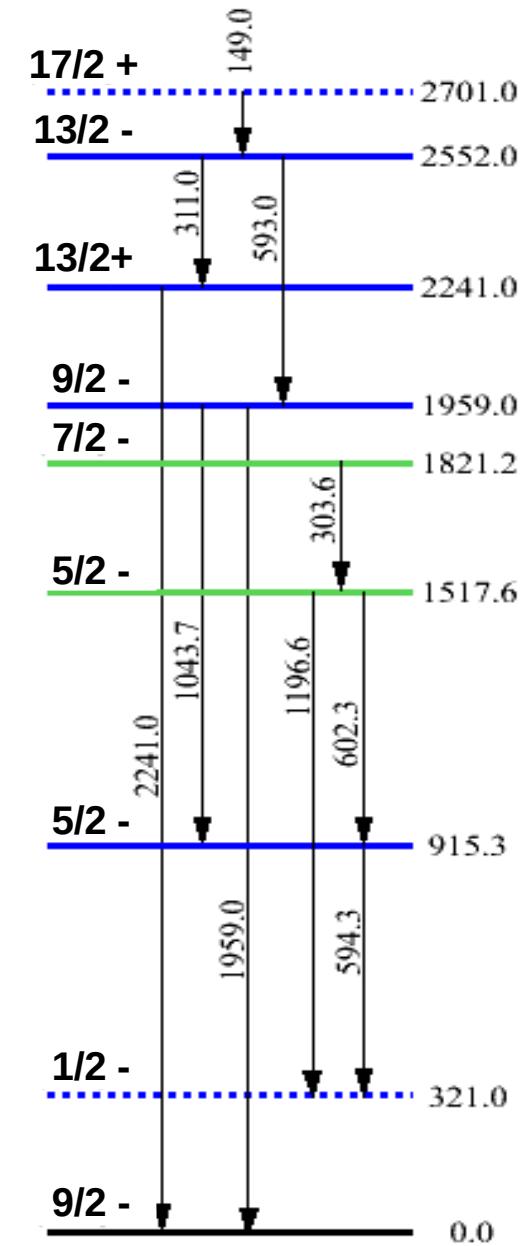
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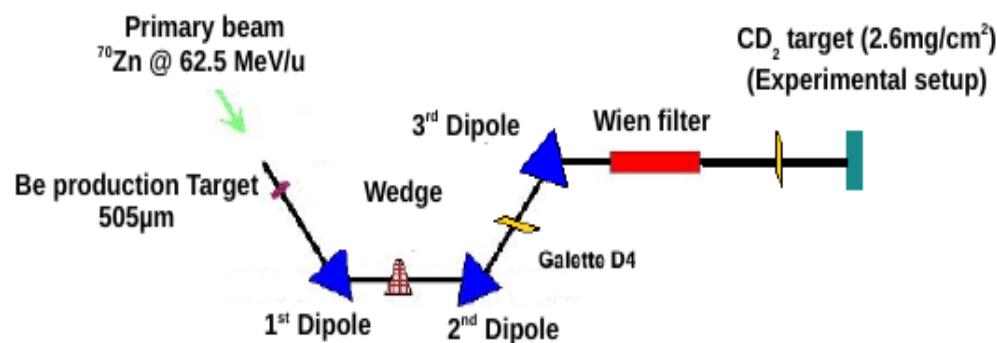
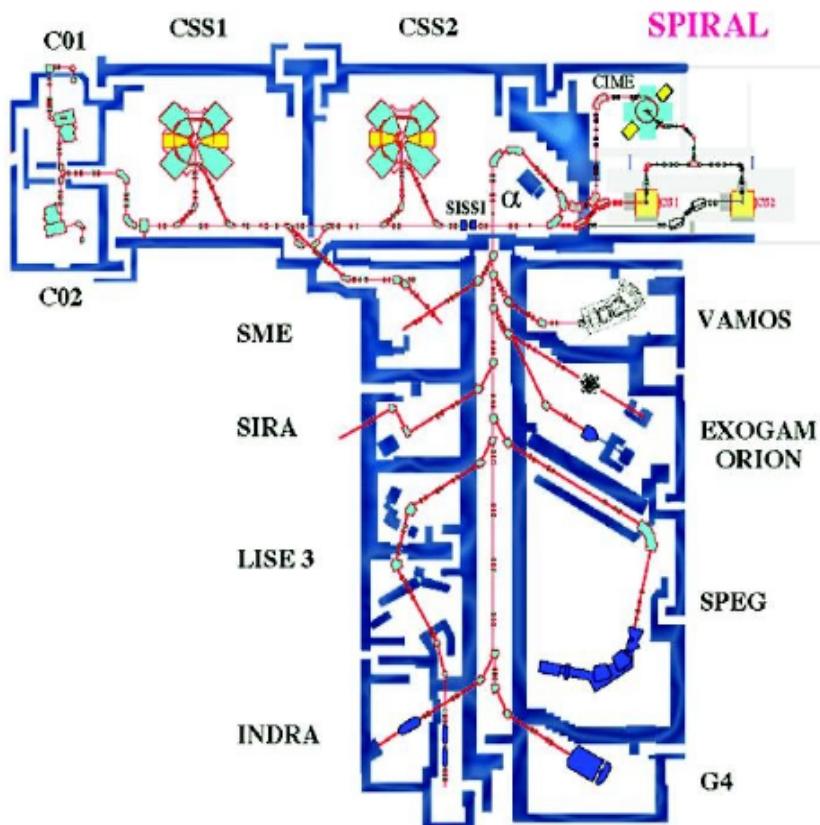
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Main goal of this work

Search for the neutron $2\text{d}_{5/2}$ orbital in ^{69}Ni
 $d(^{68}\text{Ni}, p)^{69}\text{Ni}$

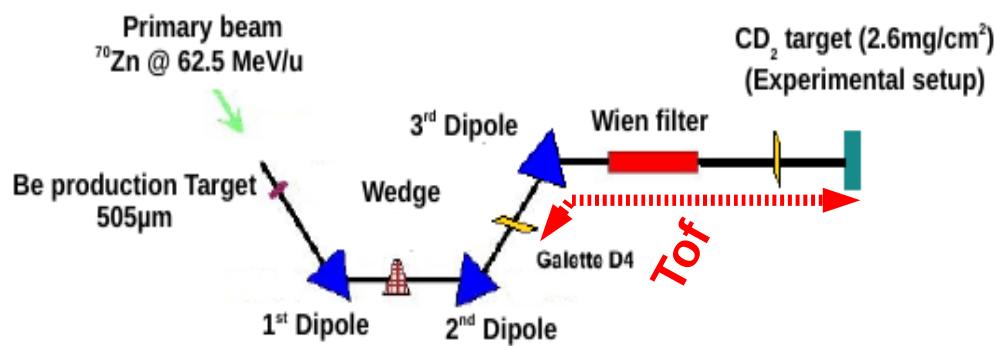
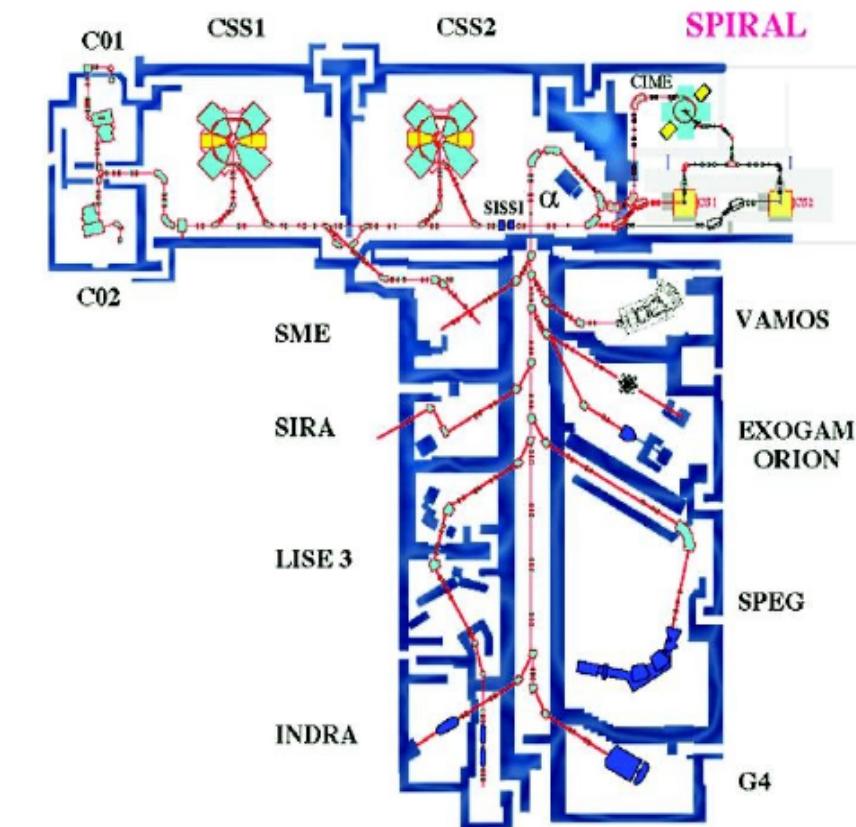


Beam production

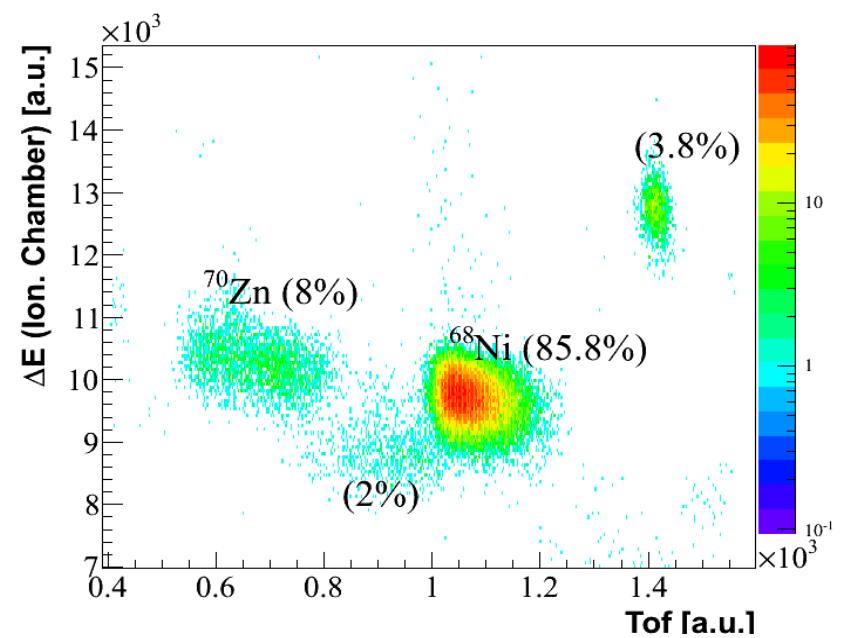


- Primary beam : ^{70}Zn (@62.5 AMeV)
- Primary beam intensity : 1.5 μAe
- Production target : ^9Be (505 μm , 0°)
- Wedge : ^9Be (1099 μm)
- Wien filter at 1kV
- Secondary beam intensity $\sim 8.10^4$ pps
@ 25.14 MeV/u
- Purity $\sim 86\%$

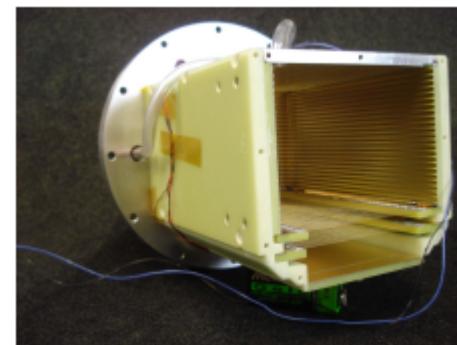
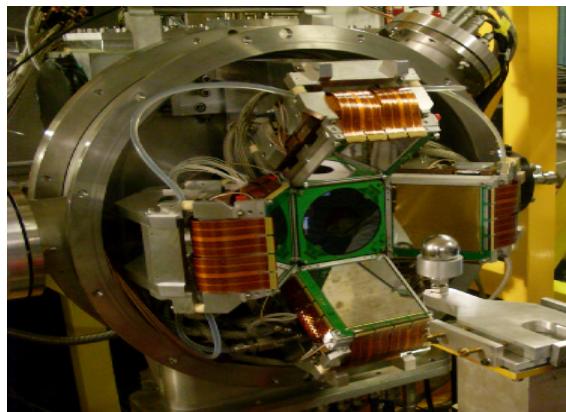
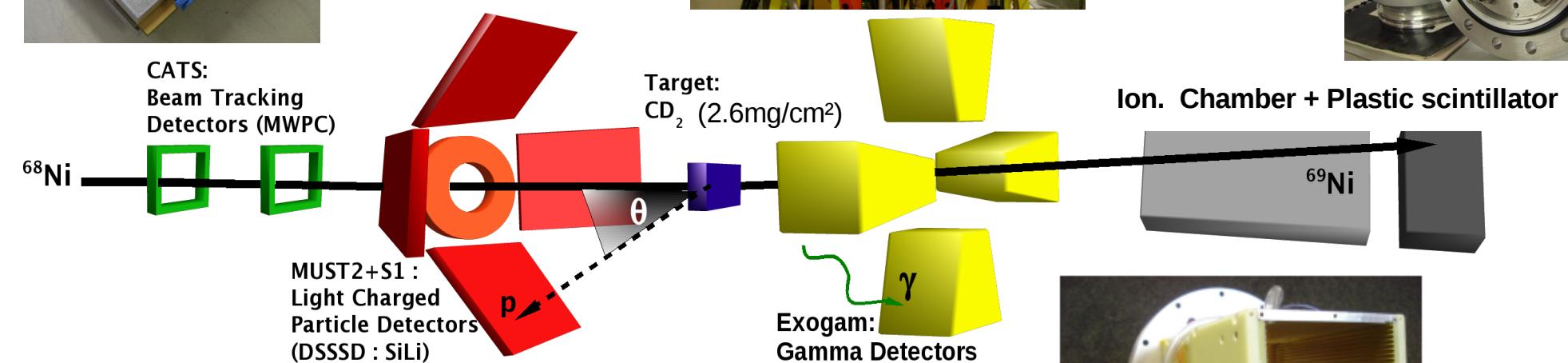
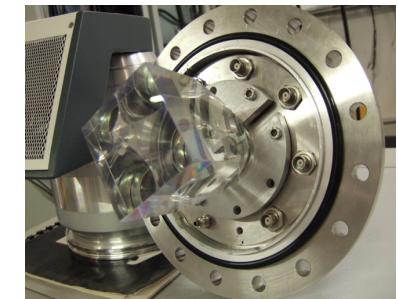
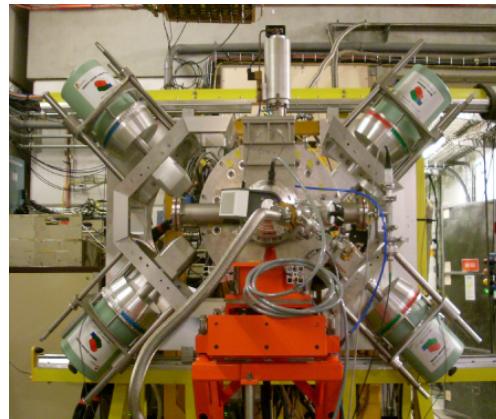
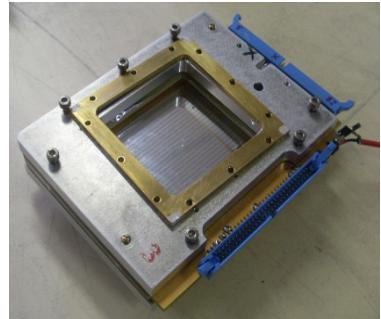
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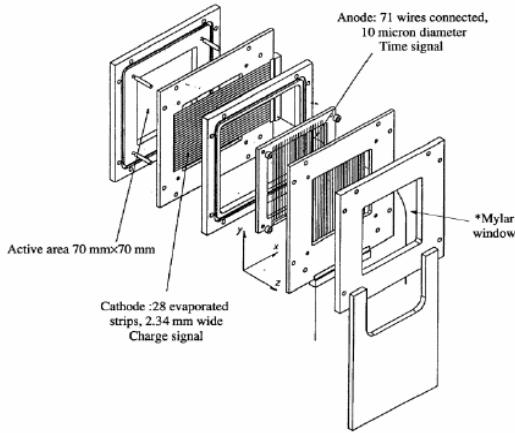
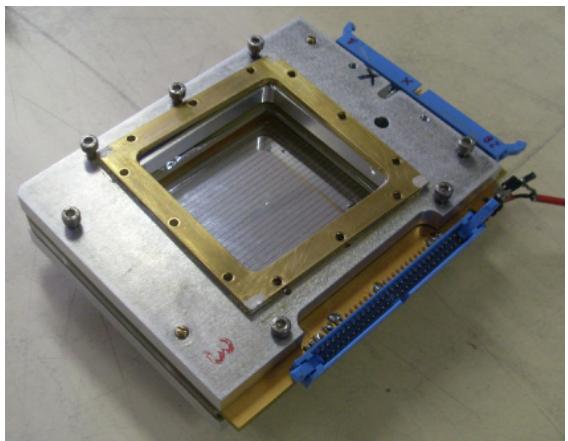
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Experimental set-up

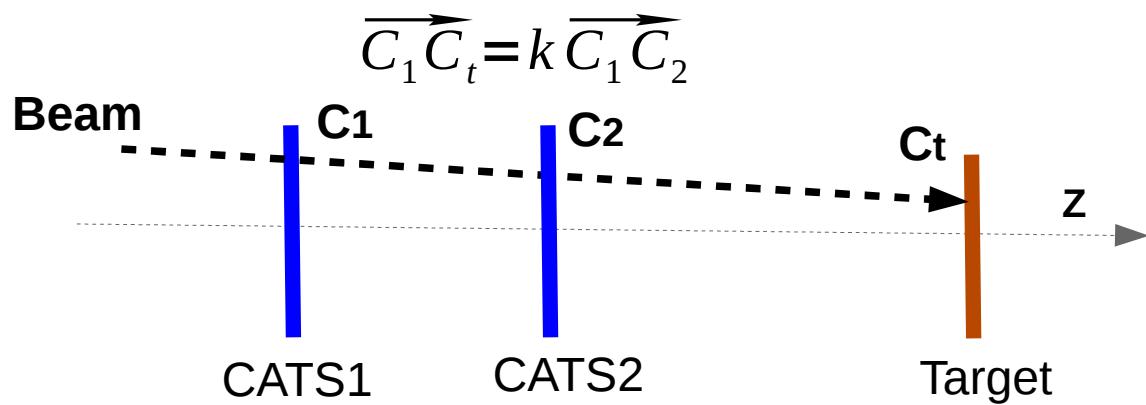


Beam trackers : CATS

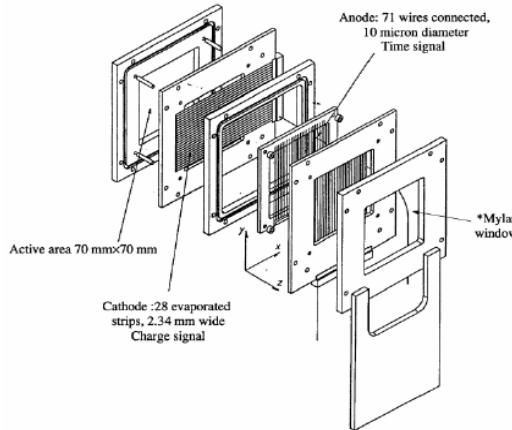
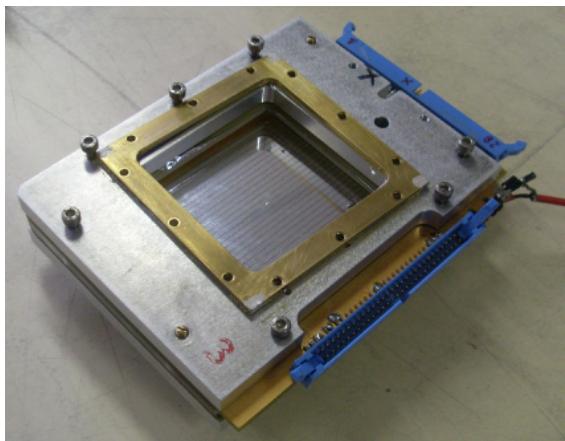


(Ottini-Hustache et al., NIM A, 431, 476, 1999)

- Event by event Reconstruction
- Reconstruction algorithms:
 - Center of gravity
 - **Analytical fit**
- Uncert. $\sim 0.65\text{mm}$ (X)
 $\sim 0.40\text{ mm}$ (Y)
- Uncert. on incidence angle
 $\sim 0.1^\circ$
- CATS2 : Time of flight stop signal

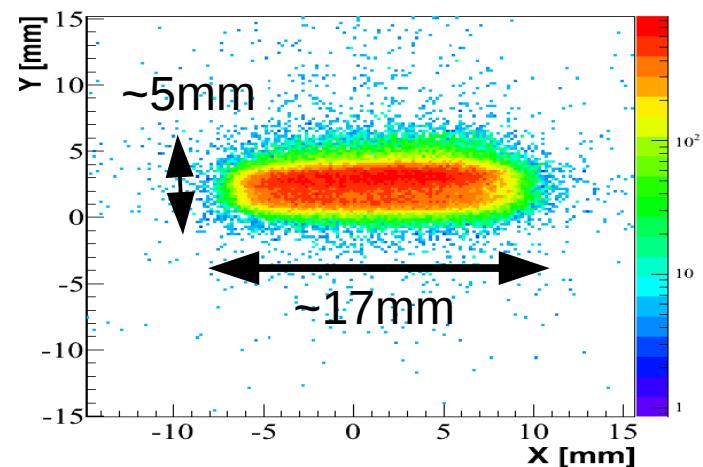
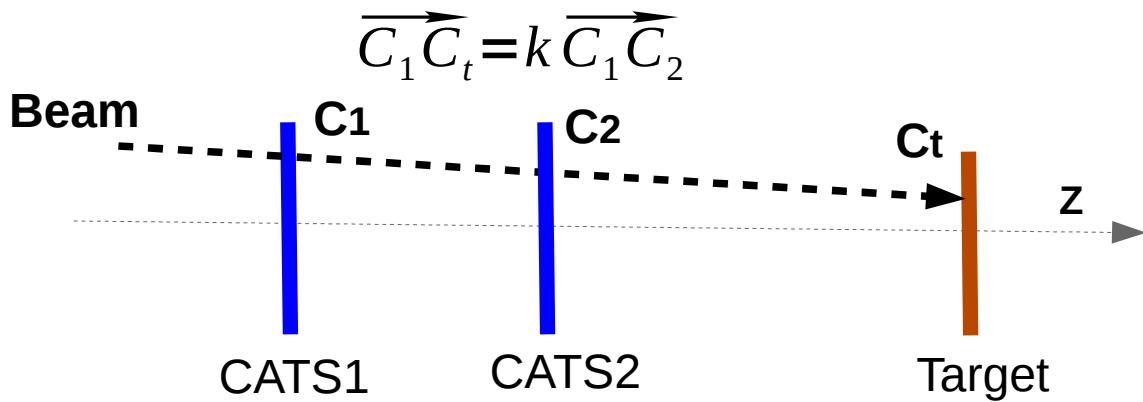


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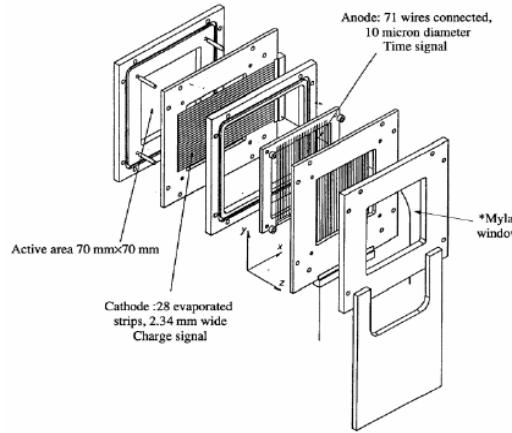
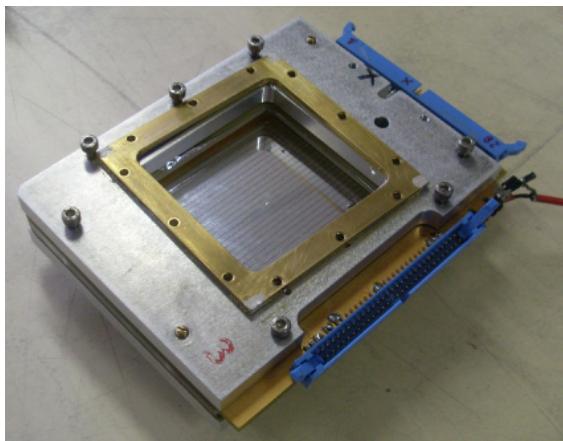
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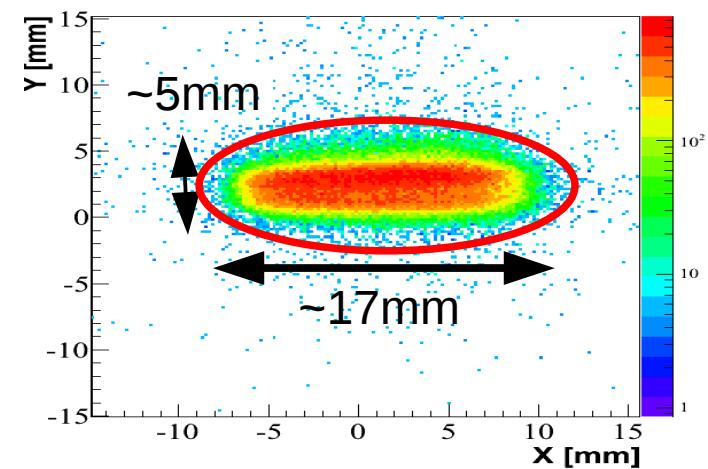
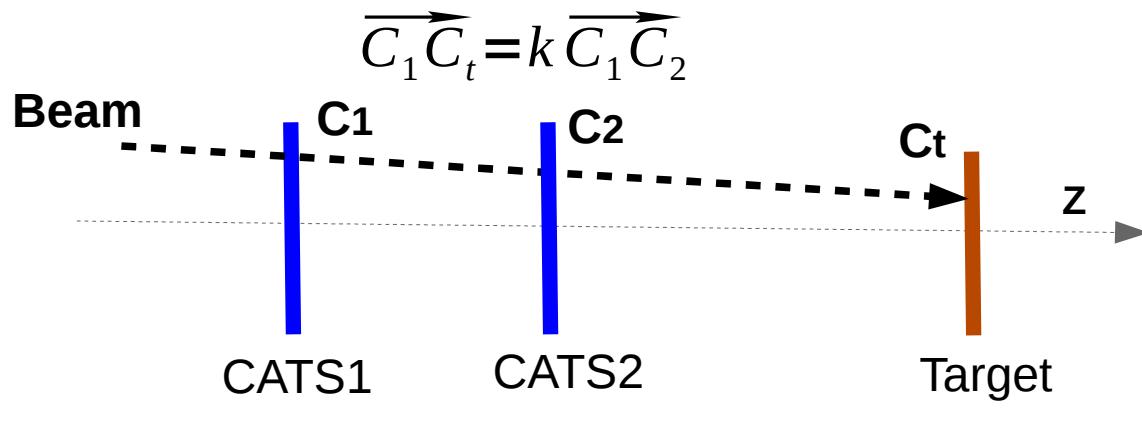
**Beam reconstruction
In the target plane**

