

Fingerprints of core polarization in two-nucleon transfer reactions in halo nuclei

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	(Foldless Srl)

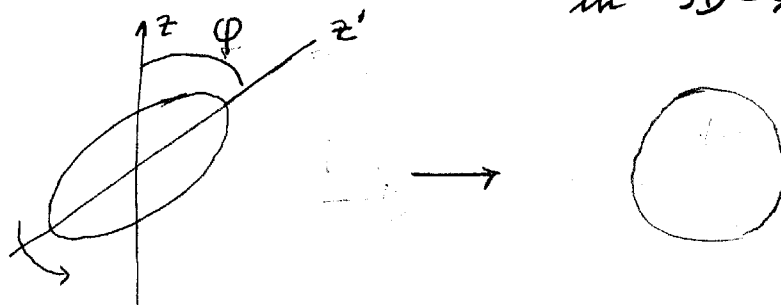
Spontaneous symmetry breaking of ① rotational invariance

$$H'_{3D} = H_{3D} - \vec{\omega}_{rot} \cdot \vec{I}$$

$$H_{3D} = H_{sp} - K \vec{Q} \cdot \vec{Q}$$

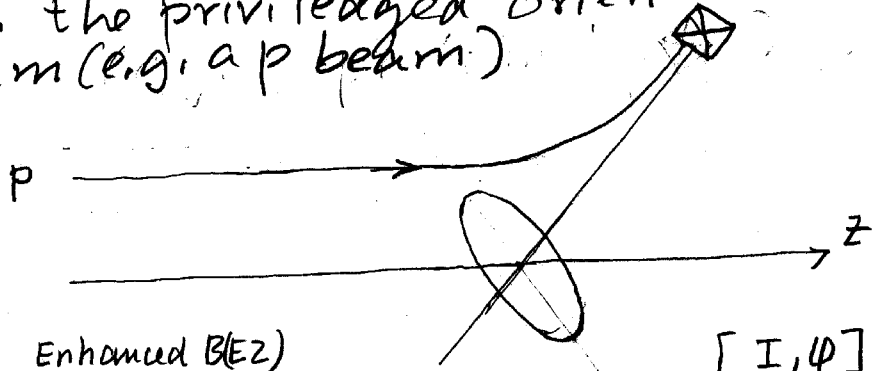
$$\langle N | Q | N \rangle = e^{i\varphi} Q_0$$

(perfect phase coherence in 3D-space)

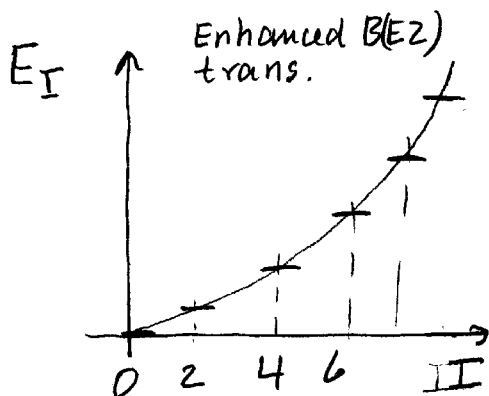


Fluctuations diverge Symm. restoration

Think of a quadrupole vibration of freq. $\omega = (C/D)^{1/2}$, $C \rightarrow 0$, D finite ($\neq 0$). To "pin down" quantal fluc., and thus to measure deformation one has to use an external field which itself violates the given symmetry (rot. inv. in the present case). Easy; essentially all instruments in the lab do so, e.g. the privileged orientation defined by a beam (e.g. a p beam).



Fingerprint of deformation in finite many-body systems: rotational bands



$$[I, \varphi] = -i$$

$$\omega_{rot} = -\frac{1}{\hbar} \frac{\partial H}{\partial I}$$

$$E_I = \frac{\hbar^2}{2J} I(I+1) + \omega_{rot} I$$

Goldstone mode (AGN).

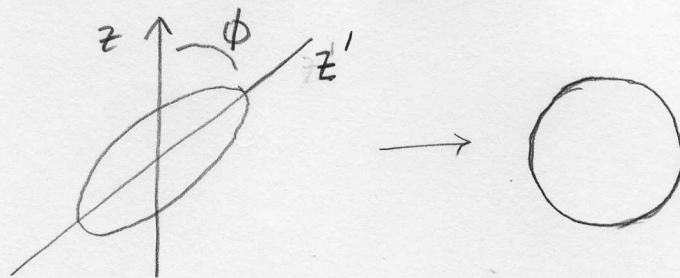
Spontaneous symmetry breaking of gauge invariance

(2)

$$H_G = H_G - \lambda N$$

$$H_G = H_{sp} - G P^\dagger P \quad ; \quad P^\dagger = \sum_{\nu > 0} a_\nu^\dagger a_\nu^\dagger$$

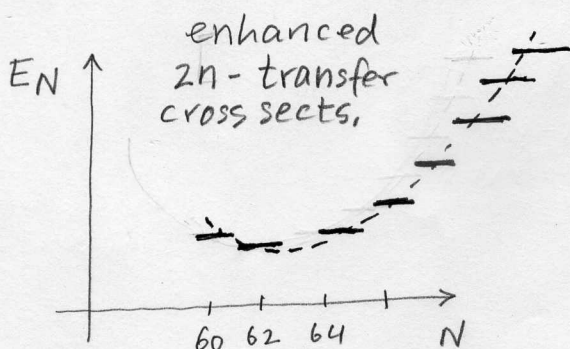
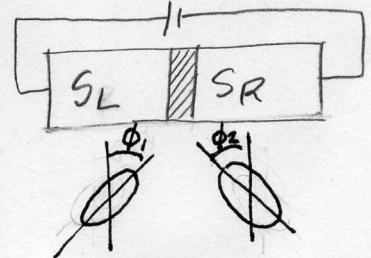
$$\langle BCS | P^\dagger | BCS \rangle = e^{i\phi} \alpha'_0 \quad ; \quad \alpha'_0 = \sum_{\nu > 0} U_\nu V_\nu$$



N, ϕ

One does not go around with objects which have perfect phase coherence in gauge space, i.e. which themselves violate gauge invariance.

This was the importance of the Josephson effect. It provided for the first time an instrument which can act like a clamp for a solid; it can pin down the gauge angle (second, measurement of electron-phonon interaction within 10% error quantitative era superconductivity)



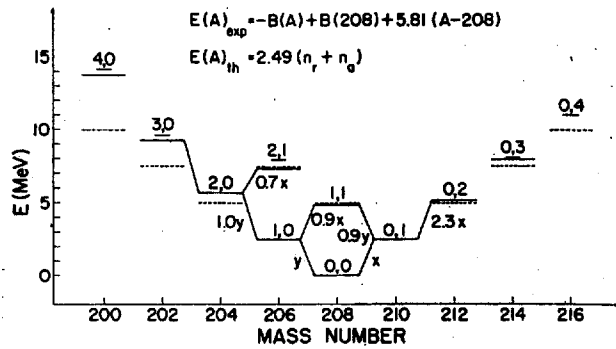
$$[N, \phi] = -i$$

$$\dot{\phi} = -\frac{1}{\hbar} \frac{\partial H}{\partial N} = \frac{\lambda}{\hbar}$$

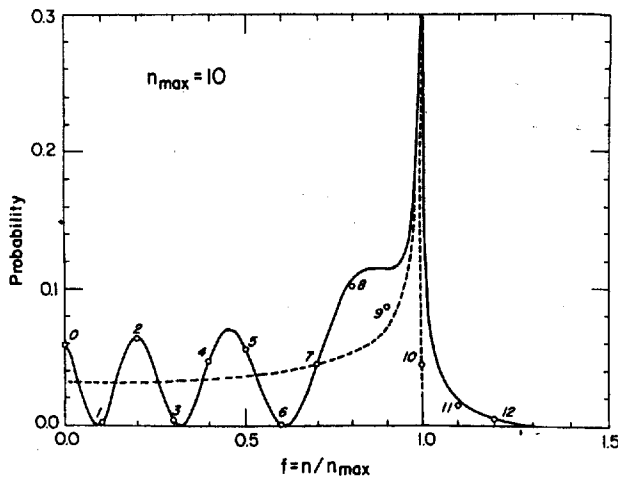
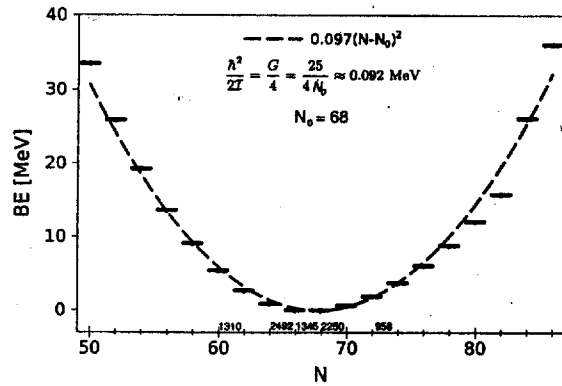
$$E_N = \frac{\hbar^2}{2J} N^2 + \lambda N$$

Goldstone mode (AGN)

