

TRIUMF

Canada's national laboratory for particle and nuclear physics
Laboratoire national canadien pour la recherche en physique nucléaire et en physique des particules

Ab initio calculations of light-ion reactions

Direct Reactions with Exotic Beams
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Petr Navratil | TRIUMF

Collaborators: Sofia Quaglioni (LLNL), Robert Roth (TU Darmstadt), W. Horiuchi (RIKEN), C. Romero-Redondo (TRIUMF), M. Kruse (UA), S. Baroni (ULB), J. Langhammer (TU Darmstadt), G. Hupin (LLNL)

Nuclear Landscape

Gamma rays

Reaction scheme: $^4\text{He} + ^4\text{He} \rightarrow ^7\text{Be} \rightarrow ^3\text{H} + ^4\text{He}$

Legend: Proton (red dot), Neutron (black dot), Gamma Ray (yellow wavy line).

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Light nuclei from first principles

- Goal:** Predictive theory of structure and reactions of light nuclei
- Needed for
 - Physics of **exotic nuclei**, tests of fundamental symmetries
 - Understanding of nuclear reactions important for **astrophysics**
 - Understanding of reactions important for **energy generation**
- From first principles or *ab initio*:
 - Nuclei as systems of nucleons interacting by nucleon-nucleon (and three-nucleon) forces that describe accurately nucleon-nucleon (and three-nucleon) systems

Diagram illustrating the fusion process: $^4\text{He} + ^4\text{He} \rightarrow ^7\text{Be} \rightarrow ^3\text{H} + ^4\text{He}$. Labels include: Alpha particle, Beta decay, Gamma ray, Tritium, Neutron, Proton.

Understanding our Sun

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No-core shell model combined with the resonating group method (NCSM/RGM)

- The NCSM:** An approach to the solution of the A -nucleon bound-state problem
 - Accurate nuclear Hamiltonian
 - Finite harmonic oscillator (HO) basis
 - Complete N_{max} model space
 - Effective interaction due to the model space truncation
 - Similarity-Renormalization-Group evolved NN+NNN potential
 - Short & medium range correlations
 - No continuum
- The RGM:** A microscopic approach to the A -nucleon scattering of clusters
 - Nuclear Hamiltonian may be simplistic
 - Cluster wave functions may be simplified and inconsistent with the nuclear Hamiltonian
 - Long range correlations, relative motion of clusters

Ab initio NCSM/RGM: Combines the best of both approaches
 Accurate nuclear Hamiltonian, consistent cluster wave functions
 Correct asymptotic expansion, Pauli principle and translational invariance

E. Jurgenson *et al.*, PRL 103, 082501 (2009)

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The *ab initio* NCSM/RGM in a snapshot

- Ansatz:** $\Psi^{(A)} = \sum_v \int d\vec{r} \varphi_v(\vec{r}) \hat{\mathcal{A}} \Phi_{v\vec{r}}^{(A-a,a)}$

$(A-a)$
 $\cdot \Psi_{1v}^{(A-a)} \Psi_{2v}^{(a)} \delta(\vec{r} - \vec{r}_{A-a,a})$

eigenstates of $H_{(A-a)}$ and $H_{(a)}$ in the *ab initio* NCSM basis

- Many-body Schrödinger equation:**

$H\Psi^{(A)} = E\Psi^{(A)}$

$T_{\text{rel}}(\vec{r}) + V_{\text{rel}} + \tilde{V}_{\text{Coul}}(\vec{r}) + H_{(A-a)} + H_{(a)}$

realistic nuclear Hamiltonian

$\sum_v \int d\vec{r} \left[\mathcal{H}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) - E \mathcal{N}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) \right] \varphi_v(\vec{r}) = 0$

Hamiltonian kernel

 $\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi_{v\vec{r}}^{(A-a,a)} \rangle$

$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}}^2 | \Phi_{v\vec{r}}^{(A-a,a)} \rangle$

Norm kernel

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Solving the RGM equations

- Input: Realistic nuclear Hamiltonian, eigenfunctions of nucleon clusters
 - Macroscopic degrees of freedom: nucleon clusters
 - Unknowns: relative wave function between the two clusters
- Non-local integral-differential coupled-channel equations:

