

Flavour and CP Violation Phenomenology in SUSY with an SU(3) Flavour Symmetry

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0804.4620 [hep-ph]
0907.4069 [hep-ph].

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Introduction

(Some) Reasons Why People Don't Like SUSY

SUSY Flavour Problem:

Generic flavoured SUSY contributions to FCNC (e.g. K mixing) are too large.

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SUSY Flavour Problem:

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Flavoured parameters cannot be generic!

$$m_{\tilde{d}_R^c}^2 \neq \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix} m_0^2$$

(Some) Reasons Why People Don't Like SUSY

SUSY CP Problem:

Large phases in flavour-independent parameters give too large contributions to EDMs.

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CP Violation cannot be flavour-independent!

$$\arg(\delta_{LR}^d)_{11} \neq 90^\circ$$

The Flavour Sector

Standard Model with Dirac Neutrinos

$$\begin{aligned}\mathcal{L} = & Q_\alpha Y_d^{\alpha\beta} d_\beta^c H - Q_\alpha Y_u^{\alpha\beta} u_\beta^c H^\dagger \\ & + L_\alpha Y_e^{\alpha\beta} e_\beta^c H - L_\alpha Y_\nu^{\alpha\beta} \nu_\beta^c H^\dagger\end{aligned}$$

The Flavour Sector

Standard Model with Dirac Neutrinos

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Four 3x3 complex Yukawa Matrices: 72 (unphysical) parameters.

$U(3)^6$ flavour symmetries reduce parameter space down to 20 physical observables: 12 masses, 6 mixings, 2 CP phases.

The Flavour Sector

Standard Model with Dirac Neutrinos

- ✗ No justification for three fermion families.
- ✗ No justification for fermion mass hierarchy.
- ✗ No justification for magnitude of fermion mixing.
- ✗ Too many arbitrary parameters.

The Flavour Sector

Standard Model: Quarks

$$Y_d = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ - & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ - & - & 1 \end{pmatrix} \quad Y_u = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ - & \varepsilon^2 & \varepsilon^2 \\ - & - & 1 \end{pmatrix}$$

$$\bar{\varepsilon} = 0.15$$

$$\varepsilon = 0.05$$

The Flavour Sector

Standard Model: Quarks

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$$\bar{\varepsilon} = 0.15$$

$$\varepsilon = 0.05$$

Standard Model Flavour Problem?

The Flavour Sector

Flavour Problems:

SUSY Flavour Problem: Arbitrary choice of parameters in Soft-Mass matrices give a wrong (too large) contribution to low-energy processes.

SM Flavour Problem: Arbitrary choice of parameters in Yukawa matrices give a wrong value of observed masses and mixing matrices.

The Flavour Sector

Do we understand CP Violation?

Standard Model: All phases come from flavour sector.

SUSY: Troublesome phases come from flavour-independent sector.

A Solution to the Flavour and CP Problems?

Devise a mechanism with which to generate the Yukawa textures, and relate CP Violation to flavour.

Extend this mechanism into SUSY models, and predict its implications on low energy phenomena.

Outline

- SU(3) Flavour Model Construction
- Consequences on Lepton Sector
- Consequences on Quark Sector
- Correlations

Flavour and CP Violation Phenomenology with Supersymmetric Flavour Symmetries

$SU(3)$ Flavour Model

S. King, G. G. Ross (hep-ph/0108112)

G. G. Ross, L. Velasco-Sevilla, O. Vives (hep-ph/0401064)

L. Calibbi, JJP, A. Masiero, J.-h. Park, W. Porod, O. Vives (0907.4069 [hep-ph])

Guides

$$Y_d = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1 \end{pmatrix} \quad Y_u = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & 1 \end{pmatrix}$$

$$\bar{\varepsilon} = 0.15$$

$$= \sqrt{\frac{m_s}{m_b}}$$

$$\varepsilon = 0.05$$

$$= \sqrt{\frac{m_c}{m_t}}$$

Assumption: Yukawas are hierarchical and symmetric

Step 1: Flavour Symmetry

$$SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_R$$

CP

Fermion Superfields:

$$Q, u^c, d^c$$

$$L, e^c, \nu^c$$

$$\mathbf{SU(3)}_F$$

3

Higgs Superfields:

$$H_u$$

$$H_d$$

1

Step 2: Flavons

$$SU(3)_F \otimes SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes Z_R$$

CP

$SU(3)_F$

$$\theta_3 \quad \theta_{23} \quad (\theta_2, \theta_{123} \dots) \quad \bar{3}$$

$$\bar{\theta}_3 \quad \bar{\theta}_{23} \quad (\bar{\theta}_2, \bar{\theta}_{123} \dots) \quad 3$$

Step 3: Couplings

$$W = \sum_{i,f,f^c} H_i f f^c \left(\sum_{\theta,\theta',n,m} y_{\theta\theta'}^{f f^c} \frac{\theta^n \theta'^m}{M_f^{n+m}} \right)$$

Step 3: Couplings

$$W = \sum_{i,f,f^c} H_i f f^c \left(\sum_{\theta,\theta',n,m} y_{\theta\theta'}^{f f^c} \frac{\theta^n \theta'^m}{M_f^{n+m}} \right)$$

MSSM Superfields

O(1) Couplings

Flavons

Messenger Mass

The diagram illustrates the decomposition of the Lagrangian W into its components. The Lagrangian is given by:

$$W = \sum_{i,f,f^c} H_i f f^c \left(\sum_{\theta,\theta',n,m} y_{\theta\theta'}^{f f^c} \frac{\theta^n \theta'^m}{M_f^{n+m}} \right)$$

Annotations with red arrows point to specific parts of the equation:

- A red arrow points from the term $H_i f f^c$ to the label "MSSM Superfields".
- A red arrow points from the coupling coefficient $y_{\theta\theta'}^{f f^c}$ to the label "O(1) Couplings".
- A red arrow points from the mass scale M_f to the label "Messenger Mass".
- A red arrow points from the term $\theta^n \theta'^m$ to the label "Flavons".

Step 3: Couplings

$$W = \sum_{i,f,f^c} H_i f f^c \left(\sum_{\theta,\theta',n,m} y_{\theta\theta'}^{f f^c} \frac{\theta^n \theta'^m}{M_f^{n+m}} \right)$$

MSSM Superfields

O(1) Couplings

Flavons

Y_{f f^c}

Messenger Mass

Step 3: Couplings

$$\begin{aligned} & H_d Q_i D_j \left(\frac{1}{M_d^2} \theta_3^i \theta_3^j + \frac{1}{M_d^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right) \\ & + H_u Q_i U_j \left(\frac{1}{M_u^2} \theta_3^i \theta_3^j + \frac{1}{M_u^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right) \end{aligned}$$

Assumption: Three messenger masses

$$M_u \sim M_Q \gg M_d$$

Step 4: Vevs

$$H_d Q_i D_j \left(\frac{1}{M_d^2} \theta_3^i \theta_3^j + \frac{1}{M_d^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right)$$

$$+ H_u Q_i U_j \left(\frac{1}{M_u^2} \theta_3^i \theta_3^j + \frac{1}{M_u^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right)$$

$$\langle \theta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u & 0 \\ 0 & a_3^d e^{i\chi} \end{pmatrix}$$

$$\langle \bar{\theta}_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u e^{i\alpha_u} & 0 \\ 0 & a_3^d e^{i\alpha_d} \end{pmatrix}$$

CP is
spontaneously
broken

Step 4: Vevs

$$H_d Q_i D_j \left(\frac{1}{M_d^2} \theta_3^i \theta_3^j + \frac{1}{M_d^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right)$$

$$+ H_u Q_i U_j \left(\frac{1}{M_u^2} \theta_3^i \theta_3^j + \frac{1}{M_u^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right)$$

$$\langle \theta_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u & 0 \\ 0 & a_3^d e^{i\chi} \end{pmatrix} \quad \left(\frac{a_3^u}{M_u} \right)^2 = y_t$$

$$\langle \bar{\theta}_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u e^{i\alpha_u} & 0 \\ 0 & a_3^d e^{i\alpha_d} \end{pmatrix} \quad \left(\frac{a_3^d}{M_d} \right)^2 = y_b$$

Step 4: Vevs

$$\begin{aligned}
 & H_d Q_i D_j \left(\frac{1}{M_d^2} \theta_3^i \theta_3^j + \frac{1}{M_d^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right) \\
 & + H_u Q_i U_j \left(\frac{1}{M_u^2} \theta_3^i \theta_3^j + \frac{1}{M_u^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right) \\
 \langle \theta_{23} \rangle = & \begin{pmatrix} 0 \\ b_{23} \\ b_{23} e^{i\beta_3} \end{pmatrix} \quad \frac{b_{23}}{M_d} = \bar{\varepsilon} \\
 \langle \bar{\theta}_{23} \rangle = & \begin{pmatrix} 0 \\ b_{23} e^{i\beta'_2} \\ b_{23} e^{i(\beta'_2 - \beta_3)} \end{pmatrix} \quad \frac{b_{23}}{M_u} = \varepsilon
 \end{aligned}$$

We require a
vacuum
alignment
mechanism

Step 4: Vevs

$$\begin{aligned} & \mathsf{H}_d Q_i D_j \left(\frac{1}{M_d^2} \theta_3^i \theta_3^j + \frac{1}{M_d^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right) \\ & + \mathsf{H}_u Q_i U_j \left(\frac{1}{M_u^2} \theta_3^i \theta_3^j + \frac{1}{M_u^4} \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \right) \end{aligned}$$

$$Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \bar{\varepsilon}^2 e^{i\sigma_d} & \bar{\varepsilon}^2 e^{i(\sigma_d + \beta_3)} \\ 0 & \bar{\varepsilon}^2 e^{i(\sigma_d + \beta_3)} & e^{2i\chi} \end{pmatrix} y_b$$

$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \varepsilon^2 e^{i\sigma_u} & \varepsilon^2 e^{i(\sigma_u + \beta_3)} \\ 0 & \varepsilon^2 e^{i(\sigma_u + \beta_3)} & 1 \end{pmatrix} y_t$$

Step 3 + 4: Couplings + Vevs

$$\frac{1}{M_d^5} H_d Q_i D_j \left(\epsilon^{ikl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^j + \epsilon^{jkl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^i \right) (\theta_{23} \bar{\theta}_3)$$

$$\frac{1}{M_d^5} H_d Q_i D_j \left(\epsilon^{ijl} \bar{\theta}_{23,l} (\theta_{23} \bar{\theta}_3)^2 \right)$$

$$\frac{1}{M_d^5} H_d Q_i D_j \left(\epsilon^{ijl} \bar{\theta}_{3,l} (\theta_{23} \bar{\theta}_3) (\theta_{23} \bar{\theta}_{23}) \right)$$

Step 5: Yukawa Structure

$$Y_d = \begin{pmatrix} 0 & \bar{\varepsilon}^3 e^{i\delta_d} & \bar{\varepsilon}^3 e^{i(\delta_d+\beta_3)} \\ \bar{\varepsilon}^3 e^{i\delta_d} & \bar{\varepsilon}^2 e^{i\sigma_d} & \bar{\varepsilon}^2 e^{i(\sigma_d+\beta_3)} \\ \bar{\varepsilon}^3 e^{i(\delta_d+\beta_3)} & \bar{\varepsilon}^2 e^{i(\sigma_d+\beta_3)} & e^{2i\chi} \end{pmatrix} y_b$$

$$Y_u = \begin{pmatrix} 0 & \varepsilon^3 e^{i\delta_u} & \varepsilon^3 e^{i(\delta_u+\beta_3)} \\ \varepsilon^3 e^{i\delta_u} & \varepsilon^2 e^{i\sigma_u} & \varepsilon^2 e^{i(\sigma_u+\beta_3)} \\ \varepsilon^3 e^{i(\delta_u+\beta_3)} & \varepsilon^2 e^{i(\sigma_u+\beta_3)} & 1 \end{pmatrix} y_t$$

Shaping Symmetries

$$H_d Q_i D_j \frac{1}{M_d^2} \theta_3^i \theta_{23}^j$$

$$Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bar{\varepsilon} y_b^{0.5} \\ 0 & \bar{\varepsilon} y_b^{0.5} & 0 \end{pmatrix}$$

Allowed by SU(3)!
Structure is spoilt!

Shaping Symmetries

$$\cancel{H_d Q_i D_j \frac{1}{M_d^2} \theta_3^i \theta_{23}^j}$$
$$Y_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bar{\varepsilon} y_b^{0.5} \\ 0 & \bar{\varepsilon} y_b^{0.5} & 0 \end{pmatrix}$$

Allowed by SU(3)!
Structure is spoilt!

