Standard Model predictions for the *W* boson mass

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FNSNF Swiss National Science Foundation

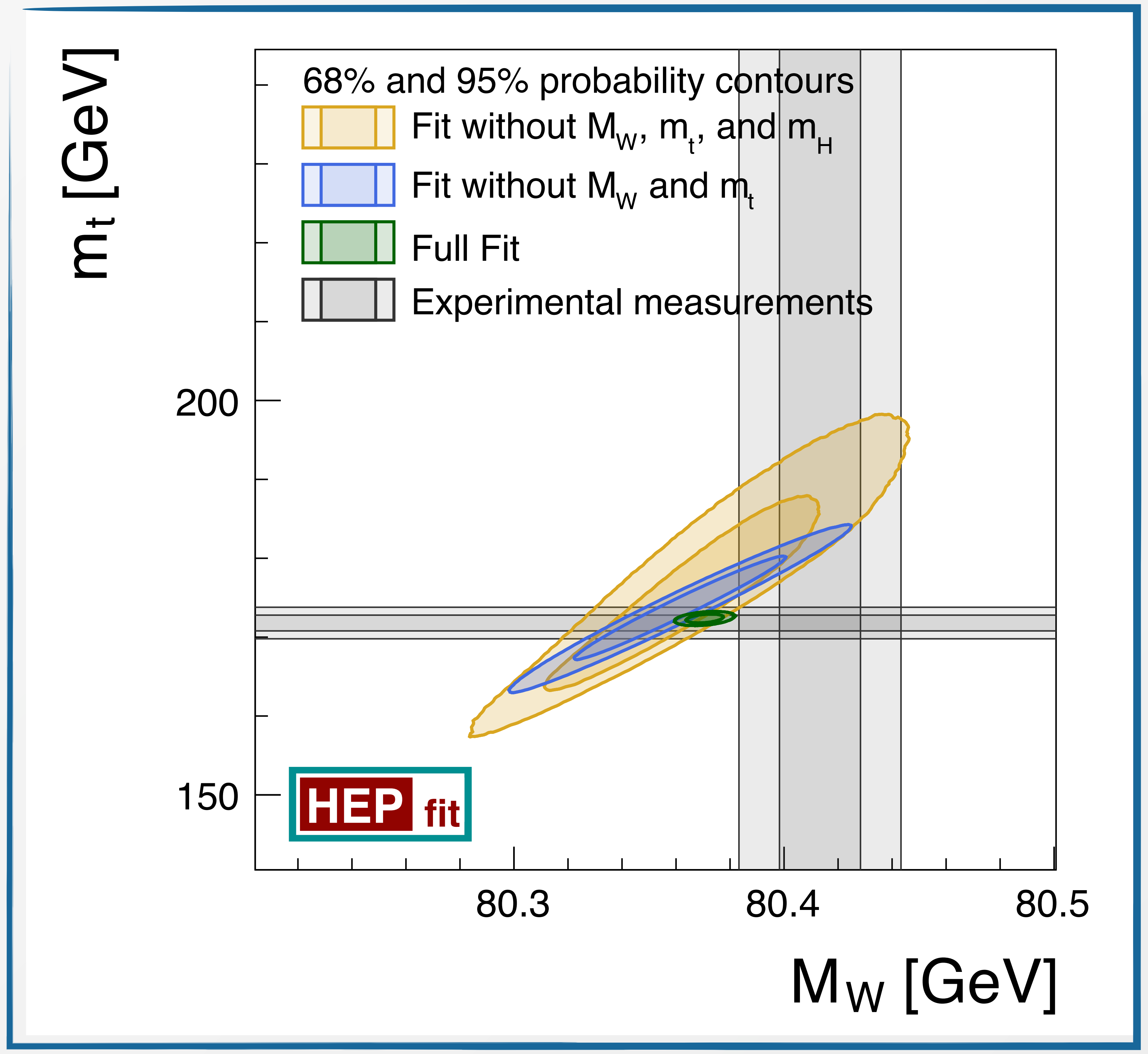
Measurements of m_W at hadron colliders

The discovery of the Higgs boson and the measurement of its mass allow for the prediction of the W mass with high precision

$$m_W = 80.350 \pm 8\,\text{GeV}$$

Which is in a 2σ agreement with the experimental average (pre-CDF II)

$$m_W = 80.385 \pm 15\,\text{GeV}$$



[de Blas, Pierini, Reina, Silvestrini ’22]

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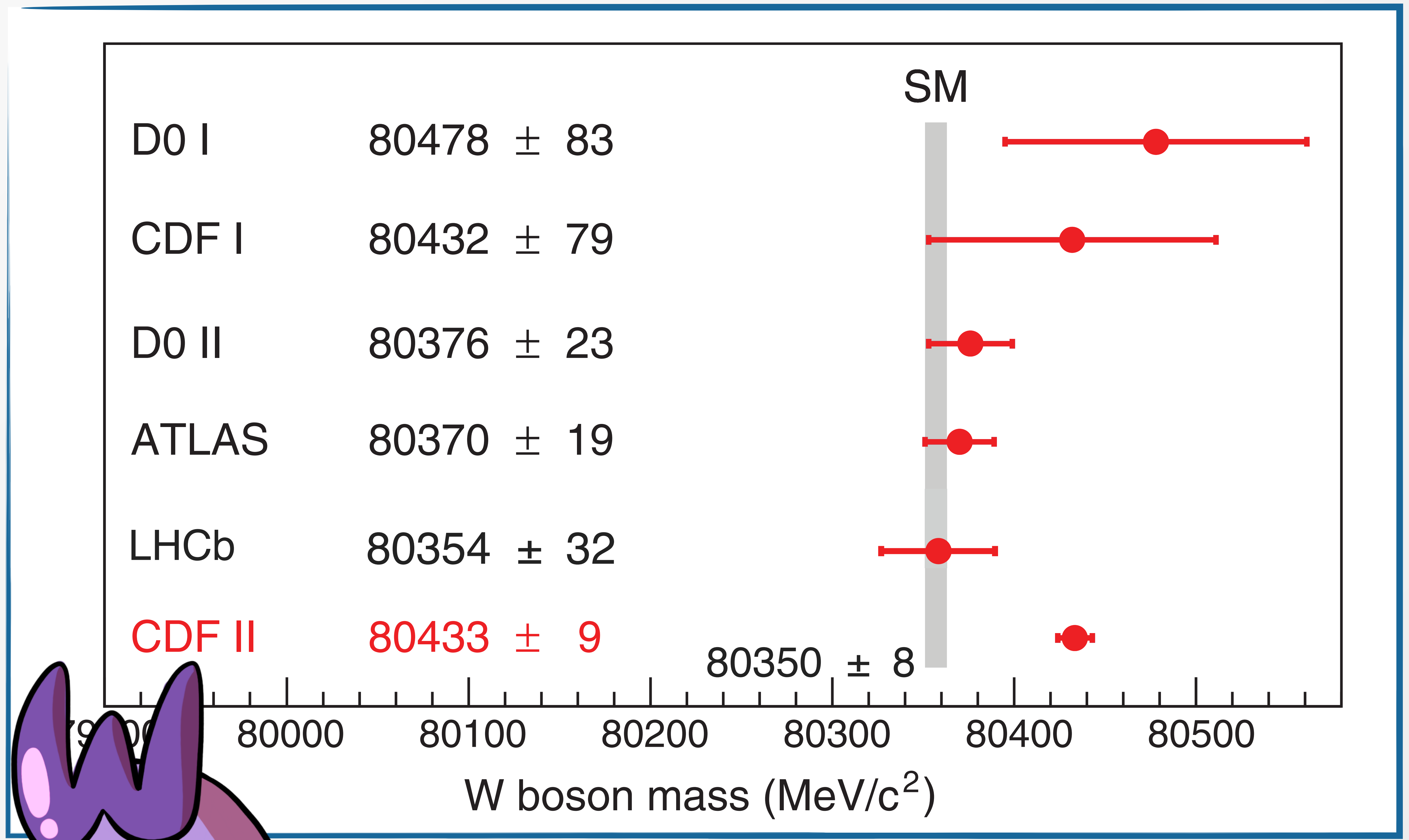
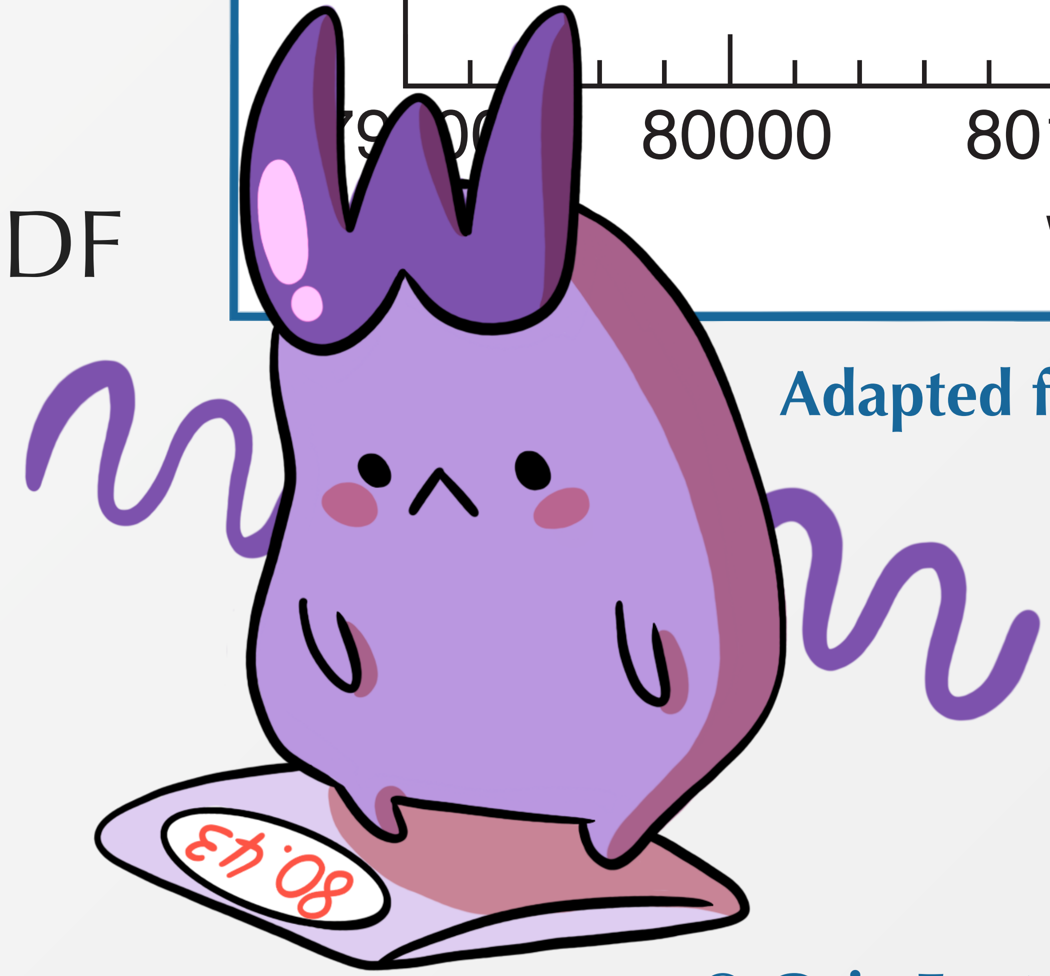
Both results are in significant tension with the CDF measurement

$$m_W = 80.433 \pm 9\,\mathrm{GeV}$$



Adapted from CDF Collaboration et al., Science 376, 170–176 (2022)



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Measurements of m_W at hadron colliders

Full kinematics of **charged DY production** is not accessible at hadron colliders; in particular, the invariant mass of the neutrino-lepton pair cannot be reconstructed

Reconstruction possible in the **transverse plane** (requires precise measurement of the **hadronic recoil**)

Precise determinations of the W mass exploit observables with high sensitivity to small variations $\mathcal{O}(10^{-4})$ of m_W, such as the lepton transverse momentum p_T^ℓ or the transverse mass $M_\perp^{\ell\nu} = \sqrt{2p_\perp^\ell p_\perp^\nu(1-\cos\Delta\phi_{\ell\nu})}$

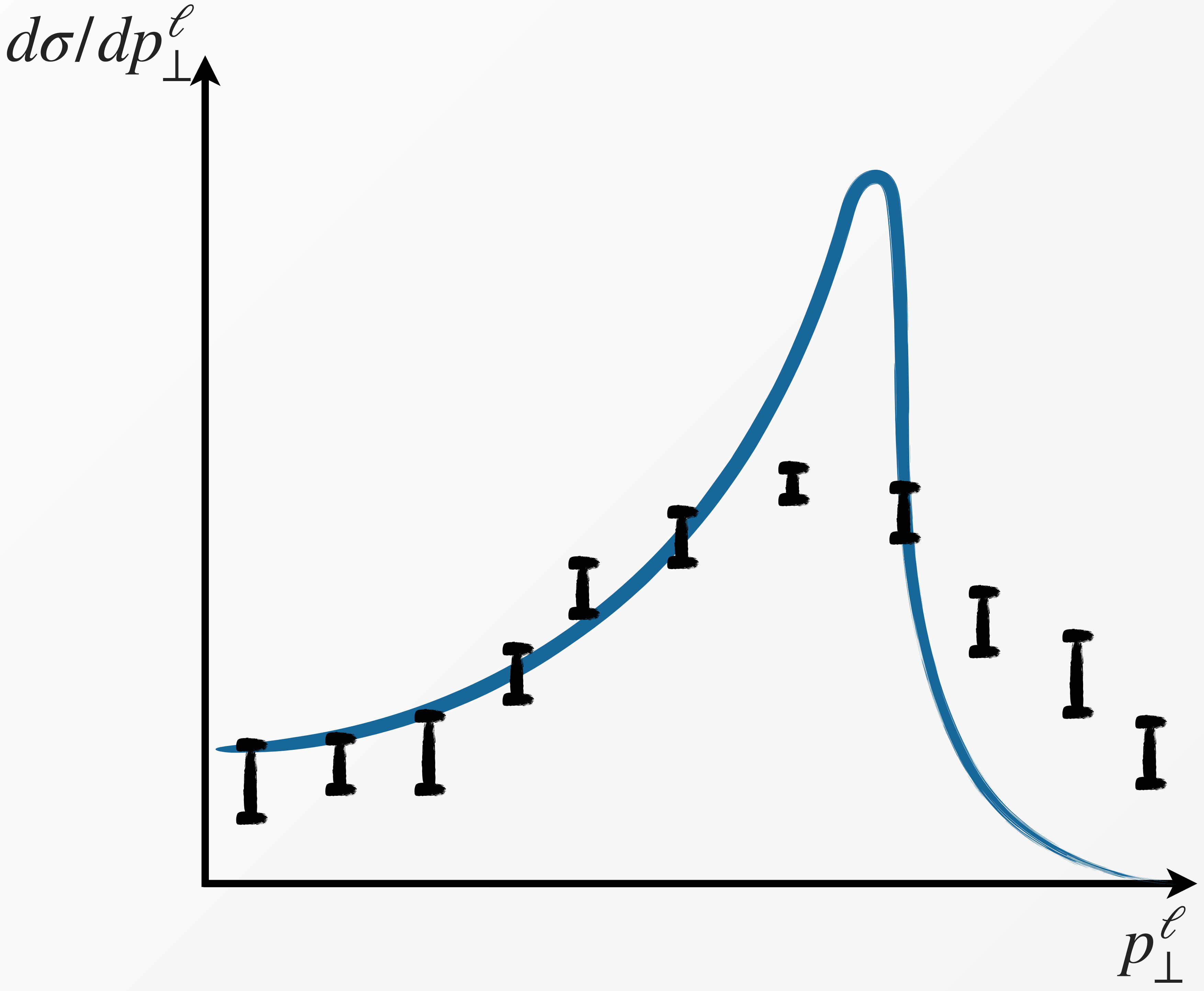
$$\frac{d\sigma}{d|p_\perp^\ell|^2} \sim \frac{1}{\sqrt{1-4\frac{|p_\perp^\ell|^2}{\hat{s}}}} \sim \frac{1}{\sqrt{1-4\frac{|p_\perp^\ell|^2}{m_W^2}}}$$

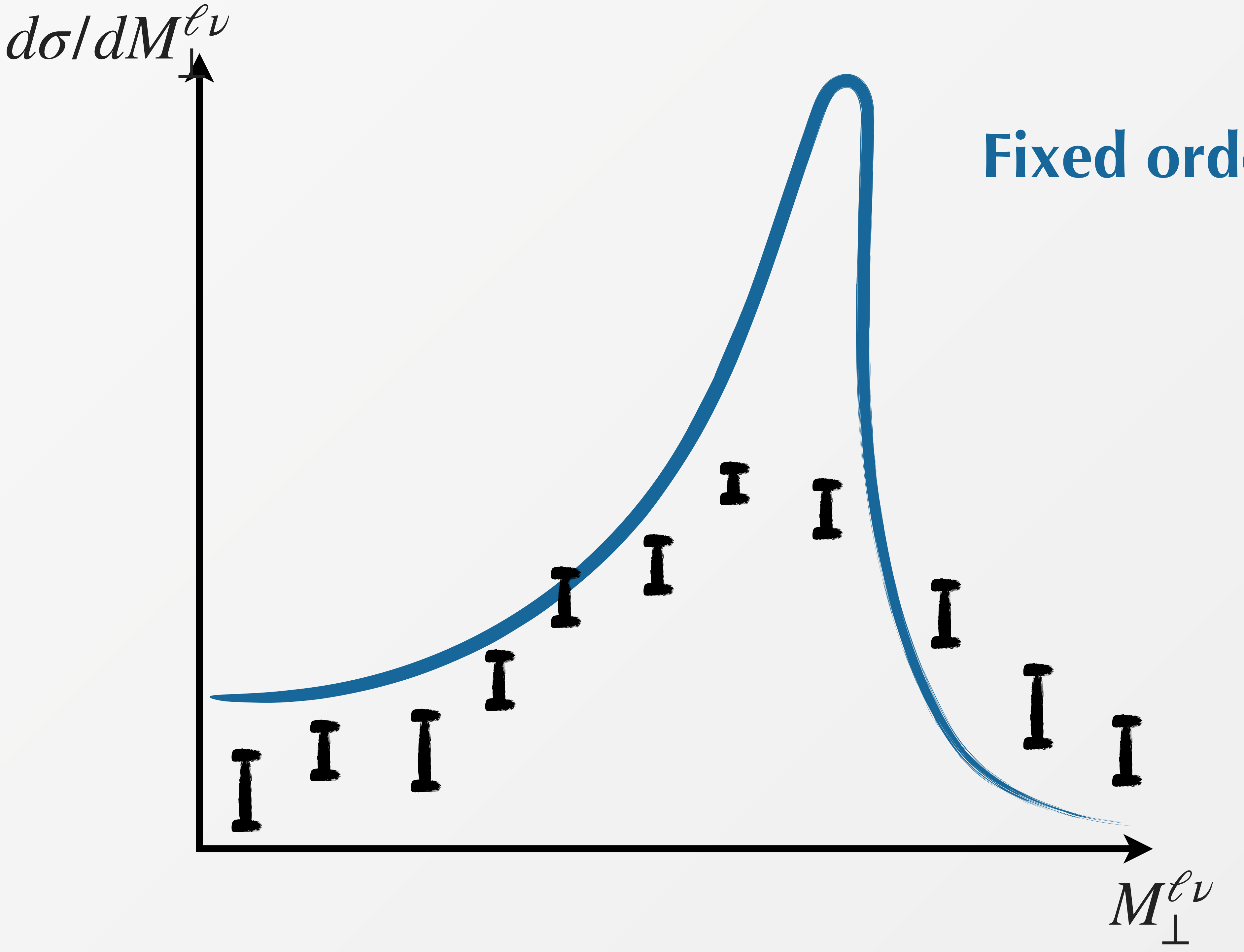
Jacobian peak at $p_\perp^\ell \sim m_W/2$

Enhanced sensitivity to m_W in both distributions at the $\mathcal{O}(10^{-3})$—$\mathcal{O}(10^{-2})$ level.

