HADRON–STRUCTURE DEPENDENT QED CORRECTIONS IN RARE EXCLUSIVE B DECAYS

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based on C. Cornella, M. König, MN: arXiv:2212.14430

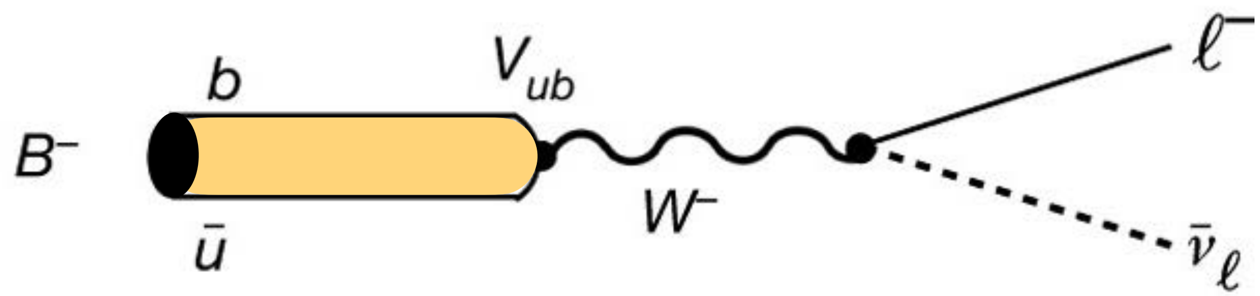
LES RENCONTRES DE PHYSIQUE DE LA VALLÉE D'AOSTE

LA THUILE, 8 MARCH 2023

MOTIVATION

Leptonic decays $B^- \to \ell^- \bar{\nu}_\ell$ are interesting for several reasons:

- **Determination of $|V_{ub}|$** largely unaffected by hadronic uncertainties



$$\Gamma \sim m_\ell^2 \, f_{B_u}^2 \, |V_{ub}|^2$$

B-meson decay constant

- Chiral suppression offers sensitive **probe of new scalar interactions**
- Comparing different lepton flavors yields **test of lepton universality**
  $\Rightarrow$ Belle II will measure $\ell = \mu, \tau$ channels with 5-7% uncertainty

[Belle II Physics Book]

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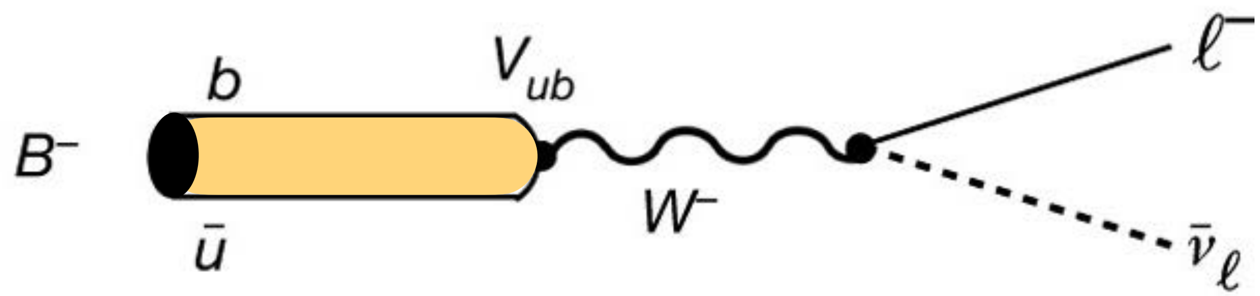

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B-meson decay constant

- QCD matrix element is known with <1% accuracy: [FNAL/MILC 2017]

$$\langle 0|\, \bar{u}\gamma^\mu\gamma_5 b\, |B^-(p)\rangle = i f_{B_u} p^\mu \quad \text{with} \quad f_{B_u} = (189.4 \pm 1.4)\,\text{MeV}$$

- QED corrections can be of similar magnitude or even larger, due to presence of **large logarithms** $\alpha \ln(m_b/m_\ell)$ and $\alpha \ln(m_\ell/E_s)$

