New Methods for Matching New Physics to EFTs

Anders Eller Thomsen

Based on work with J. Fuentes-Martín, M. König, J. Pagès, and F. Wilsch

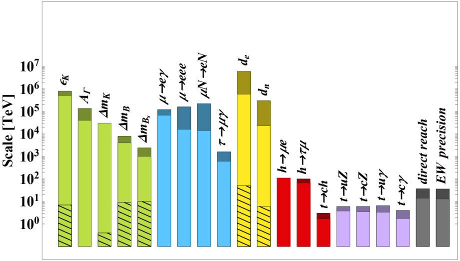
MATCHETE

University of Basel

Les Rencontres de Physique de la Vallée d'Aoste
10 March 2023

Low-energy observables

Strong NP constraints from indirect searches



Physics Briefing Book [1910.11775]

Leading NP contributions to $\Delta F = 2$, LFU, cLFV, dipoles, and EW precision often originate at loop level

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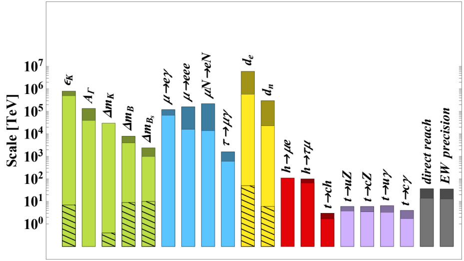

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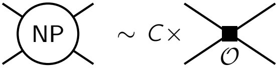

The resulting low-energy physics is captured by

$$\mathcal{L}_{\text{EFT}}(\phi) = \mathcal{L}^{d=4} + \sum_{n=5}^{\infty} \frac{C_{n,i}}{\Lambda^{n-4}} \mathcal{O}_{n,i}$$

Observables are computed *once* in $\mathcal{L}_{\text{EFT}}$ independent of the NP

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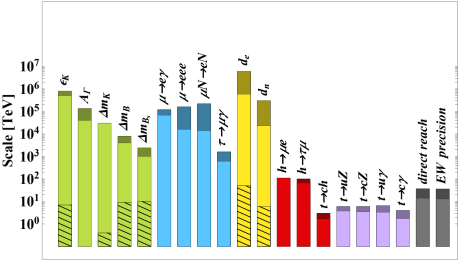

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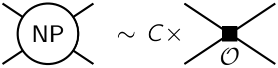

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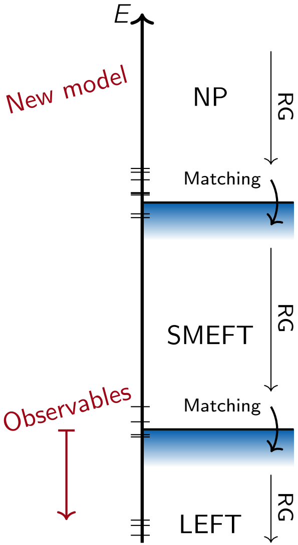
$$\mathcal{L}_{\text{UV}}(\Phi,\ \phi) \xrightarrow{\text{Matching}} \mathcal{L}_{\text{EFT}}(\phi)$$

EFT workflow



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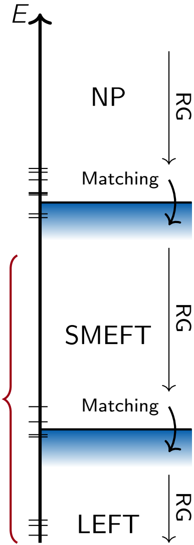

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Included in codes
wilson and **DsixTools**

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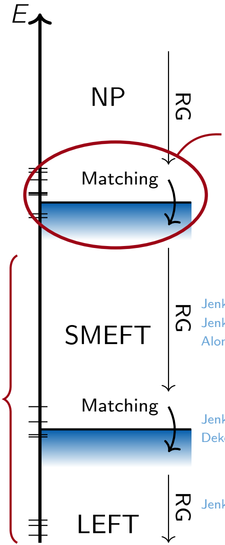

New tools @ 1-loop order: **Matchmakereft** and **Matchete**

Carmona *et al.* [2112.10787]; Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]