Leptons

- Neutrino masses: See-saw mechanism
- Not much information apart from charged lepton masses.
- Assumption: Yukawa Unification

$$y_\tau \approx y_b$$

$$y_\mu \approx 3y_s$$

$$y_e \approx \frac{1}{3}y_d$$

Leptons

- Neutrino masses: See-saw mechanism
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- Assumption: Yukawa Unification

$$y_\tau \approx y_b$$

$$y_\mu \approx 3y_s$$

$$y_e \approx \frac{1}{3}y_d$$

This depends on $\tan\beta$

G. G. Ross, M. Serna (0704.1248 [hep-ph])
S. Antusch, M. Spinrath (0902.4644 [hep-ph])

Leptons

Georgi-Jarlskog Field: Σ

$$\begin{aligned} W = & \mathsf{H}_i f_i f_j^c \left[\theta_3^i \theta_3^j + \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \Sigma \right. \\ & + \left(\epsilon^{ikl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^j + \epsilon^{jkl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^i \right) (\theta_{23} \bar{\theta}_3) + \\ & \left. \epsilon^{ijl} \bar{\theta}_{23,l} (\theta_{23} \bar{\theta}_3)^2 + \epsilon^{ijl} \bar{\theta}_{3,l} (\theta_{23} \bar{\theta}_3) (\theta_{23} \bar{\theta}_{23}) + \dots \right] \end{aligned}$$

Leptons

Georgi-Jarlskog Field: Σ

$$\begin{aligned} W = & \mathsf{H}_i f_i f_j^c \left[\theta_3^i \theta_3^j + \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \Sigma \right. \\ & + \left(\epsilon^{ikl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^j + \epsilon^{jkl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^i \right) (\theta_{23} \bar{\theta}_3) + \\ & \left. \epsilon^{ijl} \bar{\theta}_{23,l} (\theta_{23} \bar{\theta}_3)^2 + \epsilon^{ijl} \bar{\theta}_{3,l} (\theta_{23} \bar{\theta}_3) (\theta_{23} \bar{\theta}_{23}) + \dots \right] \end{aligned}$$

Leptons

Georgi-Jarlskog Field: Σ

$$\begin{aligned} W = & \mathsf{H}_i f_i f_j^c \left[\theta_3^i \theta_3^j + \theta_{23}^i \theta_{23}^j (\theta_3 \bar{\theta}_3) \Sigma \right. \\ & + \left(\epsilon^{ikl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^j + \epsilon^{jkl} \bar{\theta}_{23,k} \bar{\theta}_{3,l} \theta_{23}^i \right) (\theta_{23} \bar{\theta}_3) + \\ & \left. \epsilon^{ijl} \bar{\theta}_{23,l} (\theta_{23} \bar{\theta}_3)^2 + \epsilon^{ijl} \bar{\theta}_{3,l} (\theta_{23} \bar{\theta}_3) (\theta_{23} \bar{\theta}_{23}) + \dots \right] \end{aligned}$$

$$\langle \Sigma \rangle = (B - L + 2T_3^R) \quad \Sigma_e = 3\Sigma_d$$

Model Particles + Symmetries

Field	ψ	ψ^c	H	Σ	θ_3	θ_{23}	$\bar{\theta}_3$	$\bar{\theta}_{23}$
SU(3)	3	3	1	1	3	3	3	3
U(1)	0	0	0	1	0	-1	1	0
U'(1)	-1	-1	0	2	1	0	-1	4
U''(1)	1	1	0	-3	-1	1	0	-4

Flavoured Soft Terms

$$\begin{aligned}\mathcal{L}_{SB} = & -\tilde{Q}_i^*(M_{\tilde{Q}}^2)_{ij}\tilde{Q}_j \\ & -(\tilde{u}_R^c)_i^*(M_{\tilde{u}_R^c}^2)_{ij}(\tilde{u}_R^c)_j - (\tilde{d}_R^c)_i^*(M_{\tilde{d}_R^c}^2)_{ij}(\tilde{d}_R^c)_j \\ & -\tilde{L}_i^*(M_{\tilde{L}}^2)_{ij}\tilde{L}_j - (\tilde{e}_R^c)_i^*(M_{\tilde{e}_R^c}^2)_{ij}(\tilde{e}_R^c)_j \\ & - \left(\tilde{Q}_i(A_u)_{ij}(\tilde{u}_R^c)_j H_u - \tilde{Q}_i(A_d)_{ij}(\tilde{d}_R^c)_j H_d \right. \\ & \left. - \tilde{L}_i(A_e)_{ij}(\tilde{e}_R^c)_j H_d + h.c. \right)\end{aligned}$$

Minimal Soft Masses

$$(M_{\tilde{f}}^2)_i^j = m_0^2 \left(\delta_i^j + \left[\theta_{3,i}^\dagger \theta_3^j + \bar{\theta}_{3,i} \bar{\theta}_3^{\dagger j} + \theta_{23,i}^\dagger \theta_{23}^j + \bar{\theta}_{23,i} \bar{\theta}_{23}^{\dagger j} \right] \right. \\ \left. + (\epsilon_{ikl} \theta_3^k \theta_{23}^l) (\epsilon^{jmn} \theta_{3,m}^\dagger \theta_{23,n}^\dagger) \right. \\ \left. + (\epsilon_{ikl} \bar{\theta}_3^{\dagger k} \bar{\theta}_{23}^{\dagger l}) (\epsilon^{jmn} \bar{\theta}_{3,m} \bar{\theta}_{23,n}) + \dots \right)$$

Minimal Soft Masses

$$(M_{\tilde{f}}^2)_i^j = m_0^2 \left(\delta_i^j + \left[\theta_{3,i}^\dagger \theta_3^j + \bar{\theta}_{3,i} \bar{\theta}_3^{\dagger j} + \theta_{23,i}^\dagger \theta_{23}^j + \bar{\theta}_{23,i} \bar{\theta}_{23}^{\dagger j} \right] \right. \\ \left. + (\epsilon_{ikl} \theta_3^k \theta_{23}^l) (\epsilon^{jmn} \theta_{3,m}^\dagger \theta_{23,n}^\dagger) \right. \\ \left. + (\epsilon_{ikl} \bar{\theta}_3^{\dagger k} \bar{\theta}_{23}^{\dagger l}) (\epsilon^{jmn} \bar{\theta}_{3,m} \bar{\theta}_{23,n}) + \dots \right)$$

$$M_{\tilde{Q}}^2 = \begin{pmatrix} 1 + Q_1 \varepsilon^2 y_t & 0 & 0 \\ 0 & 1 + Q_2 \varepsilon^2 & Q_3 \varepsilon^2 e^{i\beta_3} \\ 0 & Q_3 \varepsilon^2 e^{-i\beta_3} & 1 + Q_4 y_t \end{pmatrix} m_0^2$$

RVV1

Alternative Soft Masses

Field	ψ	ψ^c	H	Σ	θ_3	θ_{23}	$\bar{\theta}_3$	$\bar{\theta}_{23}$
SU(3)	3	3	1	1	$\bar{3}$	$\bar{3}$	3	3
U(1)	-2	-2	0	-4	2	3	0	-2
U'(1)	0	0	0	1	0	-1	1	0

$$\bar{\theta}_{23,i} \theta_3^j$$

Alternative Soft Masses

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SU(3)	3	3	1	1	$\bar{3}$	$\bar{3}$	3	3
U(1)	-2	-2	0	-4	2	3	0	-2
U'(1)	0	0	0	1	0	-1	1	0

$$\bar{\theta}_{23,i} \theta_3^j$$

$$M_{\tilde{Q}}^2 = \begin{pmatrix} 1 + Q_1 \varepsilon^2 y_t & 0 & 0 \\ 0 & 1 + Q_2 \varepsilon^2 & Q_5 \varepsilon y_t^{0.5} e^{i\beta'_2} \\ 0 & Q_5 \varepsilon y_t^{0.5} e^{-i\beta'_2} & 1 + Q_4 y_t \end{pmatrix} m_0^2$$

RVV2

Alternative Soft Masses

Field	ψ	ψ^c	H	Σ	θ_3	θ_{23}	$\bar{\theta}_3$	$\bar{\theta}_{23}$
SU(3)	3	3	1	1	$\bar{3}$	$\bar{3}$	3	3
U(1)	-1	-1	0	5	1	-2	0	6
U'(1)	0	0	0	-1	0	0	1	-2

$$(\epsilon_{ikl} \theta_3^k \theta_{23}^l) \theta_3^j$$

Alternative Soft Masses

Field	ψ	ψ^c	H	Σ	θ_3	θ_{23}	$\bar{\theta}_3$	$\bar{\theta}_{23}$
SU(3)	3	3	1	1	$\bar{3}$	$\bar{3}$	3	3
U(1)	-1	-1	0	5	1	-2	0	6
U'(1)	0	0	0	-1	0	0	1	-2

$$(\epsilon_{ikl} \theta_3^k \theta_{23}^l) \theta_3^j$$

$$M_{\tilde{Q}}^2 = \begin{pmatrix} 1 + Q_1 \varepsilon^2 y_t & 0 & Q_6 \varepsilon y_t \\ 0 & 1 + Q_2 \varepsilon^2 & Q_3 \varepsilon^2 e^{i\beta_3} \\ Q_6 \varepsilon y_t & Q_3 \varepsilon^2 e^{-i\beta_3} & 1 + Q_4 y_t \end{pmatrix} m_0^2$$

RVV3

A-Terms

- Same flavour symmetries.
- Minimal SUGRA contribution: Holomorphic
- Structure is same as Yukawas
- Different origin: Different $O(1)$ Terms

$$(A_f)_{\alpha\beta} \propto (Y_f)_{\alpha\beta}$$

Phenomenology HOWTO

- Start with flavour structures at high scale.
- Apply canonical normalisation.
- Adjust Yukawa $O(1)$ s to precisely reproduce masses and mixings.
- Run RGE equations to electroweak scale.
- Rotate to SCKM basis.
- Calculate!

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Modification of O(1)s
S. King, I. Peddie, G. G. Ross,
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(hep-ph/0407012)

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SPheno
W. Porod (hep-ph/0301101)

Flavour and CP Violation Phenomenology
with Supersymmetric Flavour Symmetries

*Phenomenology in the
Lepton Sector*

L. Calibbi, JJP, O. Vives (0804.4620 [hep-ph])
L. Calibbi, JJP, A. Masiero, J.-h. Park, W. Porod, O. Vives (0907.4069 [hep-ph])

Lepton Phenomenology

- Neutrino Mixing
- Lepton Flavour Violation (LFV)
- Electric Dipole Moments (EDMs)

Lepton Phenomenology

- Lepton Flavour Violation (LFV)
- Electric Dipole Moments (EDMs)

Phenomenological Analysis

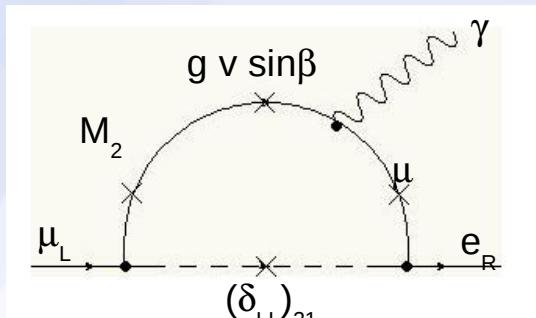
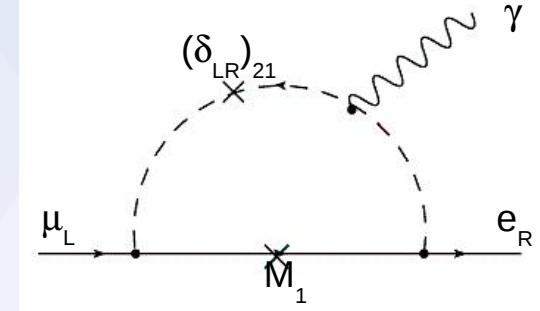
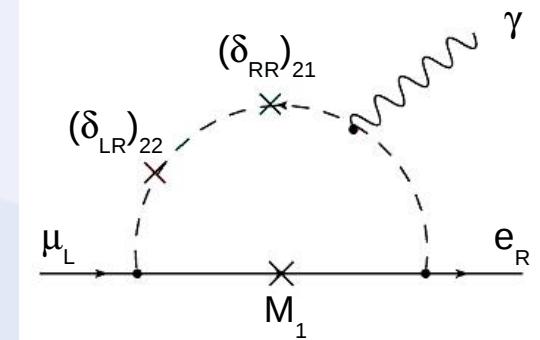
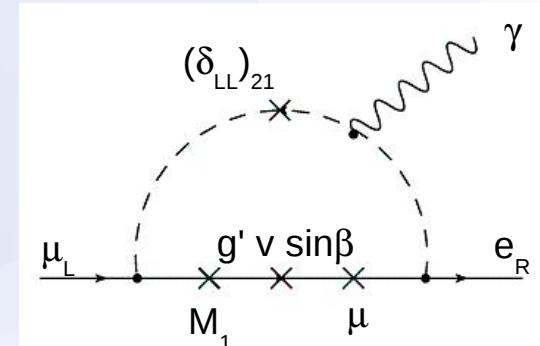
- Vary $m_0, M_{1/2}$
- Fix $\tan\beta, A_0, \mu > 0$
- Fix $O(1)$ parameters randomly

Lepton Flavour Violation

$$\mu \rightarrow e\gamma$$

$$\tau \rightarrow e\gamma$$

$$\tau \rightarrow \mu\gamma$$


$$\tilde{\chi}^\pm$$

$$\tilde{\chi}^0$$

Lepton Flavour Violation

$$\mu \rightarrow e\gamma \quad BR < 1.2 \times 10^{-11}$$

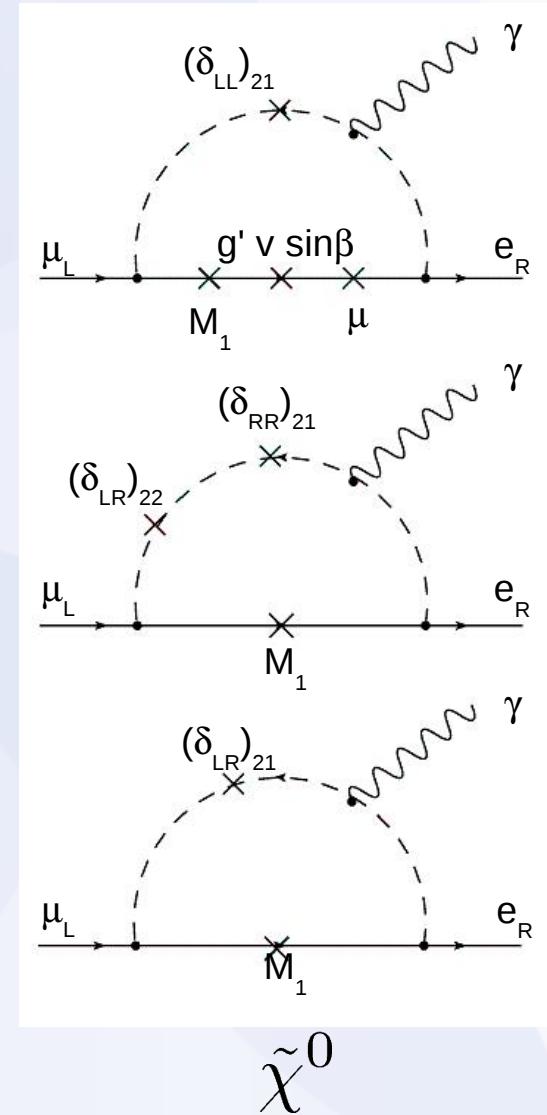
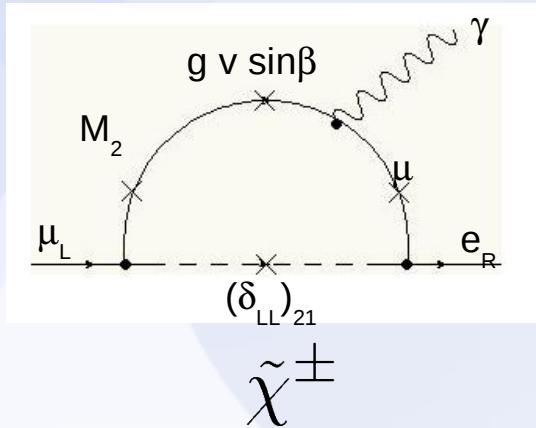
MEGA (hep-ex/9905013)

$$\tau \rightarrow e\gamma \quad BR < 3.3 \times 10^{-8}$$

BaBar (0908.2381 [hep-ex])

$$\tau \rightarrow \mu\gamma \quad BR < 1.6 \times 10^{-8}$$

Banerjee (hep-ex/0702017)
(BaBar + BELLE)