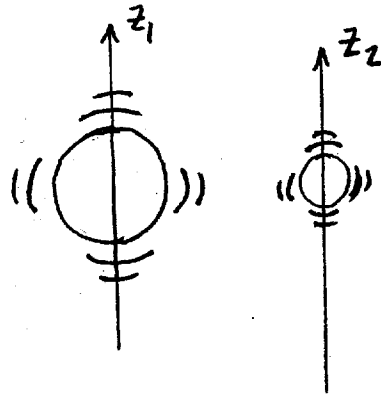
nuclei



$$\xi \approx 36 \text{ fm}$$

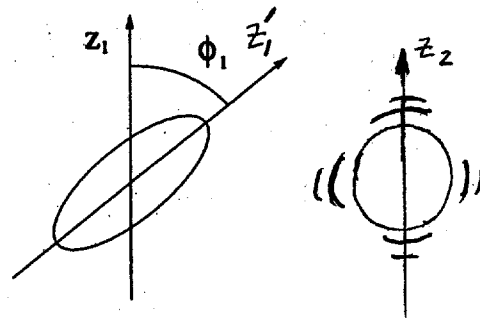


Gauge Space



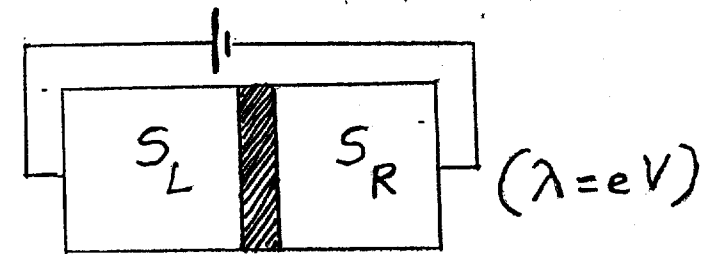
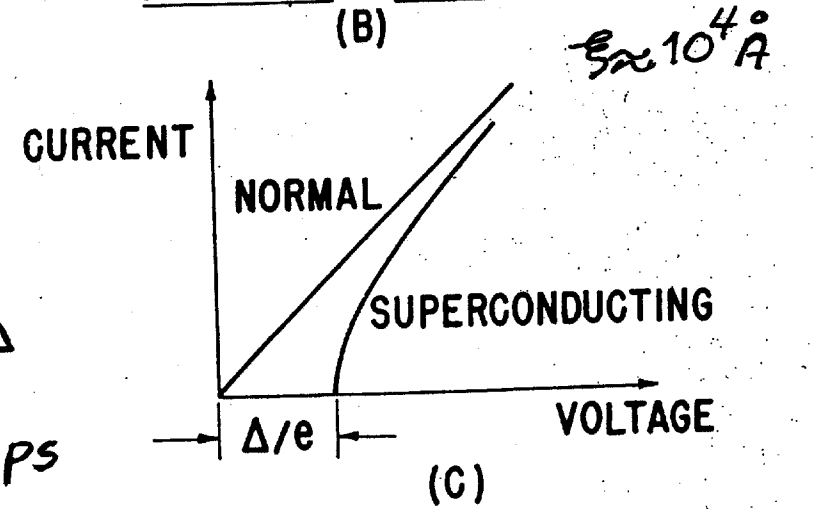
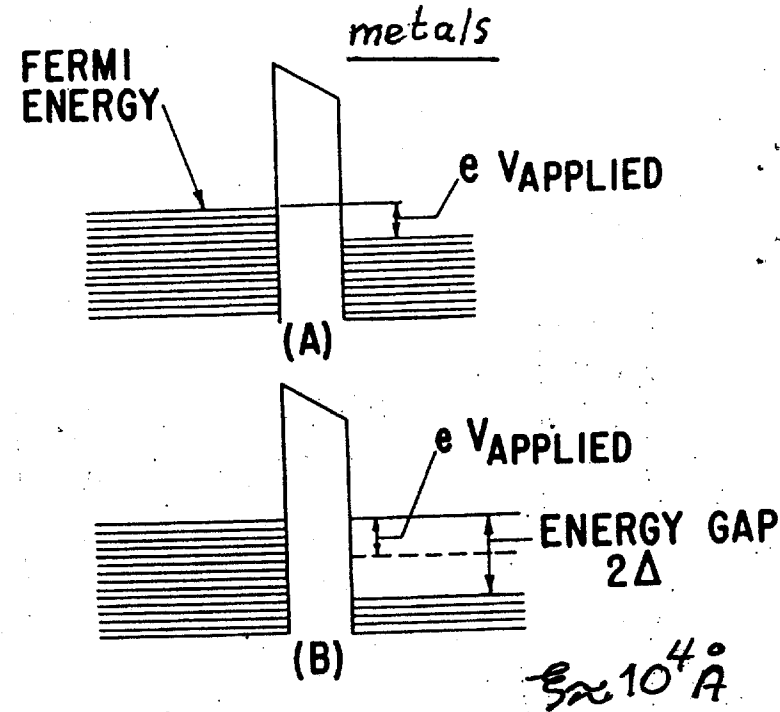
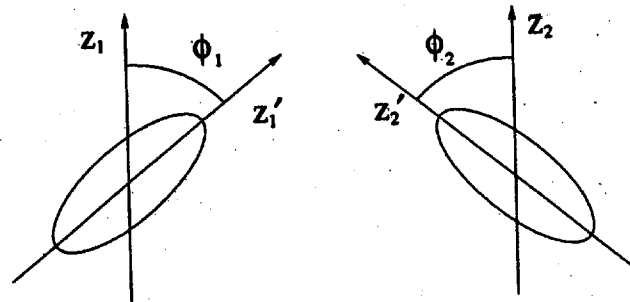
$$E = E_{\text{corr}}$$

$$\xi = \frac{\hbar v_F}{2E}$$



$$E = 2\Delta$$

succ. + sim. + north.
Cohen-Falicov-Phillips



$$N \sim \sin\left(\frac{2}{\hbar}(\lambda_R - \lambda_L)t + 2\delta\right) \quad (\omega)$$

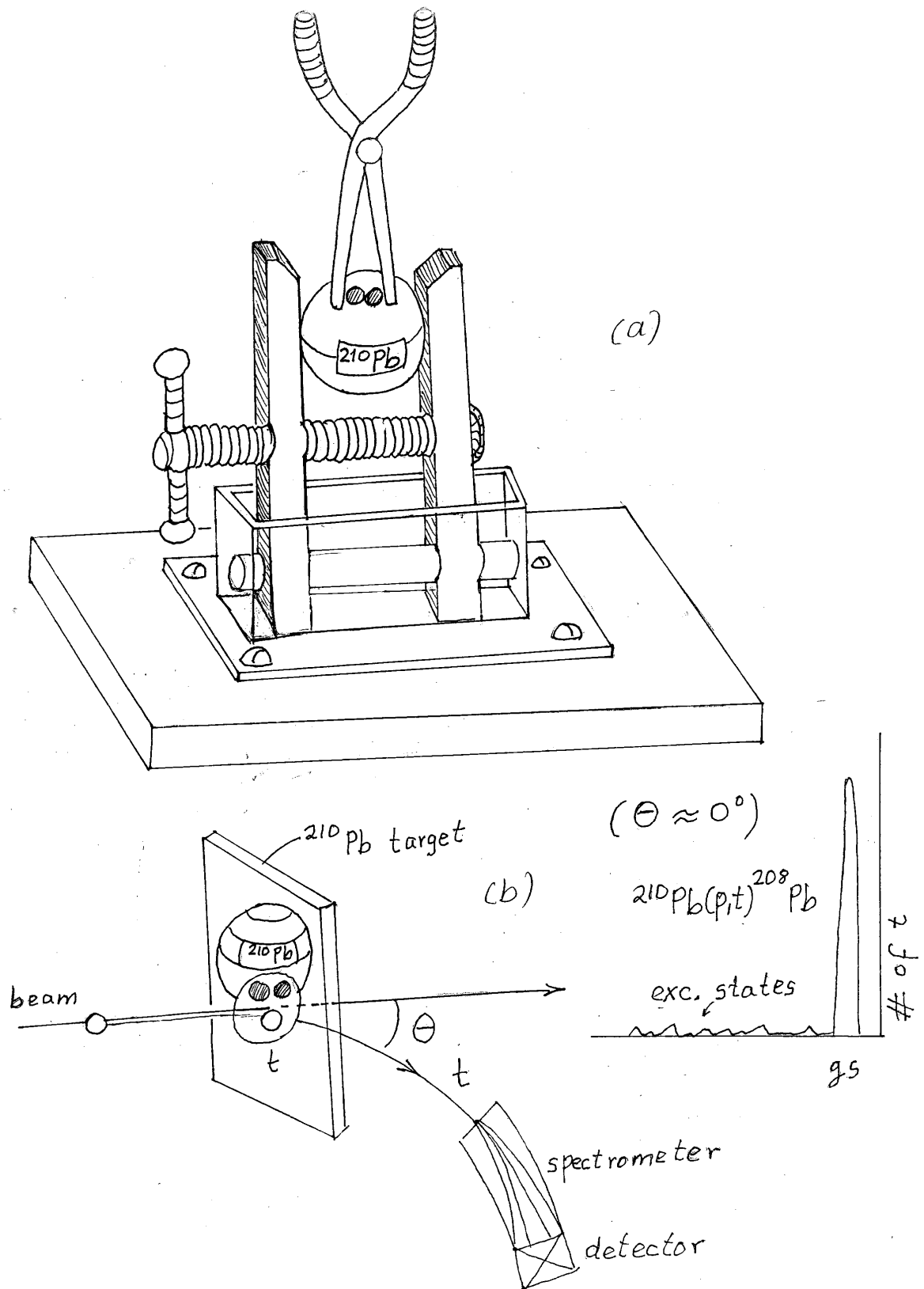


Figure 2.