Beam trackers : CATS



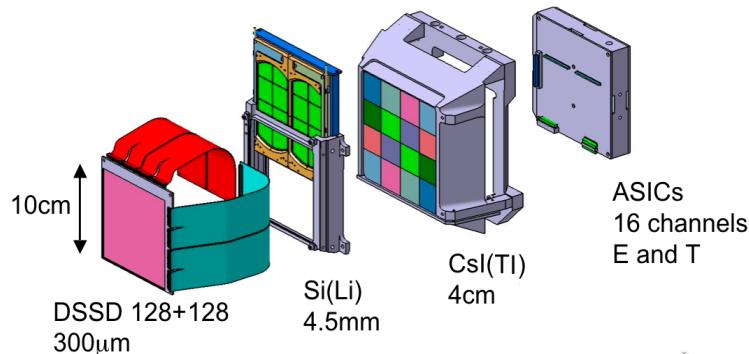
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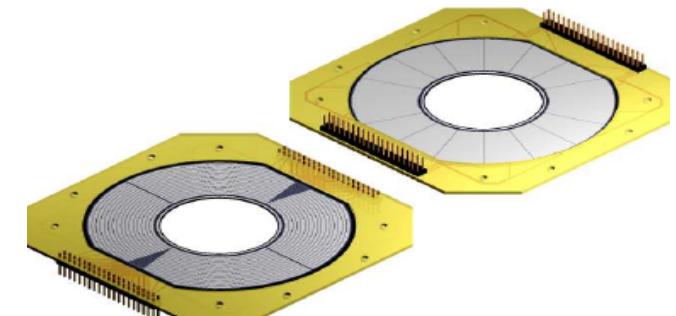
Beam reconstruction
In the target plane

Light charged particle detectors : MUST2 and S1



Pollaco et al., NIM A, 25, 287, 2005

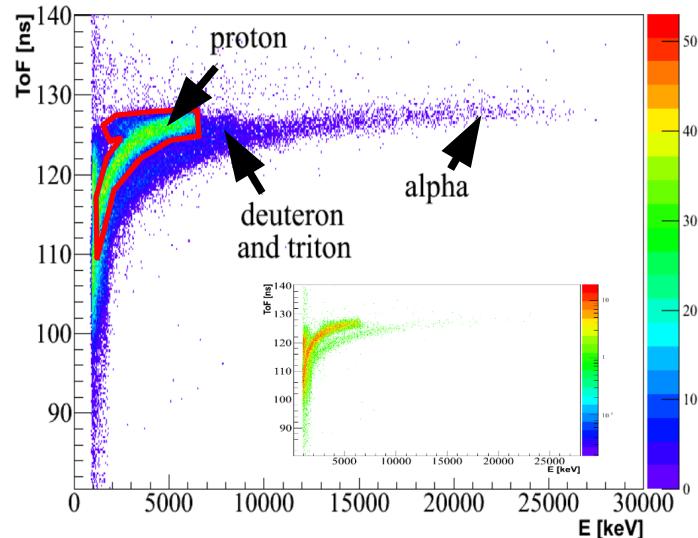
- DSSD (300 μm) and Si(Li) (4.5 mm)
- Active surface: 100mm*100mm
- 128 strips in X and Y (0.76 mm)



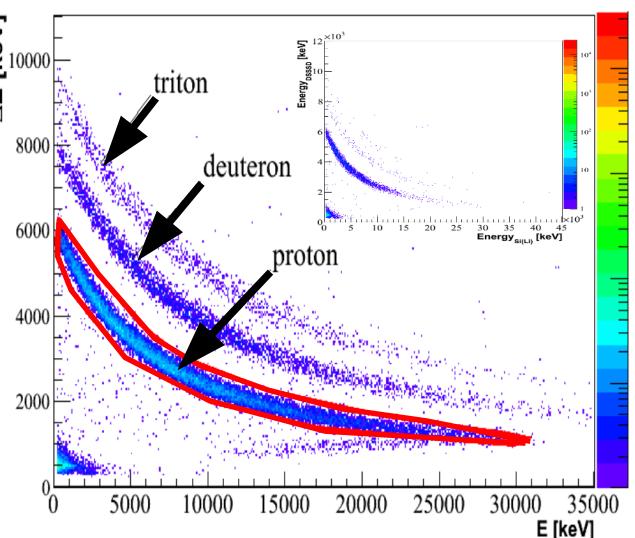
Micron semi-conductors

- DSSD (500 μm)
- Active surface : 53mm²
- 64 strip θ , 16 sectors ϕ

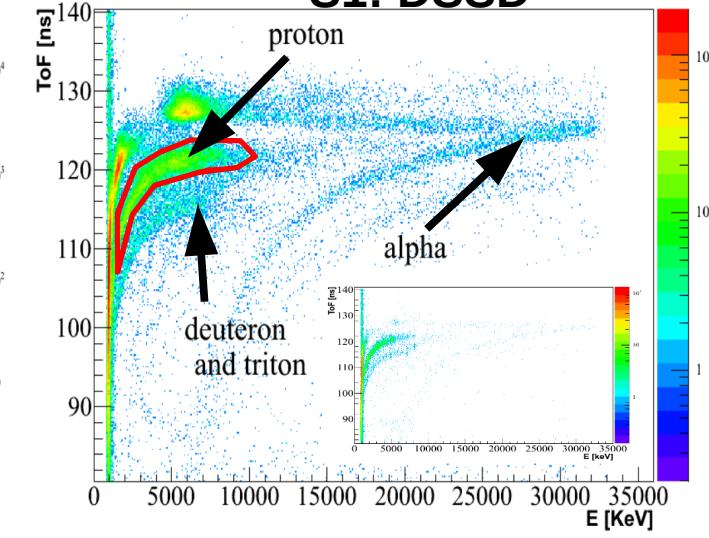
MUST2 : DSSD



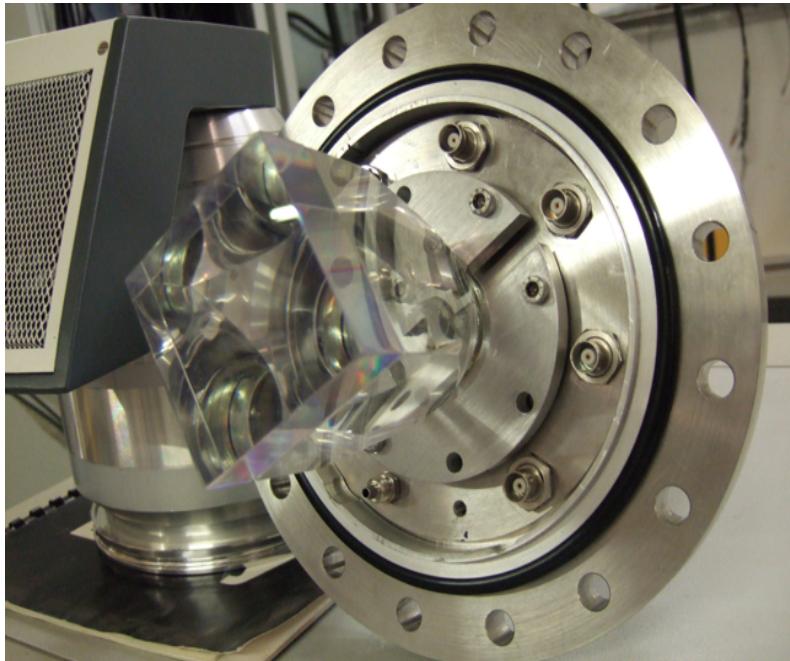
MUST2 : Si(Li)



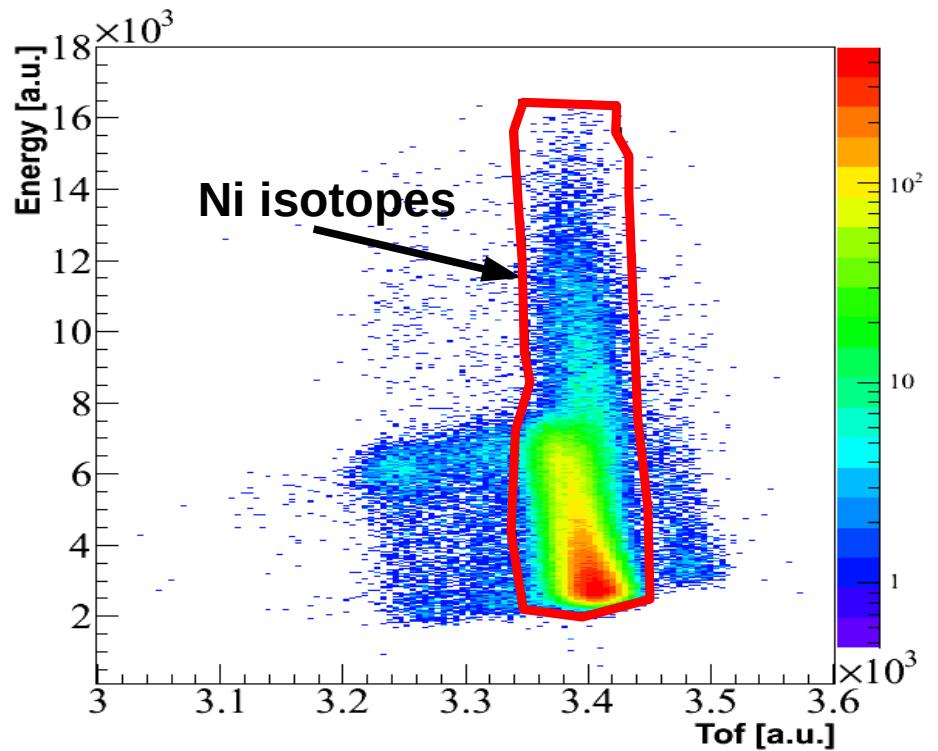
S1: DSSD



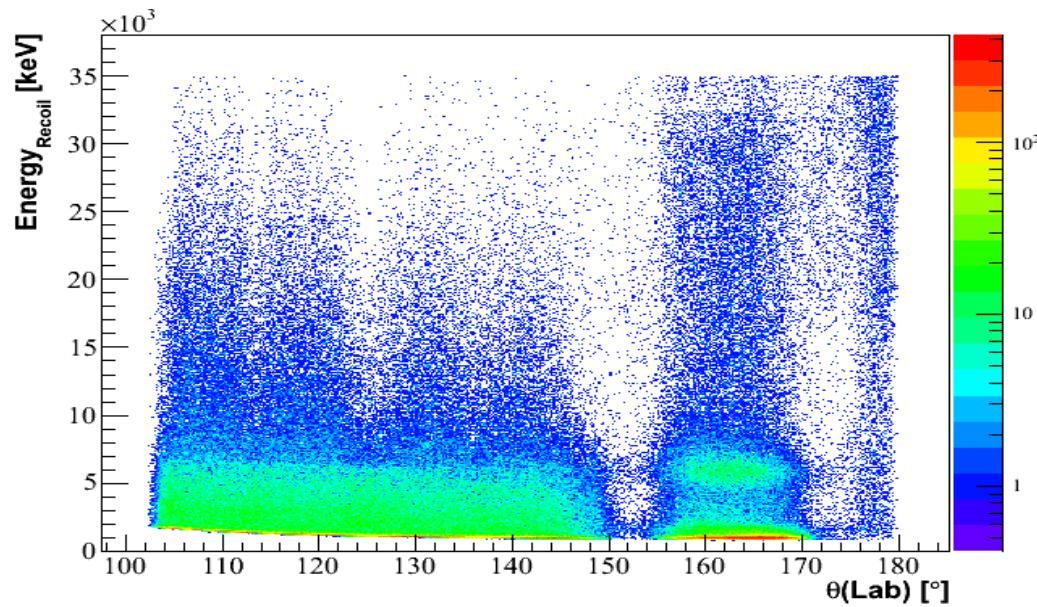
Heavy residue detector



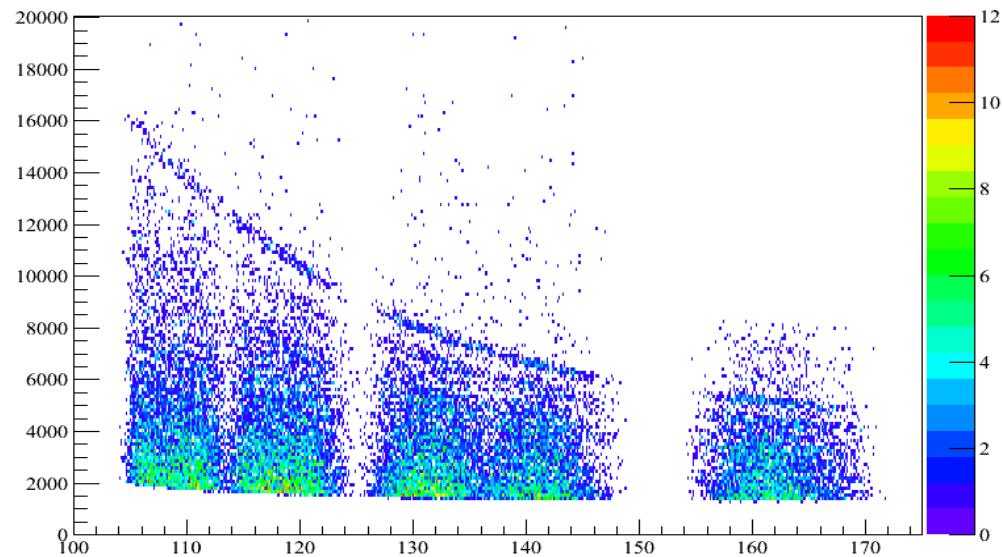
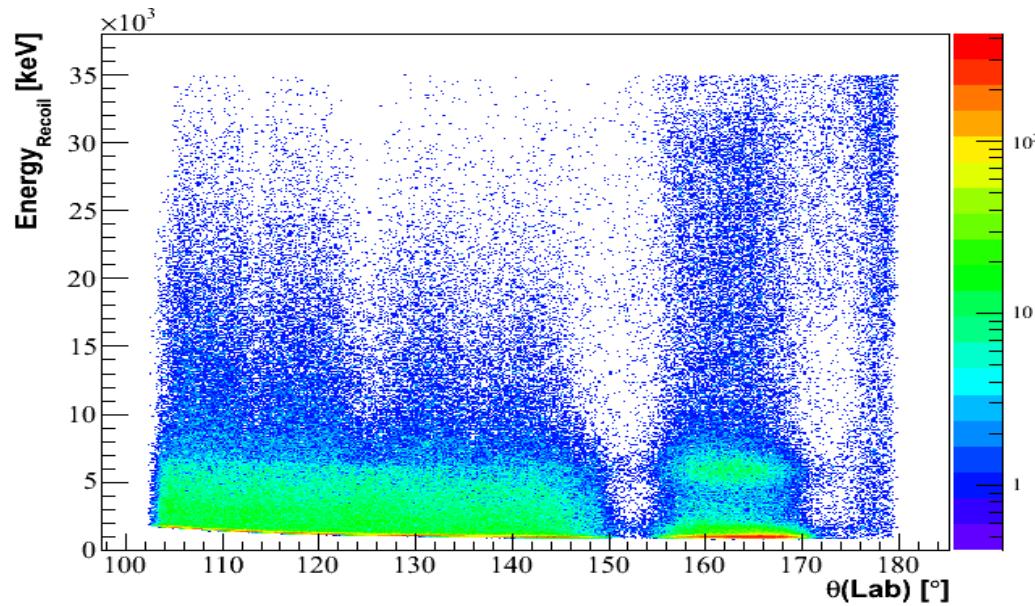
- Plastic scintillator
- Active surface : 60mm*60mm
- Energy and time (ToF)



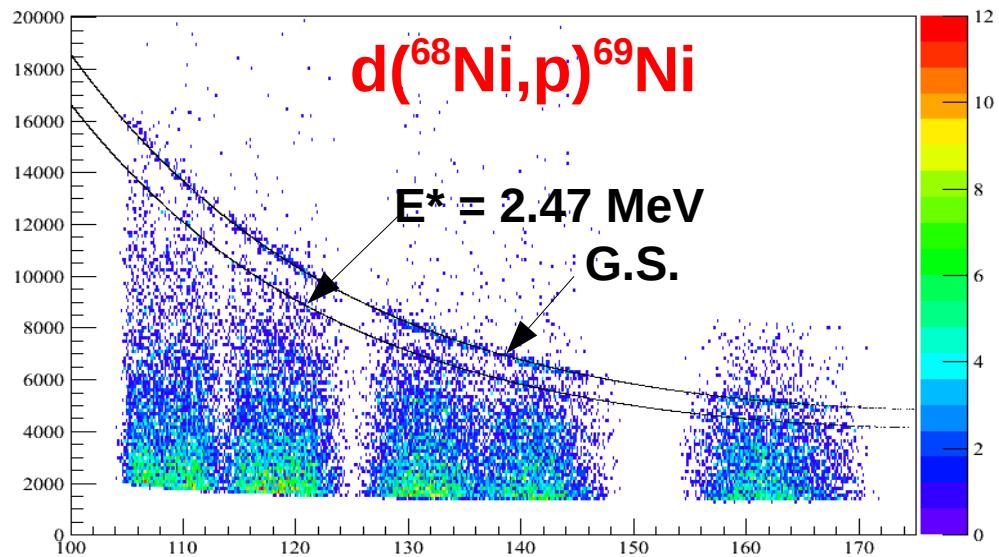
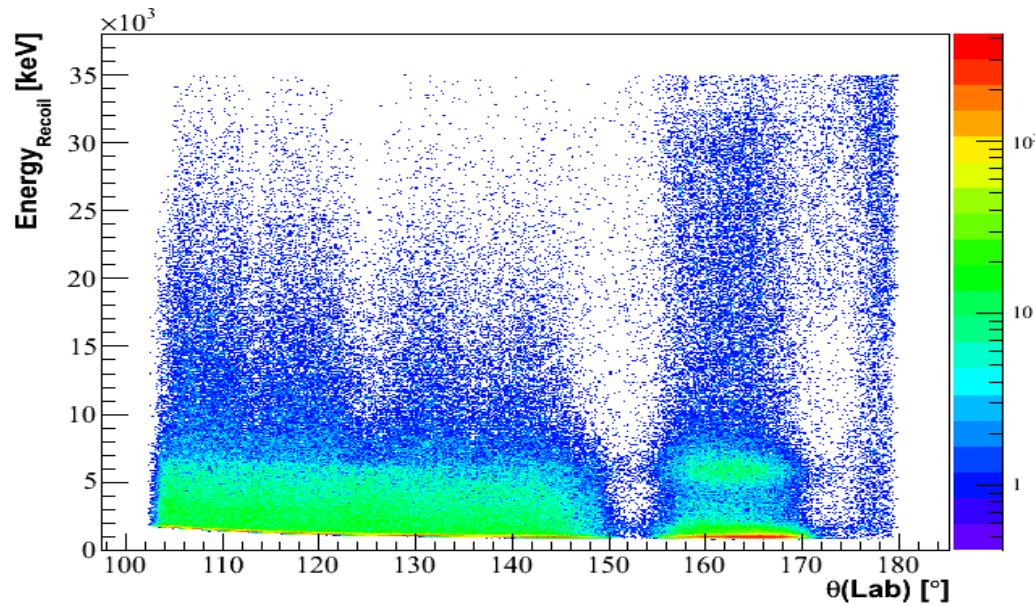
Excitation energy spectrum



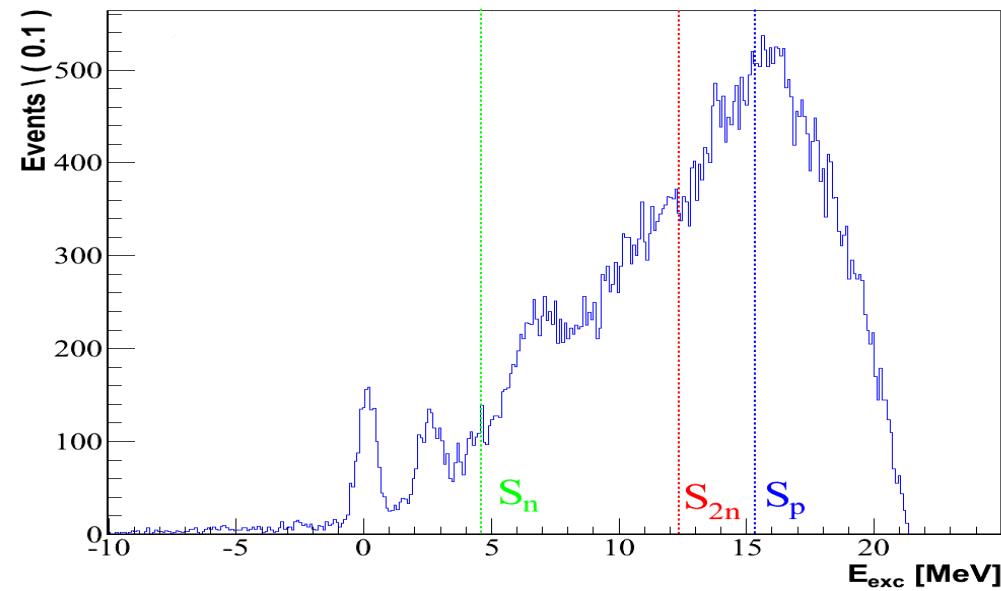
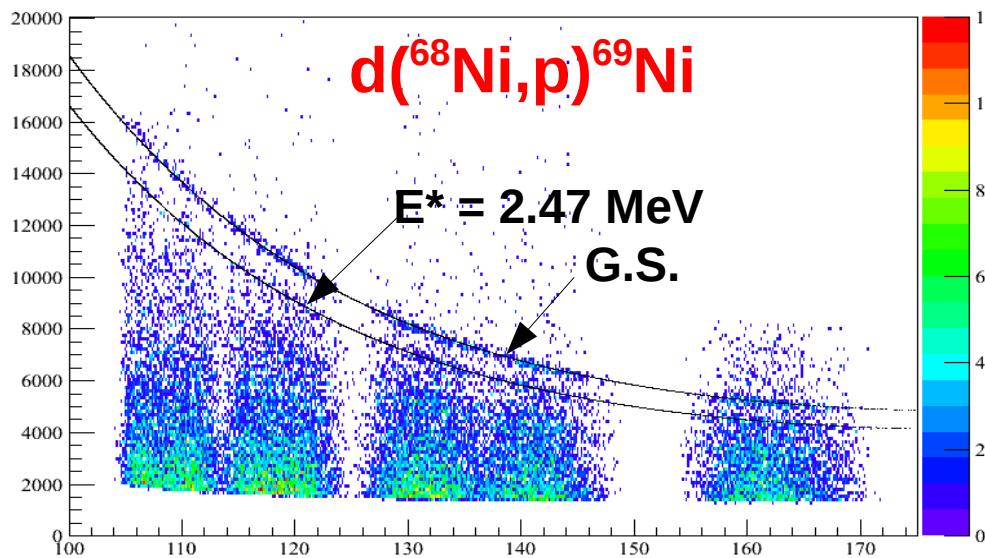
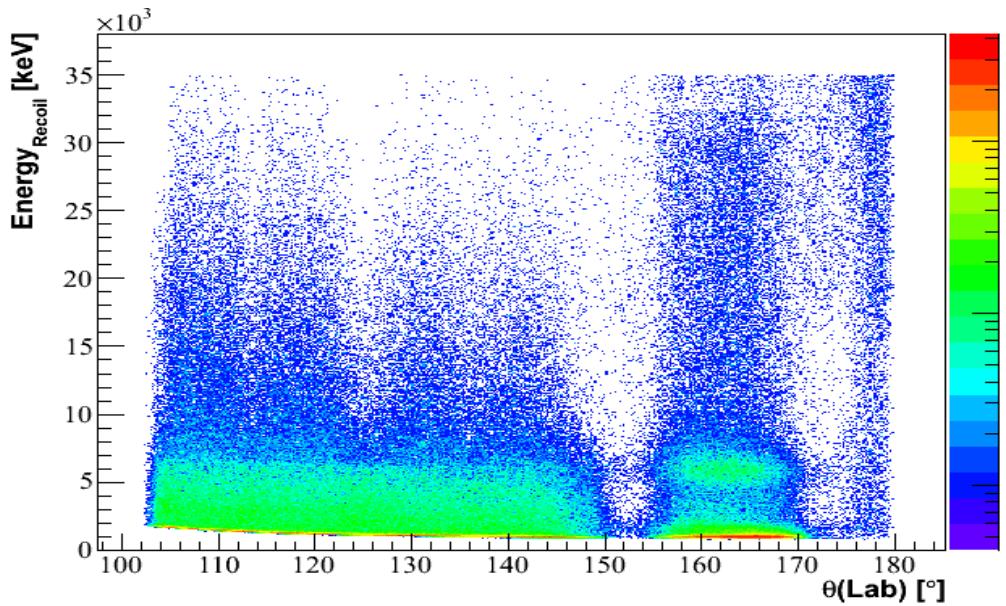
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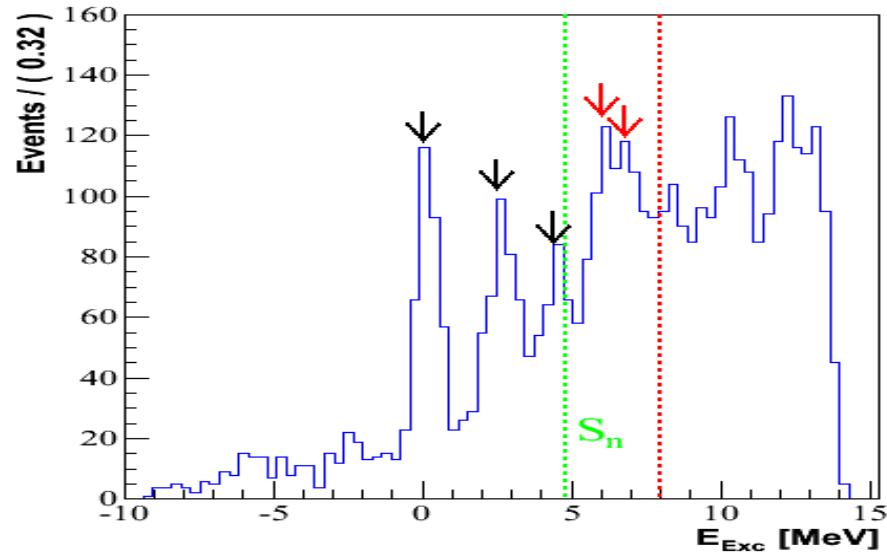


Excitation energy spectrum



- Pronounced G.S.
- 1st excited state at ~ 2.5 MeV
- Structures ~ 4 MeV and 6–7 MeV ($> S_n$)

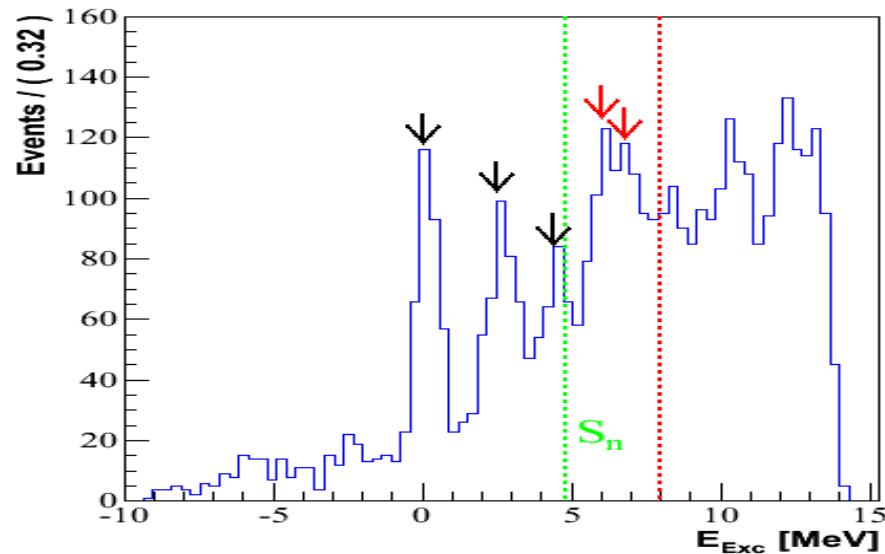
Excitation energy spectrum



- 3 bound states
- 2 resonances above S_n

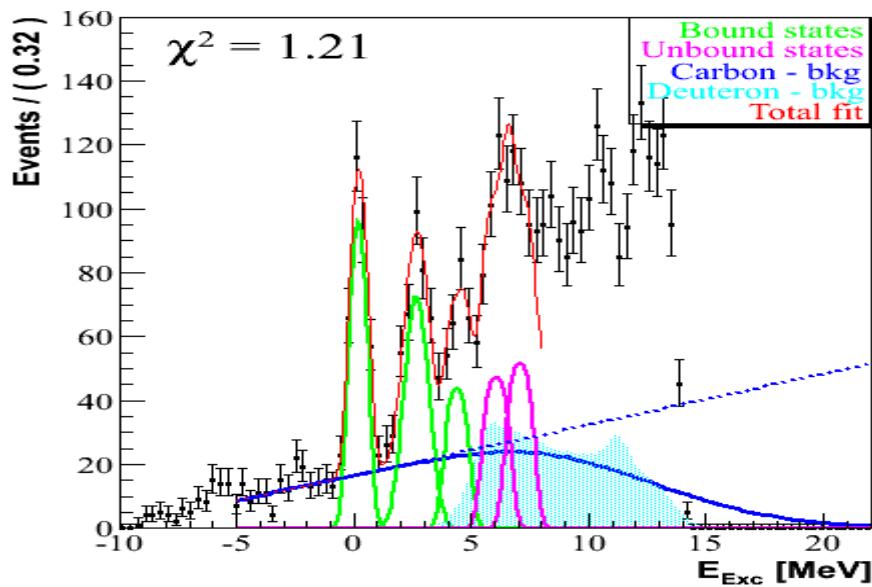
Energy reference : S1 [156°-170°]