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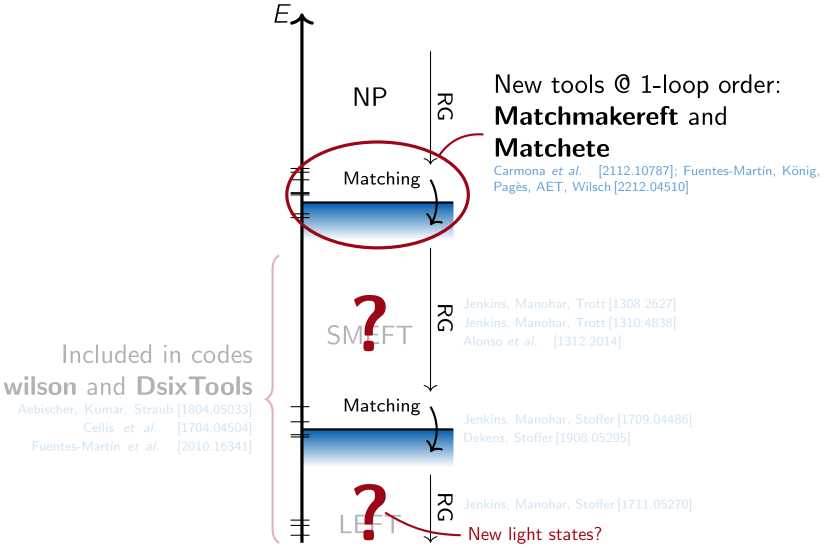
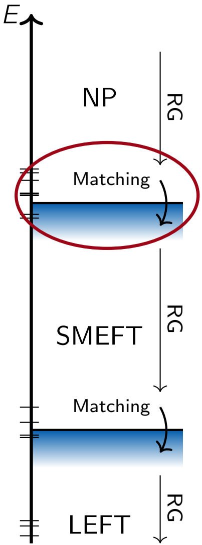


Matching weakly coupled theories



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$$\mathcal{L}_{\mathrm{EFT}}(\phi) = \mathcal{L}_{\mathrm{kin}}(\phi) + \sum_{n=2}^{\infty} \sum_{\ell=0}^{\infty} \frac{C_{n,i}^{(\ell)}}{(4\pi)^{2\ell}\, \Lambda^{n-4}} \mathcal{O}_{n,i}^{(\ell)}(\phi)$$

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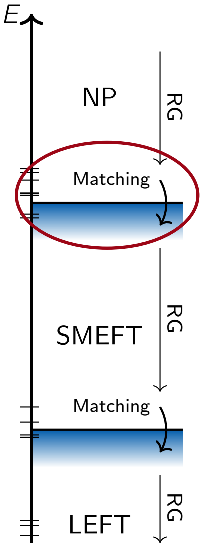

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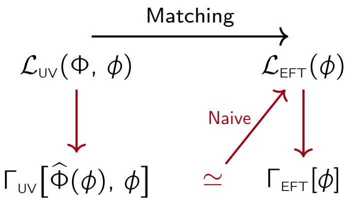

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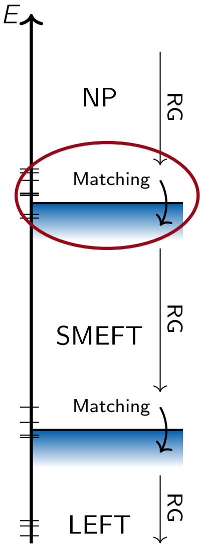

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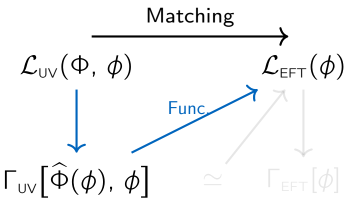

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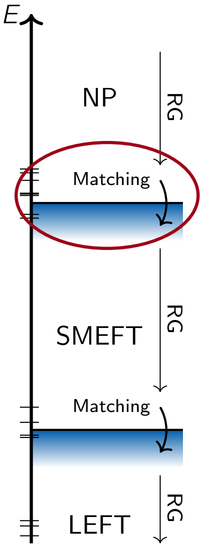

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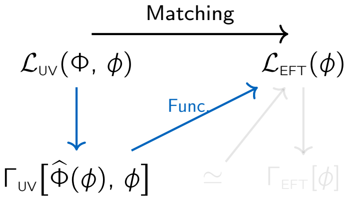