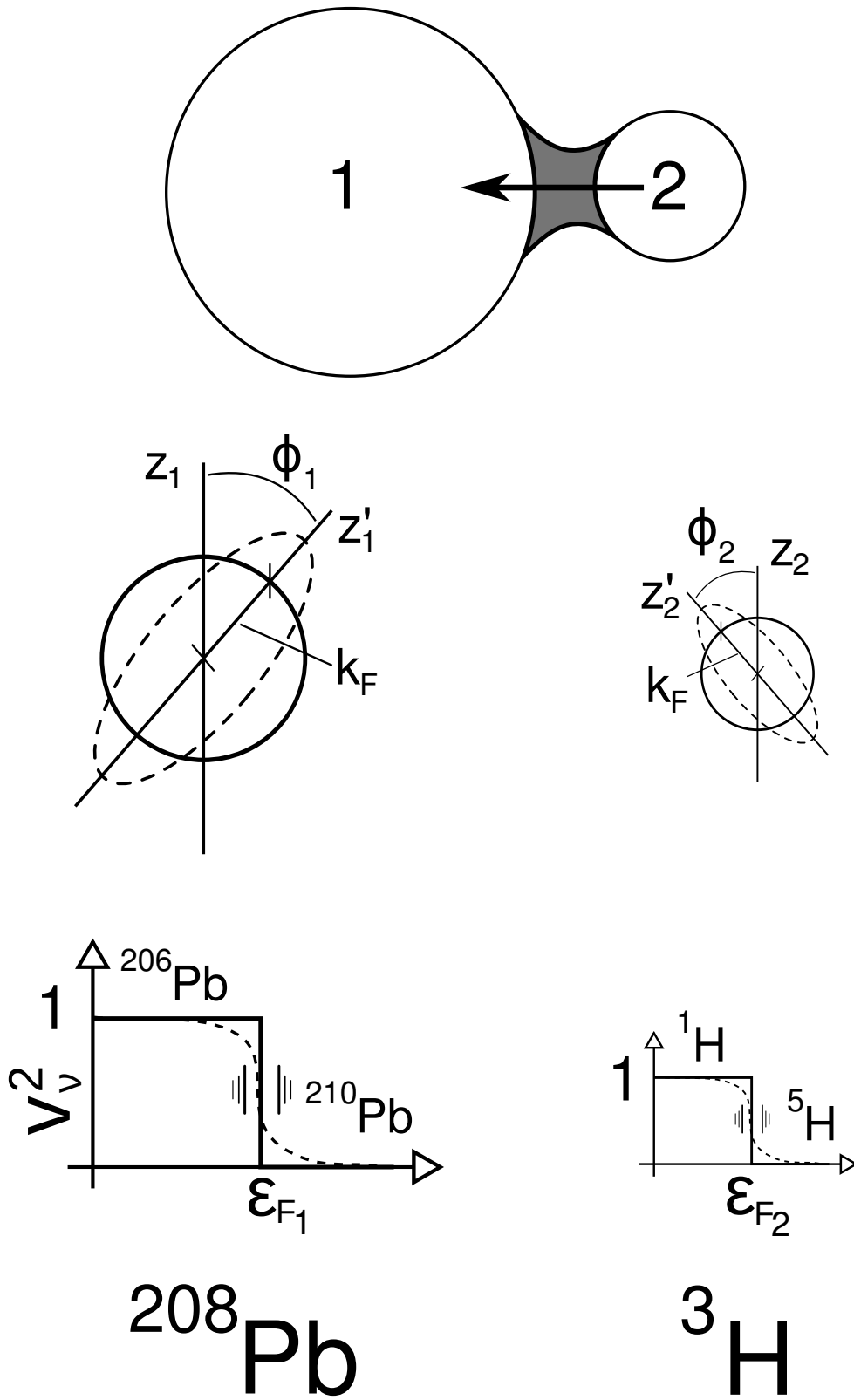


Figure 3.

Two-particle transfer
reactions: specific probe
of pairing in nuclei*)

$$\alpha_0 = \langle P^+ \rangle = \sum_{\nu > 0} U_\nu V_\nu$$

(not a gap)

*) renowned experts of high international standing...

ask Josephson and Phil Anderson, let alone Stockholm

Do we know how to
calculate it?

sub
corresponding to the
no. we know from the

reception?
two-box to transfer

Quantum Mechanics

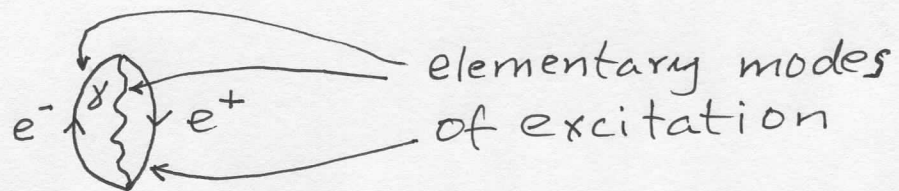
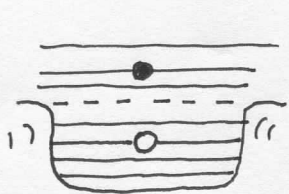
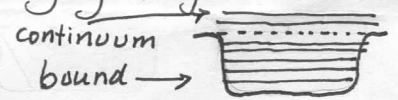
Dirac equation

$$(\mathbf{c} \boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta m c^2) \Psi = i \hbar \frac{\partial \Psi}{\partial t}$$

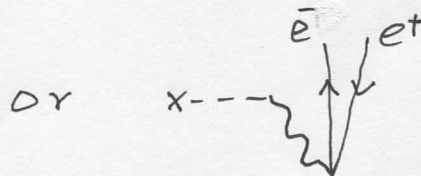
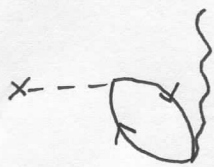
both positive and negative energy solutions.
paradox: any electron placed in a positive-energy eigenstate would decay into negative-energy eigenstates by emitting excess energy in the form of photons.

solution: conjecture (known as hole theory) that the vacuum is the many-particle quantum state in which all the negative-energy electron eigenstates are occupied. Pauli principle forbids positive-energy electrons from decaying into negative-energy eigenstates. (remember also Cooper model).

On the other hand, quantum mechanics allows for virtual processes



which can become real through the action of an external field, e.g.



pair prod.

describe explicitly only the active fermionic and bosonic degrees of freedom and their interweaving (very economic) QED, Feynman diagr.

Nuclear Structure } deeply interweaved*)
 Nuclear Reactions } aspects of the same physics

{ bound
 { continuum

their (bound, continuum) melting become essentially complete in the case of weakly bound halo nuclei, e.g. ^{11}Li or the first excited 0^+ state of ^{12}Be ($E = 2.24 \text{ MeV}$).

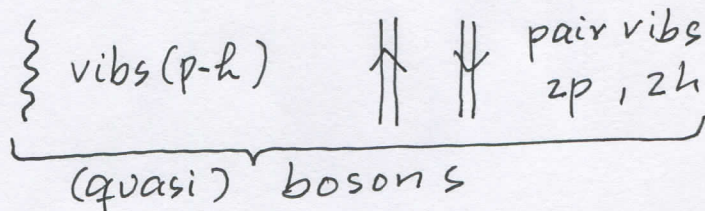
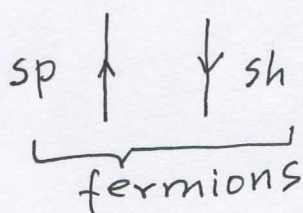
NFT (tailored after QED)

structure: Bes, Dussel, Broglia, Liotta, Mottelson PL 52B (1974)
 reactions: Broglia, Winther Heavy Ion Reactions 253
 2nd ed., Westview Press, Boulder (2004)

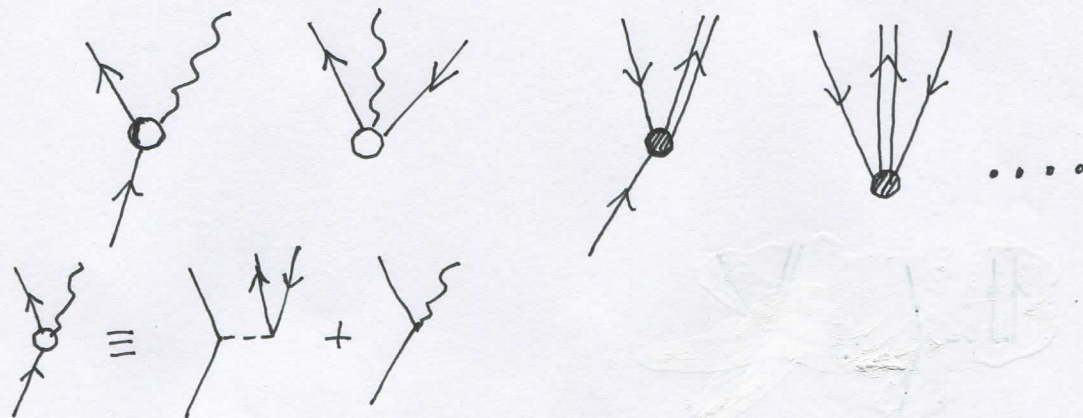
(4+1) - rules
 exact

implementation: approximations

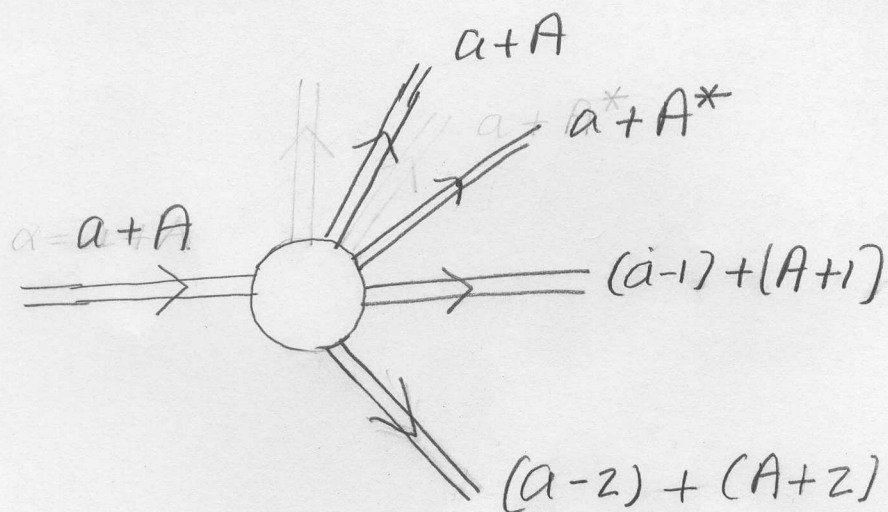
elementary modes of nuclear excitation



Overcomplete, non-orthogonal, Pauli violating



*) Reason why, strictly speaking, one cannot define a (single-particle) spectroscopic factor, let alone a two-particle spectroscopic factor: superposition, coherence, amplitudes
summed amplitudes modulus squared



Only way to compare with data
is through the calculation of
the absolute cross section.

