



**University of  
Zurich<sup>UZH</sup>**

# **Third-Family Quark–Lepton Unification and Electroweak Precision Tests**

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**In collaboration with L. Allwicher, G. Isidori, N. Selimovic, B.A. Stefanek**

**[\[2302.11584\]](#)**

**Rencontres de Physique de la Vallée d'Aoste - La Thuile - March 2023**

# 4321 model

See Joe Davighi's talk

## Third-family quark-lepton unification:

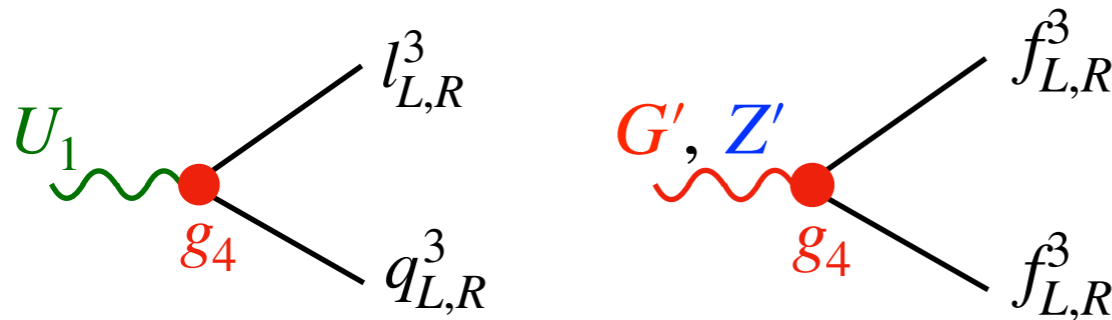
$$\Psi_{L/R} = \begin{pmatrix} Q_{L,R}^1 \\ Q_{L,R}^2 \\ Q_{L,R}^3 \\ L_{L,R} \end{pmatrix}$$

$$SU(3)_h \xrightarrow{\text{red}} SU(3)_c$$

$$SU(4)_h \times SU(3)_l \times SU(2)_L \times U(1)_{R+\frac{(B-L)_l}{2}} \xrightarrow{\sim \text{TeV}} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$U(1)_{(B-L)_h} \xrightarrow{\text{blue}} U(1)_Y$$

$$+ \begin{cases} U_1 \sim (3, 1)_{2/3} \\ G' \sim (8, 1)_0 \\ Z' \sim (1, 1)_0 \end{cases}$$

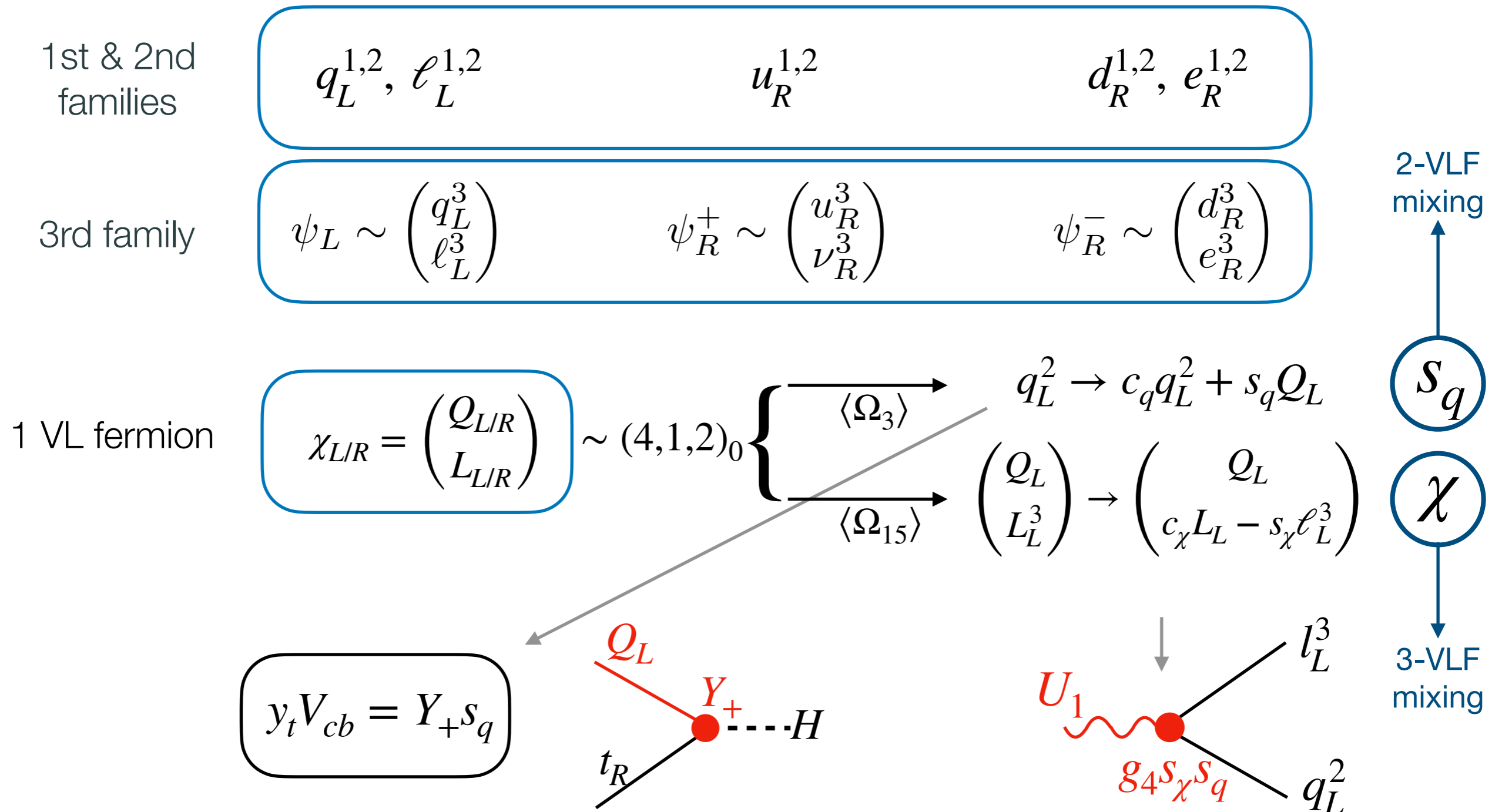


$$\begin{aligned} g_4 &\gg g_3 \\ &\downarrow \\ g_s &\approx g_3 \\ g_h^{G'} &\approx g_4 \\ g_l^{G'} &\approx g_s^2 / g_4 \end{aligned}$$

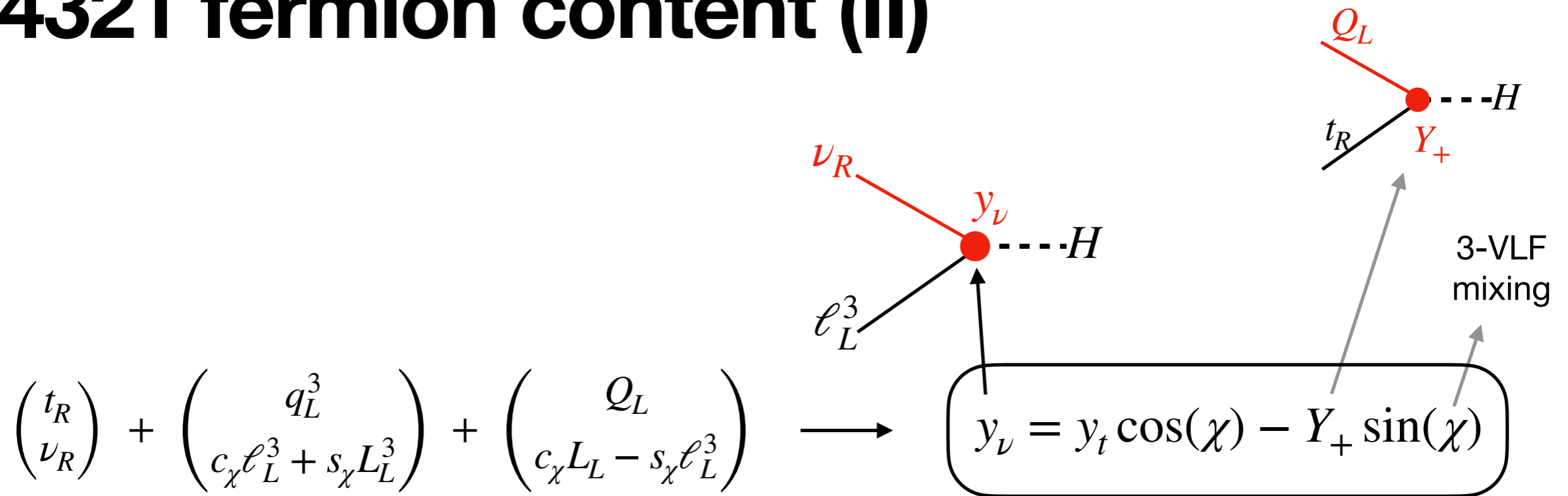
LHC bounds:  
 $M_{G'} \gtrsim 3 - 3.5 \text{ TeV}$

Accidental  $U(2)$  flavor symmetry

# 4321 fermion content (I)



# 4321 fermion content (II)

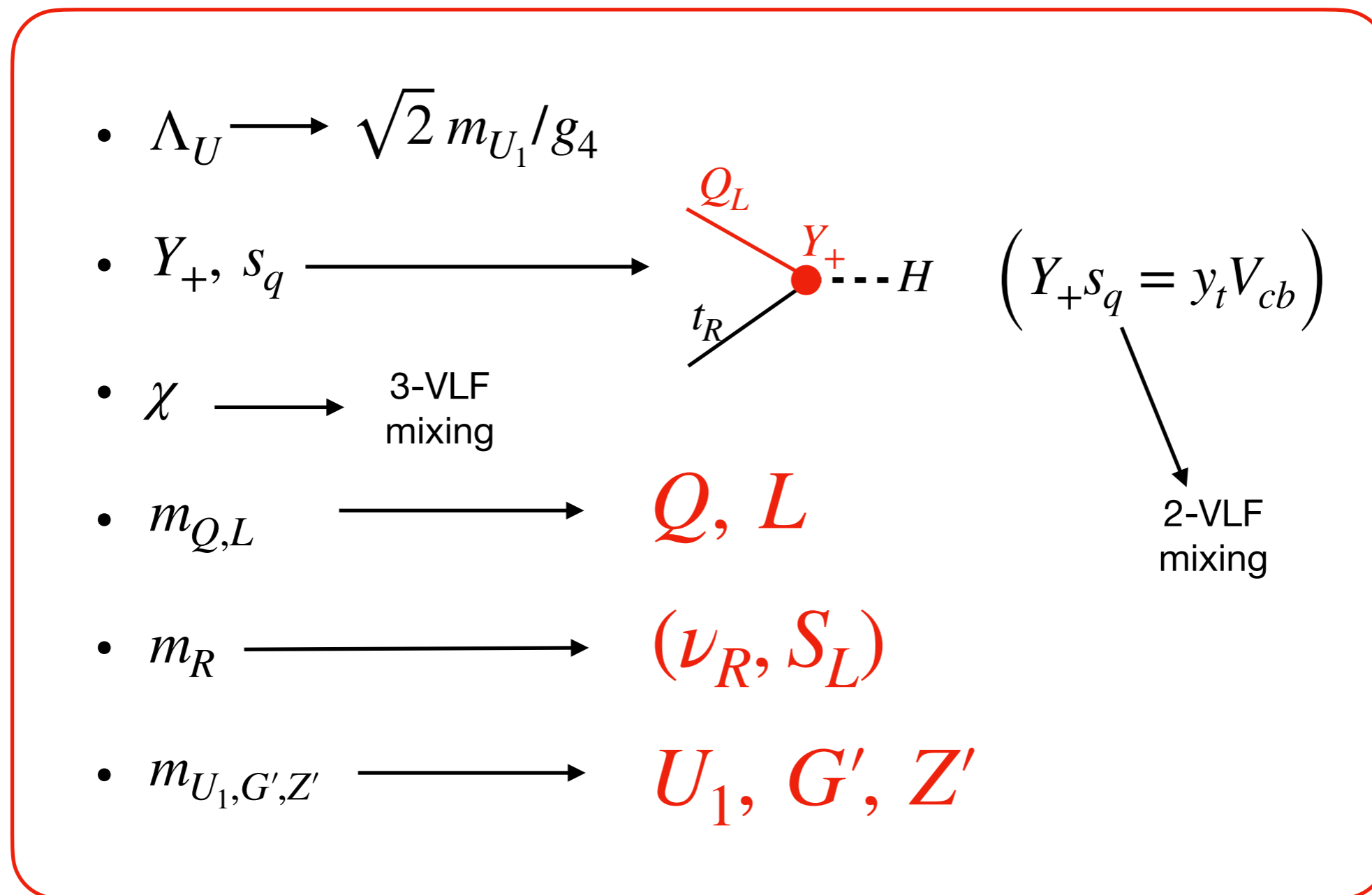


1 singlet fermion  $S_L \sim (1,1,1)_0 \quad \mathcal{L} \supset \lambda_S \bar{S}_L \Omega_1 \Psi_R \xrightarrow{\langle \Omega_1 \rangle} \mathcal{L} \supset m_R \bar{S}_L \nu_R$

$$\nu_L^3 \rightarrow c_\nu \nu_L^3 + s_\nu S_L$$

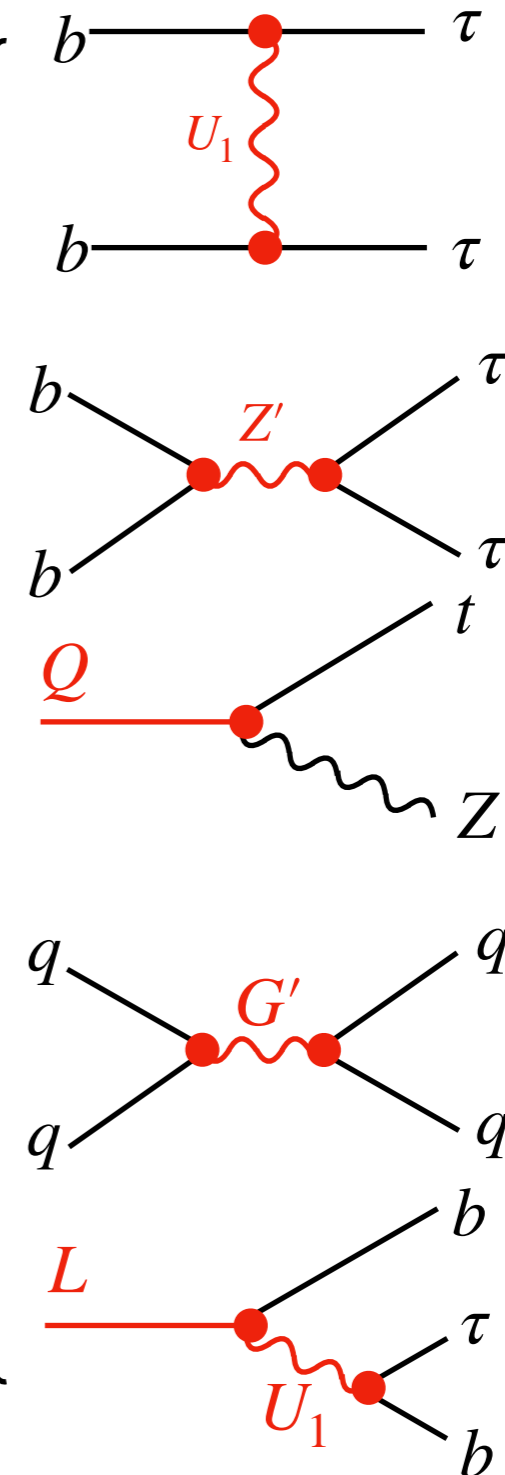
$$(s_\nu = y_\nu v_{EW} / m_R)$$

# Relevant parameters for this analysis



# Phenomenology

- $b \rightarrow c\tau\nu$  physics ( $R_{D^{(*)}}, R_{\Lambda_c}$ )
- EWPO
- LFUV in  $\tau$  decays  
[Allwicher, Isidori, Selimović, [2109.03833](#)]
- High  $p_T$  at LHC
- Other  $q_3 \rightarrow q_2$  transitions:
  - $B_s \rightarrow \tau\tau, B \rightarrow K\nu\nu, B \rightarrow K\tau\tau, B_s$  mixing, etc ...  
[Cornella, Faroughy, Fuentes-Martin, Isidori, Neubert, [2103.16558](#)]
- $q_2 \rightarrow q_1$  transitions:
  - $K \rightarrow \pi\nu\nu, K, D$  mixing  
[Crosas, Isidori, JML, Selimović, Stefanek, [2203.01952](#)]

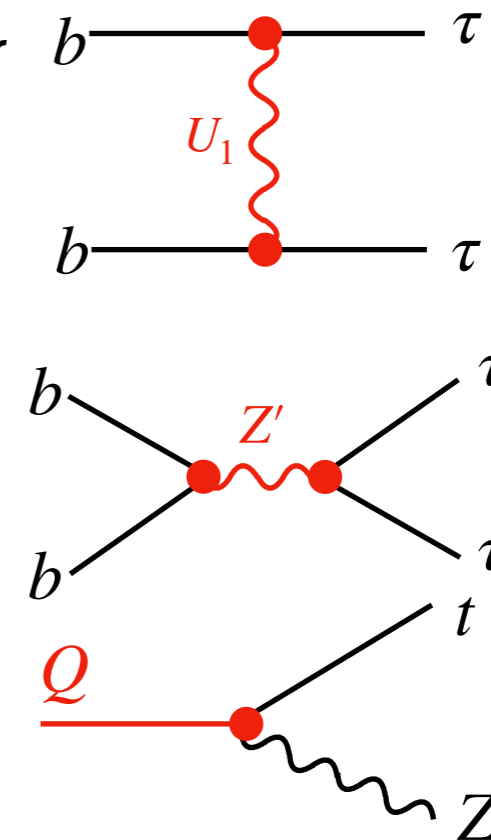


$m_Q > 1.5$  TeV  
95 % CL

# Phenomenology

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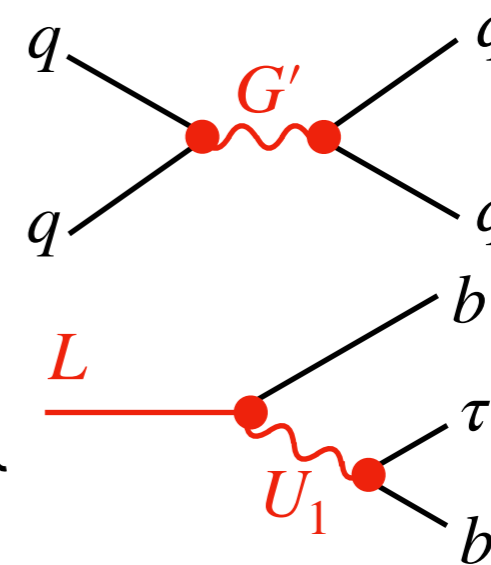
- Other  $q_3 \rightarrow q_2$  transitions:
  - $B_s \rightarrow \tau\tau$ ,  $B \rightarrow K\nu\nu$ ,  $B \rightarrow K\tau\tau$ ,  $B_s$  mixing, etc ...  
[Cornella, Faroughy, Fuentes-Martin, Isidori, Neubert, [2103.16558](#)]
- $q_2 \rightarrow q_1$  transitions:
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[Crosas, Isidori, JML, Selimović, Stefanek, [2203.01952](#)]