Excitation energy spectrum



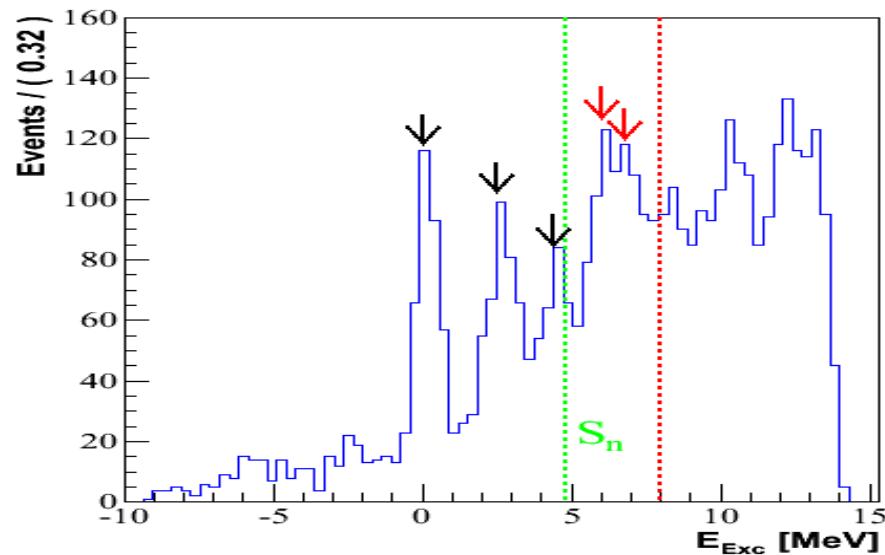
- 3 bound states
- 2 resonances above S_n
- Background reactions

Energy reference : S1 [156°-170°]



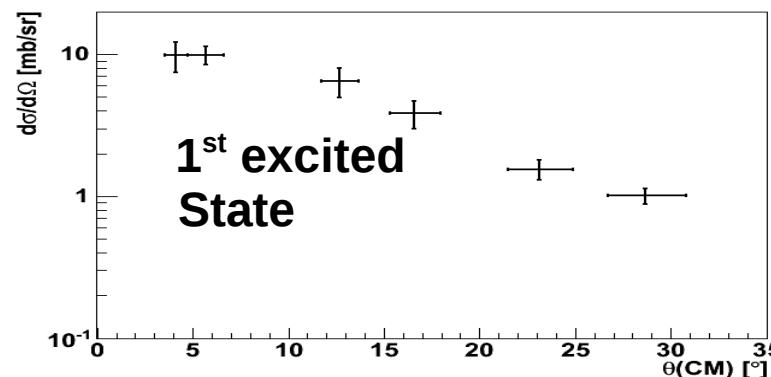
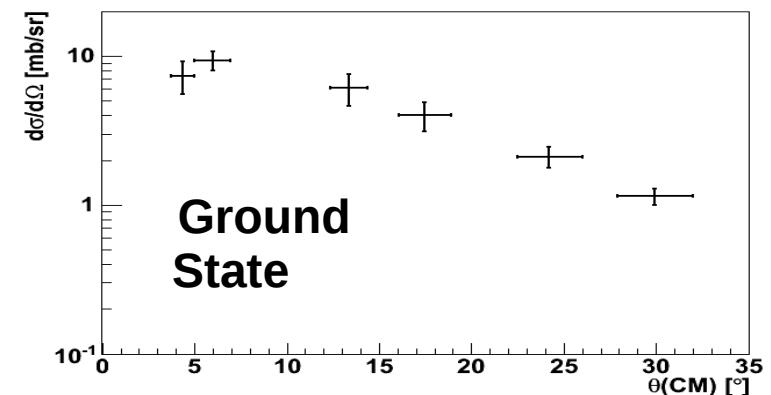
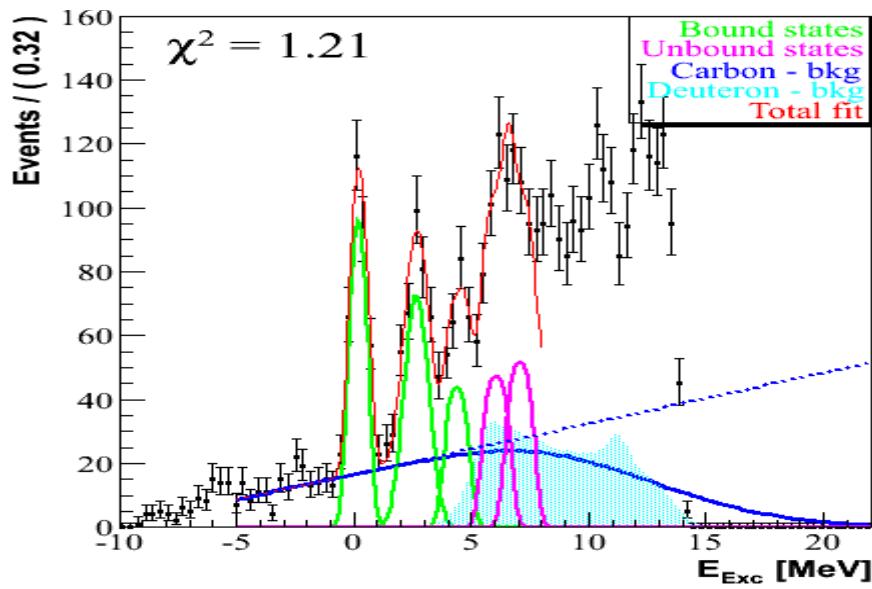
Pic #	Energy [MeV]	FWHM [MeV]
G.S	0.00	1.04
1	2.47	1.43
2	4.19	1.27
3	5.88	1.39
4	6.89	1.39

Excitation energy spectrum



- 3 bound states
 - 2 resonances above S_n
 - Background reactions
- ⇒ 6 angular ranges

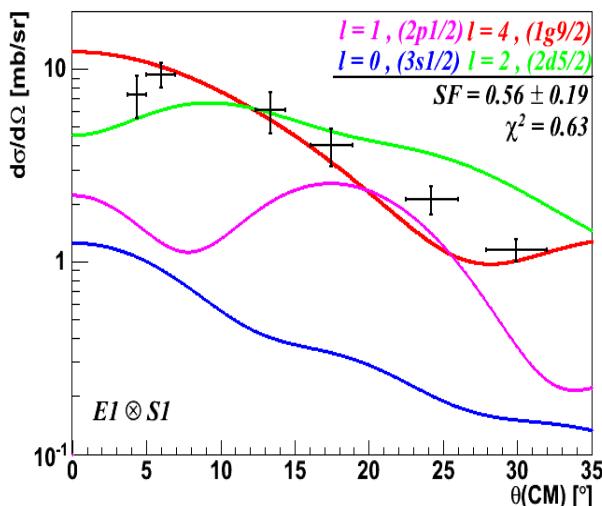
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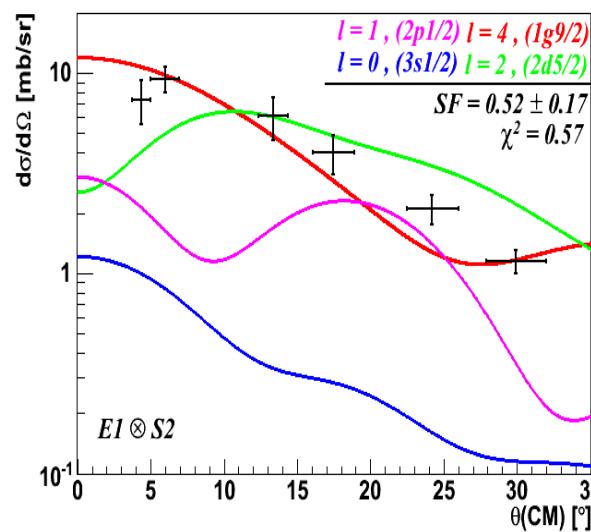
Distorted Waves Born Approximation (DWBA)

- **Code DWUCK4, Zero-Range approximation**
- **Elastic channel \Rightarrow Global optical potentials**
 - **$^{68}\text{Ni} + d$:**
 - » **E1:** $27 < A < 238$, $12 \text{ MeV} < E_d < 90 \text{ MeV}$
Daehnick et al. PRC, 21, 2253, 1981
 - » **E2:** Adiabatic Distorted Waves Approximation (ADWA) for the deuteron break-up
pour $E_d > 20 \text{ MeV}$: $V = V_p + V_n + V_{pn}$ ($E_n = E_p = E_d/2$)
Johnson and Soper, PRC, 1, 976, 1970
 - **$^{69}\text{Ni} + p$:**
 - » **S1:** $40 < A < 209$, $16 \text{ MeV} < E_p < 65 \text{ MeV}$
Varner et al. Phys. Reports, 201, 57, 1991
 - » **S2:** $24 < A < 209$, $1 \text{ keV} < E_p < 200 \text{ MeV}$
Koning and Delaroche. Nucl. Phys. A, 713, 231, 2003

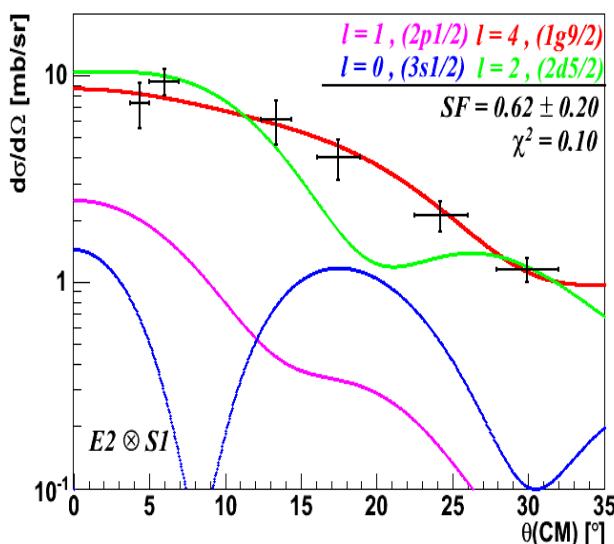
Analysed differential cross sections : Ground state



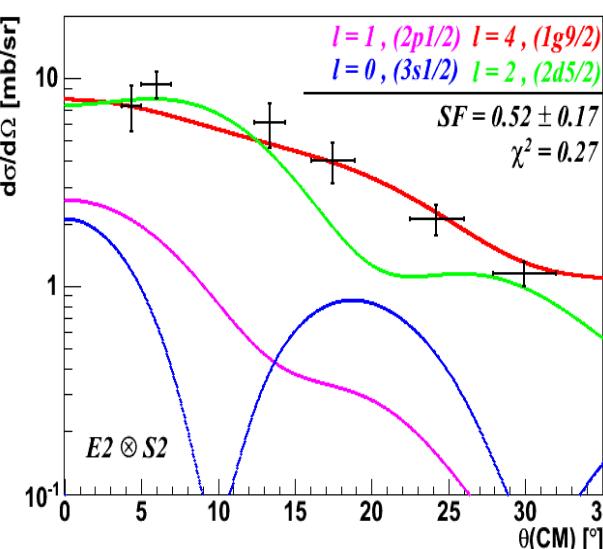
$E1 \otimes S1$



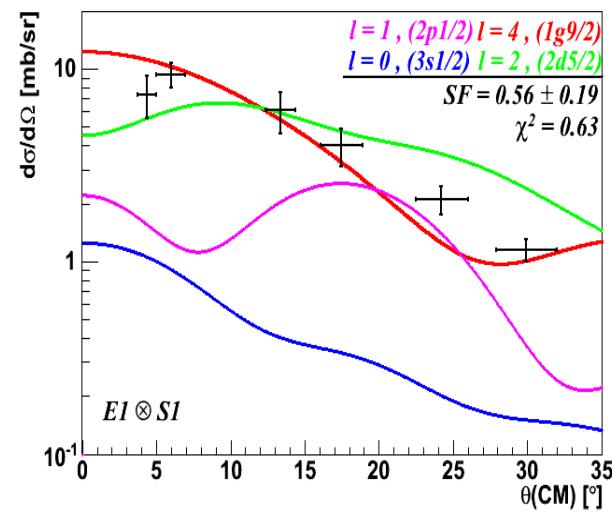
- 4 combinations of optical pot.
- G.S. : L = 0, 1, 2 and 4
 $\Rightarrow L = 4, SF = 0.56 \pm 0.19$



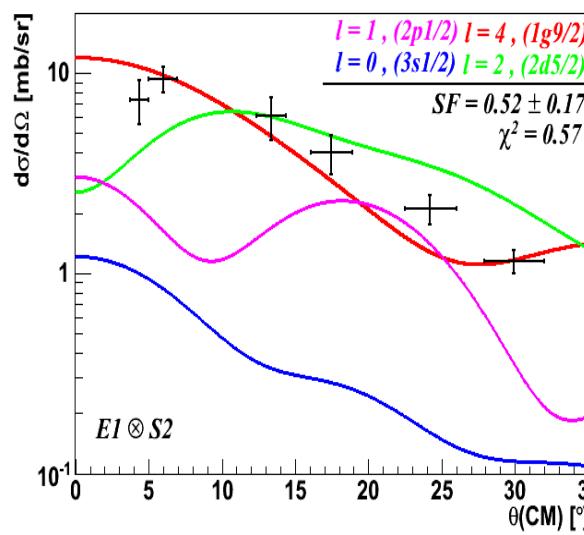
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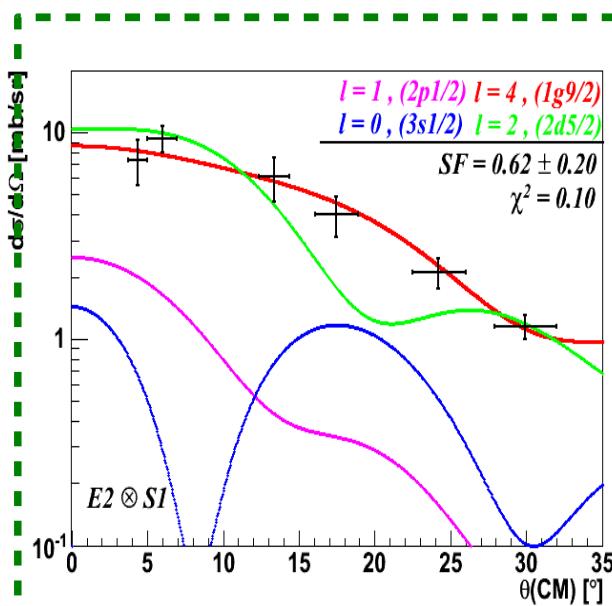
Analysed differential cross sections : Ground state



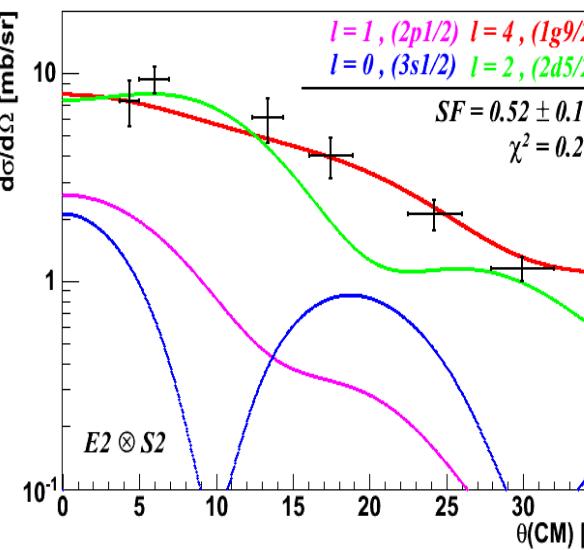
$E1 \otimes S1$



$E1 \otimes S2$



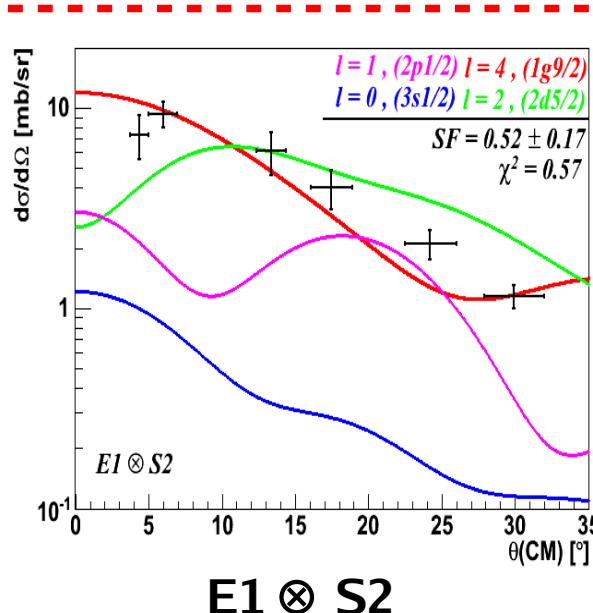
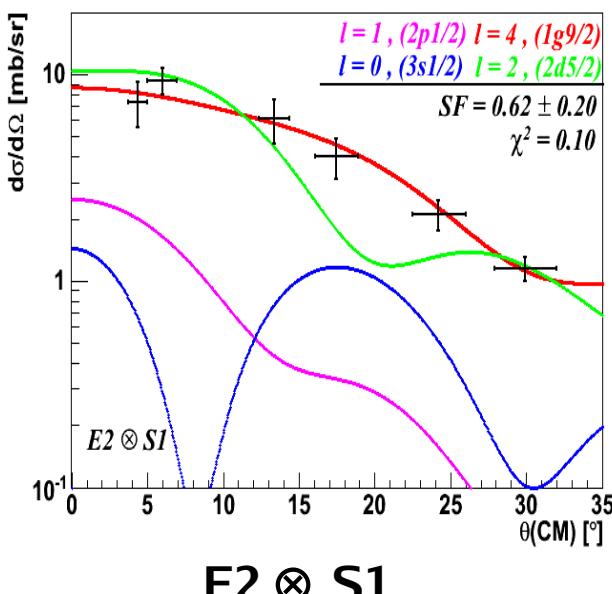
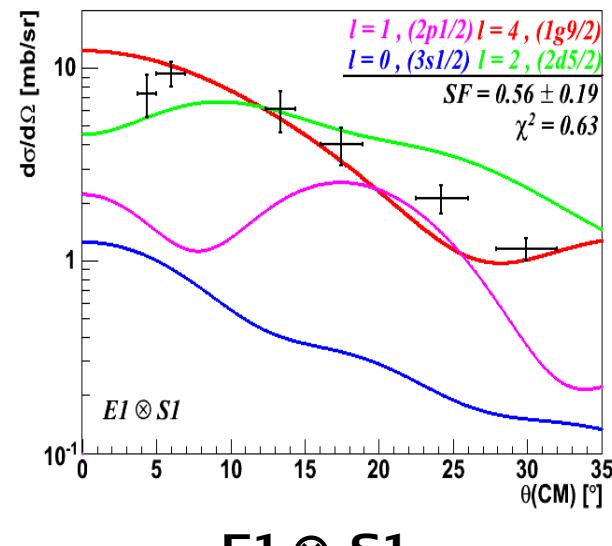
$E2 \otimes S1$



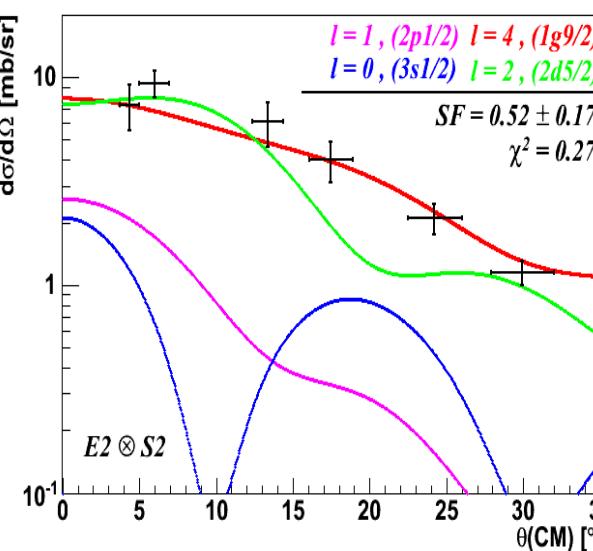
- 4 combinations of optical pot.
- G.S. : L = 0, 1, 2 and 4
 $\rightarrow L = 4, SF = 0.56 \pm 0.19$

- Weak dependency according to the exit pot. parameterization
- Significant dependency according to the entrance pot. parameterization

Analysed differential cross sections : Ground state



$E1 \otimes S2$



$E2 \otimes S2$

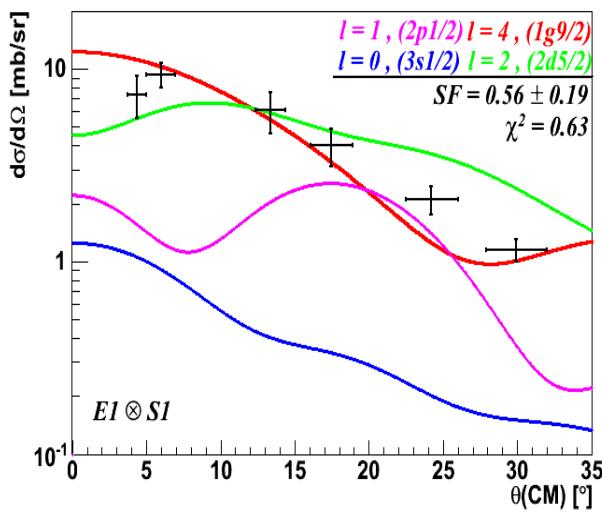
- 4 combinations of optical pot.

- G.S. : $L = 0, 1, 2$ and 4
 $\rightarrow L = 4, SF = 0.56 \pm 0.19$

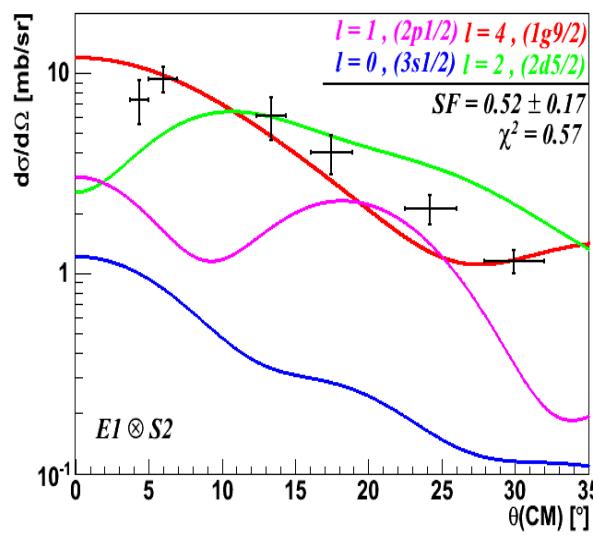
- Weak dependency according to the exit pot. parameterization

- Significant dependency according to the entrance pot. parameterization

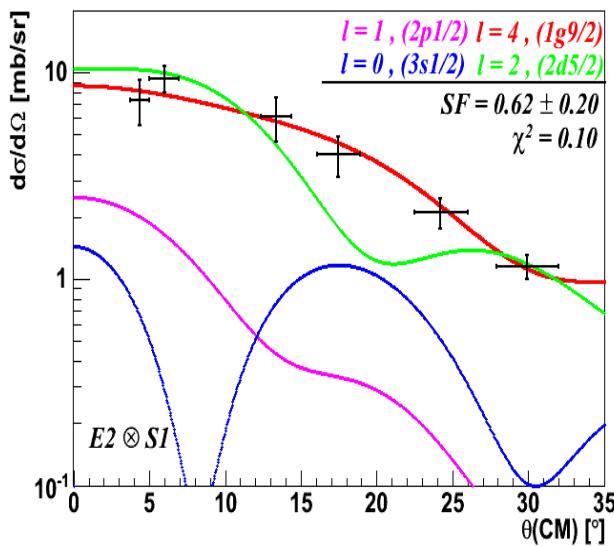
Analysed differential cross sections : Ground state



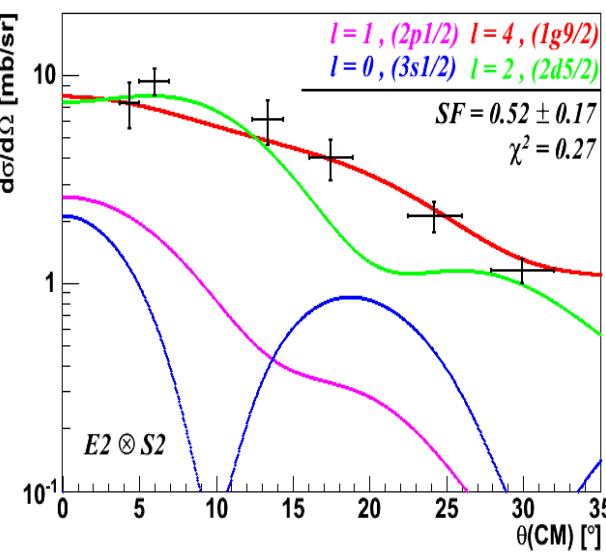
$E1 \otimes S1$



$E1 \otimes S2$



$E2 \otimes S1$



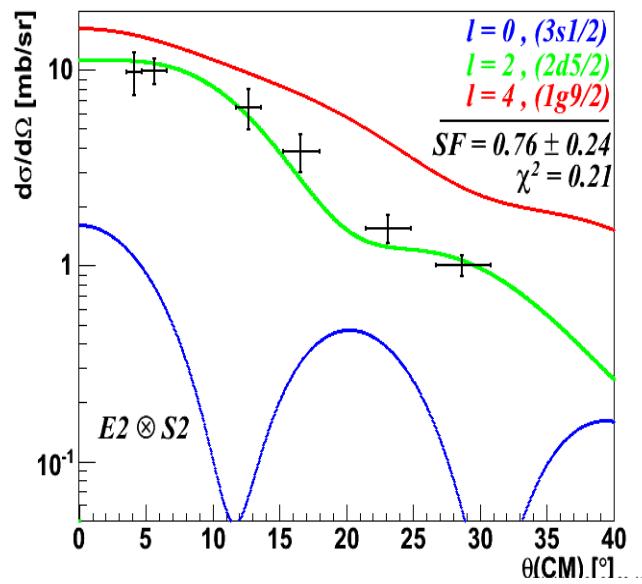
$E2 \otimes S2$

- 4 combinations of optical pot.
- G.S. : $L = 0, 1, 2$ and 4
 $\Rightarrow L = 4, SF = 0.52 \pm 0.17$

- Weak dependency according to the exit pot. parameterization
- Significant dependency according to the entrance pot. parameterization