Example

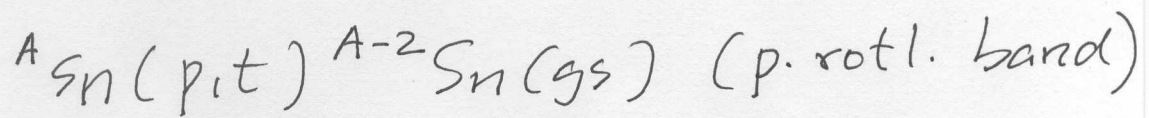
$$\frac{d\sigma(p + {}^{208}\text{Pb} \rightarrow p + {}^{208}\text{Pb}^*(3-))}{d\sigma} = \beta_3^2 \frac{d\sigma(\text{elastic})}{d\Omega}$$

only approximat

$\delta\rho$: transition density

Rules (4+1)

Implementation : approximations



$$B_v \sim U_v V_v$$

essentially exact

coherent state

optical potentials

Phys. Rev. Lett. 107, 092501 (2011) [5 pages]

Calculation of the Transition from Pairing Vibrational to Pairing Rotational Regimes between Magic Nuclei ^{100}Sn and ^{132}Sn via Two-Nucleon Transfer Reactions

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Absolute values of two-particle transfer cross sections along the Sn-isotopic chain are calculated. They agree with measurements within errors and without free parameters. Within this scenario, the predictions concerning the absolute value of the two-particle transfer cross sections associated with the excitation of the pairing vibrational spectrum expected around the recently discovered closed shell nucleus $^{50}_{132}\text{Sn}_{82}$ and the very exotic nucleus $^{50}_{100}\text{Sn}_{50}$ can be considered quantitative, opening new perspectives in the study of pairing in nuclei.

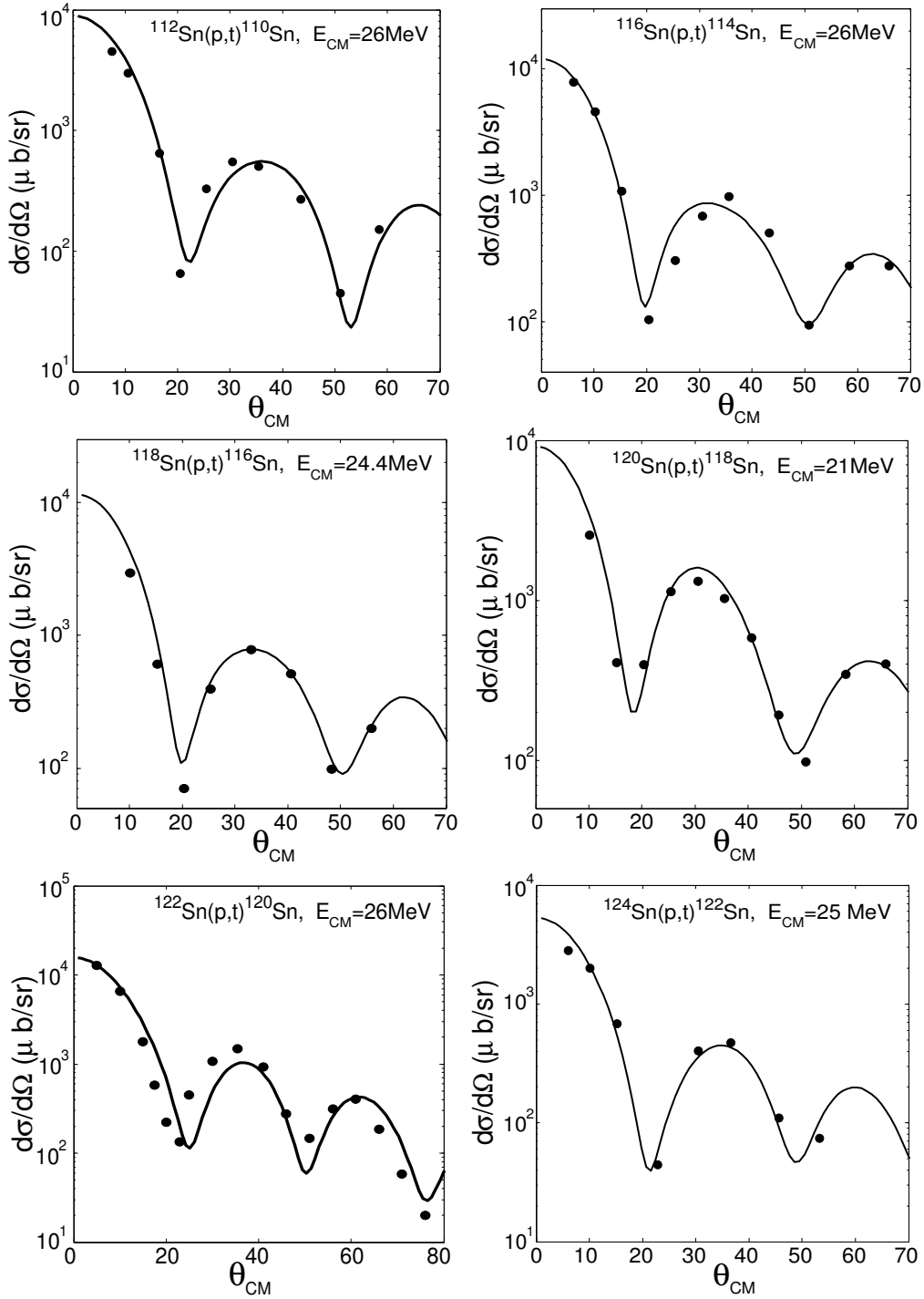
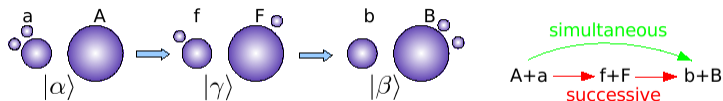


Figure 3: Absolute calculated cross sections compared with the experimental results of (21; 22; 23; 24)

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

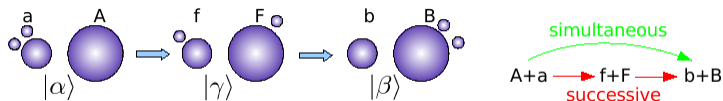
$$\frac{d\sigma}{d\Omega} = \frac{\mu_i \mu_f}{(4\pi \hbar^2)^2} \frac{k_f}{k_i} |T_{2NT}|^2$$

Simultaneous transfer

$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) \\ \times v(\mathbf{r}_{b1}) [\psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_{\mu}^{\Lambda} \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

Two particle transfer in second order DWBA

Some details of the calculation of the differential cross section for two-nucleon transfer reactions



$$T_{2NT} = \sum_{j_f j_i} B_{j_f} B_{j_i} \left(T^{(1)}(j_i, j_f) + T_{succ}^{(2)}(j_i, j_f) - T_{NO}^{(2)}(j_i, j_f) \right)$$

Successive transfer

$$\begin{aligned} T_{succ}^{(2)}(j_i, j_f) = & 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\ & \times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\ & \times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\ & \times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{b2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_\mu^\Lambda \chi_{aA}^{(+)}(\mathbf{r}'_{aA}) \end{aligned}$$

One knows how to calculate the absolute differential two-nucleon transfer cross sections within experimental errors
(quantitative era)

One can then use this probe to test nuclear structure predictions, by direct comparison with experimental data.

In particular a new embodiment of the Bardeen-Frölich-Pines mechanism to bind Cooper pairs^{*)} and eventually break gauge symmetry and thus give rise to superfluidity or superconductivity.

".... It has become fashionable... to assert... that once gauge symmetry is broken the properties of superconductors follow, with no need to inquire into the mechanism by which the symmetry is broken.... in 1957... the major problem was to show... how... gauge-invariant symmetry of the Lagrangian could be spontaneously broken due to interactions which were themselves gauge invariant". L. Cooper in BCS: 50 Years.

*) 1S_0 NN-force $V(r_{12}) = \sum_{\lambda} V_{\lambda}(r_{12}) P_{\lambda}(\cos \theta_{12})$, $V_{\lambda} = \frac{2\lambda+1}{4\pi r_1^2} \delta(r_1 - r_2)$
contributions come from high λ -terms ($r_{12} < R/\lambda$)
In the case of e.g. ^{11}Li small.