See Uli Haisch's talk

HighPT [\[2207.10756\]](#)  
(see Lukas's talk)

$m_Q > 1.5$  TeV  
95 % CL

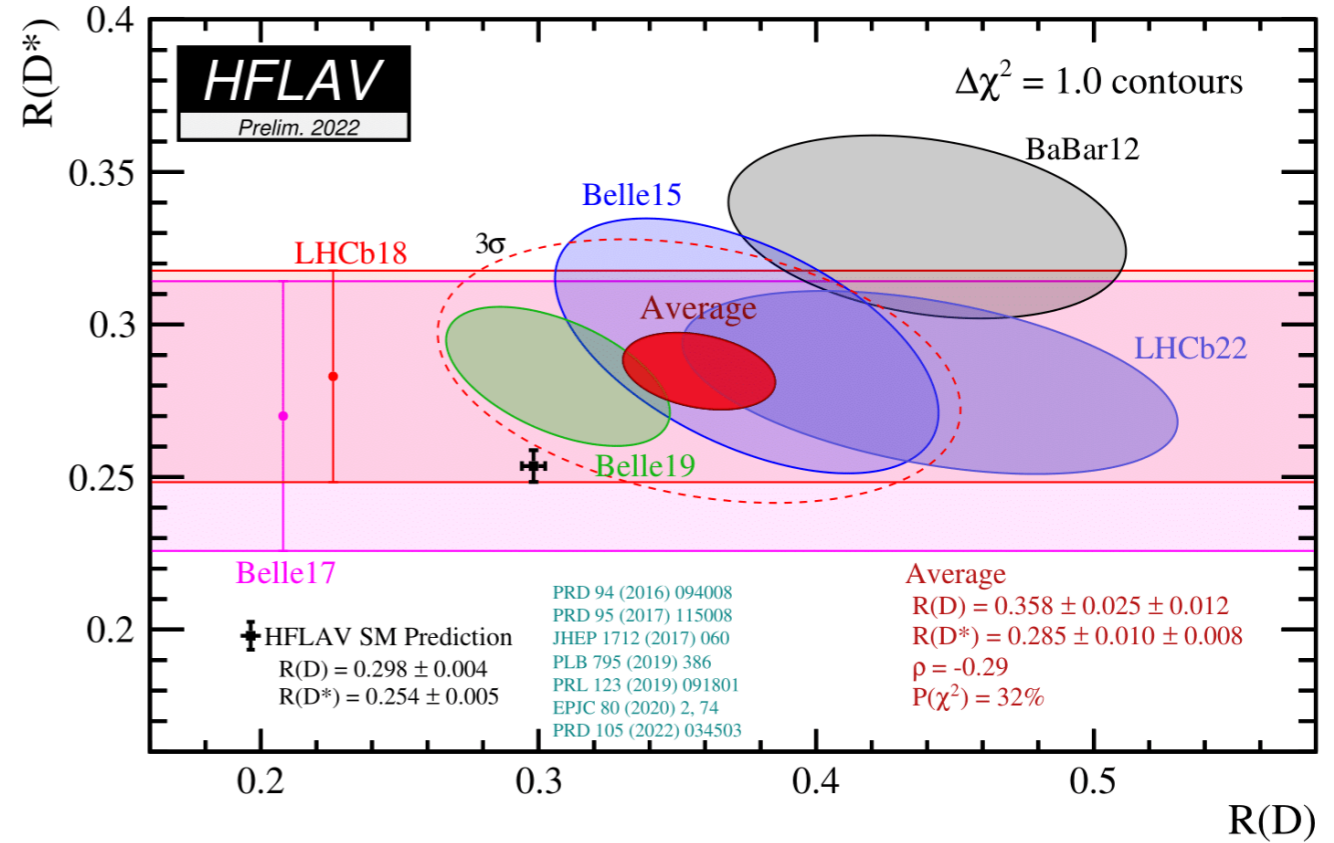


# $b \rightarrow c\tau\nu$ data

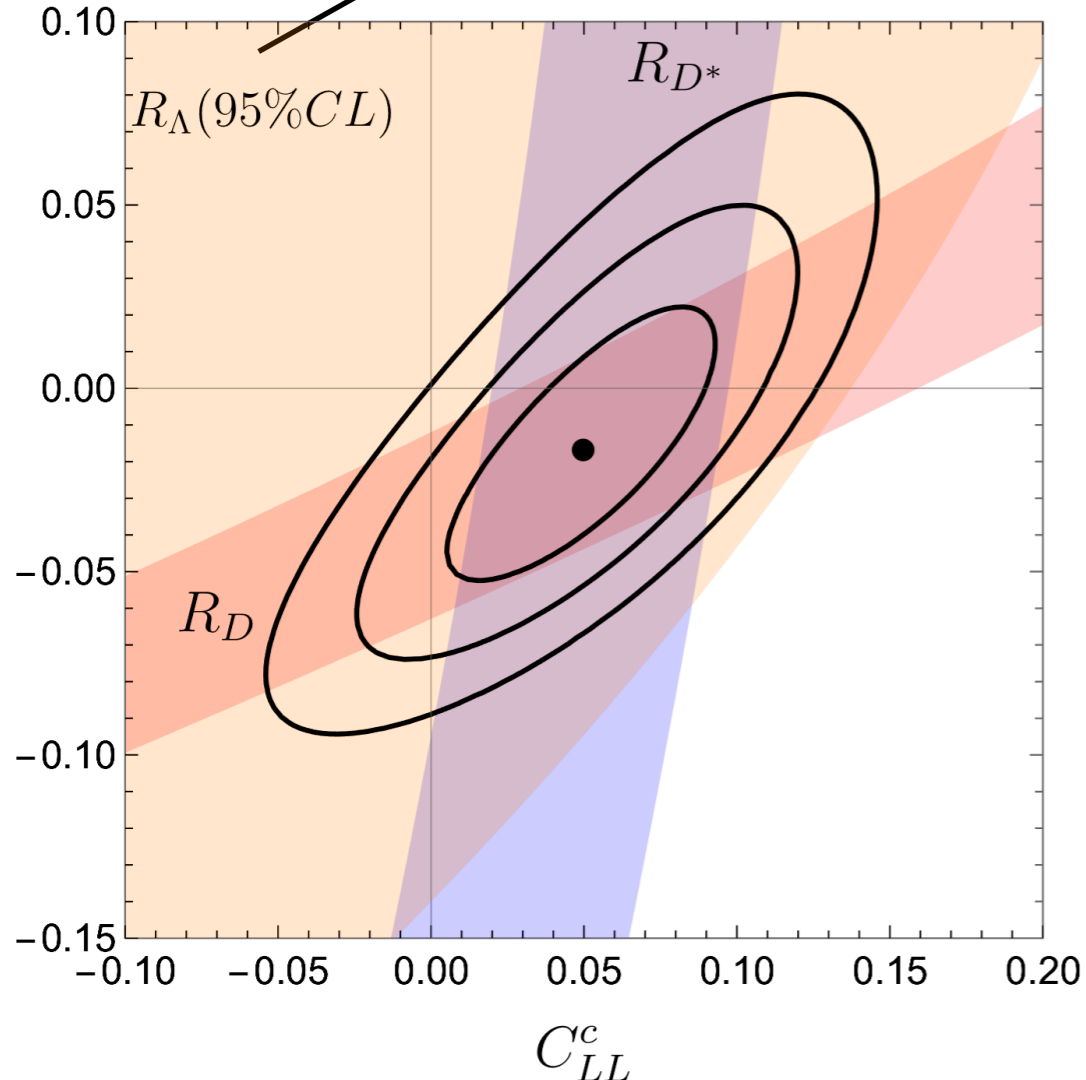
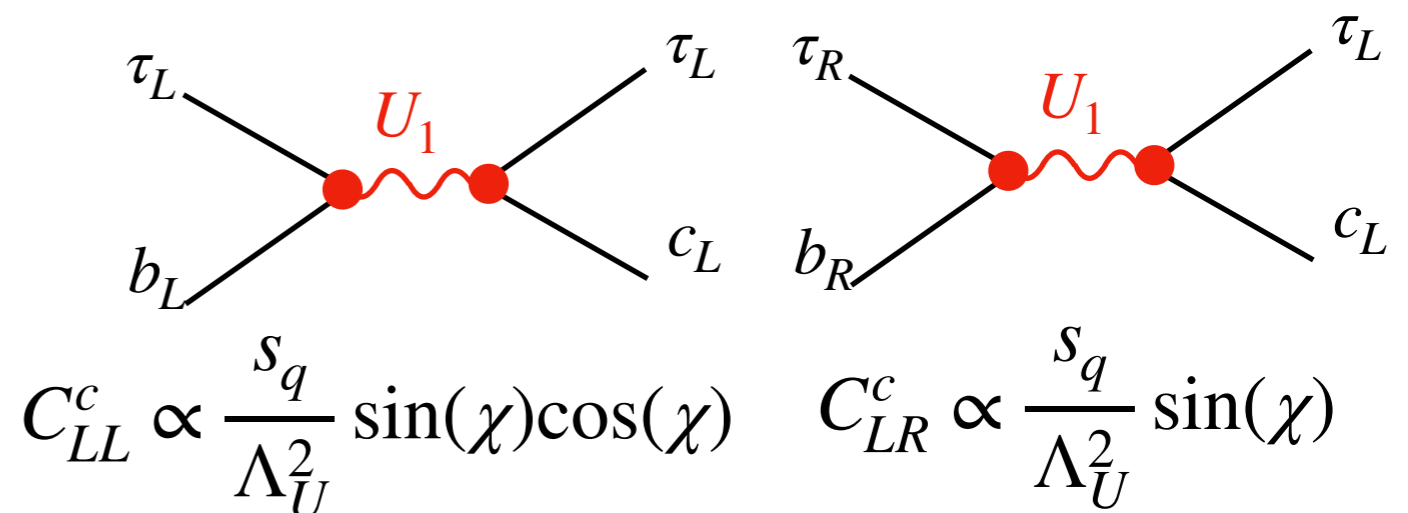
$$R_{D^{(*)}} = \frac{Br(B \rightarrow D^{(*)}\tau\nu)}{Br(B \rightarrow D^{(*)}l\nu)}$$

$\sim 3.2\sigma$

$\Lambda_b \rightarrow \Lambda_c\tau\nu$



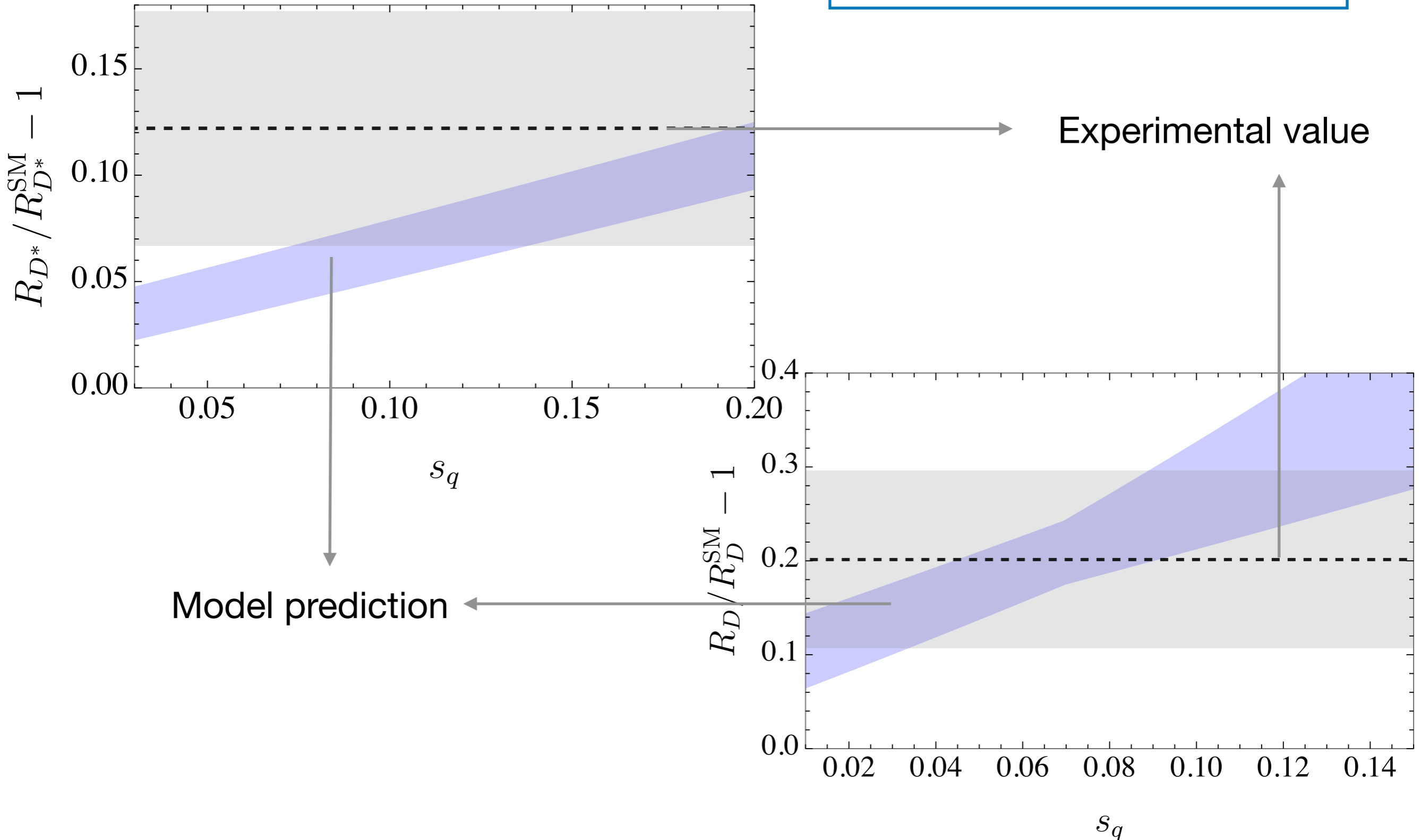
$$\mathcal{L} \supset \frac{2}{v^2} V_{cb} \left[ (1 + C_{LL}^c) (\bar{c}_L \gamma_\mu b_L) (\bar{\tau}_L \gamma^\mu \nu_L) - 2C_{LR}^c (\bar{c}_L b_R) (\bar{\tau}_L \nu_L) \right]$$





# $b \rightarrow c\tau\nu$ data

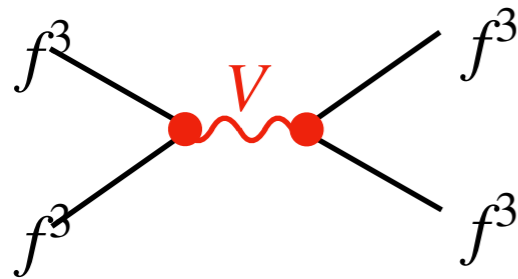
$$\Lambda_U = \sqrt{2} m_{U_1} / g_4 = 1.5 \text{ TeV}$$



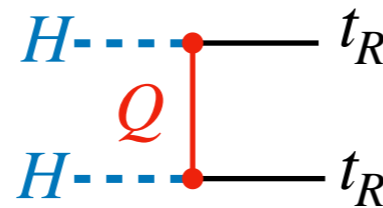
# SMEFT

- One-loop matching in  $g_4$ ,  $Y_+$ ,  $y_t$ ,  $g_s$  to the relevant SMEFT operators:

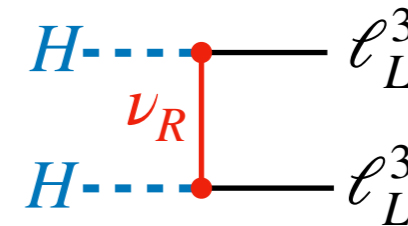
Tree level matching:



$$\rightarrow C_{ff} (\bar{f}^3 f^3)(\bar{f}^3 f^3)$$

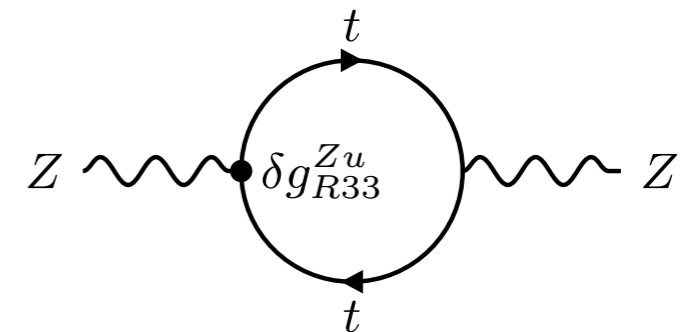
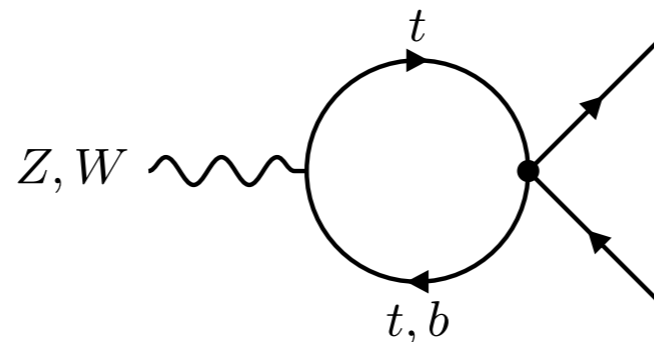
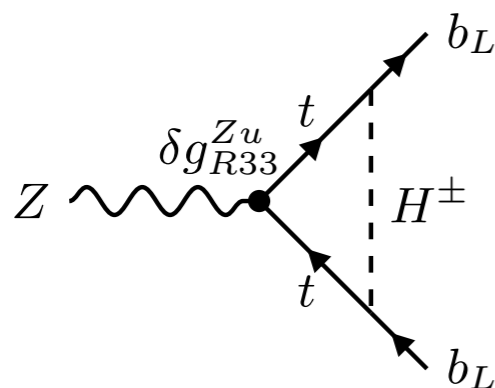


$$\rightarrow C_{Hu} (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{t}_R \gamma^\mu t_R)$$