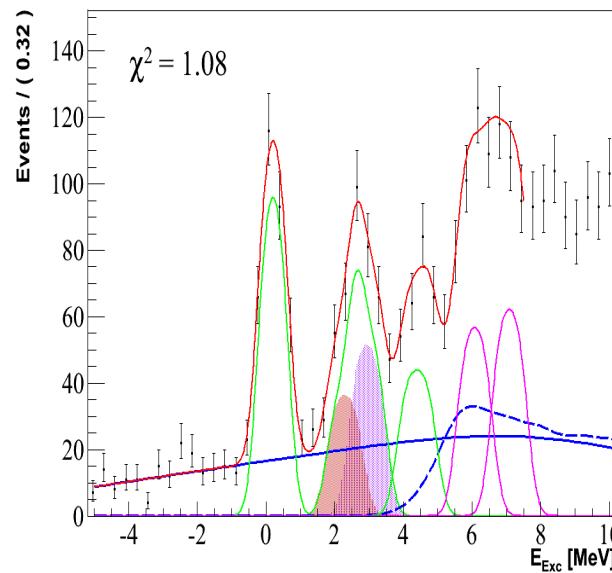
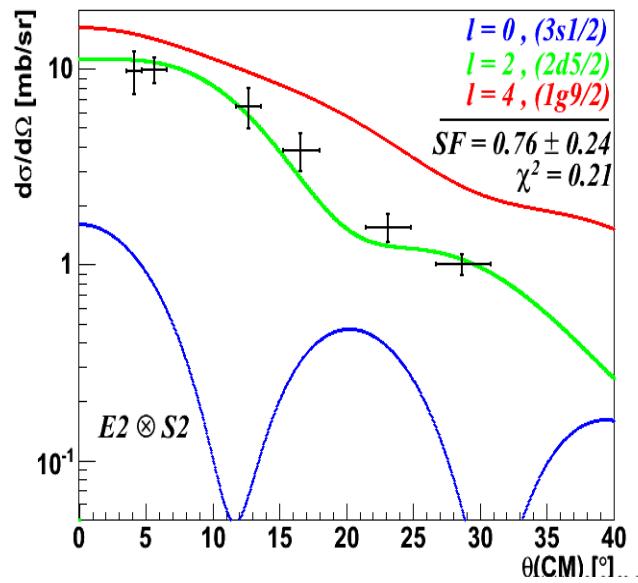
⇒ Adiabatic potential
 $(E2 \otimes S2)$

Analysed differential cross sections : 1st excited state



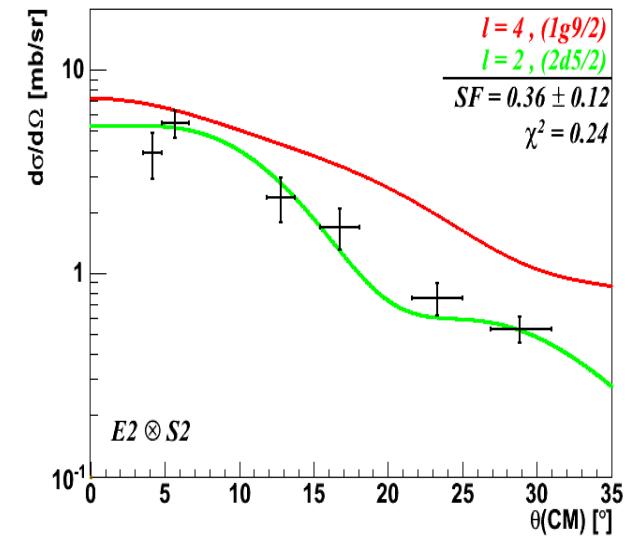
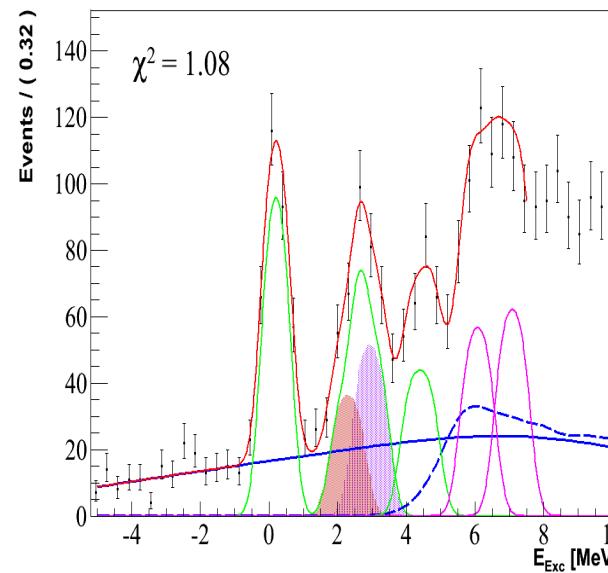
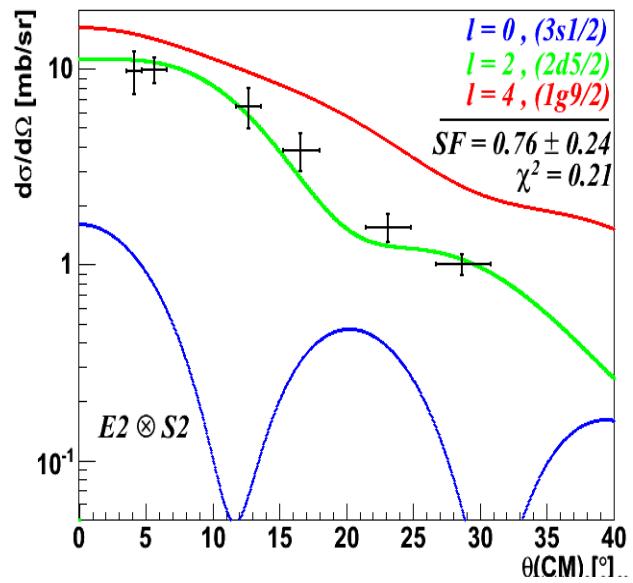
- $L = 0, 2$ and 4
- $E_{\text{exc}} = 2.47 \text{ MeV}$
- $\Rightarrow L = 2, SF = 0.76 \pm 0.24$

Analysed differential cross sections : 1st excited state



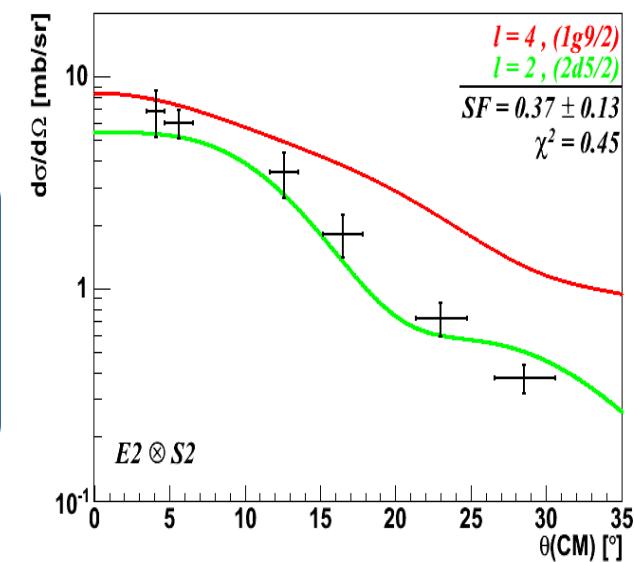
- $L = 0, 2$ and 4
- $E_{\text{exc}} = 2.47 \text{ MeV}$
- $\Rightarrow L = 2, SF = 0.76 \pm 0.24$

Analysed differential cross sections : 1st excited state

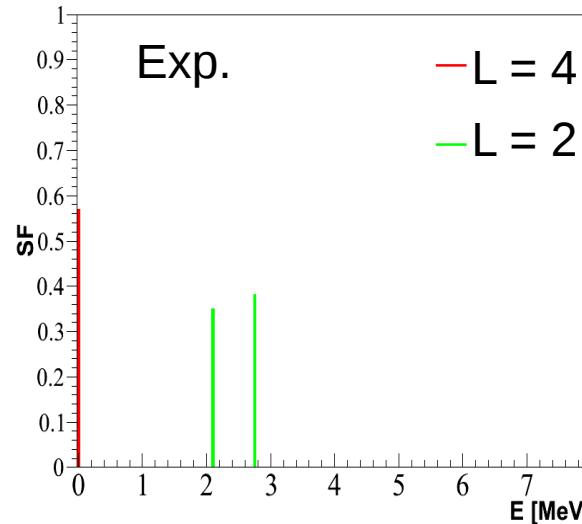
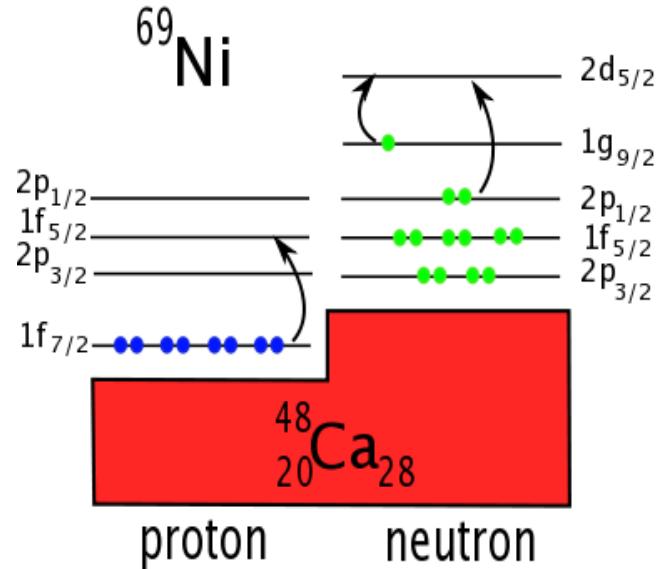


- $L = 0, 2$ and 4
- $E_{\text{exc}} = 2.47 \text{ MeV}$
- $\rightarrow L = 2, SF = 0.76 \pm 0.24$

- $E_{\text{exc}} = 2.11 \text{ MeV}$
- $L = 2, SF = 0.36 \pm 0.12$
- $E_{\text{exc}} = 2.76 \text{ MeV}$
- $L = 2, SF = 0.37 \pm 0.13$



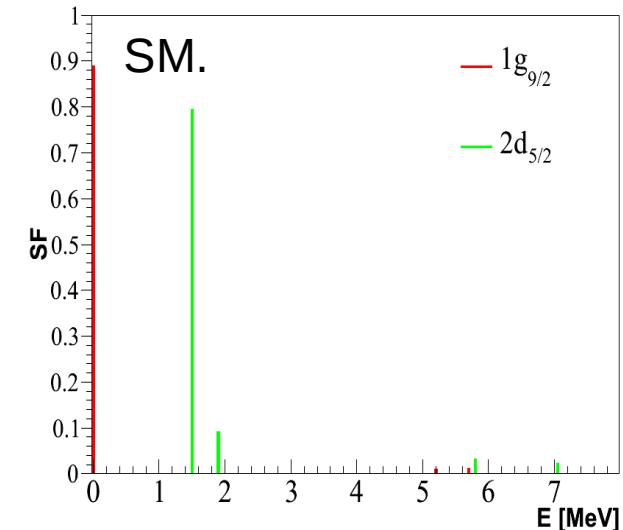
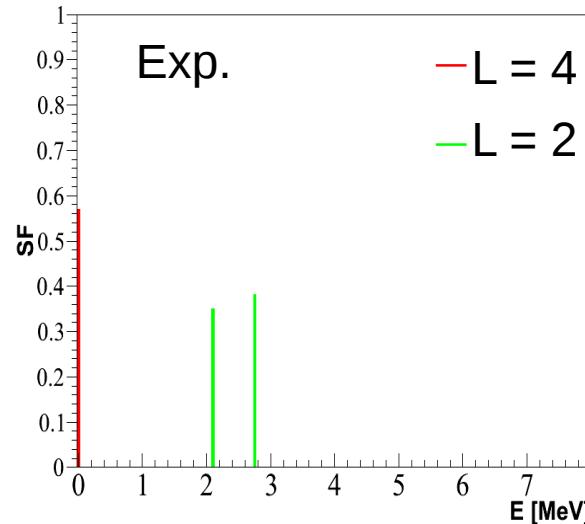
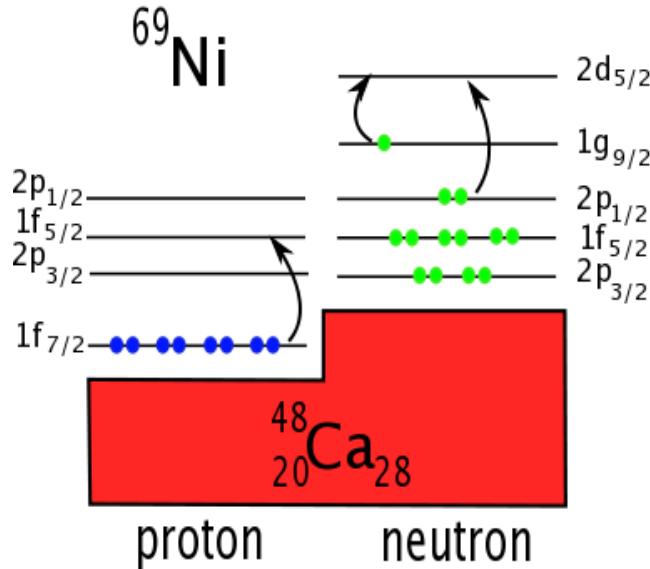
Shell-model calculations



- LNPS Interaction
- fp shell + $1g_{9/2}$ and $2d_{5/2}$

Lenzi et al., PRC 82, 054301, 2010
Sieja and Nowacki, submitted

Shell-model calculations

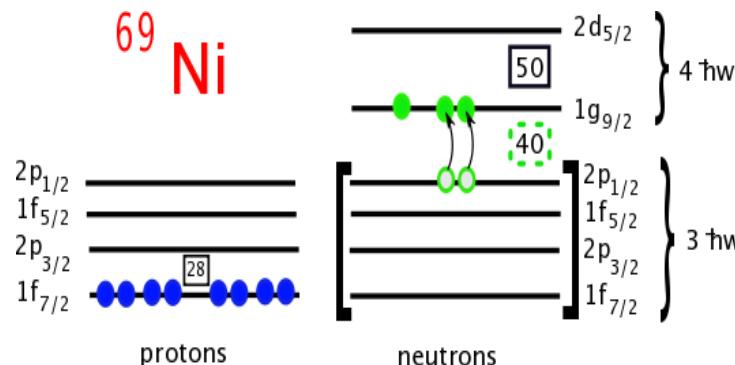
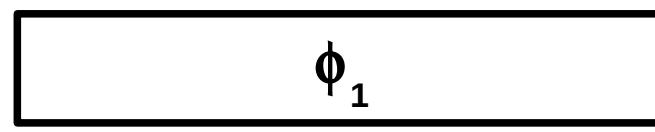
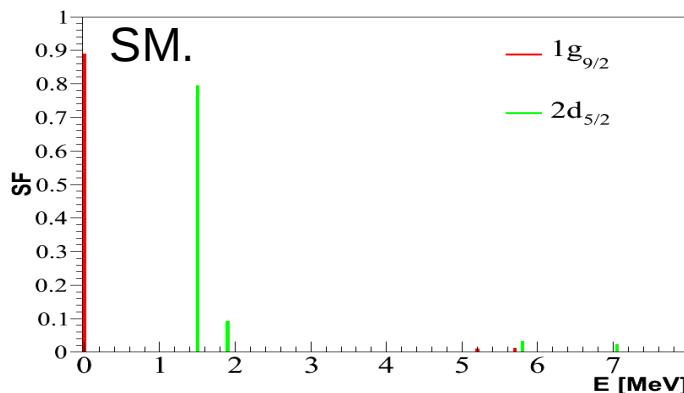
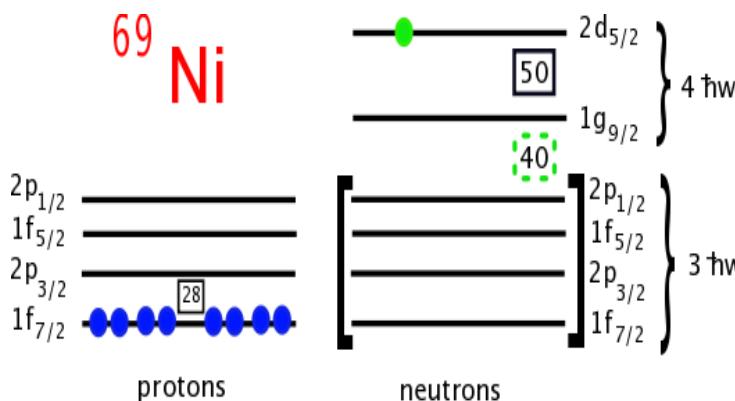
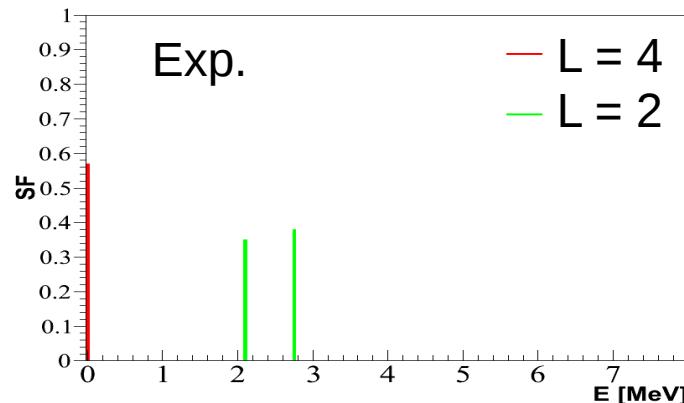


- LNPS Interaction
- fp shell + $1g_{9/2}$ and $2d_{5/2}$

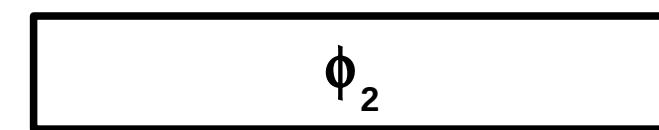
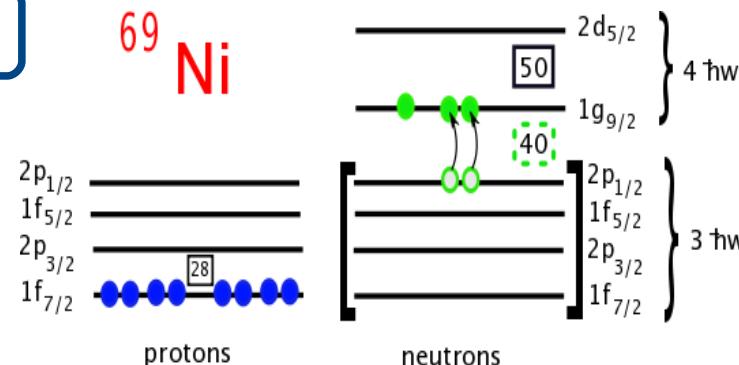
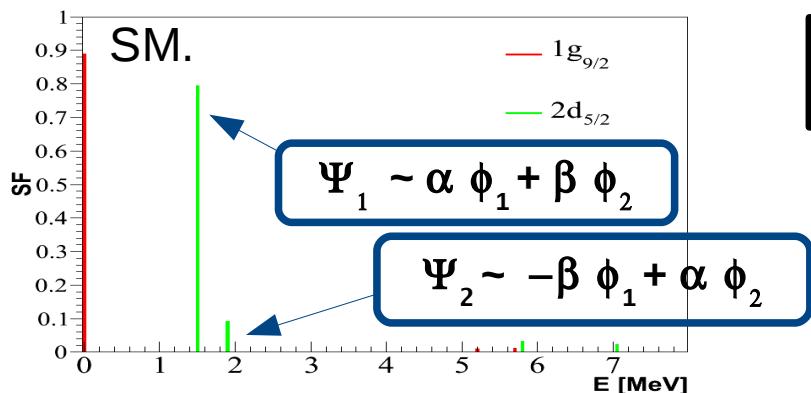
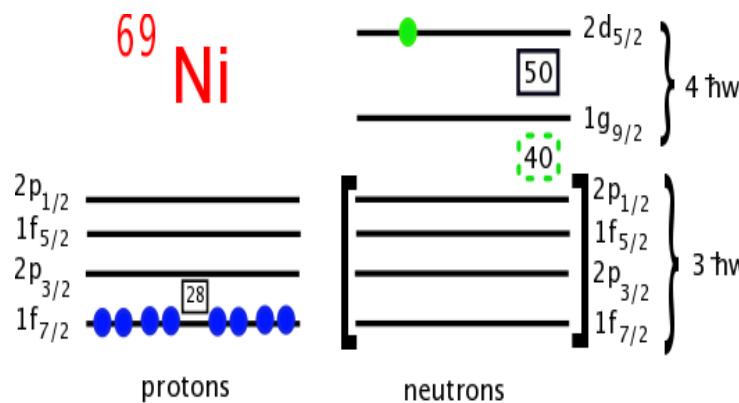
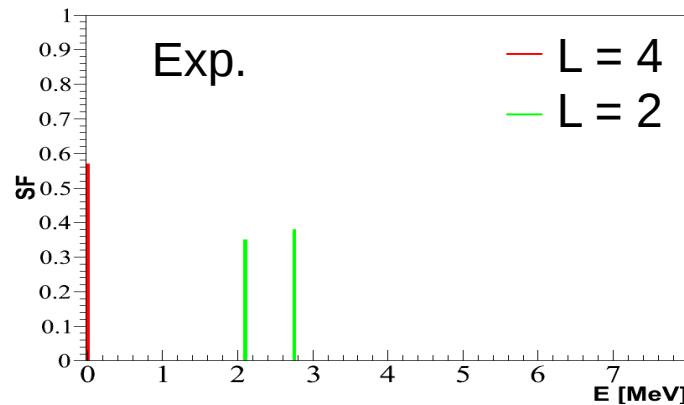
*Lenzi et al., PRC 82, 054301, 2010
Sieja and Nowacki, submitted*

- Good overall agreement
- $1g_{9/2}$: Large SF at 0 MeV
- $2d_{5/2}$: Doublet of $5/2^+$ states

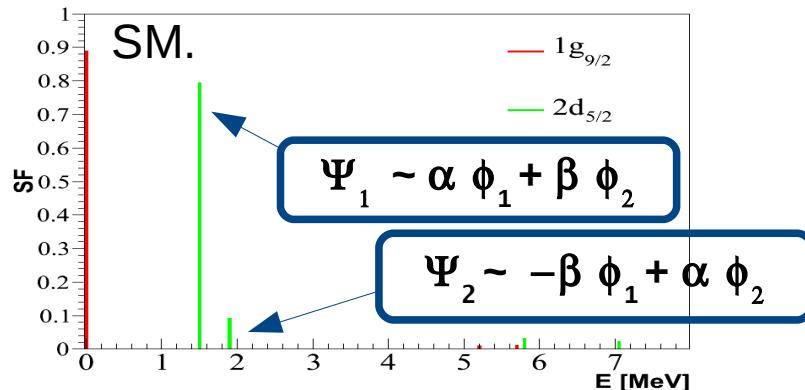
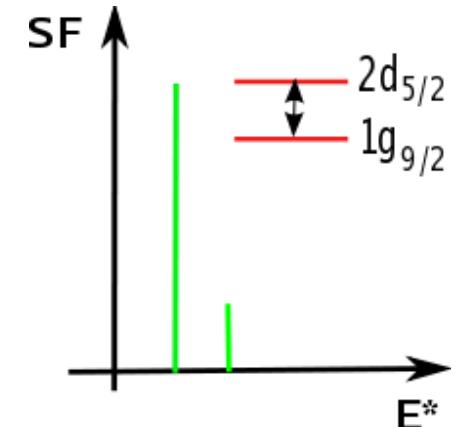
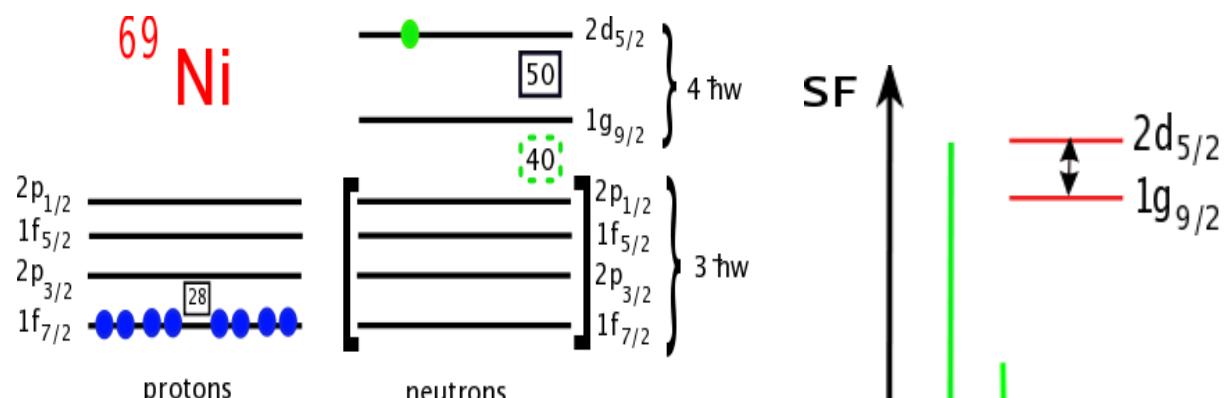
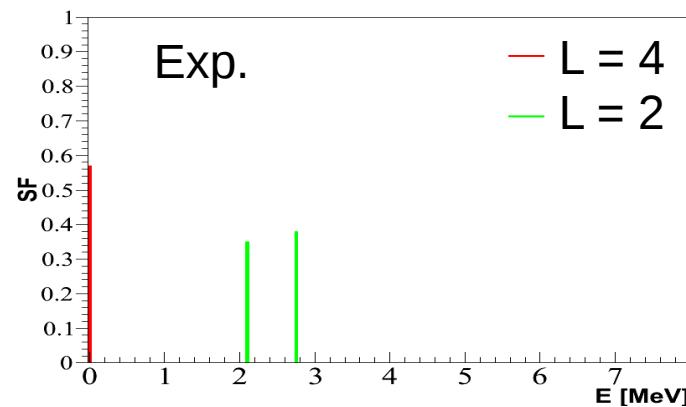
Shell-model calculations



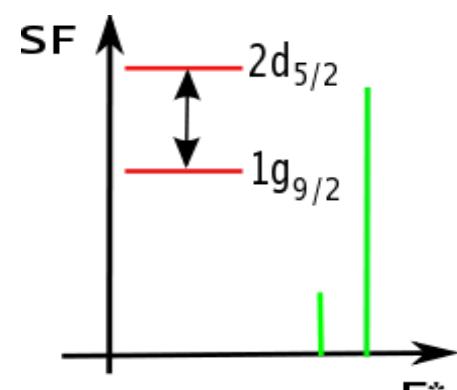
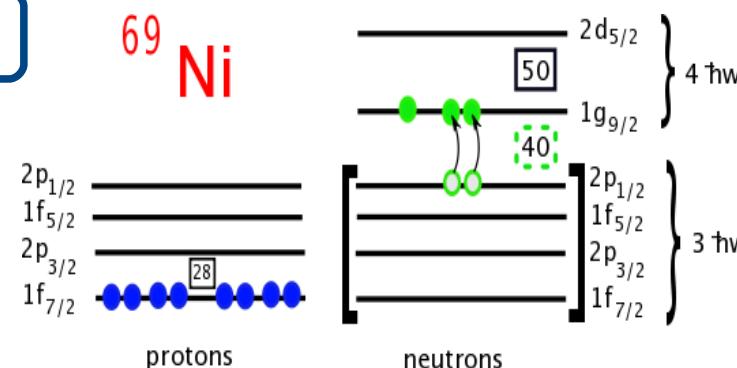
Shell-model calculations