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling

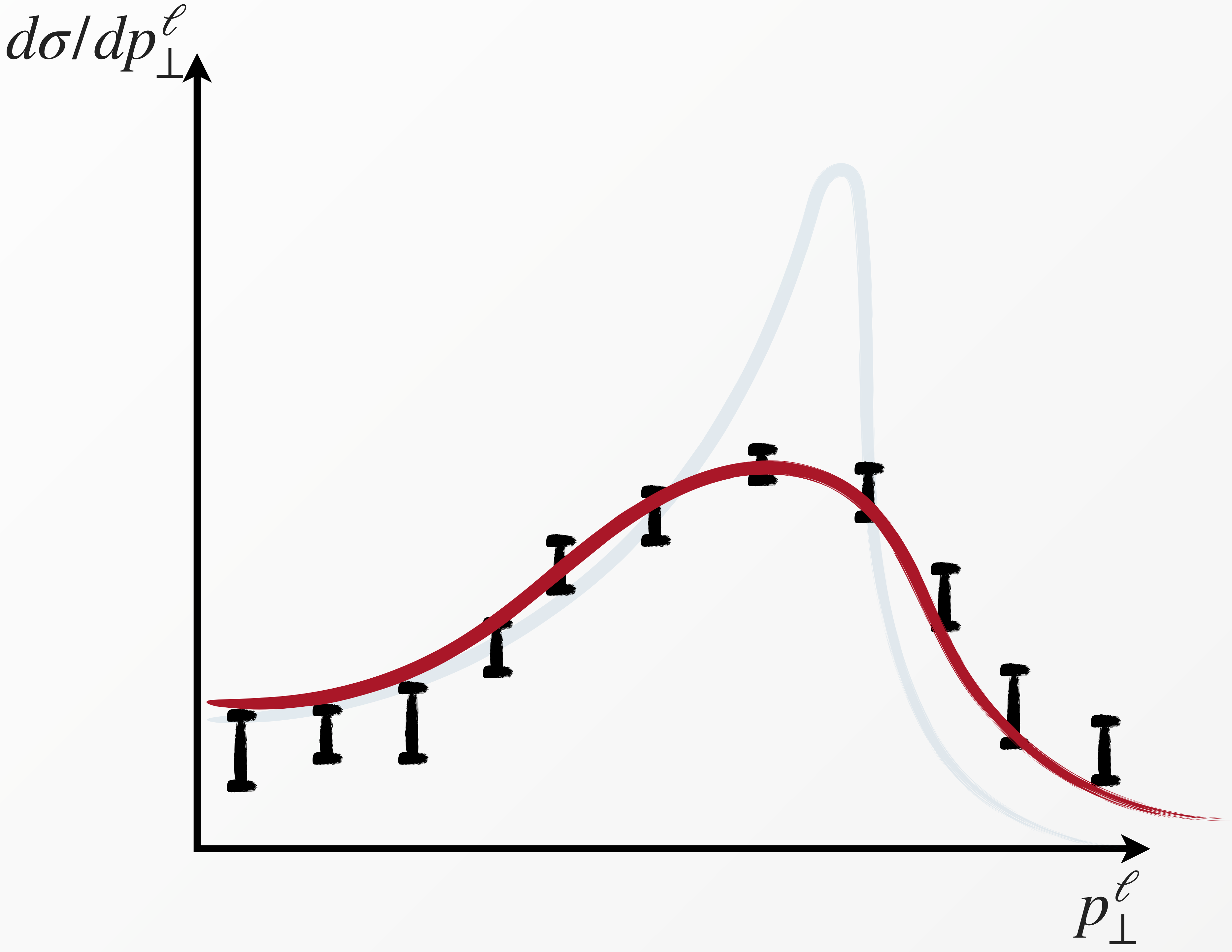


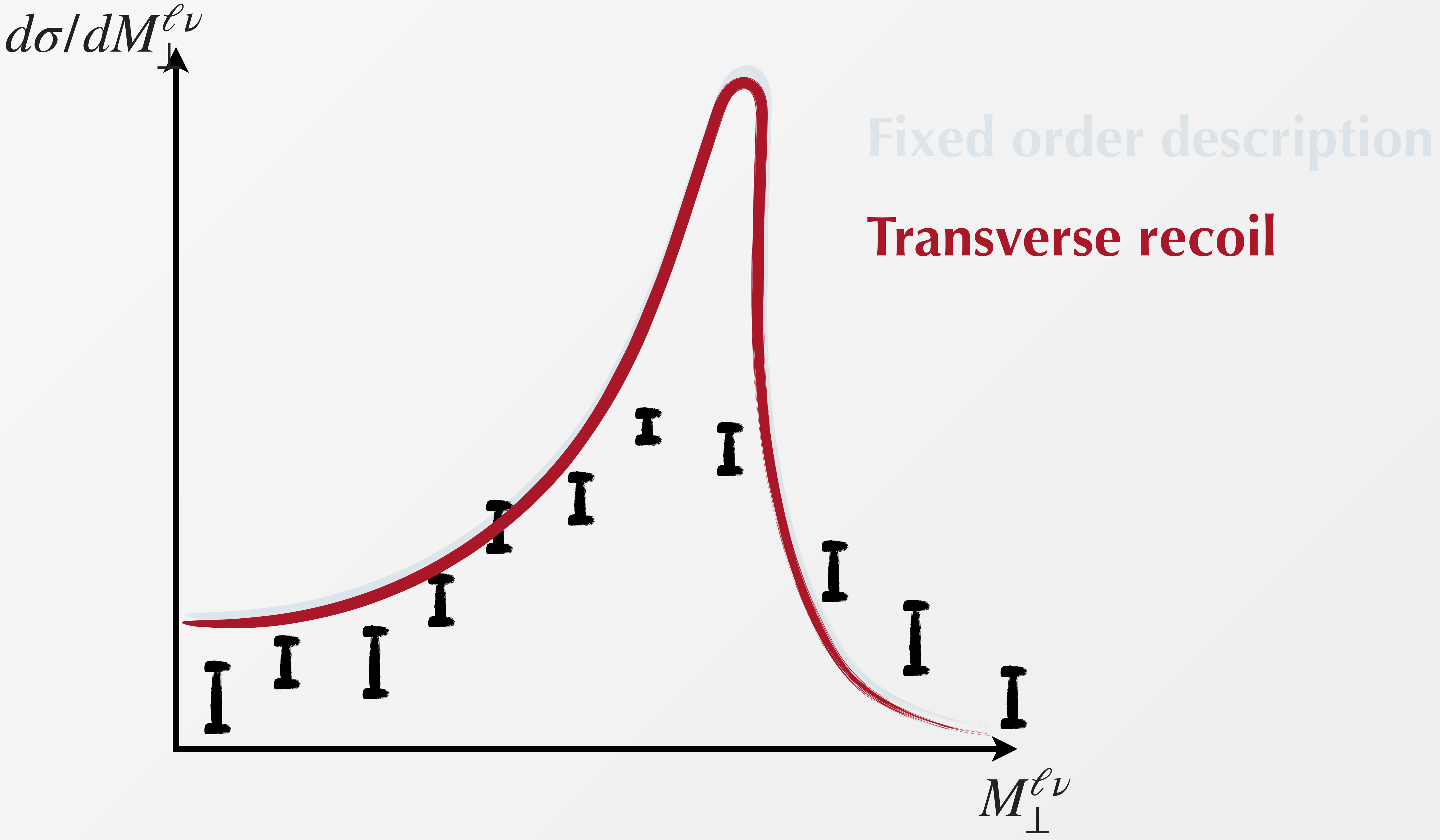


Fixed order description

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling



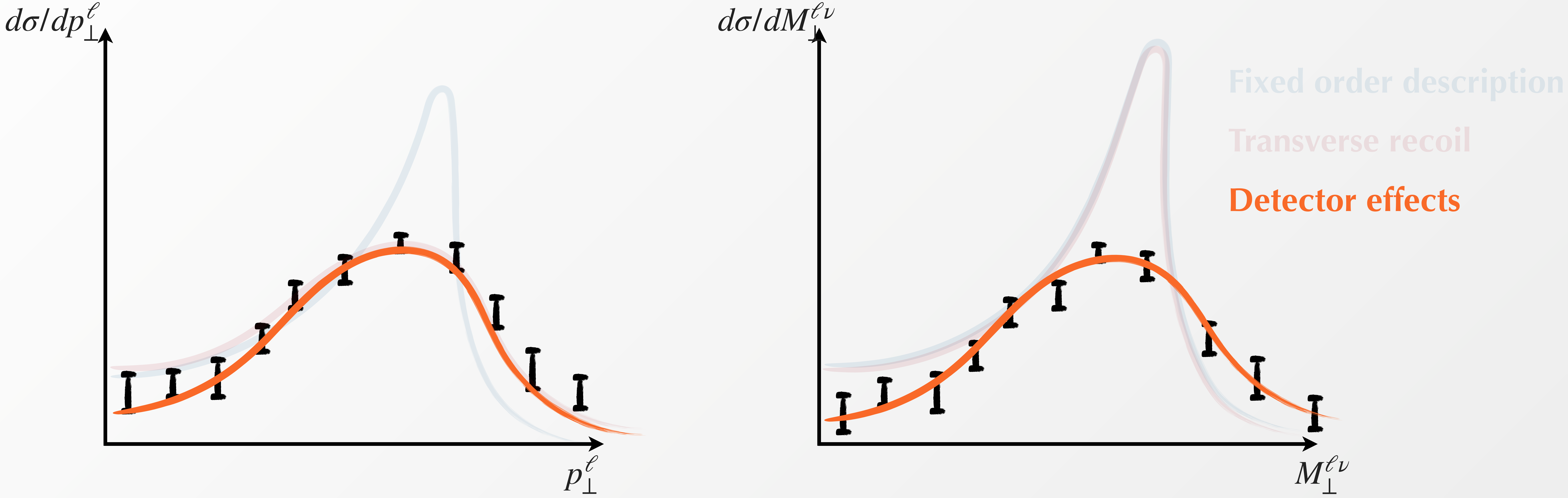


Description of the data requires:

- Modelling of **IS QCD + FS QED radiation**

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling

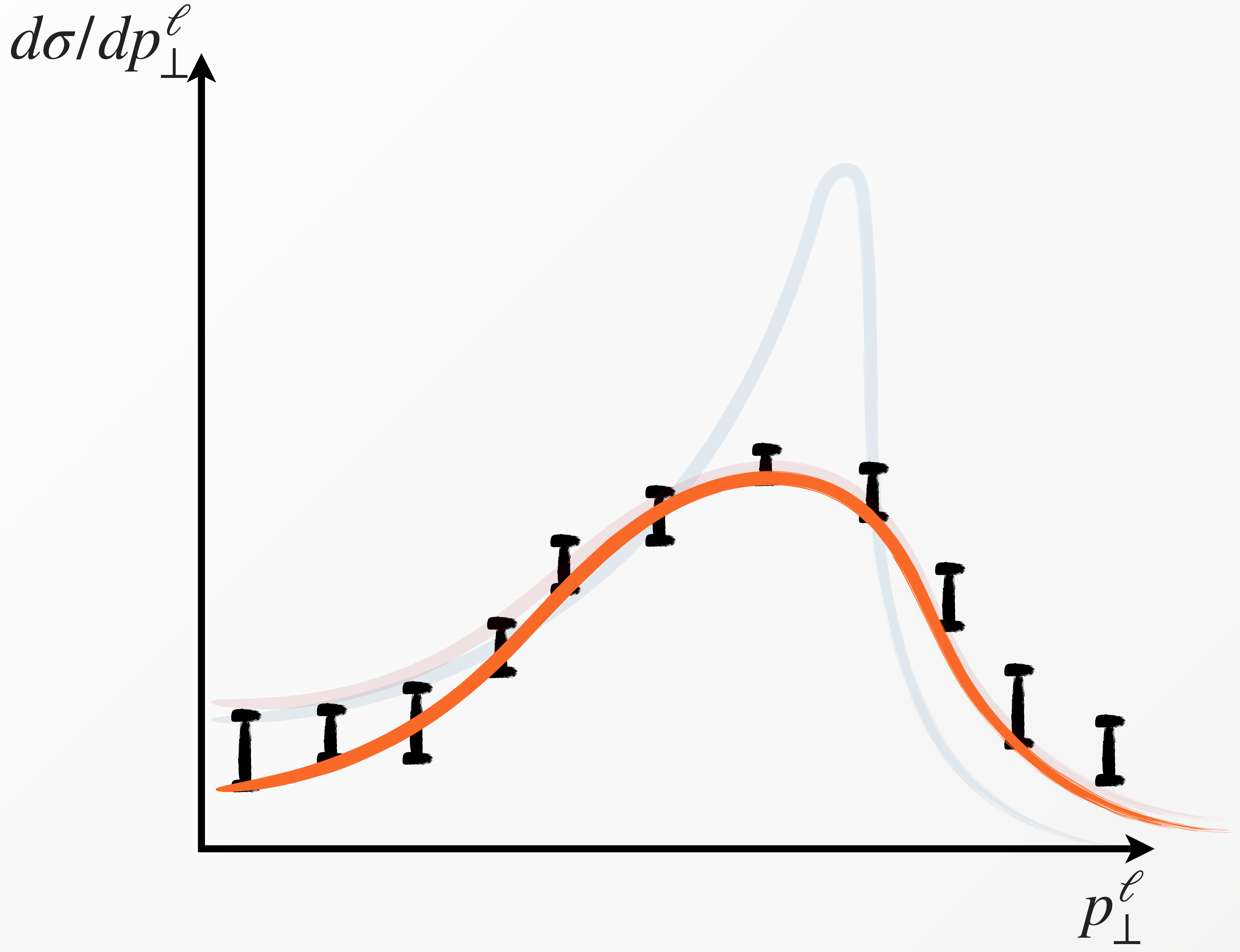


Description of the data requires:

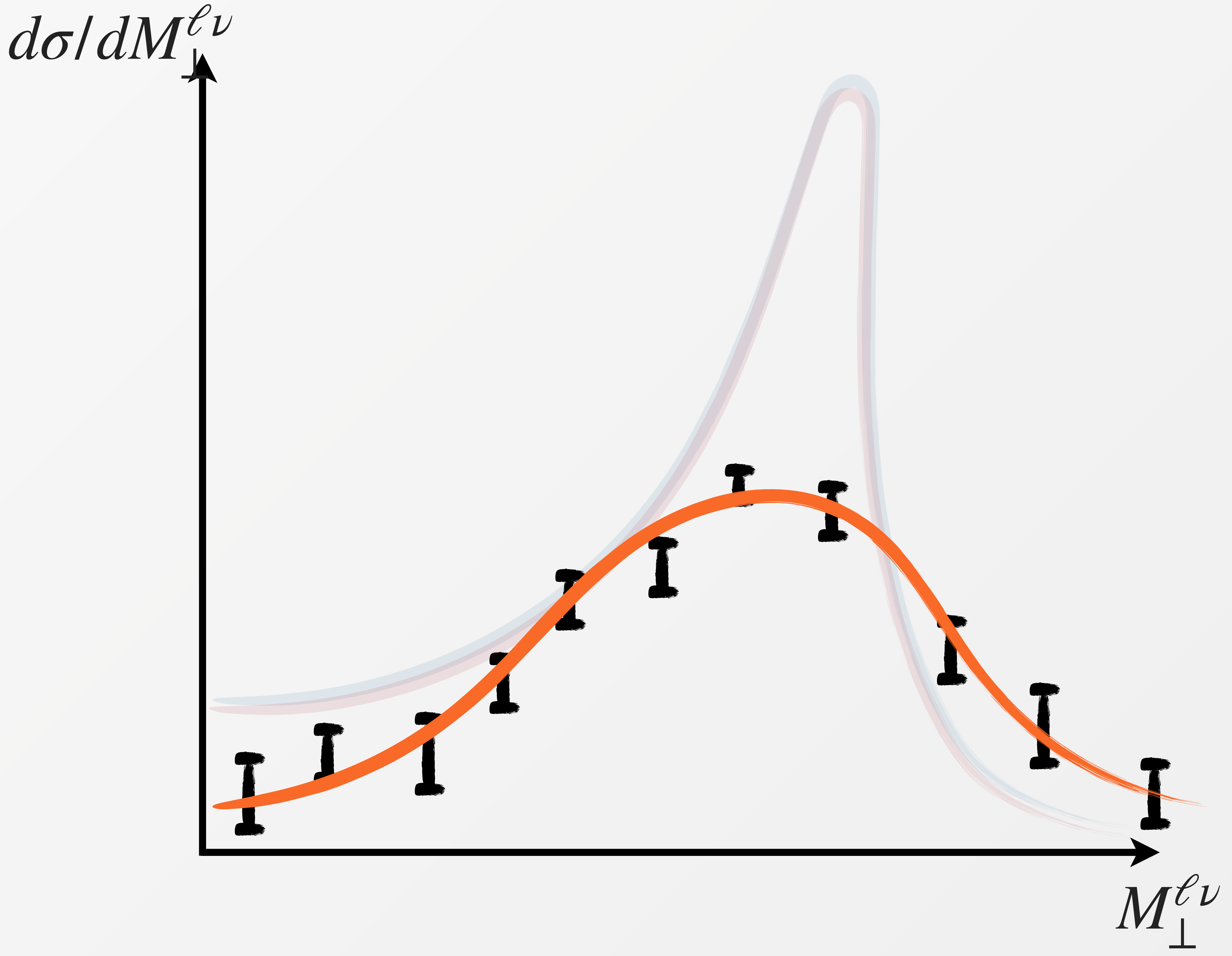
- Modelling of **IS QCD + FS QED radiation**
- Modelling of the **smearing** of the distributions due to the reconstruction of the neutrino in the transverse plane

Measurements of m_W at hadron colliders

Different sensitivity to experimental uncertainties and quality of theoretical modelling



Mostly **QCD + QED radiation**



Mostly **detector effects**

$$M_\perp^{\ell\nu} = \sqrt{2p_\perp^\ell p_\perp^\nu(1-\cos\Delta\phi_{\ell\nu})}$$

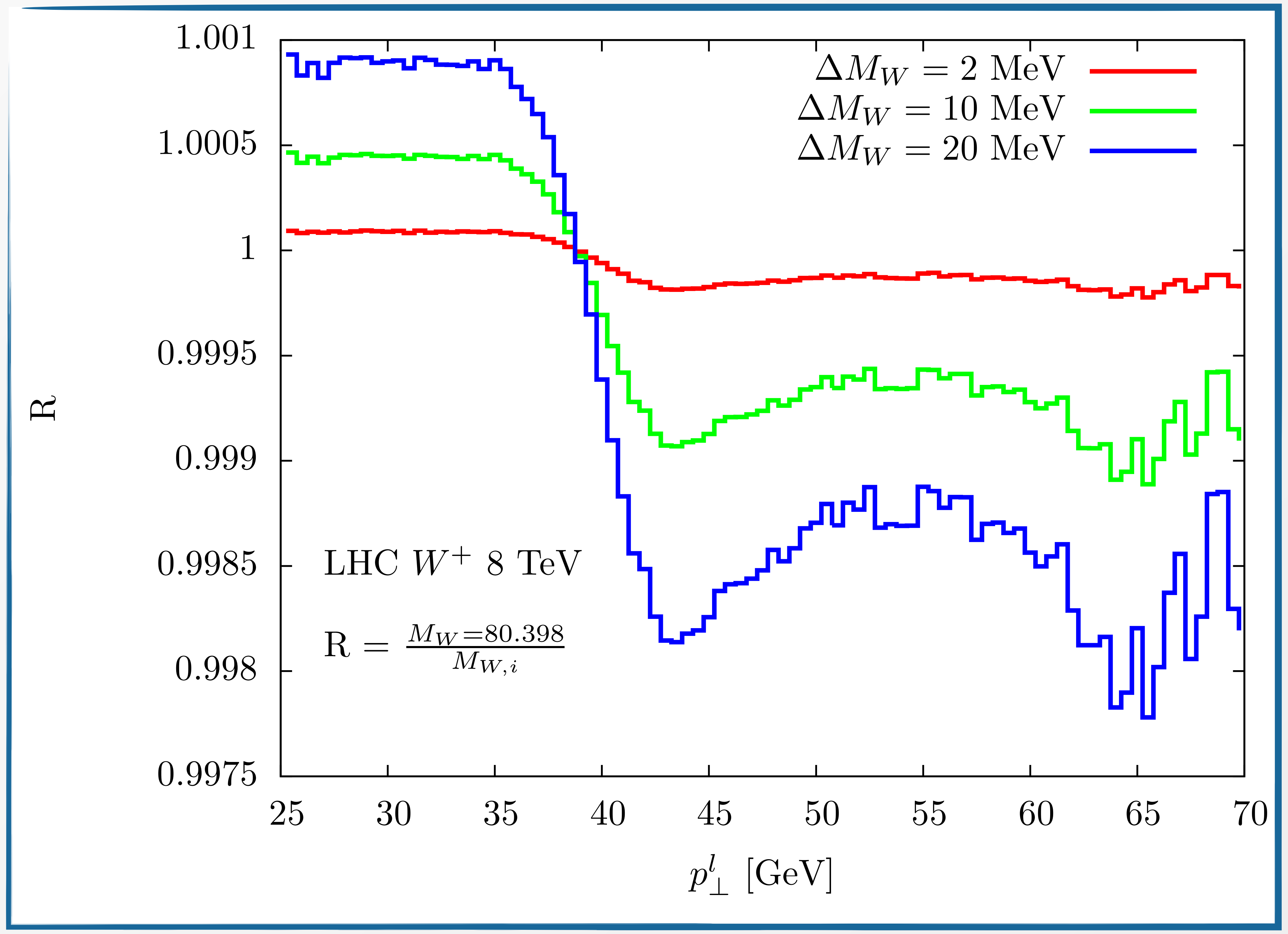
Requires precise determination of the neutrino transverse momentum: **challenging at the LHC** due to worse control of the hadronic recoil

Measurements of m_W at hadron colliders: template fitting

Extraction of m_W performed by template fittings of relevant kinematic observables e.g. lepton transverse momentum $p_\perp^\ell$

1. Compute theoretical distributions for different values of $m_W^{(k)}$
2. For each hypothesis, compute a figure of merit χ_k^2 for a defined window in $p_\perp^\ell$
3. The minimum value of χ_k^2 defines the experimental value of m_W

Permille-level control of the shape is necessary to obtain m_W with 10^{-4} precision



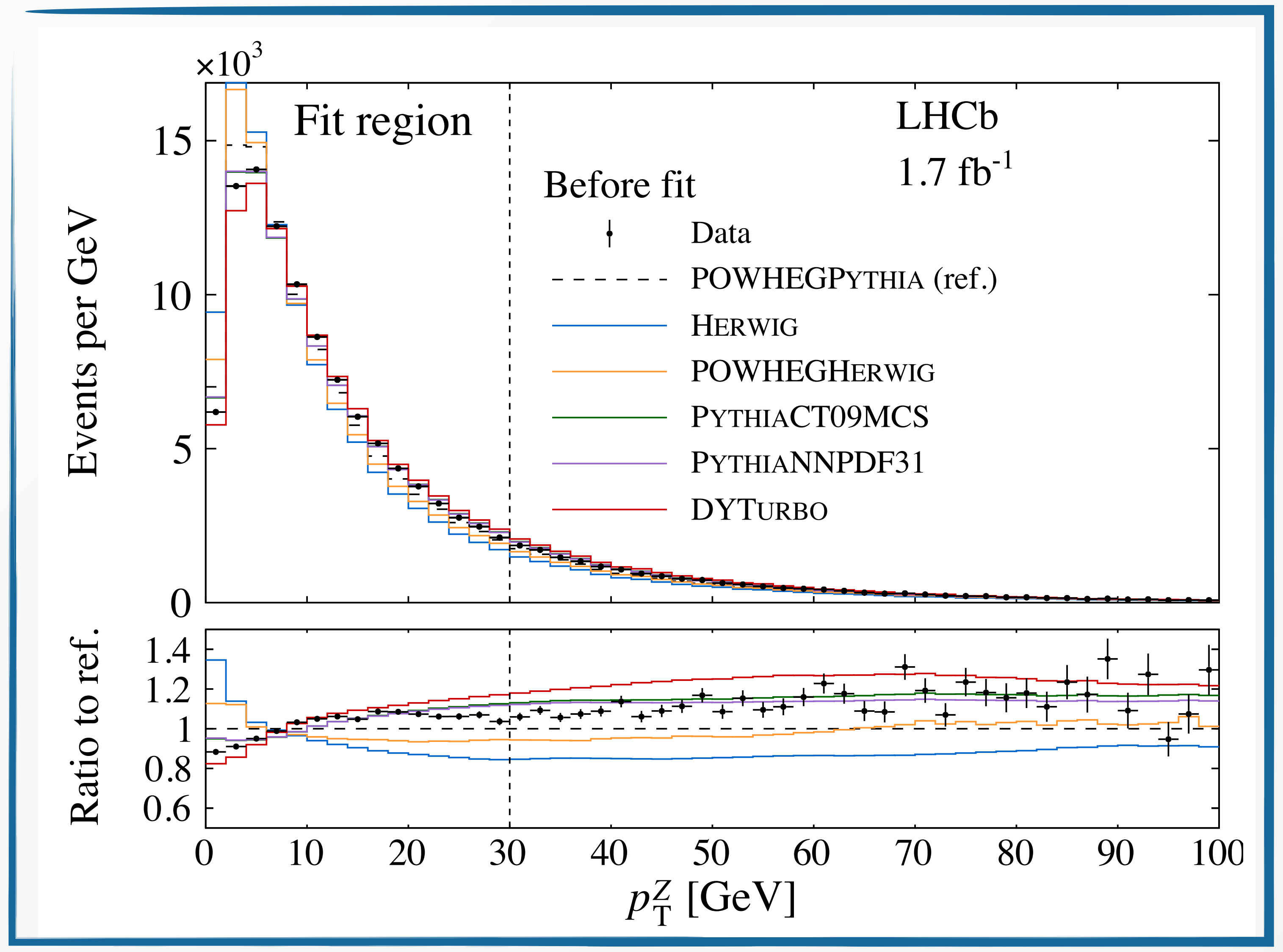
The **description of experimental data** plays a crucial role

Precise control of the **associated theoretical uncertainties** needed to assess the **theoretical systematic error** on m_W