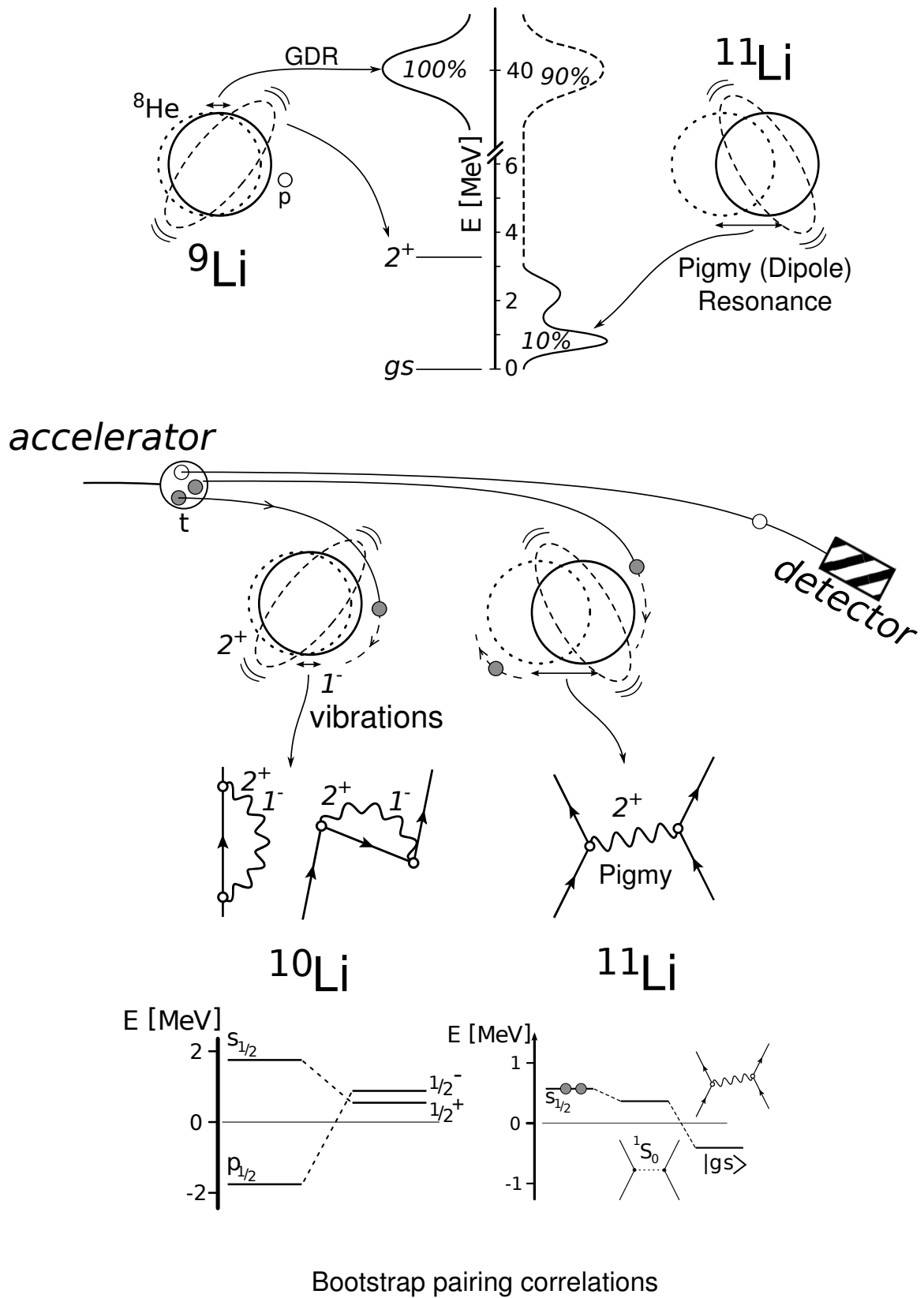


Figure 4.

Example of (pairing) LRO in nuclei

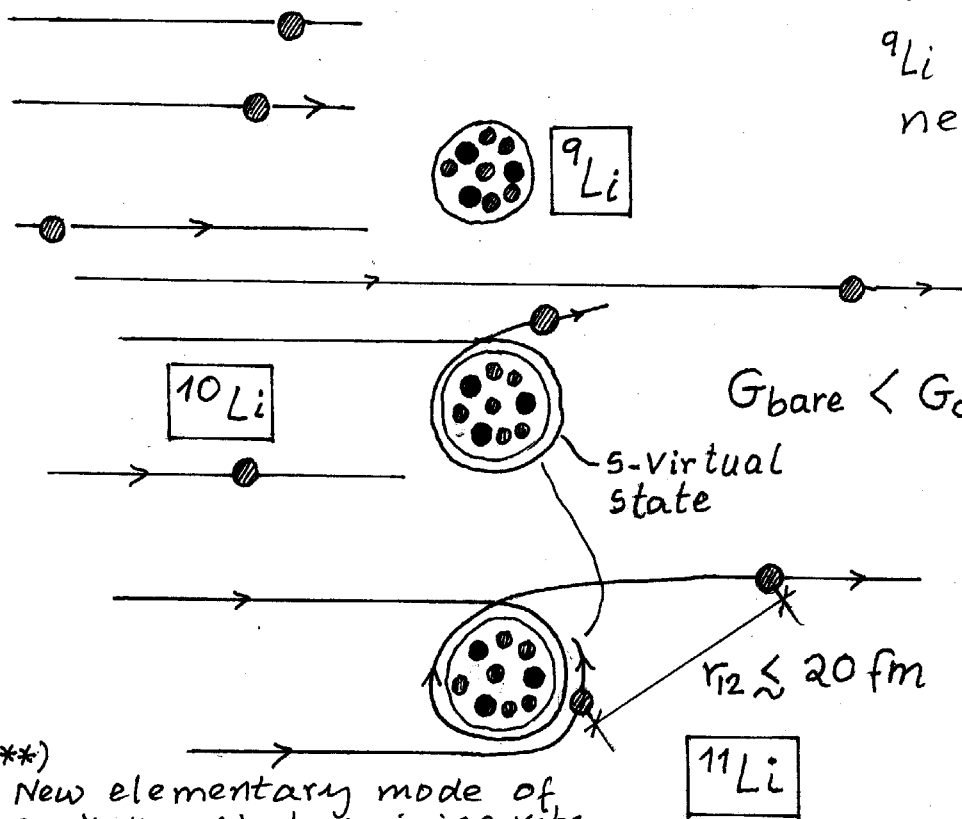
Pairing in nuclei is, as a rule, related to a gap $\Delta (= G\alpha_0, \alpha_0 = \sum_{\nu>0} U_\nu V_\nu = \langle P^+ \rangle)$ and thus to the OEMD
 Now,

${}^9\text{Li}_6$ bound; ${}^{10}\text{Li}_7$ not bound; ${}^{11}\text{Li}_8$ bound

How does one define OEMD in this case?
 Dynamically*)

$$\xi \approx \frac{\hbar v_F}{2E_{\text{corr}}} \approx 20 \text{ fm} ; v_F \approx 0.1c, E_{\text{corr}} \approx 0.5 \text{ MeV}$$

Dynamical, bootstrap, mechanism of Cooper pair condensation***) (pairing at the edge of stability)



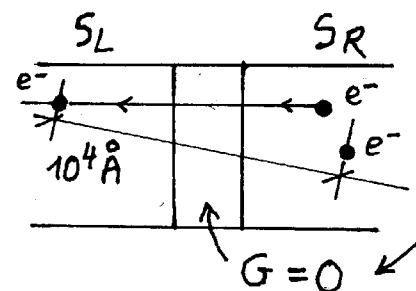
${}^9\text{Li}$ subject to an intense neutron flux.

$$G_{\text{bare}} < G_{\text{cr}} \text{ (i.e. } G=0, \infty\text{-syst.)}$$

s-virtual state

$$r_{12} \lesssim 20 \text{ fm}$$

$$G = G_{\text{bare}} + G_{\text{ind}} > G_{\text{cr}}$$



***)
 New elementary mode of excitation: halo pairing vibs.
 Very fragile (think of High T_c ceramics)

*) Thus, the Borromean (static) picture of light halo nuclei does not seem appropriate.

**) The basic requirement for superconductivity (superfluidity) to exist in a many-fermionic system is not $\Delta \neq 0$, but $\alpha_0 \neq 0$ (G can be $< G_{\text{cr}}$ (finite system or $G=0, \infty$ system))