Shell-model calculations

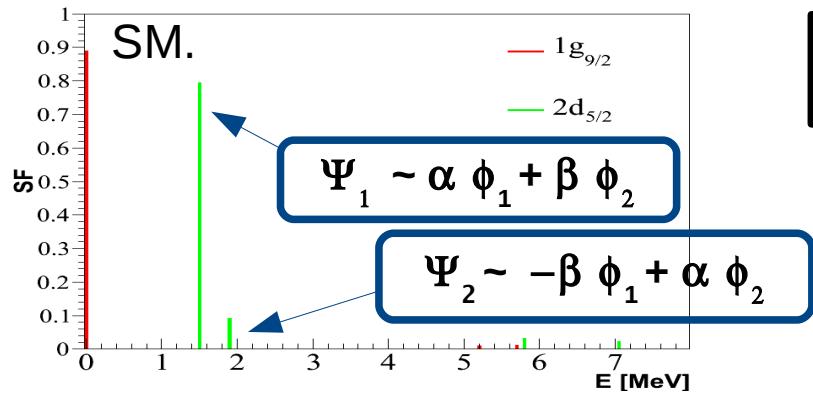
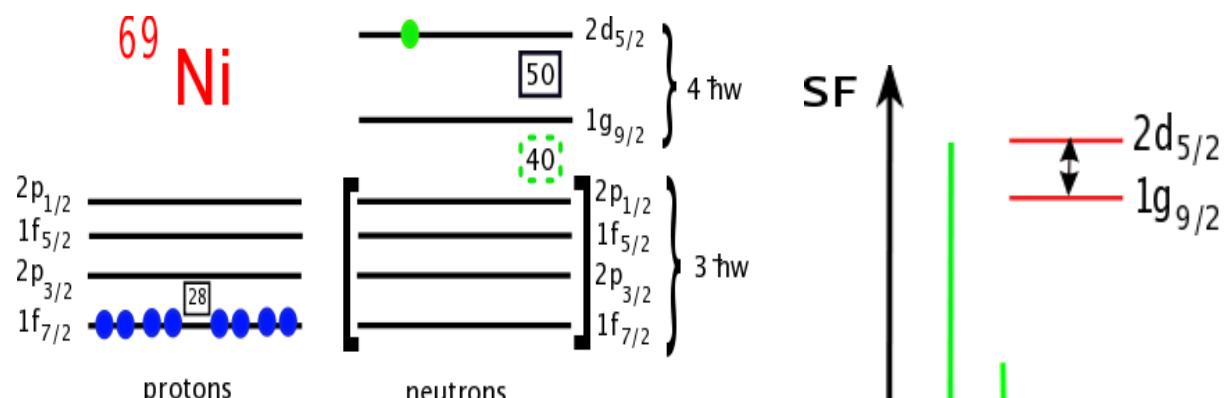
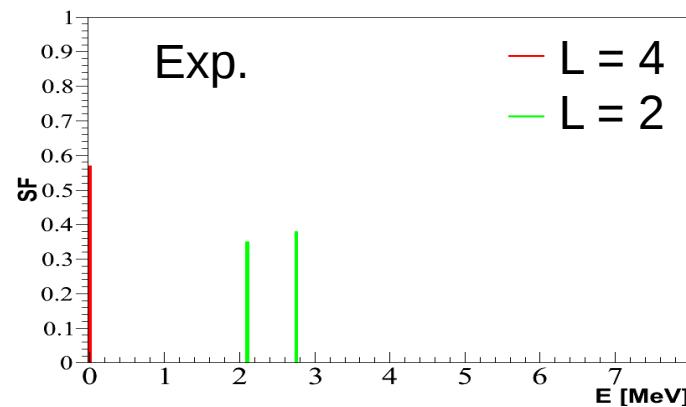


ϕ_1

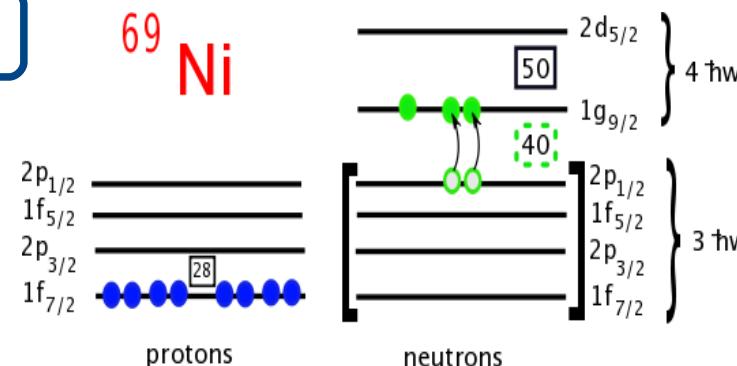
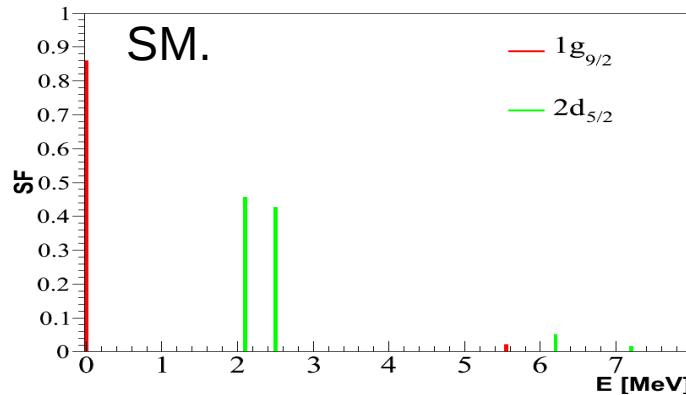


ϕ_2

Shell-model calculations



ϕ_1



ϕ_2

Conclusion and perspective

- ◆ Search for the $2d_{5/2}$ neutron orbital in the ^{69}Ni nucleus,
- ◆ Study of the $d(^{68}\text{Ni}, p)^{69}\text{Ni}$ transfer reaction at GANIL,
- ◆ Experimental set-up : CATS/MUST2-S1/Plastic,
- ◆ Spin and parity assignement $9/2^+$ for the G.S. and $5/2^+$ for the doublet at 2.47 MeV, with an important spectroscopic factor.

Energy [MeV]	L	Jπ	SF (E2⊗S1)	χ²	SF (E2⊗S2)	χ²	SF average
0.00	4	$9/2^+$	0.62 ± 0.20	0,10	0.52 ± 0.17	0,28	0.57 ± 0.19
2.11	2	$5/2^+$	0.37 ± 0.13	0,41	0.36 ± 0.12	0,24	0.36 ± 0.13
2.76	2	$5/2^+$	0.39 ± 0.14	0,58	0.37 ± 0.13	0,45	0.38 ± 0.14

- ◆ Good agreement with the shell model calculations,
- ◆ Possibility to further constrain of the $2d_{5/2}$ orbital position,
- ◆ Validation of the hypothesis postulated by Strasbourg on the small energy difference between the $1g_{9/2}$ and $2d_{5/2}$ orbitals, (*Caurier et al. EPJ, A, 15, 2002, 145*)
- ◆ Identification of a neutron state at 4.2 MeV and two resonances at ~5.9 and ~6.9 MeV.
- ◆ **Perspective : Data analysis of gamma rays (EXOGAM) for more accurate determination of the excitation energies.**

Collaborators

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O. Sorlin, C. Stoedel, J.C. Thomas
(GANIL - Caen, FRANCE)**

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(SPhN - Saclay, FRANCE)**

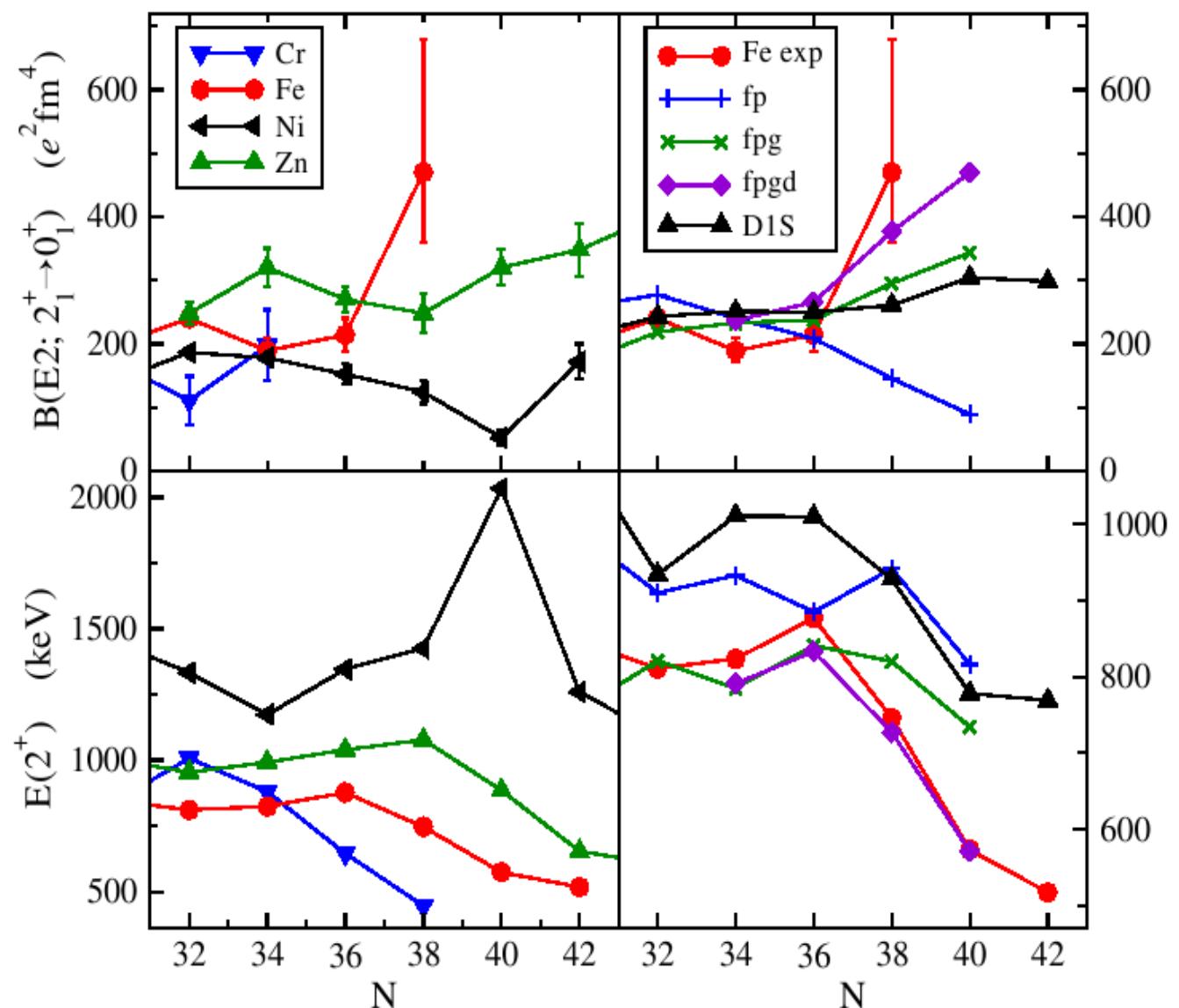
**J. Gibelin
(LPC - Caen, FRANCE)**

**K. Kemper
(Florida State University, USA)**

**M. Harakeh
(GSI - Darmstadt, Allemagne)**

Back up

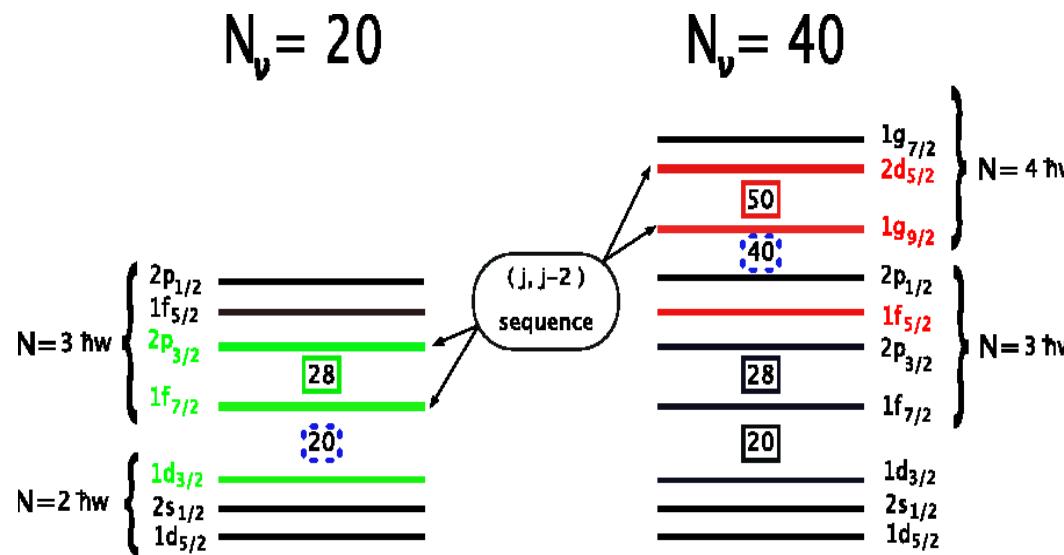
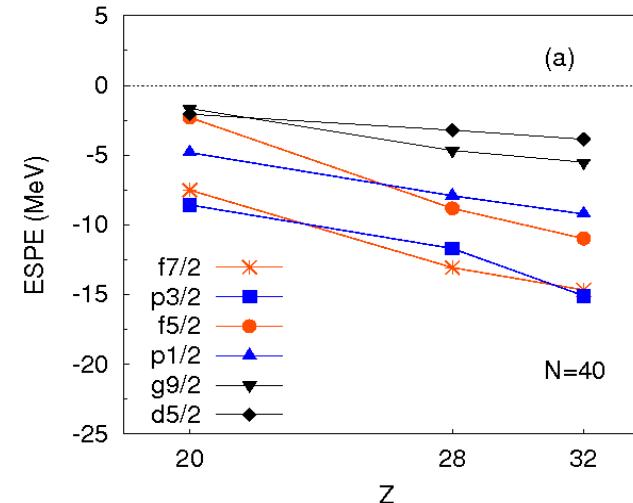
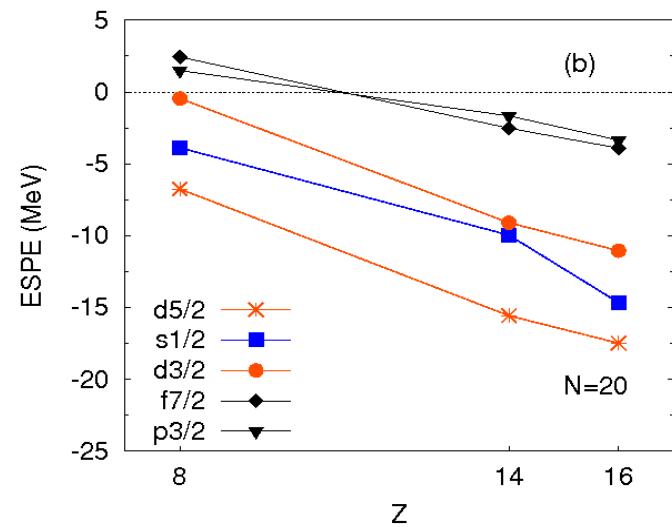
Island of inversion at N = 40 : Deformation in Fe



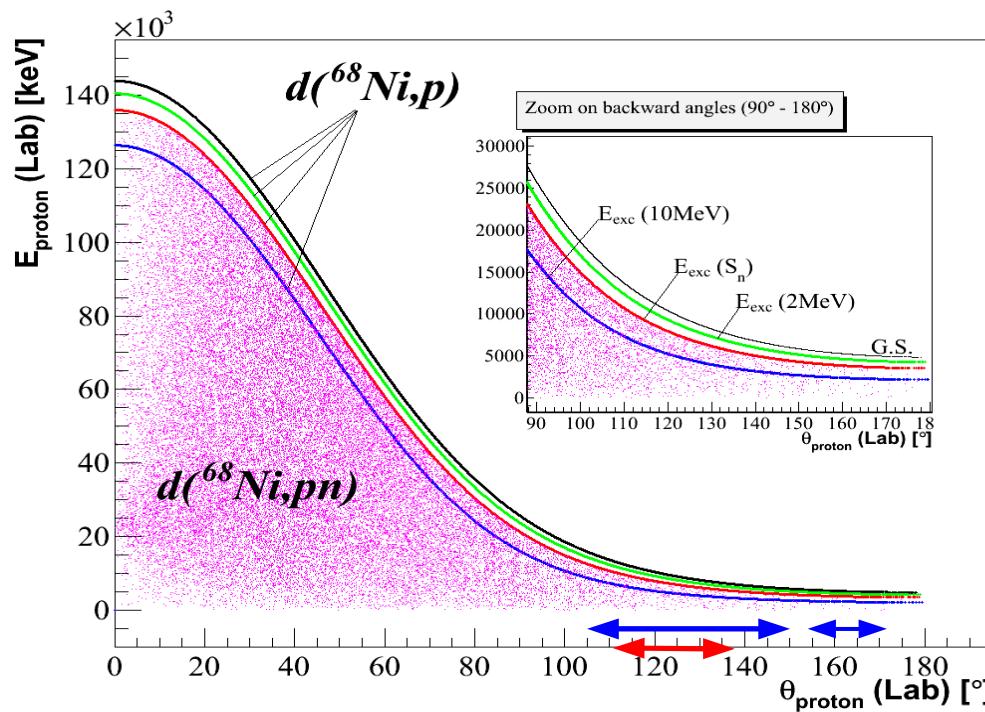
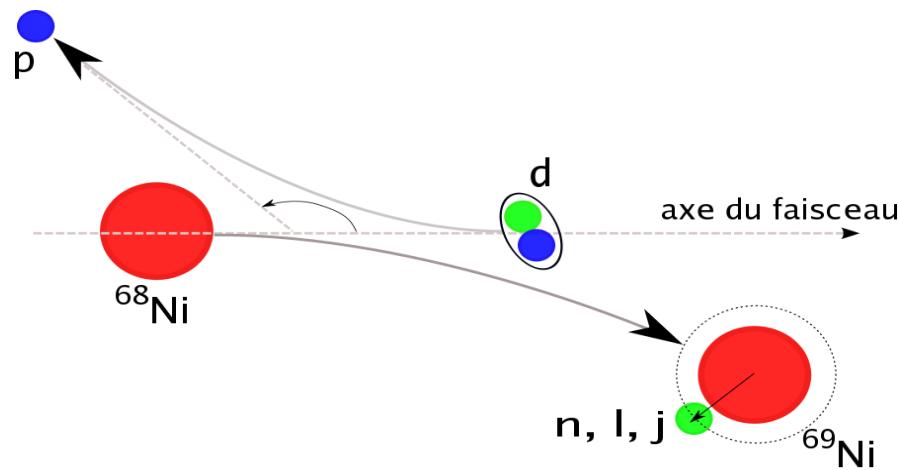
- fp
- fp + 1g9/2
- fp + 1g9/2 + 2d5/2

Ljungval et al. Physical Review C 81 (2010) 061301

Island of inversion - N=20 and N=40



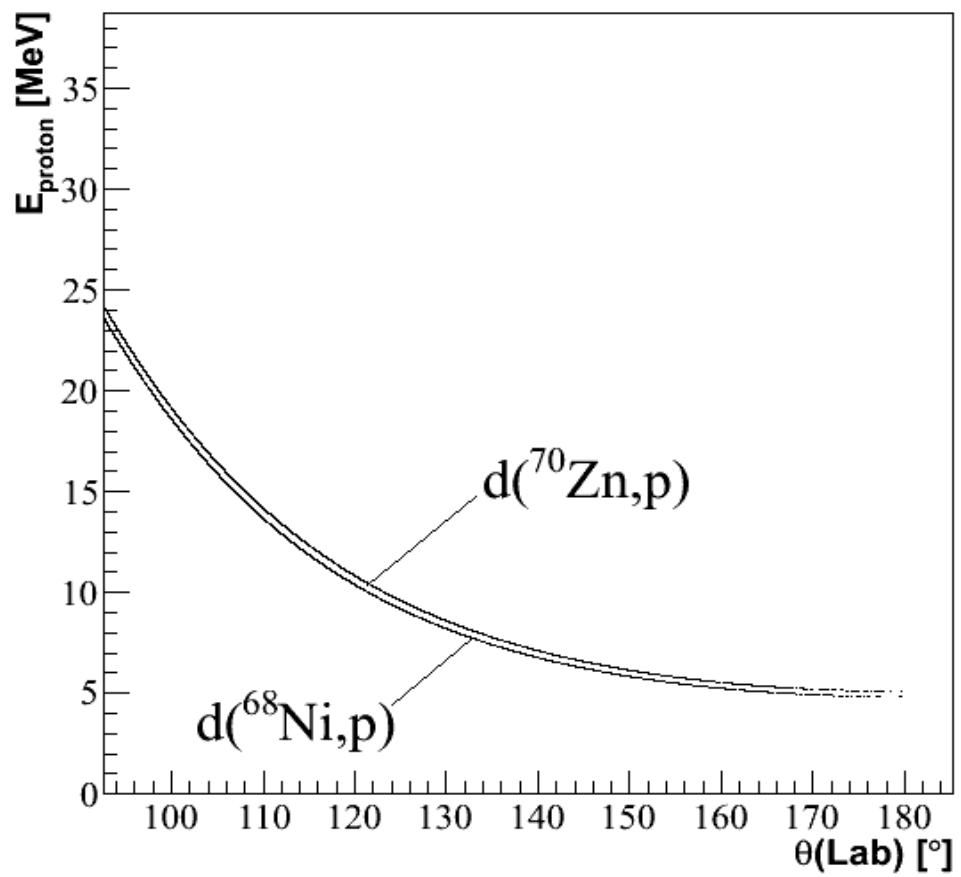
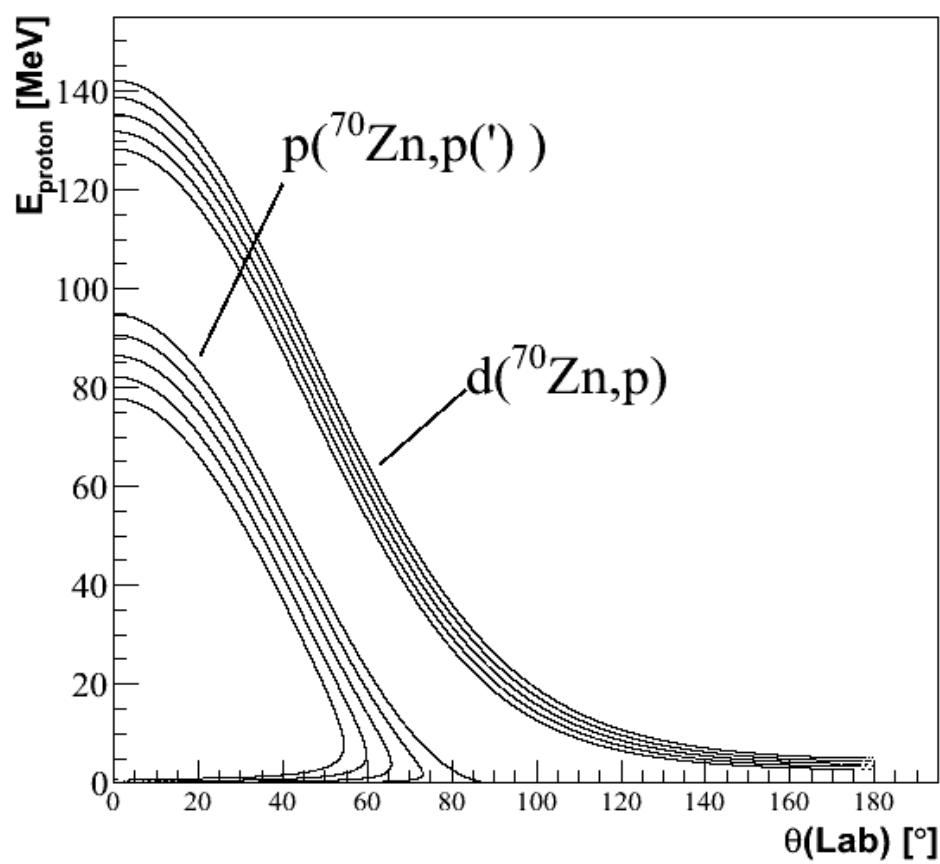
Experimental setup : Inverse kinematics



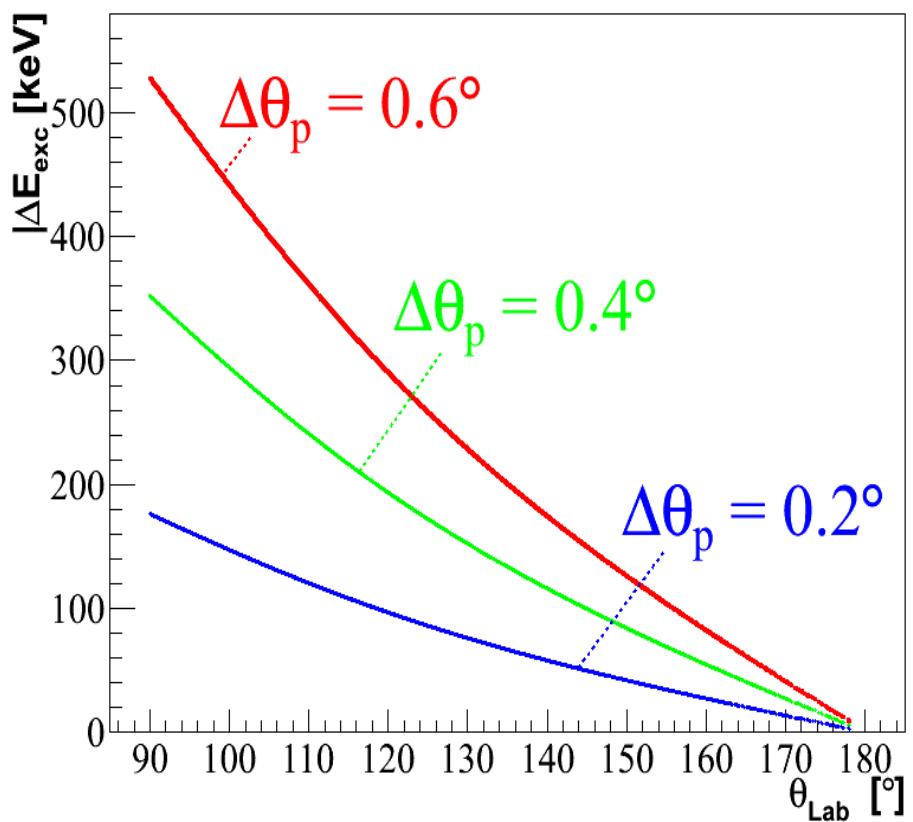
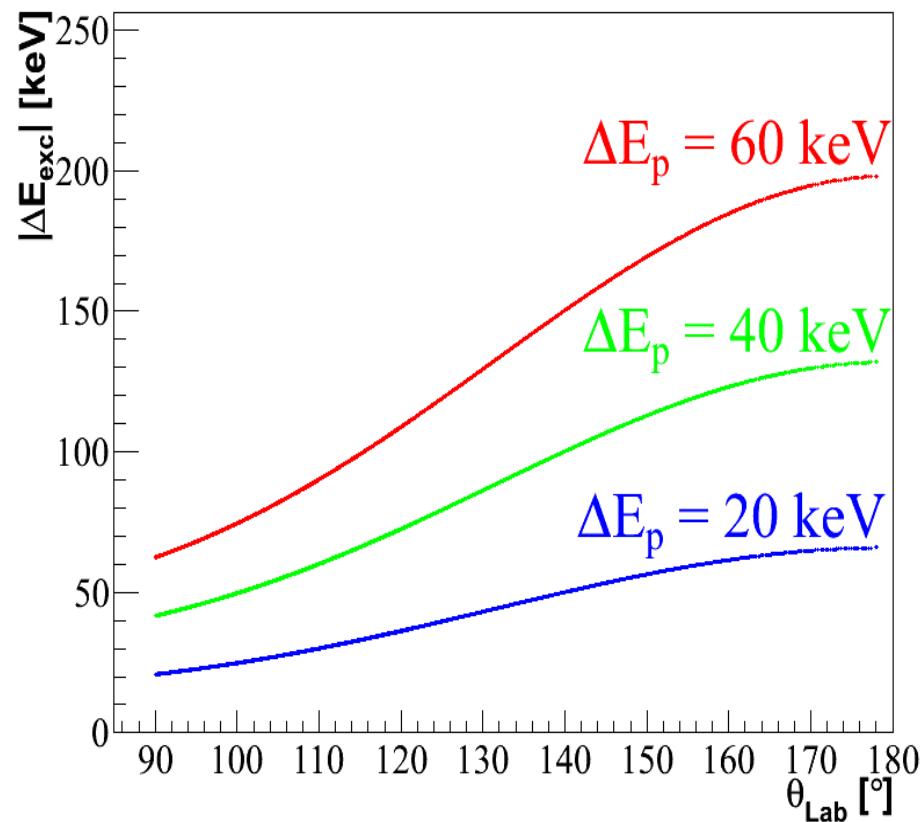
^{68}Ni ($T = 29\text{s}$)
⇒ Inverse kinematics
Forward CM angles
↔
Backward lab angles

Interesting angular range :
 $12^\circ - 30^\circ$ (CM) ↔ $110^\circ - 140^\circ$ (Lab.)