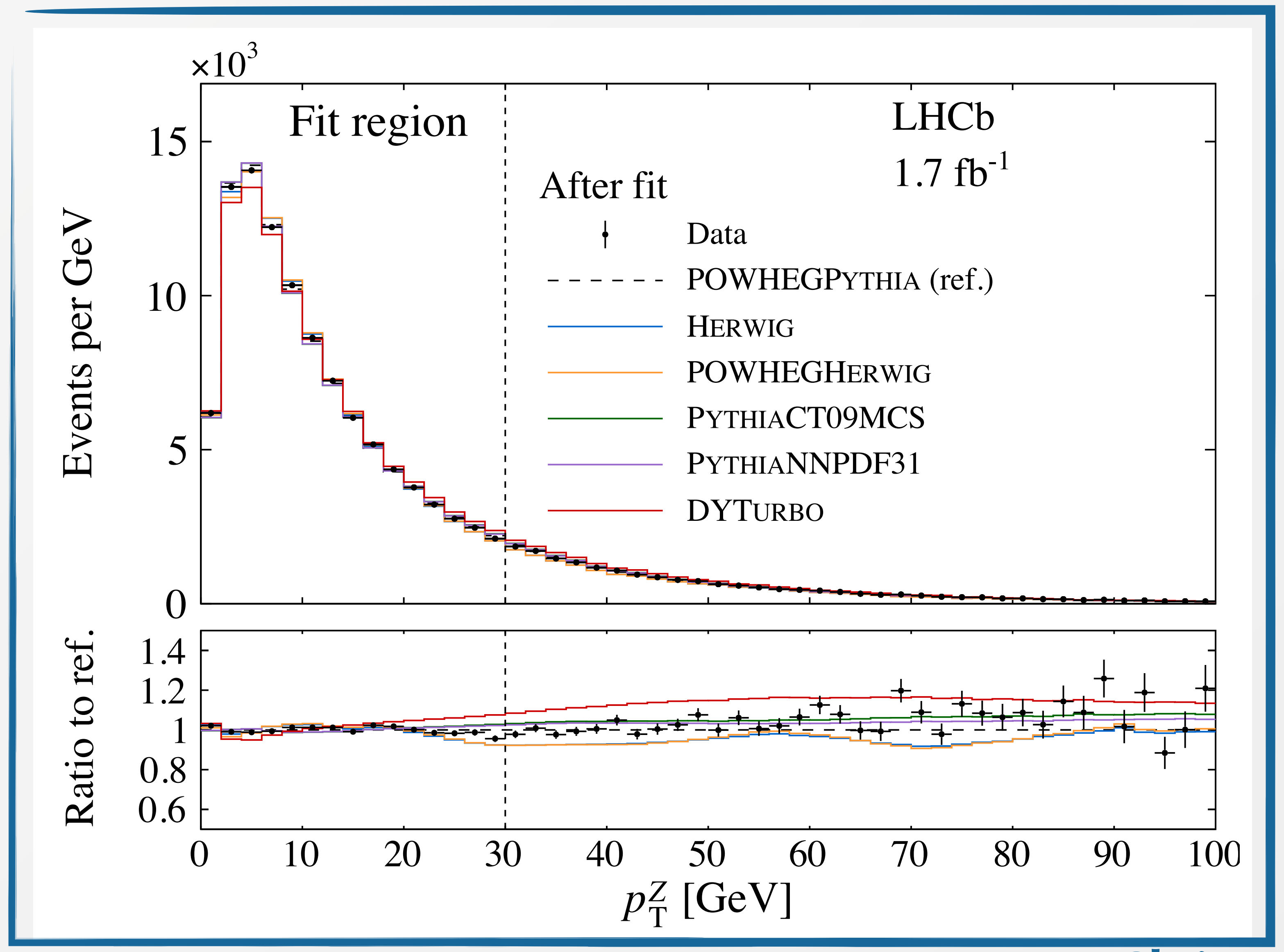
Template fitting and tuning

Template fitting procedure requires that the theoretical distribution can describe the data with high quality

Data-driven approach: Monte Carlo event generators tuned with NCDY data exploiting astonishing precision of $p_{\perp}^{\ell\ell}$ spectrum

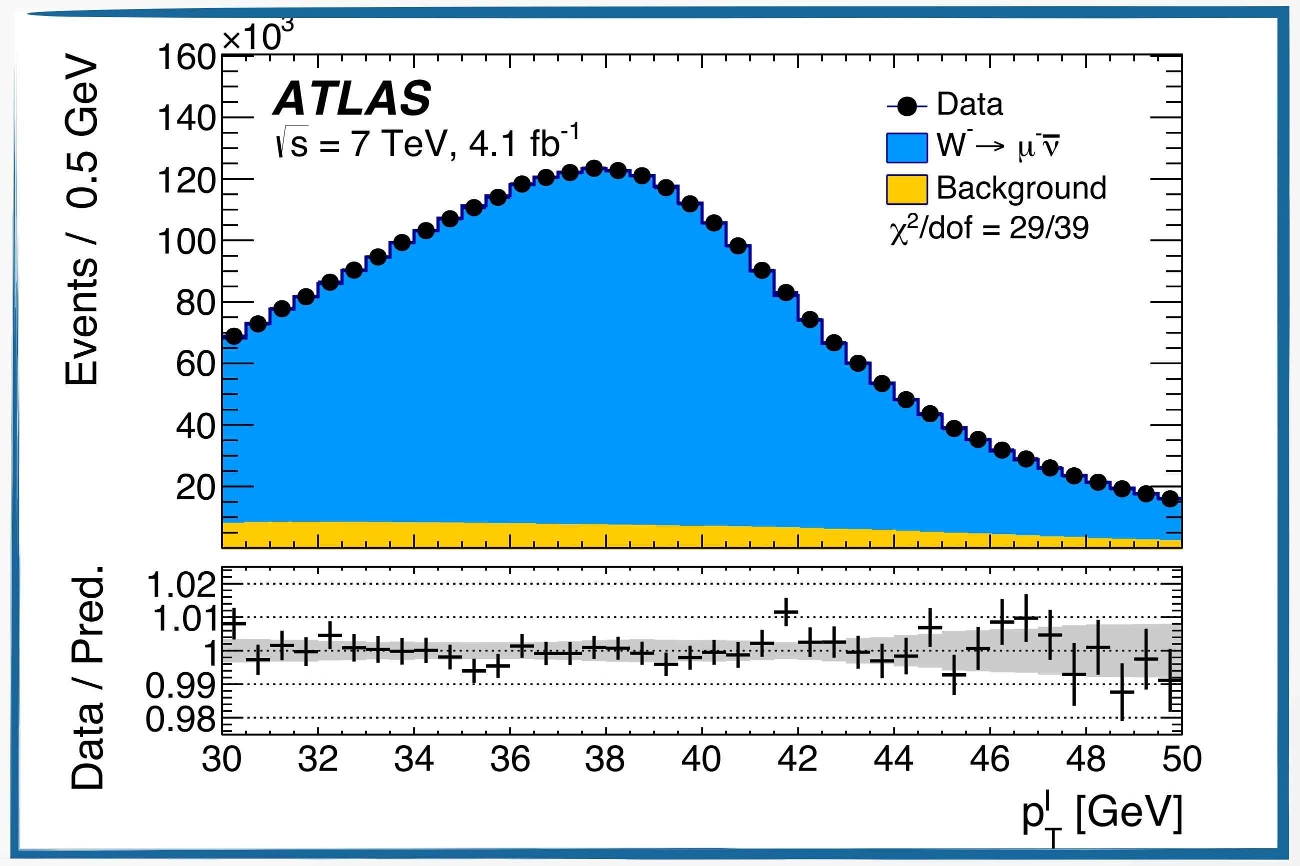


[LHCb '21]

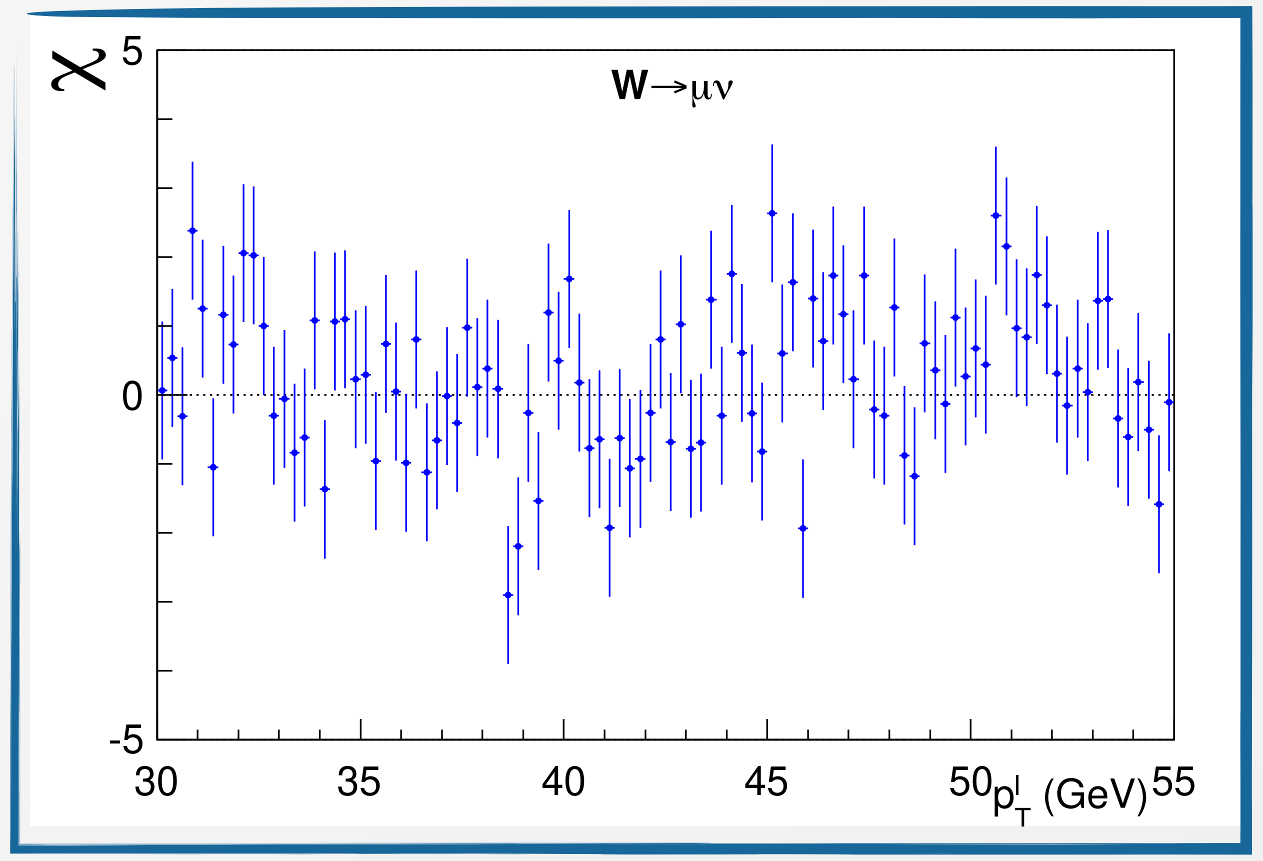


[LHCb '21]

Templates prepared for CCDY using the same tuned parameters



[ATLAS '17]



[CDF '22]

Procedure heavily relies on the similarities between NC and CC DY, and assumes that the information obtained from the data is fully correlated between the two processes

Precision physics at the LHC: theory

$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2) \hat{\sigma}_{ab\to X}(Q^2, x_1 x_2 s) + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p)$$

Input parameters:

strong coupling α_s

PDFs f

few percent uncertainty; improvable

Non-perturbative effects

percent effect; not yet under full control

Precision physics at the LHC: fixed order computations

$$\sigma(s,Q^2) = \sum_{a,b} \int dx_1 dx_2\, f_{a/h_1}(x_1,Q^2) f_{b/h_2}(x_2,Q^2)\, \hat{\sigma}_{ab\to X}(Q^2, x_1x_2 s) + \mathcal{O}(\Lambda^p_{QCD}/Q^p)$$

$$\tilde{\sigma} = 1 + \alpha_s\tilde{\sigma}_1 + \alpha_s^2\tilde{\sigma}_2 + \alpha_s^3\tilde{\sigma}_3 + \dots$$

LO_{QCD} NLO_{QCD} $NNLO_{QCD}$ N^3LO_{QCD}

$\alpha_s \sim 0.1$

δ~10-20% NLO_{QCD}

δ~1-5% $NNLO_{QCD}$ (or even N^3LO_{QCD})

$NNLO_{QCD}$ available for a larger number of processes, 2→3 computations current frontier

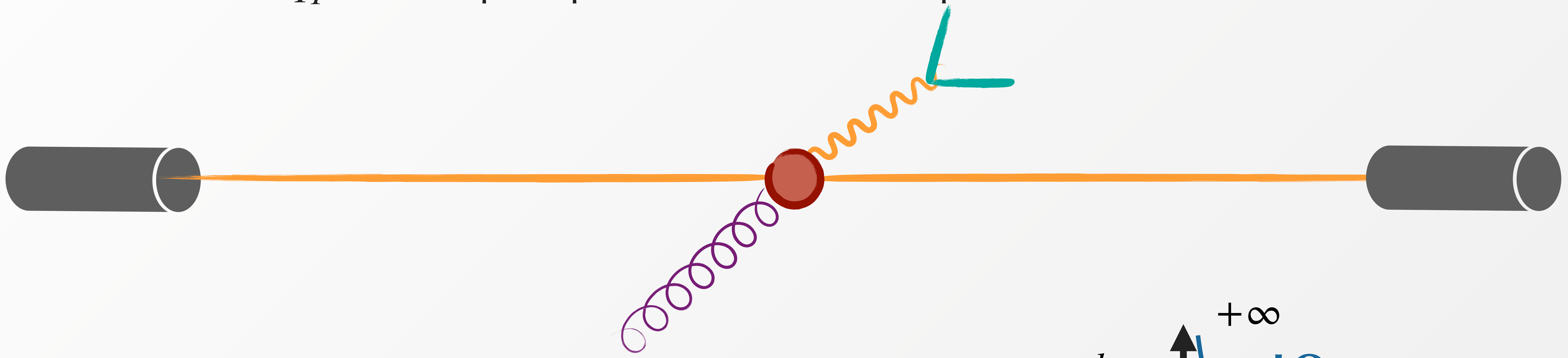
N^3LO_{QCD} available for few LHC processes

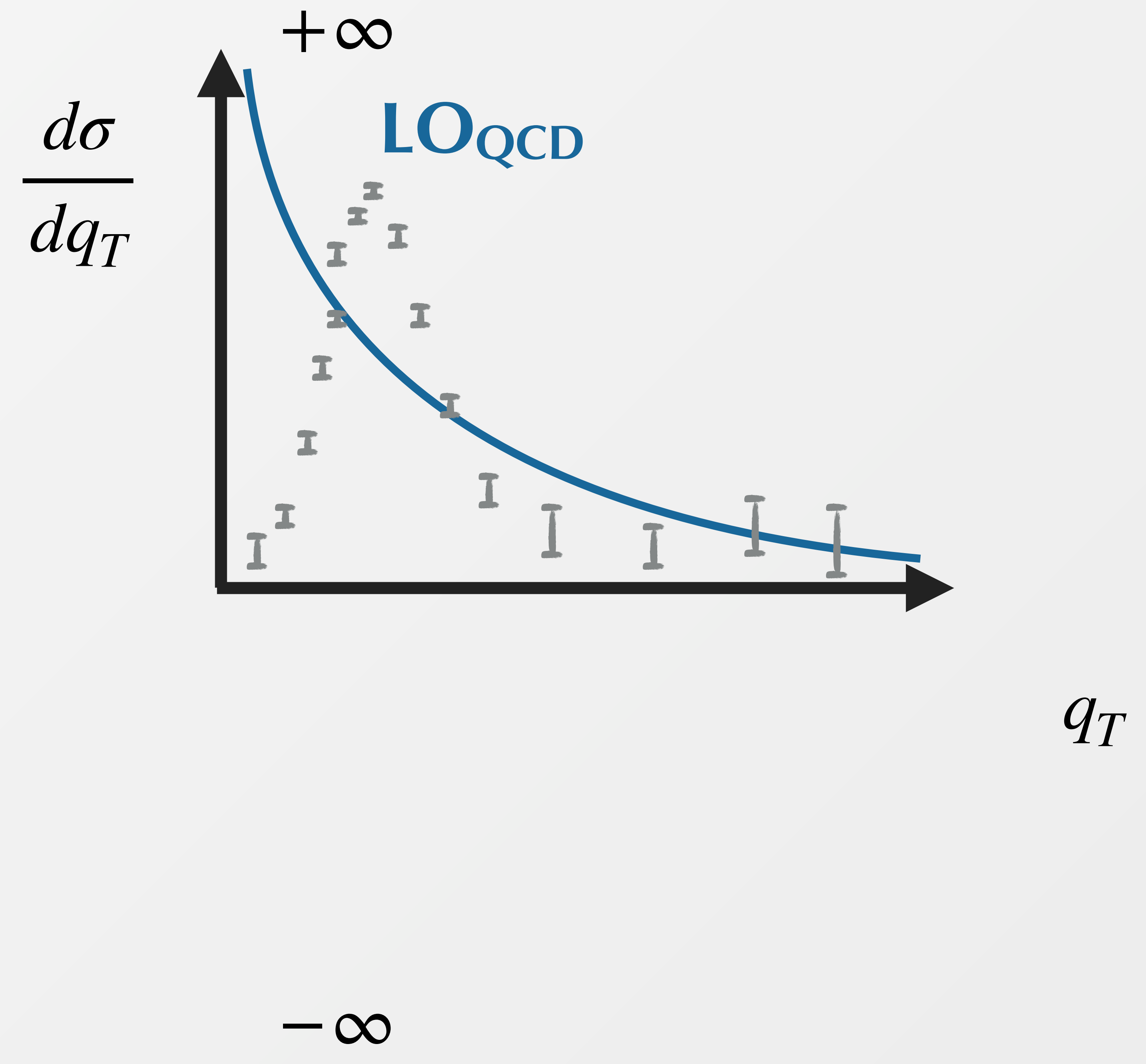
Calculation of NLO_{EW} and of mixed $NNLO_{QCD\text{-}EW}$ corrections relevant for precise phenomenology (especially for candle processes such as DY production) $\alpha \sim 0.01$

Precision physics at the LHC: all-order resummation

Fixed-order description not sufficient for observable sensitive to soft / collinear radiation

E.g transverse momentum q_T of the lepton pair in NC Drell-Yan production





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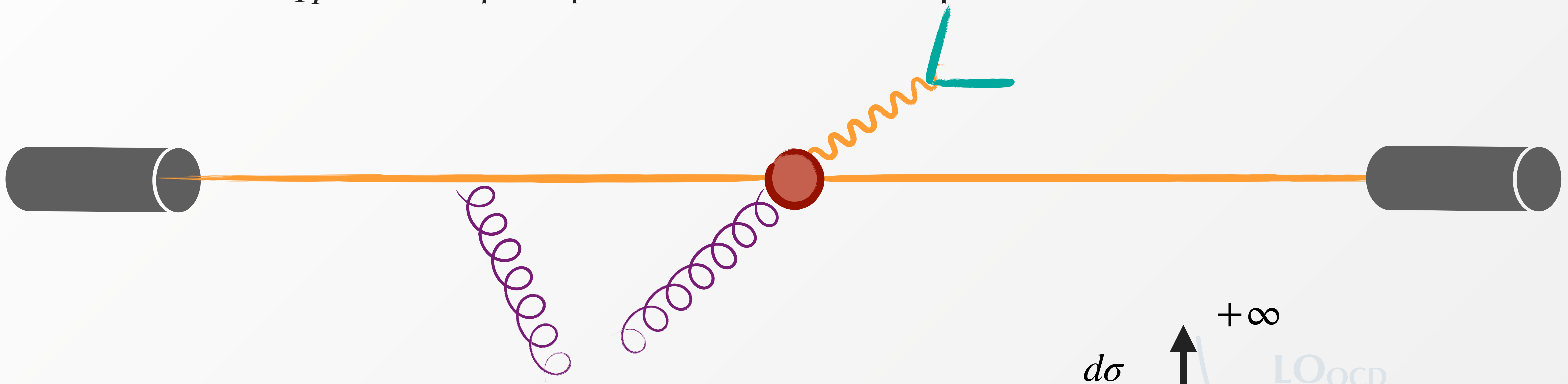

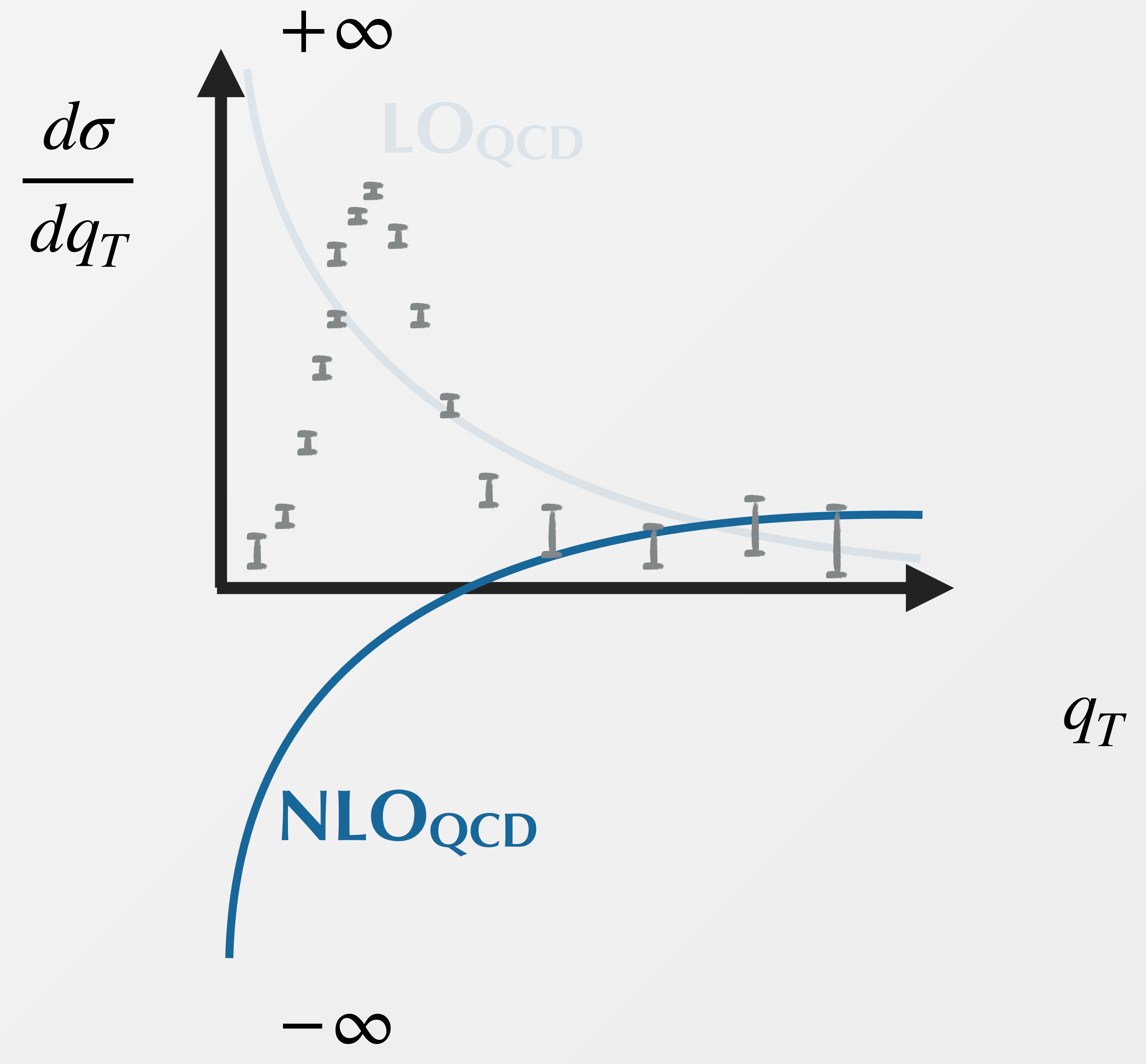

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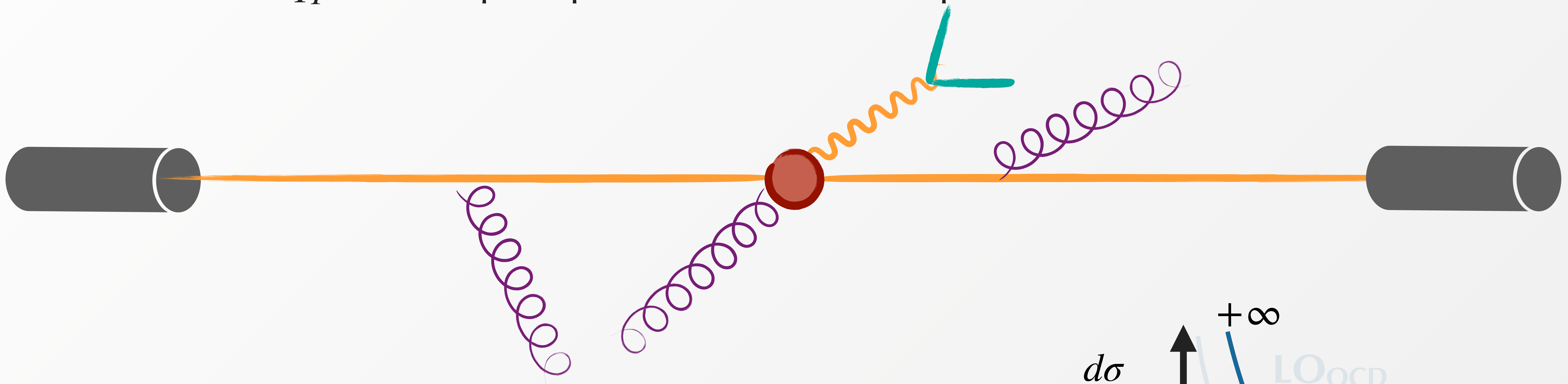