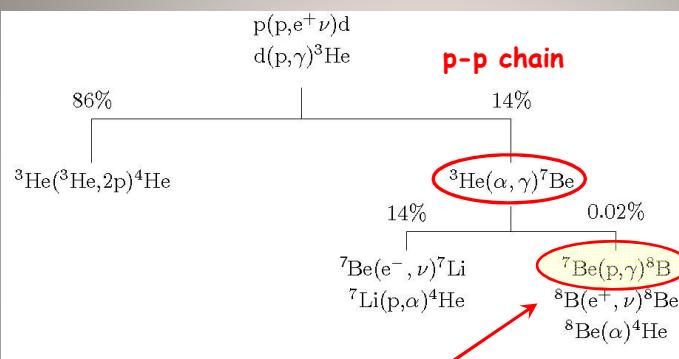
$$\left[T_{\text{rel}}(r) + V_C(r) + E_{\alpha_1}^{(A-a)} + E_{\alpha_2}^{(a)} \right] U_\nu^{(A-a,a)}(r) + \sum_{a' \nu'} \int dr' r' W_{a\nu, a'\nu'}(r, r') U_{\nu'}^{(A-a',a')}(r') = 0$$

- Solve with R-matrix theory on Lagrange mesh imposing
 - Bound state boundary conditions → eigenenergy + eigenfunction