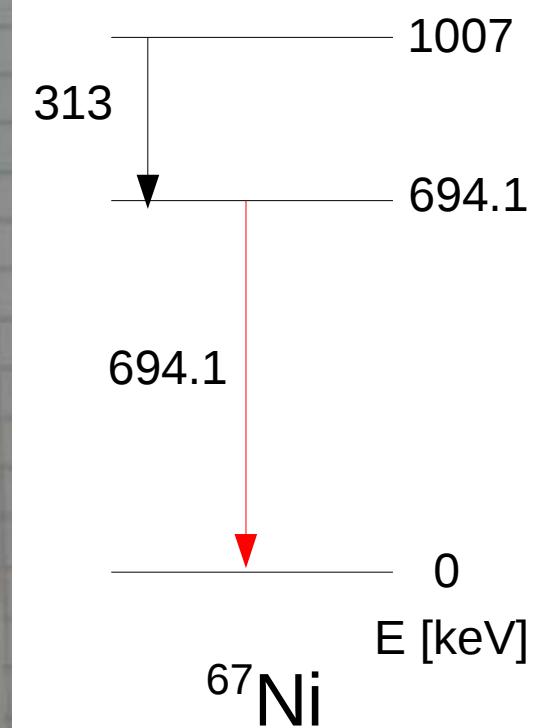
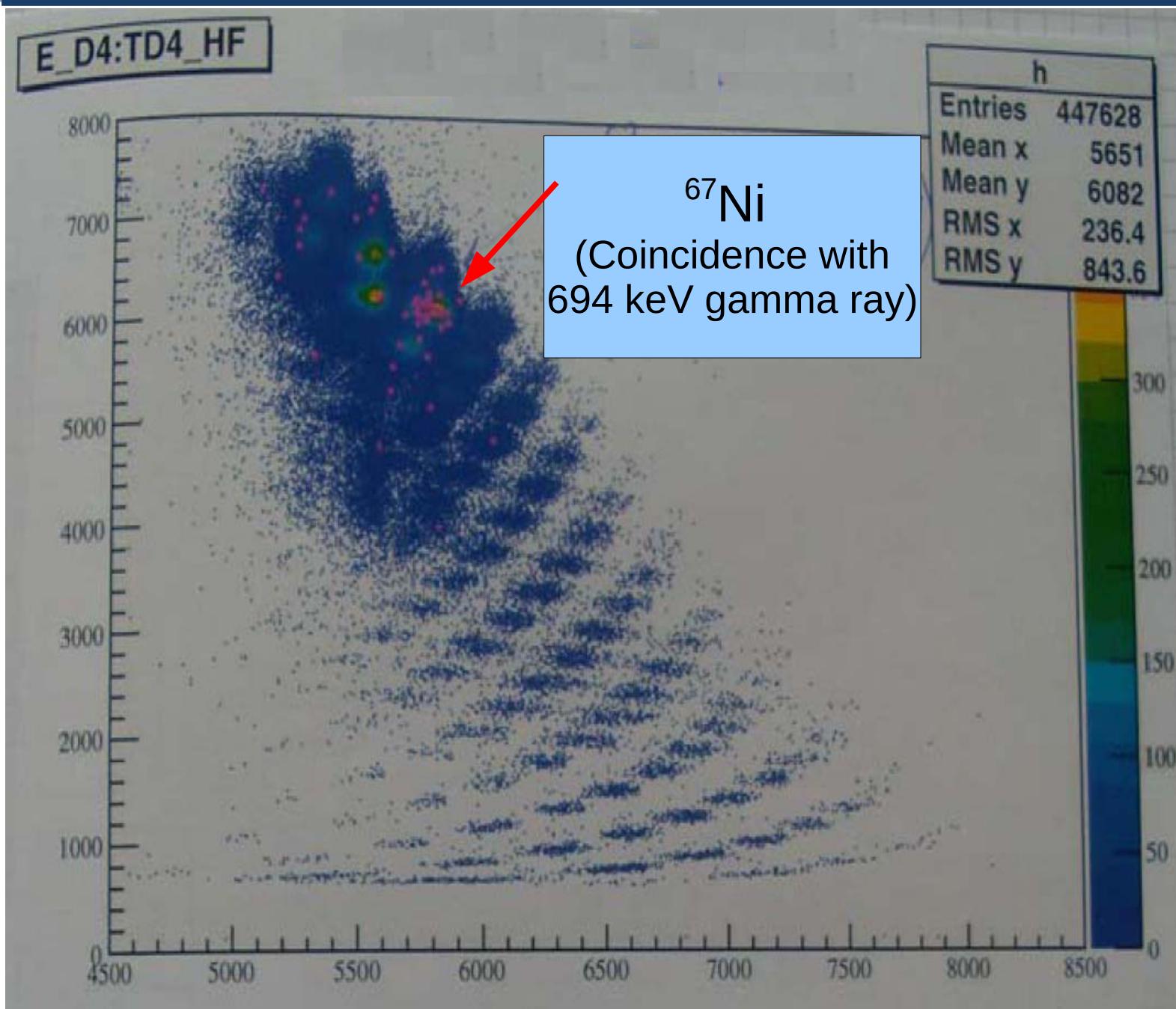
Kinematic lines



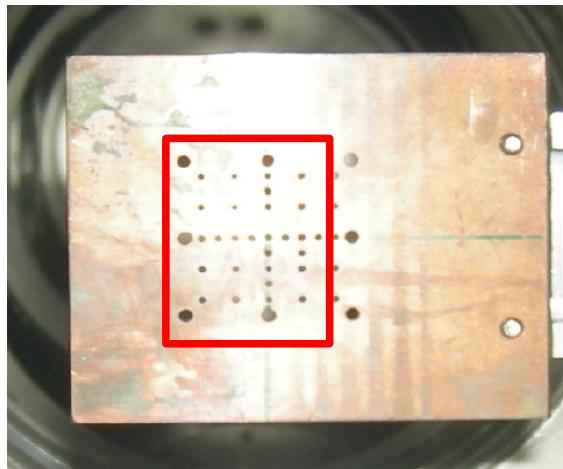
Variation of E_{exc} with respect to proton's energy and angle of emission



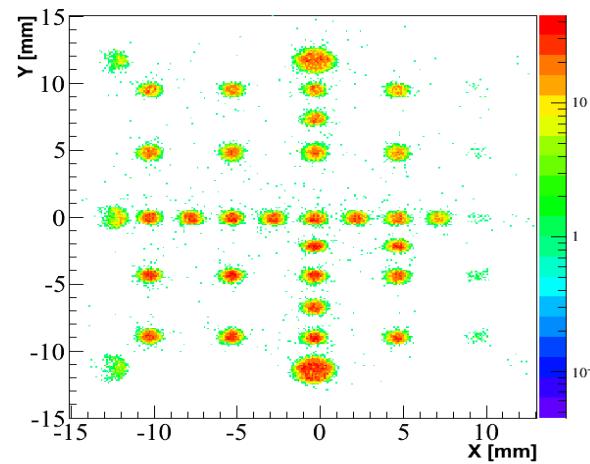
LISE (D4) beam identification



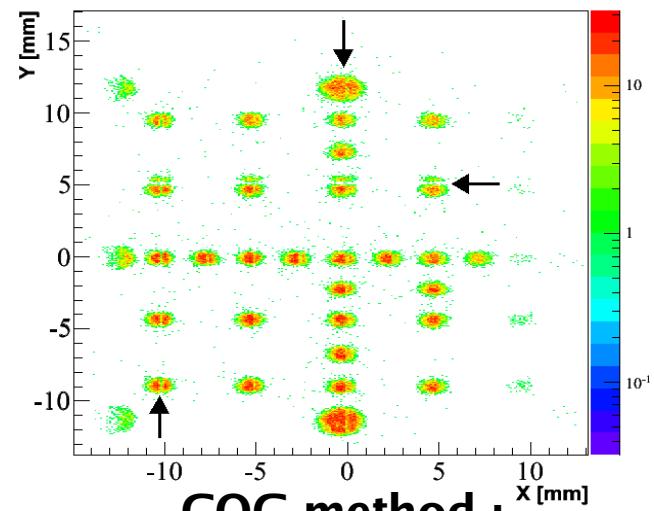
Beam tracker : CATS



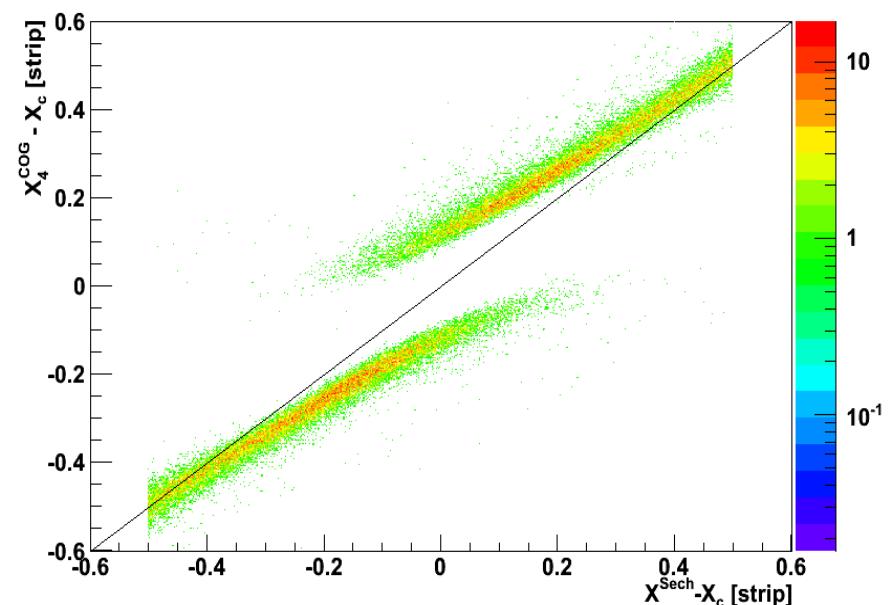
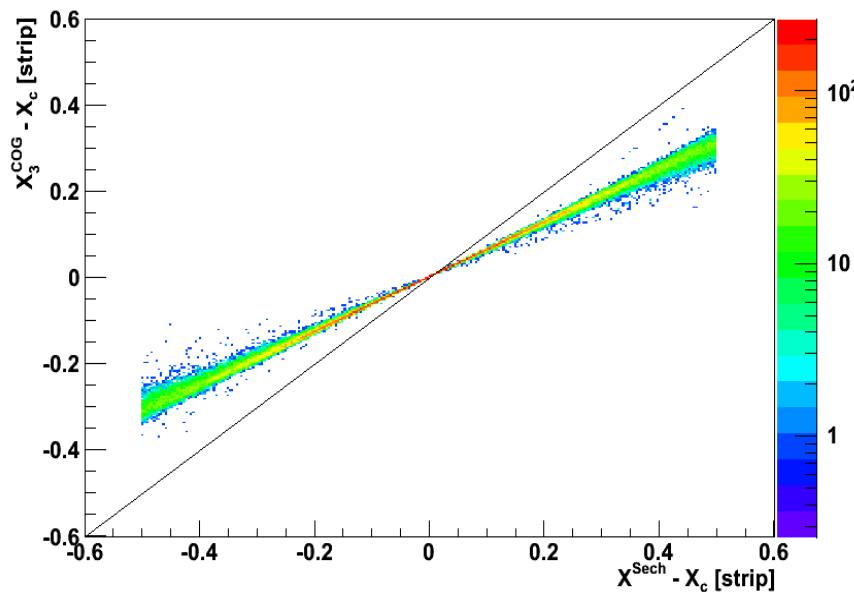
CATS1 mask



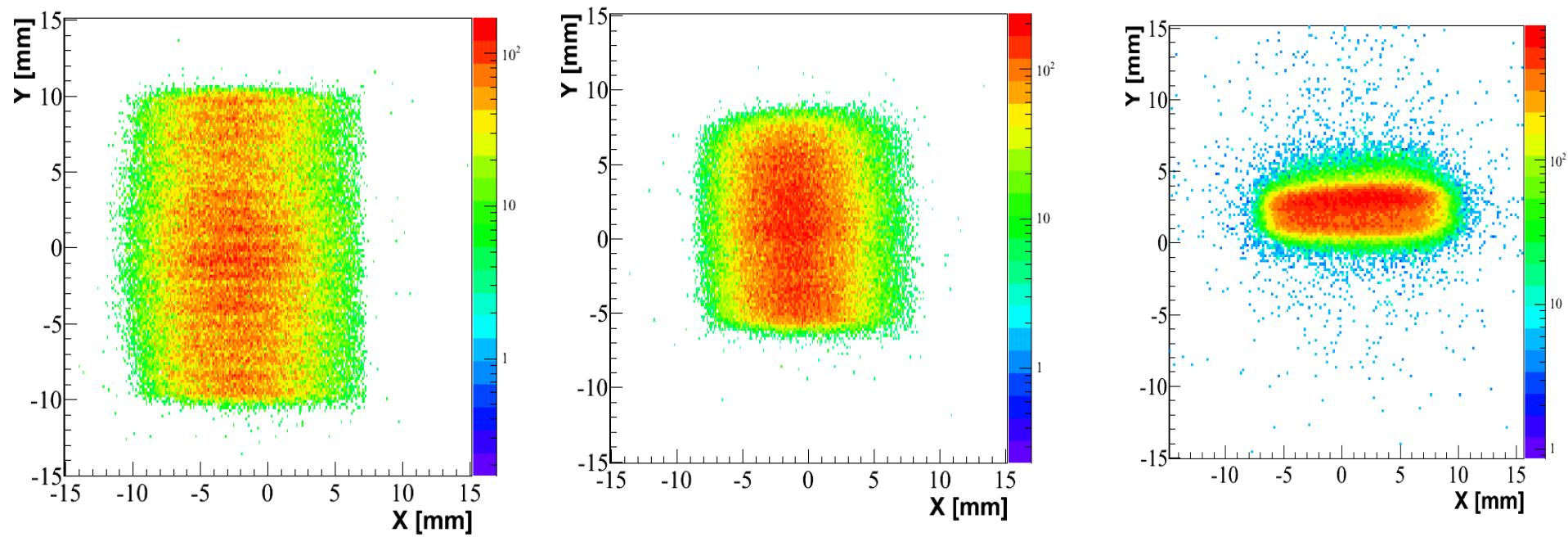
Analytical method :
(Hyperbolic secant squared)



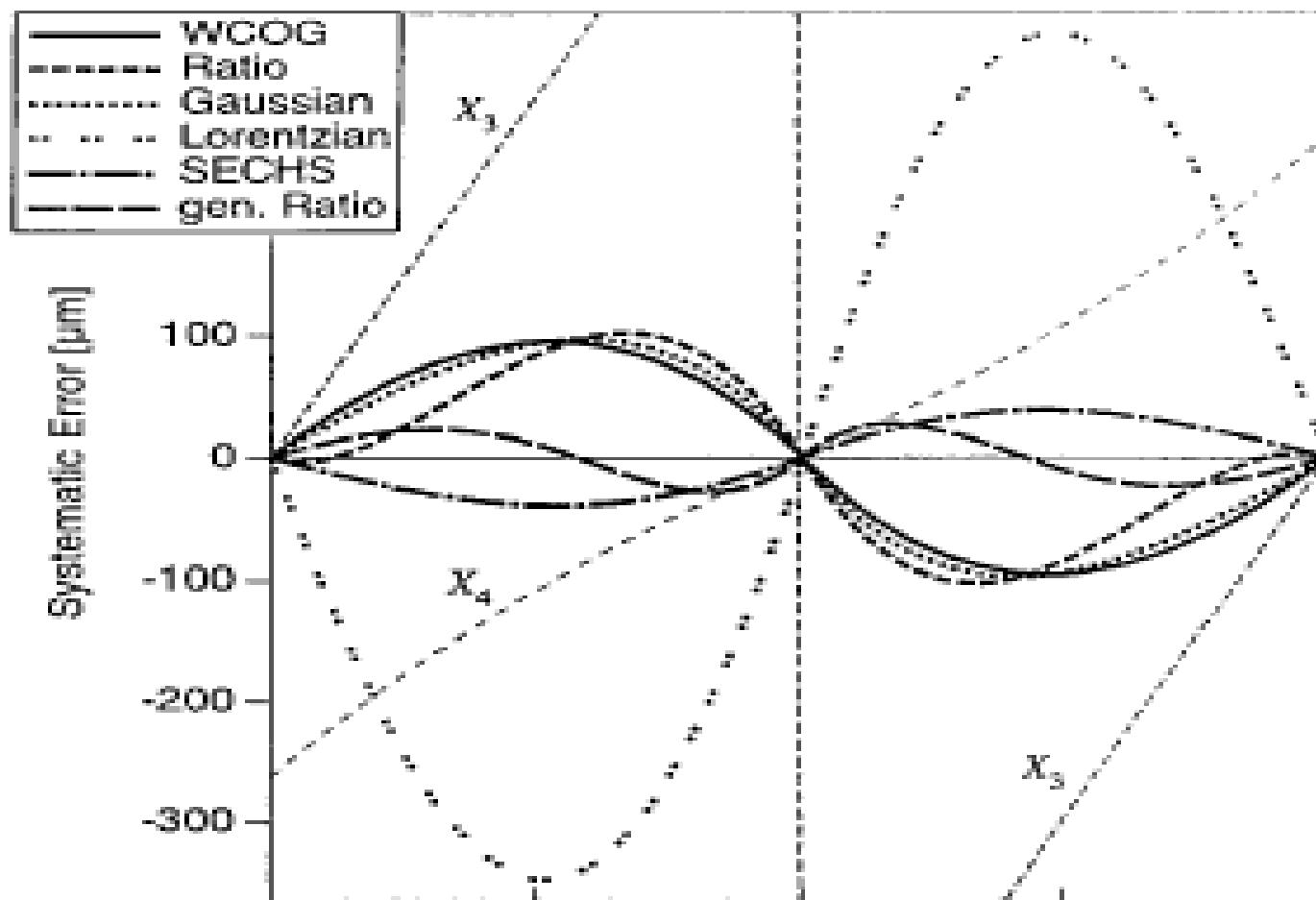
COG method :
(3 strips)



Beam tracker : CATS

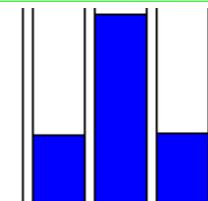
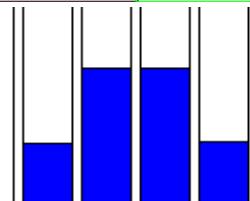
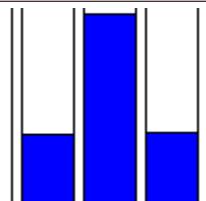


Construction with CATS : systematic errors

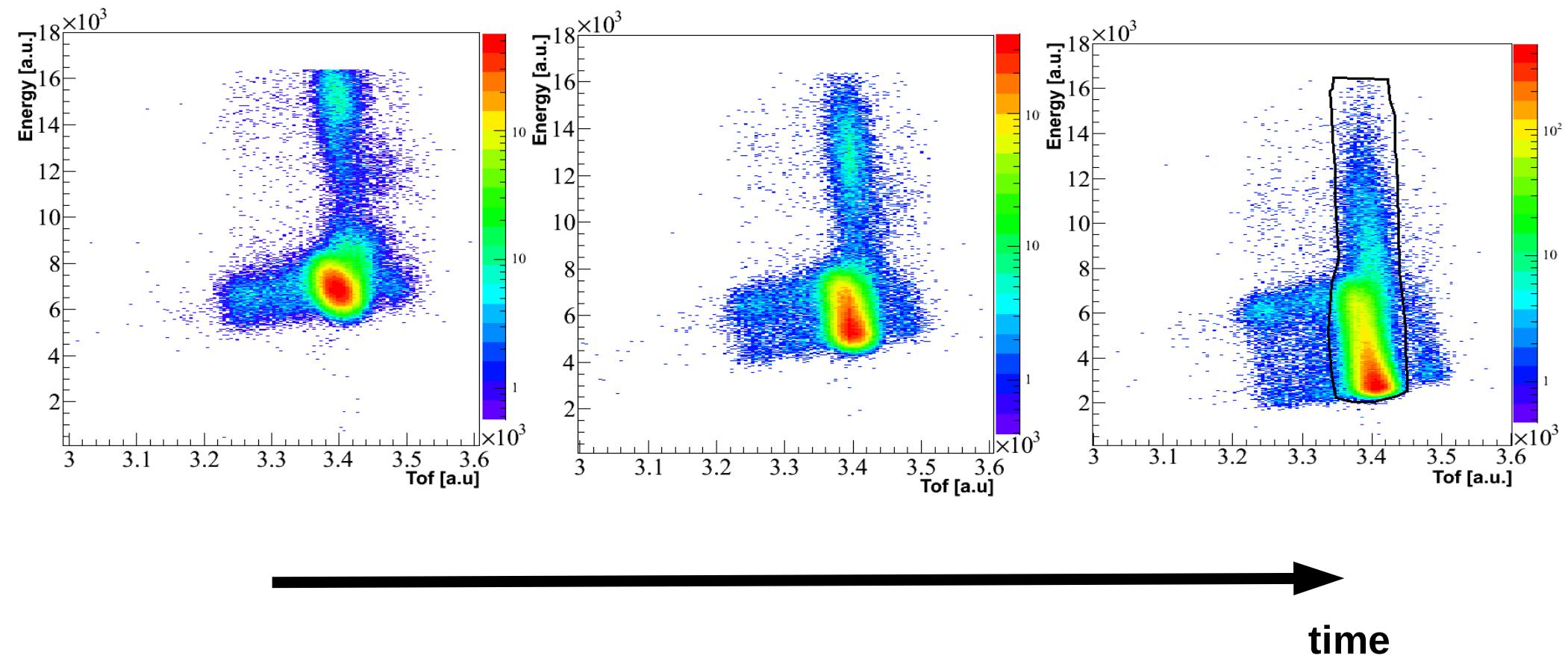


Strip (n) center

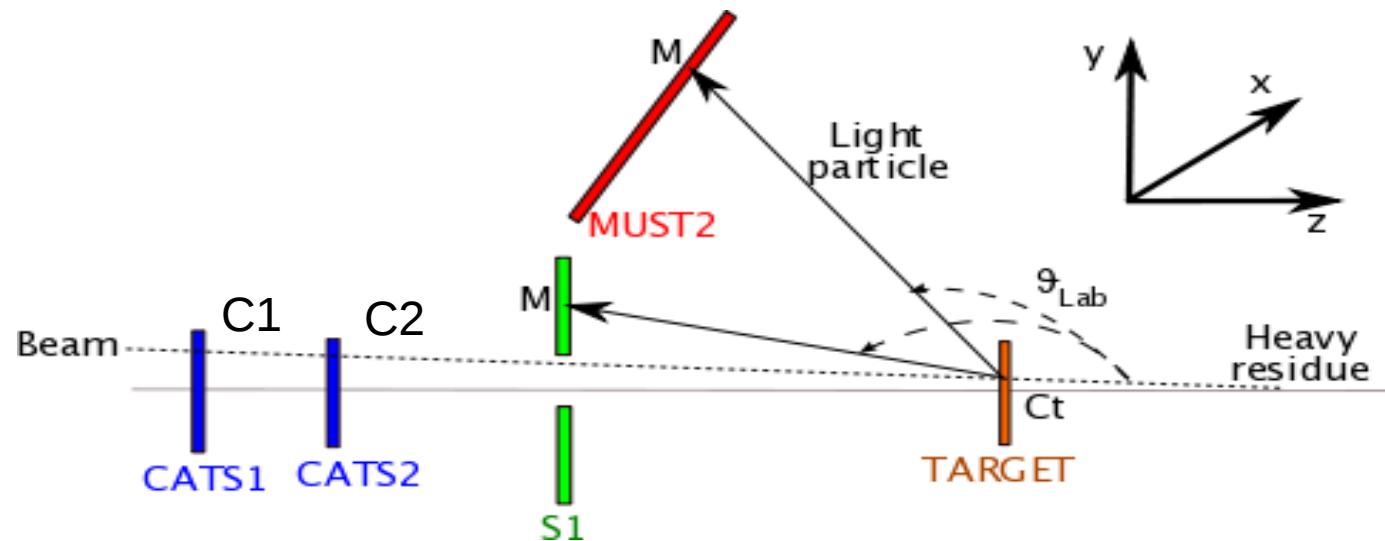
Strip (n+1) center



Plastic deterioration

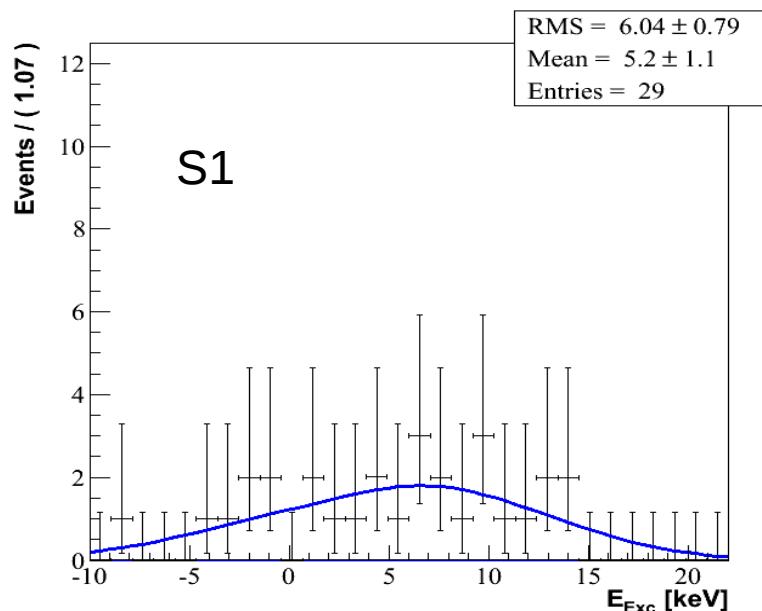
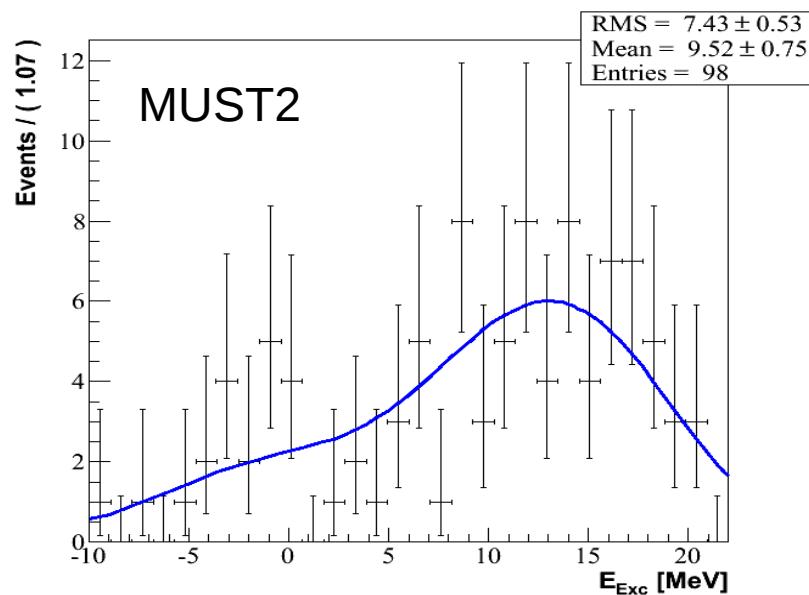