Josephson-Bardeen
 Cohen-Falicov-Phillips

LRO: Long Range Order; OEMD: Odd-Even Mass Difference. $H_P = -G P^+ P$

Bootstrapping or booting

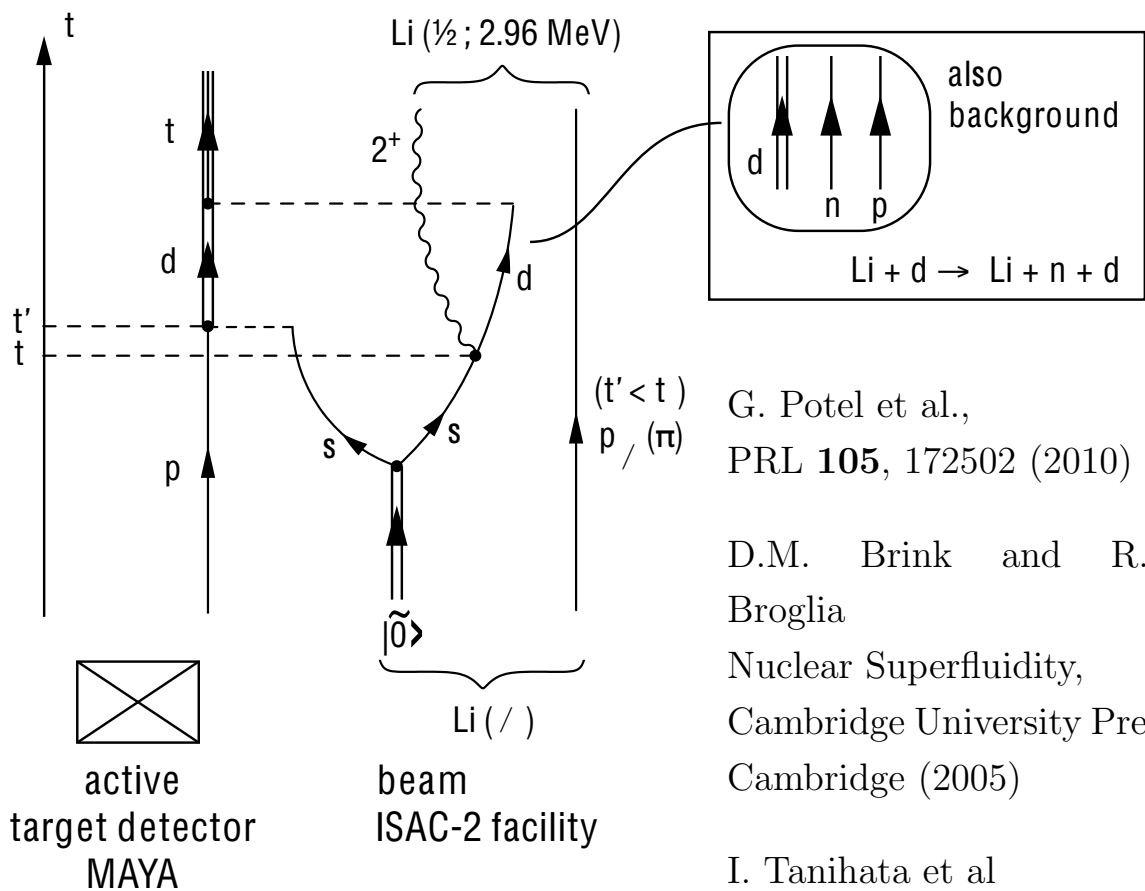
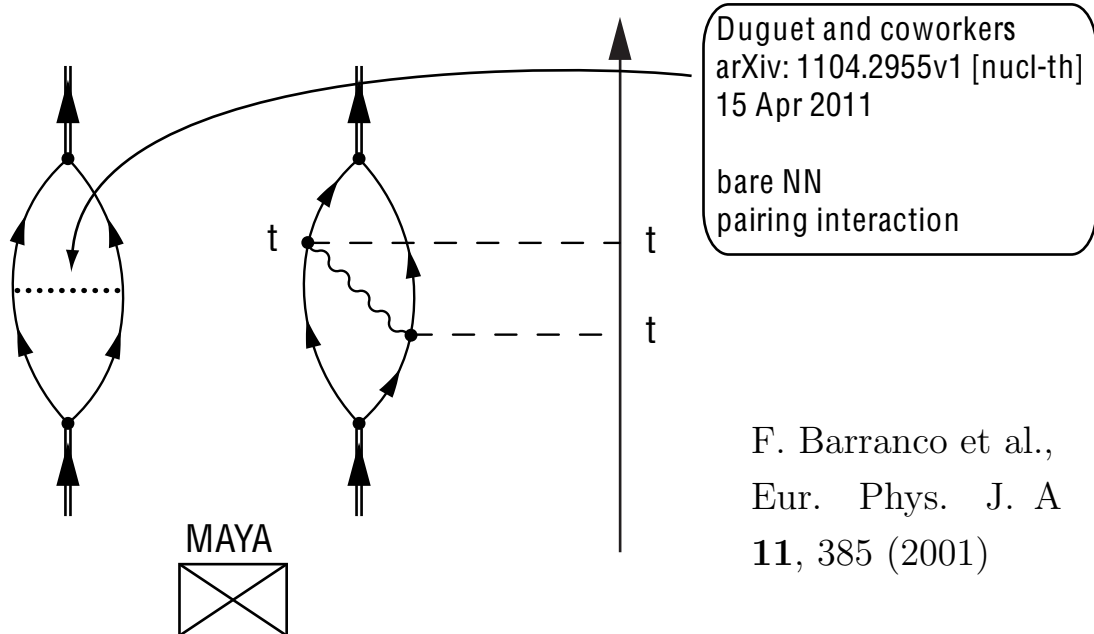
The term is often attributed to Rudolf Erich Raspe's story *The surprising Adventures of Baron Münchhausen*, where the main character pulls himself out of a swamp by his hair.

early 19th century USA : "pull oneself over a fence by one's bootstraps"

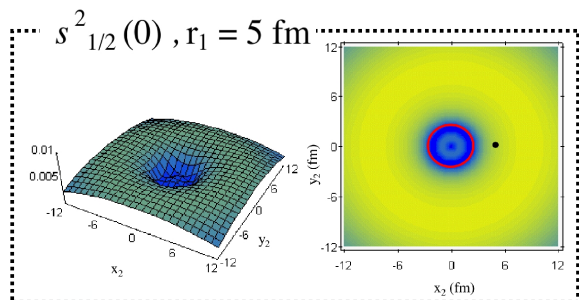
NFT



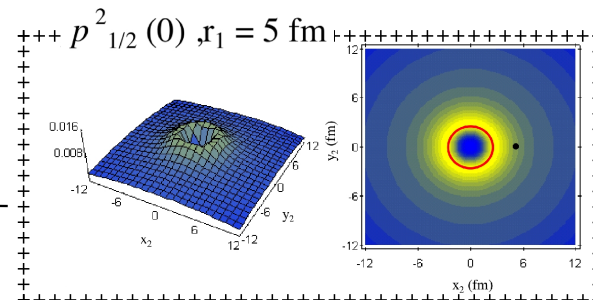
Virtual processes become real: II



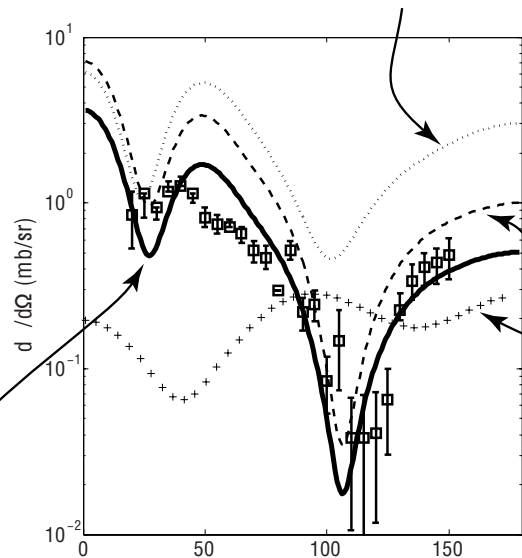
beam ISAC-2 facility



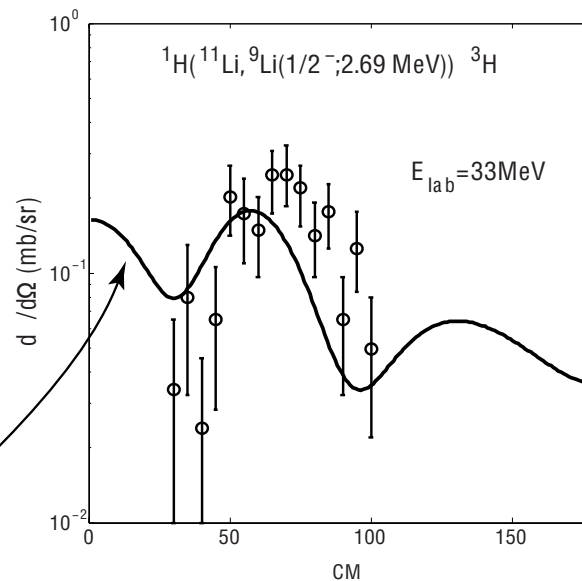
Barranco et al
EPJ, A11 (2001) 305



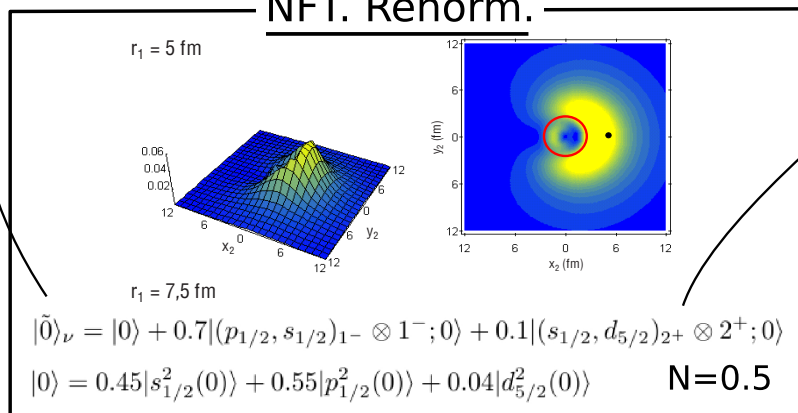
Tanihata et al
PRL, 100 (2008) 192502



Potel et al
PRL, 105 (2010) 172502



NFT. Renorm.



Barranco et al
EPJ, A11 (2001) 305

$$|\tilde{0}\rangle_\nu = |0\rangle$$

$$|0\rangle = 0.63|s^2_{1/2}(0)\rangle + 0.77|p^2_{1/2}(0)\rangle + 0.06|d^2_{5/2}(0)\rangle$$

N=1

Phys. Rev. C 69, 041302(R) (2004) [5 pages]