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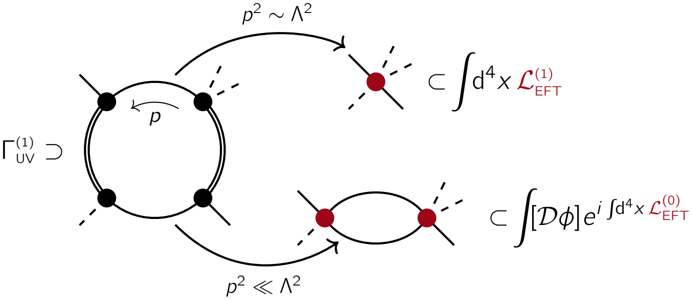
Advantages of functional matching:

- Does not require prior knowledge of EFT basis
- Well-suited for algorithmic approach
- Computations are manifestly gauge covariant

Separation of scales

Mixed (heavy–light) loop example:



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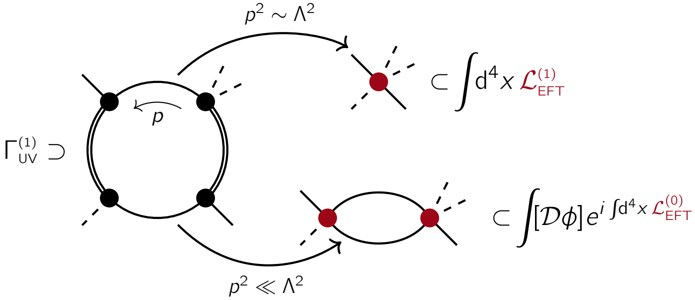

Expansion by regions allows for separating scales in dimensional regularization:
Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

$$\Gamma^{(1)}_{\text{UV}} = \Gamma^{(1)}_{\text{UV}}\Big|_{\text{hard}} + \Gamma^{(1)}_{\text{UV}}\Big|_{\text{soft}}$$

Heavy field loops → $\Gamma^{(1)}_{\text{UV}}|_{\text{hard}}$; Mixed field loops → $\Gamma^{(1)}_{\text{UV}}|_{\text{hard}}$ and $\Gamma^{(1)}_{\text{UV}}|_{\text{soft}}$; Light field loops → $\Gamma^{(1)}_{\text{UV}}|_{\text{soft}}$

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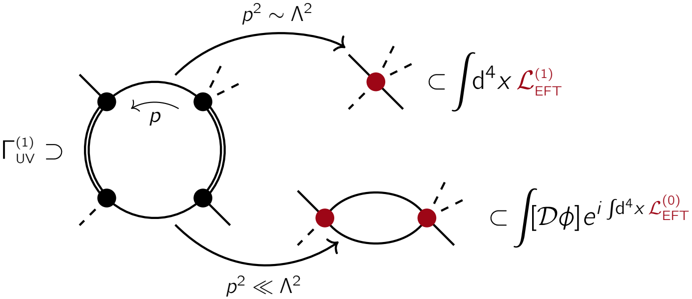

- $\Gamma_{\text{UV}}^{(1)}\big|_{\text{soft}}$: long-distance contributions included in 1-loop matrix elements of tree-level EFT operators

$$\Gamma_{\text{UV}}^{(1)}\Big|_{\text{soft}} = \Gamma_{\text{EFT}}^{(1)}$$

- $\Gamma_{\text{UV}}^{(1)}\big|_{\text{hard}}$: short-distance contributions going into the EFT operators

Fuentes-Martin *et al.* [1607.02142]; Zhang [1610.00710]

$$\Gamma_{\text{UV}}^{(1)}\Big|_{\text{hard}} = \int d^dx\, \mathcal{L}_{\text{EFT}}^{(1)}$$

Functional matching

Master formula for 1-loop matching

$$\int \mathrm{d}^d x \, \mathcal{L}^{(1)}_{\mathrm{EFT}} = \frac{i}{2} \, \mathrm{STr} \ln \Delta^{-1} \Big|_{\mathrm{hard}} - \frac{i}{2} \sum_{n=1}^{\infty} \frac{1}{n} \mathrm{STr}\big[(\Delta X)^n\big]\Big|_{\mathrm{hard}}$$

where $$\frac{\delta^2 \mathcal{L}_{\mathrm{UV}}}{\delta\eta\,\delta\eta}[\hat{\eta}] = \Delta^{-1}(\hat{P}, M) - X(\hat{P}, \hat{\eta}), \qquad \Lambda^2 \sim \Delta^{-1} \gg X$$

Cohen, Lu, Zhang [2011.02484] [2012.07851]
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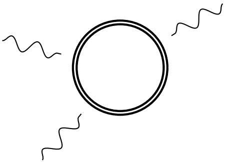