MOTIVATION

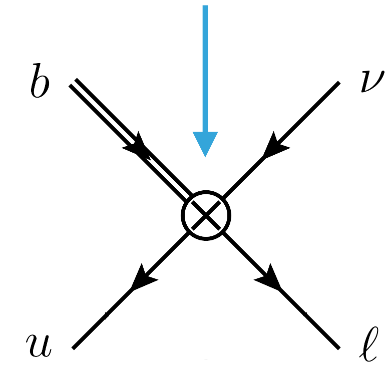
QED effects are well under control for $\mu > m_b$ and $\mu \ll \Lambda_{\rm QCD}$:

- Effective weak Hamiltonian contains all short-distance effects ($\mu > m_b$)

$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} K_{\rm EW}(\mu) V_{ub}\, (\bar{u}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu P_L \nu_\ell)$$

- Very soft photons see *B* meson as point-like particle

effective 4-fermion interaction from *W*-boson exchange



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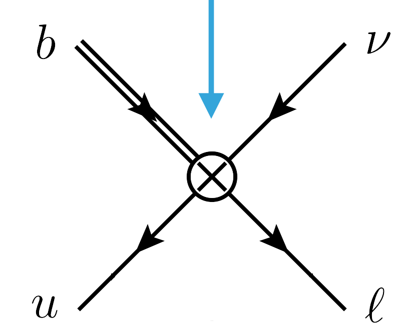

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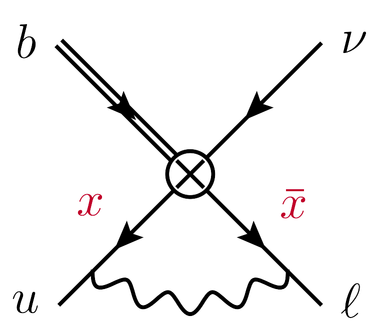
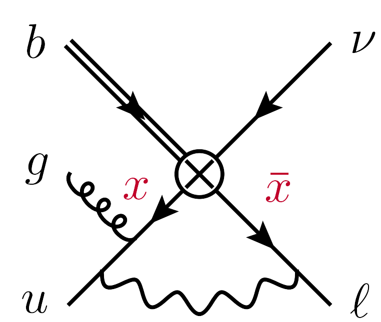
- Intermediate scale range $\Lambda_{\rm QCD} < \mu < m_b$ gives rise to more intricate effects, as virtual photons can resolve **inner structure of *B* meson**

[Cornella, König, MN 2022]





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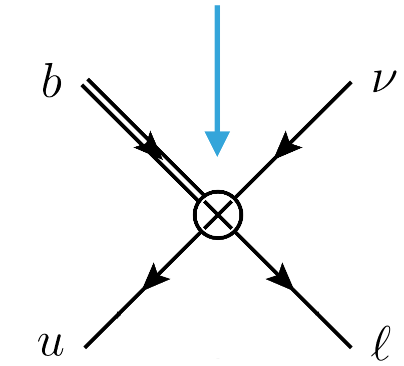
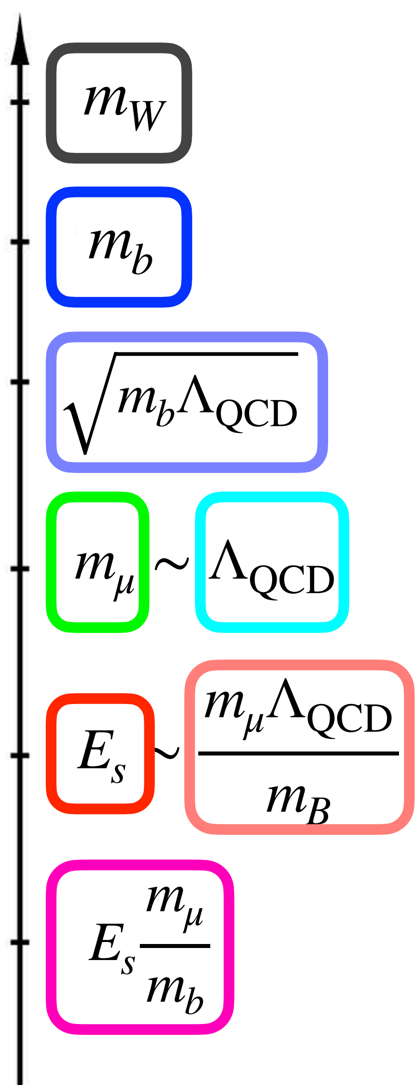
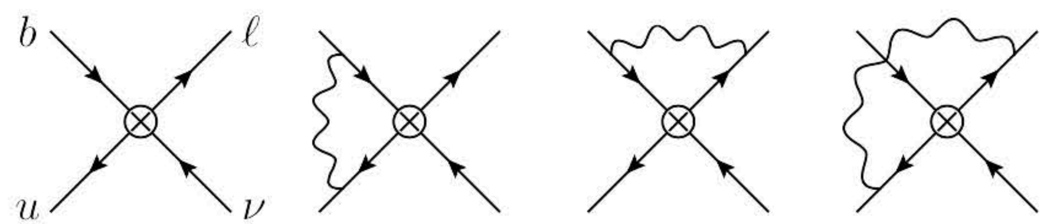