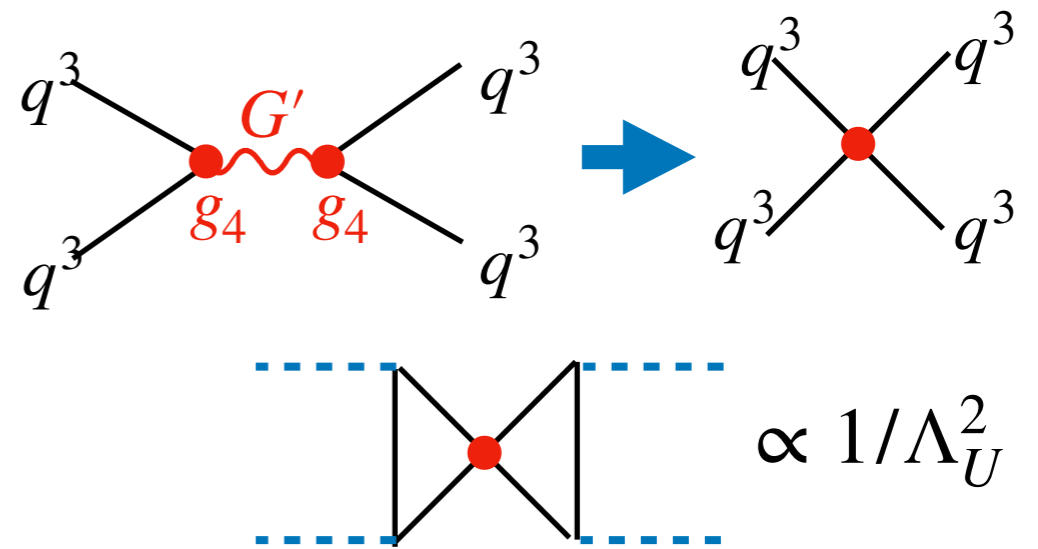
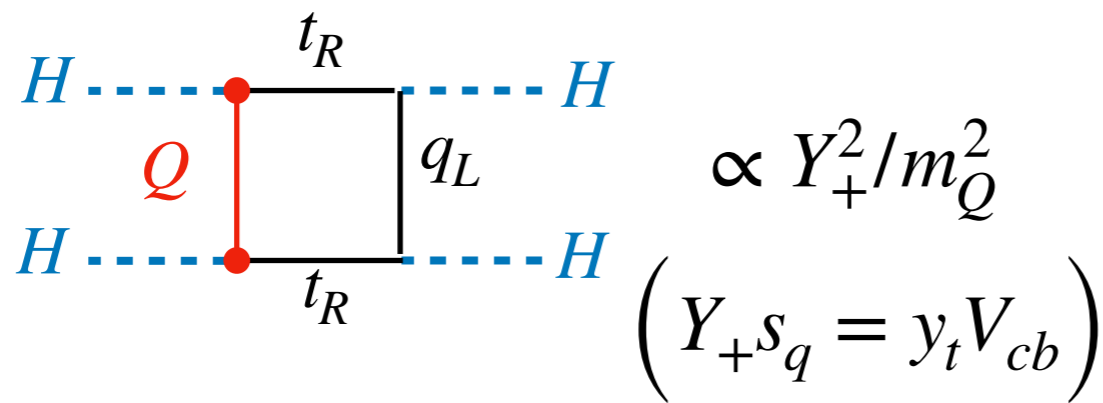
$$\rightarrow C_{H\ell} \left( (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{\ell}_L^3 \gamma^\mu \ell_L^3) - (H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{\ell}_L^3 \tau_I \gamma^\mu \ell_L^3) \right)$$

- Running at one loop from UV scale to EW.
- To keep consistency with the one-loop matching, calculation of the EW observables at one-loop in  $y_t$ ,  $g_s$ :



# EW: Universal contributions

$$\mathcal{L} \supset C_{HD} |H^\dagger D_\mu H|^2$$

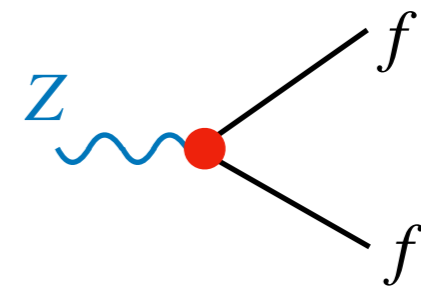
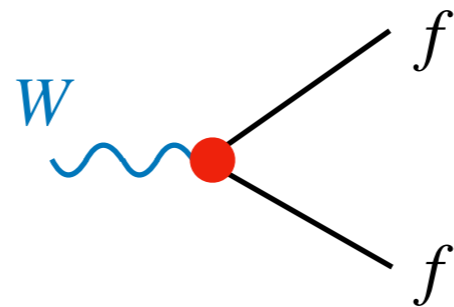


$(C_{Hu} \rightarrow C_{HD}) + 1\text{-loop matching}$

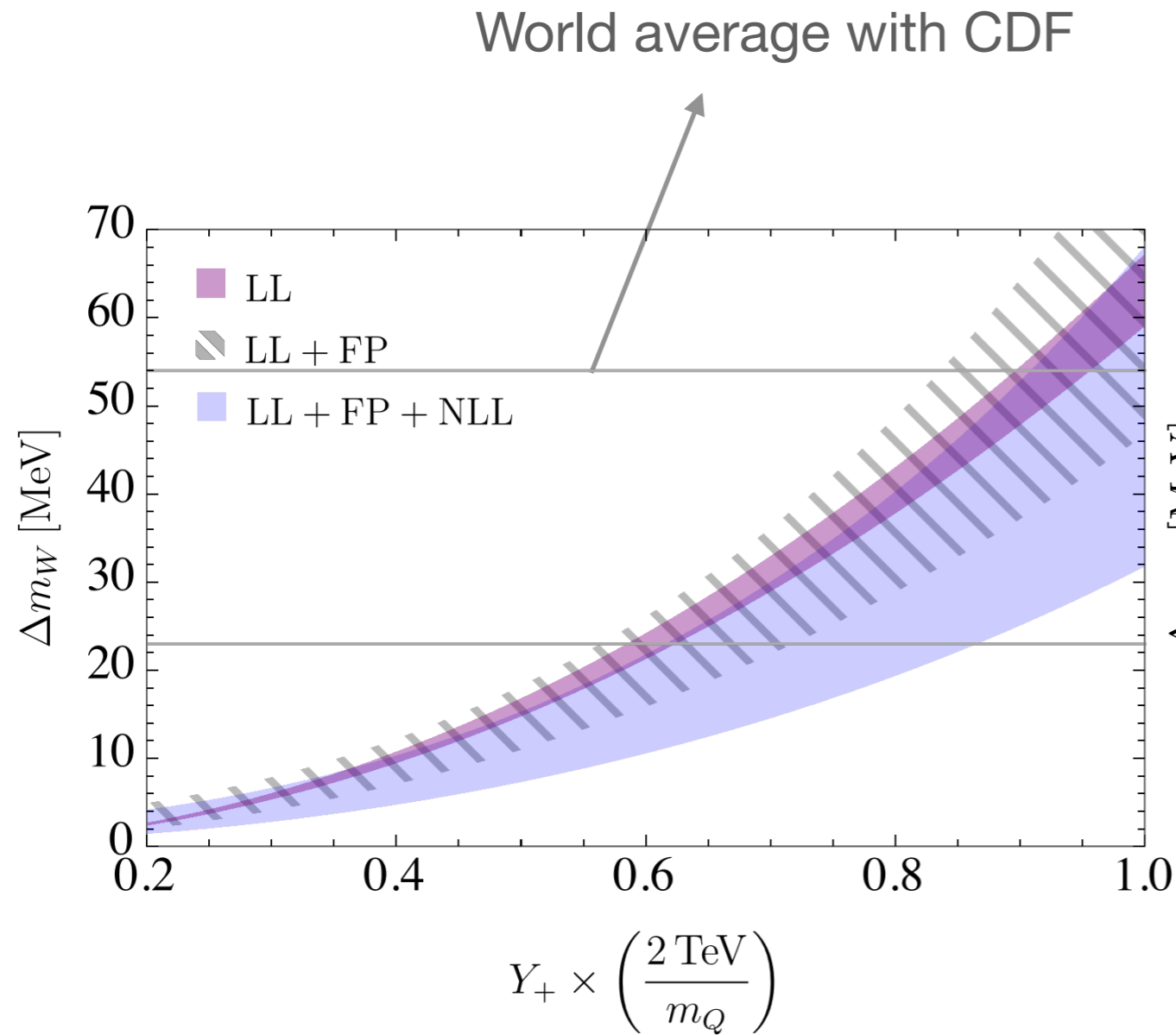
$$C_{uu} + C_{qq} \rightarrow C_{Hu} + C_{Hq}^{(1)} \rightarrow C_{HD}$$

$$m_W$$

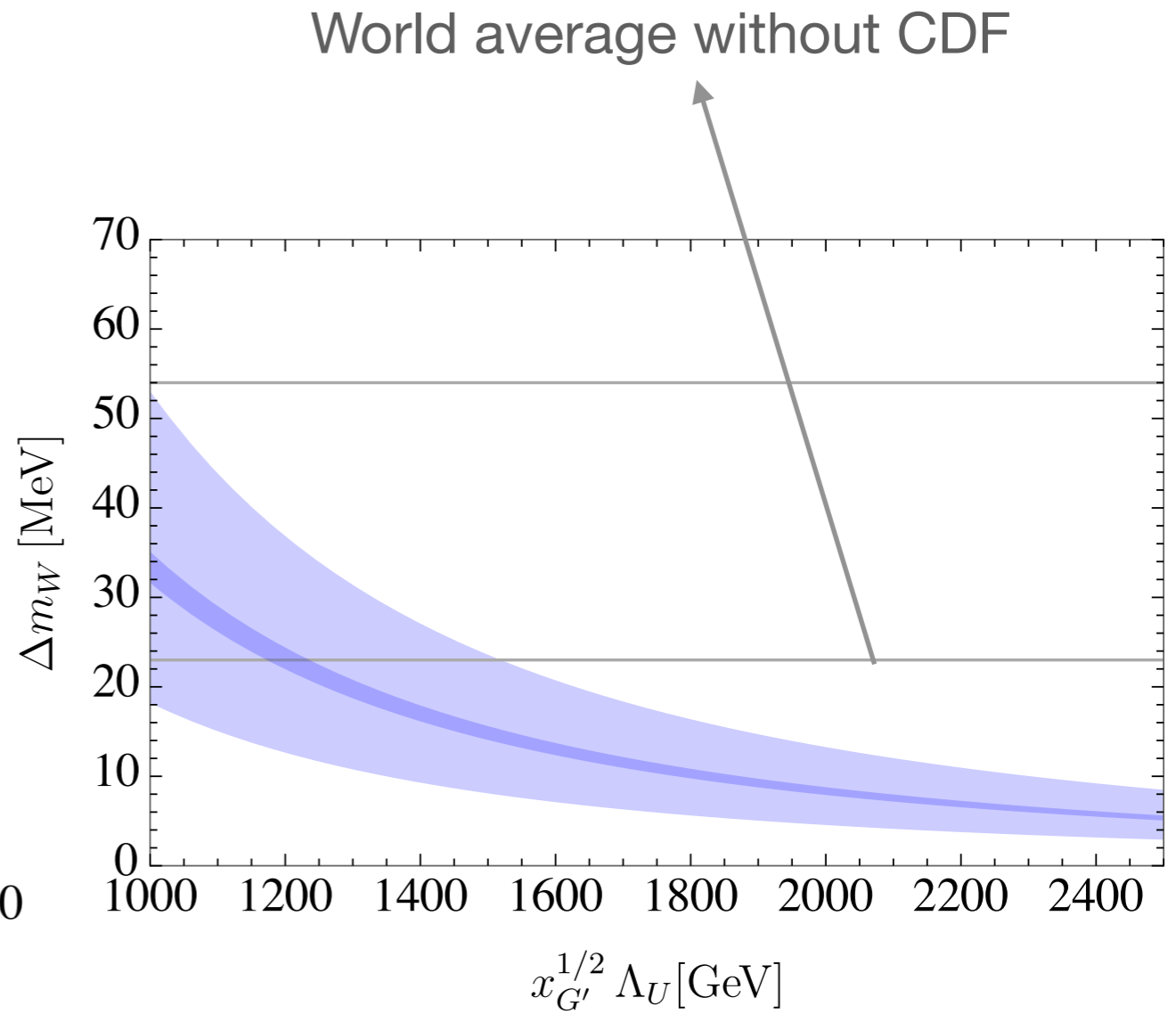
Universal coupling modification



# EW: Universal contributions



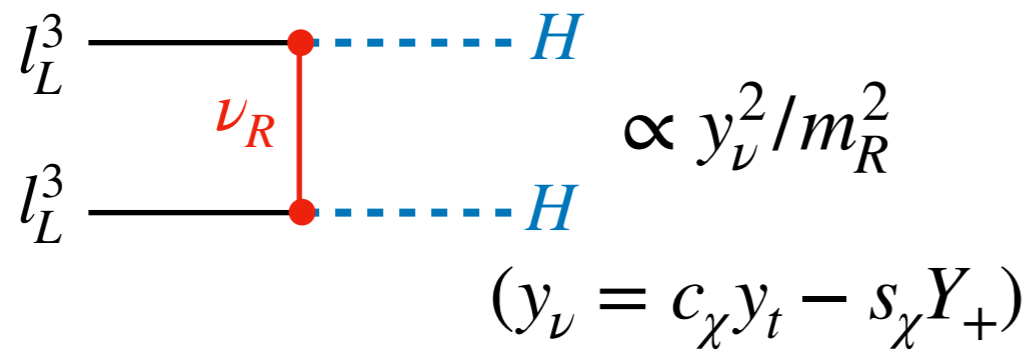
VLF sector



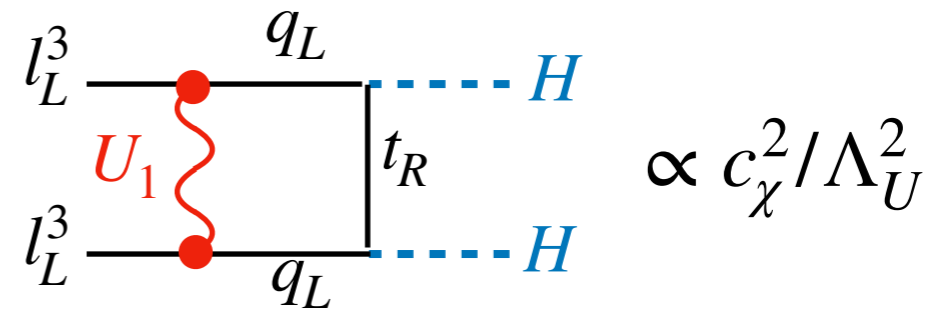
Coloron sector

# EW: Non-Universal contributions

$$\mathcal{L} \supset C_{Hl}^{(1)} (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{\ell}_L^3 \gamma^\mu \ell_L^3) + C_{Hl}^{(3)} (H^\dagger i \overleftrightarrow{D}_\mu^I H) (\bar{\ell}_L^3 \tau_I \gamma^\mu \ell_L^3)$$



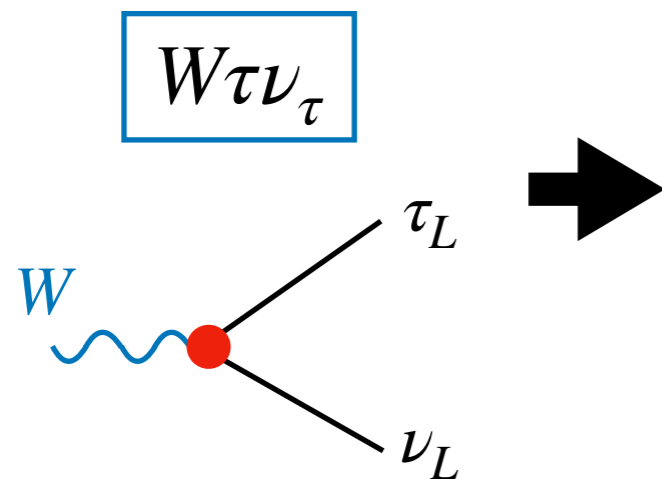
Tree-level + LL running ( $C_{Hl} \rightarrow C_{H\ell}$ )



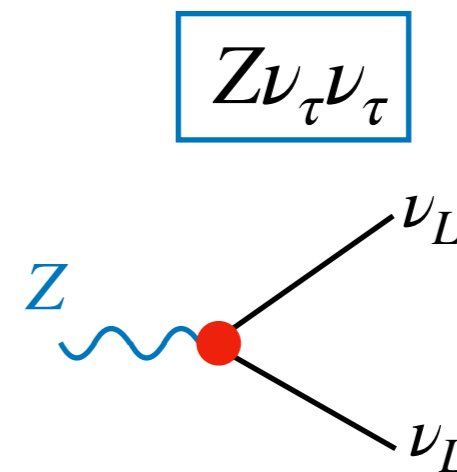
( $C_{lq} \rightarrow C_{H\ell}$ ) + 1-loop matching (+ NLL)

$$C_{H\ell}^{(1)} = - C_{H\ell}^{(3)}$$

(Only broken by small  $g_2$  effects and negligible NLL)