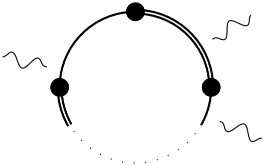

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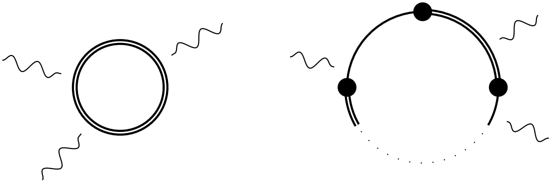

Part of a functional trace (scalar theory, $n = 1$):

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{(k_\mu + i\widetilde{G}_{\mu\nu}\partial_k^\nu)^2 - M^2} \left(\tfrac{\lambda}{2}\phi^\dagger\phi\right)$$

Simplification and basis reduction

Number of SMEFT generators (1 gen., dim. 6):

$$80 \quad (1986) \quad \longrightarrow \quad 59 \quad (2017)$$

Buchmüller, Wyler '86 — Grzadkowski *et al.* [1008.4884]

Simplification and basis reduction

$$\mathcal{L} = -\tfrac{1}{2}\phi\partial^2\phi - \tfrac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4 + \frac{C_1}{\Lambda^2}\phi^6 + \frac{C_2}{\Lambda^2}\phi^3\partial^2\phi + \frac{C_3}{\Lambda^2}\phi^2(\partial_\mu\phi)^2$$

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IBP, Dirac identities, group identities, commutation relations,...

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On-shell equivalence (non-linear):

$$\text{Field redefinition:} \qquad \phi \longrightarrow \phi + \frac{3C_2 - C_3}{3\Lambda^2}\phi^3$$

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Removal of evanescent operators: (in application of fermion Fierz identities)
See talk by *Javier Fuentes-Martín*

Linear simplifications

Example: Integrating out heavy fermion in the fundamental representation of SU(3)

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Out[12]//NiceForm=

$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

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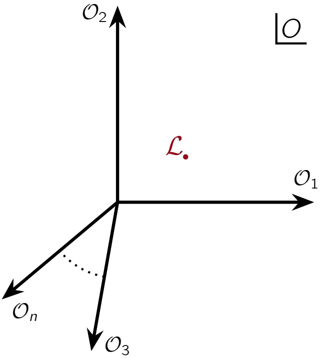

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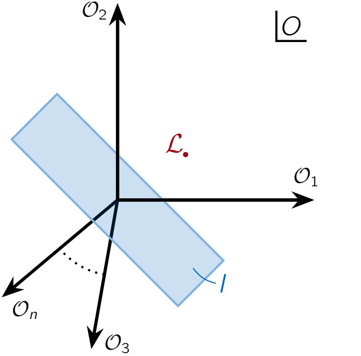

$I \subseteq O$ is the space of all operator identities, e.g., IBP. Relations such as

$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

is interpreted as

$$\mathcal{O}_1 + 2\mathcal{O}_3 \in I$$

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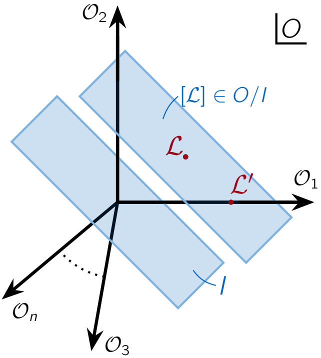


$I \subseteq O$ is the space of all operator identities, e.g., IBP. Relations such as

$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

is interpreted as

$$\mathcal{O}_1 + 2\mathcal{O}_3 \in I$$

Linear simplifications

Example: Integrating out heavy fermion in the fundamental representation of SU(3)

In[12]:= `LEFT // NiceForm`

Out[12]//NiceForm=

$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

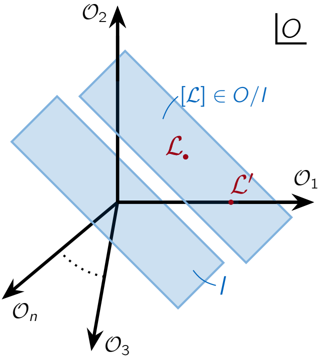
$$\mathcal{L}_{\text{EFT}} = \sum_i C_i \mathcal{O}_i \in O$$

With linear algebra on the basis of I we find a simple representative element for $[\mathcal{L}_{\text{EFT}}] \in O/I$:

In[13]:= `LEFT // GreensSimplify // NiceForm`

Out[13]//NiceForm=

$$-\frac{1}{15}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$



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$$\mathcal{O}_1 + 2\mathcal{O}_3 = 0$$