Lepton Flavour Violation

	$ (\delta_{LL}^e)_{12} $	$ (\delta_{LL}^e)_{13} $	$ (\delta_{LL}^e)_{23} $
RVV1	$\frac{1}{3}\varepsilon^2\bar{\varepsilon}$	$y_t\bar{\varepsilon}^3$	$3y_t\bar{\varepsilon}^2$
RVV2	$\frac{1}{3}\varepsilon^2\bar{\varepsilon}$	$\frac{1}{3}\sqrt{y_t}\varepsilon\bar{\varepsilon}$	$\sqrt{y_t}\varepsilon$
RVV3	$3y_t\varepsilon\bar{\varepsilon}^2$	$y_t\varepsilon$	$3y_t\bar{\varepsilon}^2$

	$ (\delta_{RR}^e)_{12} $	$ (\delta_{RR}^e)_{13} $	$ (\delta_{RR}^e)_{23} $
RVV1	$\frac{1}{3}\bar{\varepsilon}^3$	$\frac{1}{3}\bar{\varepsilon}^3$	$\bar{\varepsilon}^2$
RVV2	$\frac{1}{3}\bar{\varepsilon}^3$	$\frac{1}{3}\sqrt{y_b}\bar{\varepsilon}^2$	$\sqrt{y_b}\bar{\varepsilon}$
RVV3	$\frac{1}{3}\bar{\varepsilon}^3$	$y_b\bar{\varepsilon}$	$\bar{\varepsilon}^2$

Lepton Flavour Violation

	$ (\delta_{LL}^e)_{12} $	$ (\delta_{LL}^e)_{13} $	$ (\delta_{LL}^e)_{23} $
RVV1	$\frac{1}{3}\varepsilon^2\bar{\varepsilon}$	$y_t\bar{\varepsilon}^3$	$3y_t\bar{\varepsilon}^2$
RVV2	$\frac{1}{3}\varepsilon^2\bar{\varepsilon}$	$\frac{1}{3}\sqrt{y_t}\varepsilon\bar{\varepsilon}$	$\sqrt{y_t}\varepsilon$
RVV3	$3y_t\varepsilon\bar{\varepsilon}^2$	$y_t\varepsilon$	$3y_t\bar{\varepsilon}^2$

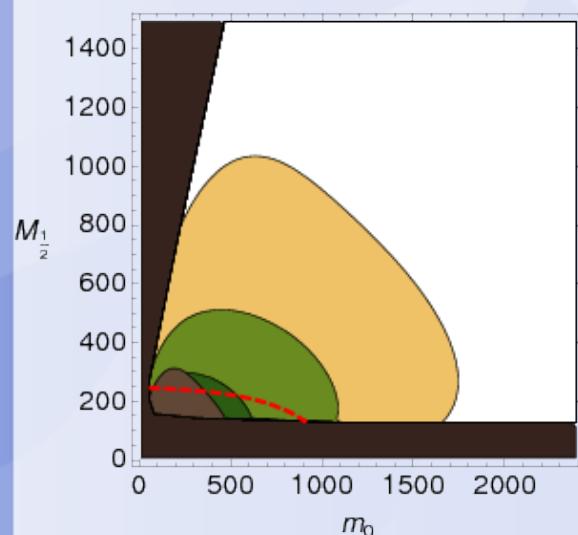
	$ (\delta_{RR}^e)_{12} $	$ (\delta_{RR}^e)_{13} $	$ (\delta_{RR}^e)_{23} $
RVV1	$\frac{1}{3}\bar{\varepsilon}^3$	$\frac{1}{3}\bar{\varepsilon}^3$	$\bar{\varepsilon}^2$
RVV2	$\frac{1}{3}\bar{\varepsilon}^3$	$\frac{1}{3}\sqrt{y_b}\bar{\varepsilon}^2$	$\sqrt{y_b}\bar{\varepsilon}$
RVV3	$\frac{1}{3}\bar{\varepsilon}^3$	$y_b\bar{\varepsilon}$	$\bar{\varepsilon}^2$

Lepton Flavour Violation

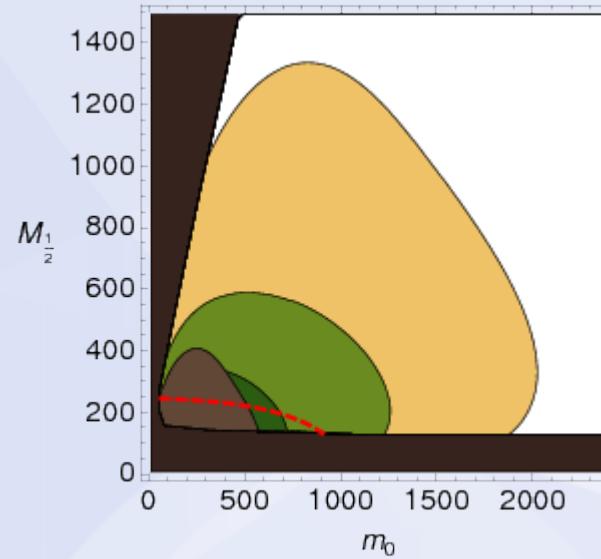
	$ (\delta_{LL}^e)_{12} $	$ (\delta_{LL}^e)_{13} $	$ (\delta_{LL}^e)_{23} $
RVV1	$\frac{1}{3}\varepsilon^2\bar{\varepsilon}$	$y_t\bar{\varepsilon}^3$	$3y_t\bar{\varepsilon}^2$
RVV2	$\frac{1}{3}\varepsilon^2\bar{\varepsilon}$	$\frac{1}{3}\sqrt{y_t}\varepsilon\bar{\varepsilon}$	$\sqrt{y_t}\varepsilon$
RVV3	$3y_t\varepsilon\bar{\varepsilon}^2$	$y_t\varepsilon$	$3y_t\bar{\varepsilon}^2$

	$ (\delta_{RR}^e)_{12} $	$ (\delta_{RR}^e)_{13} $	$ (\delta_{RR}^e)_{23} $
RVV1	$\frac{1}{3}\bar{\varepsilon}^3$	$\frac{1}{3}\bar{\varepsilon}^3$	$\bar{\varepsilon}^2$
RVV2	$\frac{1}{3}\bar{\varepsilon}^3$	$\frac{1}{3}\sqrt{y_b}\bar{\varepsilon}^2$	$\sqrt{y_b}\bar{\varepsilon}$
RVV3	$\frac{1}{3}\bar{\varepsilon}^3$	$y_b\bar{\varepsilon}$	$\bar{\varepsilon}^2$

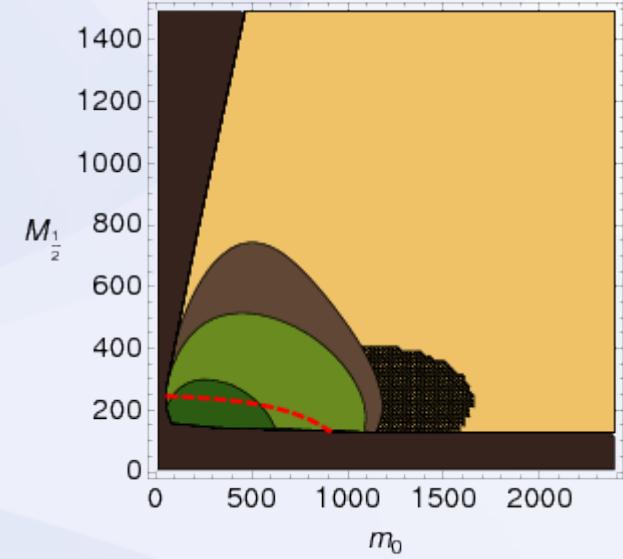
Lepton Flavour Violation



RVV1



RVV2



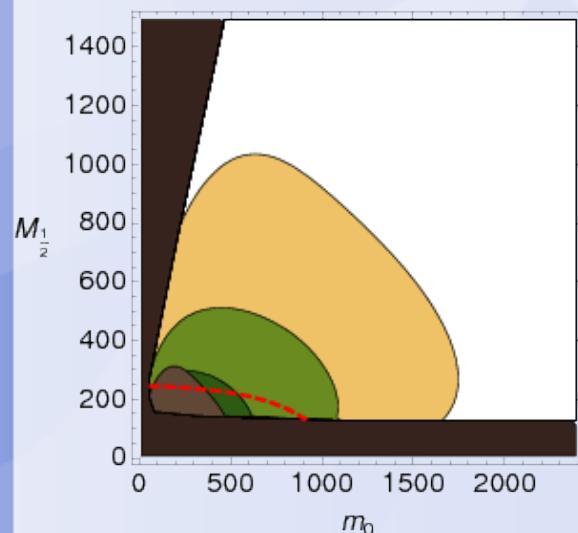
RVV3

- Direct Search + LSP Bound
- MEGA ($\mu \rightarrow e \gamma$)
- BaBar + BELLE ($\tau \rightarrow \mu \gamma$)
- MEG (10^{-13})
- Super Flavour Factory (10^{-9})

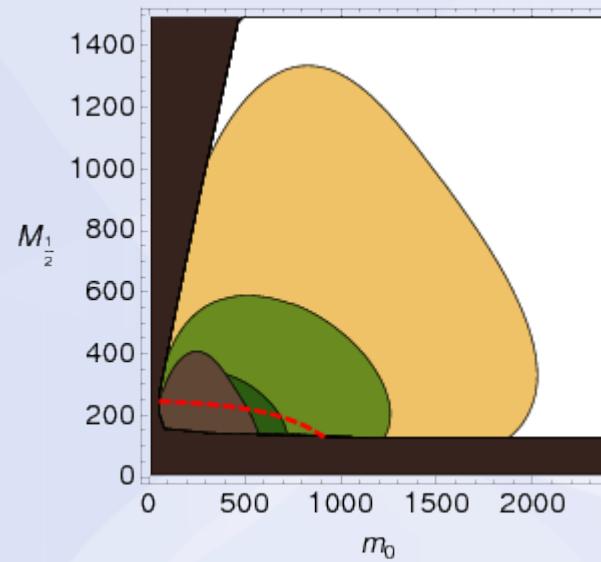
- Higgs Bound
- Meson Bound

$$\tan\beta = 10$$
$$A_0 = 0$$

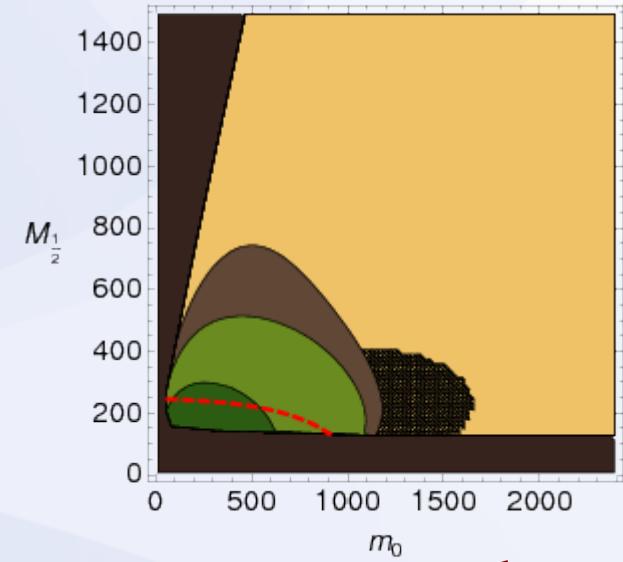
Lepton Flavour Violation



RVV1



RVV2



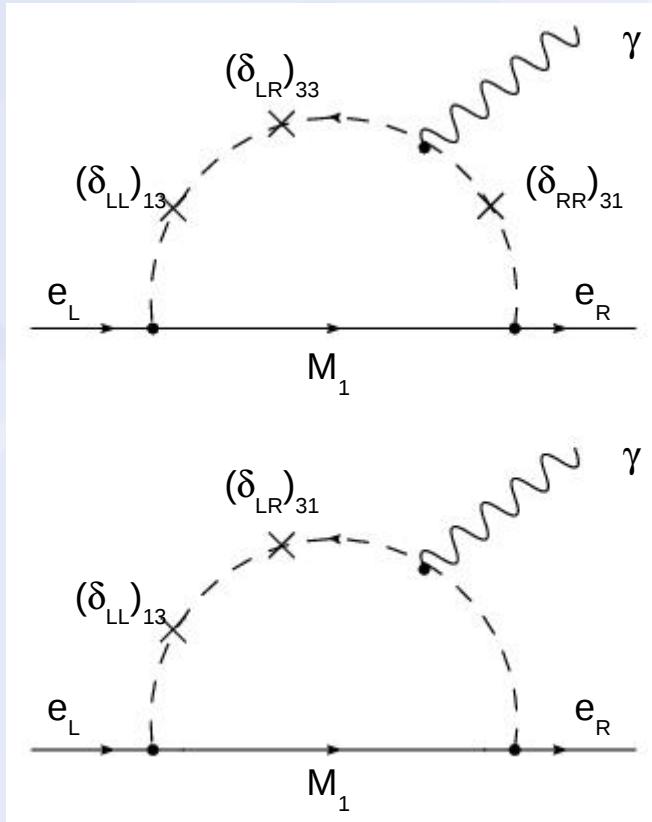
- Direct Search + LSP Bound
- MEGA ($\mu \rightarrow e \gamma$)
- BaBar + BELLE ($\tau \rightarrow \mu \gamma$)
- MEG (10^{-13})
- Super Flavour Factory (10^{-9})