- Very soft photons see *B* meson as point-like particle
- Intermediate scale range $\Lambda_{\rm QCD} < \mu < m_b$ gives rise to more intricate effects, as virtual photons can resolve **inner structure of B meson**
  - $B_s \to \mu^+\mu^-$ [Beneke, Bobeth, Szafron 2017 & 2019]
  - $B \to K\pi, D\pi$ [Beneke, Böer, Toelstede, Vos 2020; Beneke, Böer, Finauri, Vos 2021]

RELEVANT SCALES

In the presence of QED effects, the decay $B^- \to \mu^- \bar{\nu}_\mu$ is sensitive to eight different energy scales:





Fock-state description of *B* meson: $|\bar{u}b\rangle + |\bar{u}gb\rangle + \dots$

[see: Beneke, Bobeth, Szafron 2019]

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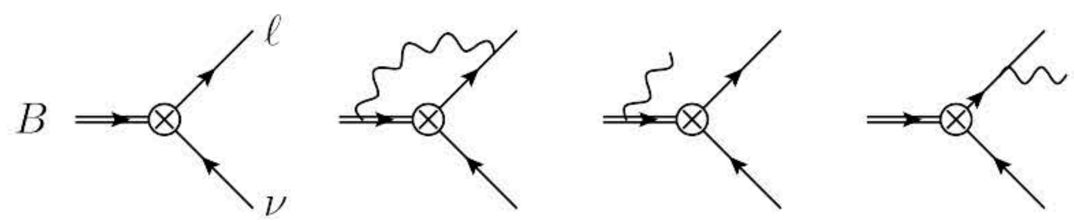
description of *B* meson as a point-like pseudo-scalar boson



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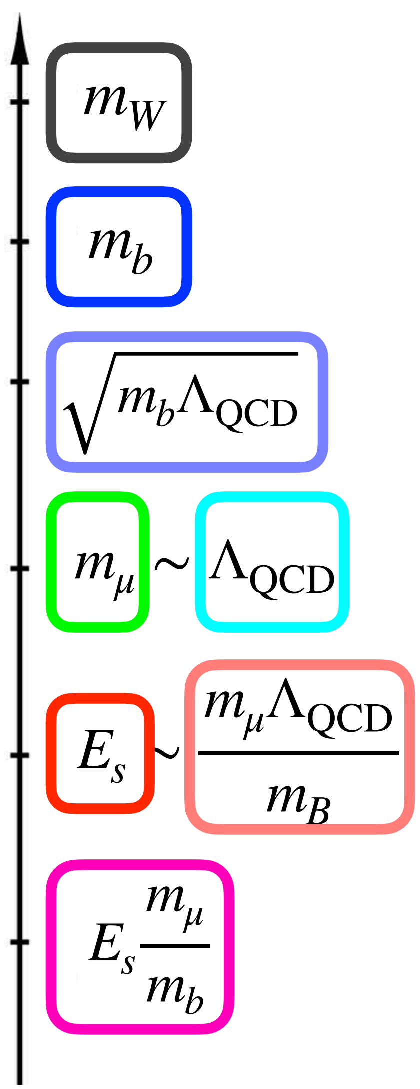