Parity inversion and breakdown of shell closure in Be isotopes

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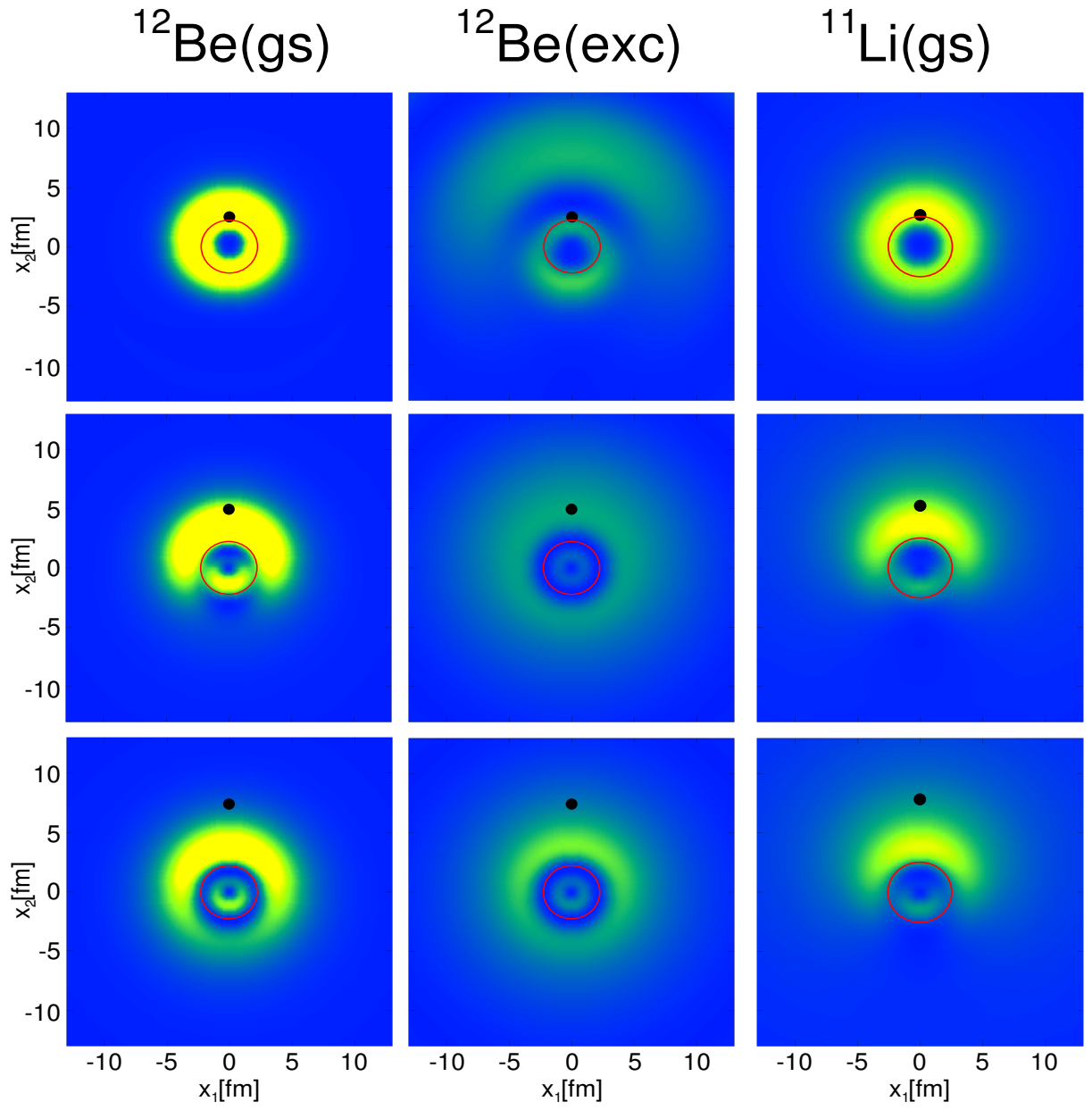
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The coupling of single-particle motion and of vibrations in ${}_4^{11}\text{Be}$ produces dressed neutrons which spend only a fraction of the time in pure single-particle states, and which weighing differently from the bare neutrons lead to parity inversion. The interaction of the two least bound neutrons in the ground state of ${}_4^{12}\text{Be}$ mediated by the v_{14} Argonne nucleon-nucleon potential and by the exchange of surface vibrations of the core ${}^{10}\text{Be}$ gives rise to a strongly correlated state, where the two valence neutrons are distributed over s^2 , p^2 , and d^2 configurations, resulting in the breaking of the $N=8$ shell closure.



$$|0\rangle_\nu = |0\rangle + \alpha|(p, s)_{1-} \otimes 1^-; 0\rangle + \beta|(s, d)_{2+} \otimes 2^+; 0\rangle + \gamma|(p, d)_{3-} \otimes 3^-; 0\rangle$$

$$|0\rangle_\nu = a|s^2(0)\rangle + b|p^2(0)\rangle + c|d^2(0)\rangle$$

	$^{11}\text{Li}(gs)$	$^{12}\text{Be}(gs)$	$^{12}\text{Be}(exc)$
α	0.7	0.10	0.08
β	0.1	0.30	-0.39
γ	—	0.37	-0.1
a	0.45	0.37	0.89
b	0.55	0.50	0.17
c	0.04	0.60	0.19

Appendix A: Mean square radius of ^{11}Li

(see e.g. Brink and Broglia, **Nuclear Superfluidity**, Cambridge University press, Cambridge (2010) 2nd Ed.)

Making use of the relation $\langle r^2 \rangle \approx (3/5)R^2$, one can write the mean square radius of ^{11}Li as

$$\langle r^2 \rangle_{^{11}\text{Li}} \approx \frac{3}{5} R_{eff}^2(^{11}\text{Li}), \quad (\text{A1})$$

with

$$R_{eff}^2(^{11}\text{Li}) = \left(\frac{9}{11} R_0^2(^9\text{Li}) + \frac{2}{11} \left(\frac{\xi}{2} \right)^2 \right), \quad (\text{A2})$$

where

$$R_0(^9\text{Li}) = 2.7\text{fm}, \quad (\text{A3})$$

is the ^9Li radius ($R_0 \approx r_0 A^{1/3}$, $r_0 = 1.2$ fm), while ξ is the correlation length of the halo neutron Cooper pair. A fair estimate of this quantity is provided by the relation

$$\xi = \frac{\hbar v_F}{2E_{corr}} \approx 20\text{fm}, \quad (\text{A4})$$

in keeping with the fact that in ^{11}Li , $(v_F/c) \approx 0.1$ and $E_{corr} \approx 0.5$ MeV. Consequently, $\langle r^2 \rangle_{^{11}\text{Li}}^{1/2} \approx 3.8$ fm ($R_{eff}(^{11}\text{Li}) \approx 4.9$ fm), in overall agreement with the experimental value $\langle r^2 \rangle^{1/2} = 3.55 \pm 0.1$ fm (Kobayashi et al. (1989) Phys. Lett. **B232**: 51)).

Appendix B: Centroid pigmy resonance

(Bortignon, Bracco and Broglia, **Giant Resonances: Nuclear Structure at Finite Temperatures**, Harwood Academic Publishers, Amsterdam (1998))

From the dispersion relation given in Eq. (3.30) p.55, and the fact that $\varepsilon_{\nu_k} - \varepsilon_{\nu_i} = \varepsilon_{2s_{1/2}} - \varepsilon_{1p_{1/2}} \approx 0.5$ MeV (see Fig. 11.1 p. 264 Brink and Broglia (2010)), and that the EWSR associated with the ^{11}Li pigmy resonance is $\approx 10\%$ of the total Thomas-Reiche-Kuhn sum rule one can write,

$$0.1 \frac{\hbar^2 A}{2m} = \frac{1}{\kappa_1} [(0.5\text{MeV})^2 - (\hbar\omega_{pigmy})^2], \quad (\text{B1})$$

$$(\hbar\omega_{pigmy})^2 = (0.5\text{MeV})^2 - 0.1 \frac{\hbar^2 A}{2m} \kappa_1, \quad (\text{B2})$$

where (see Eq.(3.51) Bortignon et. al. (1998)),

$$\kappa_1 = -\frac{5V_1}{A(\xi/2)^2} = -\frac{125\text{MeV}}{A \times 100\text{fm}^2} \approx -\frac{1.25}{A} \text{fm}^{-2} \text{ MeV}, \quad (\text{B3})$$

the ratio in parenthesis reflecting the fact that only 2 out of 11 nucleons, slosh back and forth in an extended configuration with little overlaps with the core nucleons.

From the above relation one obtains,

$$-0.1 \frac{\hbar^2 A}{2m} \kappa_1 = 2.5 \text{ MeV}^2 = (1.6 \text{ MeV})^2. \quad (\text{B4})$$

Consequently,

$$\hbar\omega_{\text{pigmy}} = \sqrt{0.5^2 + 1.6^2} \text{ MeV} \approx 1.7 \text{ MeV}, \quad (\text{B5})$$

in overall agreement with the experimental findings (Zinser et al (1997), Nucl. Phys. **A619**:151). It is of notice that the centroid of the pigmy resonance calculated in the RPA with the help of a separable interaction is $\approx (0.8 \text{ MeV} + 2.0 \text{ MeV})/2 \approx 1.4 \text{ MeV}$ (see Fig. 11.3(a) p.269, Brink and Broglia (2010)).

Appendix C: Estimate considering Self energy renormalization

The two neutrons must create the correlations that binds them together and to the core. Self Energy renormalization (see e.g. Mahaux et al. Phys. Rep. **120**(1985)287) is of the order of

$$\Delta V \simeq U_1 U_2 \frac{h^2}{\varepsilon_1 - \varepsilon_2 - \hbar\omega} \approx -U_1 U_2 \frac{h^2}{\hbar\omega}, \quad (\text{C1})$$

using the fact that the splitting between $p_{1/2}$ and $s_{1/2}$ states of ^{11}Li is little compared to the phonon energy. We can use as h , the coupling matrix element, the average pairing binding constant $G \approx 22/A$ which in the case of ^{11}Li is $G \approx 22/11 \text{ MeV} = 2 \text{ MeV}$. U_1 and U_2 being the occupation factors of s and p states, that we can assume being both 0.5 (More precise calculations (see e.g. Brink and Broglia, Nuclear Superfluidity, Cambridge University press, Cambridge (2010) 2nd Ed.) give 0.40 for s and 0.58 for p states). That gives us

$$\hbar\omega \approx -U_1 U_2 \frac{h^2}{\Delta V} \approx (0.5)^2 \frac{2^2}{0.6} \text{ MeV} = 1.67 \text{ MeV} \quad (\text{C2})$$

Phys. Rev. C 50, 1355–1359 (1994)