- Higgs Bound
- Meson Bound

$$\tan\beta = 10$$
$$A_0 = 0$$

Electric Dipole Moments

$$-\frac{d_e}{2} [\bar{e} \sigma^{\mu\nu} \gamma^5 e] F_{\mu\nu}$$

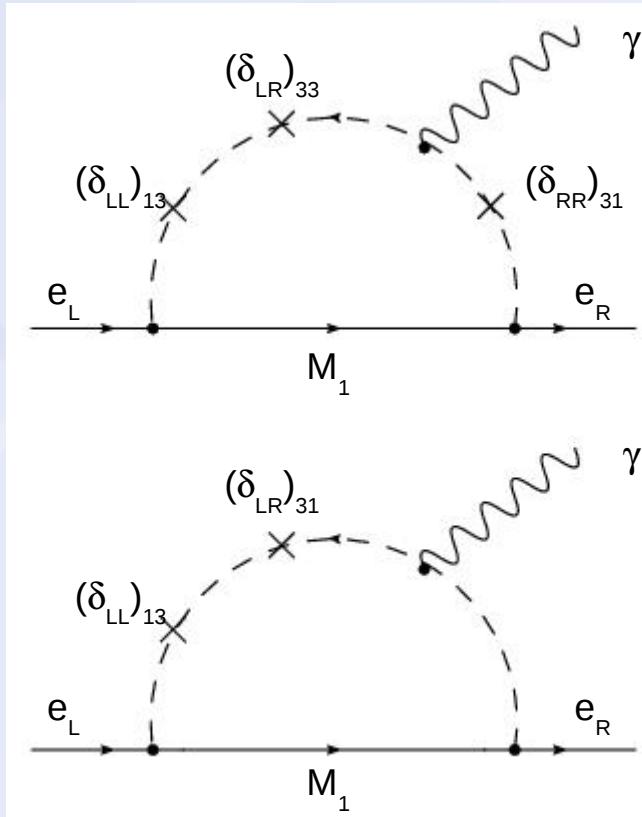
 $\tilde{\chi}^0$

Electric Dipole Moments

$$-\frac{d_e}{2} [\bar{e} \sigma^{\mu\nu} \gamma^5 e] F_{\mu\nu}$$

$$|d_e| < 1.4 \times 10^{-27}$$

B. C. Regan, E.D. Commins,
C.J. Schmidt, D. DeMille
(Phys.Rev.Lett.88:071805,2002)



$\tilde{\chi}^0$

Electric Dipole Moments

$$(\delta_{LL}^e)_{13} (\delta_{LR}^e)_{33} (\delta_{RR}^e)_{31}$$

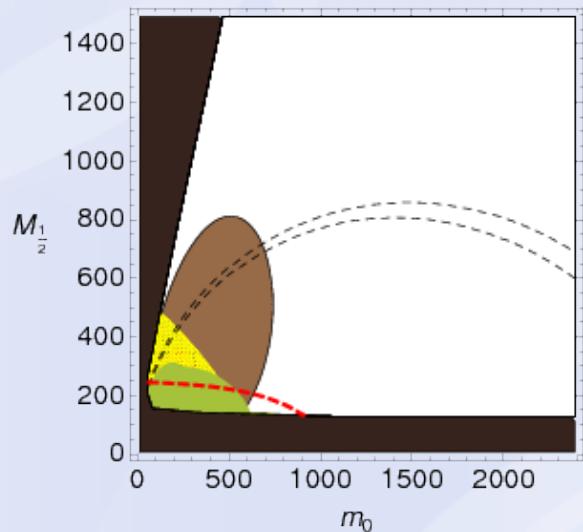
$$(\delta_{LL}^e)_{13} (\delta_{LR}^e)_{31}$$

Flavour Suppression:

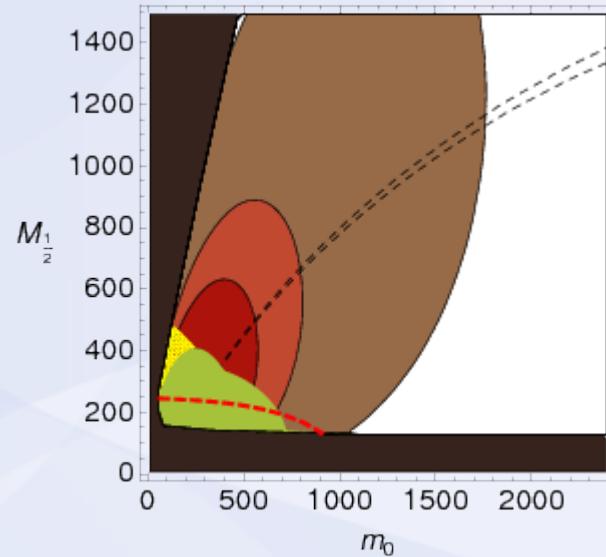
$$(d_e)_{\text{RVV1}} \sim \bar{\varepsilon}^6 y_t \sim 1 \times 10^{-5}$$

$$(d_e)_{\text{RVV2}} \sim \frac{\varepsilon^{1.5} \bar{\varepsilon}^3}{\sqrt{3}} y_t^{0.5} (\tan \beta)^{0.5} \sim 7 \times 10^{-5}$$

Electric Dipole Moments



RVV1



RVV2

- Direct Search + LSP Bound
- LFV Bounds
- $d_e > 10^{-28}$
- $d_e > 5 \times 10^{-29}$
- $d_e > 10^{-29}$

- Higgs Bound
 - $(g-2)_\mu$ Region
 - ε_K Strip
- $\tan\beta = 10$
 $A_0 = 0$

Flavour and CP Violation Phenomenology
with Supersymmetric Flavour Symmetries

*Phenomenology in the
Quark Sector*

Quark Phenomenology

- K Sector (ε_K)
- B_s Sector (Φ_{Bs})
- Neutron Electric Dipole Moment

K Mesons

- Tension: ε_K , $\sin 2\beta$, $\Delta M_s / \Delta M_d$

A. Buras, D. Guadagnoli (0805.3887 [hep-ph])

W. Altmannshofer, A. Buras, S. Gori, P. Paradisi, D. Straub (0909.1333 [hep-ph])

$$\varepsilon_K^{exp} = (2.228 \pm 0.011) \times 10^{-3} \quad \varepsilon_K^{th} = (1.78 \pm 0.25) \times 10^{-3}$$

K Mesons

- Tension: ε_K , $\sin 2\beta$, $\Delta M_s / \Delta M_d$

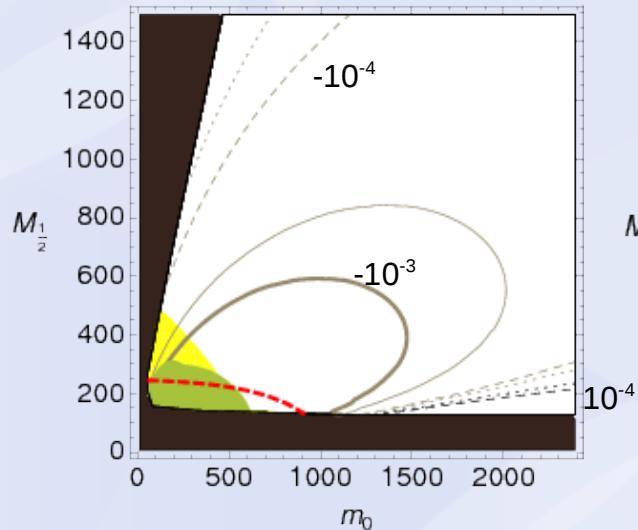
A. Buras, D. Guadagnoli (0805.3887 [hep-ph])

W. Altmannshofer, A. Buras, S. Gori, P. Paradisi, D. Straub (0909.1333 [hep-ph])

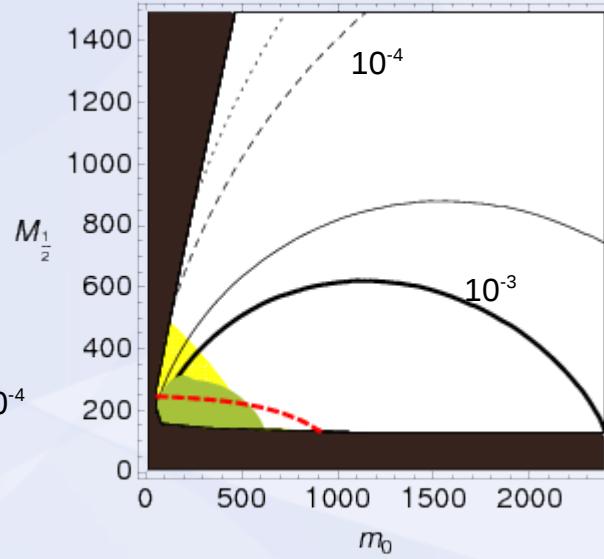
$$\varepsilon_K^{exp} = (2.228 \pm 0.011) \times 10^{-3} \quad \varepsilon_K^{th} = (1.78 \pm 0.25) \times 10^{-3}$$

- RVV: No significant contribution to ΔM_i or $\sin 2\beta$.
- Sizeable contribution to ε_K .

K Mesons: RVV1



$$O(1)_{RR} > 0$$



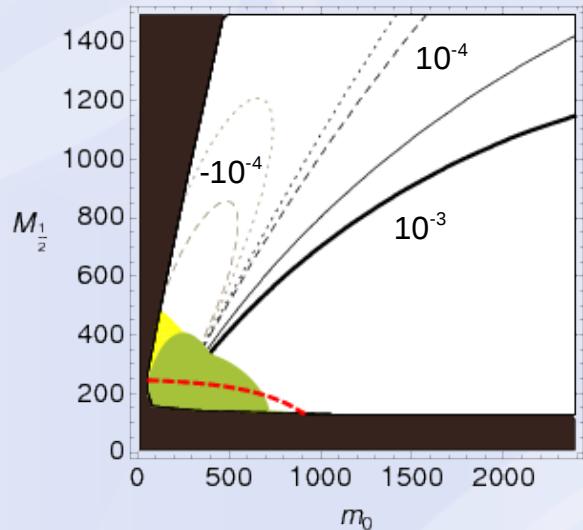
$$O(1)_{RR} < 0$$

■ Direct Search + LSP Bound
■ LFV Bounds

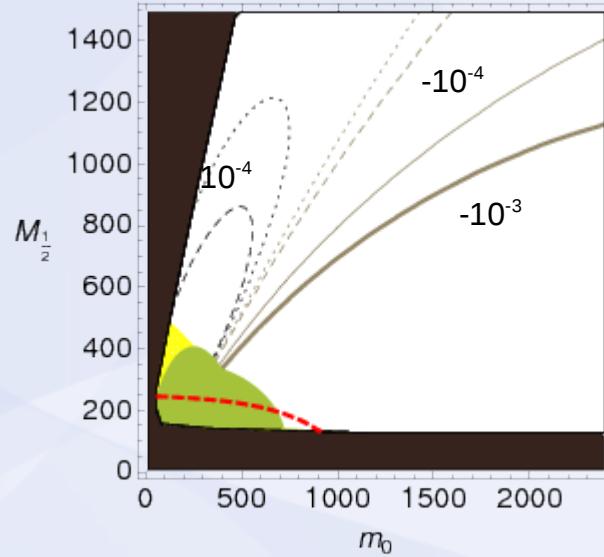
○ Higgs Bound
■ $(g-2)_\mu$ Region

$\tan\beta = 10$
 $A_0 = 0$

K Mesons: RVV2



$O(1)_{RR} > 0$



$O(1)_{RR} < 0$

Direct Search + LSP Bound
 LFV Bounds

Higgs Bound
 $(g-2)_\mu$ Region

$\tan\beta = 10$
 $A_0 = 0$

B_s Mesons

$$C_{B_s} e^{2i\phi_{B_s}} = \frac{M_{s12}^{\text{SM}} + M_{s12}^{\text{SUSY}}}{M_{s12}^{\text{SM}}} = 1 + \frac{M_{s12}^{\text{SUSY}}}{M_{s12}^{\text{SM}}}.$$

B_s Mesons

$$C_{B_s} e^{2i\phi_{B_s}} = \frac{M_{s12}^{\text{SM}} + M_{s12}^{\text{SUSY}}}{M_{s12}^{\text{SM}}} = 1 + \frac{M_{s12}^{\text{SUSY}}}{M_{s12}^{\text{SM}}}.$$

- Tension with Φ_{B_s} phase.