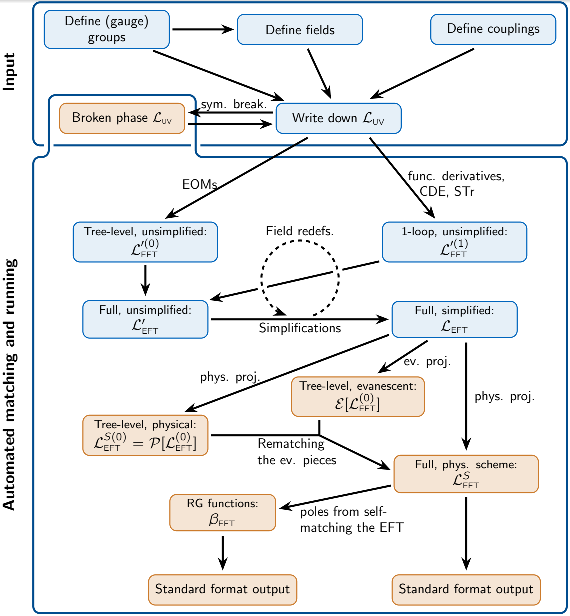
is interpreted as

$$\mathcal{O}_1 + 2\mathcal{O}_3 \in I$$

MATCHETE

To make your way through the BSM jungle

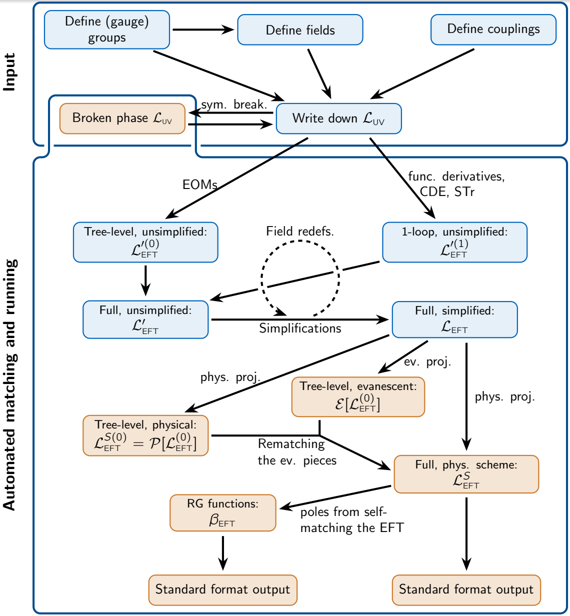
Automated EFT matching



Fuentes-Martín, König, Pagès, AET, Wilsch [2212.04510]

- **Matchete v0.1** is a Mathematica package
- Matching of *any* model with heavy scalars/fermions
- Simple and intuitive input/output
- Handles all group theory
- Simplifies to EFT basis*

Automated EFT matching



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Future plans:

- Handling of evanescent contribution
- Symmetry breaking and heavy vectors
- Interface with EFT tool chain
- 1-loop RG computations

Example: SM + Vector-like lepton

Setup

SM Lagrangian

In[3]:=
```
LSM = LoadModel["SM"];
```

Define new field

In[4]:=
```
DefineField[EE, Fermion, Charges → {U1Y[-1]}, Mass → {Heavy, ME}]
```

Define new coupling

In[5]:=
```
DefineCoupling[yE, EFTOrder → 0, Indices → {Flavor}]
```

Write interactions

In[6]:=
```
Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\overline{yE}^{p}\,\bar{H}_{i}\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Define full UV Lagrangian

In[8]:=
```
LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+D_{\mu}\bar{H}_{i}\,D_{\mu}H^{i}+\mu^{2}\,\bar{H}_{i}\,H^{i}+i\left(\bar{d}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}d^{ap}\right)+i\left(\bar{e}^{p}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}e^{p}\right)+$$
$$i\left(\overline{EE}\cdot\gamma_{\mu}\cdot D_{\mu}EE\right)-ME\left(\overline{EE}\cdot EE\right)+i\left(\bar{l}^{p}_{i}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}l^{ip}\right)+i\left(\bar{q}^{p}_{ai}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}q^{aip}\right)+i\left(\bar{u}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}u^{ap}\right)-$$
$$\frac{1}{2}\lambda\,\bar{H}_{i}\,\bar{H}_{j}\,H^{i}\,H^{j}-\overline{Yd}^{pr}\,\bar{H}_{i}\left(\bar{d}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)-\overline{Ye}^{pr}\,\bar{H}_{i}\left(\bar{e}^{r}\cdot P_{L}\cdot l^{ip}\right)-Ye^{pr}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot e^{r}\right)-Yd^{pr}\,H^{i}\left(\bar{q}^{p}_{ai}\cdot P_{R}\cdot d^{ar}\right)-$$
$$Yu^{pr}\,\bar{H}_{i}\left(\bar{q}^{p}_{aj}\cdot P_{R}\cdot u^{ar}\right)\varepsilon^{ji}-\overline{Yu}^{pr}\,H^{j}\left(\bar{u}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)\bar{\varepsilon}_{ij}-\overline{yE}^{p}\,\bar{H}_{i}\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Example: SM + Vector-like lepton