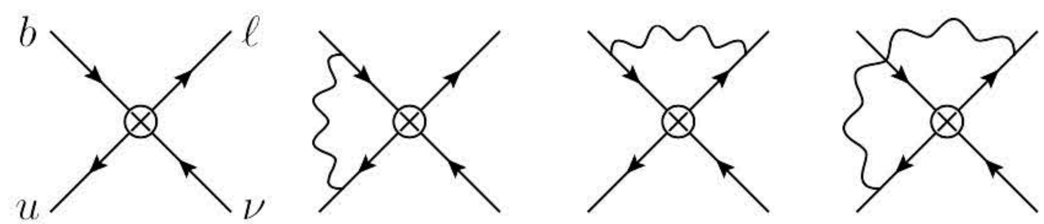

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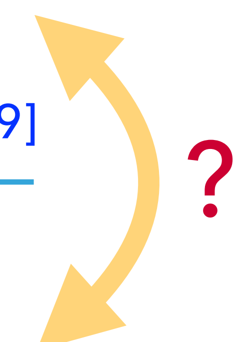

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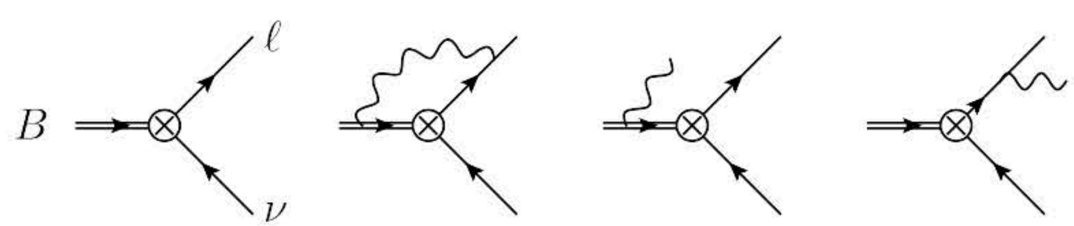

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QED CORRECTIONS IN LEPTONIC B DECAY

- Fact that external particles are charged under QED invalidates naive factorization, but scale separation can still be studied using SCET
- For power-suppressed processes such as $B^- \to \ell^- \bar{\nu}_\ell$, derivation of SCET factorization theorems is far more complicated than at leading power and has been understood only recently

  [Liu, MN 2019; Liu, Mecaj, MN, Wang 2020; Beneke *et al.* 2022; Liu, MN, Schnubel, Wang 2022]
- Our work is one of the first derivations of a **subleading-power factorization theorem** for a process involving **nonperturbative hadronic dynamics** [along with: Feldmann, Gubernari, Huber, Seitz 2022; see also: Hurth, Szafron 2023]

QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

- Quark current $\bar{u}\,\gamma^\mu P_L\, b$ is not gauge invariant under QED
  $\Rightarrow$ add a Wilson line to account for soft photon interactions with charged lepton
- One option: light-like Wilson line $\bar{u}\,\gamma^\mu P_L\, b\, S_n^{(\ell)\dagger}$ [Beneke, Bobeth, Szafron 2019]
  - anomalous dimension sensitive to IR regulators 😟
  - matching onto point-like meson theory not yet understood 😟

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  - matching onto point-like meson theory not yet understood 😟
- Our choice: **time-like Wilson line** $\bar{u}\,\gamma^\mu P_L\,b\,S_{v_\ell}^{(\ell)\dagger}$ [Cornella, König, MN 2022]
  - well-defined anomalous dimension 😌
  - consistent matching onto point-like meson theory 😌

QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

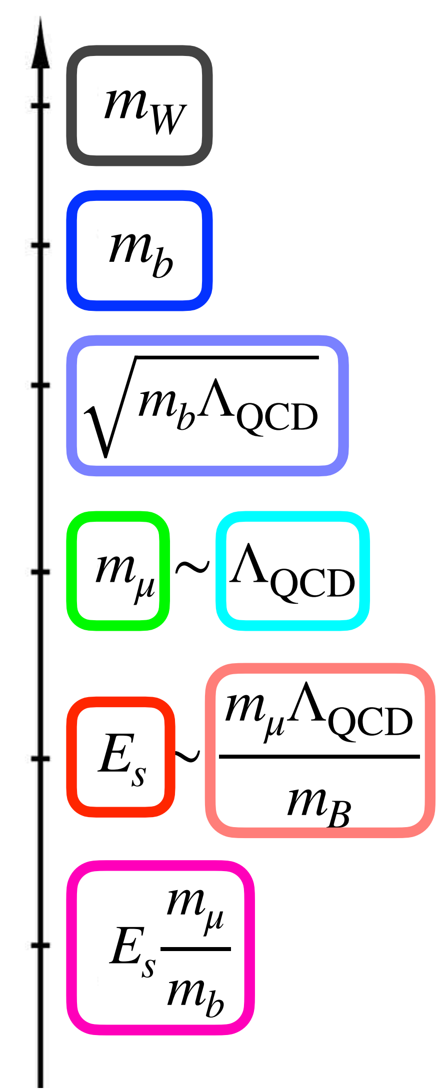
Have analyzed the factorization properties (scale separation) of the $B^- \to \mu^- \bar{\nu}_\mu$ amplitude including QED corrections in SCET

[Cornella, König, MN 2022]

- Relevant modes for **virtual QED corrections**:
  - hard
  - *hard-collinear*
  - soft
  - *collinear*
  - *soft-collinear*

resolve the light-cone structure of the *B* meson

- Relevant modes for **real QED corrections**:
  - ultra-soft
  - ultra-soft-collinear



QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

Subleading-power SCET factorization theorems suffer from endpoint-divergent convolution integrals [Liu, MN 2019; Liu, Mecaj, MN, Wang 2020]

Two most important contributions:

soft & soft-collinear modes

$$\mathcal{A}^{\text{virtual}}_{B\to\ell\bar\nu} = -\frac{4G_F}{\sqrt{2}}\,K_{\text{EW}}(\mu)\,V_{ub}\,\frac{m_\ell}{m_b}\,K_A(m_\ell)\,\bar u(p_\ell)P_L\,v(p_\nu)$$
$$\cdot\left[H_A(m_b)\,S_A + \int d\omega \int_0^1 dx\,H_B(m_b,x)\,J_B(m_b\omega,x)\,S_B(\omega)\right]$$

$\sim x^{-n\epsilon}$ $\quad\sim x^{-1-n\epsilon}$

where $S_{A,B} \propto \langle 0\,|\,O_{A,B}\,|\,B\rangle$ and (with $\bar n \parallel p_\nu$):

$$O_A = \bar n_\mu\,\bar u_s \gamma^\mu P_L h_{v_B}\,S^{(\ell)\dagger}_{v_\ell}$$

$$O_B(\omega) = \int \frac{dt}{2\pi}\,e^{i\omega t}\,\bar u_s(tn)\,[tn,0]\,\not{n}\,P_L h_{v_B}(0)\,S^{(\ell)\dagger}_{v_\ell}(0)$$

quark fields at light-like separation
⇒ ***B*-meson light-cone distribution amplitudes**

QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

Remove endpoint divergence using the refactorization-based subtraction (RBS) scheme [Liu, MN 2019; Liu, Mecaj, MN, Wang 2020]