Calculated parameters

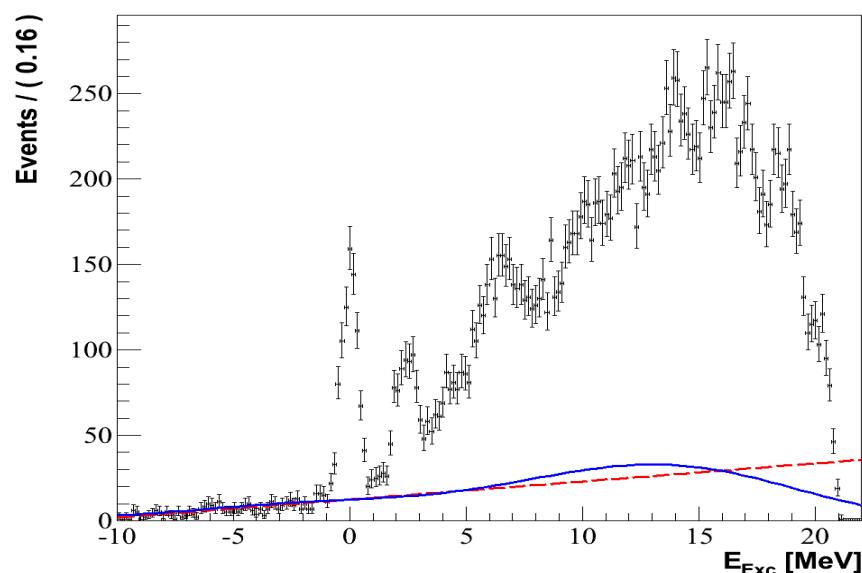


- Reconstruction of angle of emission $\theta(\text{Lab})$ (uncert. $\sim 0.5^\circ$)
- Beam energy correction taking into account the energy losses in the:
 - Beam trackers CATS
 - Target
- Proton energy correction taking into account the energy losses in the :
 - Target
 - Detector dead layers
- Reconstruction of excitation energy using missing mass method

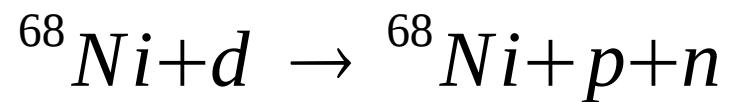
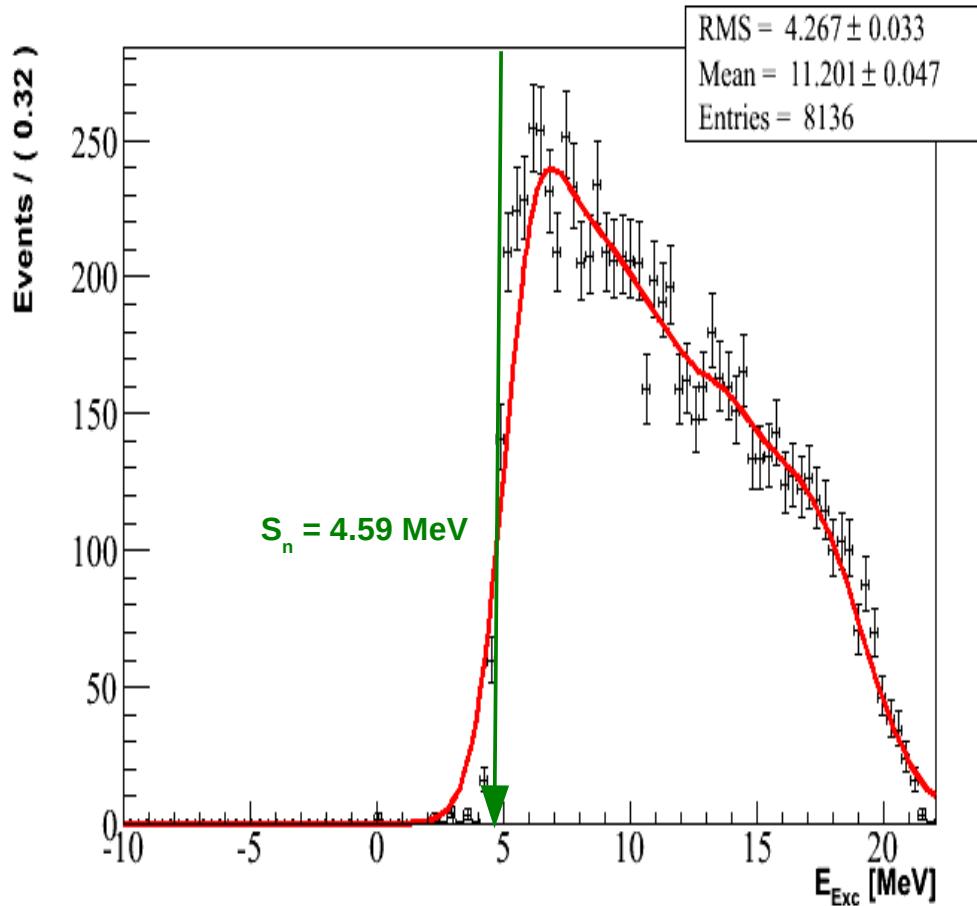
Background reactions : Carbon background



- CD₂ Target \Rightarrow Carbon background
- Poor statistics
- Estimated with:
 - Linear back-ground
 - « Kernel estimation »

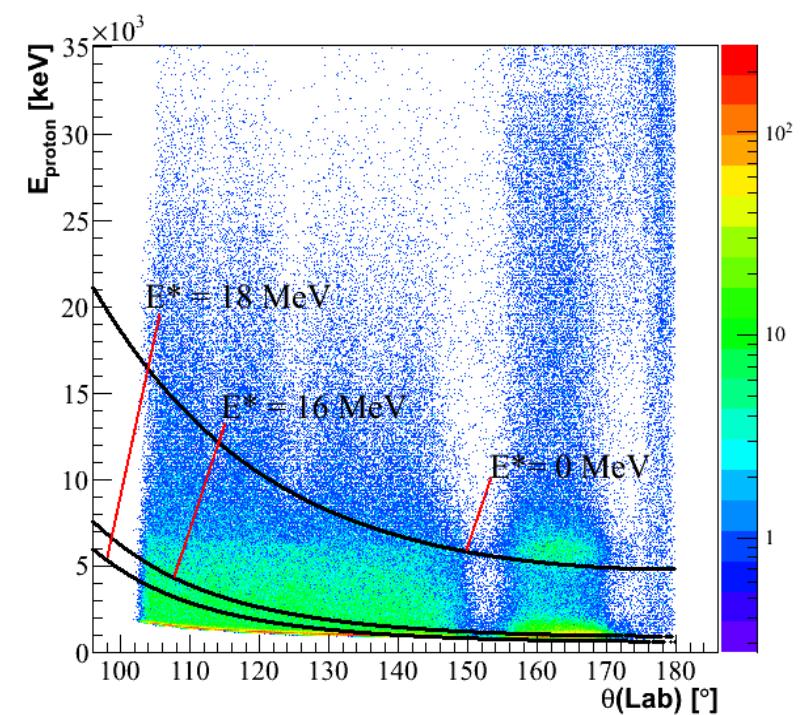
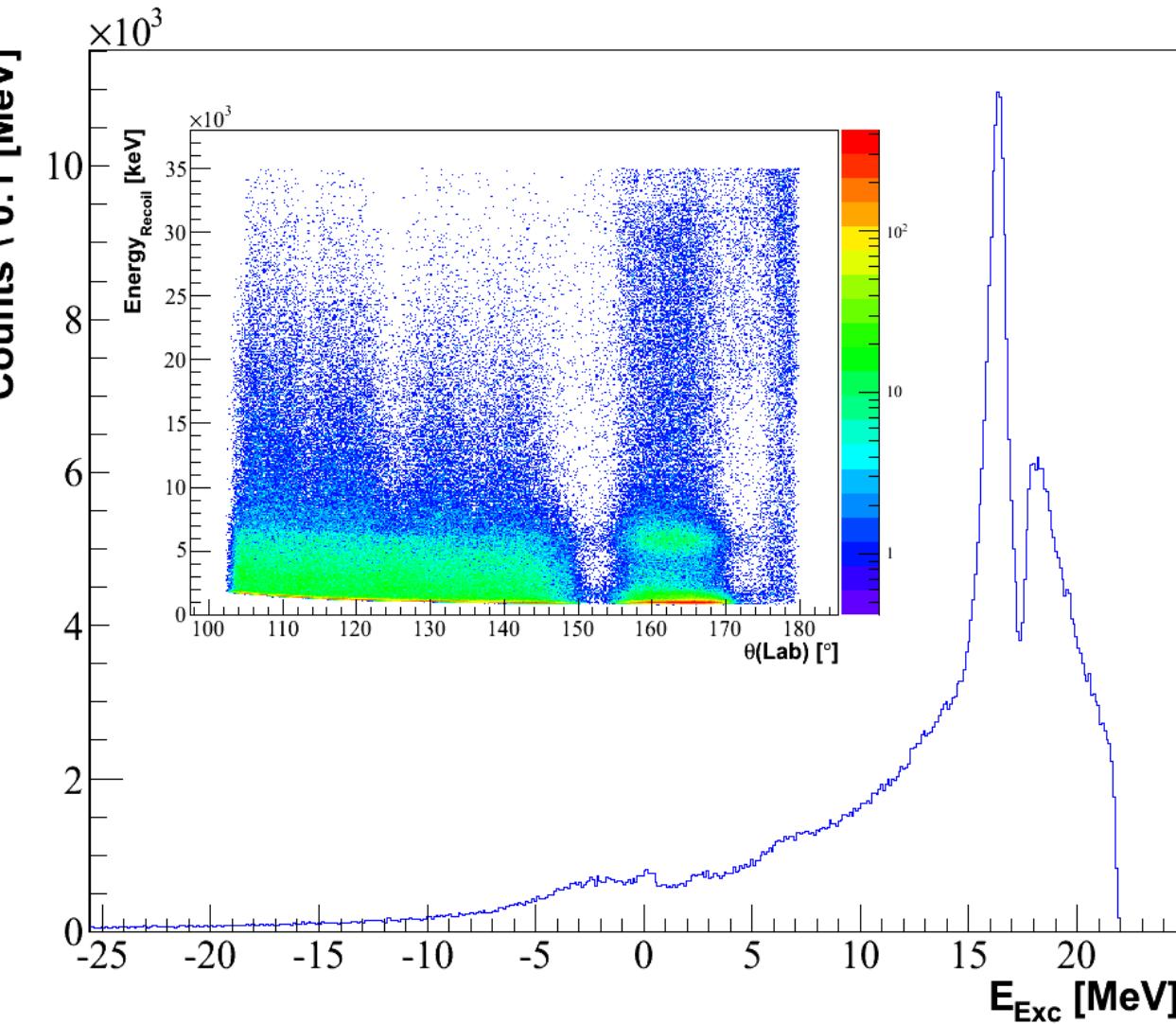


Background reactions : Deuteron break-up

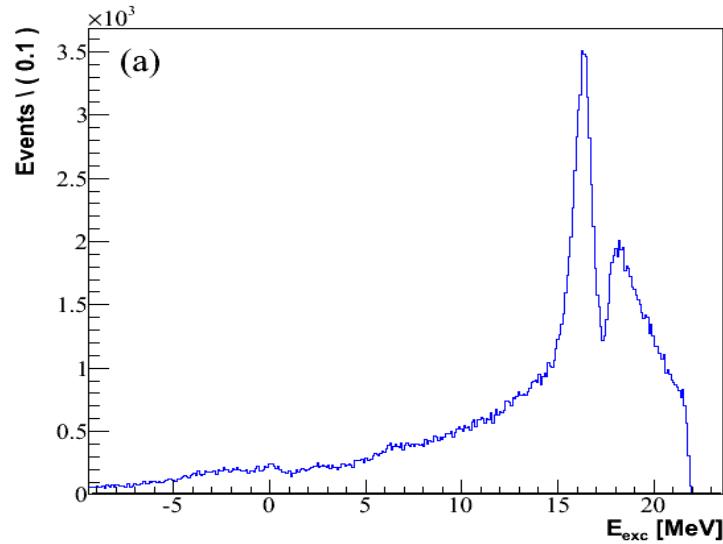


- Deuteron break-up \Rightarrow 3-body kinematics
- Phase-space calculations
- « Kernel estimation »

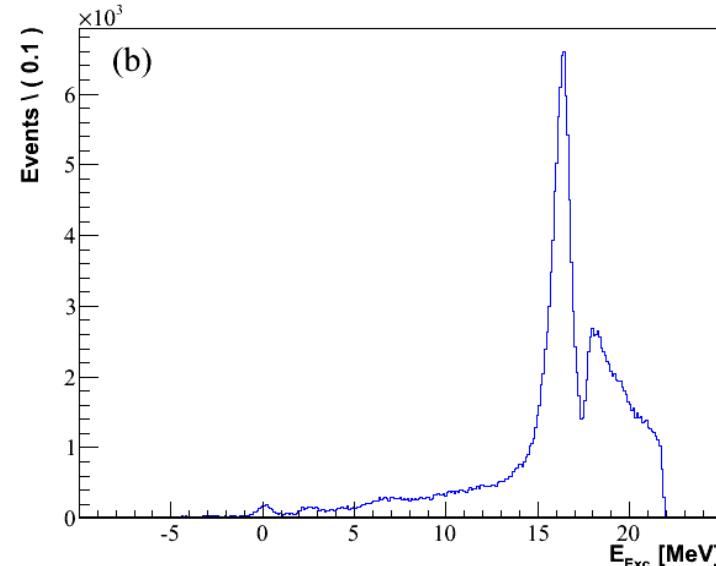
Excitation energy spectrum : no cuts



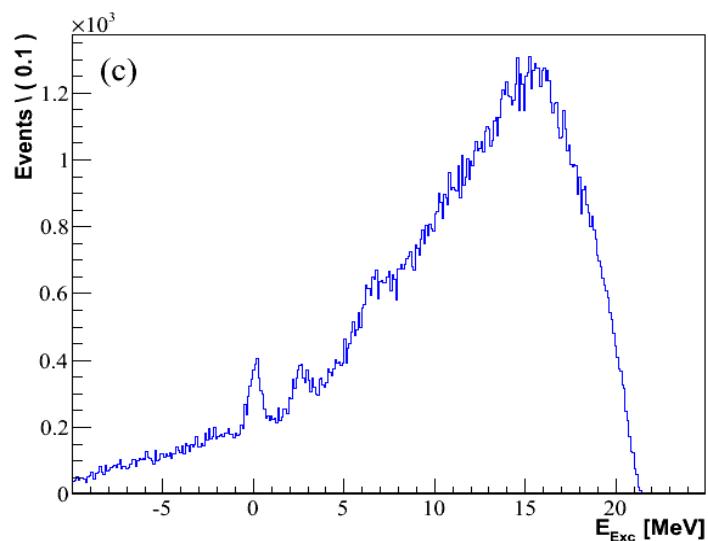
Excitation energy spectrum : gated



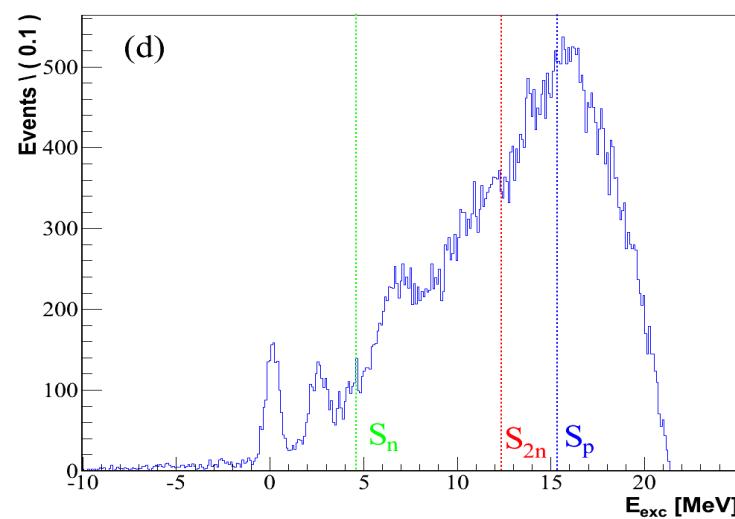
Target gate



Heavy residue gate

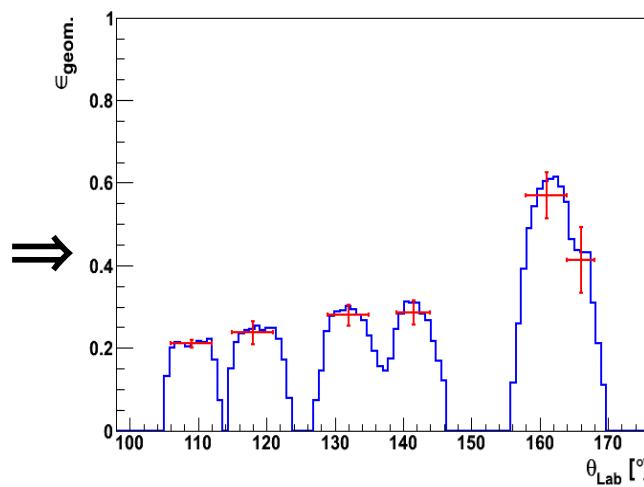
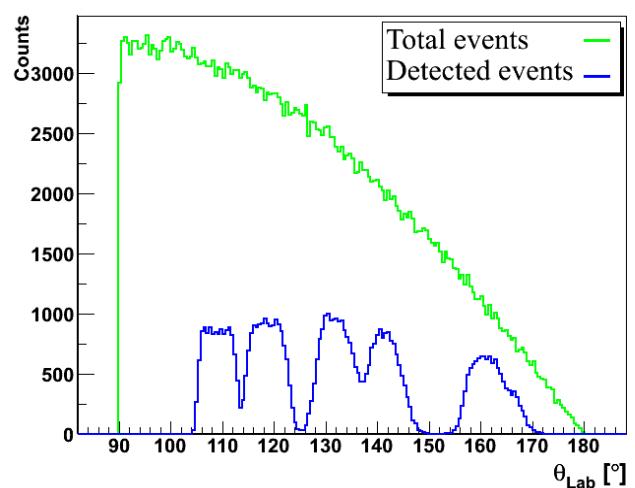


Proton gate



All gates

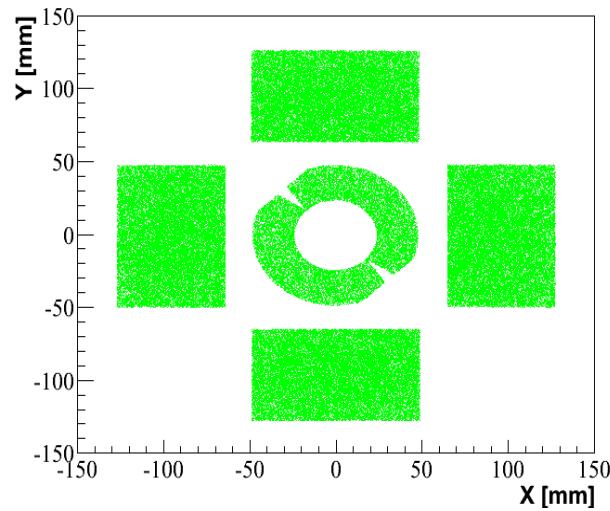
Angular efficiency



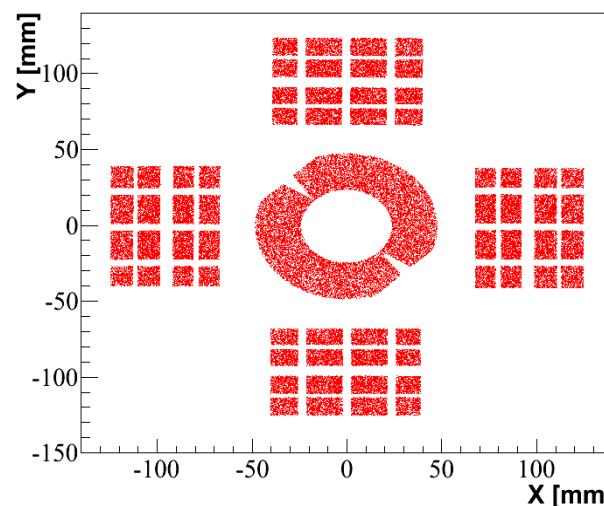
Angular efficiency :

- Detectors geometry
- Position
- Pad/strip state
- Analysis method

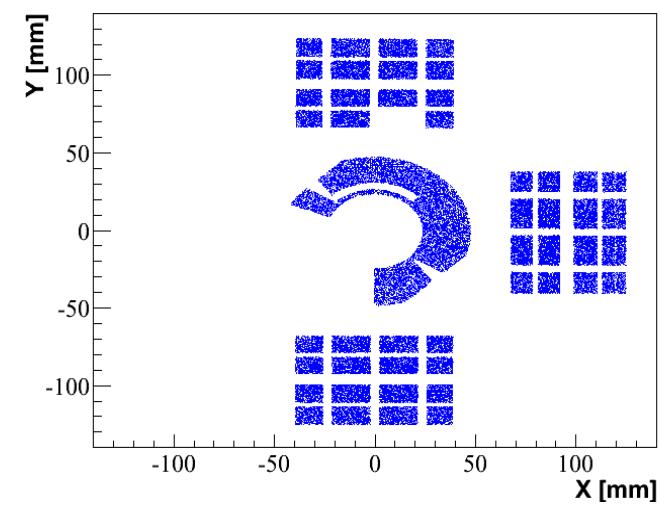
⇒ Simulation



Flawless detector



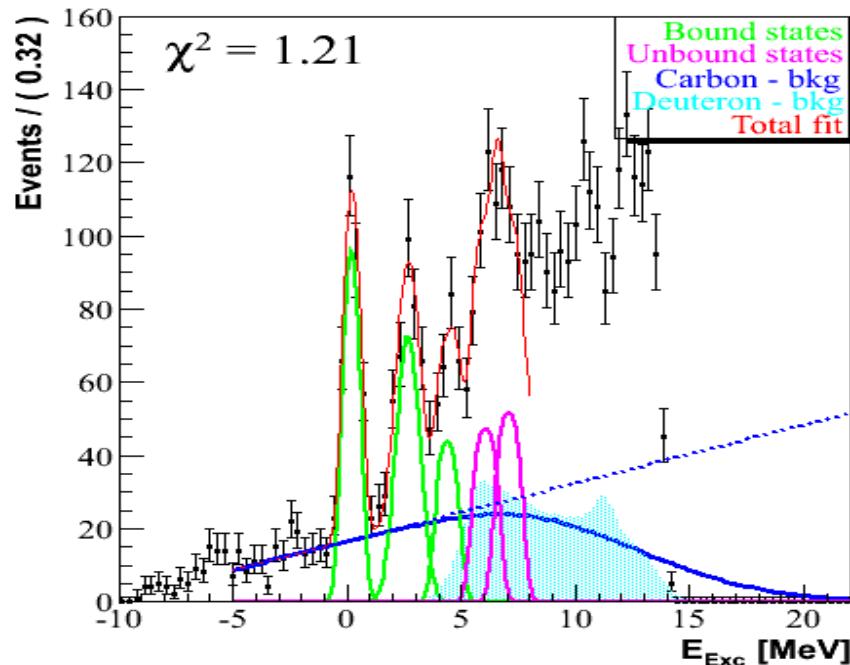
Flawless detector
+ matching



Flawless detector + matching
+ non-operational strip/pad

Excitation energy spectrum : fit

[156.0° - 170.0°]

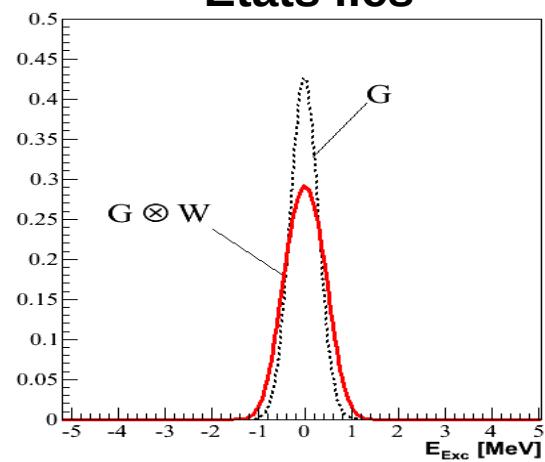


S1 référence en énergie :
[156°-170°]

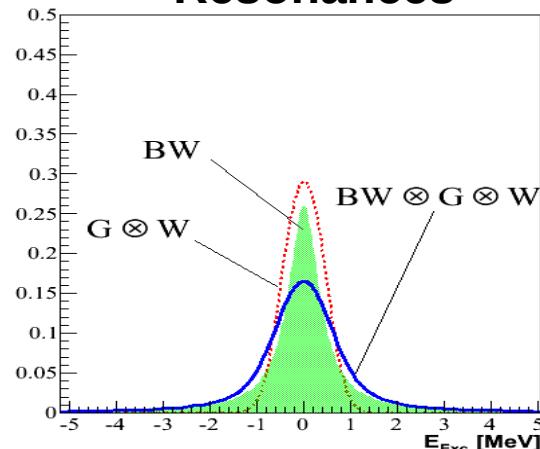
- Less dependant on the constructed angle of proton
- Transfer cross section is maximum

- 3 bound states
- 2 resonances above S_n

États liés

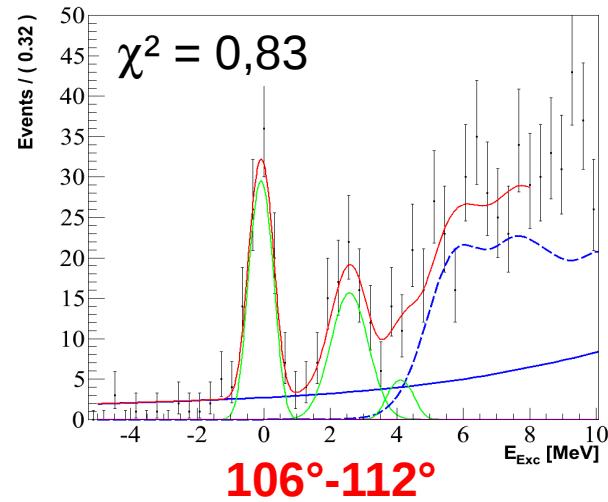


Résonances

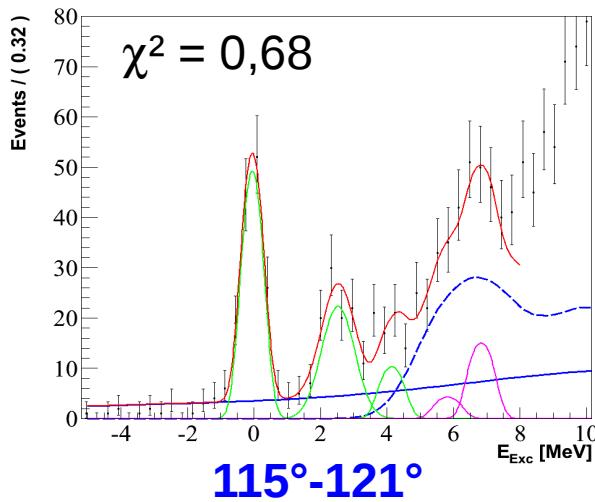


- C back-grd : $E_{\text{Exc}} < 0$
- Target thickness \Rightarrow window function $\equiv W$
- Bound states : $G \otimes W$
- Resonances : $BW \otimes G \otimes W$
- D back-grd : free param.