 - Scattering state boundary conditions → Scattering matrix
 - Phase shifts
 - Cross sections
 - ...

The R-matrix theory on Lagrange mesh is an elegant and powerful technique, particularly for calculations with non-local potentials

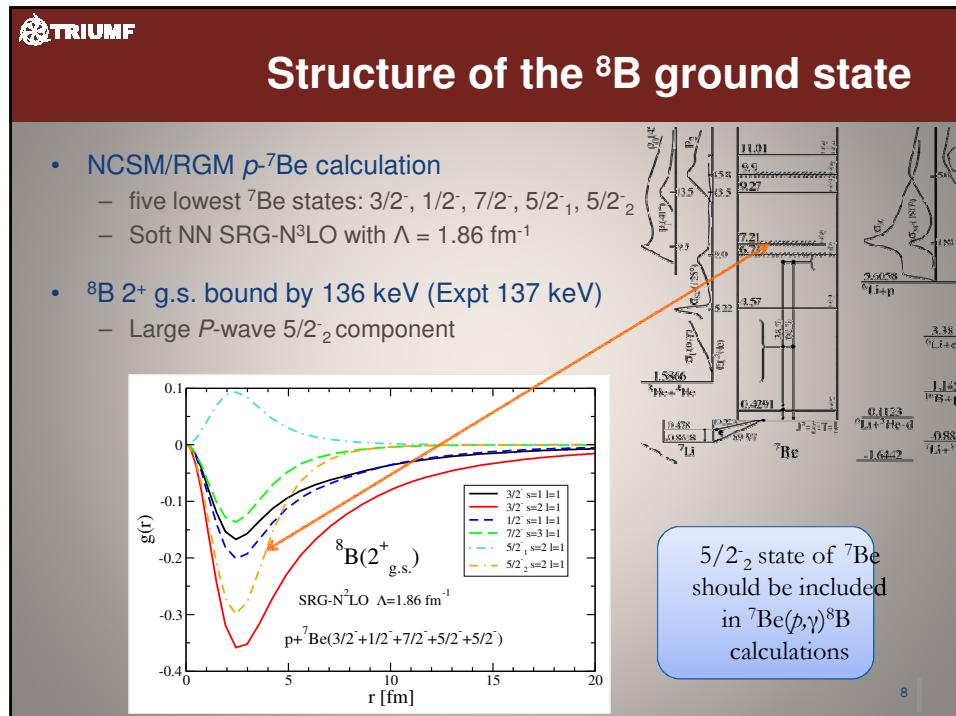
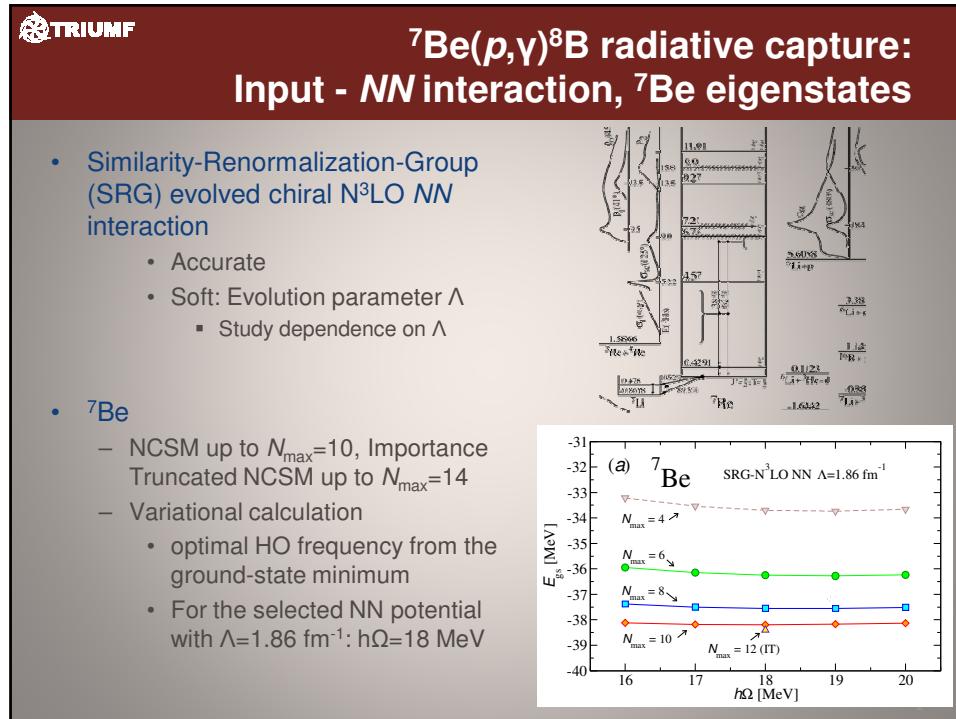