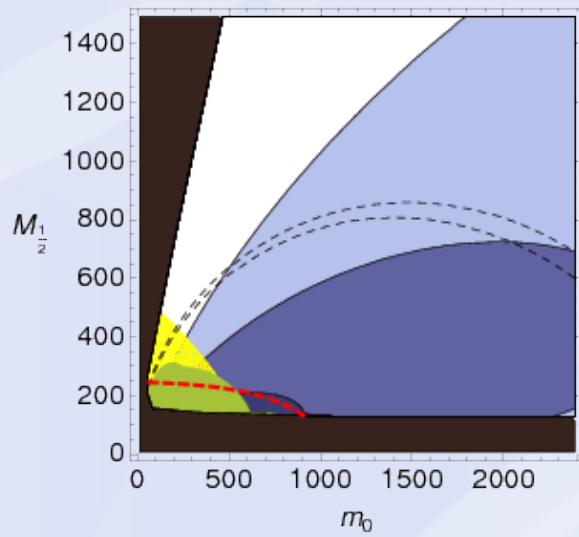
UTFit Collaboration (0803.0659 [hep-ph])

$$\phi_{B_s} \in [-0.62, -0.086] \cup [-1.46, -0.93].$$

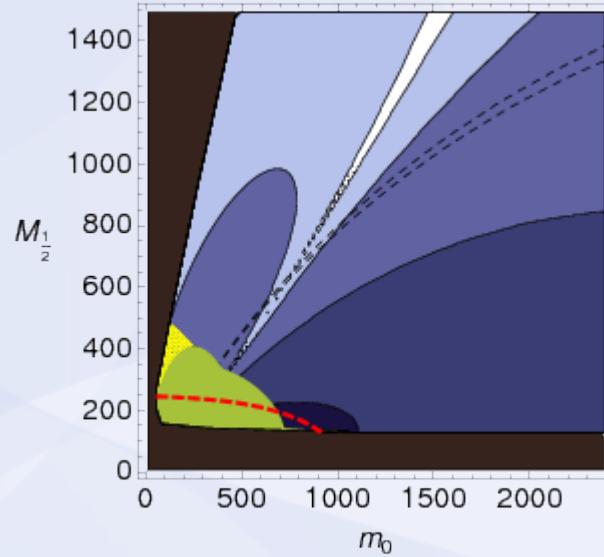
- Related to D0 same-sign muon anomaly.

D0 Collaboration (1005.2757 [hep-ex])

Φ_{Bs}



RVV1



RVV2

- Direct Search + LSP Bound
- LFV Bounds
- $\Phi_{Bs} > 10^{-2}$
- $\Phi_{Bs} > 10^{-3}$
- $\Phi_{Bs} > 10^{-4}$
- $\Phi_{Bs} > 10^{-5}$

- Higgs Bound
- (g-2) $_\mu$ Region
- ε_K Strip

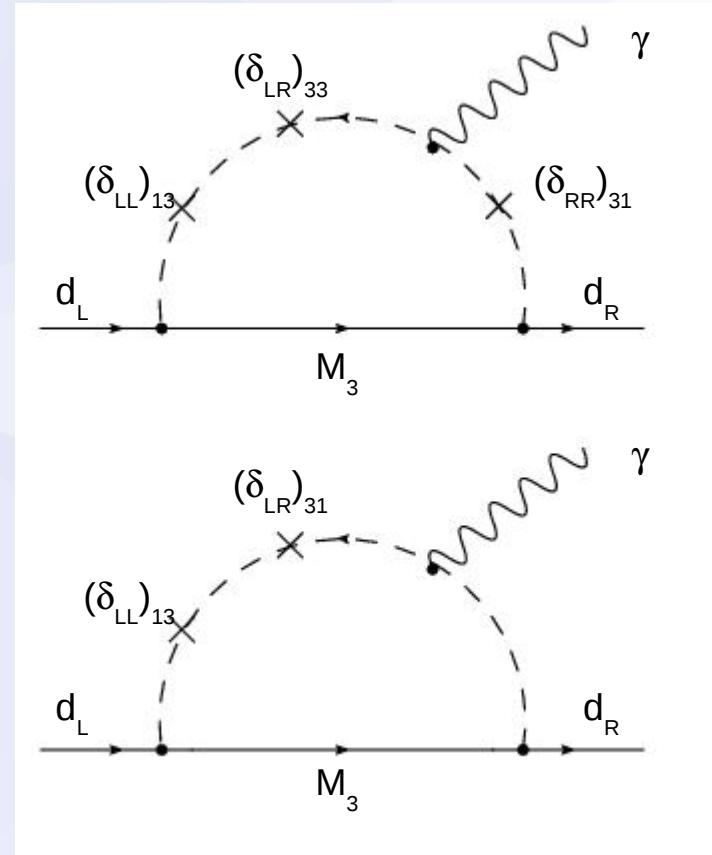
$\tan\beta = 10$
 $A_0 = 0$

Neutron Electric Dipole Moment

$$-\frac{d_n}{2} \left[\bar{n} \sigma^{\mu\nu} \gamma^5 n \right] F_{\mu\nu}$$

$$|d_n| < 2.9 \times 10^{-26}$$

C. A. Baker *et al* (hep-ex/0602020)



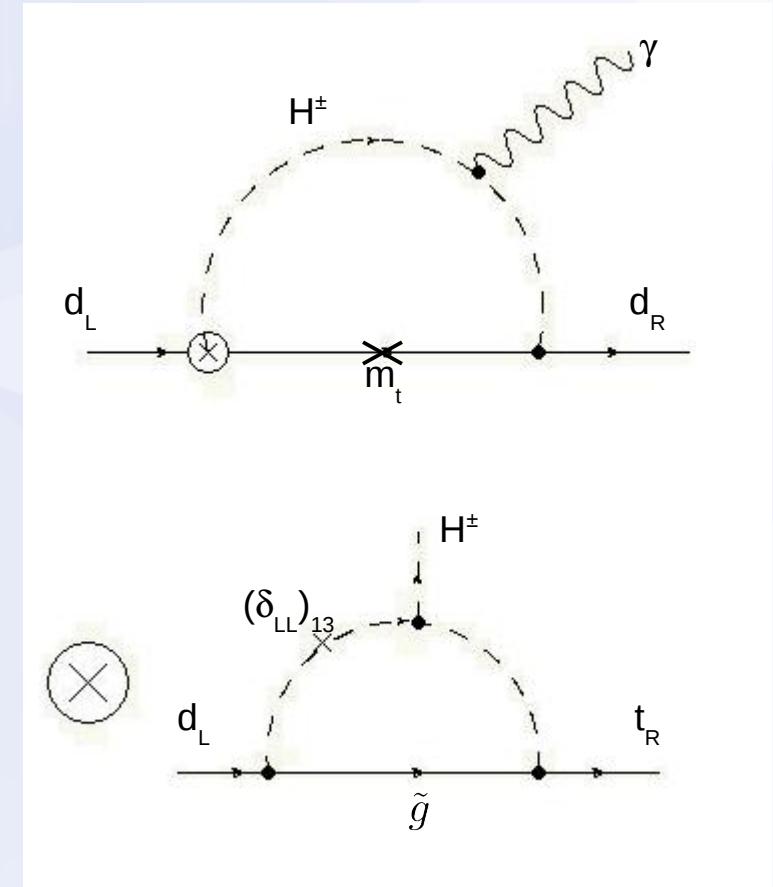
\tilde{g}

Neutron Electric Dipole Moment

$$-\frac{d_n}{2} \left[\bar{n} \sigma^{\mu\nu} \gamma^5 n \right] F_{\mu\nu}$$

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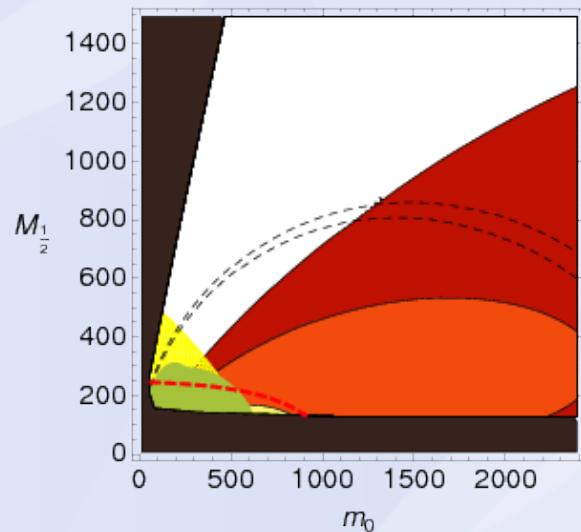
C. A. Baker *et al* (hep-ex/0602020)



J. Hisano, M. Nagai, P. Paradisi
(0812.4283 [hep-ph])

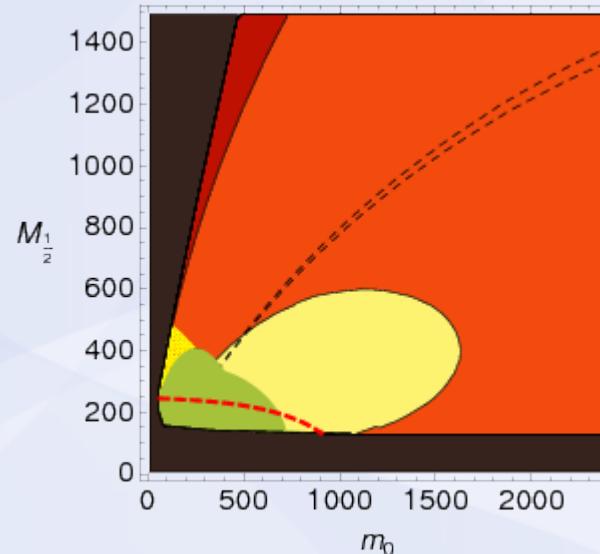
Neutron Electric Dipole Moment Quark-Parton Model

J. Ellis, R. Flores
(hep-ph/9602211)



RVV1

- Direct Search + LSP Bound
- LFV Bounds
- $d_n > 10^{-27}$
- $d_n > 10^{-28}$
- $d_n > 10^{-29}$



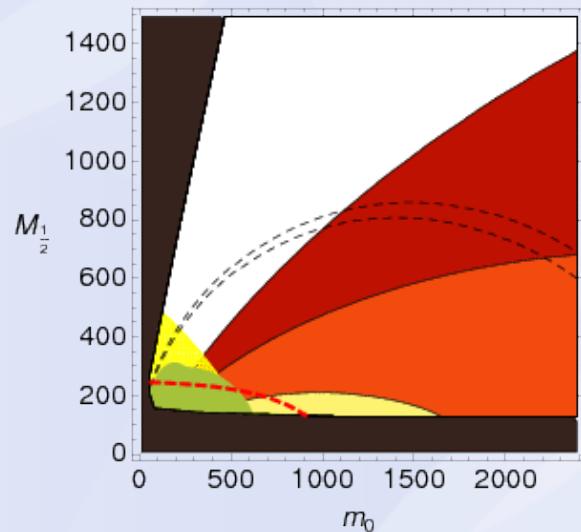
RVV2

- Higgs Bound
- $(g-2)_\mu$ Region
- ε_K Strip

$$\tan\beta = 10$$
$$A_0 = 0$$

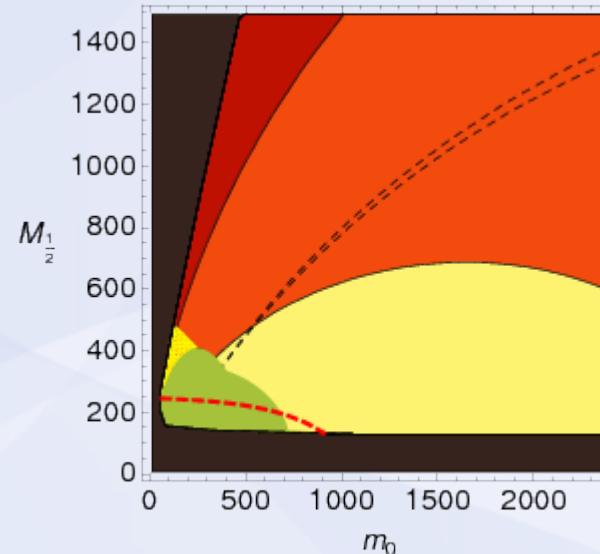
Neutron Electric Dipole Moment Chiral Quark Model

A. Manohar, G. Georgi
(Nucl.Phys.B234:189)



RVV1

- Direct Search + LSP Bound
- LFV Bounds
- $d_n > 10^{-27}$
- $d_n > 10^{-28}$
- $d_n > 10^{-29}$



RVV2

- Higgs Bound
- $(g-2)_\mu$ Region
- ε_K Strip

$$\tan\beta = 10$$
$$A_0 = 0$$

Flavour and CP Violation Phenomenology with Supersymmetric Flavour Symmetries

*Correlations between
Observables*

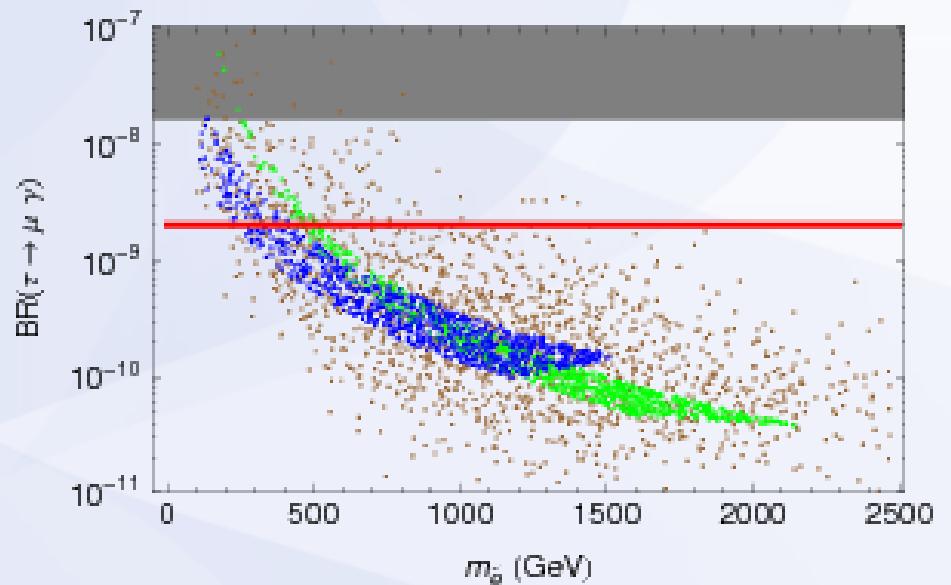
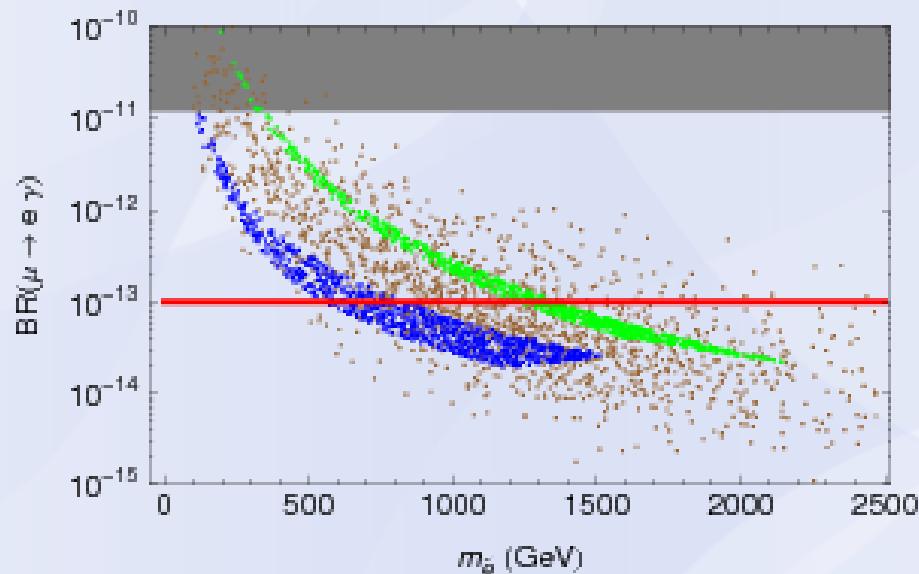
How to Differentiate the Models?