Matching

```
In[10]:= LEFT = Match[LUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 → 0;

In[11]:= LEFTOnShell = LEFT // EOMSimplify;
         Length@%
```

EOMSimplify: The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

```
Out[12]= 67

In[13]:= SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

Out[13]//NiceForm=

$$\frac{i}{360}\hbar\frac{1}{ME^2}\left(48\,gY^4\,\delta^{pr}+5\,\overline{yE}^{s}\left(3\,yE^{t}\,\overline{Ye}^{tr}\,Ye^{sp}\left(1+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)-2\,yE^{s}\,gY^2\left(13+6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)\delta^{pr}\right)\right)$$
$$\left(-D_\mu\bar{H}_i\,H^i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot e^p\right)+\bar{H}_i\,D_\mu H^i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot e^p\right)\right)$$

Example: SM + Vector-like lepton

```
LEFTOnShell // NiceForm
```

liceForm=

$$-\frac{1}{4}G^{\mu\nu A2}-\frac{1}{4}W^{\mu\nu I2}+\left(-\frac{1}{4}-\frac{1}{3}\hbar\,gY^2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)B^{\mu\nu 2}+D_\mu\bar{H}_i\,D_\mu H^i+$$

$$\left(\text{cHH}+\frac{1}{6}\hbar\,\overline{yE}^p\,yE^p\,\text{cHH}\,\frac{1}{\text{ME}^2}\left(2\,\text{cHH}-3\,\text{ME}^2\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\bar{H}_i H^i+i\left(\bar{d}^r_a\cdot\gamma_\mu P_R\cdot D_\mu d^{ap}\right)\delta^{pr}+$$

$$i\left(\bar{e}^r\cdot\gamma_\mu P_R\cdot D_\mu e^p\right)\delta^{pr}+i\left(\bar{l}^r_i\cdot\gamma_\mu P_L\cdot D_\mu l^{ip}\right)\delta^{pr}+i\left(\bar{q}^r_{ai}\cdot\gamma_\mu P_L\cdot D_\mu q^{aip}\right)\delta^{pr}+i\left(\bar{u}^r_a\cdot\gamma_\mu P_R\cdot D_\mu u^{ap}\right)\delta^{pr}+$$

$$\left(-\frac{1}{2}\lambda+\hbar\left(-\frac{1}{2}\overline{yE}^p\left(4\,yE^r\,\overline{Ye}^{rs}\,Ye^{ps}\left(1+\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)-yE^p\left(-2\,\overline{yE}^r\,yE^r\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]+\lambda\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)-\right.\right.$$

$$\frac{1}{180}\text{cHH}\,\frac{1}{\text{ME}^2}\left(12\,gY^4-5\,\overline{yE}^p\,yE^p\,gY^2\left(13+6\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)+\right.$$

$$\left.\left.\left.5\,\overline{yE}^p\left(-12\left(\overline{yE}^r\,yE^p\,yE^r+6\,yE^r\,\overline{Ye}^{rs}\,Ye^{ps}-2\,yE^p\lambda\right)+yE^p\,gL^2\left(5+6\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\right)\right)\bar{H}_i\bar{H}_j H^i H^j+$$

$$\left(-\overline{Yd}^{pr}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,\overline{Yd}^{pr}\,\frac{1}{\text{ME}^2}\left(-4\,\text{cHH}+3\,\text{ME}^2\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\bar{H}_i\left(\bar{d}^r_a\cdot P_L\cdot q^{aip}\right)+$$

$$\left(-\overline{Ye}^{pr}+\frac{1}{24}\hbar\,yE^s\,\frac{1}{\text{ME}^2}\left(-3\,\overline{yE}^p\,\overline{Ye}^{sr}\left(2\,\text{cHH}-\text{ME}^2\right)\left(3+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)+2\,\overline{yE}^s\,\overline{Ye}^{pr}\left(-4\,\text{cHH}+3\,\text{ME}^2\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\right)$$

$$\bar{H}_i\left(\bar{e}^r\cdot P_L\cdot l^{ip}\right)+$$