LFUV in  $\tau$ -decays



# Global fit

- Global likelihood:

$$\chi^2 = \chi_{b \rightarrow c\tau\nu}^2 + \chi_{\text{EWPO}}^2 + \chi_{\tau\text{-LFU}}^2 + \chi_{\text{high-}p_T}^2$$

- Fixed parameters:

$$\chi = 60^\circ, m_L = 1 \text{ TeV}, m_R = 1.5 \text{ TeV}, m_U = 3 \text{ TeV}, m_{G'} = 3.5 \text{ TeV}, m_{Z'} = 3 \text{ TeV}$$

- Two fits:

$m_W$  without CDF

$$m_W^{\text{exp}} = (80.379 \pm 0.012)\text{GeV}$$

Parameter	Best-fit point	$1\sigma$ interval
$\Lambda_U$	1.61 TeV	[1.46, 1.86] TeV
$m_Q$	$m_Q \rightarrow \infty$	[2.31, $\infty$ ) TeV
$Y_+$	0.36	[0.26, 0.56]

$$\chi_{\text{SM}}^2 - \chi_{\text{BFP}}^2 = 12.3 (2.4\sigma)$$

$m_W$  with CDF

$$m_W^{\text{exp}} = (80.410 \pm 0.015)\text{GeV}$$

Parameter	Best-fit point	$1\sigma$ interval
$\Lambda_U$	1.46 TeV	[1.32, 1.68] TeV
$m_Q$	2.08 TeV	[1.43, 4.72] TeV
$Y_+$	0.65	[0.43, 0.83]

$$\chi_{\text{SM}}^2 - \chi_{\text{BFP}}^2 = 15.4 (2.9\sigma)$$

# Global fit

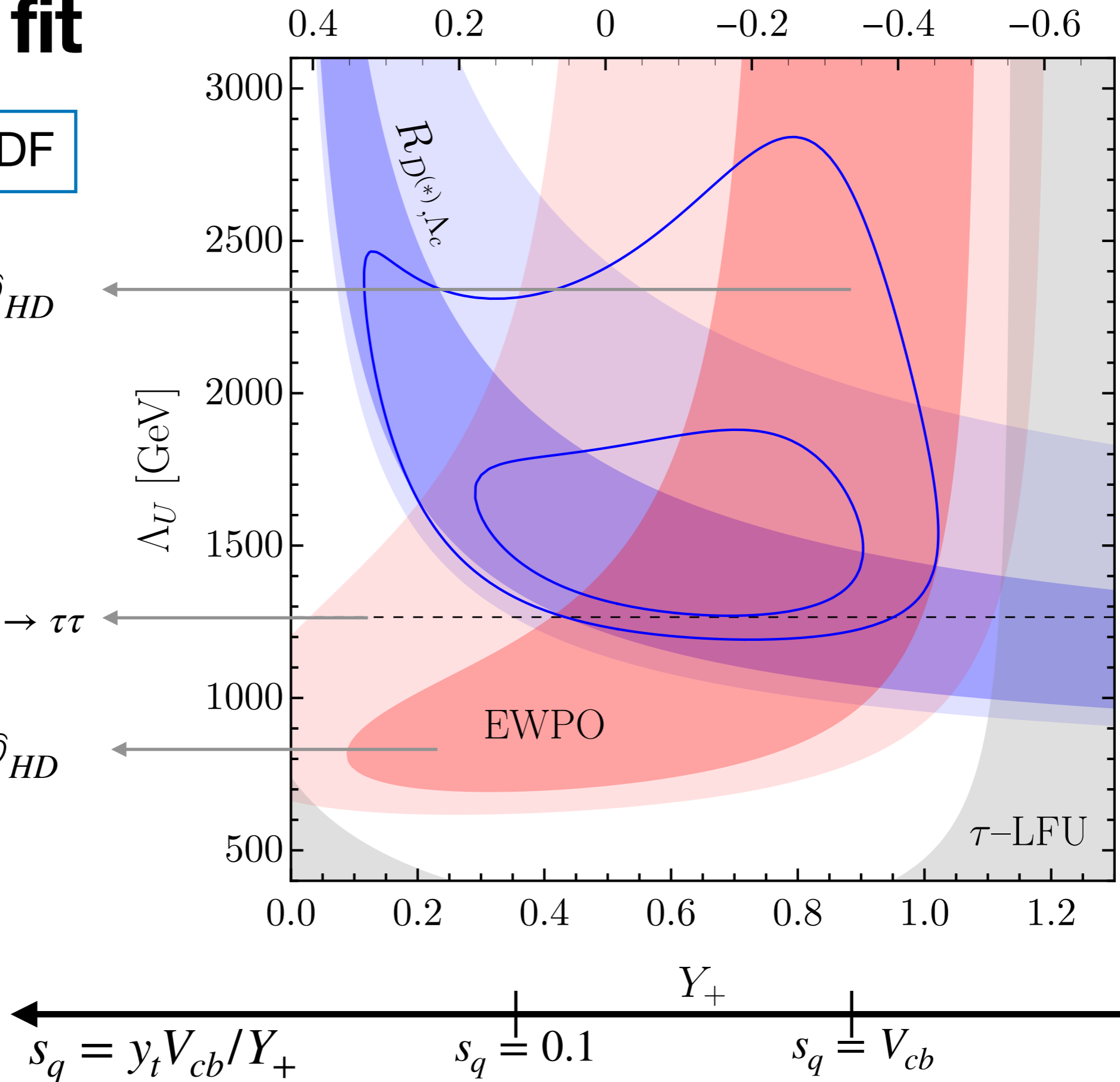
$$y_\nu = y_t \cos(\chi) - Y_+ \sin(\chi)$$

$m_W$  with CDF

$Q \rightarrow \mathcal{O}_{HD}$

95 % CL CMS  $pp \rightarrow \tau\tau$

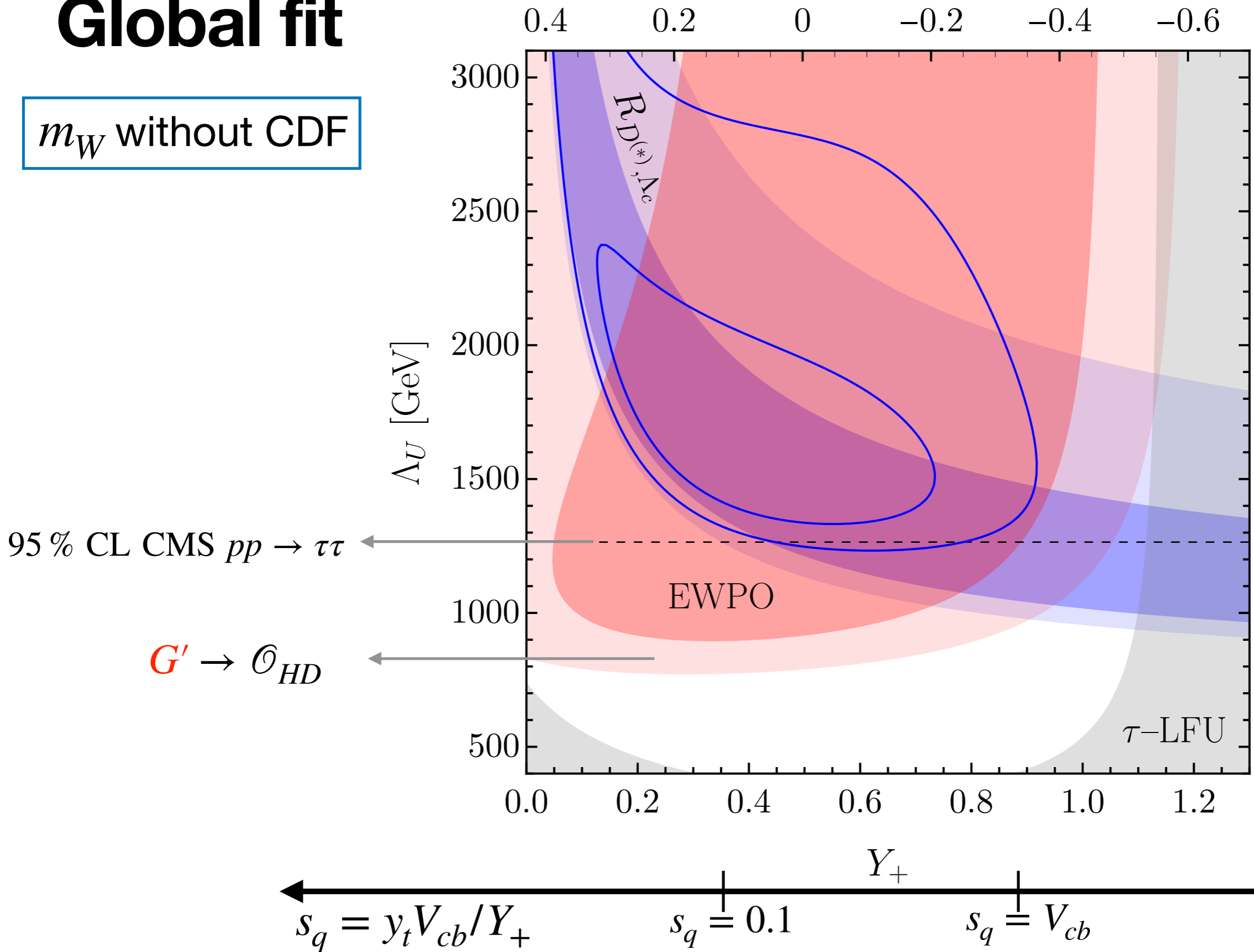
$G' \rightarrow \mathcal{O}_{HD}$



# Global fit

$m_W$  without CDF

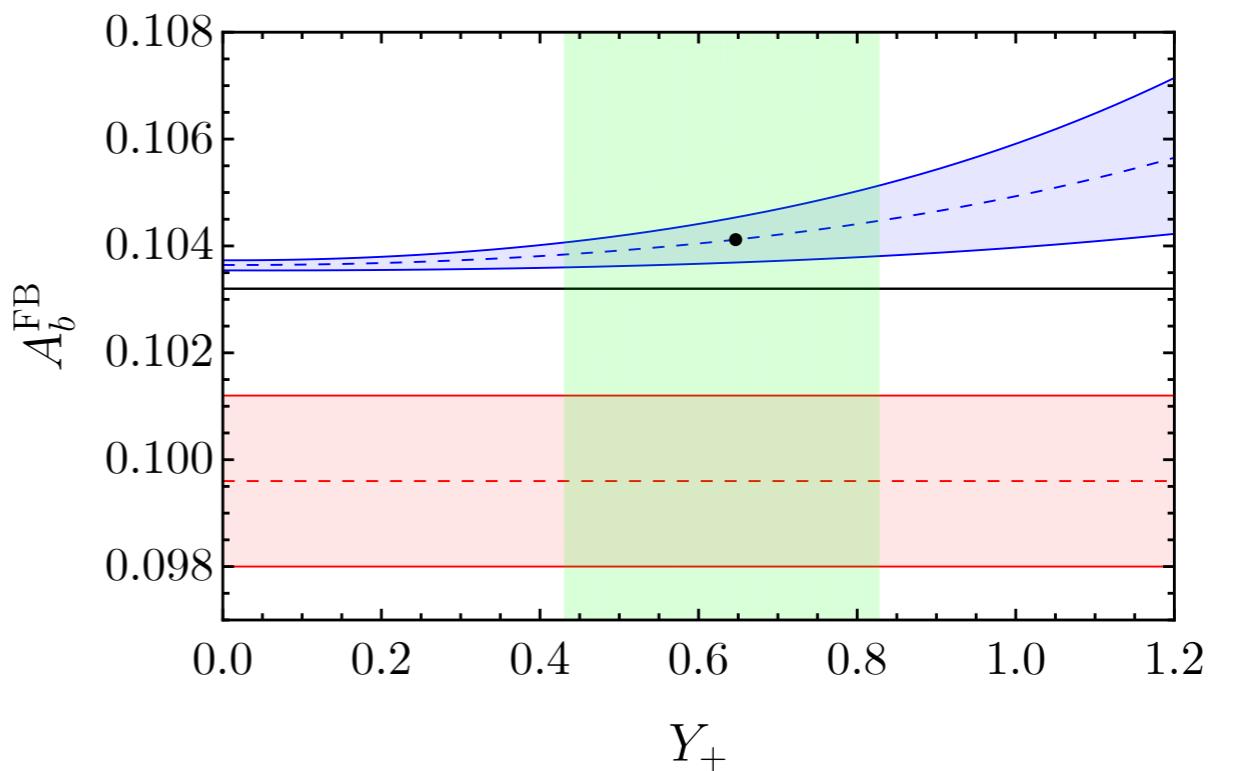
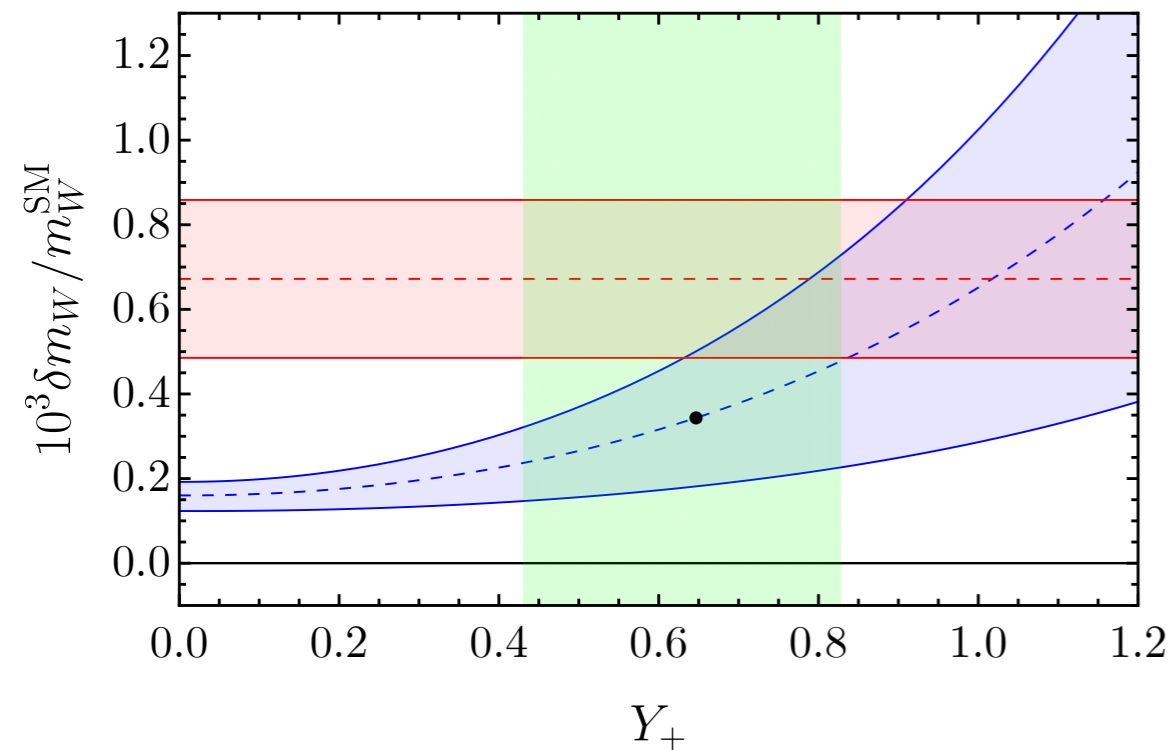
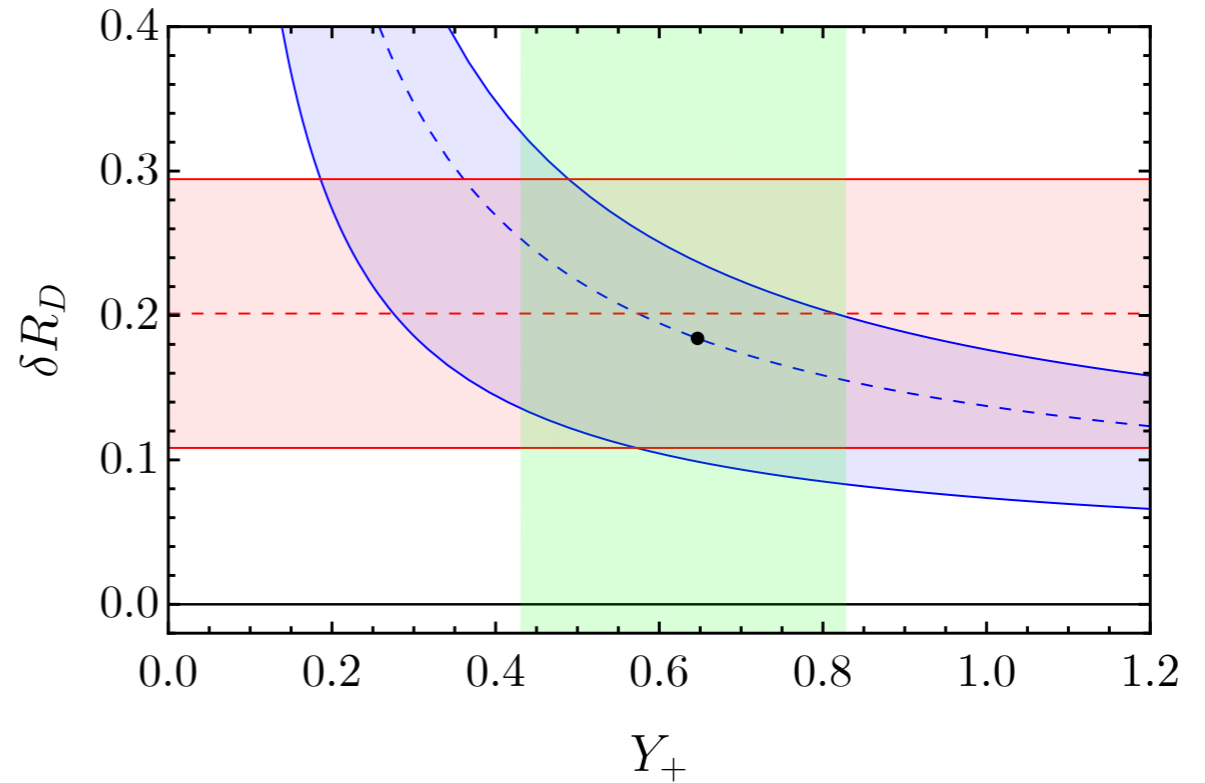
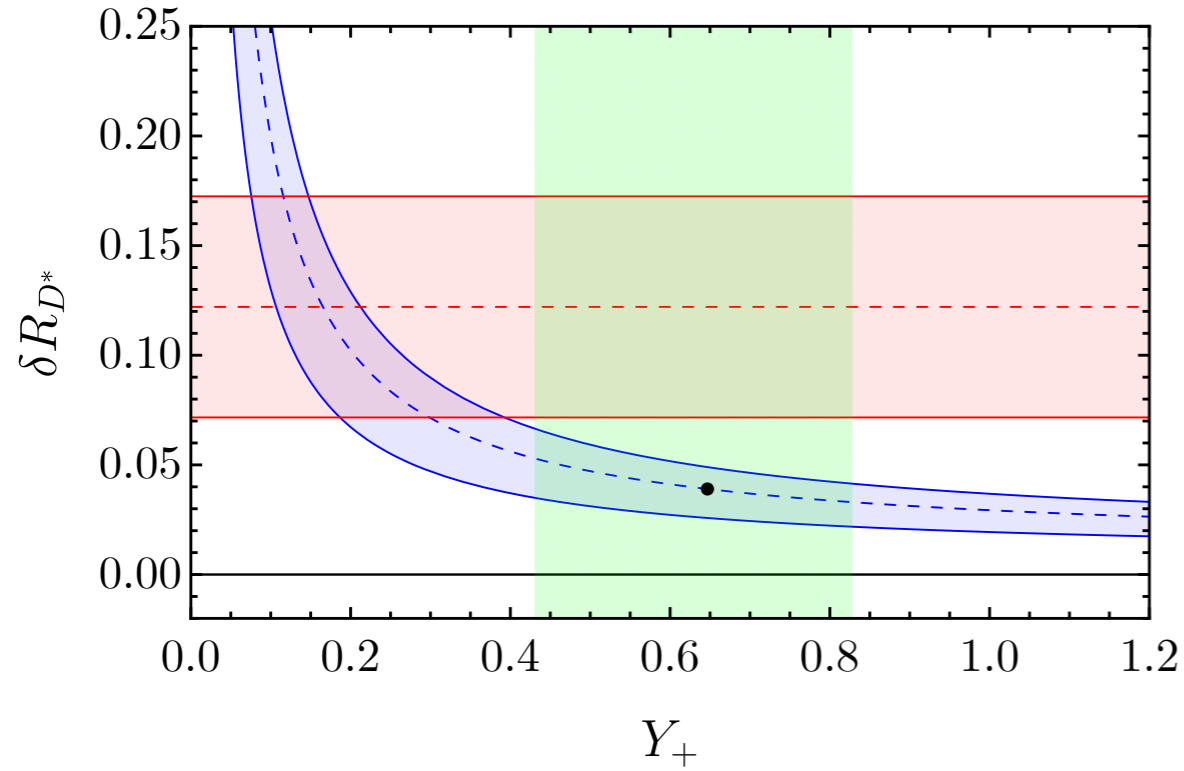
$$y_\nu = y_t \cos(\chi) - Y_+ \sin(\chi)$$