- Different models could predict the same observation for different values of m_0 , $M_{1/2}$, $O(1)$ s, etc...
- How do we disentangle the information?

How to Differentiate the Models?

- Different models could predict the same observation for different values of m_0 , $M_{1/2}$, $O(1)$ s, etc...
- How do we disentangle the information?
- Correlations help!
- Enhance correlations by demanding a solution to the ε_K puzzle.

Correlation with Masses

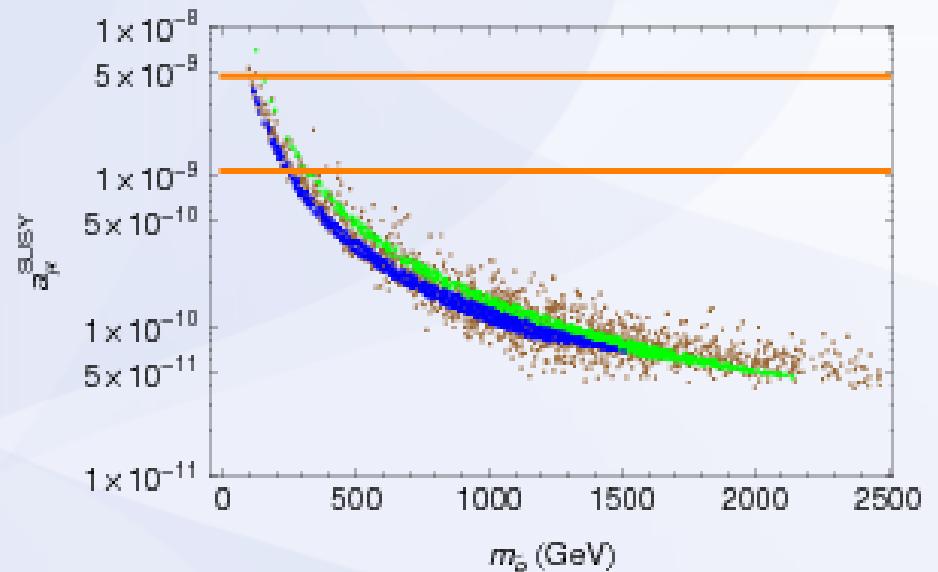
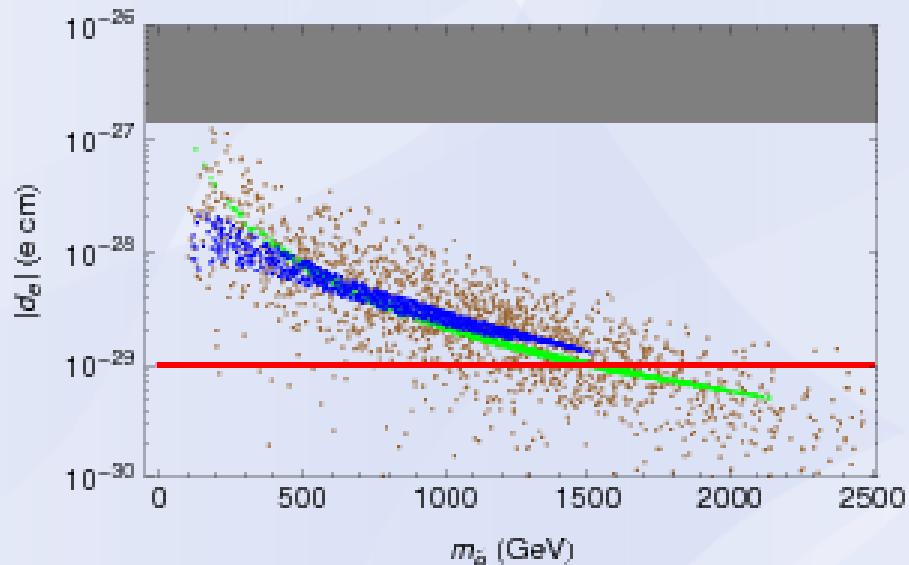


RVV2

- O(1) = 1
- O(1) = -1
- Random O(1)

- Future Bound
 - Current Bound
- $\tan\beta = 10$
 $A_0 = 0$

Correlation with Masses

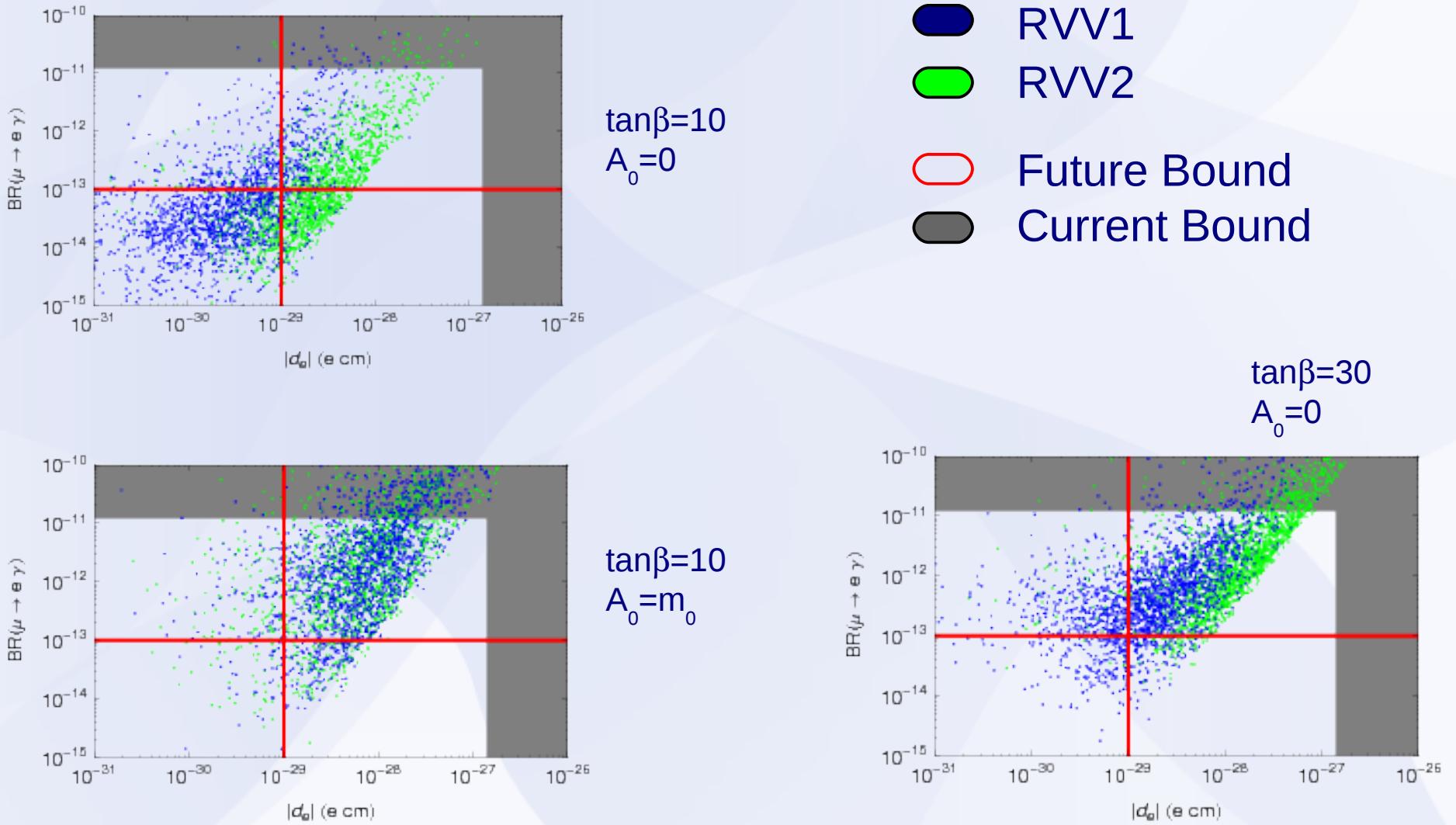


RVV2

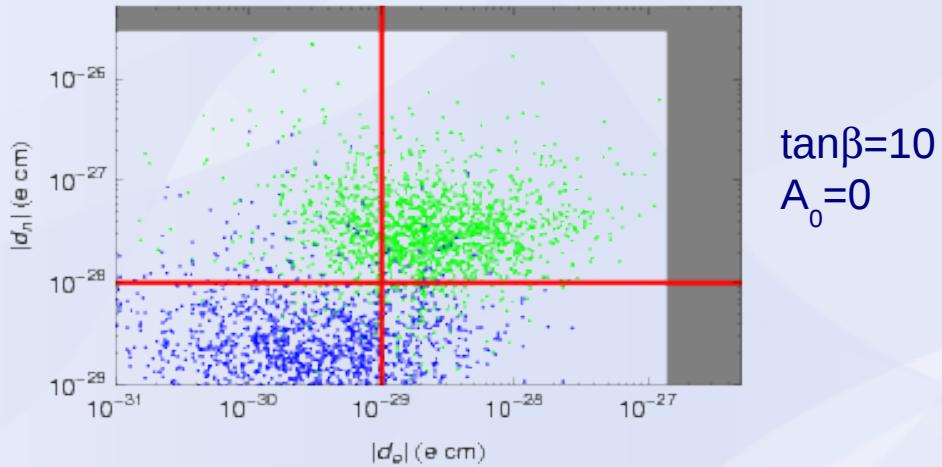
- $O(1) = 1$
- $O(1) = -1$
- Random $O(1)$

- Future Bound
- Current Bound
- $(g-2)_\mu^{2\sigma} \quad \tan\beta = 10$
 $A_0 = 0$