$$\left(-Ye^{rp}+\frac{1}{24}\hbar\,\overline{yE}^s\,\frac{1}{\text{ME}^2}\left(3\,\text{ME}^2\left(2\,yE^s\,Ye^{rp}\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)+yE^r\,Ye^{sp}\left(3+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)-\right.\right.$$

$$\left.\left.2\,\text{cHH}\left(4\,yE^s\,Ye^{rp}+3\,yE^r\,Ye^{sp}\left(3+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\right)H^i\left(\bar{l}^r_i\cdot P_R\cdot e^p\right)+$$

$$\left(-Yd^{rp}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,Yd^{rp}\,\frac{1}{\text{ME}^2}\left(-4\,\text{cHH}+3\,\text{ME}^2\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)H^i\left(\bar{q}^r_{ai}\cdot P_R\cdot d^{ap}\right)+$$

$$\left(-Yu^{rp}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,Yu^{rp}\,\frac{1}{\text{ME}^2}\left(-4\,\text{cHH}+3\,\text{ME}^2\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\bar{H}_i\left(\bar{q}^r_{aj}\cdot P_R\cdot u^{ap}\right)\varepsilon^{ji}+$$

$$\left(-\overline{Yu}^{pr}+\frac{1}{12}\hbar\,\overline{yE}^s\,yE^s\,\overline{Yu}^{pr}\,\frac{1}{\text{ME}^2}\left(-4\,\text{cHH}+3\,\text{ME}^2\left(1+2\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)H^j\left(\bar{u}^r_a\cdot P_L\cdot q^{aip}\right)\bar{\varepsilon}_{ij}+$$

$$\frac{1}{180}\hbar\,\frac{1}{\text{ME}^2}\left(12\,\lambda\,gY^4+\right.$$

$$5\,\overline{yE}^p\left(12\,\overline{yE}^r\,yE^p\left(\overline{yE}^s\,yE^r\,yE^s+6\,yE^s\,\overline{Ye}^{st}\,Ye^{rt}-yE^r\lambda\right)-72\,yE^r\,\overline{Ye}^{rs}\left(Ye^{ps}\lambda+\overline{Ye}^{tu}\,Ye^{pu}\,Ye^{ts}\left(1+\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)+\right.$$

$$\left.\left.yE^p\lambda\left(12\,\lambda+gL^2\left(5+6\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)-gY^2\left(13+6\,\text{Log}\left[\frac{\bar{\mu}^2}{\text{ME}^2}\right]\right)\right)\right)\right)\bar{H}_i\bar{H}_j\bar{H}_k H^i H^j H^k+$$

Outlook

- (Automatic) EFT matching is crucial to BSM phenomenology
- A vast array of techniques is required for efficient matching
- Functional matching provides a direct approach to automated matching
- **Matchete** is a public code for EFT matching. It already greatly simplifies the matching task and many more features are planned!

https://gitlab.com/matchete/matchete

MATCHETE

Backup

Expansion by regions: an example

Find the result of a multi-scale integral as a series in $m^2/M^2 \ll 1$:

$$I = \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{\ell^2 - m^2} \frac{1}{\ell^2 - M^2} = \frac{i}{16\pi^2} \left(\frac{1}{\epsilon} + 1 + \log \frac{\bar{\mu}^2}{M^2} + \frac{m^2}{M^2} \log \frac{m^2}{M^2} \right)$$

$\ell^2 \sim M^2$

$$I_h = \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{\ell^2} \left(1 + \frac{m^2}{\ell^2} + \ldots \right) \frac{1}{\ell^2 - M^2} = \frac{i}{16\pi^2} \frac{m^2 + M^2}{M^2} \left(\frac{1}{\epsilon} + 1 + \log \frac{\bar{\mu}^2}{M^2} \right)$$

$\ell^2 \sim m^2$

$$I_s = \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \frac{1}{\ell^2 - m^2} \frac{-1}{M^2} \left(1 - \frac{\ell^2}{M^2} + \ldots \right) = \frac{-i}{16\pi^2} \frac{m^2}{M^2} \left(\frac{1}{\epsilon} + 1 + \log \frac{\bar{\mu}^2}{m^2} \right)$$

In dimensional regularization, integrals equal the sum of their *hard* and *soft* parts

Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

$$I = I_h + I_s$$

Matchete showcase (SM implementation)

Gauge Groups