# Global fit

$m_W$  with CDF



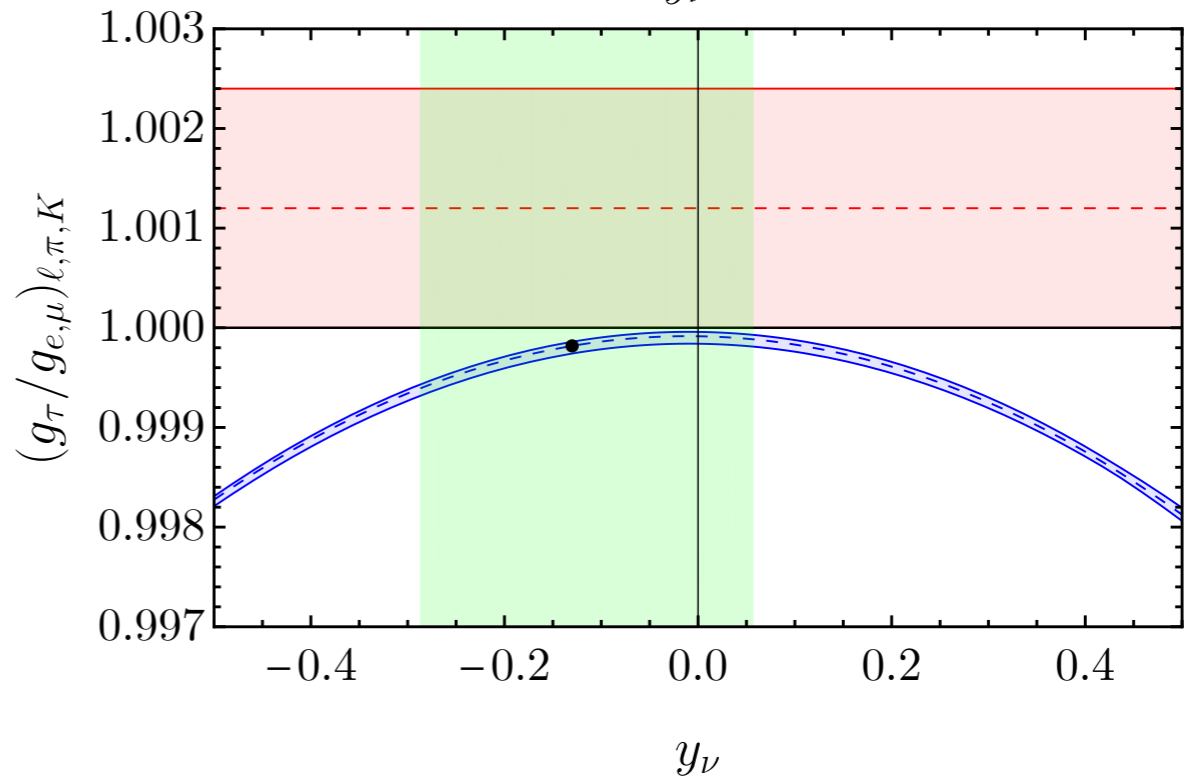
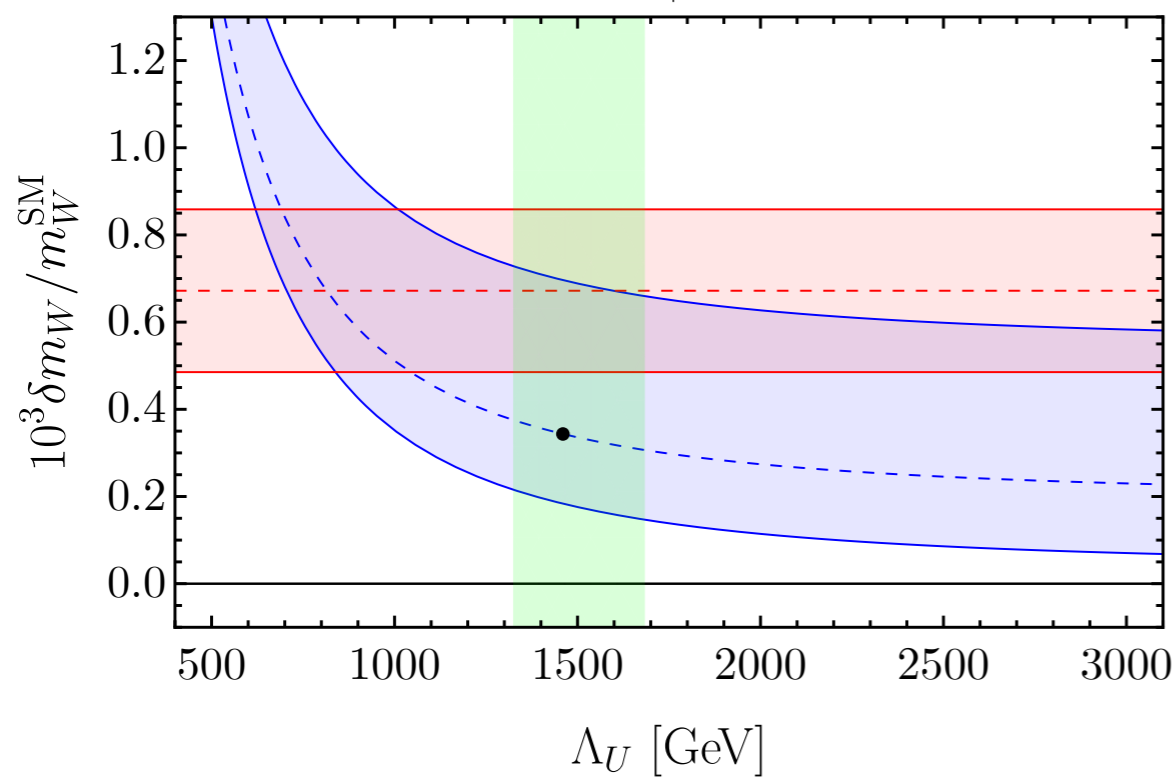
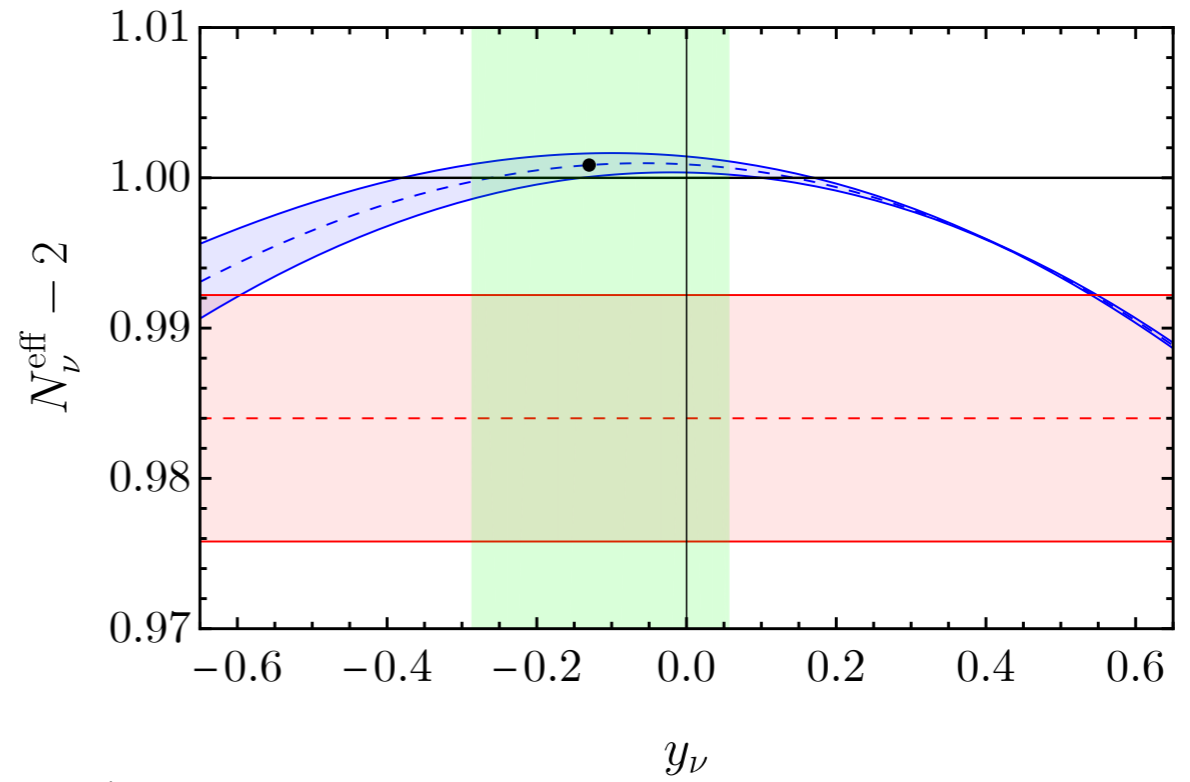
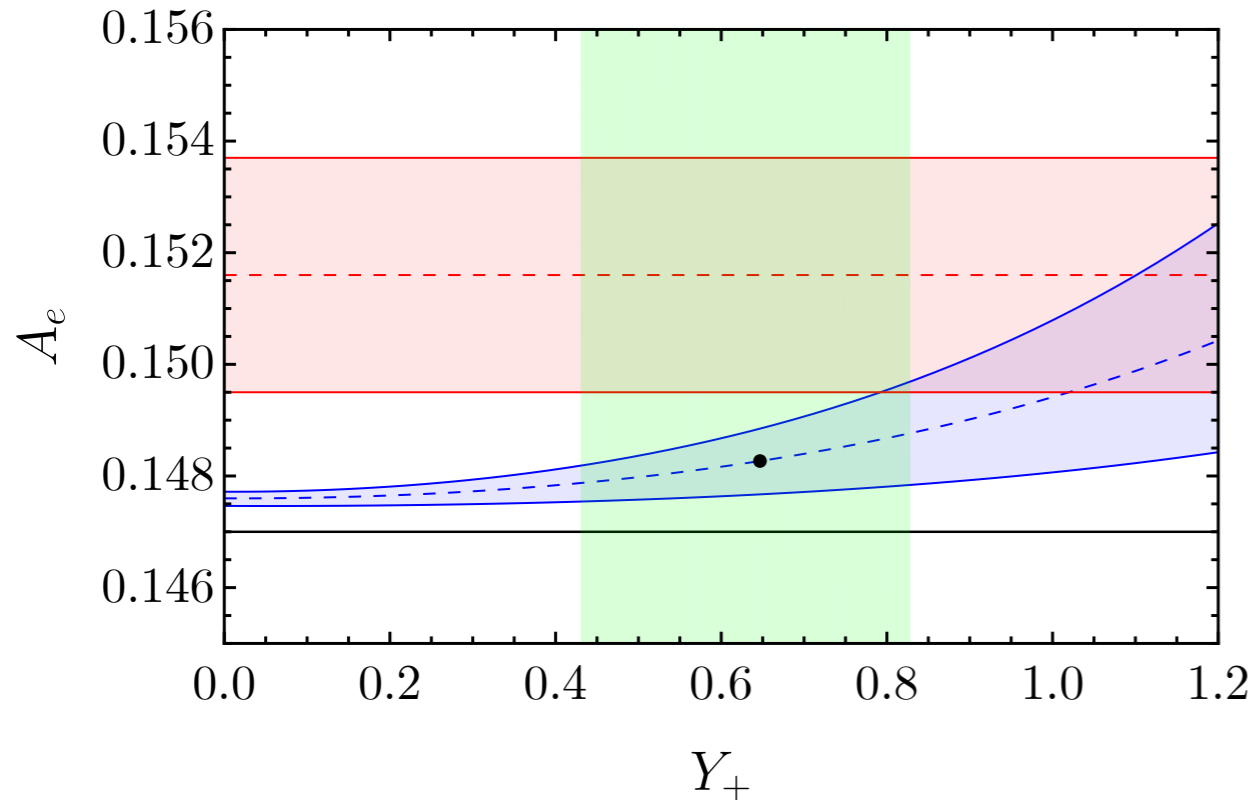
# Conclusions

- Quark-lepton unification of the third family at the TeV scale could be the infrared limit of a natural solution to the flavor puzzle.
- Apart from a rich B-physics pheno, the model has an interesting impact on EW physics.
- We find that the new colored states generate large universal contributions at the loop level.
- Higher order effects can play a key role.
- These results have a wider range of applicability: VLF, extended gauge groups, or inverse-seesaw mechanism.

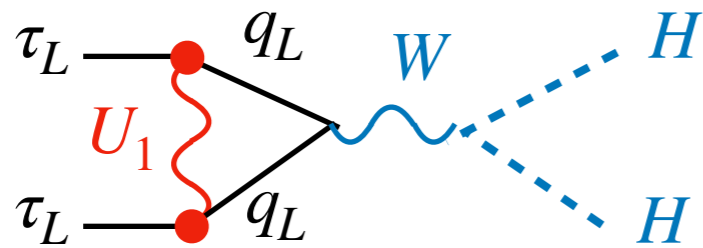
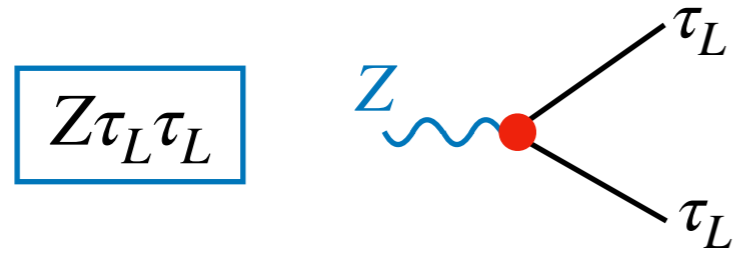
Thank you!