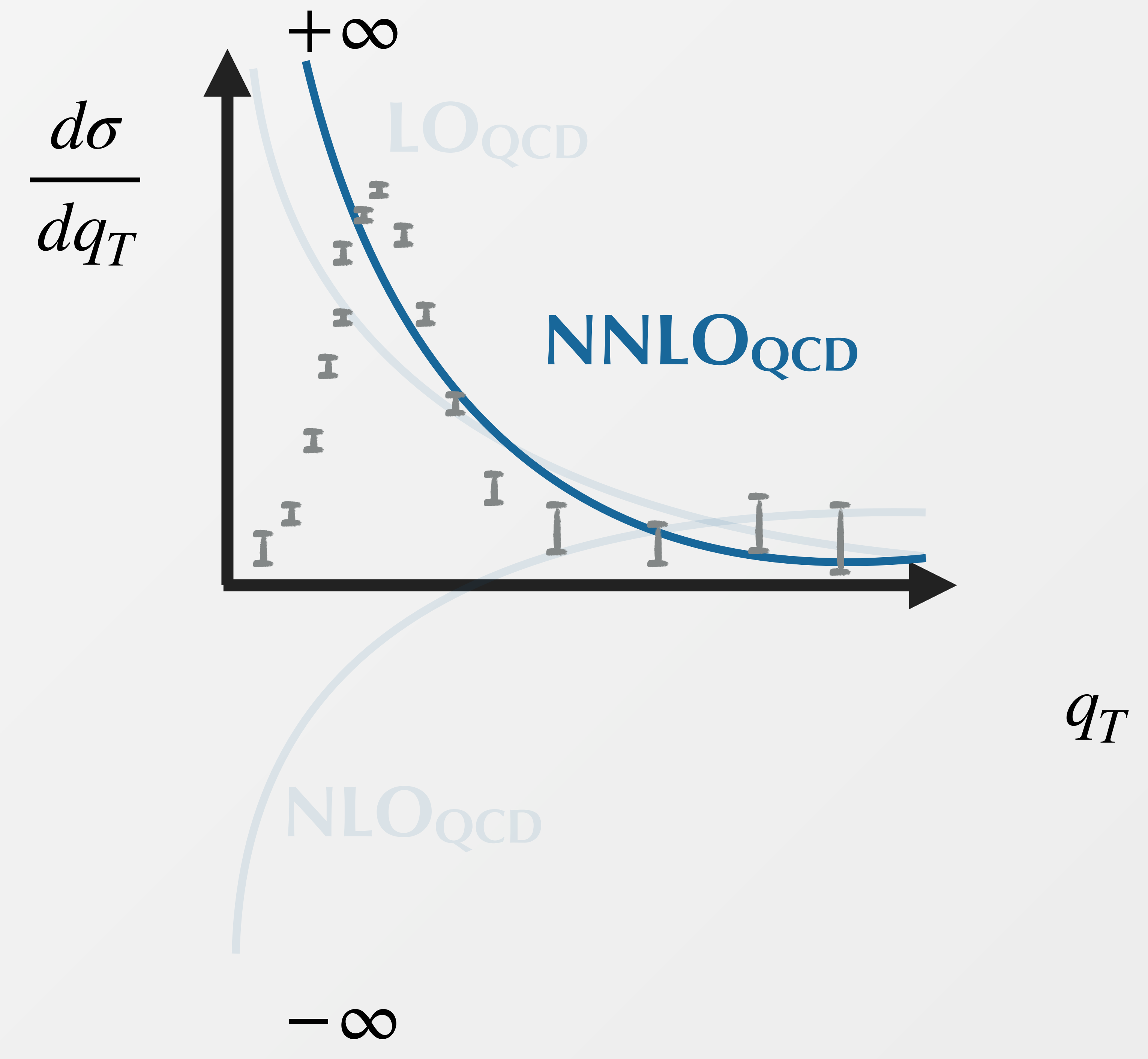

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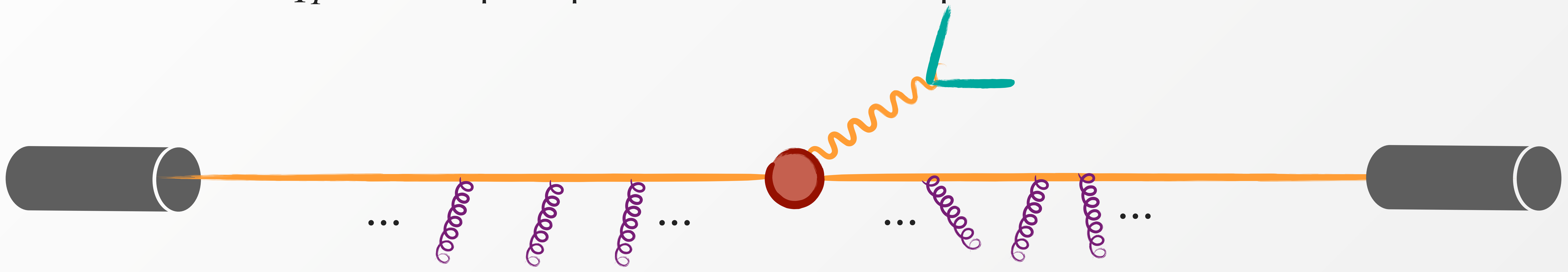
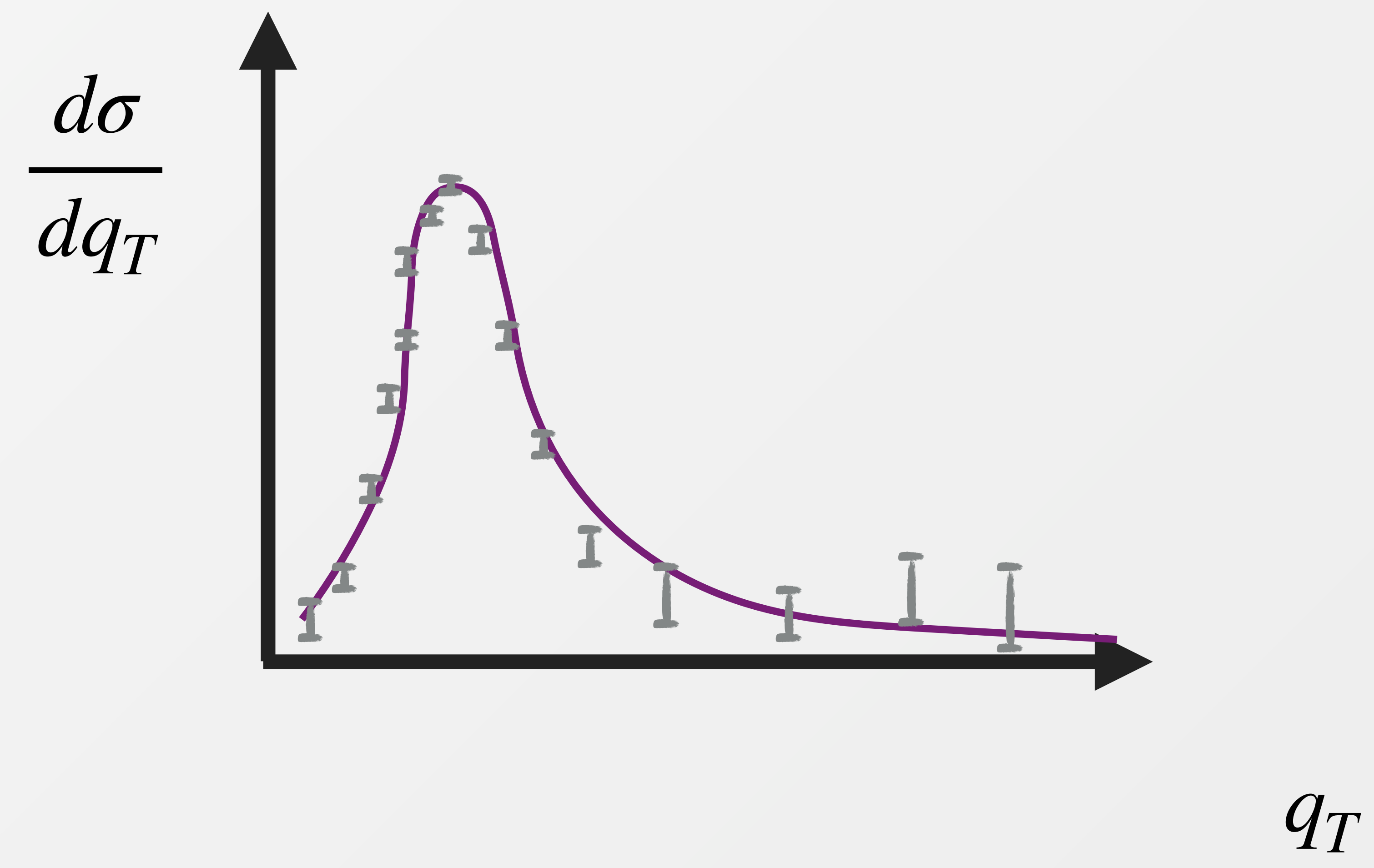


Need to consider infinite number of soft / collinear emissions



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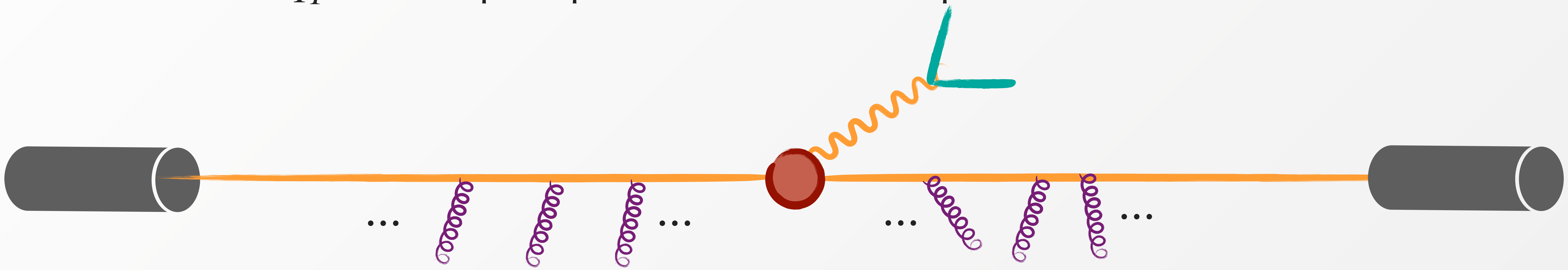
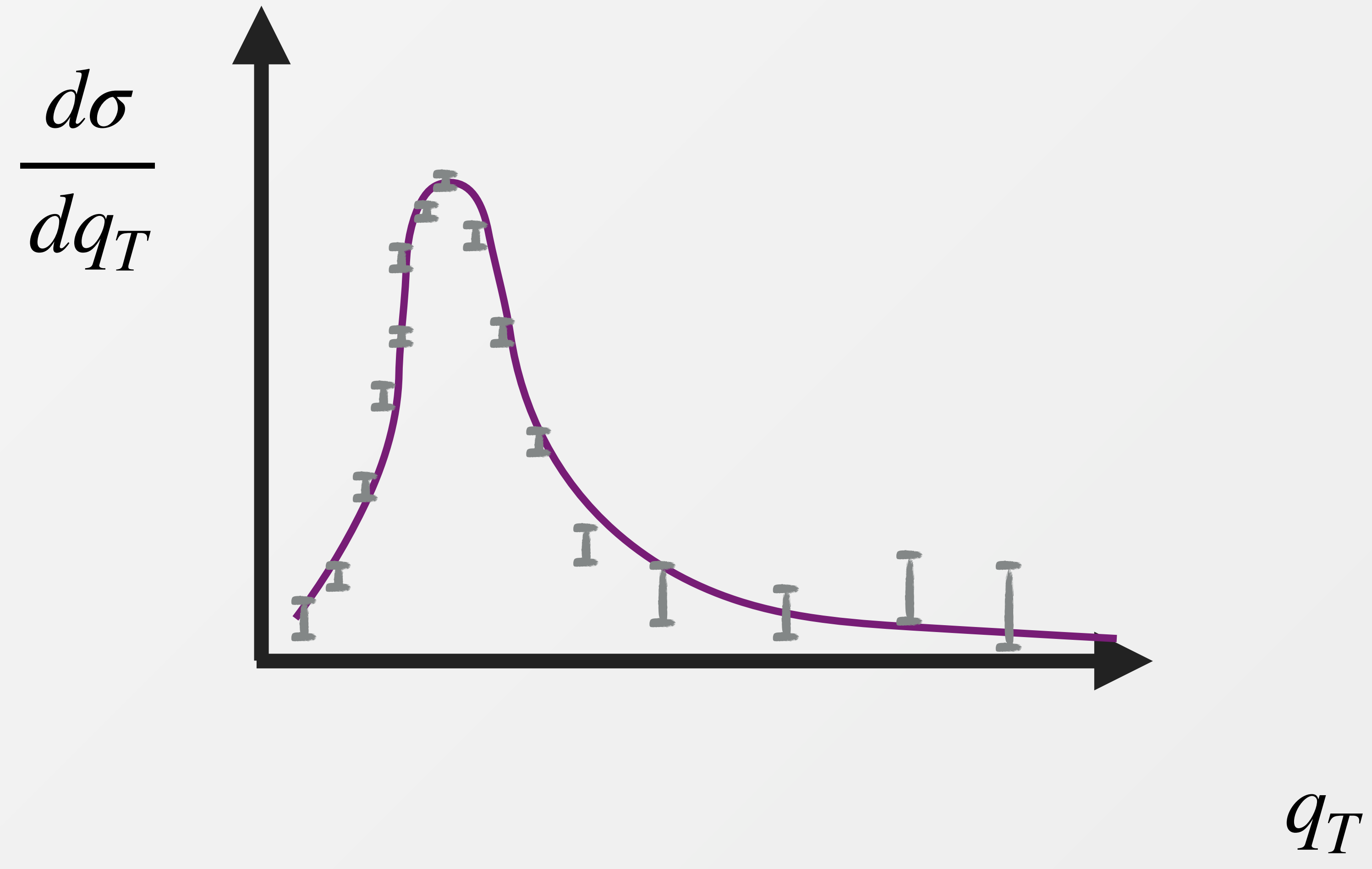


Need to consider infinite number of soft / collinear emissions

Many independent **soft-collinear gluons** with comparable angles and transverse momenta



LL$_{\text{QCD}}$ NLL$_{\text{QCD}}$ NNLL$_{\text{QCD}}$

$v \to 0$

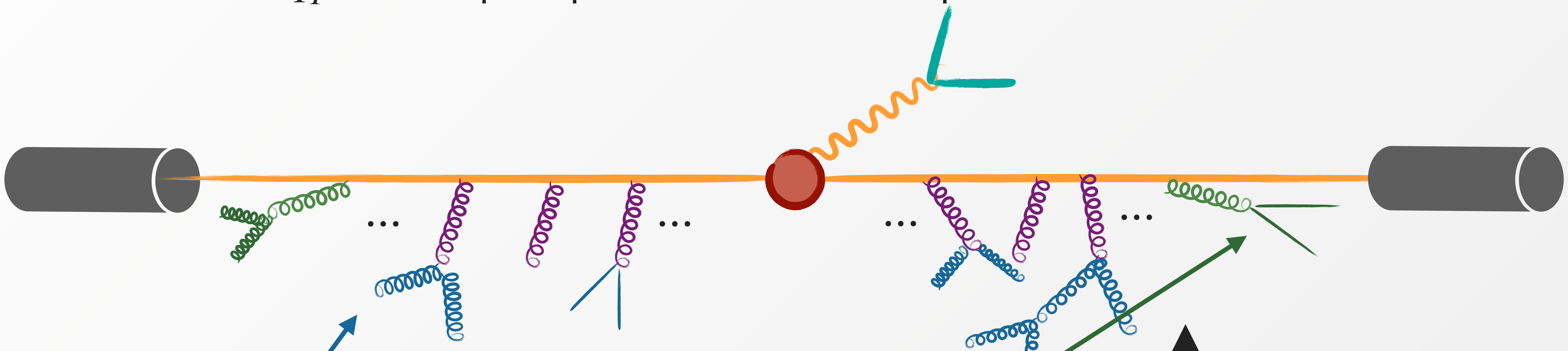
$L = \ln(1/v)$

$$\tilde{\sigma}(v) = \exp\left[\sum_n \left(\mathcal{O}(\alpha_s^n L^{n+1}) + \mathcal{O}(\alpha_s^n L^n) + \mathcal{O}(\alpha_s^n L^{n-1}) + \ldots\right)\right]$$

Precision physics at the LHC: all-order resummation

Fixed-order description not sufficient for observable sensitive to soft / collinear radiation

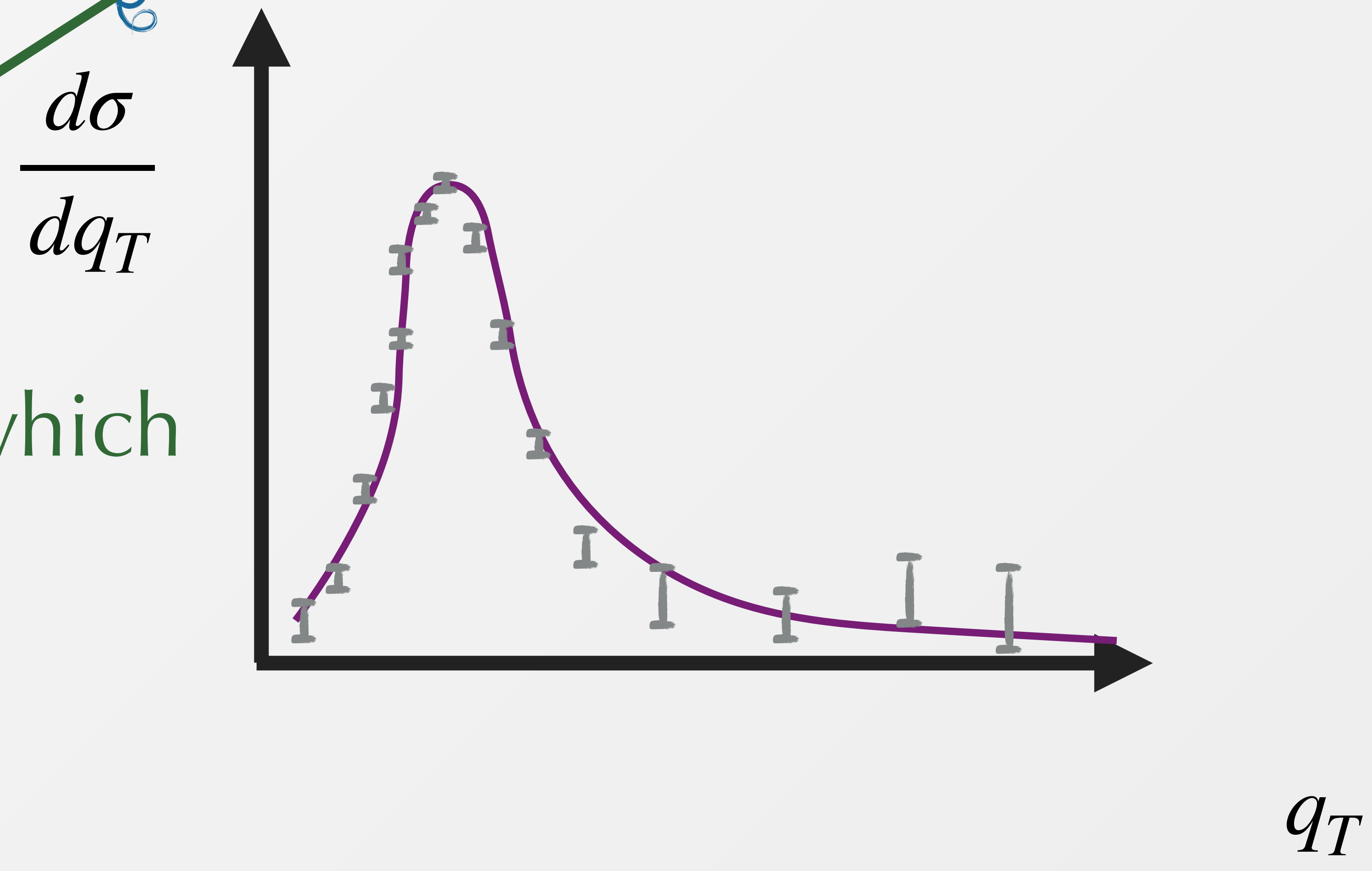
E.g transverse momentum q_T of the lepton pair in NC Drell-Yan production



Need to consider infinite number of soft / collinear emissions

Further, less singular, soft splittings

Hard-collinear emission which can also split



$v \to 0$

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LL$_{QCD}$ NLL$_{QCD}$ NNLL$_{QCD}$

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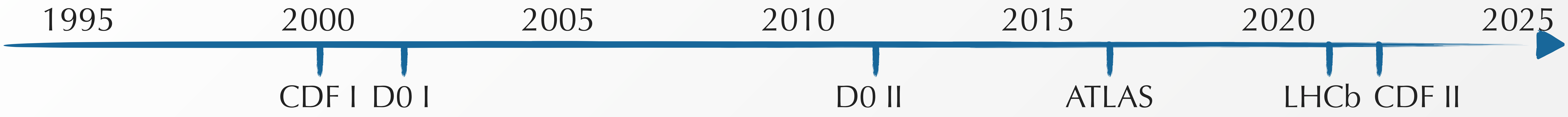
Progress in theoretical calculations



$(N)NLL_{QCD}+NLO_{QCD}$
[Balasz, Yuan '97]

$NNLL'_{QCD}+NNLO_{QCD}$
[Bozzi, Catani, Ferrera, De Florian, Grazzini '10]

$N^3LL_{QCD}+NNLO_{QCD}$
[Bizon, Monni, Re, LR, Torrielli '17]

$N^3LL'_{QCD}+N^3LO_{QCD}$
[Camarda, Cieri, Ferrera '21]
[Re, LR, Torrielli '21]
[Chen, Gehrmann, Glover, Huss, Monni, Re, LR, Torrielli '22]
[Neumann, Campbell '22]