Subtracted amplitude:

$$\mathcal{A}_{B\to\ell\bar\nu}^{\text{virtual}} = -\frac{4G_F}{\sqrt{2}}\,K_{\text{EW}}(\mu)\,V_{ub}\,\frac{m_\ell}{m_b}\,K_A(m_\ell)\,\bar u(p_\ell)P_L\,v(p_\nu)$$
$$\cdot\Big[H_A(m_b)\,S_A^{(\Lambda)} + \int d\omega\int_0^1 dx\,\Big[H_B(m_b,x)\,J_B(m_b\omega,x)$$
$$-\,\theta(\lambda-x)\,[\![H_B(m_b,x)]\!]\,[\![J_B(m_b\omega,x)]\!]\Big]\,S_B(\omega)\Big]$$

[Cornella, König, MN 2022]

where:

$$O_A^{(\Lambda)} = \bar u_s\,\not{n}\,P_L h_{v_B}\,S_{v_\ell}^{(\ell)\dagger}$$
$$\times\left[1 + Q_\ell Q_u\,\frac{\alpha}{2\pi}\,\frac{e^{\epsilon\gamma_E}\,\Gamma(\epsilon)}{\epsilon\,(1-\epsilon)}\int d\omega\,\phi_-(\omega)\left(\frac{\mu^2}{\omega\Lambda}\right)^{\epsilon}\right]$$

$$\Lambda = \lambda m_b$$

QED CORRECTIONS IN LEPTONIC B DECAY – CHALLENGES

Generalization of the decay "constant" in presence of QED effects

- Matching relation (with X_γ an *n*-soft-photon state):

$$\langle X_\gamma | O_A^{(\Lambda)} | B^- \rangle = -\frac{i}{2} \sqrt{m_B}\, F(\mu, \Lambda, w)\, \langle X_\gamma | S_{v_B}^{(B)} S_{v_\ell}^{(\ell)\dagger} | 0 \rangle \quad \text{with} \quad w \equiv v_B \cdot v_\ell \approx \frac{m_B}{2m_\ell}$$

⇒ a **form factor** (like the Isgur-Wise function in $B \to D^{(*)}$ transitions)

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  [Cornella, König, MN 2022]

- Defining *F* as a Wilson coefficient implements the **nonperturbative matching of SCET onto the point-like meson effective theory** envisioned in [Beneke, Bobeth, Szafron 2019]

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- Evolution equations:

$$\frac{d \ln F}{d \ln \mu} = C_F \frac{3\alpha_s}{4\pi} - \frac{3\alpha}{4\pi} \left(Q_\ell^2 - Q_b^2 + \frac{2}{3} Q_\ell Q_u \ln \frac{\Lambda^2}{\mu^2} \right)$$

$$\frac{d \ln F}{d \ln \Lambda} = Q_\ell Q_u \frac{\alpha}{2\pi} \left[\int d\omega\, \phi_-(\omega) \ln \frac{\omega \Lambda}{\mu^2} - 1 + \dots \right]$$

well-defined and insensitive to IR regulators

VIRTUAL QED CORRECTIONS IN LEPTONIC B DECAY

Decay amplitude including virtual QED corrections at $\mathcal{O}(\alpha)$:

$$\mathcal{A}_{B\to\ell\bar{\nu}}^{\text{virtual}} = i\sqrt{2}G_F K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b}\sqrt{m_B}\, F(\mu, m_b, w)\, \bar{u}(p_\ell) P_L v(p_\nu) \Big[\mathcal{M}_{2p}(\mu) + \mathcal{M}_{3p}(\mu)\Big]$$

with:

$$\mathcal{M}_{2p}(\mu) = 1 + \frac{C_F \alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right]$$
$$+ \frac{\alpha}{4\pi}\left\{Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right] - Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2} + 2\ln\frac{m_b^2}{\mu^2} - 3\ln\frac{m_\ell^2}{\mu^2} + 1 + \frac{5\pi^2}{12}\right]\right.$$
$$\left. + 2Q_\ell Q_u \int_0^\infty d\omega\, \phi_-(\omega) \ln\frac{m_b\omega}{\mu^2} + Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2} - 2\right) + \frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2} - \frac{1}{2}\ln\frac{m_\ell^2}{\mu^2} + 2 + \frac{5\pi^2}{12}\right]\right\}$$

$$\mathcal{M}_{3p}(\mu) = \frac{\alpha}{\pi} Q_\ell Q_u \int_0^\infty d\omega \int_0^\infty d\omega_g\, \phi_{3g}(\omega, \omega_g)\left[\frac{1}{\omega_g}\ln\left(1 + \frac{\omega_g}{\omega}\right) - \frac{1}{\omega + \omega_g}\right]$$

[Cornella, König, MN 2022]

⇒ significant hadronic uncertainties in $\mathcal{O}(\alpha)$ terms!