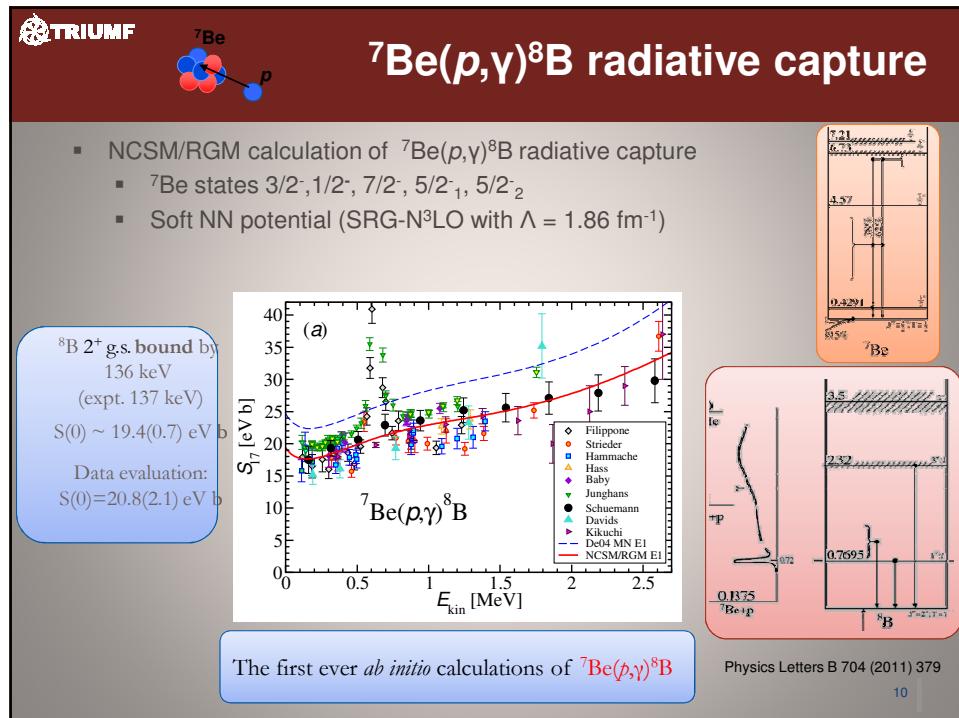
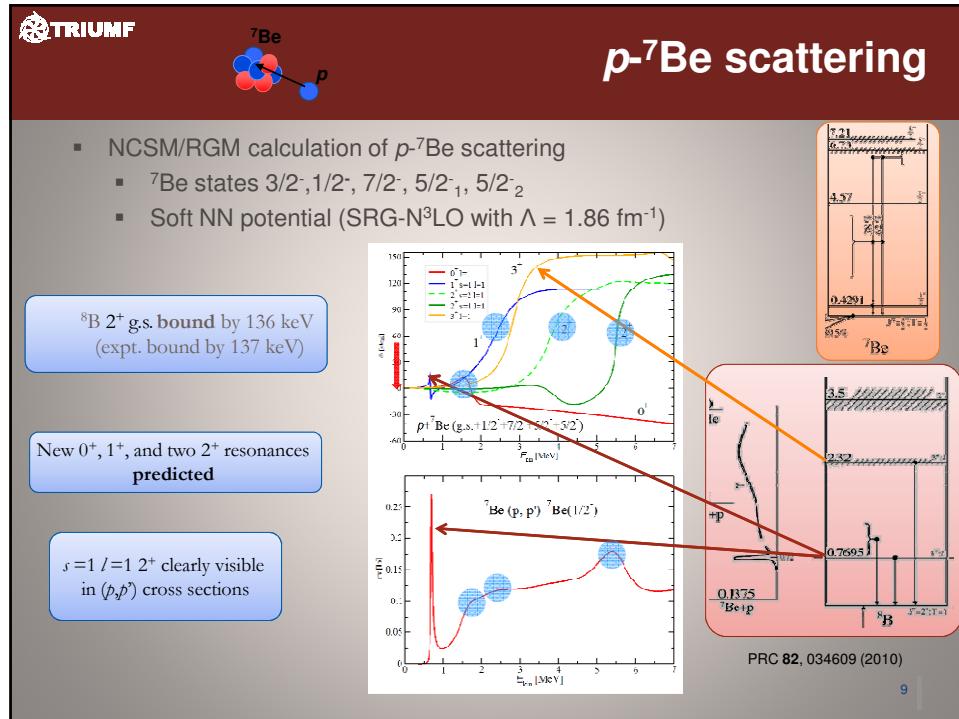
Solar $p-p$ chain