Low Energy Correlations

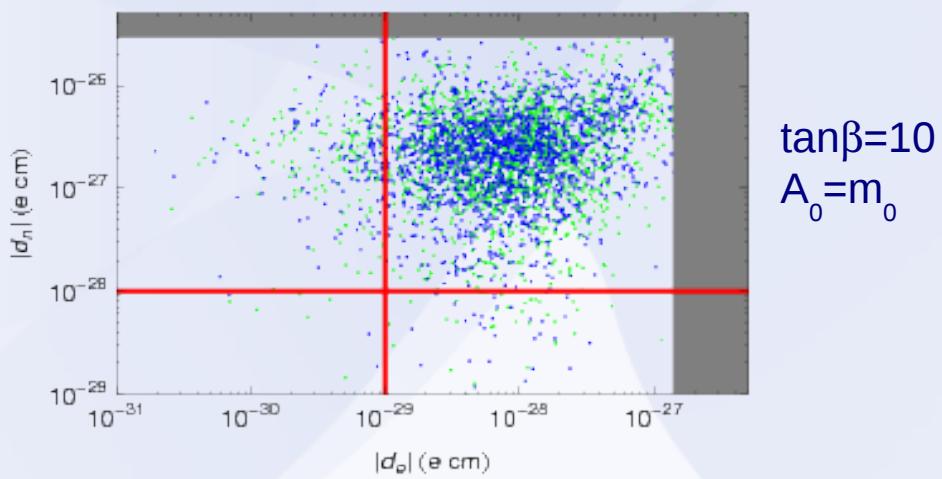


Low Energy Correlations

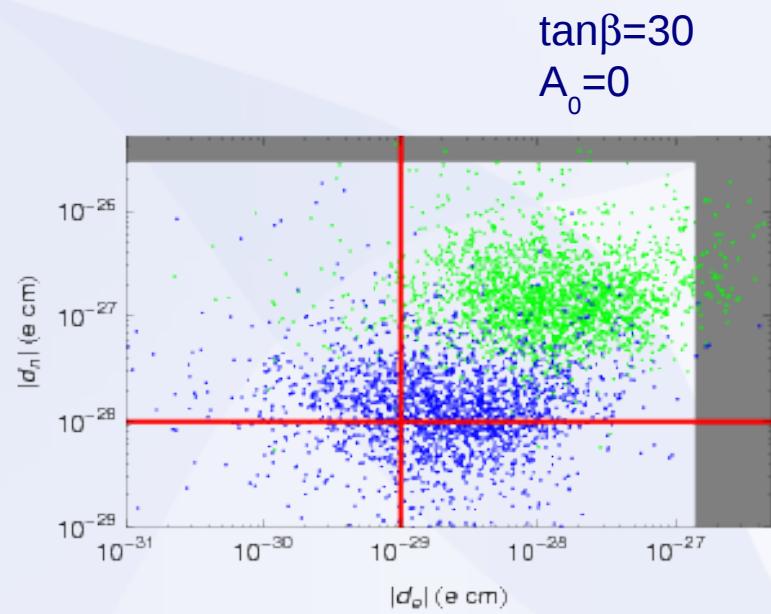


$\tan\beta=10$
 $A_0=0$

- RVV1
- RVV2
- Future Bound
- Current Bound

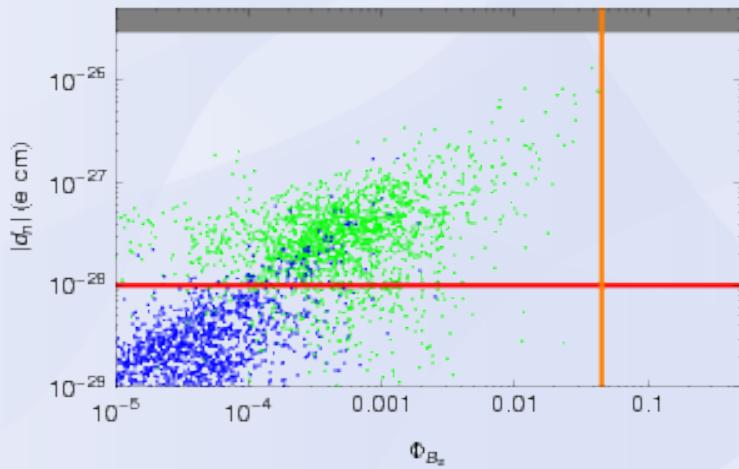


$\tan\beta=10$
 $A_0=m_0$



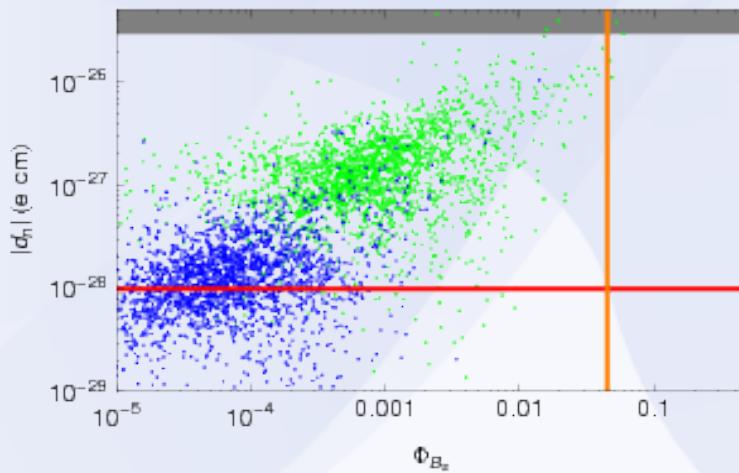
$\tan\beta=30$
 $A_0=0$

Low Energy Correlations

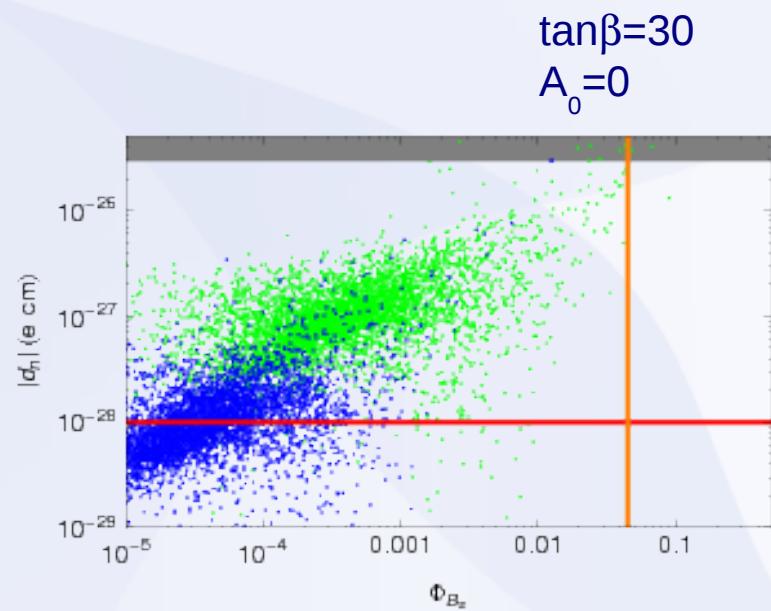


$\tan\beta=10$
 $A_0=0$

- RVV1
- RVV2
- Future Bound
- Current Bound
- Φ_{Bs} Bound



$\tan\beta=10$
 $A_0=m_0$



$\tan\beta=30$
 $A_0=0$

Flavour and CP Violation Phenomenology with Supersymmetric Flavour Symmetries

Conclusions

SU(3) Model

- Explains SM flavour sector.
- Generates SUSY flavour structures.
- Addresses CP Violation.
- Testable soon.

SU(3) Phenomenology

Leptons

- ◆ LFV: $\mu \rightarrow e \gamma$ is crucial
- ◆ Electron EDM: Very important

SU(3) Phenomenology

Leptons

- LFV: $\mu \rightarrow e \gamma$ is crucial
- Electron EDM: Very important

Quarks

- ε_K : Imposes strong constraints
- Φ_{B_s} : Must go away
- Neutron EDM: Very important

Flavour and CP Violation Phenomenology with Supersymmetric Flavour Symmetries

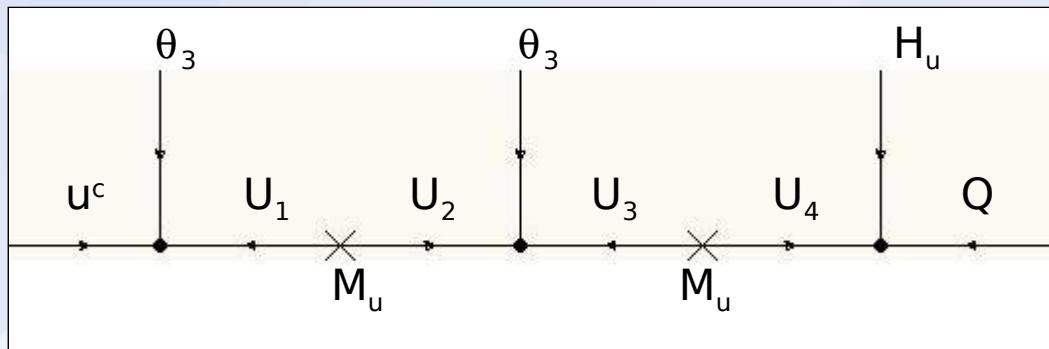
Backup Slides

Renormalizability

$$\begin{aligned} W_1 = & u_R^c U_1 \theta_3 + U_2 U_3 \theta_3 + U_4 Q_L H_u \\ & + d_R^c D_1 \theta_3 + D_2 D_3 \theta_3 + D_4 Q_L H_u \\ & + M_u (U_1 U_2 + U_3 U_4) \\ & + M_d (D_1 D_2 + D_3 D_4) \end{aligned}$$

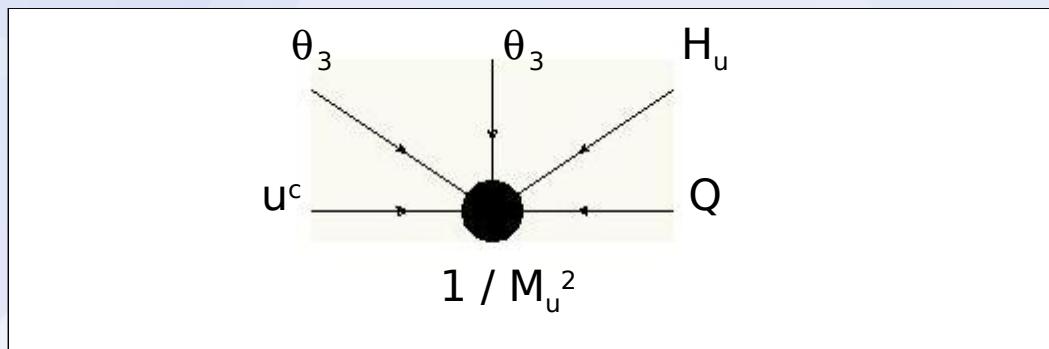
Renormalizability

$$\begin{aligned} W_1 = & u_R^c U_1 \theta_3 + U_2 U_3 \theta_3 + U_4 Q_L H_u \\ & + d_R^c D_1 \theta_3 + D_2 D_3 \theta_3 + D_4 Q_L H_u \\ & + M_u (U_1 U_2 + U_3 U_4) \\ & + M_d (D_1 D_2 + D_3 D_4) \end{aligned}$$



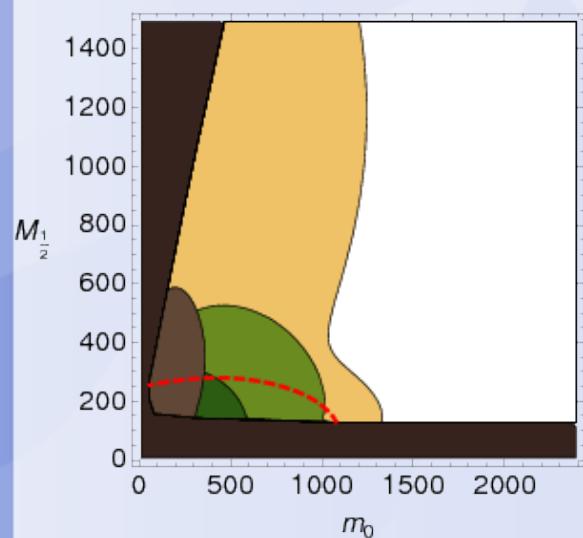
Renormalizability

$$\begin{aligned} W_1 = & u_R^c U_1 \theta_3 + U_2 U_3 \theta_3 + U_4 Q_L H_u \\ & + d_R^c D_1 \theta_3 + D_2 D_3 \theta_3 + D_4 Q_L H_u \\ & + M_u (U_1 U_2 + U_3 U_4) \\ & + M_d (D_1 D_2 + D_3 D_4) \end{aligned}$$

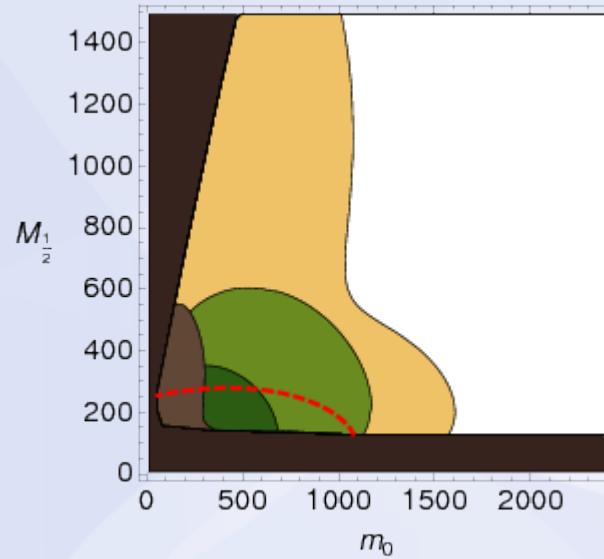


$$Q_L u_R^c H_u \frac{\theta_3 \theta_3}{M_u^2}$$

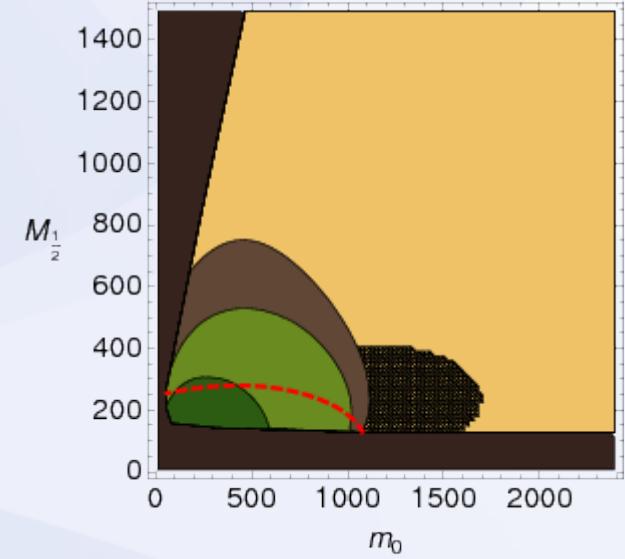
Lepton Flavour Violation



RVV1



RVV2



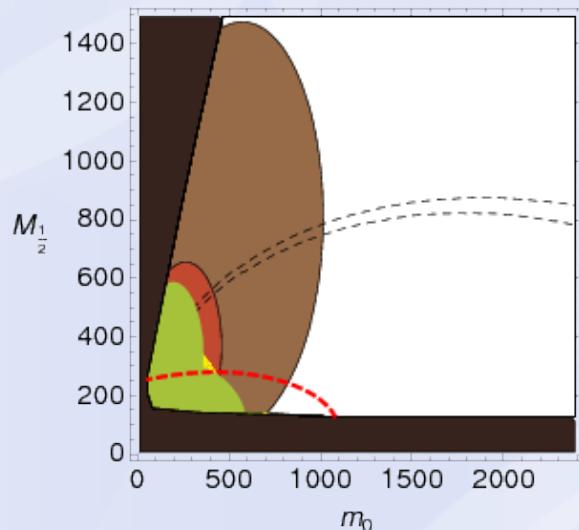
RVV3

- Direct Search + LSP Bound
- MEGA ($\mu \rightarrow e \gamma$)
- BaBar + BELLE ($\tau \rightarrow \mu \gamma$)
- MEG (10^{-13})
- Super Flavour Factory (10^{-9})