```
DefineGaugeGroup[SU3c, SU@3, gs, G,
    FundAlphabet → CharacterRange["a", "f"],
    AdjAlphabet → CharacterRange["A", "F"]]
DefineGaugeGroup[SU2L, SU@2, gL, W,
    FundAlphabet → CharacterRange["i", "n"],
    AdjAlphabet → CharacterRange["I", "N"]]
DefineGaugeGroup[U1Y, U@1, gY, B]
```

Generation index

```
DefineFlavorIndex[Flavor, 3,
    IndexAlphabet → {"p", "r", "s", "t", "u", "v"}]
```

Fermions

```
DefineField[q, Fermion,
    Indices → {SU3c@fund, SU2L@fund, Flavor},
    Charges → {U1Y[1/6]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[u, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[2/3]},
    Chiral → RightHanded,
    Mass → 0]
DefineField[d, Fermion,
    Indices → {SU3c@fund, Flavor},
    Charges → {U1Y[-1/3]},
    Chiral → RightHanded,
    Mass → 0]
```

```
DefineField[l, Fermion,
    Indices → {SU2L@fund, Flavor},
    Charges → {U1Y[-1/2]},
    Chiral → LeftHanded,
    Mass → 0]
DefineField[e, Fermion,
    Indices → {Flavor},
    Charges → {U1Y[-1]},
    Chiral → RightHanded,
    Mass → 0]
```

Higgs

```
DefineField[H, Scalar,
    Indices → {SU2L@fund},
    Charges → {U1Y[1/2]},
    Mass → 0]
```

Couplings

```
DefineCoupling[λ, SelfConjugate → True]
DefineCoupling[μ, SelfConjugate → True,
  EFTorder → 1];
DefineCoupling[Ye,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yu,
    Indices → {Flavor, Flavor}]
DefineCoupling[Yd,
    Indices → {Flavor, Flavor}]
```

Matchete showcase (SM implementation)

Lagrangian

```
ℒSM = FreeLag[] +
    -μ[]^2 Bar@H[i] × H[i] -
    λ[]/2 Bar@H[i] × H[i] × Bar@H[j] × H[j] +
    PlusHc[
        -Yu[p, r] × CG[eps@SU2L, {i, j}] ×
       Bar@H@i × Bar@q[a, j, p] ** u[a, r]
        - Yd[p, r] × H@i × Bar@q[a, i, p] ** d[a, r]
        - Ye[p, r] × H@i × Bar@l[i, p] ** e[r]
    ] // RelabelIndices;
```

```
ℒSM // NiceForm
```

Form=

$$-\frac{1}{4} B^{\mu\nu 2} - \frac{1}{4} G^{\mu\nu A 2} - \frac{1}{4} W^{\mu\nu I 2} + D_\mu \bar{H}_i \, D_\mu H^i - \mu^2 \bar{H}_i H^i + i \left(\bar{d}^p_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap}\right) + i \left(\bar{e}^p \cdot \gamma_\mu P_R \cdot D_\mu e^p\right) + i \left(\bar{l}^p_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip}\right) + i \left(\bar{q}^p_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip}\right) + i \left(\bar{u}^p_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap}\right) - \frac{1}{2} \lambda \bar{H}_i \bar{H}_j H^i H^j - \overline{Yd}^{pr} \bar{H}_i \left(\bar{d}^r_a \cdot P_L \cdot q^{aip}\right) - \overline{Ye}^{pr} \bar{H}_i \left(\bar{e}^r \cdot P_L \cdot l^{ip}\right) - Ye^{pr} H^i \left(\bar{l}^p_i \cdot P_R \cdot e^r\right) - Yd^{pr} H^i \left(\bar{q}^p_{ai} \cdot P_R \cdot d^{ar}\right) - Yu^{pr} \bar{H}_i \left(\bar{q}^p_{aj} \cdot P_R \cdot u^{ar}\right) \varepsilon^{ij} - \overline{Yu}^{pr} H^i \left(\bar{u}^r_a \cdot P_L \cdot q^{ajp}\right) \bar{\varepsilon}_{ij}$$