Solar neutrinos

$E_\nu < 15 \text{ MeV}$





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Structure of the unbound ${}^9\text{He}$ nucleus

- ${}^9\text{He}$ offers the opportunity to study the evolution of nuclear structure as a function of increasing number of neutrons
- Does the ground state of ${}^9\text{He}$ present the same parity inversion observed in the neighboring ${}^{11}\text{Be}$ and ${}^{10}\text{Li}$?
- Controversy on the nature of $S_{1/2}$ contribution to the ${}^9\text{He}$ spectrum

Experiments:
 $a_0 < -10 \text{ fm}$ (Chen et al.) [PLB 505, 2001]
 $a_0 \sim -3 \text{ fm}$ (Al Falou, et al.) [arXiv: 1008:0543]

- Here:
 - $n + {}^8\text{He}(\text{g.s.}, 2^+, 1^-)$, $N_{\max} = 13$
 - SRG-N³LO NN pot. ($\lambda=2.02 \text{ fm}^{-1}$)

g.s. parity inv. for exotic $N=7$ nuclei, well established in ${}^{11}\text{Be}$ and ${}^{10}\text{Li}$, disappears for ${}^9\text{He}$?
 More NCSM/RGM calculations under way...

$n\text{-}{}^8\text{He}$ scattering phase shifts

$n + {}^8\text{He}(\text{g.s.}, 2^+, 1^-)$