$NNLO_{QCD\text{-}EW}$
[Armadillo, Bonciani, Buonocore, Devoto, Grazzini, Kallweit, Rana, Savoini, Tramontano, Vicini '21, '22]
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile '22]

Huge progress in the theoretical description of NC and CC Drell-Yan processes in the last few years

Fixed-order description now reaches $\mathcal{O}(\alpha_s^3)$ (N^3LO_{QCD})

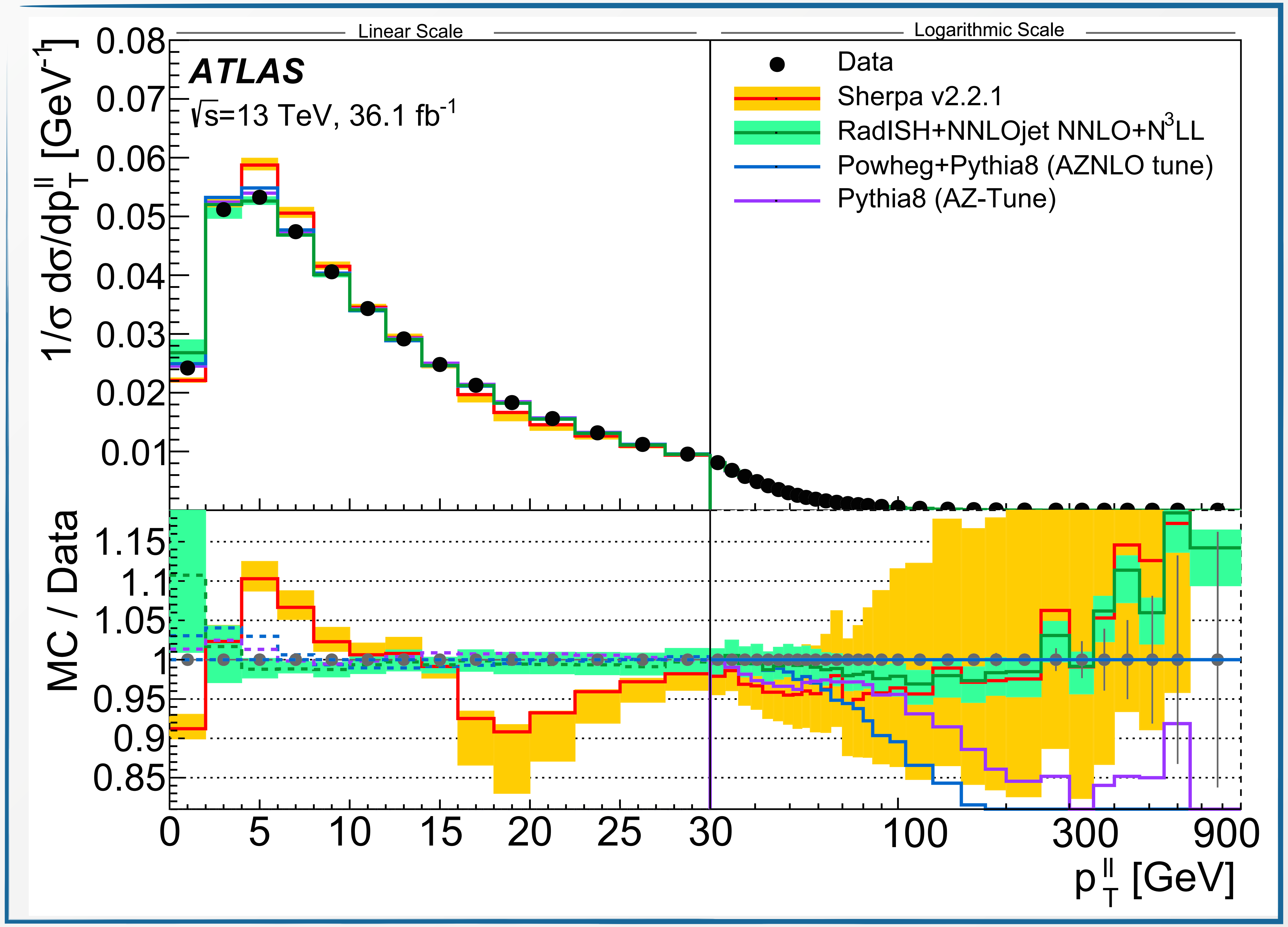
All-order resummation up to $N^3LL'_{QCD}$

QCD-EW correction at order $\mathcal{O}(\alpha_s\alpha)$ $NNLO_{QCD\text{-}EW}$

Description of experimental data at N^3LO_{QCD}+N^3LL_{QCD}

Theoretical predictions now are capable of describing the data **precisely** across a wide range of scales

green: N^3LL_{QCD}+N^3LO_{QCD}



[ATLAS '20]

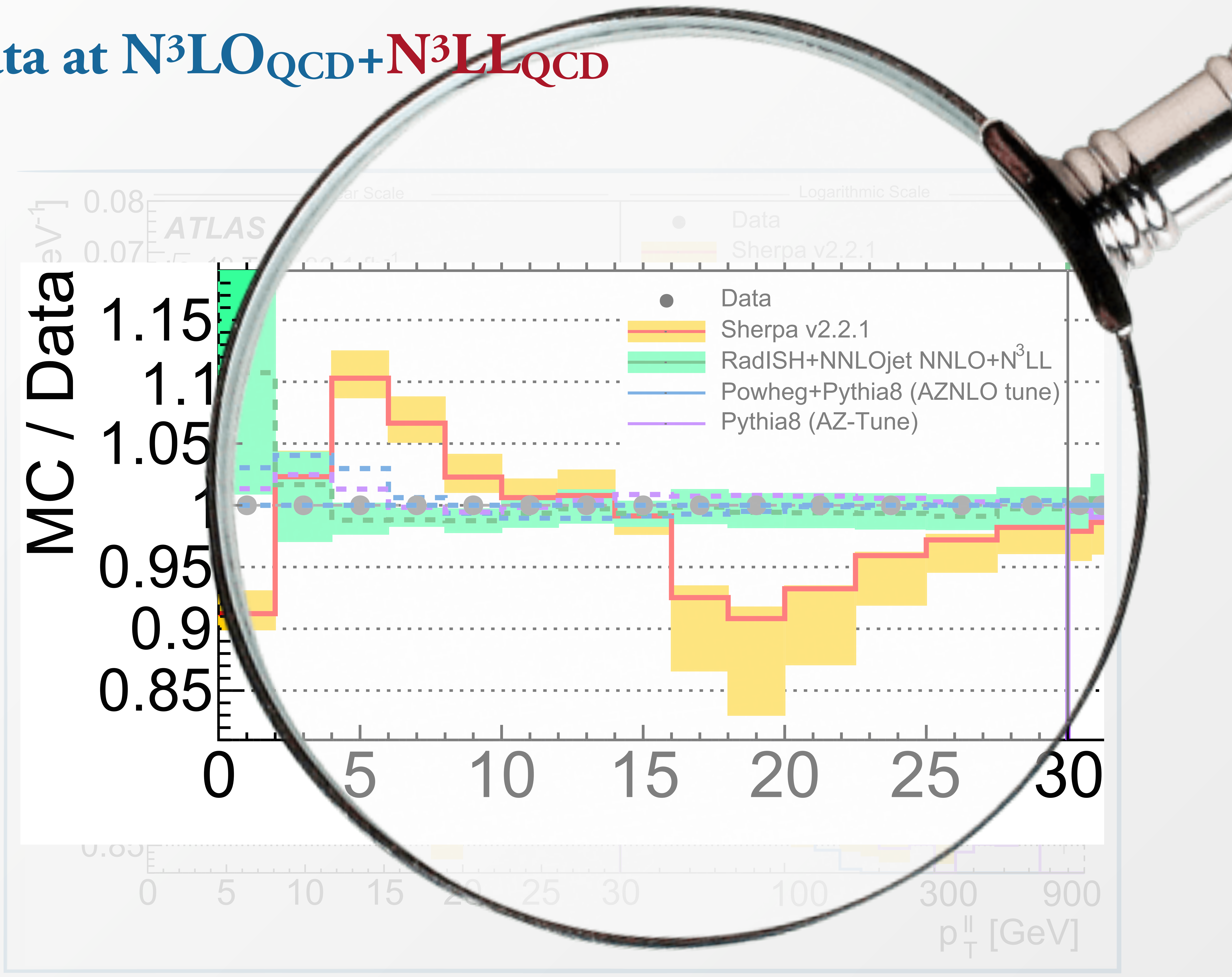
Description of experimental data at N^3LO_{QCD}+N^3LL_{QCD}

Theoretical predictions now are capable of describing the data **precisely** across a wide range of scales

green: N^3LL_{QCD}+N^3LO_{QCD}

And are on par, if not better, to parton showers predictions that have been **tuned to experimental data**

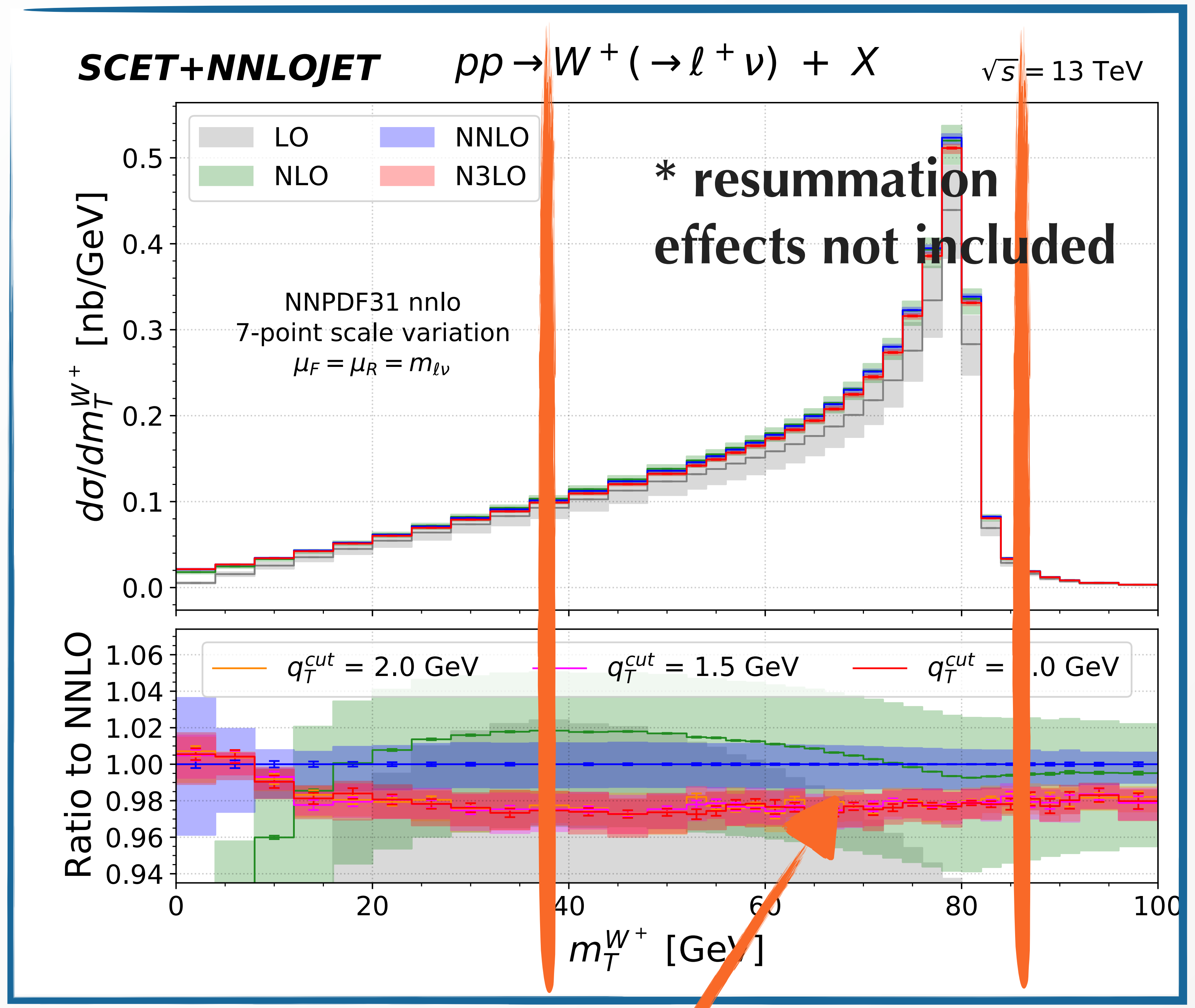
N.B. : `RadISH+NNLOJET` predictions **do not** include any non-perturbative modelling at low q_T



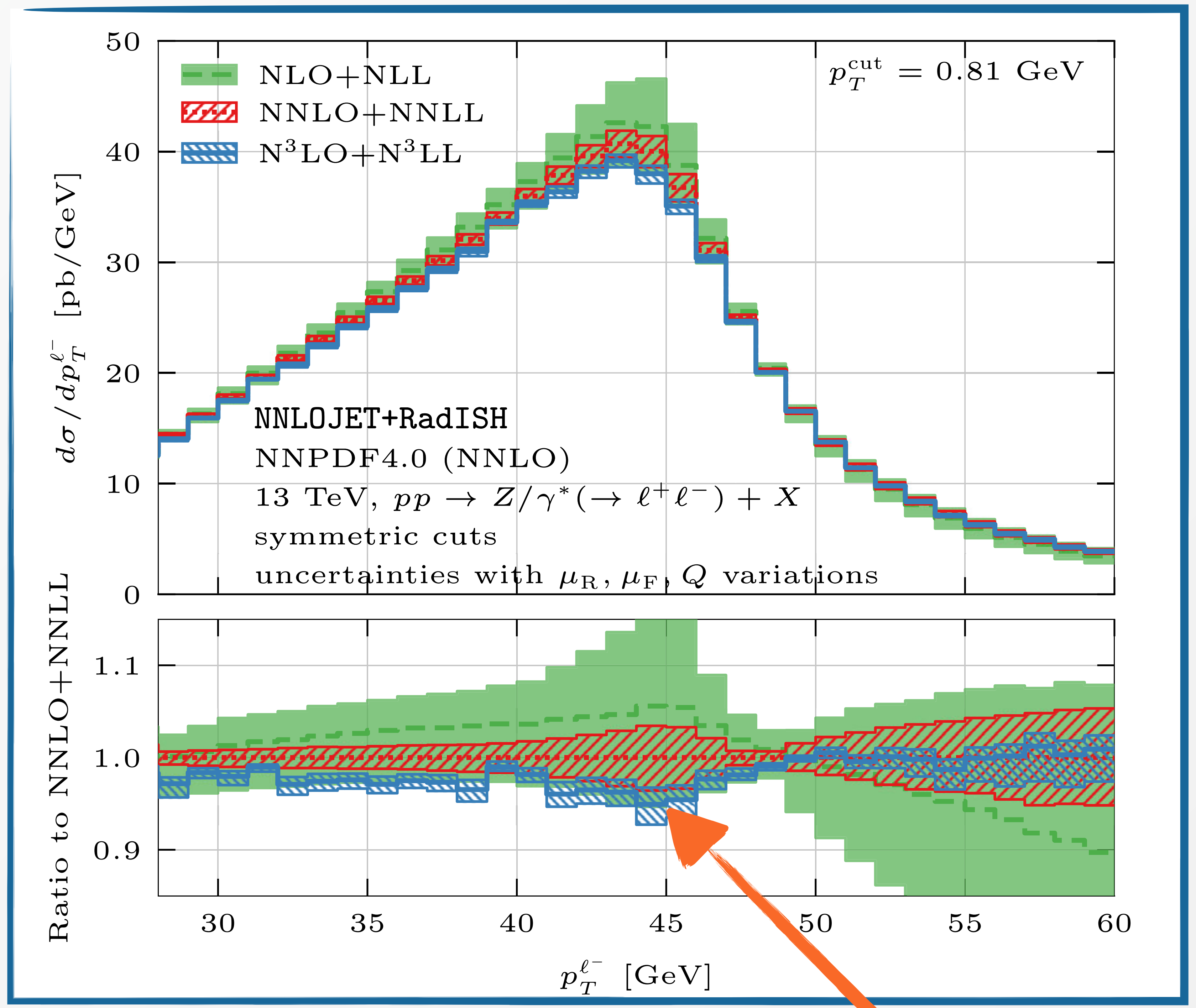
[ATLAS '20]

Control of the differential distributions in DY production

Shape of differential spectra is affected by higher order predictions



[Gehrmann, Glover, Huss, Chen, Yang, Zhu 2205.11426]



[Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Torrielli, 2203.01565]

Residual uncertainties at **N³LO$_{QCD}$** are at the $\mathcal{O}(1-2\%)$ level

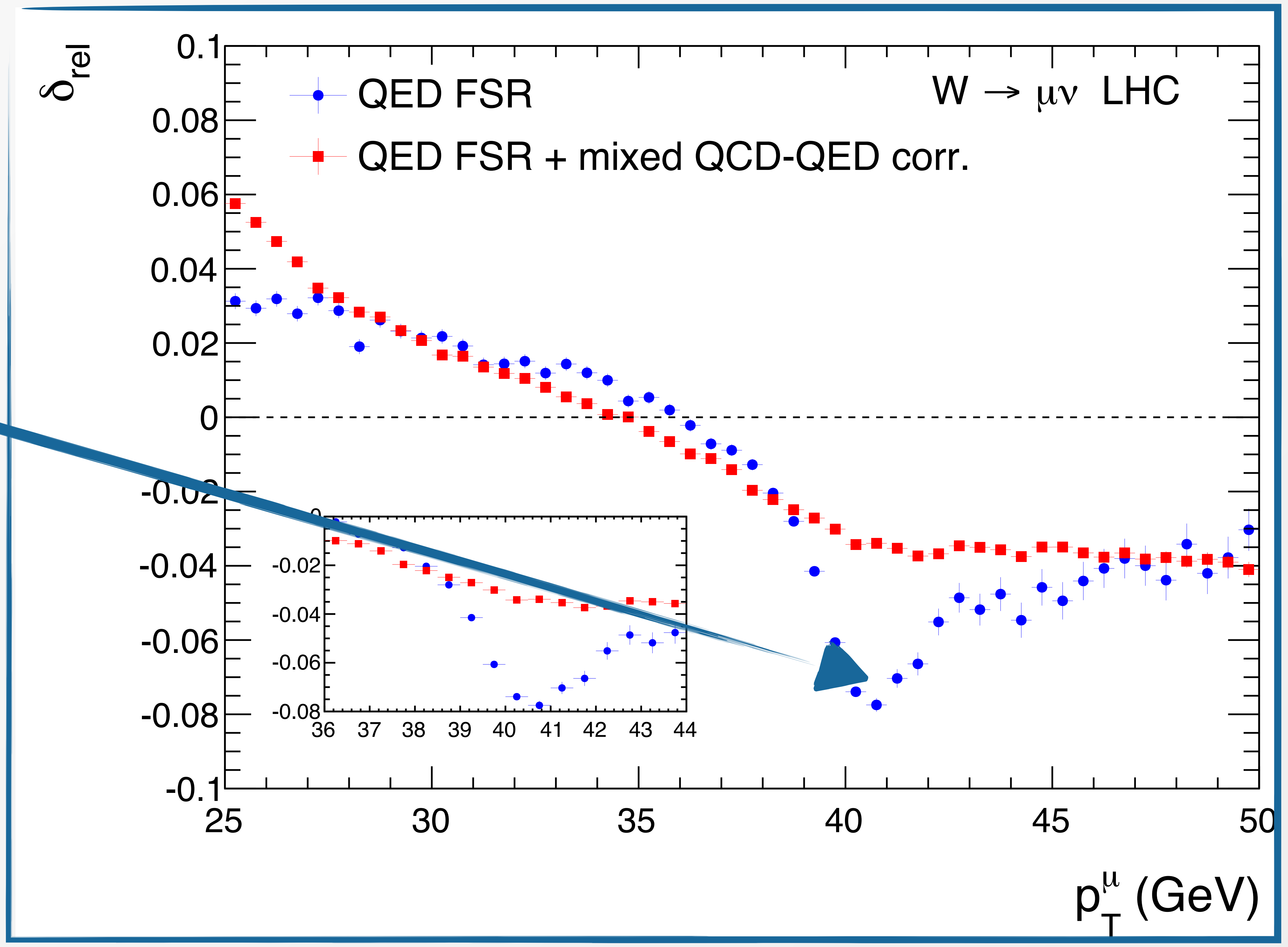
Impact of **N³LO$_{QCD}$** corrections relatively flat in the fit window for $M_\perp$

N³LL′$_{QCD}$+N³LO$_{QCD}$ modifies the shape after the Jacobian peak for $p_\perp^\ell$

Interplay of QCD and EW corrections further modify the shape of the differential distributions

Impact of QED and mixed QCD×QED corrections

Both $p_\perp^\ell$ and $M_\perp$ features large radiative corrections due to **QED final state radiation** at the **jacobian peak**



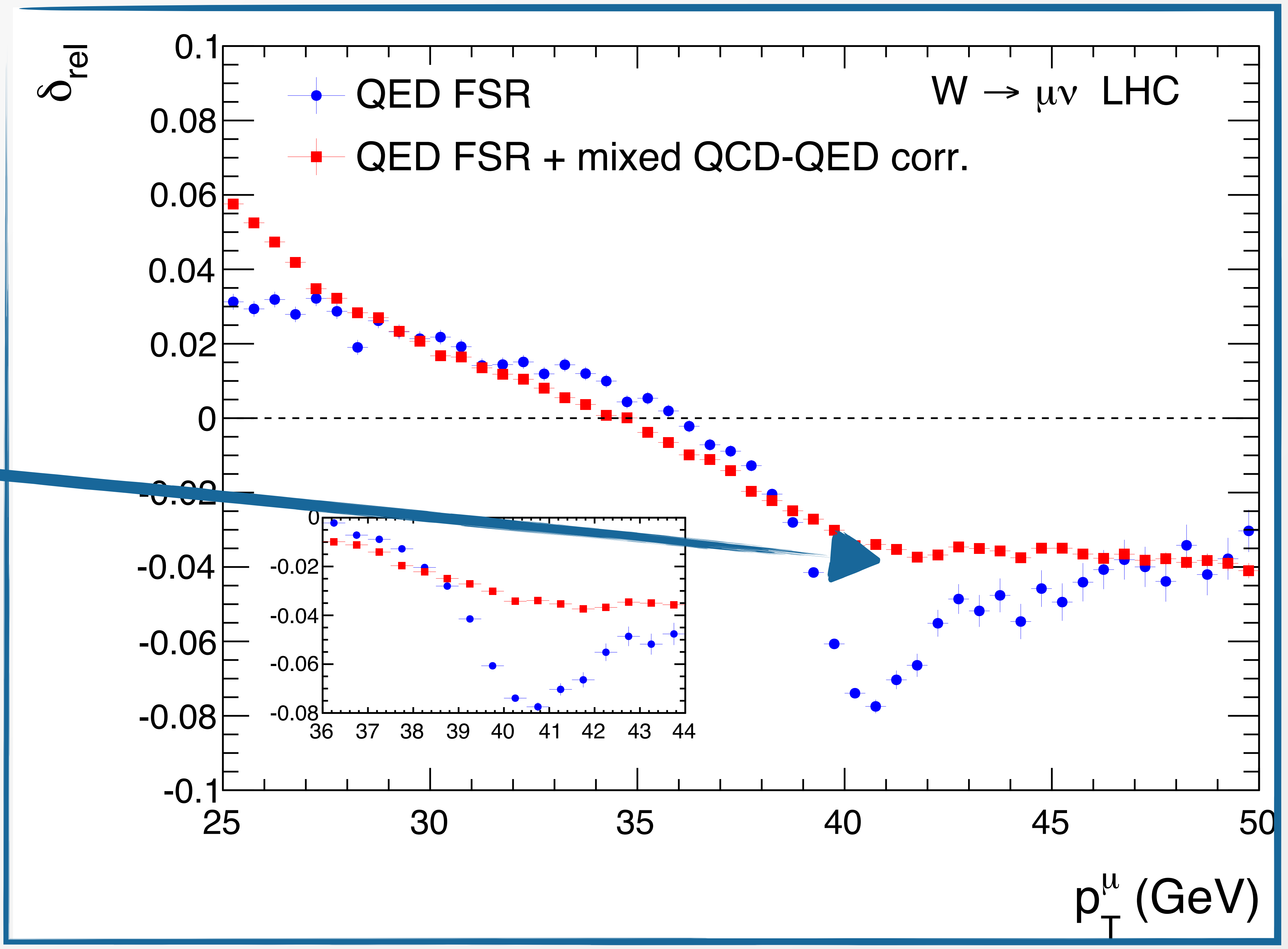
[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Impact of QED and mixed QCD×QED corrections