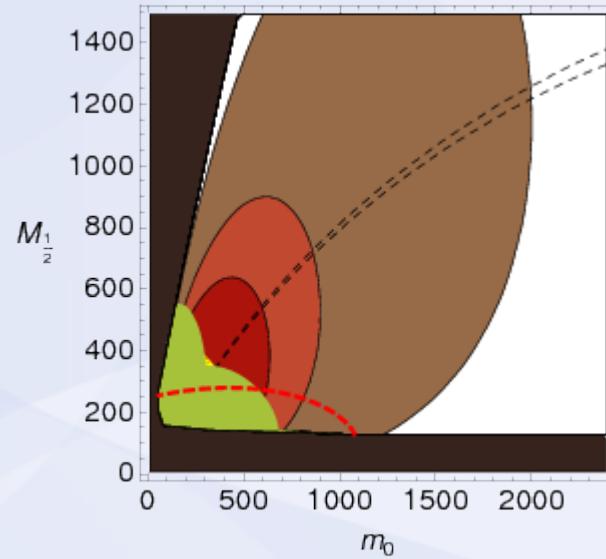
- Higgs Bound
- ▨ Meson Bound

$$\tan\beta = 10$$
$$A_0 = m_0$$

Electric Dipole Moments



RVV1

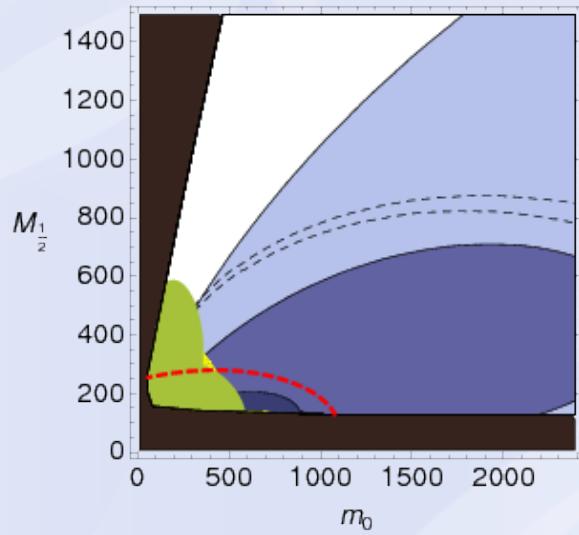


RVV2

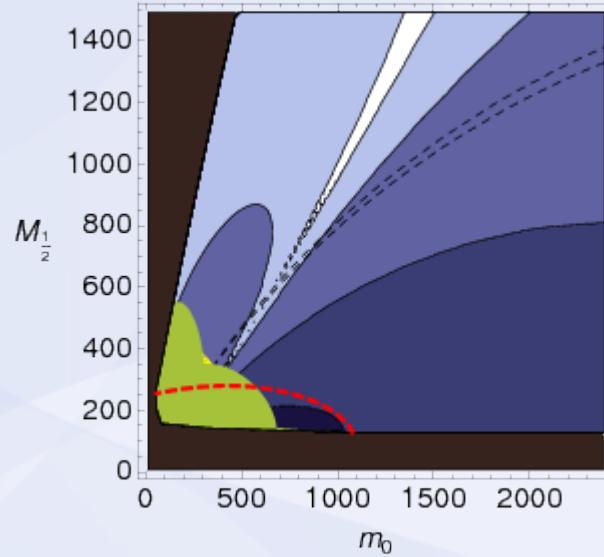
- Direct Search + LSP Bound
- LFV Bounds
- $d_e > 10^{-28}$
- $d_e > 5 \times 10^{-29}$
- $d_e > 10^{-29}$

- Higgs Bound
 - $(g-2)_\mu$ Region
 - ε_K Strip
- $\tan\beta = 10$
 $A_0 = m_0$

Φ_{Bs}



RVV1



RVV2

- Direct Search + LSP Bound
- LFV Bounds
- $\Phi_{Bs} > 10^{-2}$
- $\Phi_{Bs} > 10^{-3}$
- $\Phi_{Bs} > 10^{-4}$
- $\Phi_{Bs} > 10^{-5}$

- Higgs Bound
- (g-2) $_\mu$ Region
- ε_K Strip

$\tan\beta = 10$
 $A_0 = m_0$

Leptons

$$\langle \Sigma \rangle = (B - L + 2T_3^R)$$

$$\Sigma_d = \frac{\langle \Sigma \rangle_d}{M_d} \propto \frac{1}{M_d} \left(\frac{1}{3} - 0 - 1 \right) = -\frac{2}{3} \frac{1}{M_d}$$

$$\Sigma_u = \frac{\langle \Sigma \rangle_u}{M_u} \propto \frac{1}{M_u} \left(\frac{1}{3} - 0 + 1 \right) = \frac{4}{3} \frac{1}{M_u}$$

$$\Sigma_e = \frac{\langle \Sigma \rangle_e}{M_d} \propto \frac{1}{M_d} (0 - 1 - 1) = -2 \frac{1}{M_d}$$

$$\Sigma_\nu = \frac{\langle \Sigma \rangle_\nu}{M_u} \propto \frac{1}{M_u} (0 - 1 + 1) = 0$$

$$\Sigma_e = 3\Sigma_d$$

Neutrino Mixing

Assumption: Type 1 See-Saw mechanism

- Introduce L-violating flavon, θ_ν .
- Generate Majorana mass matrix.
- Rotate Y_ν to Y_e -diagonal basis.
- Build neutrino mass matrix.

Neutrino Mixing

$$\begin{aligned} M_\nu &= v_u^2 (Y_\nu^e) (M_R)^{-1} (Y_\nu^e)^T \\ &= \frac{v_u^2}{M_\nu} \begin{pmatrix} -2 \left(\frac{x_{12}^e (x_{12}^\nu)^2}{y_t \Sigma_e} \right) \bar{\varepsilon} \varepsilon & \frac{(x_{12}^\nu)^2}{y_t} \varepsilon & \left(\frac{x_{12}^\nu x_{13}^\nu}{y_t} \right) \varepsilon \\ \frac{(x_{12}^\nu)^2}{y_t} \varepsilon & 2 \left(\frac{x_{12}^e (x_{12}^\nu)^2}{y_t \Sigma_e} \right) \bar{\varepsilon} \varepsilon & -x_{23}^e \Sigma_e \bar{\varepsilon}^2 \\ \left(\frac{x_{12}^\nu x_{13}^\nu}{y_t} \right) \varepsilon & -x_{23}^e \Sigma_e \bar{\varepsilon}^2 & 1 \end{pmatrix} y_t^2 \end{aligned}$$

Anarchic Mixing
Difficult to satisfy 3σ bounds.

Neutrino Mixing

$$\begin{aligned} M_\nu &= v_u^2 (Y_\nu^e) (M_R)^{-1} (Y_\nu^e)^T \\ &= \frac{v_u^2}{M_\nu} \begin{pmatrix} -2 \left(\frac{x_{12}^e (x_{12}^\nu)^2}{y_t \Sigma_e} \right) \bar{\varepsilon} \varepsilon & \frac{(x_{12}^\nu)^2}{y_t} \varepsilon & \left(\frac{x_{12}^\nu x_{13}^\nu}{y_t} \right) \varepsilon \\ \frac{(x_{12}^\nu)^2}{y_t} \varepsilon & 2 \left(\frac{x_{12}^e (x_{12}^\nu)^2}{y_t \Sigma_e} \right) \bar{\varepsilon} \varepsilon & -x_{23}^e \Sigma_e \bar{\varepsilon}^2 \\ \left(\frac{x_{12}^\nu x_{13}^\nu}{y_t} \right) \varepsilon & -x_{23}^e \Sigma_e \bar{\varepsilon}^2 & 1 \end{pmatrix} y_t^2 \end{aligned}$$

Anarchic Mixing

Difficult to satisfy 3σ bounds.

Not a characteristic feature of RVV Models

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I. Varzielas, G. G. Ross (hep-ph/0507176)

Fit to Quark Masses and Mixings

$$Y_d = \begin{pmatrix} 0 & x_{12}^d \bar{\varepsilon}^3 e^{i\delta_d} & x_{13}^d \bar{\varepsilon}^3 e^{i(\delta_d + \beta_3)} \\ x_{12}^d \bar{\varepsilon}^3 e^{i\delta_d} & \Sigma_d \bar{\varepsilon}^2 e^{i\sigma_d} & x_{23}^d \Sigma_d \bar{\varepsilon}^2 e^{i(\sigma_d + \beta_3)} \\ x_{13}^d \bar{\varepsilon}^3 e^{i(\delta_d + \beta_3)} & x_{23}^d \Sigma_d \bar{\varepsilon}^2 e^{i(\sigma_d + \beta_3)} & e^{2i\chi} \end{pmatrix} y_b$$

$$Y_u = \begin{pmatrix} 0 & x_{12}^u \varepsilon^3 e^{i\delta_u} & x_{13}^u \varepsilon^3 e^{i(\delta_u + \beta_3)} \\ x_{12}^u \varepsilon^3 e^{i\delta_u} & \Sigma_u \varepsilon^2 e^{i\sigma_u} & x_{23}^u \Sigma_u \varepsilon^2 e^{i(\sigma_u + \beta_3)} \\ x_{13}^u \varepsilon^3 e^{i(\delta_u + \beta_3)} & x_{23}^u \Sigma_u \varepsilon^2 e^{i(\sigma_u + \beta_3)} & 1 \end{pmatrix} y_t$$

$$\begin{pmatrix} x_{12}^d \\ x_{13}^d \\ x_{23}^d \end{pmatrix} = \begin{pmatrix} 1.80 \\ 0.4 \\ 1.68 \end{pmatrix};$$

$$\begin{pmatrix} x_{12}^u \\ x_{13}^u \\ x_{23}^u \end{pmatrix} = \begin{pmatrix} 1.52 \\ 2.0 \\ 0.5 \end{pmatrix}$$

Fit to Quark Masses and Mixings

$$V_{CKM} = \begin{pmatrix} \left| 1 - \frac{1}{2} \left(\frac{x_{12}^d}{\Sigma_d} \right)^2 \Lambda_{ud} \bar{\varepsilon}^2 \right| & \frac{x_{12}^d}{\Sigma_d} |\Lambda_{us}| \bar{\varepsilon} & x_{13}^d |\Lambda_{ub}| \bar{\varepsilon}^3 e^{-i\delta_{CKM}} \\ -\frac{x_{12}^d}{\Sigma_d} |\Lambda_{us}| \bar{\varepsilon} & \left| 1 - \frac{1}{2} \left(\frac{x_{12}^d}{\Sigma_d} \right)^2 \Lambda_{ud} \bar{\varepsilon}^2 \right| & x_{23}^d \Sigma_d |\Lambda_{cb}| \bar{\varepsilon}^2 \\ x_{\delta}^d \bar{\varepsilon}^3 e^{i(\omega_{cb} + \omega_{us})} & -x_{23}^d \Sigma_d |\Lambda_{cb}| \bar{\varepsilon}^2 e^{i\omega_{ud}} & 1 \end{pmatrix}$$

Fit to Quark Masses and Mixings

$$V_{CKM} = \begin{pmatrix} \left| 1 - \frac{1}{2} \left(\frac{x_{12}^d}{\Sigma_d} \right)^2 \Lambda_{ud} \bar{\varepsilon}^2 \right| & \frac{x_{12}^d}{\Sigma_d} |\Lambda_{us}| \bar{\varepsilon} & x_{13}^d |\Lambda_{ub}| \bar{\varepsilon}^3 e^{-i\delta_{CKM}} \\ -\frac{x_{12}^d}{\Sigma_d} |\Lambda_{us}| \bar{\varepsilon} & \left| 1 - \frac{1}{2} \left(\frac{x_{12}^d}{\Sigma_d} \right)^2 \Lambda_{ud} \bar{\varepsilon}^2 \right| & x_{23}^d \Sigma_d |\Lambda_{cb}| \bar{\varepsilon}^2 \\ x_\delta^d \bar{\varepsilon}^3 e^{i(\omega_{cb} + \omega_{us})} & -x_{23}^d \Sigma_d |\Lambda_{cb}| \bar{\varepsilon}^2 e^{i\omega_{ud}} & 1 \end{pmatrix}$$

$$\Lambda_{ub} = 1 - x_{23}^d \left(\frac{x_{12}^u}{x_{13}^d} \right) \left(\frac{\Sigma_d}{\Sigma_u} \right) \frac{\varepsilon}{\bar{\varepsilon}} e^{-i((\delta_u - \delta_d) - (\sigma_u - \sigma_d))}$$

$$\Lambda_{us} = 1 - \left(\frac{x_{12}^u}{x_{12}^d} \right) \left(\frac{\Sigma_d}{\Sigma_u} \right) \frac{\varepsilon}{\bar{\varepsilon}} e^{-i((\delta_u - \delta_d) - (\sigma_u - \sigma_d))}$$

$$+ \frac{x_\delta^d x_{23}^d}{x_{12}^d} \Sigma_d \bar{\varepsilon}^2 e^{i(2(\chi - \beta_3) - \sigma_d)}$$

$$\Lambda_{cb} = 1 - \left(\frac{x_{23}^u}{x_{23}^d} \right) \left(\frac{\Sigma_u}{\Sigma_d} \right) \frac{\varepsilon^2}{\bar{\varepsilon}^2} e^{-i(2\chi + (\sigma_u - \sigma_d))}$$

$$\Lambda_{ud} = 1 - 2 x_{23}^d \left(\frac{x_{12}^u}{x_{12}^d} \right) \left(\frac{\Sigma_d}{\Sigma_u} \right) \frac{\varepsilon}{\bar{\varepsilon}} e^{-i((\delta_u - \delta_d) - (\sigma_u - \sigma_d))}$$

Fit to Quark Masses and Mixings

$$V_{CKM} = \begin{pmatrix} \left| 1 - \frac{1}{2} \left(\frac{x_{12}^d}{\Sigma_d} \right)^2 \Lambda_{ud} \bar{\varepsilon}^2 \right| & \frac{x_{12}^d}{\Sigma_d} |\Lambda_{us}| \bar{\varepsilon} & x_{13}^d |\Lambda_{ub}| \bar{\varepsilon}^3 e^{-i\delta_{CKM}} \\ -\frac{x_{12}^d}{\Sigma_d} |\Lambda_{us}| \bar{\varepsilon} & \left| 1 - \frac{1}{2} \left(\frac{x_{12}^d}{\Sigma_d} \right)^2 \Lambda_{ud} \bar{\varepsilon}^2 \right| & x_{23}^d \Sigma_d |\Lambda_{cb}| \bar{\varepsilon}^2 \\ x_\delta^d \bar{\varepsilon}^3 e^{i(\omega_{cb} + \omega_{us})} & -x_{23}^d \Sigma_d |\Lambda_{cb}| \bar{\varepsilon}^2 e^{i\omega_{ud}} & 1 \end{pmatrix}$$

Flavon Phase	α_u	α_d	χ	β_3	β'_2	σ_d
Allowed Values	1.87	2.38	-0.628	1.12	0	1.752