${}^2\text{P}_{1/2}$ (blue curve)
 ${}^2\text{S}_{1/2}$ (red curve)

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Ab initio calculation of the ${}^3\text{H}(d,n){}^4\text{He}$ fusion

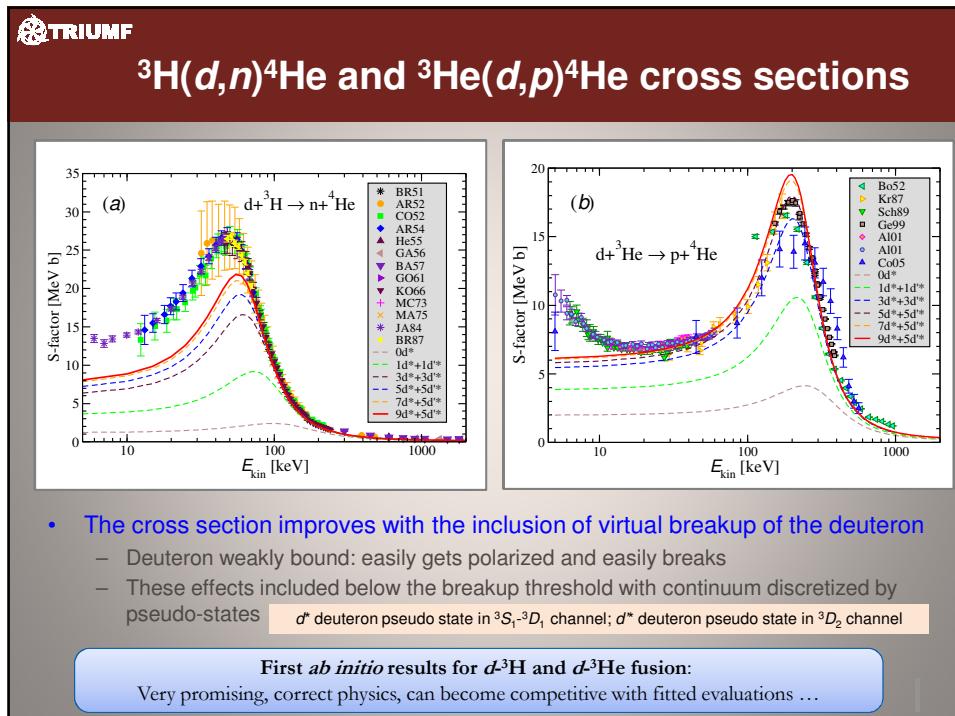
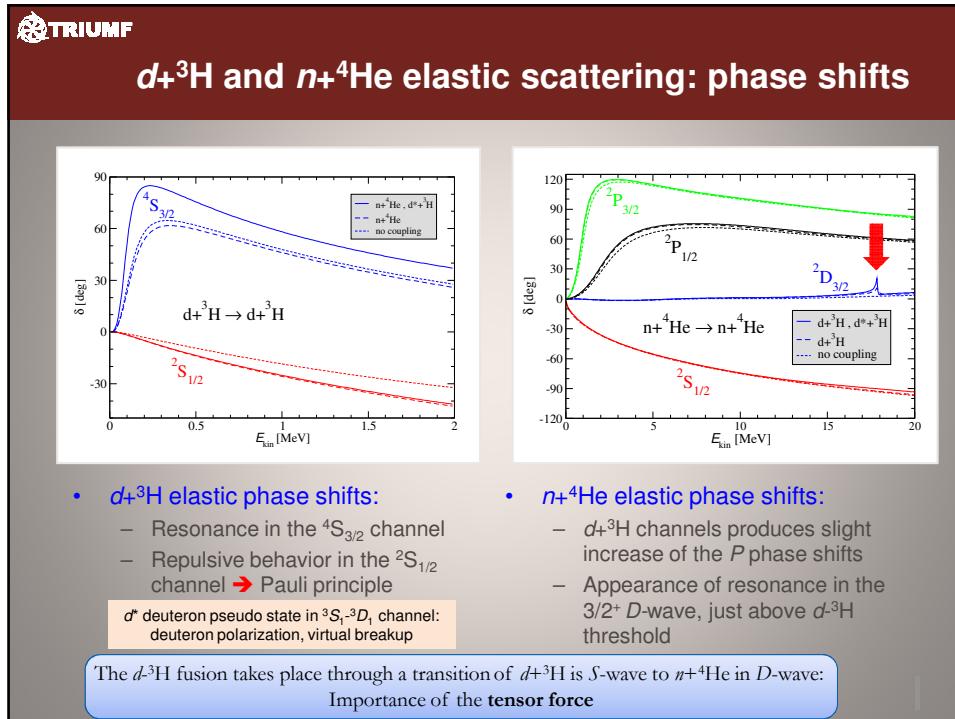
$$\int dr r^2 \left(\begin{array}{l} \left\langle \begin{array}{c} r' \\ n \end{array} \alpha \left| \hat{A}_1(H-E)\hat{A}_1 \right| \begin{array}{c} r \\ n \end{array} \alpha \right\rangle \quad \left\langle \begin{array}{c} r' \\ n \end{array} \alpha \left| \hat{A}_1(H-E)\hat{A}_2 \right| \begin{array}{c} r \\ {}^3\text{H} \end{array} d \right\rangle \\ \left\langle \begin{array}{c} r' \\ d \end{array} {}^3\text{H} \left| \hat{A}_2(H-E)\hat{A}_1 \right| \begin{array}{c} r \\ n \end{array} \alpha \right\rangle \quad \left\langle \begin{array}{c} r' \\ d \end{array} {}^3\text{H} \left| \hat{A}_2(H-E)\hat{A}_2 \right| \begin{array}{c} r \\ d \end{array} \right\rangle \end{array} \right) \left(\begin{array}{l} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{array} \right) = 0$$