# Backup: Global fit

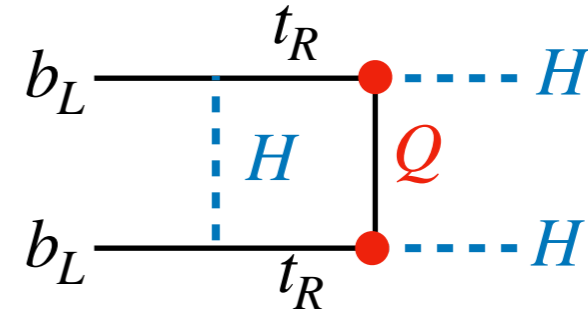
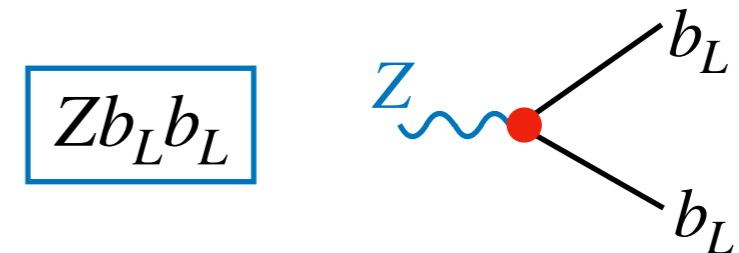
$m_W$  with CDF



# Backup: Other EW observables

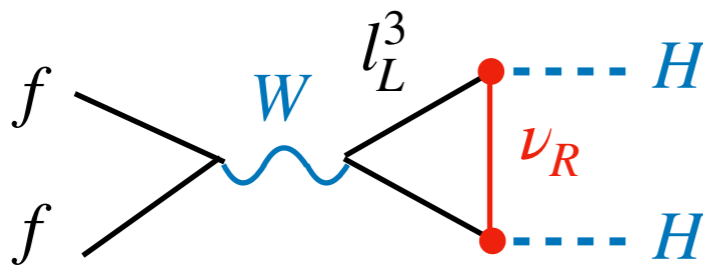


$$\mathcal{L} \supset C_{H\ell}^{(3)} (H^\dagger i\overleftrightarrow{D}_\mu^I H) (\bar{\ell}_L^3 \tau_I \gamma^\mu \ell_L^3)$$



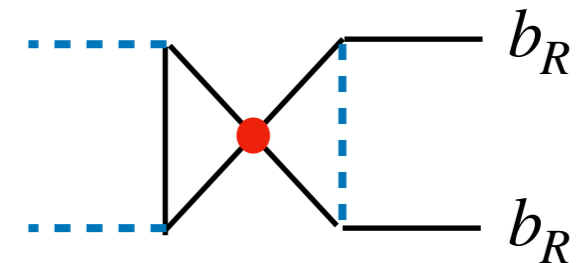
$$\mathcal{L} \supset C_{Hq}^{(1)} (H^\dagger i\overleftrightarrow{D}_\mu H) (\bar{q}_L^3 \gamma^\mu q_L^3)$$

Universal coupling modification



$$\mathcal{L} \supset C_{H\ell}^{(3)U} (H^\dagger i\overleftrightarrow{D}_\mu^I H) (\bar{\ell}_L \tau_I \gamma^\mu \ell_L)$$

$Zb_R b_R$



$$\mathcal{L} \supset C_{Hb} (H^\dagger i\overleftrightarrow{D}_\mu H) (\bar{b}_R \gamma^\mu b_R)$$

# Backup: EW observables

Observable	Experimental value	SM prediction	Definition
$\Gamma_Z$ [GeV]	$2.4955 \pm 0.0023$ [4, 28]	2.4941	$\sum_f \Gamma(Z \rightarrow f\bar{f})$
$\sigma_{\text{had}}$ [nb]	$41.4802 \pm 0.0325$ [4, 28]	41.4842	$\frac{12\pi}{m_Z^2} \frac{\Gamma(Z \rightarrow e^+e^-)\Gamma(Z \rightarrow q\bar{q})}{\Gamma_Z^2}$
$R_e$	$20.804 \pm 0.050$ [4]	20.734	$\frac{\sum_q \Gamma(Z \rightarrow q\bar{q})}{\Gamma(Z \rightarrow e^+e^-)}$
$R_\mu$	$20.785 \pm 0.033$ [4]	20.734	$\frac{\sum_q \Gamma(Z \rightarrow q\bar{q})}{\Gamma(Z \rightarrow \mu^+\mu^-)}$
$R_\tau$	$20.764 \pm 0.045$ [4]	20.781	$\frac{\sum_q \Gamma(Z \rightarrow q\bar{q})}{\Gamma(Z \rightarrow \tau^+\tau^-)}$
$A_{\text{FB}}^{0,e}$	$0.0145 \pm 0.0025$ [4]	0.0162	$\frac{3}{4}A_e^2$
$A_{\text{FB}}^{0,\mu}$	$0.0169 \pm 0.0013$ [4]	0.0162	$\frac{3}{4}A_e A_\mu$
$A_{\text{FB}}^{0,\tau}$	$0.0188 \pm 0.0017$ [4]	0.0162	$\frac{3}{4}A_e A_\tau$
$R_b$	$0.21629 \pm 0.00066$ [4]	0.21581	$\frac{\Gamma(Z \rightarrow b\bar{b})}{\sum_q \Gamma(Z \rightarrow q\bar{q})}$
$R_c$	$0.1721 \pm 0.0030$ [4]	0.17222	$\frac{\Gamma(Z \rightarrow c\bar{c})}{\sum_q \Gamma(Z \rightarrow q\bar{q})}$
$A_b^{\text{FB}}$	$0.0996 \pm 0.0016$ [4, 29]	0.1032	$\frac{3}{4}A_e A_b$
$A_c^{\text{FB}}$	$0.0707 \pm 0.0035$ [4]	0.0736	$\frac{3}{4}A_e A_c$
$A_e$	$0.1516 \pm 0.0021$ [4]	0.1470	$\frac{\Gamma(Z \rightarrow e_L^+e_L^-) - \Gamma(Z \rightarrow e_R^+e_R^-)}{\Gamma(Z \rightarrow e^+e^-)}$
$A_\mu$	$0.142 \pm 0.015$ [4]	0.1470	$\frac{\Gamma(Z \rightarrow \mu_L^+\mu_L^-) - \Gamma(Z \rightarrow \mu_R^+\mu_R^-)}{\Gamma(Z \rightarrow \mu^+\mu^-)}$
$A_\tau$	$0.136 \pm 0.015$ [4]	0.1470	$\frac{\Gamma(Z \rightarrow \tau_L^+\tau_L^-) - \Gamma(Z \rightarrow \tau_R^+\tau_R^-)}{\Gamma(Z \rightarrow \tau^+\tau^-)}$
$A_e$	$0.1498 \pm 0.0049$ [4]	0.1470	$\frac{\Gamma(Z \rightarrow e_L^+e_L^-) - \Gamma(Z \rightarrow e_R^+e_R^-)}{\Gamma(Z \rightarrow e^+e^-)}$
$A_\tau$	$0.1439 \pm 0.0043$ [4]	0.1470	$\frac{\Gamma(Z \rightarrow \tau_L^+\tau_L^-) - \Gamma(Z \rightarrow \tau_R^+\tau_R^-)}{\Gamma(Z \rightarrow \tau^+\tau^-)}$
$A_b$	$0.923 \pm 0.020$ [4]	0.935	$\frac{\Gamma(Z \rightarrow b_L b_L) - \Gamma(Z \rightarrow b_R b_R)}{\Gamma(Z \rightarrow b\bar{b})}$
$A_c$	$0.670 \pm 0.027$ [4]	0.668	$\frac{\Gamma(Z \rightarrow c_L \bar{c}_L) - \Gamma(Z \rightarrow c_R \bar{c}_R)}{\Gamma(Z \rightarrow c\bar{c})}$
$A_s$	$0.895 \pm 0.091$ [30]	0.936	$\frac{\Gamma(Z \rightarrow s_L \bar{s}_L) - \Gamma(Z \rightarrow s_R \bar{s}_R)}{\Gamma(Z \rightarrow s\bar{s})}$
$R_{uc}$	$0.166 \pm 0.009$ [9]	0.1722	$\frac{\Gamma(Z \rightarrow u\bar{u}) + \Gamma(Z \rightarrow c\bar{c})}{2 \sum_q \Gamma(Z \rightarrow q\bar{q})}$

Observable	Experimental value	SM prediction
$m_W$ [GeV]	$80.379 \pm 0.012$ [9]	80.356
$\Gamma_W$ [GeV]	$2.085 \pm 0.042$ [9]	2.088
$\text{Br}(W \rightarrow e\nu)$	$0.1071 \pm 0.0016$ [5]	0.1082
$\text{Br}(W \rightarrow \mu\nu)$	$0.1063 \pm 0.0015$ [5]	0.1082
$\text{Br}(W \rightarrow \tau\nu)$	$0.1138 \pm 0.0021$ [5]	0.1081
$\text{Br}(W \rightarrow \mu\nu)/\text{Br}(W \rightarrow e\nu)$	$0.982 \pm 0.024$ [32]	1.000
$\text{Br}(W \rightarrow \mu\nu)/\text{Br}(W \rightarrow e\nu)$	$1.020 \pm 0.019$ [12]	1.000
$\text{Br}(W \rightarrow \mu\nu)/\text{Br}(W \rightarrow e\nu)$	$1.003 \pm 0.010$ [13]	1.000
$\text{Br}(W \rightarrow \tau\nu)/\text{Br}(W \rightarrow e\nu)$	$0.961 \pm 0.061$ [9, 31]	0.999
$\text{Br}(W \rightarrow \tau\nu)/\text{Br}(W \rightarrow \mu\nu)$	$0.992 \pm 0.013$ [14]	0.999
$R_{Wc} \equiv \frac{\Gamma(W \rightarrow cs)}{\Gamma(W \rightarrow ud) + \Gamma(W \rightarrow cs)}$	$0.49 \pm 0.04$ [9]	0.50

[V. Breso-Pla, A. Falkowski, M. Gonzalez-Alonso, [2103.12074](#)]

# Backup: Running

$$\mathcal{A}(\mu) = \frac{y_t(\mu)^2}{16\pi^2} \mathcal{A}_t + \frac{g_s(\mu)^2}{16\pi^2} \mathcal{A}_s + \frac{g_L(\mu)^2}{16\pi^2} \mathcal{A}_L + \dots$$

- RGE:  $\mu \frac{d}{d\mu} \mathcal{C}(\mu) = \mathcal{A}(\mu) \mathcal{C}(\mu)$

- Integration:

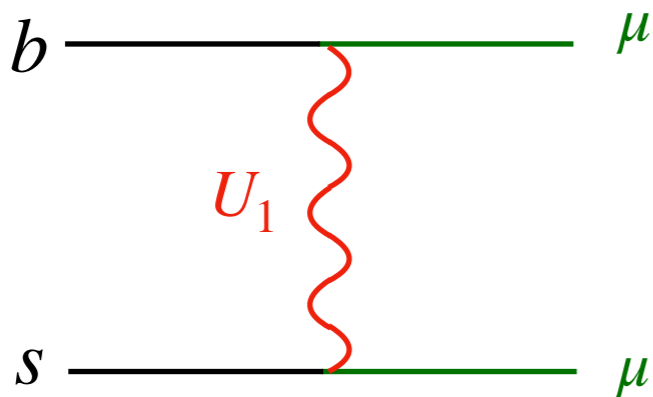
$$\begin{aligned} \mathcal{C}(\mu) &= \mathcal{P} \int_{\mu_0}^{\mu} \exp \mathcal{A}(\mu) d \log \mu \mathcal{C}(\mu_0) \\ &= \left( \mathbb{1} + \int_{\mu_0}^{\mu} d \log \mu \mathcal{A}(\mu) + \int_{\mu_0}^{\mu} d \log \mu_1 \int_{\mu_0}^{\mu_1} d \log \mu_2 \mathcal{A}(\mu_1) \mathcal{A}(\mu_2) + \dots \right) \mathcal{C}(\mu_0) \end{aligned}$$

- Top Yukawa running: 
$$\begin{aligned} \mathcal{C}(\mu) - \mathcal{C}(\mu_0) &= \frac{1}{16\pi^2} \mathcal{A}_t \mathcal{C}(\mu_0) \int_{\mu_0}^{\mu} y_t(\mu)^2 d \log \mu \\ &= \frac{\bar{y}_t^2}{16\pi^2} \mathcal{A}_t \mathcal{C}(\mu_0) \log \frac{\mu}{\mu_0}, \end{aligned}$$

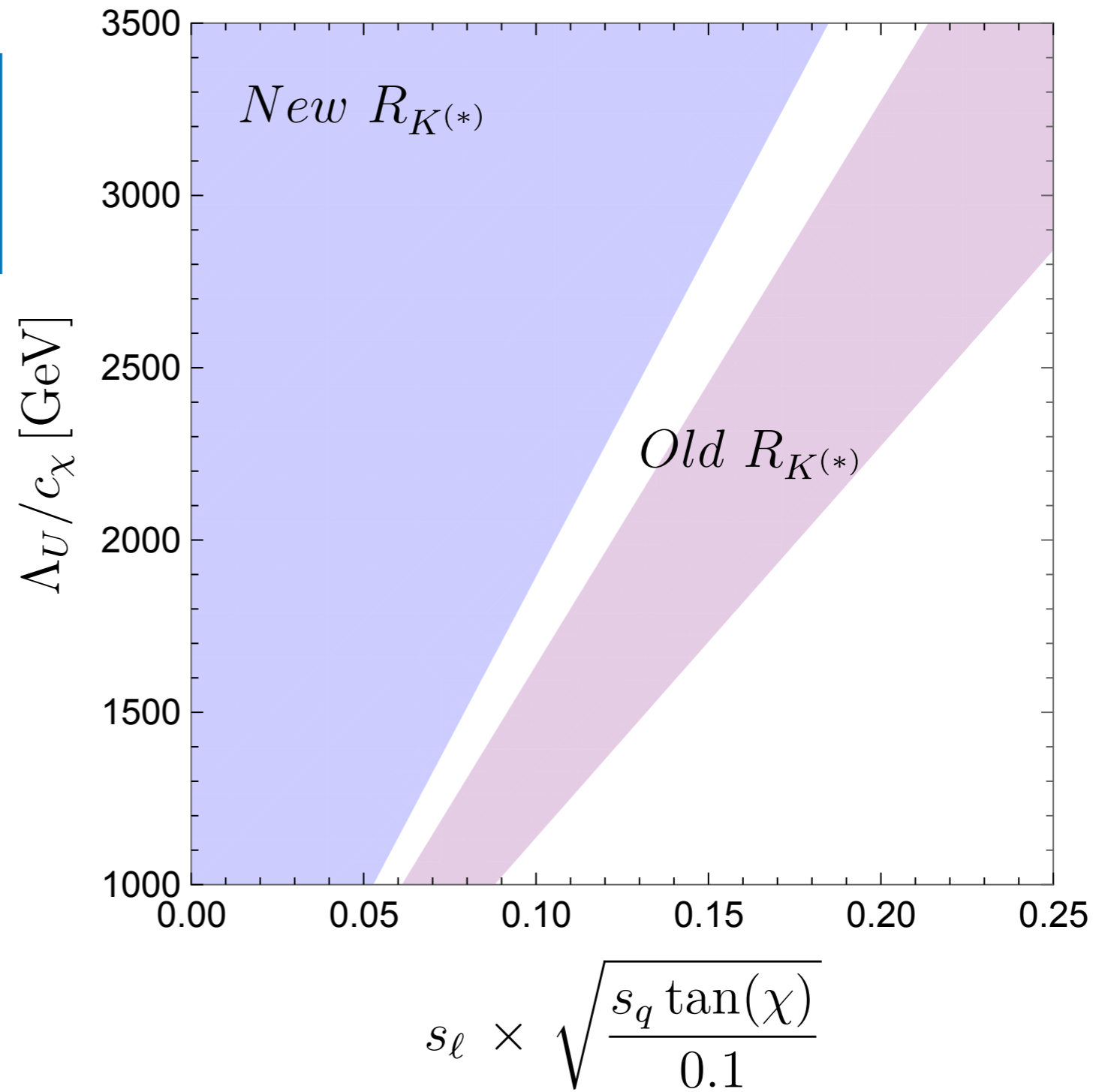
$$\bar{y}_t^2 = \frac{1}{\log \frac{\mu}{\mu_0}} \int_{\mu_0}^{\mu} d \log \mu' y_t^2(\mu') \quad \longrightarrow \quad \bar{y}_t \approx 0.87$$

# Backup: $R_{K^{(*)}}$

$$R_{K^{(*)}} = \frac{Br(B \rightarrow K^{(*)}\mu\mu)}{Br(B \rightarrow K^{(*)}ee)}$$