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IR divergence cancels against real soft photon emission

[Cornella, König, MN 2022]

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with:

large double logarithms

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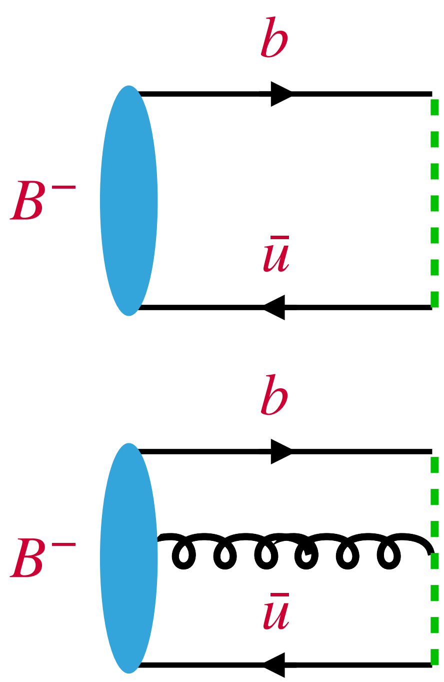
HADRONIC UNCERTAINTIES

Nonperturbative hadronic matrix elements:

$F(\mu, m_b, v_B \cdot v_\ell)$ — a **form factor** rather than a decay "constant" [Cornella, König, MN 2022]

LCDAs



$\phi_\pm(\omega)$ [Grozin, MN 1996]

$\phi_{3g}(\omega, \omega_g)$ [Kawamura, Kodaira, Qiao, Tanaka 2001; Braun, Ji, Manashov 2017]

Several model LCDAs have been proposed, e.g.:

$$\phi_-(\omega) = \frac{1}{\omega_0}\, e^{-\omega/\omega_0}\,, \qquad \phi_{3g}(\omega, \omega_g) = \frac{\lambda_E^2 - \lambda_H^2}{3\omega_0^5}\, \omega\, \omega_g\, e^{-(\omega+\omega_g)/\omega_0}$$

HADRONIC UNCERTAINTIES

Model-independent results for the form factor $F(\mu, m_b, v_B \cdot v_\ell)$

- Relation to lattice QCD results for the *B*-meson decay constant:

$$\sqrt{m_B}\, f_B^{\rm QCD} = \left[1 - C_F\,\frac{\alpha_s(m_b)}{2\pi}\right] F(m_b, m_b, w)\Big|_{\alpha\to 0}$$

- Structure of QED corrections:

$$F(\mu, \Lambda, w) \propto f_{B_u}^{\rm QCD} \left\{1 + \frac{\alpha}{4\pi}\left[\underbrace{-2Q_\ell^2 \ln^2(2w)}_{-1.9\%} + c_1 \ln(2w) + c_0(\Lambda, \mu)\right]\right\}$$

$\ln^2(2w) \to \ln^2 \frac{m_B}{m_\ell}$, $\quad \ln(2w) \to \ln \frac{m_B}{m_\ell}$

with nonperturbative parameters $c_0(\Lambda, \mu)$ and c_1 that can likely be determined using lattice QCD

CONCLUSIONS

- First subleading-power factorization theorem in which endpoint divergences are subtracted in a nonperturbative context
- First consistent matching of SCET onto point-like meson theory
- Structure-dependent QED corrections a generic feature resulting from contributions of (hard-, soft-) collinear modes in SCET
  - important source of large double-logarithmic corrections
  - missed in previous treatments based on point-like meson model
- Results provide basis for consistent analyses of QED effects in rare *B* decays and allow for high-precision determination of $|V_{ub}|$