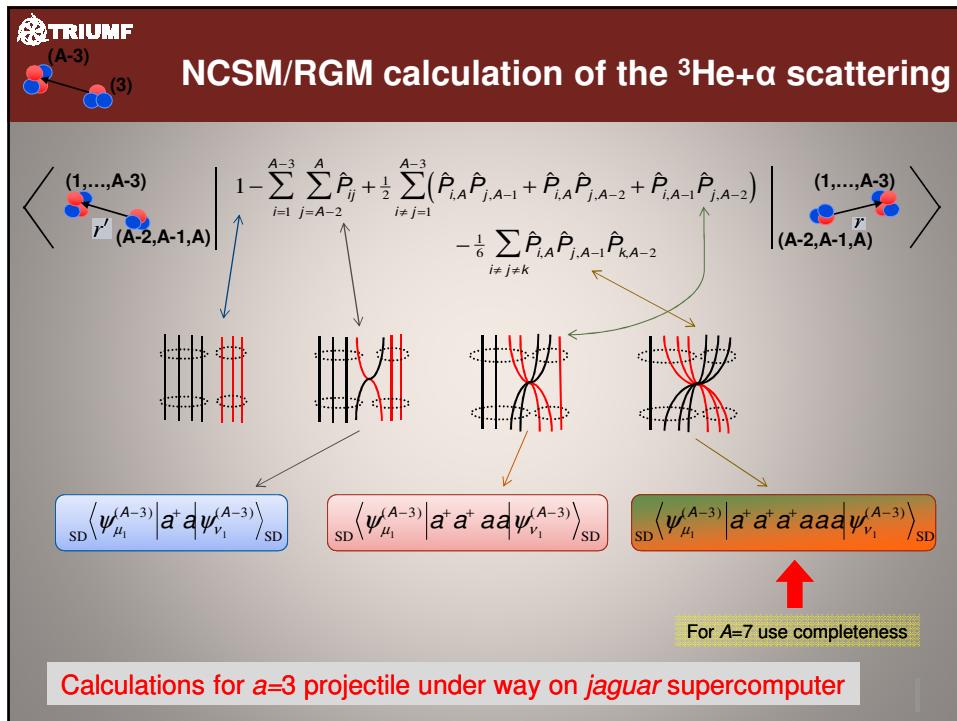
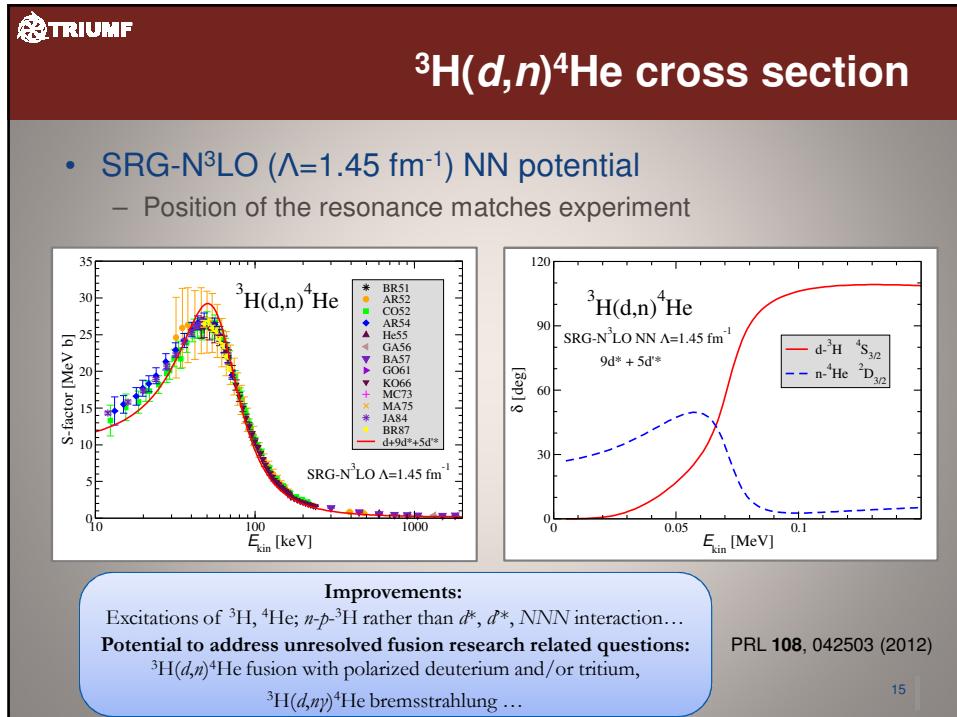
Bringing Star Power to Earth
NIF