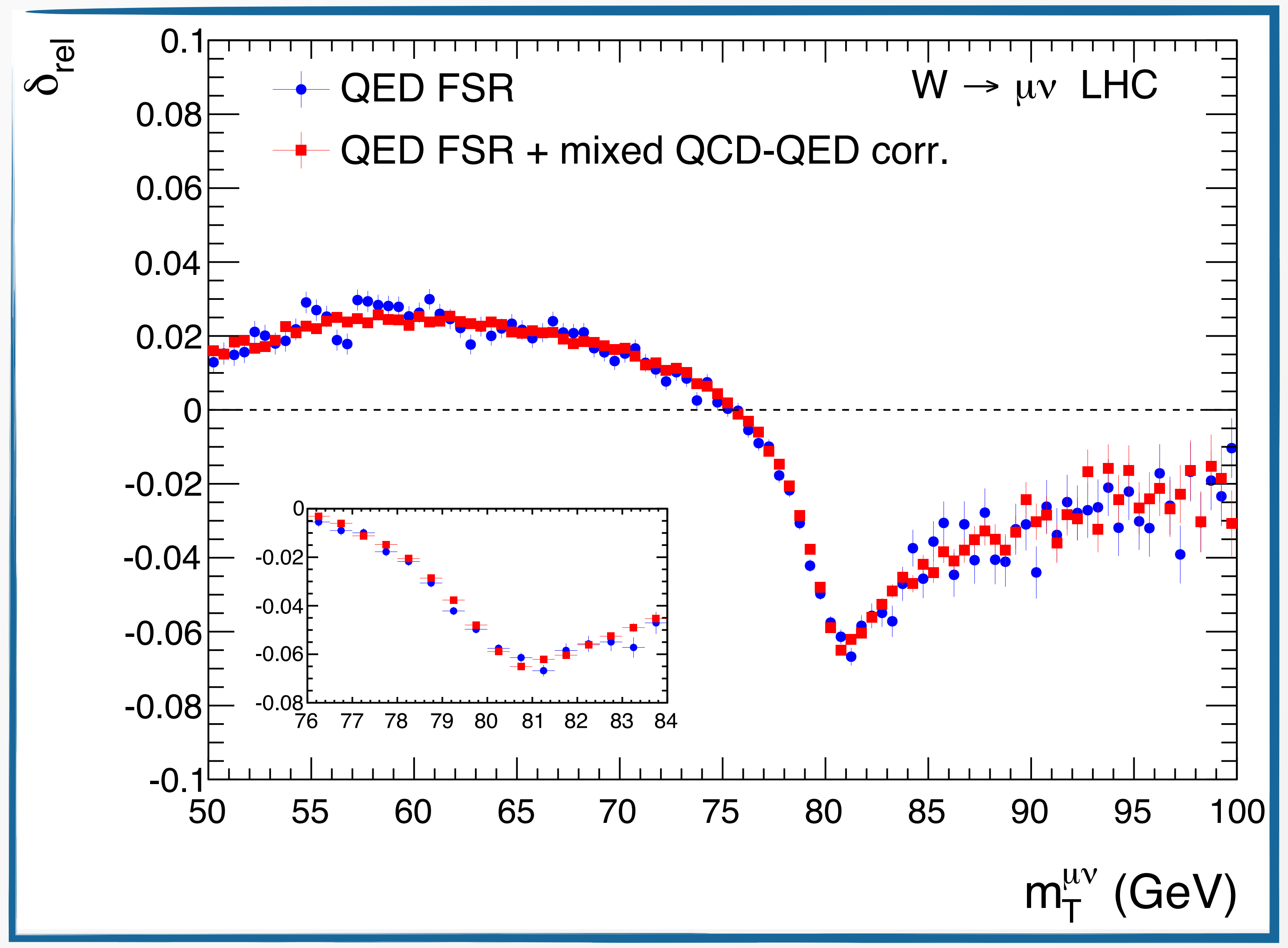
Both $p_\perp^\ell$ and $M_\perp$ features large radiative corrections due to **QED final state radiation** at the **jacobian peak**

The precise shape of $p_\perp^\ell$ at the Jacobian peak is determined by the **interplay of QCD and QED corrections**

Impact of mixed corrections minor for $M_\perp$



[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

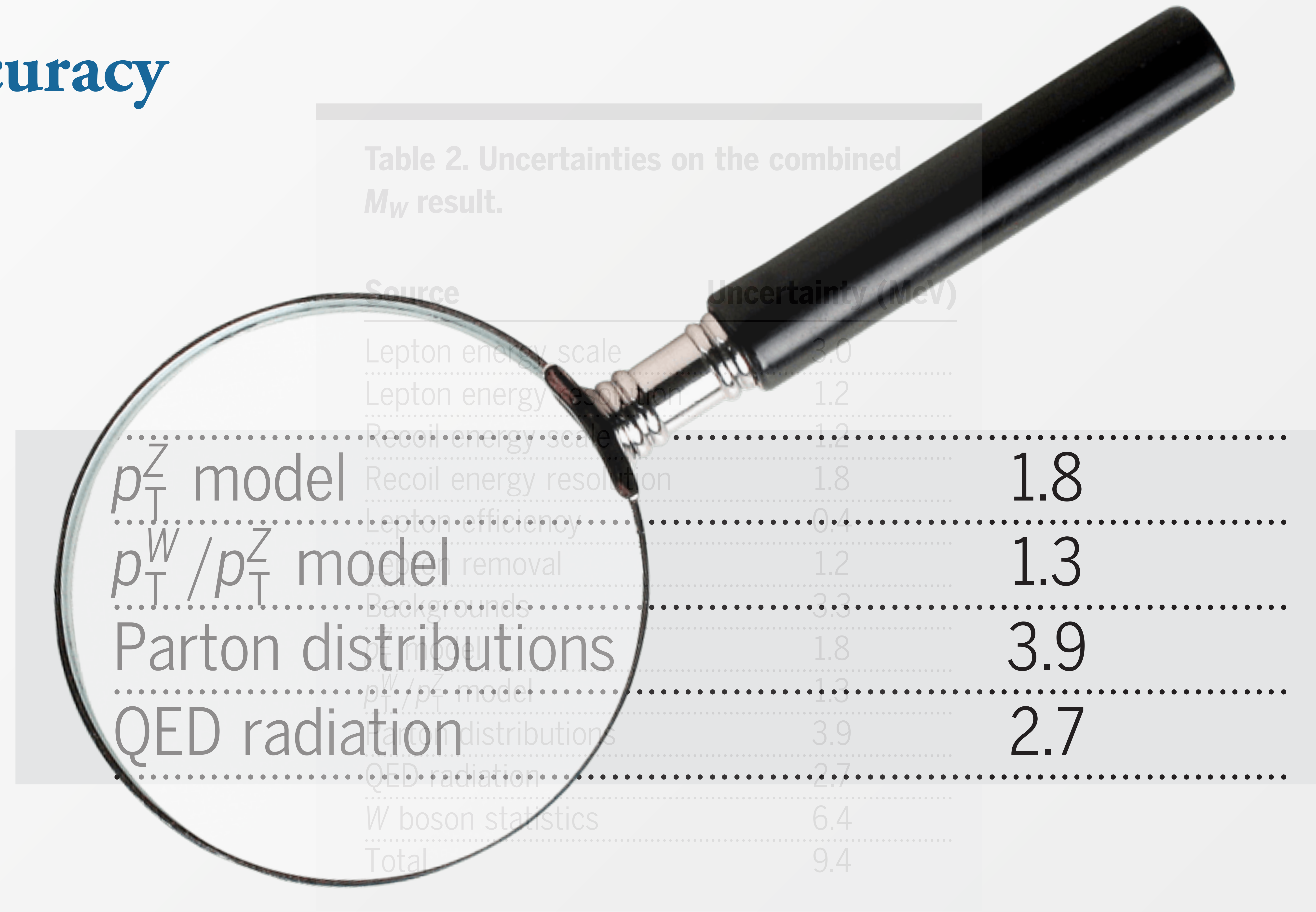


Analyses do include the bulk of the (factorised) QCD×QED corrections

The impact on the m_W shifts of the mixed QCD×QED corrections strongly depends on the underlying QCD model

CDF measurement and theoretical accuracy

CDF II measurement features **very aggressive** estimates for theory uncertainties, especially when compared to CDF I results with lower luminosity, as all the errors are reduced by a factor 2-3



$p_T(W)$ model	5
Parton distributions	10
QED radiation	4

[CDF collaboration, 1203.0275]

CDF II, 2.2 fb^{-1} (2012) $m_W = 80.387 \pm 19$

CDF II, 8.8 fb^{-1} (2022) $m_W = 80.433 \pm 9$

Do these error reflect the **improved theoretical understanding** of Z/W production at hadron colliders?

CDF measurement and theoretical accuracy

Not really: despite being published 10 years apart, the two analyses share most of the same underlying **theoretical model**

CDF II, 2.2 fb^{-1} (2012)

`ResBos` (private '03 version) **[Balasz, Yuan '97]**
$(N)NLL_{QCD}+NLO_{QCD}$
CTEQ6.6 NLO PDFs
QED modelling with `PHOTOS+HORACE`

CDF II, 8.8 fb^{-1} (2022)

`ResBos` (private '03 version) **[Balasz, Yuan '97]**
$(N)NLL_{QCD}+NLO_{QCD}$
NNPDF3.1 NNLO PDFs
QED modelling with `PHOTOS+HORACE`

Reduction of the theoretical error obtained via additional **data constraint** and use of **more modern PDF sets**

CDF measurement and theoretical accuracy

$(N)NLL_{QCD}+NLO_{QCD}$

CDF measurement and theoretical accuracy

(N)NLL$_{QCD}$+NLO$_{QCD}$

ResBos

[Balasz, Yuan '97]

"RESBOS offers one of the most accurate theoretical calculations available for these processes"

CDF Collaboration et al., Science 376, 170–176 (2022)

CDF measurement and theoretical accuracy

1995 2000 2005 2010 2015 2020 2025

CDF I D0 I D0 II ATLAS LHCb CDF II

$\mathrm{(N)NLL_{QCD}+NLO_{QCD}}$ $\mathrm{NNLL'_{QCD}+NNLO_{QCD}}$ $\mathrm{N^3LL_{QCD}+N^3LO_{QCD}}$ $\mathrm{N^3LL'_{QCD}+N^3LO_{QCD}}$

`ResBos`
[Balasz, Yuan '97]

`DYQT`
[Bozzi, Catani, Ferrera, De Florian, Grazzini '10]

`CUTE`
[Becher, Neubert , Wilhelm '11]

`DYRES`
[Catani, Ferrera, De Florian, Grazzini '15]

`ResBos2`
[Isaacson '17]

`RadISH`
[Bizon, Monni, Re, LR, Torrielli '17]

`DYTURBO`
[Camarda et al,'19]

`MATRIX+RadISH`
[Kallweit, Wiesemann, Re, LR, '20]

`MCFM+CUTE`
[Becher, Neumann '20]

> "RESBOS offers one of the most accurate theoretical calculations available for these processes"

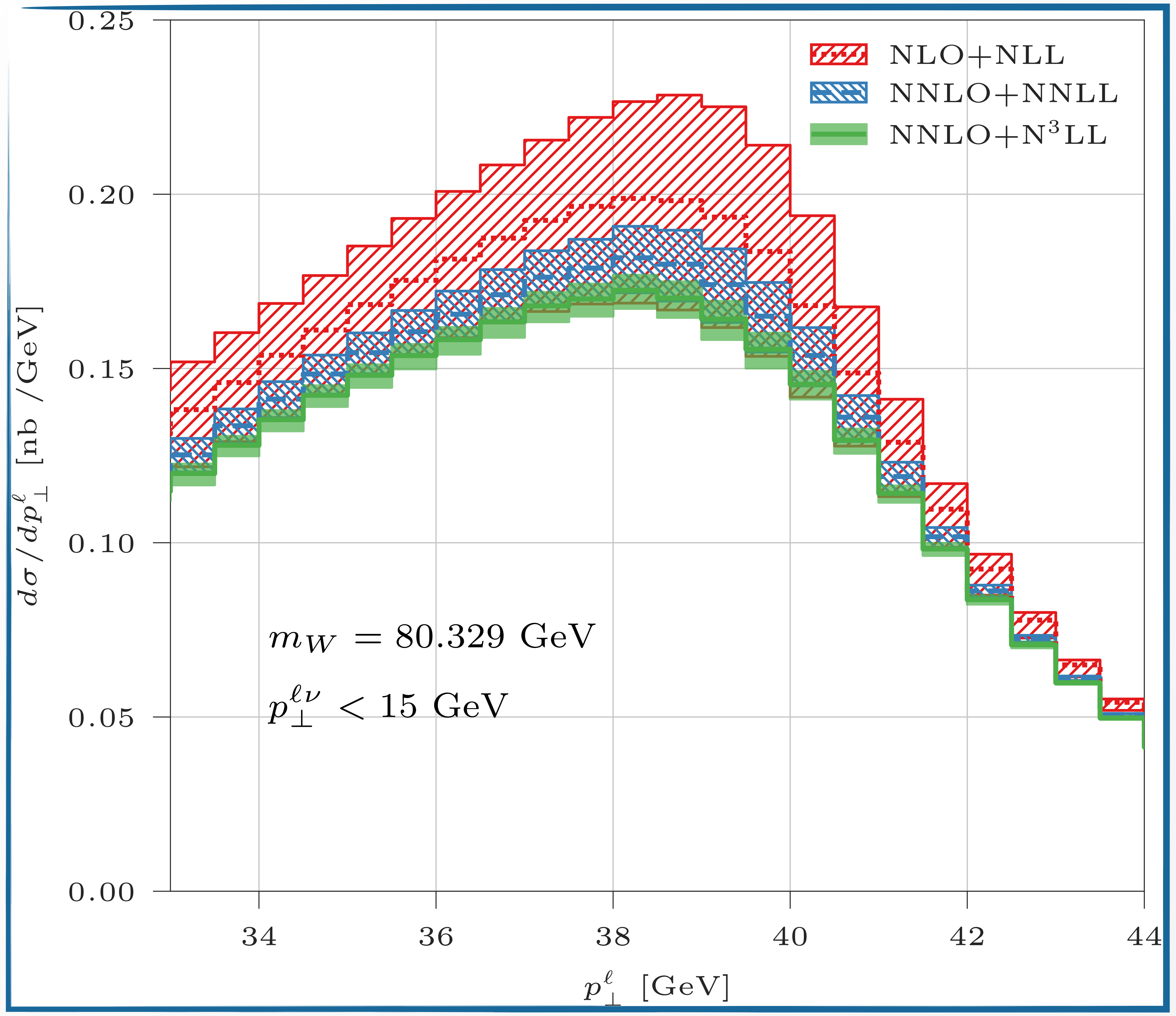
CDF Collaboration et al., Science 376, 170–176 (2022)

CDF measurement and theoretical accuracy

$(N)NLL_{QCD}+NLO_{QCD}$ ⟷ $N^3LL'_{QCD}+N^3LO_{QCD}$

How to **systematically** study the theoretical error associated to the use of predictions at a given accuracy?

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$



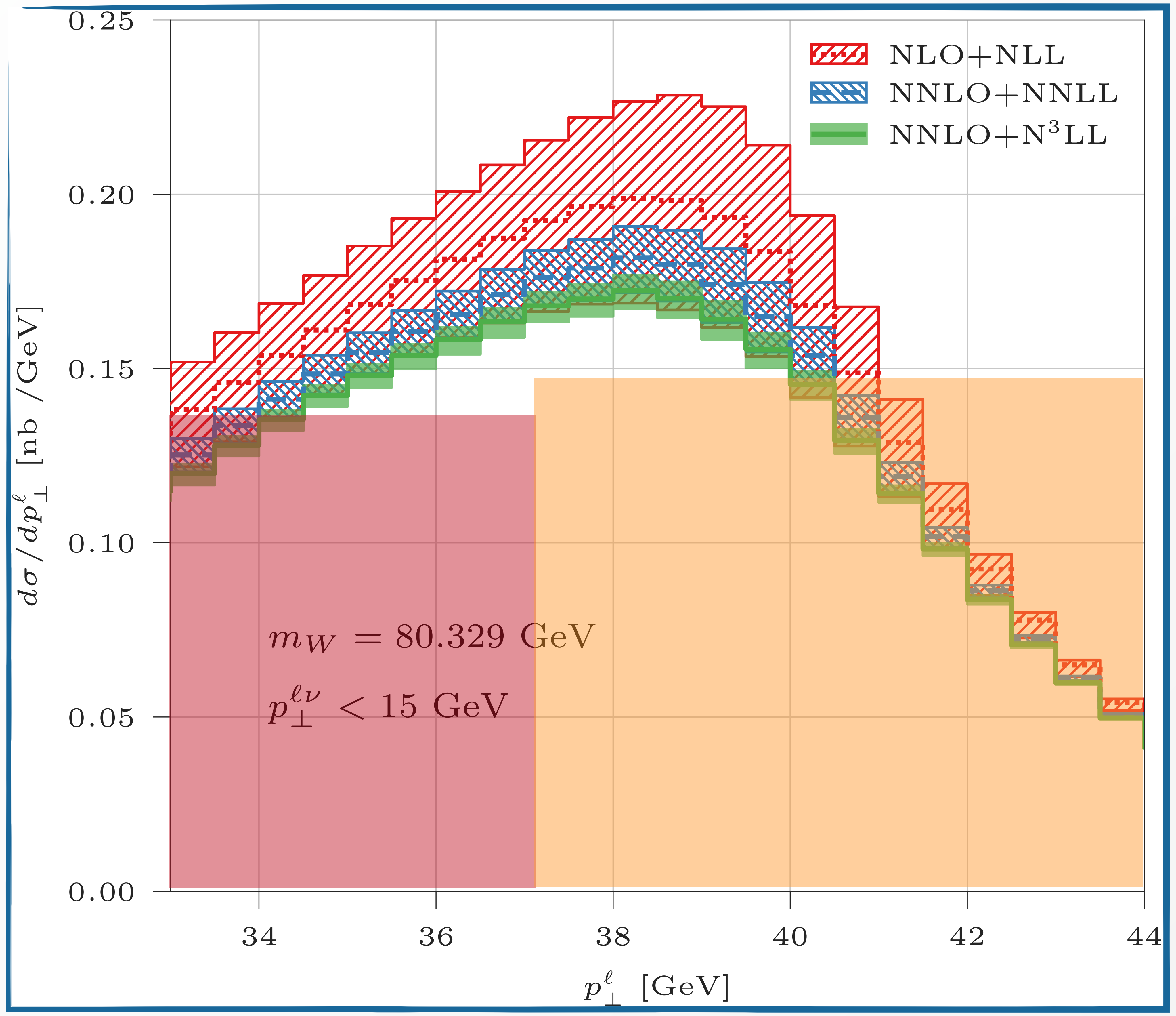
[LR, P. Torrielli, A. Vicini '23]

The lepton transverse momentum distribution features a **Jacobian peak** at $p_T^\ell \sim m_W/2$

Presence of the endpoint makes the distribution particularly sensitive to m_W

Study of covariance matrix constructed from the bins and considering various m_W hypothesis suggests bulk of m_{W_w} sensitivity captured by a single bin combination

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$



[LR, P. Torrielli, A. Vicini '23]

$$L = \int_{p^\ell_{\perp,\mathrm{min}}}^{p^\ell_{\perp,\mathrm{mid}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

$$U = \int_{p^\ell_{\perp,\mathrm{mid}}}^{p^\ell_{\perp,\mathrm{max}}} dp_\perp^\ell \frac{d\sigma}{dp_\perp^\ell}$$

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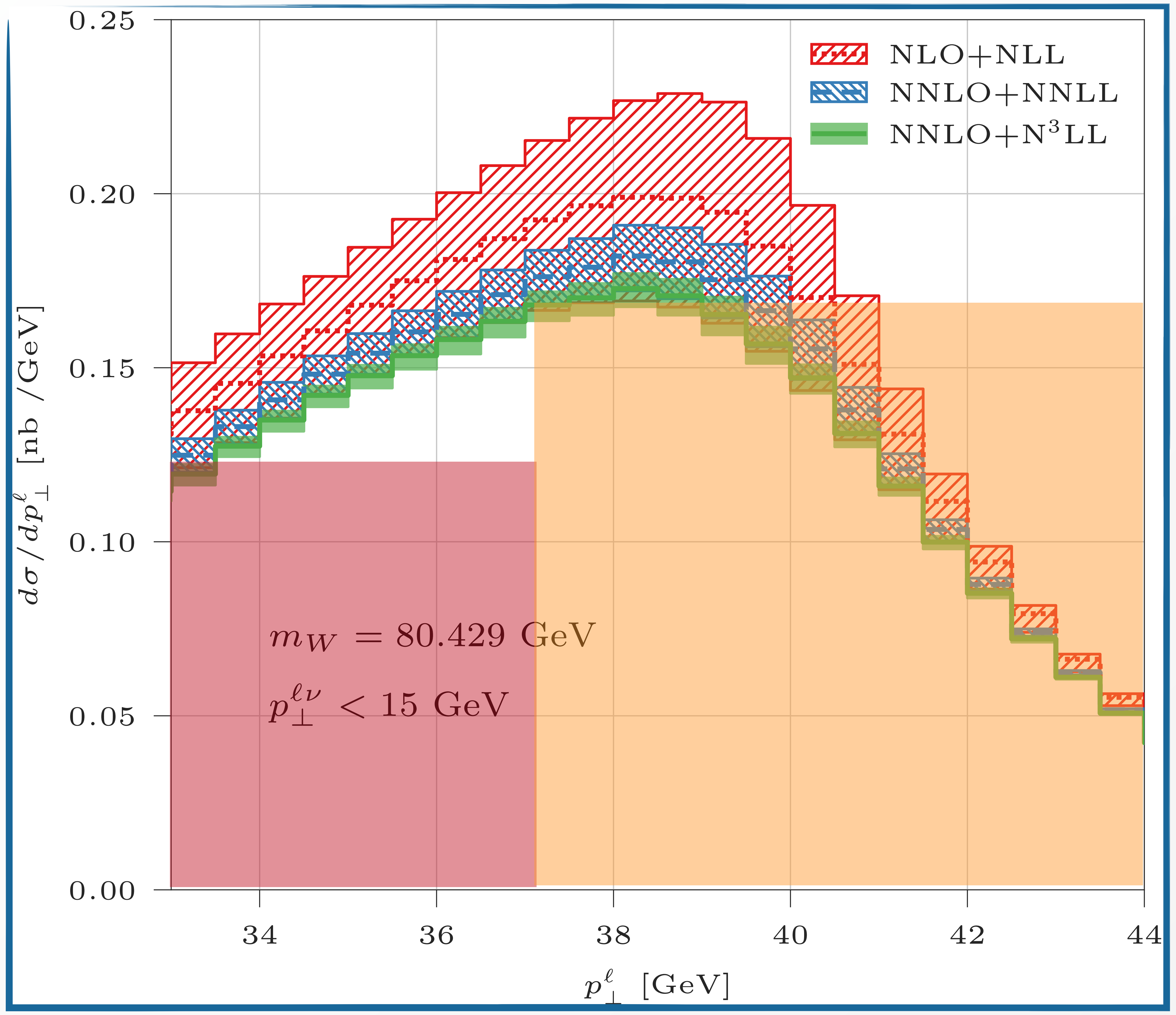