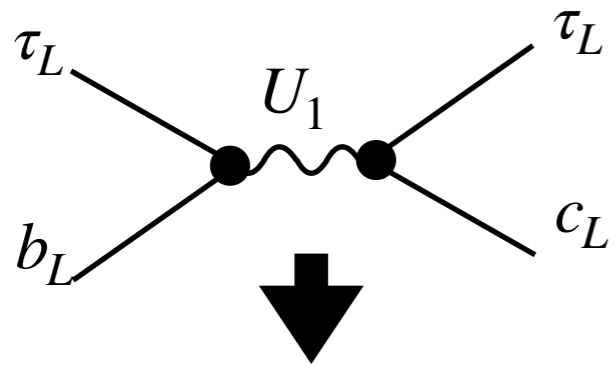


$$\propto \frac{s_q s_l^2}{\Lambda_U^2} \sin(\chi) \cos(\chi)$$

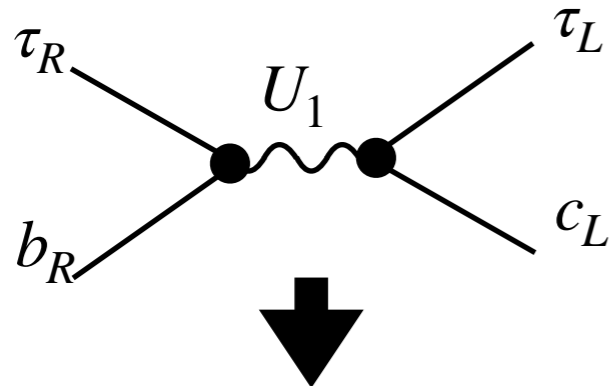


# Backup: $R_D^{(*)}$

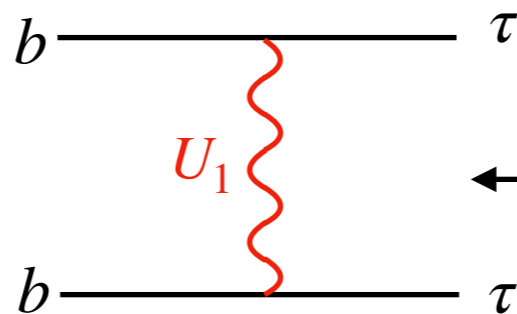
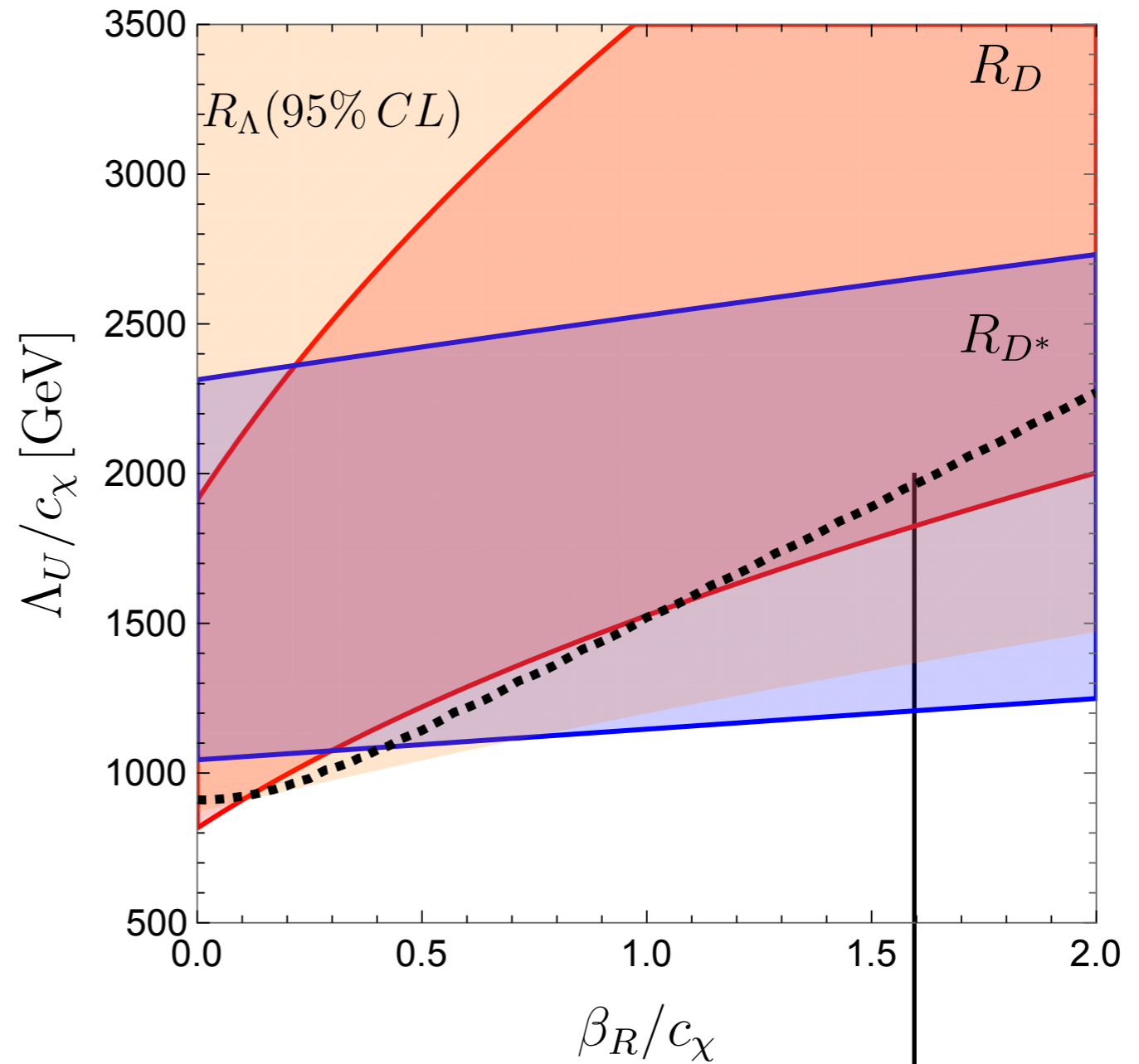
$$s_q \tan(\chi) = 0.1 \approx 2.4 V_{cb}$$



$$C_{LL}^c \propto \frac{s_q}{\Lambda^2}$$



$$C_{LR}^c \propto \frac{\beta_R s_q}{\Lambda_U^2}$$



95% CL CMS exclusion limits  
on  $pp \rightarrow \tau\tau$

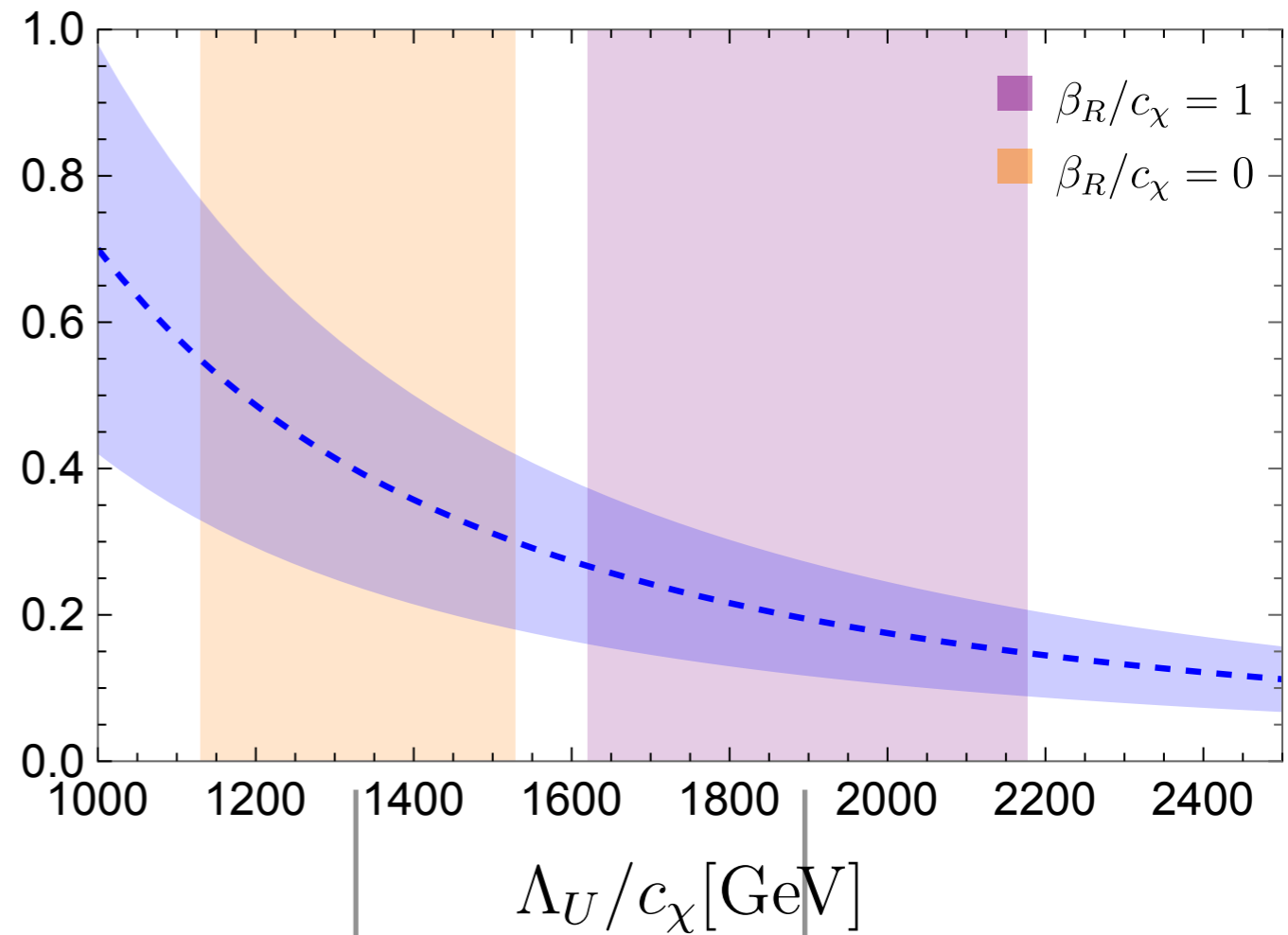
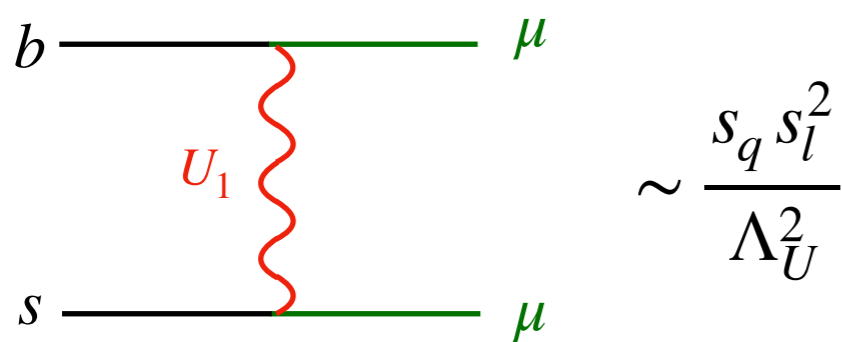
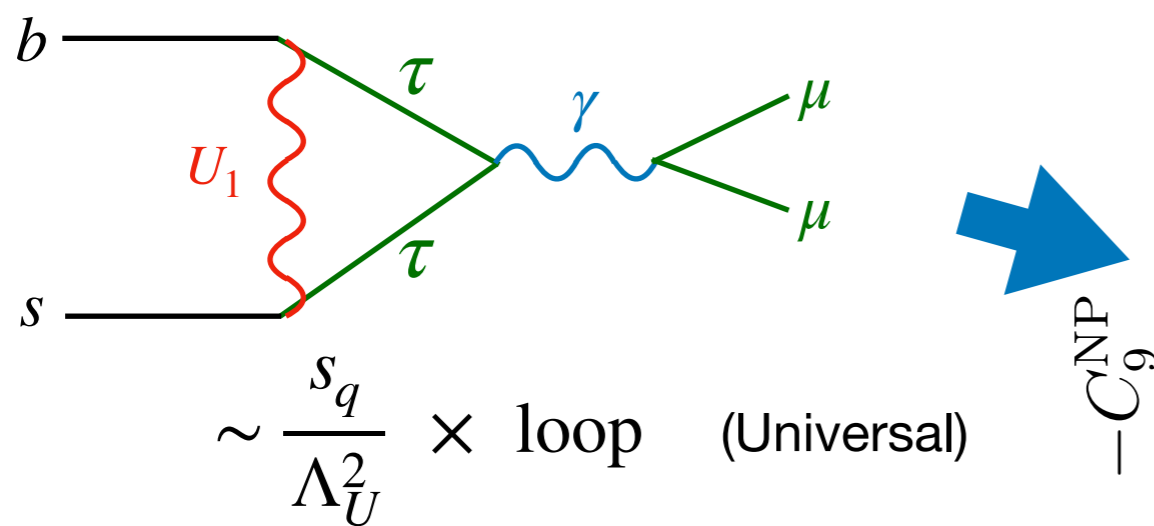


# Backup: $b \rightarrow s\mu\mu$

$$B \rightarrow K^* \mu\mu$$

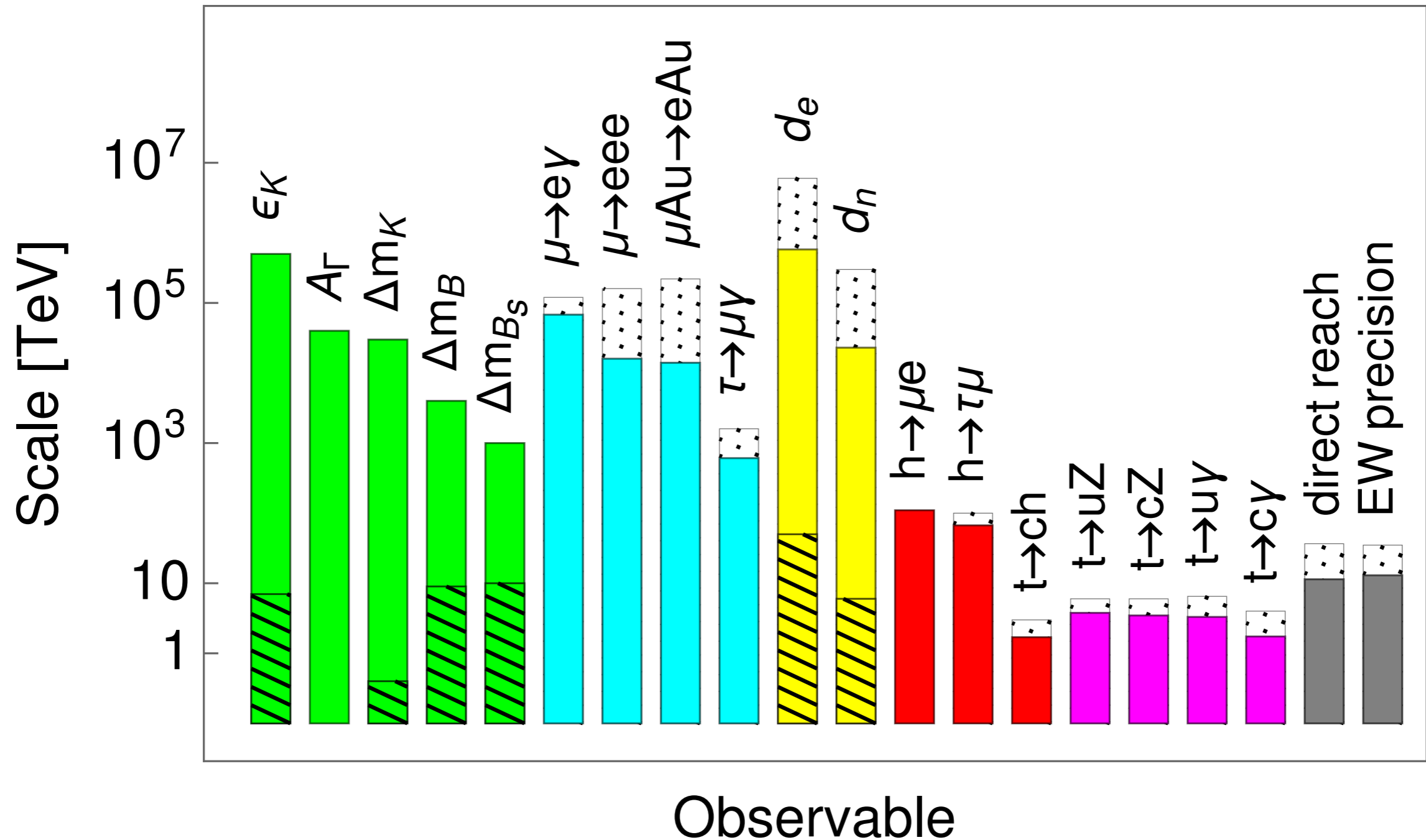
$$\mathcal{L} \supset \frac{2}{v^2} V_{ts}^* V_{tb} C_9 (\bar{s}_L \gamma^\mu b_L) (\mu \gamma_\mu \mu)$$

$$C_9^{\text{NP}} = -0.75 \pm 0.23 \quad (\sim 3.4\sigma)$$



$b \rightarrow c\tau\nu$  preferred regions for  $s_q \tan(\chi) = 0.1$

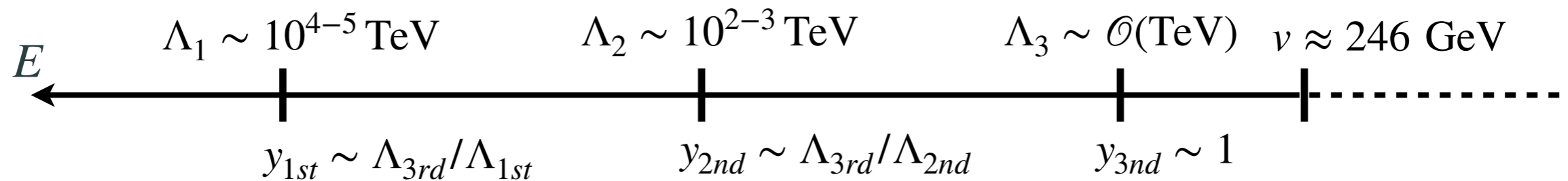
# Backup: Flavor bounds on NP



[Physics Briefing Book, [1910.11775](#)]

# Backup: Multiscale flavor

- Safe solution to the flavor puzzle: multiscale origin of the flavor hierarchies.