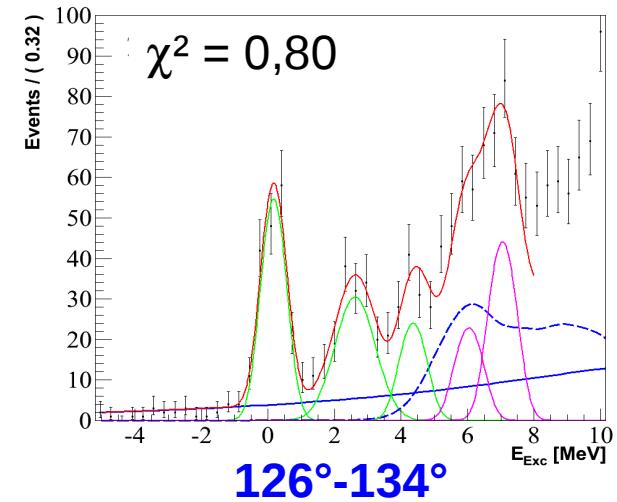
Excitation energy : spectra analysis



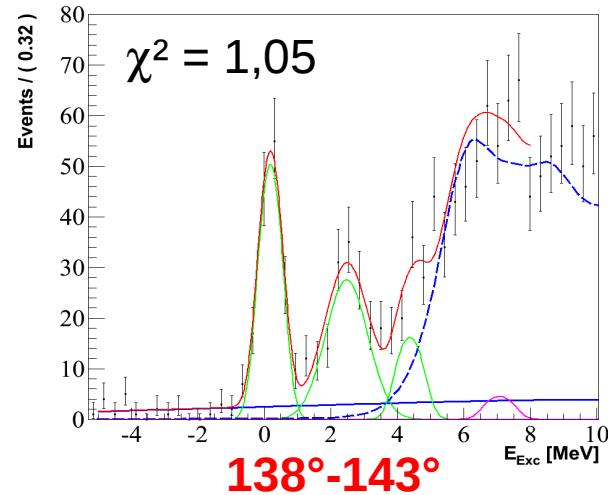
$106^\circ - 112^\circ$



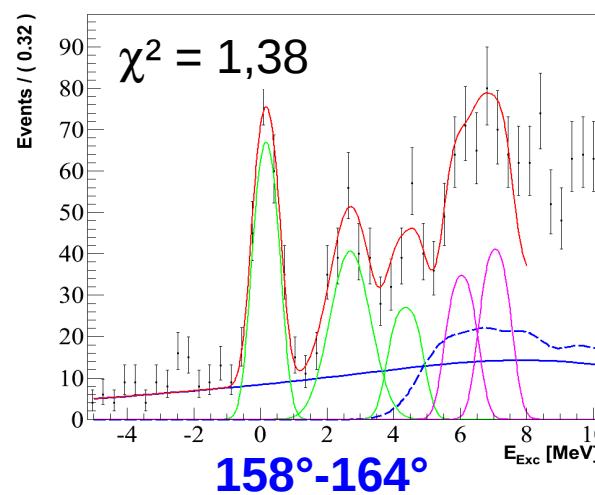
$115^\circ - 121^\circ$



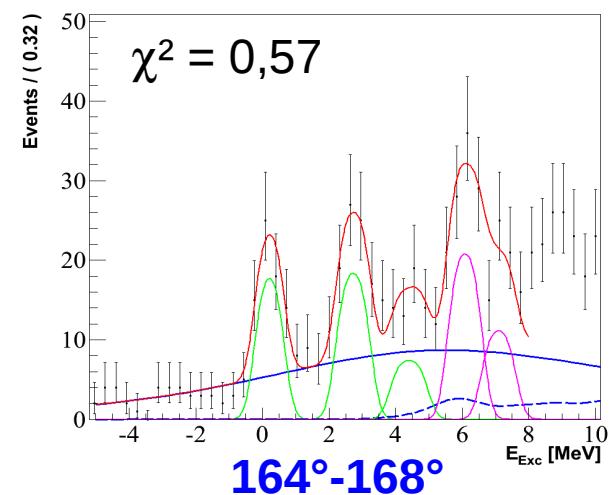
$126^\circ - 134^\circ$



$138^\circ - 143^\circ$

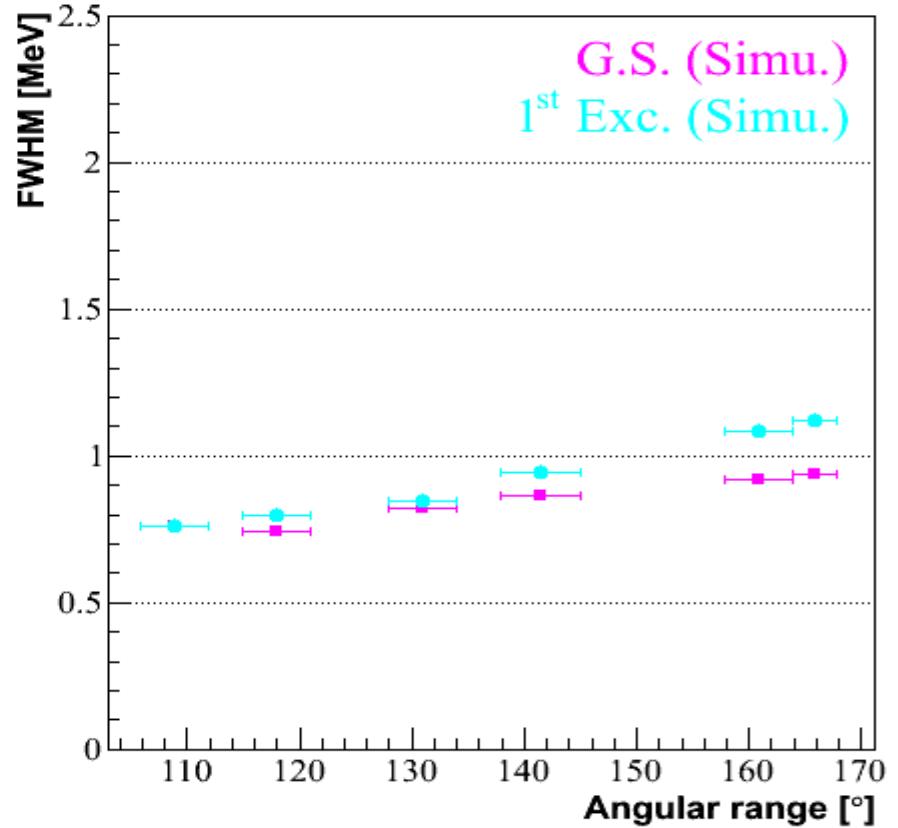
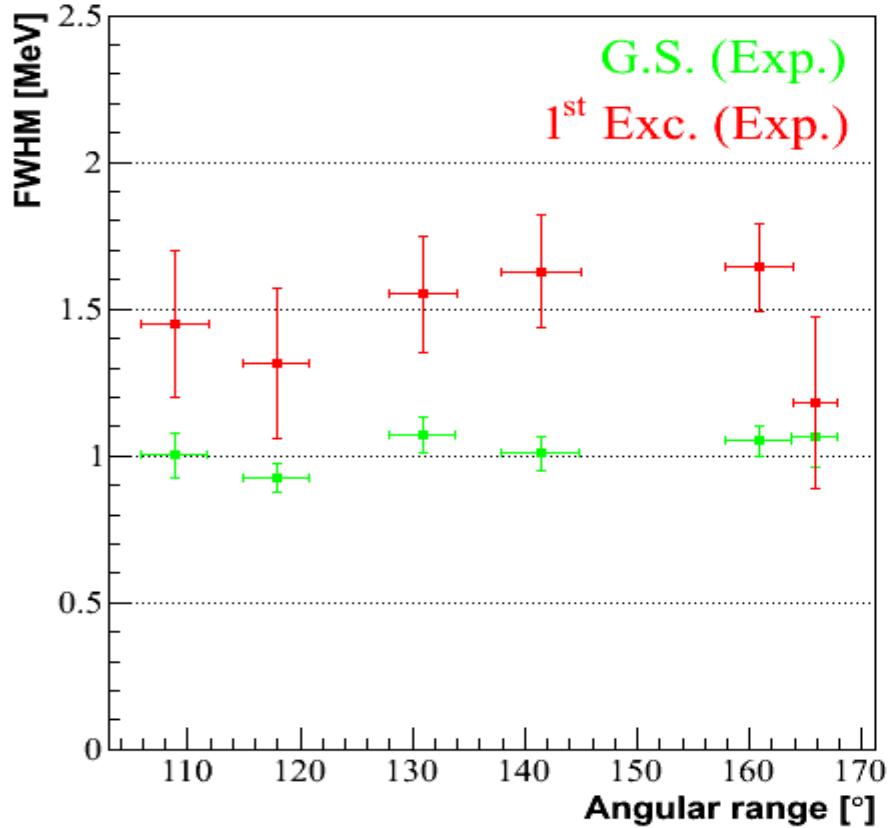


$158^\circ - 164^\circ$



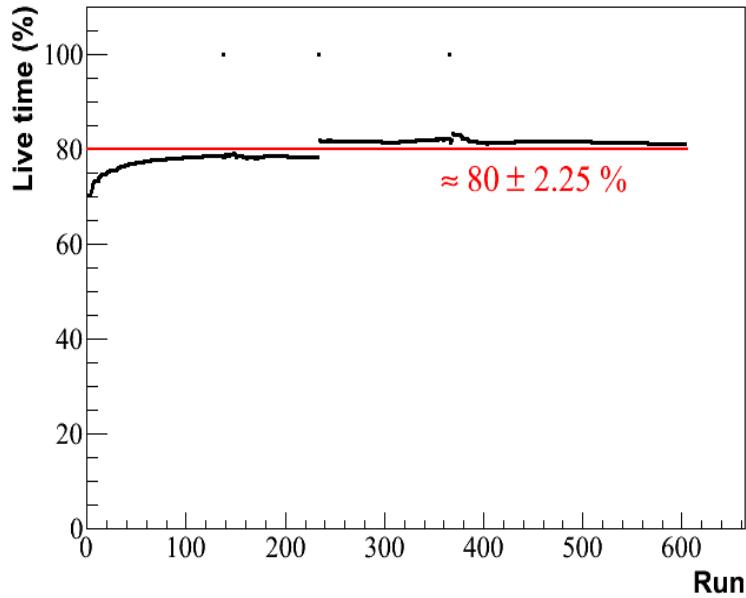
$164^\circ - 168^\circ$

First excited state at 2.47 MeV : doublet structure



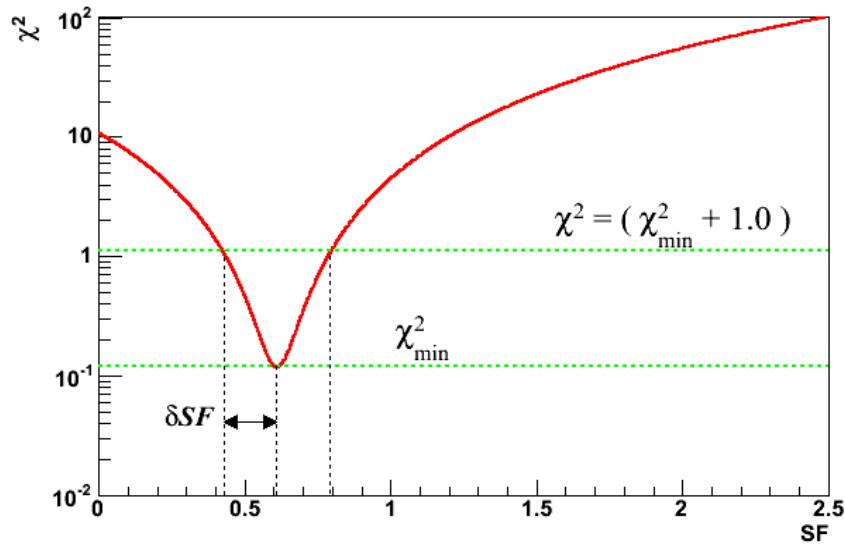
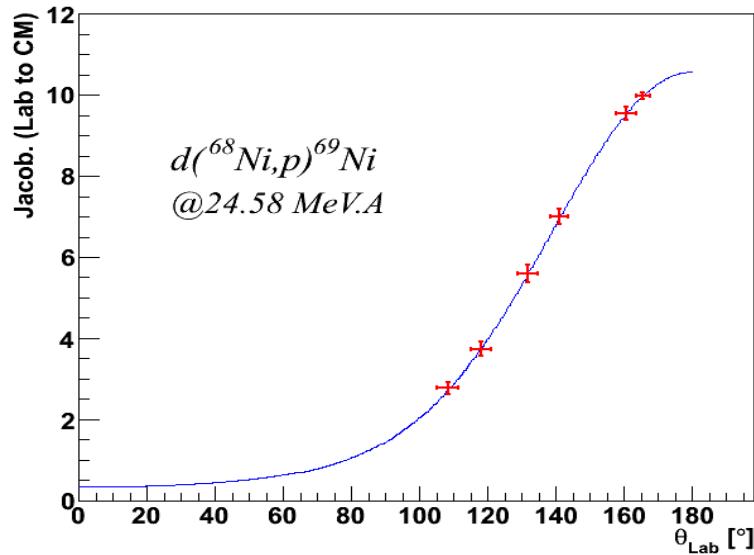
FWHM (1st excited) \sim 1.5 FWHM (G.S.) \Rightarrow doublet at 2.47 MeV

Differential cross sections

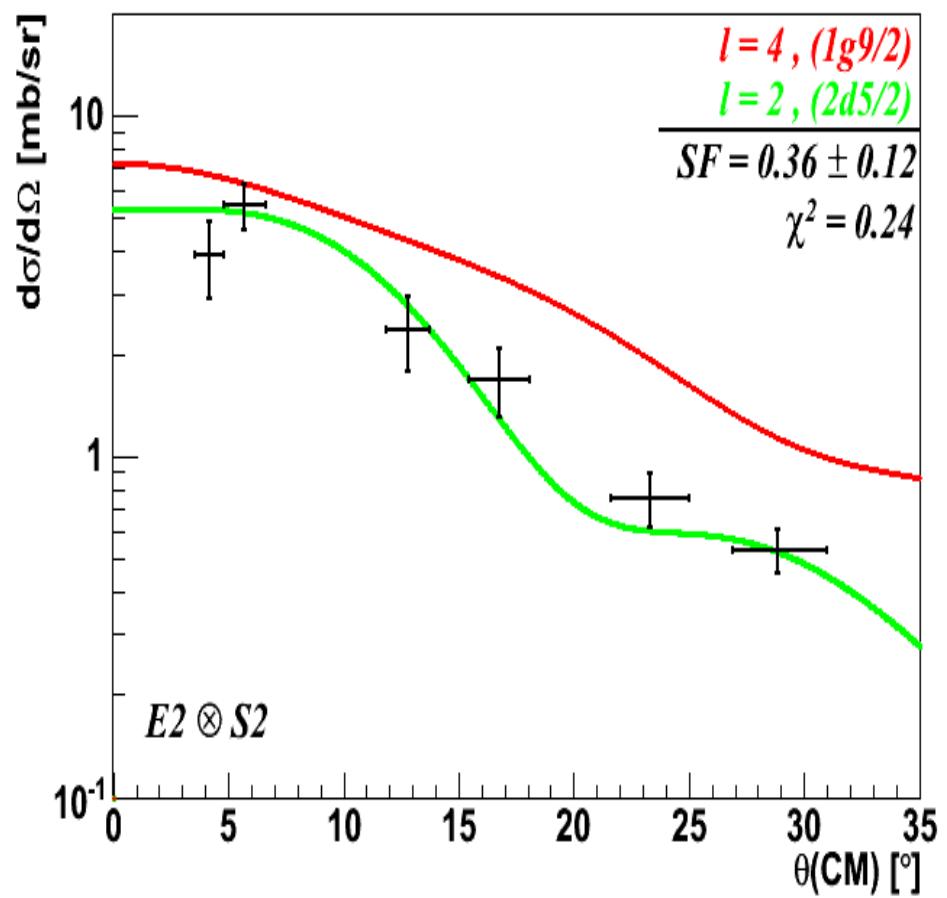
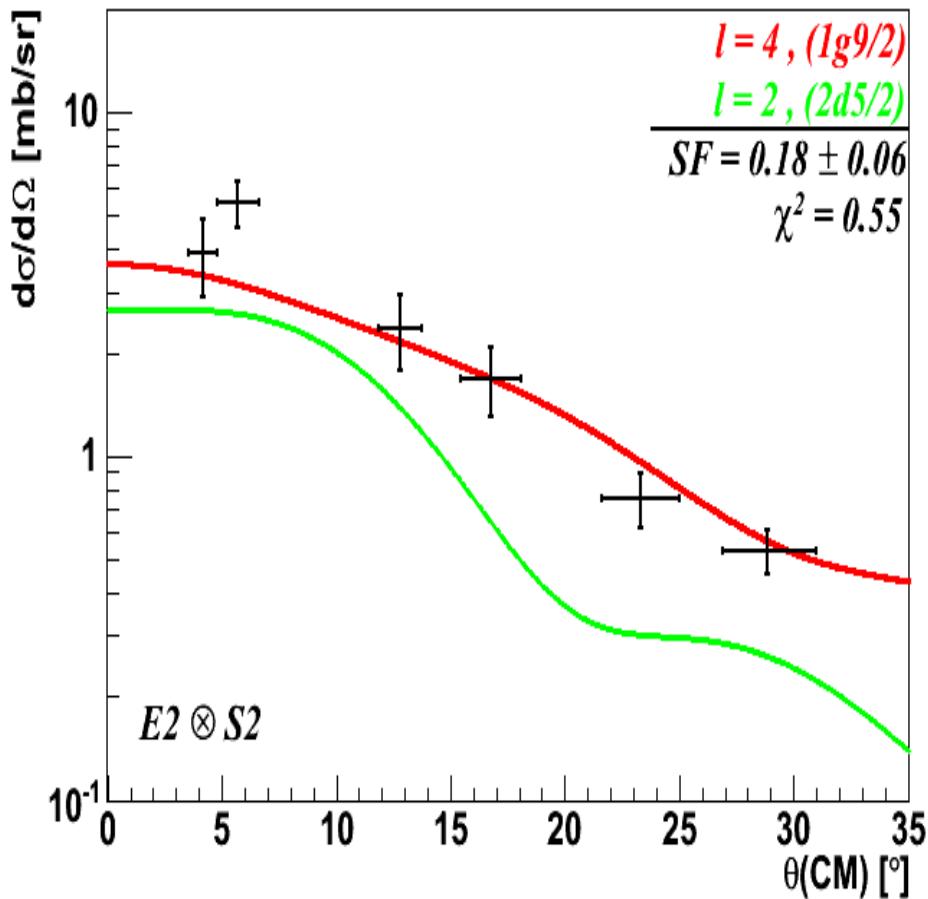


$$\frac{d\sigma}{d\Omega}(\theta_{Lab}) = \frac{N_{det}(\theta_{Lab})}{N_{faisceau}} \frac{(1+\epsilon_{temps mort})}{N_{Cible} \Delta\Omega(\theta_{Lab}) \epsilon_{MUST2,S1}}$$

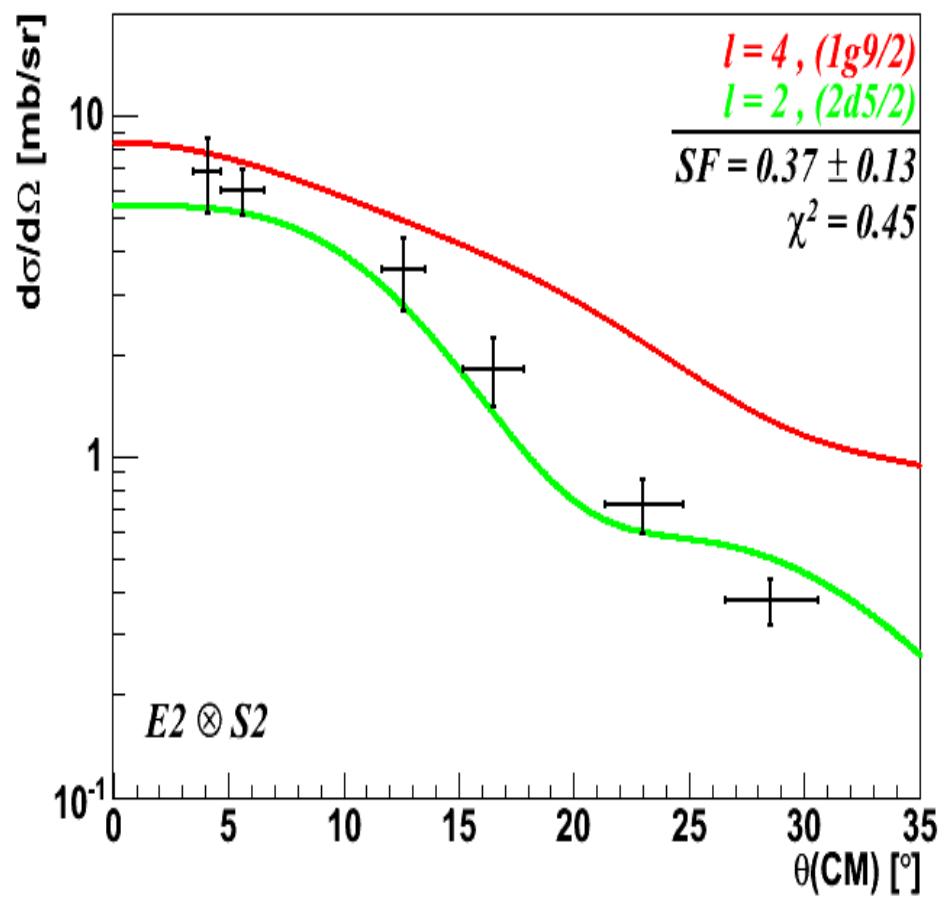
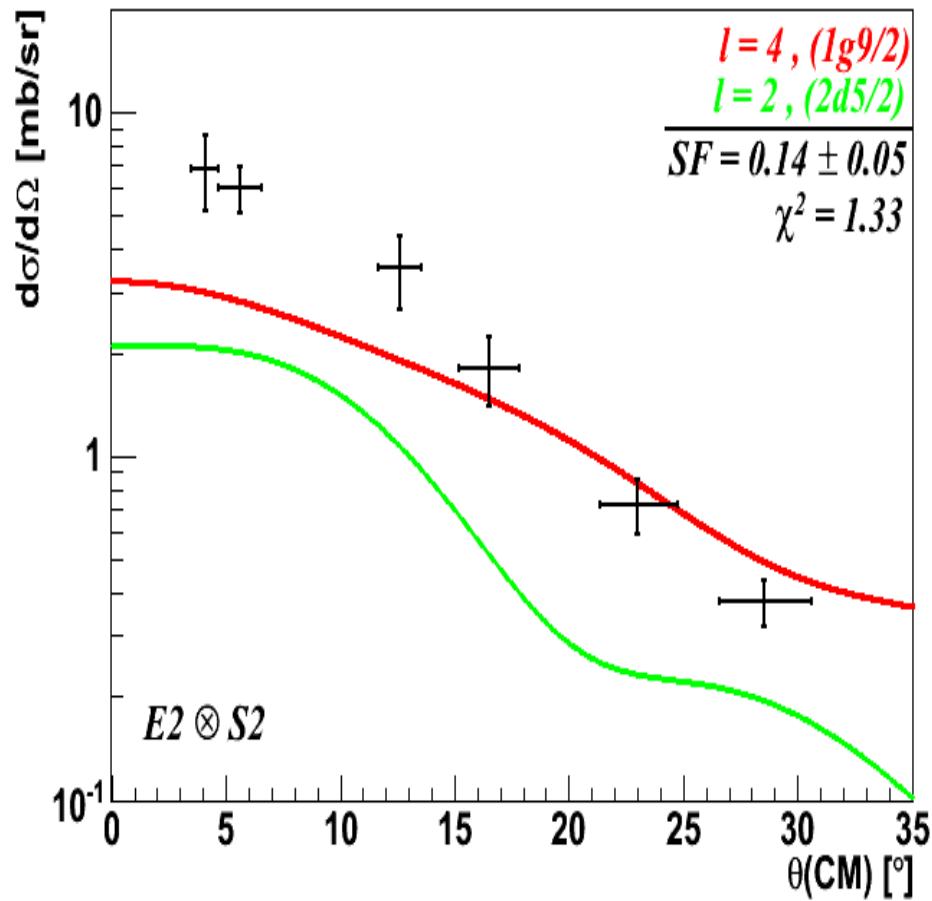
$$\frac{d\sigma}{d\Omega}(\theta_{CM}) = Jacob.(\theta_{Lab}) \frac{d\sigma}{d\Omega}(\theta_{Lab})$$



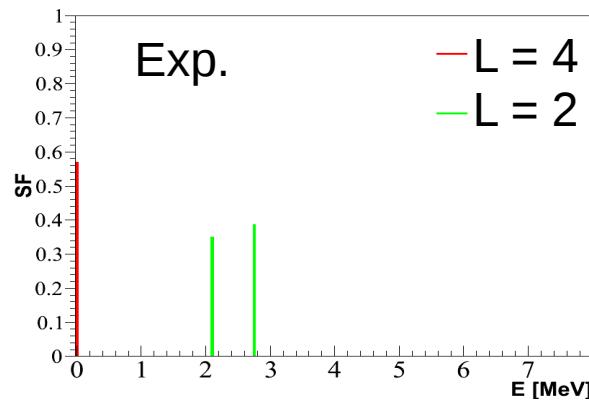
Diff. cross sections analysis : state @ 2.11 MeV



Diff. cross sections analysis : state @ 2.76 MeV



Experiment Vs Shell model calculations



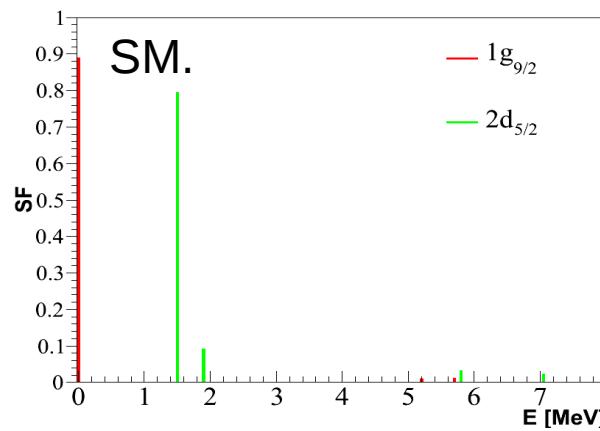
$$E_{\text{exc}}(\text{Exp.}) = 2.44 \text{ MeV}$$

$$\rightarrow E_{\text{exc}} = 2.11 \text{ MeV}$$

$$L = 2, \text{SF} = 0.36 \pm 0.13$$

$$\rightarrow E_{\text{exc}} = 2.76 \text{ MeV}$$

$$L = 2, \text{SF} = 0.38 \pm 0.14$$



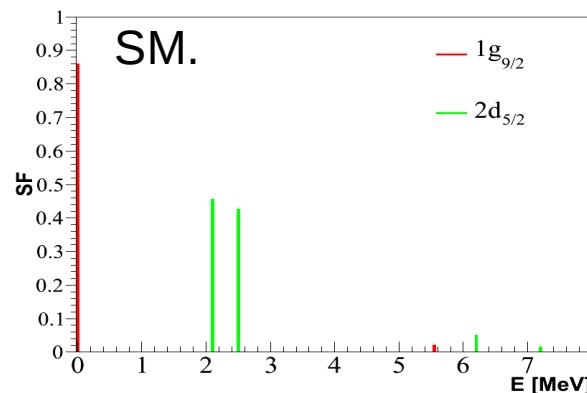
$$E_{\text{exc}}(\text{SM}) = 1.53 \text{ MeV}$$

$$\rightarrow E_{\text{exc}} = 1.49 \text{ MeV}$$

$$L = 2, \text{SF} = 0.79$$

$$\rightarrow E_{\text{exc}} = 1.92 \text{ MeV}$$

$$L = 2, \text{SF} = 0.09$$



$$E_{\text{exc}}(\text{SM}) = 2.30 \text{ MeV}$$

$$\rightarrow E_{\text{exc}} = 2.12 \text{ MeV}$$

$$L = 2, \text{SF} = 0.46$$

$$\rightarrow E_{\text{exc}} = 2.50 \text{ MeV}$$

$$L = 2, \text{SF} = 0.43$$