$(sd)^2$ states in ^{12}Be

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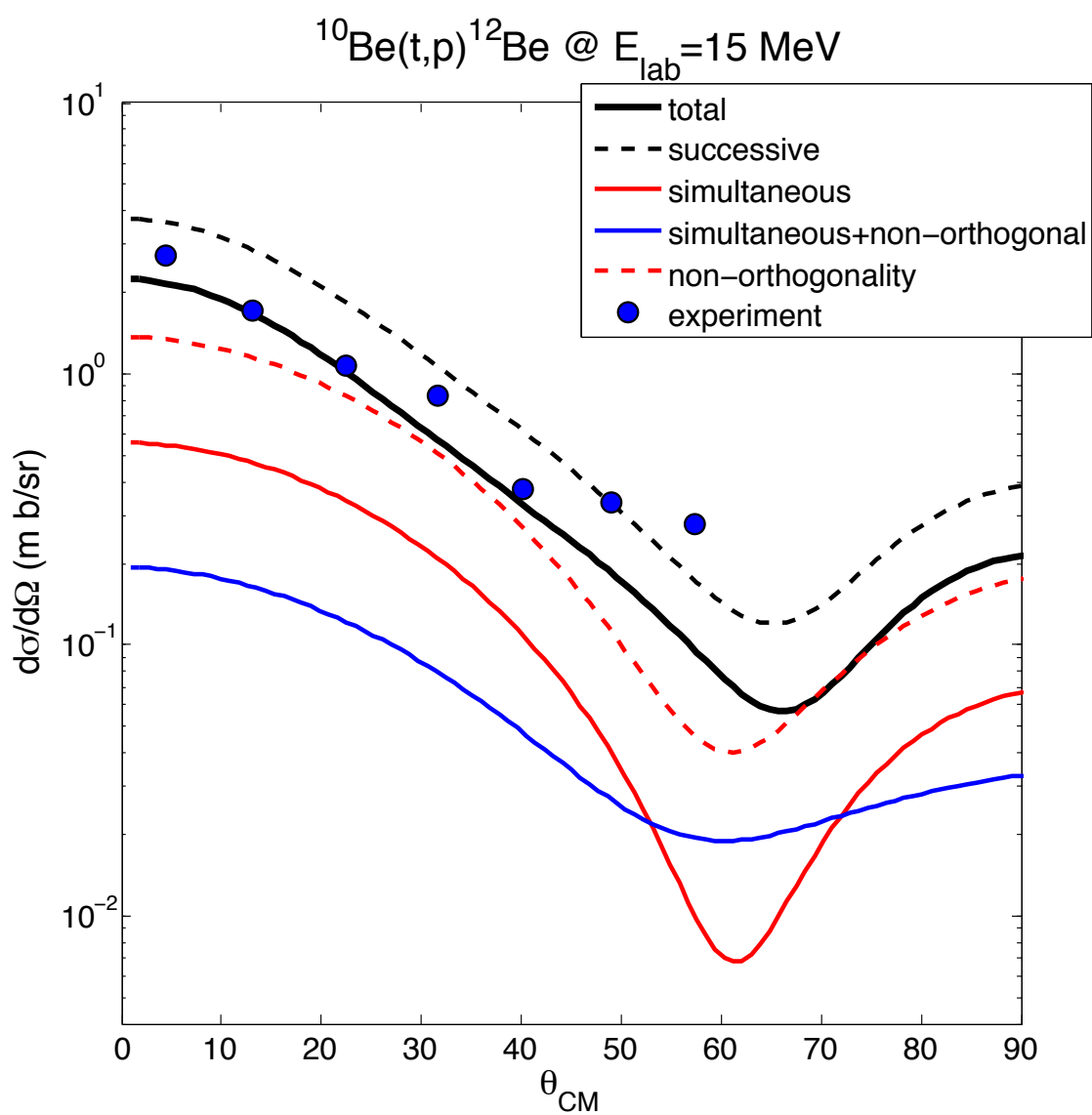
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The $^{10}\text{Be}(t,p)^{12}\text{Be}$ reaction has been studied with 15- and 17-MeV triton beams. At 17 MeV, angular distributions were measured for five low-lying states, and distorted-wave Born-approximation calculations were used to analyze the data. Contributions from $^{10}\text{Be}(\text{g.s.}) \otimes (sd)^2$ and complete $1p$ -shell wave functions were investigated. Comparison is made with $(sd)^2$ states in ^{14}C and ^{16}C .



$$\begin{aligned}
 T_{1/2-} = \sum_{n p_{1/2}} \tilde{\xi}_{n p_{1/2}} \left\{ \sum_{\substack{p'' \\ p p'}} \xi_{p''} \xi_{p p'} \times \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} a_{n p_{1/2}} \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \quad (0.85) \\
 + \sum_{\substack{p'', k \\ d d'}} \xi_{p''} \xi_{d d'} \times \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ d d' \end{array} \begin{array}{c} a_{n p_{1/2}} \\ \uparrow \\ \text{---} \\ \uparrow \\ d d' \end{array} \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ d d' \end{array} \quad (-0.01) \\
 + \sum_{\substack{p'', k \\ m, p p'}} \xi_{p''} \xi_{p p'} \times \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ m d_{5/2} \end{array} \begin{array}{c} a_{n p_{1/2}} \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \quad (0.01) \\
 + \sum_{\substack{p'', k \\ p p'}} \xi_{p''} \xi_{p p'} \times \left[\begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} a_{n p_{1/2}} \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} 1 p_{3/2} \\ \uparrow \\ \text{---} \\ \uparrow \\ k 2^+ \end{array} \quad (0.01) \\
 + \sum_{\substack{p'', k \\ p p'}} \xi_{p''} \xi_{p p'} \times \left[\begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} a_{n p_{1/2}} \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} p'' \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \begin{array}{c} p_{3/2} \\ \uparrow \\ \text{---} \\ \uparrow \\ k 2^+ \end{array} \begin{array}{c} a_{n p_{1/2}} \\ \uparrow \\ \text{---} \\ \uparrow \\ p p' \end{array} \right] \quad (-1.61) \end{aligned}$$

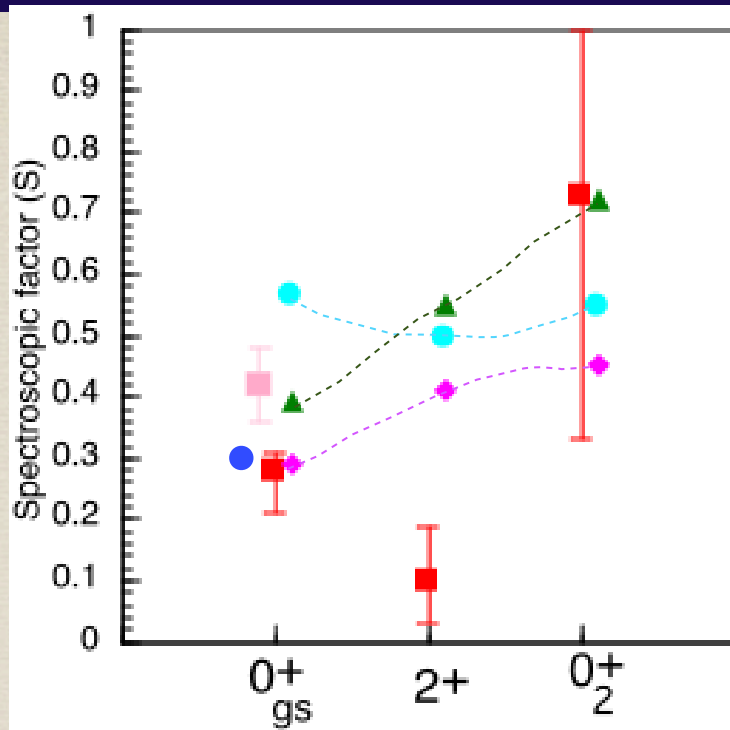
Summing up the different contributions (numbers in brackets), one obtains the spectroscopic factor $|T_{1/2-}|^2 = 0.57$. Similarly, one obtains $|T_{1/2+}|^2 = 0.31$ (cf. Table I). Also the spectroscopic factors associated with ^{11}Be (Table I) were calculated following the same scheme.

Diagonalizing the matrix shown in Fig. 2, we also obtain the energies of the excited 0^+ states in ^{12}Be . In particular, we obtain that the energy of the first excited state is 2.04 MeV, to be compared with the experimental value of 2.24 MeV [31].

We conclude that the main nuclear structure properties of both ^{11}Be and ^{12}Be may be understood in terms of the self-energy and induced interaction processes associated with the dynamic polarization of the nuclear surface. The similarity of NFT results with those of large shell model calculations reported in Ref. [7] for ^{12}Be and in Ref. [32] for ^{11}Be , indicates that a proper treatment of single particle and of collective degrees of freedom and of their interweaving provides an essentially complete description of the nuclear structure of these nuclei as was already found in the case of nuclei lying along the stability valley.

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Spectroscopic factors: *comparison to model predictions*



^{12}Be halo in 0^+_2 state

R. Kanungo et al., Phy. Lett. B 682 (2010) 391.

- ◆ $\Delta_{\text{psd}} = -1.2$, 42%N=0 + 58%N=2, $d_{5/2}$ lower 0.8 MeV
 - ▲ $\Delta_{\text{psd}} = -1.85$, 50%N=0 + 50%N=2
 - $\Delta_{\text{psd}} = -1.85$, 32%N=0 + 68%N=2
 - G. Gori et al., PRC 69 ('04) 041302R (particle vibration + Av14)
- WBP
B.A. Brown

Questions

Is there a pigmy resonance built on top of the $|^{12}\text{Be}(0^+; 2.42 \text{ MeV})\rangle$ excited state^{*)}?

we think so,

Generally, this is likely to happen for nuclei with neutron excess in situations in which $s^2(0)$ -configurations at threshold, thus allowing for extended halo, which sloshing back and forth with respect to protons in core, lead to a dipole phonon which can glue the (Cooper) pair.
bootstrap mechanism^{**))}

- inhomogeneous damping
- vibrations built on excited states (PhD thesis D. Brink); interest to learn about temperature dependence of vibrations^{***)}

-
- two-nucleon transfer as a function of bomb. energy (strength functions)

resonant effects related to phonon frequencies as in sup-sup tunneling

quantitative era based on absolute cross sections.

^{*)} Likely yes, in keeping with the fact that $^1\text{S}_0$ NN-force $V(r_{12}) = \sum_{\lambda} V_{\lambda}(r_{12}) P_{\lambda}(\cos \theta_{12})$, $V_{\lambda} = \frac{2\lambda+1}{4\pi r_{12}^2} \delta(r_1 - r_2)$ come from all the high λ terms, ($r_{12} < R/\lambda$).
(see e.g. Brink and Broglia, Nuclear Superfluidity, Cambridge Univ. Press (2010) p. 39)

^{**))} Usefulness of strings in physics

^{***)} Bortignon, Bracco, Broglia, Giant Resonances: nuclear structure at finite temperature, Harwood ac. press, Amsterdam (1998)