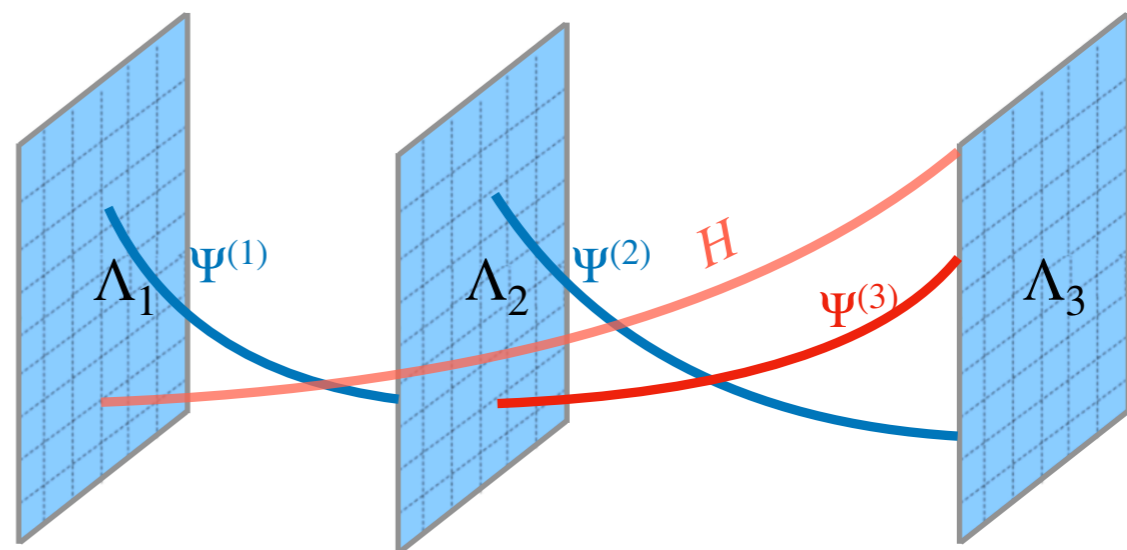
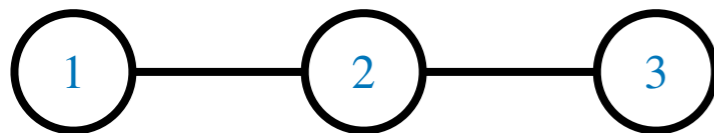


Gauge deconstruction

Extra-dimensions

dual  
↔

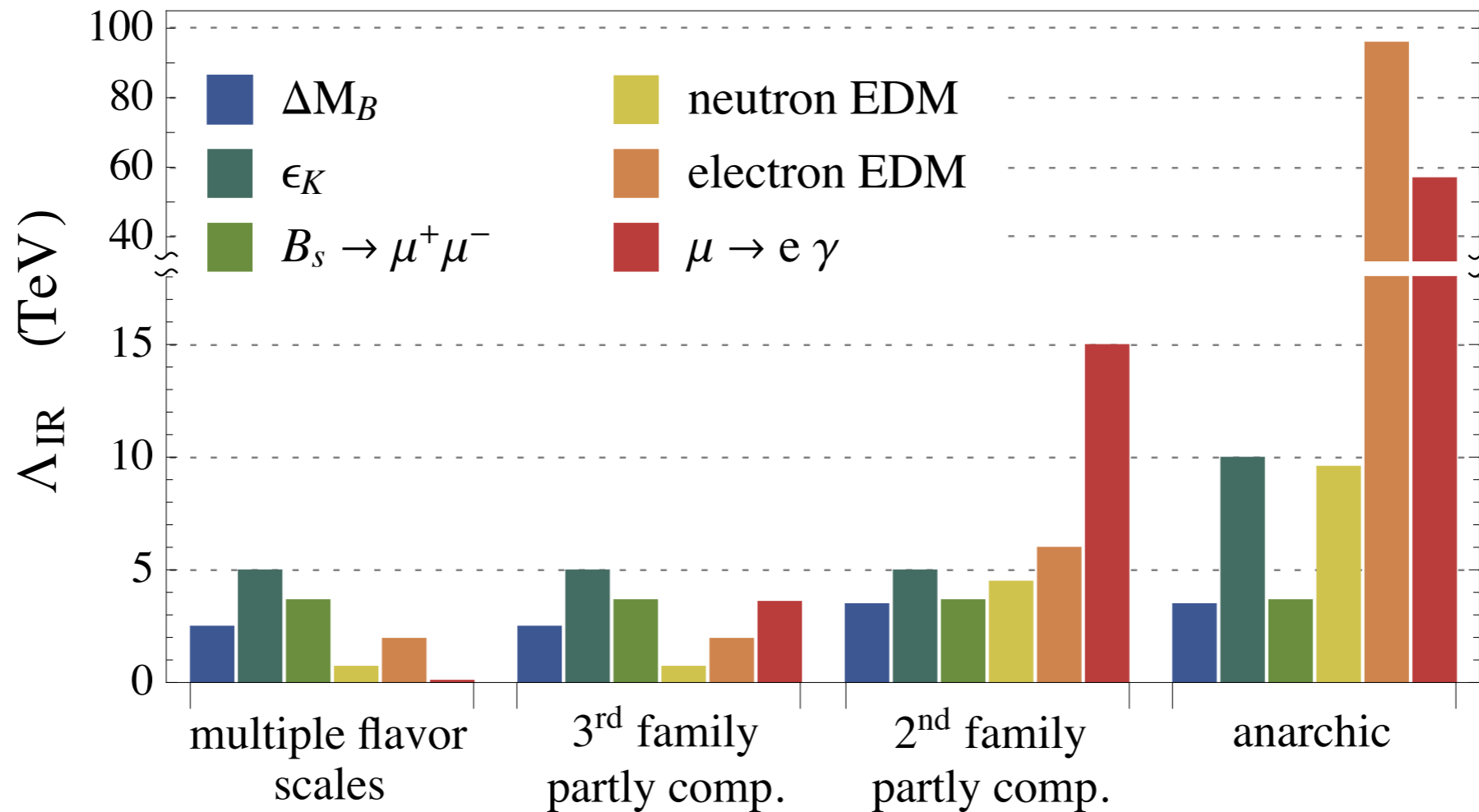
Composite dynamics



# Backup: Composite models

[Panico, Pomarol, [1603.06609](#)]

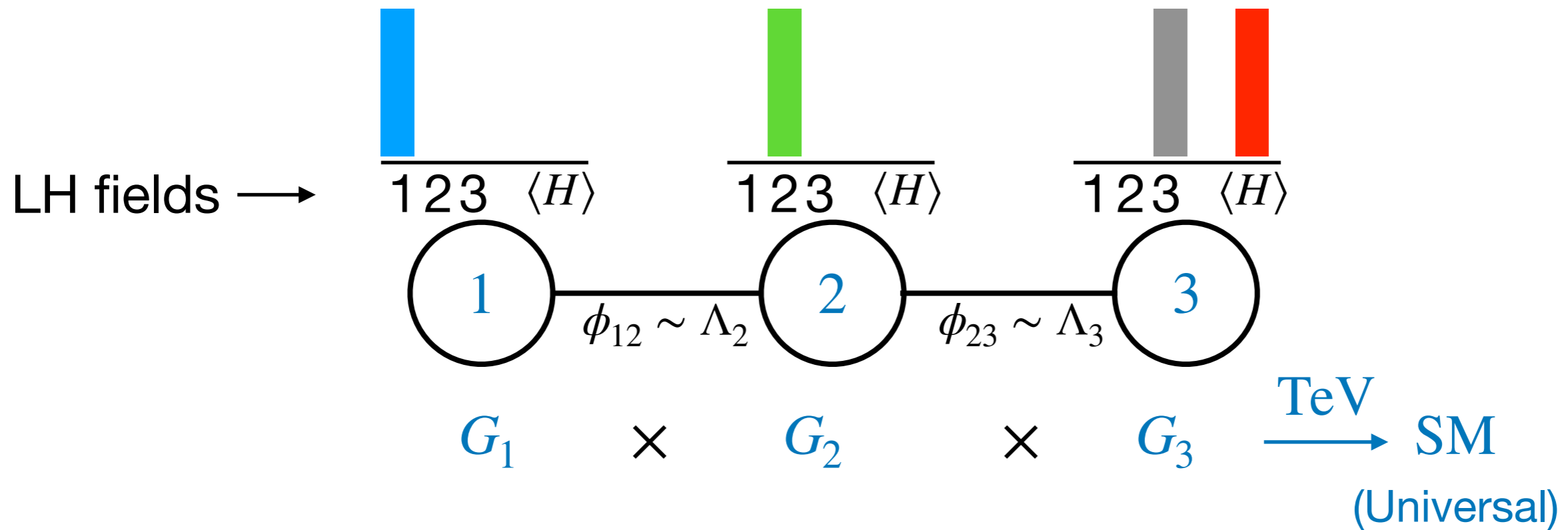
- Example in composite models/RS:



Dangerous dipoles (among others)  
generated at the IR scale

$$\sim \frac{g_*^2}{16\pi^2} \frac{m_e}{\Lambda_{\text{IR}}^2} \bar{e}_L \sigma_{\mu\nu} e_R F^{\mu\nu}$$

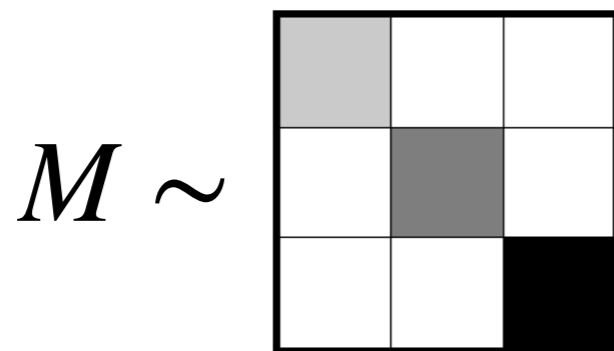
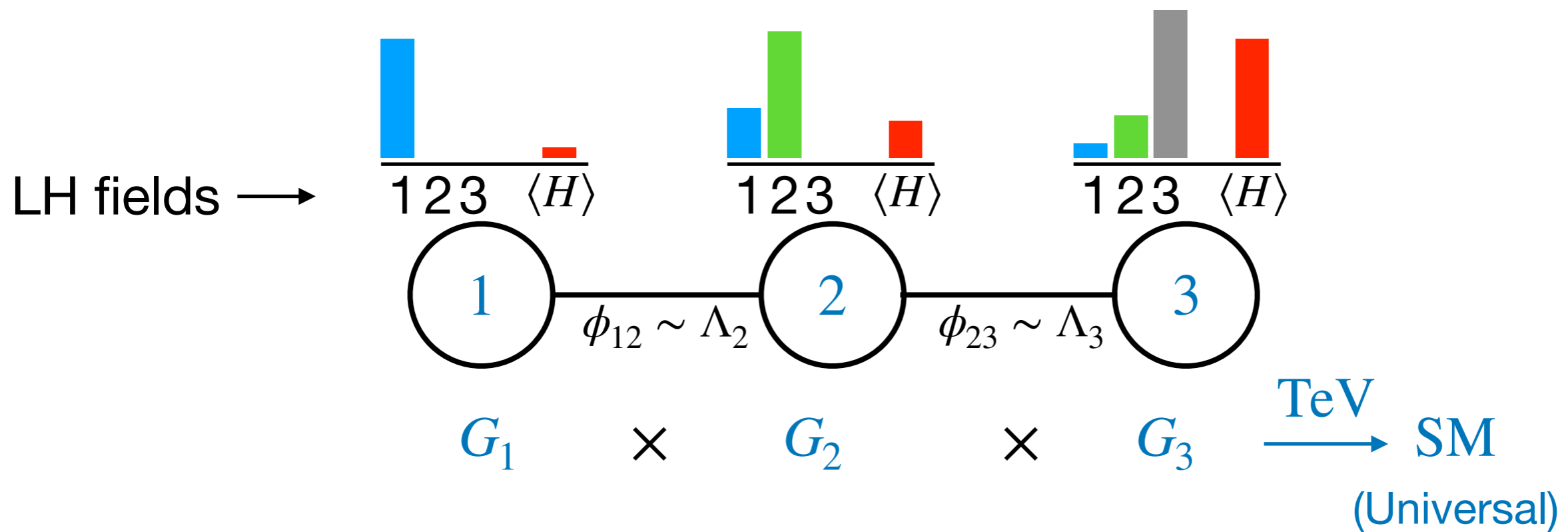
# Backup: Deconstructing flavor



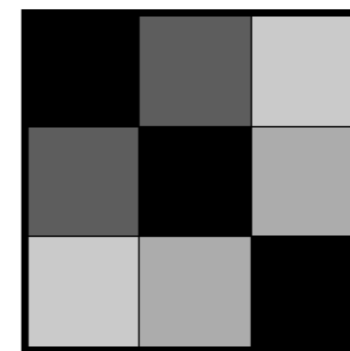
$$M \sim \begin{array}{|c|c|c|} \hline & & \\ \hline & & \\ \hline & & \blacksquare \\ \hline \end{array}$$

$$V_{\text{CKM}} \sim \begin{array}{|c|c|c|} \hline \blacksquare & & \\ \hline & \blacksquare & \\ \hline & & \blacksquare \\ \hline \end{array}$$

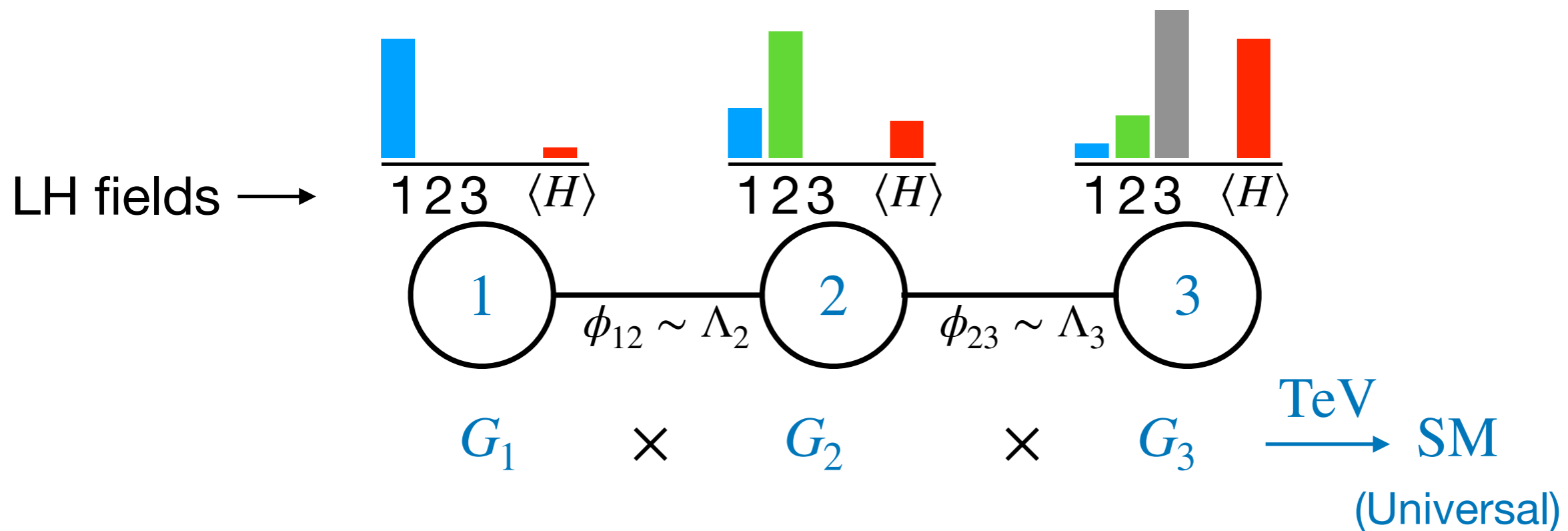
# Backup: Deconstructing flavor



$V_{\text{CKM}} \sim$



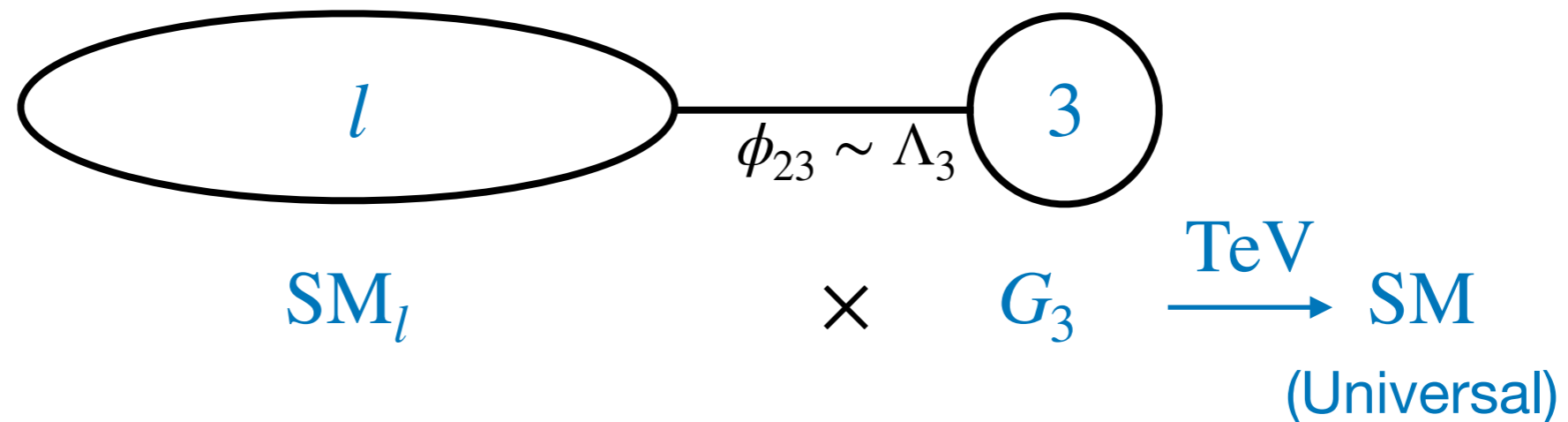
# Backup: Deconstructing flavor



- Only rotations in the LH sector → **No RH or scalar FCNC** ✓

# Backup: Gauge deconstruction

- From the TeV scale, we see...



- Emerging flavor symmetry:

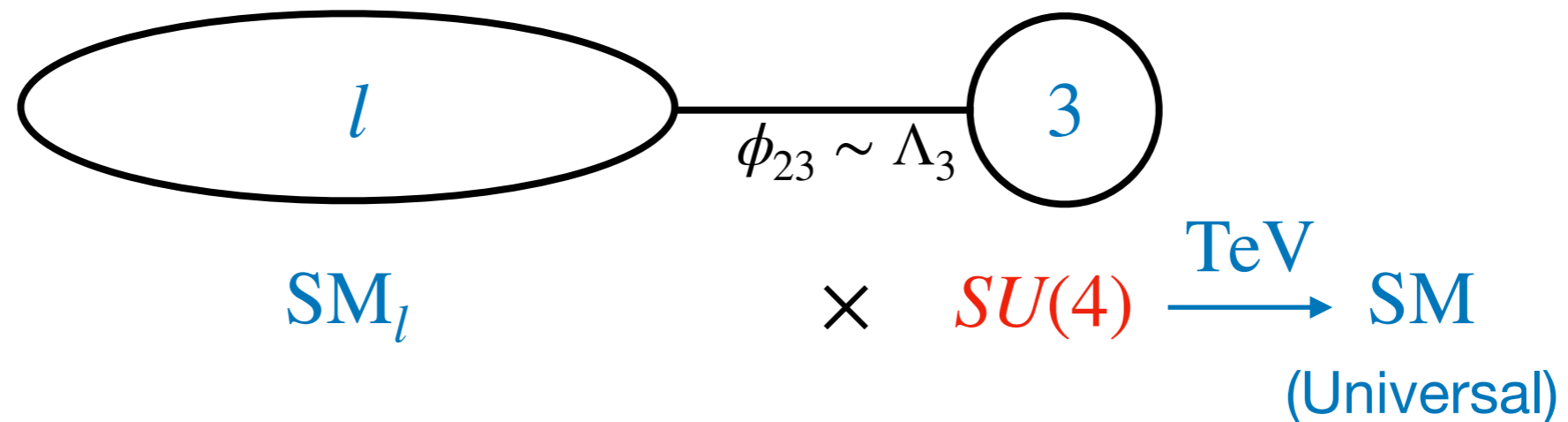
$$U(2)$$

(Only broken minimally in the LH sector)



# Backup: Gauge deconstruction

- From the TeV scale, we see...



- Emerging flavor symmetry:

$$U(2)$$

(Only broken minimally in the LH sector)