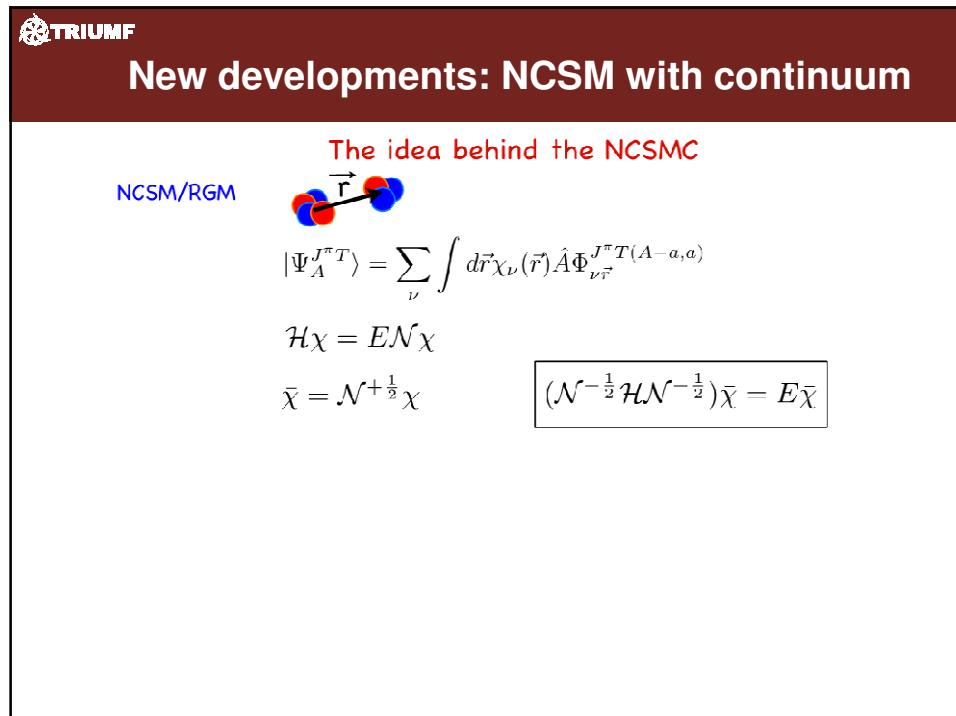
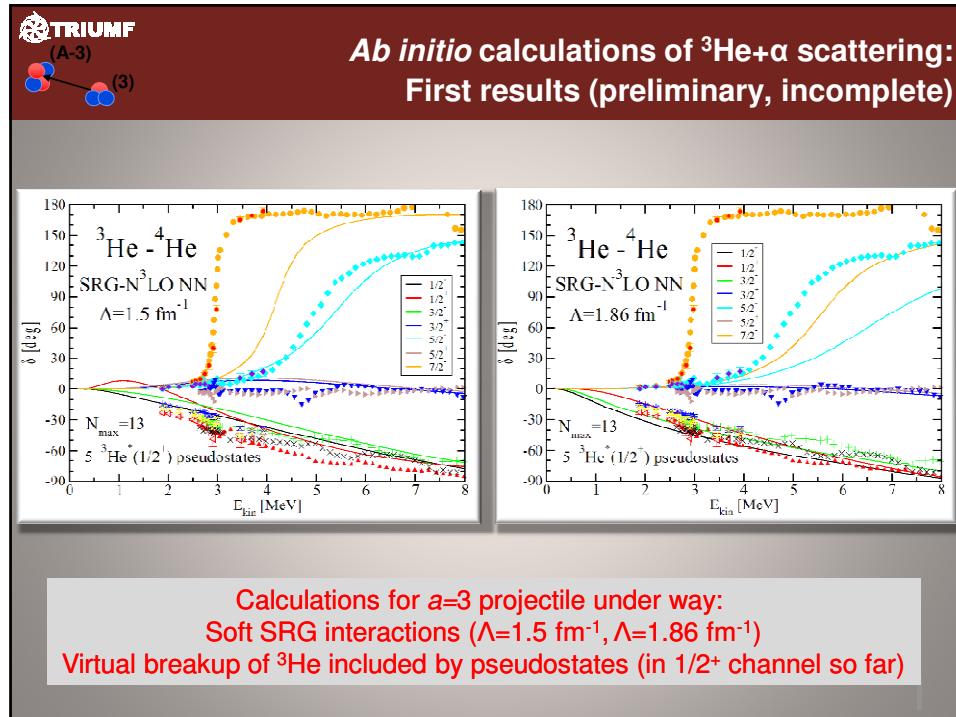
LLNL Open House 2009

ITER

energy generation







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New developments: NCSM with continuum

The idea behind the NCSMC

NCSM/RGM

$$|\Psi_A^{J^\pi T}\rangle = \sum_\nu \int d\vec{r} \chi_\nu(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{\frac{1}{2}}\chi$$

$$(\mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}})\bar{\chi} = E\bar{\chi}$$

NCSMC

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda |A\lambda J^\pi T\rangle + \sum_\nu \int d\vec{r}' \left(\sum_{\nu'} \int d\vec{r}'' \mathcal{N}_{\nu\nu'}^{-\frac{1}{2}}(\vec{r}, \vec{r}') \bar{\chi}_{\nu'}(\vec{r}'') \right) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \mathcal{N}^{-\frac{1}{2}}\mathcal{H}\mathcal{N}^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix}$$

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NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He} + n$

NCSM/RGM with up to three ${}^6\text{He}$ states

Expt.

Preliminary

NCSMC with up to three ${}^6\text{He}$ states and three ${}^7\text{He}$ eigenstates
More 7-nucleon correlations
Fewer target states needed

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