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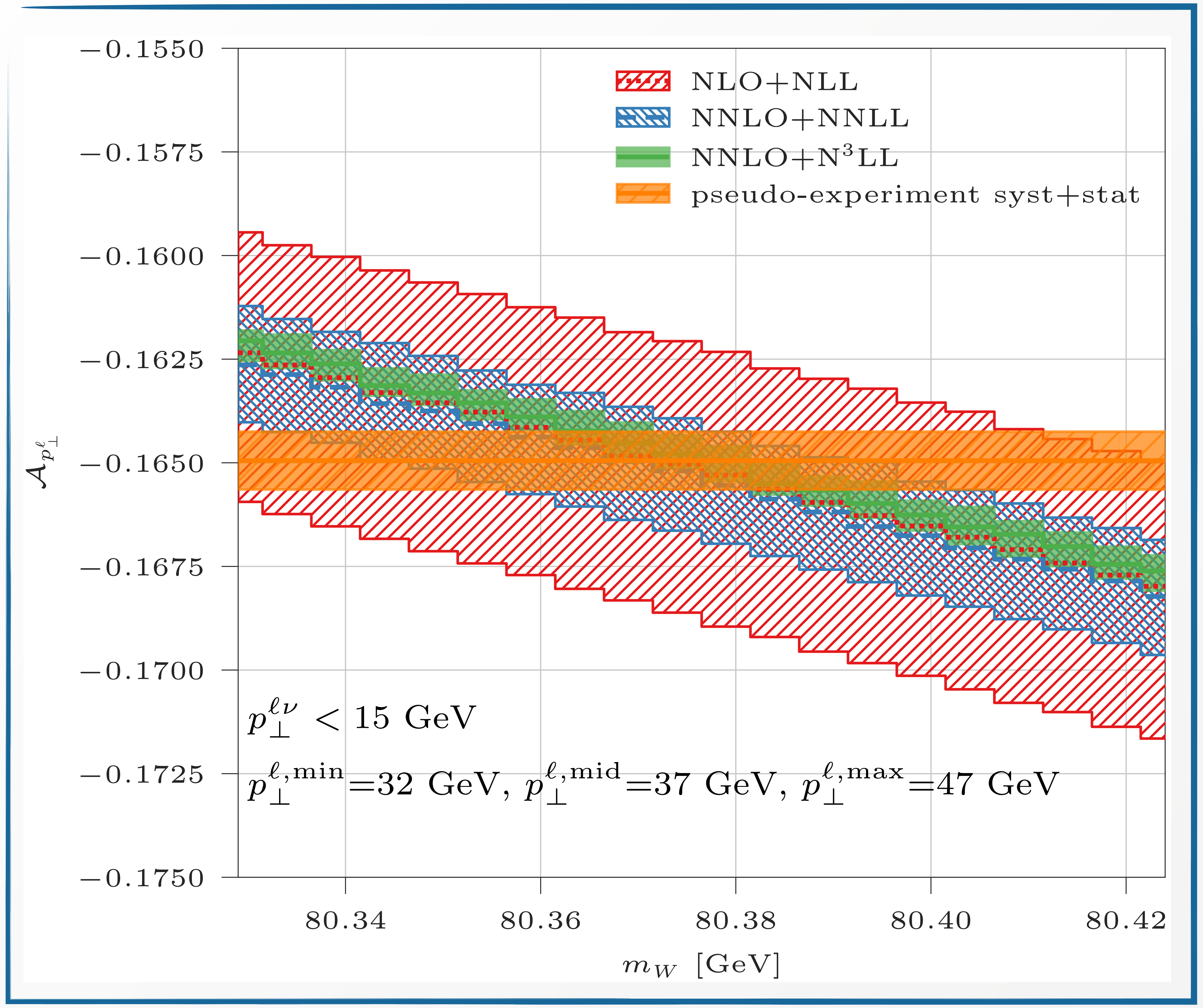
Increasing m_W shifts the peak to the right

Orange bin gets more populated → asymmetry decreases

Analogous observable can be defined for the transverse mass

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ @ the LHC



[LR, P. Torrielli, A. Vicini '23]

Sensitivity to m_W expressed through the slope in each $(p^\ell_{\perp,\mathrm{min}}, p^\ell_{\perp,\mathrm{mid}}, p^\ell_{\perp,\mathrm{max}})$ window

Slope **independent** on the QCD approximation

Experimental result and theoretical predictions can be directly compared by looking at the intersection between the lines

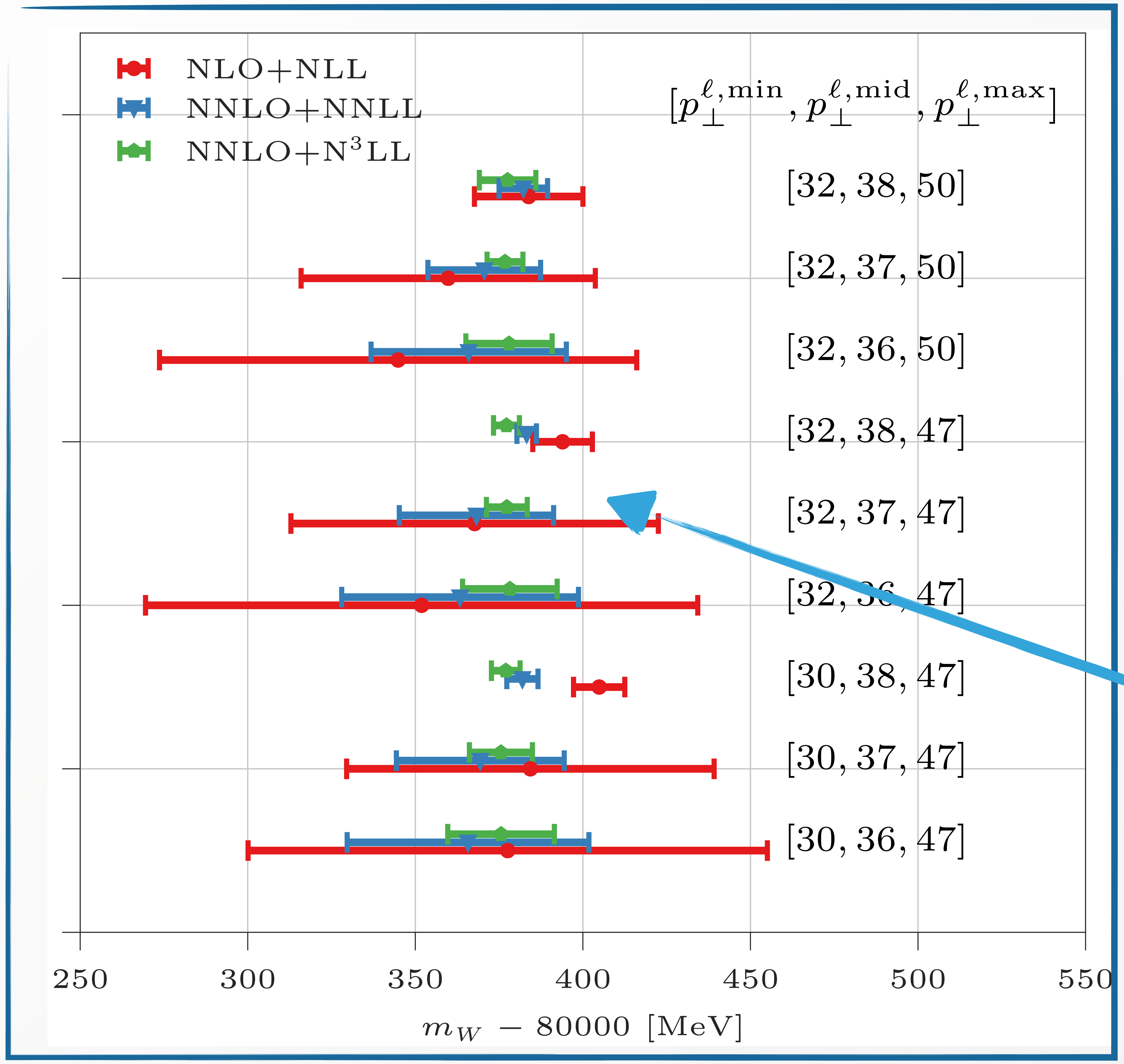
Bin size $\mathcal{O}(10\,\mathrm{GeV})$ has threefold advantage

1. Small statistical error
2. Perturbative stability of the QCD result
3. Unfolding to particle level viable

The main systematics on the two fiducial cross sections is related to the lepton momentum scale resolution

Determination at the LHC at the ±15 MeV level from the experimental side seems possible ($\delta A_{p_\perp^\ell}$=0.0007 with 140 fb^{-1} and 0.001 systematic error on U, L)

The jacobian asymmetry $\mathcal{A}_{p_\perp^\ell}$ and its theoretical uncertainty



[LR, P. Torrielli, A. Vicini '23]

For each interval choice the QCD scale-variation band determines a given m_W interval

N³LL corrections play an important role in reducing uncertainty band

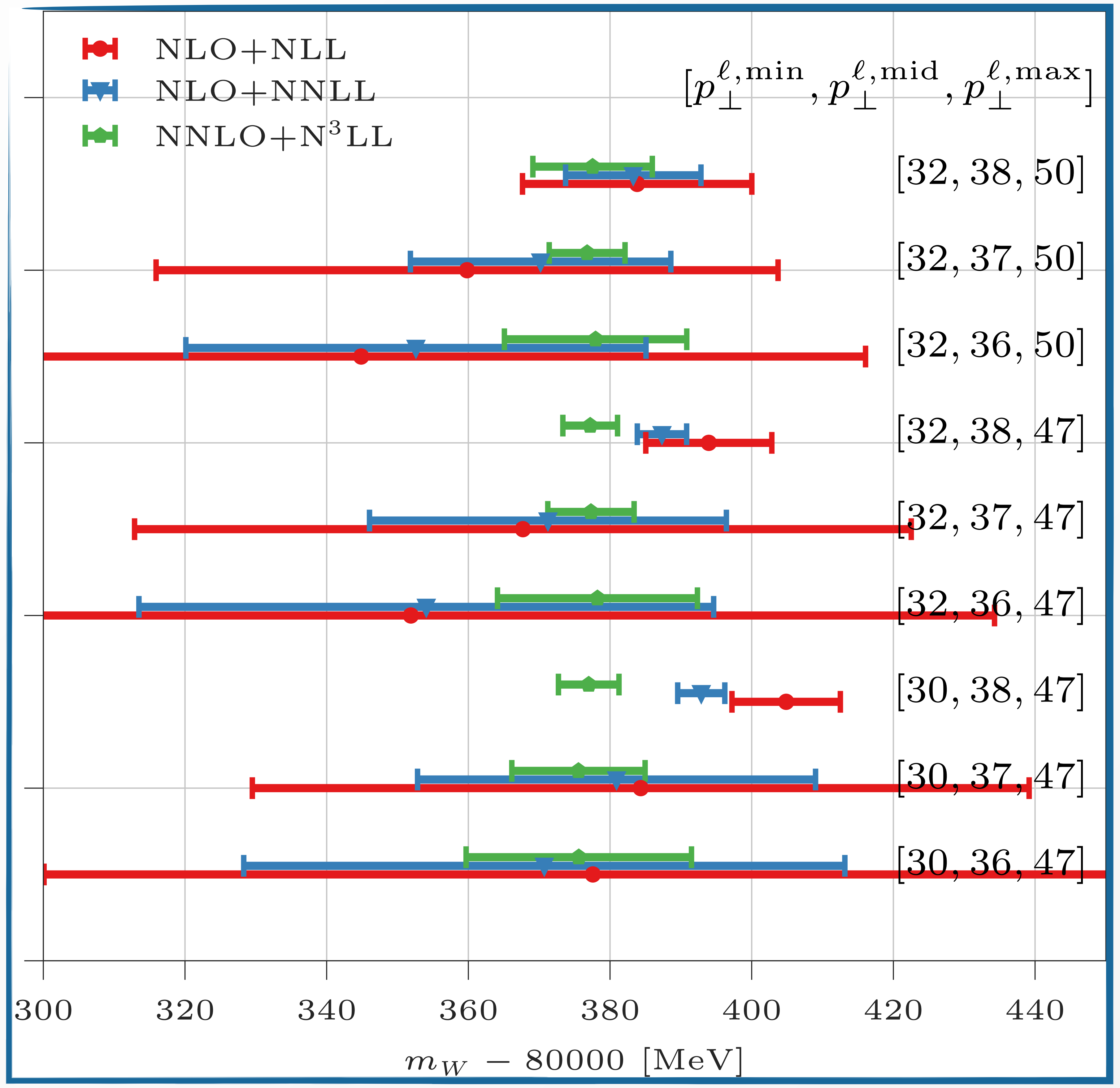
We check the convergence order-by-order. If we observe convergence the size of the m_W interval provides an estimate of the QCD uncertainty

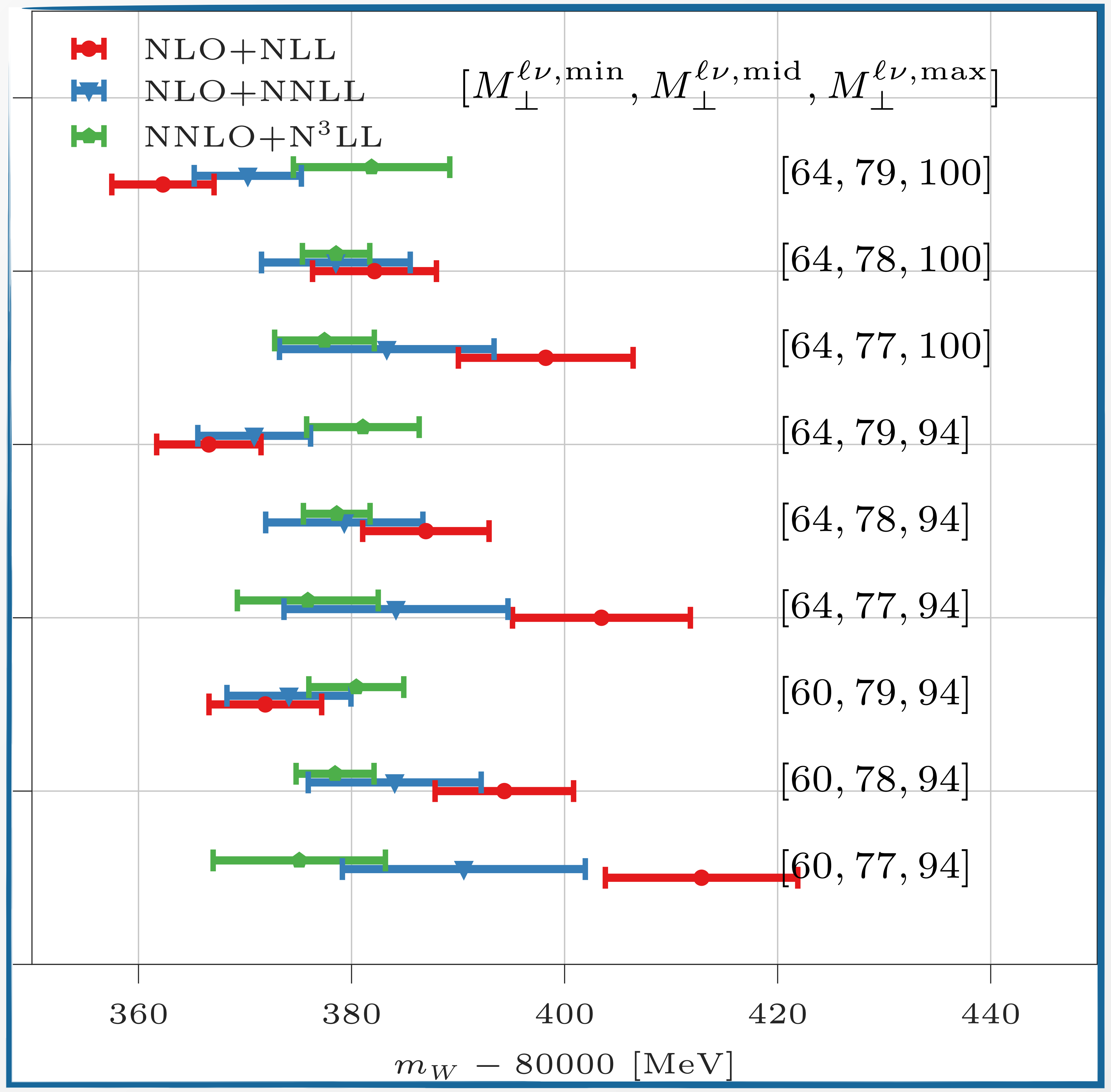
A perturbative QCD uncertainty at the ± 5 MeV level is achievable **using CC DY data alone**

The choice of the midpoint is important to identify two regions with excellent QCD convergence (see regions with $p^\ell_{\perp,\mathrm{mid}} = 38$ GeV)

Yet to be included: EW corrections; NP effects; detector modelling (smearing)

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$ and its theoretical uncertainty





Asymmetry good starting point to investigate size of the QCD uncertainty at a given accuracy (**without tuning**)

QCD uncertainty at lower accuracy considerably larger than state-of-the-art predictions for $p_\perp^\ell$ (more than ±80 MeV for some combinations), shifts between central values smaller in size

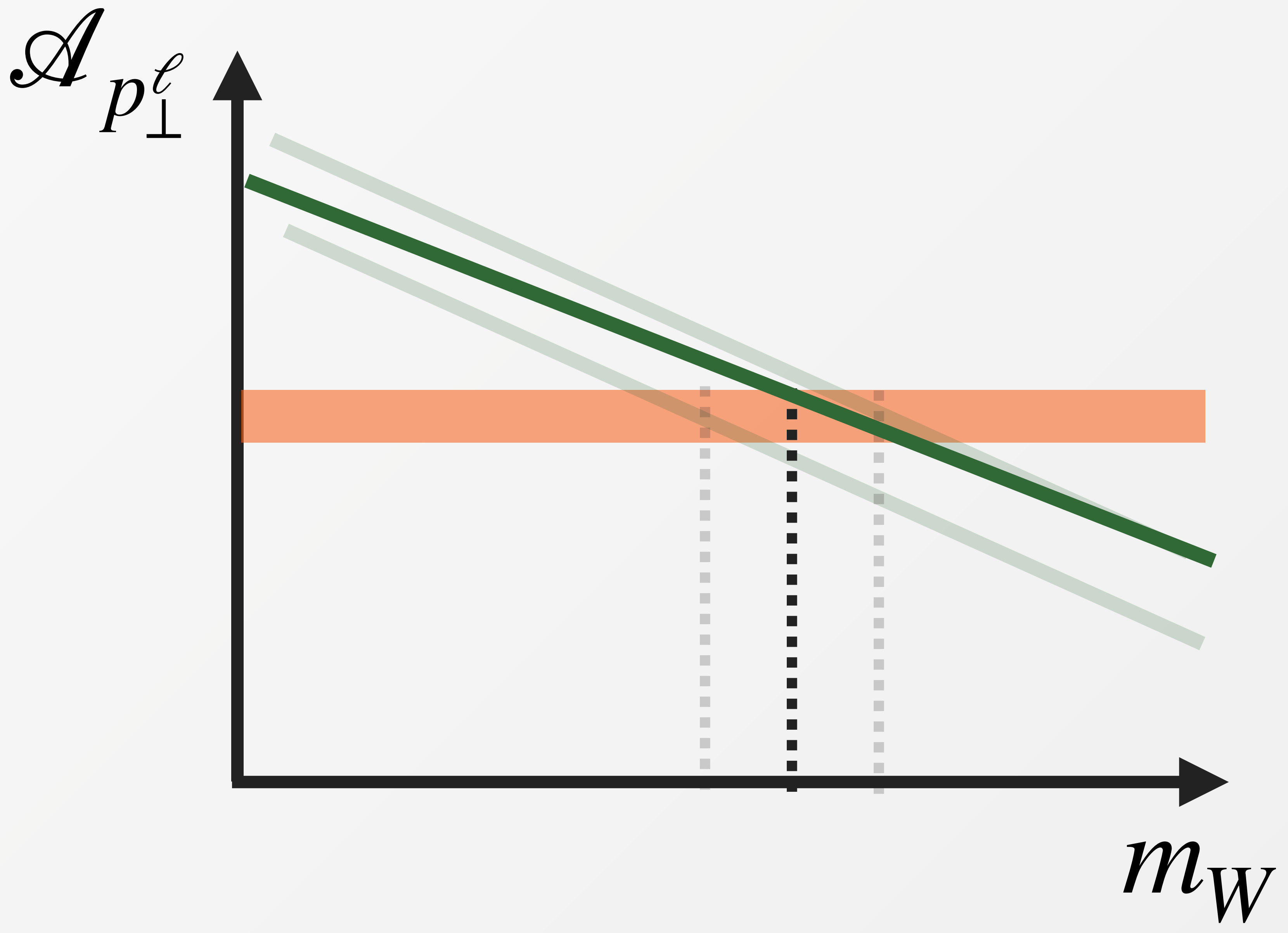
QCD uncertainties smaller for transverse mass (5-10 MeV), but shifts can be larger

See also [Isaacson, Fu, Yuan 2205.02788] [CERN-LPCC-2022-06]

LHC kinematics

The jacobian asymmetry $\mathscr{A}_{p_\perp^\ell}$: additional effects and uncertainties

Excellent convergence properties of the asymmetry in perturbative QCD are a good starting point to discuss additional effects we did not include:



Impact on the central m_W value of

- missing perturbative corrections (QED, QCDxEW)
- non-perturbative effects

Each effect yields a vertical offset on the asymmetry

QED corrections might also change the shape

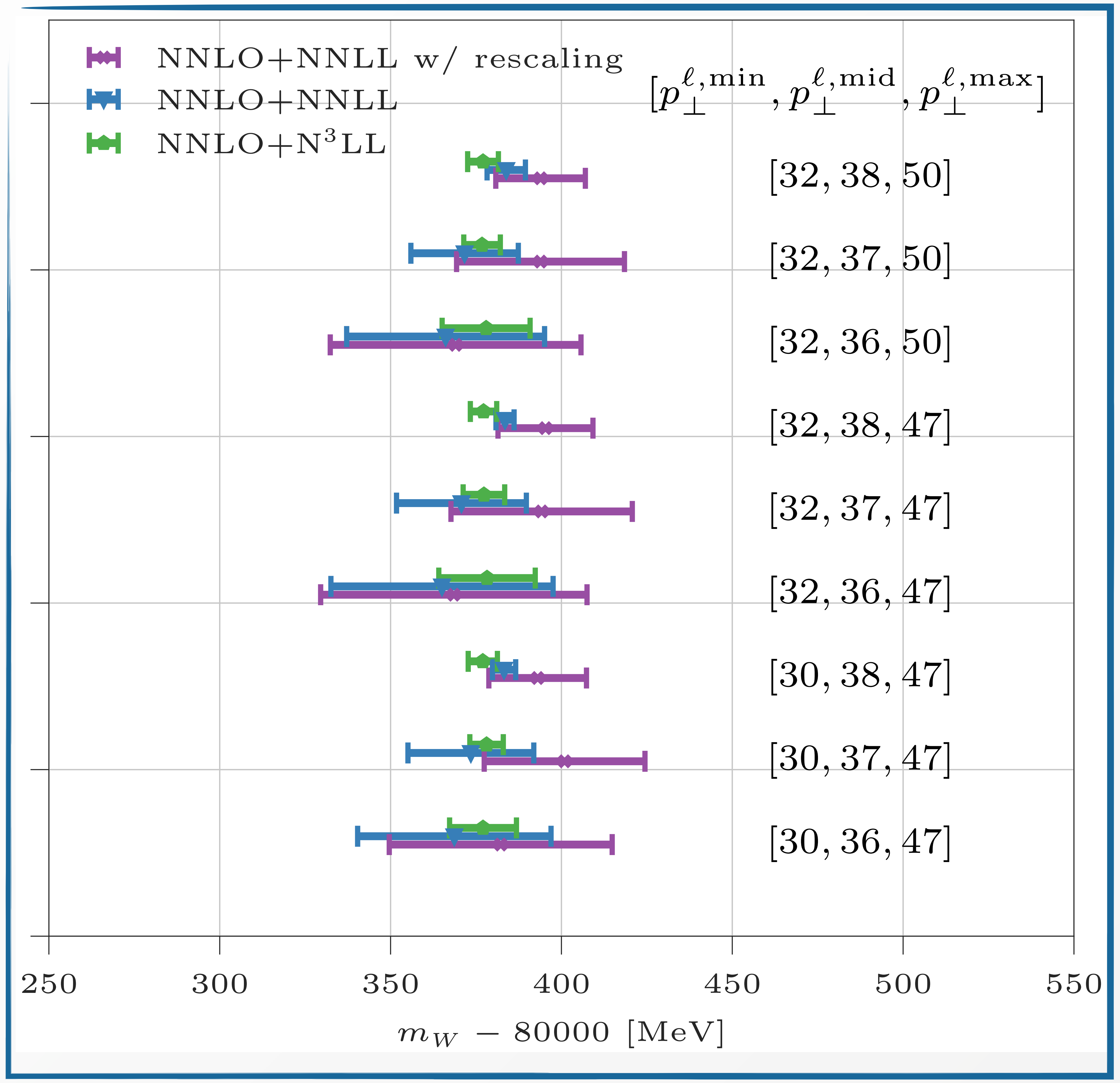
→ shift on m_W

Impact of non-perturbative corrections expected to reduce when using NNLO+N^{3}LL predictions with respect to results with lower accuracy: **interplay of NP QCD model and perturbative accuracy**

Parton distribution functions are an additional source of theoretical uncertainty

Linearity of the dependence on m_W allows an easy propagation of each uncertainty source

Information transfer from NCDY to CCDY



NNLO+NNLL taken as our theory model

NNLO+N^{3}LL with central scales as our MC truth

- pseudo-data generated both for NCDY and CCDY
- reweighting function computed from NNLO+NNLL to the pseudo-data in NC **for each scale**
- same reweighing function applied in CC DY

The $p_\perp^{\ell\nu}$ and the $p_\perp^{\ell}$ distributions get closer to the CCDY pseudodata but **still maintain some shape differences**

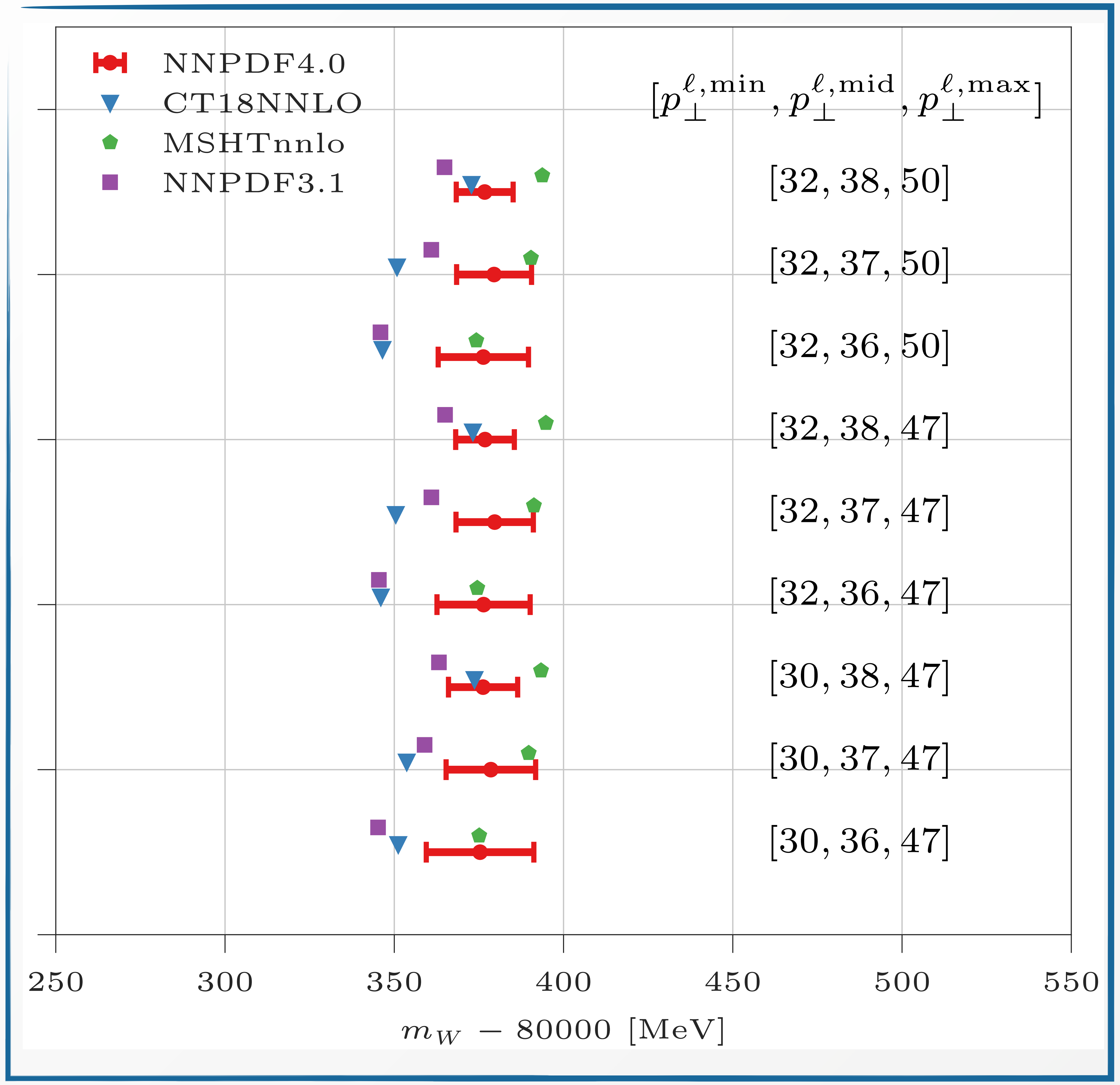
→delicate to assume that $p_\perp^{\ell\nu}$ rescaling applies equally well to $p_\perp^{\ell}$

Perturbative QCD uncertainty on m_W estimated **with or without reweighing is of similar size**

Usage of the **highest available perturbative order is recommended** to minimize the systematics in the transfer from Z to W

Similar conclusions for $m_\perp$ (where uncertainties are smaller)

The lepton transverse momentum distribution in CCDY



PDF uncertainties on m_W evaluated conservatively using the 100 replicas of the NNPDF4.0 set at NLO+NLL

$$\delta m_W = \pm\, 11\,\mathrm{MeV}$$

Spread of the central values of CT18NNLO, MSHTnnlo, NNPDF4.0 of $\sim 30\,\mathrm{MeV}$

(For $M_\perp^{\ell\nu}$: $\delta m_W \simeq \pm\, 5\,\mathrm{MeV}$, spread $\simeq 10\,\mathrm{MeV}$)

Size of the uncertainty expected, as the asymmetry is a single scalar observable particularly sensitive to PDF variations

More information needed to mitigate PDF uncertainty, e.g. profiling using additional bins of the $p_\perp^\ell$ distribution

PDF uncertainty can be reduced to the **few MeV level** thanks to the **strong anti correlated behaviour** of the two tails of $p_\perp^\ell$

Summary and outlook

- **Huge amount of theoretical work** in the last few years in the computation of **higher-order corrections** (**QCD resummation, mixed QCD and QED corrections)**, which now allow for a precise and accurate description of neutral and charged DY production

- Future measurement of m_W should exploit these computation for a **reliable estimate** of the theoretical uncertainties using state-of-the-art predictions

- Shape of the p_T^ℓ distribution and presence of the Jacobian peak motivates the definition of a **scalar observable** which **maximises the sensitivity** on m_W and has several advantages

  - excellent pQCD convergence
  - more systematic study of theoretical uncertainties
  - large linear dependence on $m_W \rightarrow$ sensitivity for a precision measurement

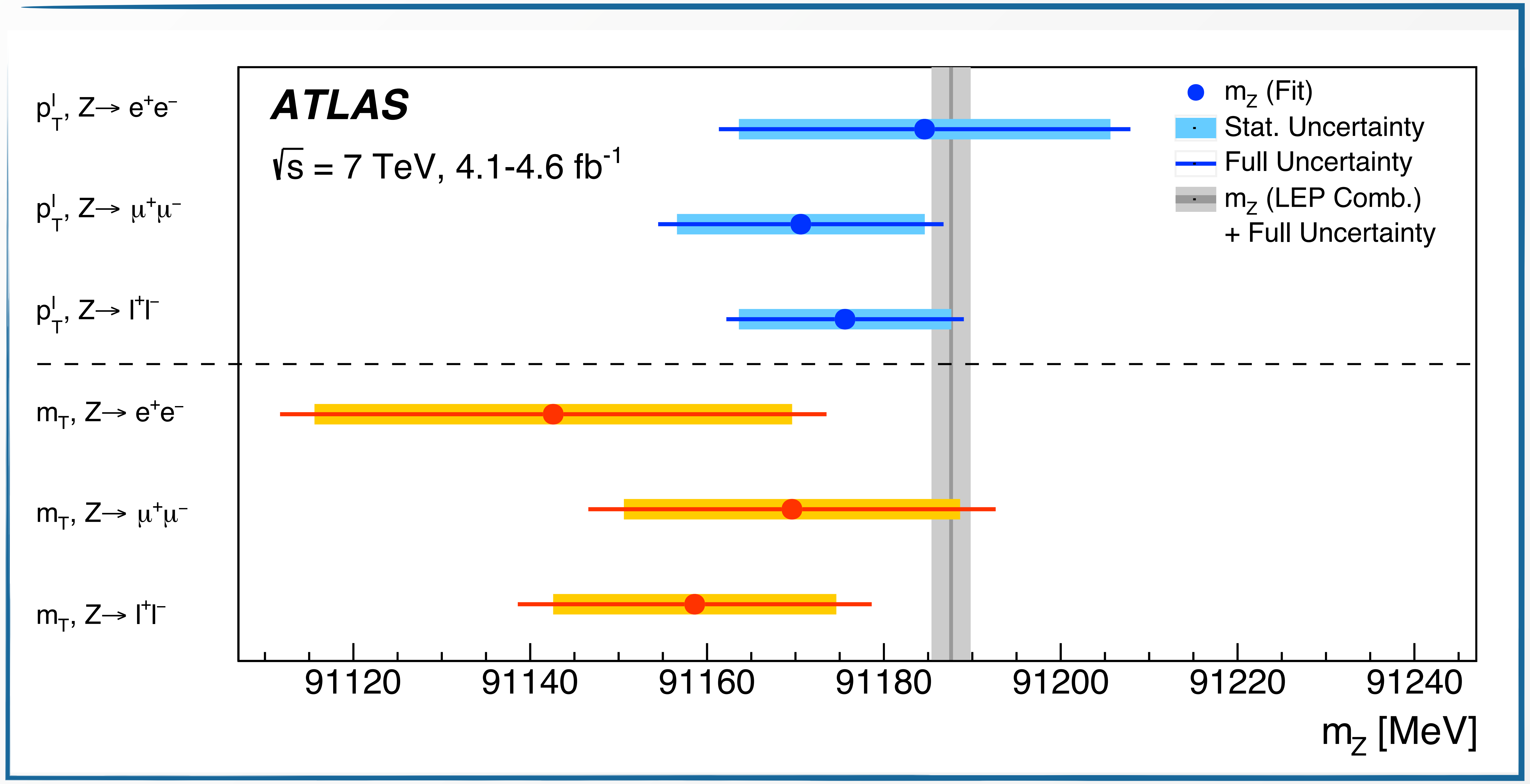
- Determination at the $\pm 15\,\mathrm{MeV}$ level from the experimental side seems possible; perturbative QCD uncertainty at the $\pm 5\,\mathrm{MeV}$ level is achievable using CC DY data alone

- Use of theoretical predictions at lower accuracy and reliance on tuning might **underestimate theory uncertainties** due to **interplay** of non perturbative effects and missing higher orders corrections

- For the future: thorough phenomenological study, including all the available SM radiative corrections

Backup

Controlling systematics



[ATLAS '17]

Robust check of many underlying systematics (although not sensitive to modelling of p_T^Z/p_T^W ratio) can be performed by extracting the Z mass using template fit technique

"Quia vidisti me, credidisti; beati, qui non viderunt et crediderunt"

Johannes 20, 29

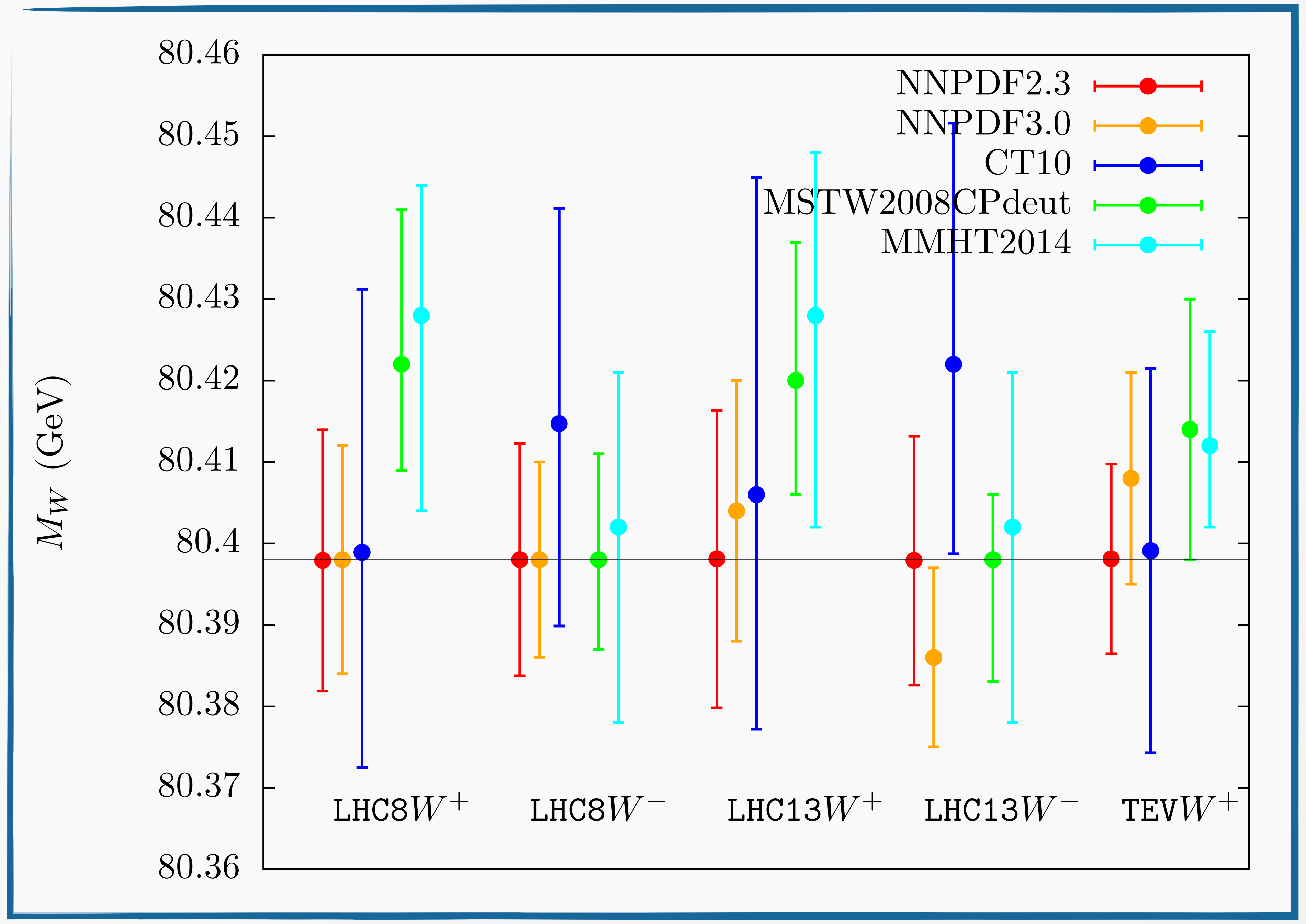
[because thou hast seen me, thou hast believed: blessed *are* they that have not seen, and *yet* have believed]

PDFs and their uncertainties: template fits

PDF-induced uncertainty typically computed by generating templates with a given PDF member *i* for various values of m_W, and subsequently fitting all other members *j* defining a proper figure of merit

$$\chi^2_{i,j} = \sum_{k \in \text{bins}} \frac{(T^j_k - D^i_k)^2}{\sigma^2_k}$$

Once the preferred value for m_W for each member has been determined by minimising the figure of merit, compute PDF-induced uncertainty



[Bozzi, Citelli, Vicini 1501.05587]

PDF uncertainties with this strategy are **relatively large at the LHC**, with a resulting uncertainty larger than 10 MeV and considerably large spreads between different PDF sets

Cfr. ~ 4 MeV quoted by CDF II with NNLO PDFs

4 MeV also claimed by CDF II to be the shift between NNPDF3.1 NNLO and ~**15 years old** NLO CTEQ6.6 PDFs

THE STANDARD MODEL AND MISSING E_T

OR

THE MANY ROADS TO PARADISE

Stephen D. Ellis‡

CERN - Geneva

cuts. We are thus able to sum the contributions of MANY SMALL sources which were absent in previous studies and which can, in the sum, yield a sizeable result. These are the "many roads" of the title. [This concern, that many small numbers can yield a large sum, was colourfully voiced at the meeting by G. Altarelli who described his vision of a mixture of small effects -- the Altarelli cocktail -- leading to the observed signal.] Before proceeding to the

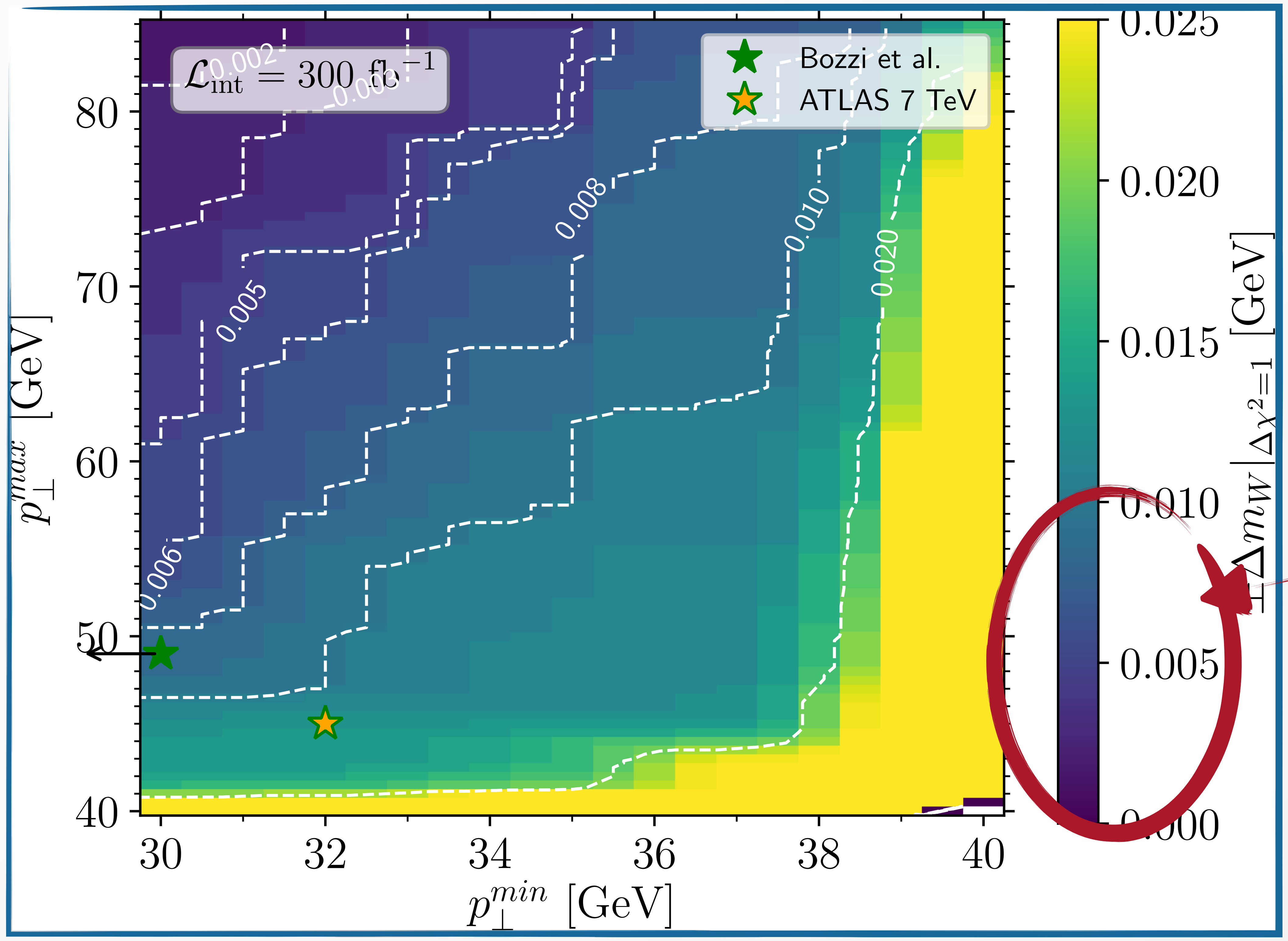
PDFs and their uncertainties: bin-by-bin correlations

Bin-by-bin correlations between PDF replicas can be taken into account inserting the information about PDFs in the covariance matrix

$$(\Sigma_{\text{PDF}})_{ij} = \langle (\mathcal{T} - \langle \mathcal{T} \rangle_{\text{PDF}})_i (\mathcal{T} - \langle \mathcal{T} \rangle_{\text{PDF}})_j \rangle_{\text{PDFs}}$$

Compute χ^2 using full covariance matrix in the definition

$$\chi^2_{i,\text{min}} = \sum_{k,l \in \text{bins}} (T_{0,i} - D)_k (C^{-1})_k (T_{0,i} - D)_l \qquad \forall m_{W,i} \qquad C = \Sigma_{\text{PDF}} + \Sigma_{\text{MC}} + \Sigma_{\text{stat}} + \Sigma_{\text{exp,syst}}$$

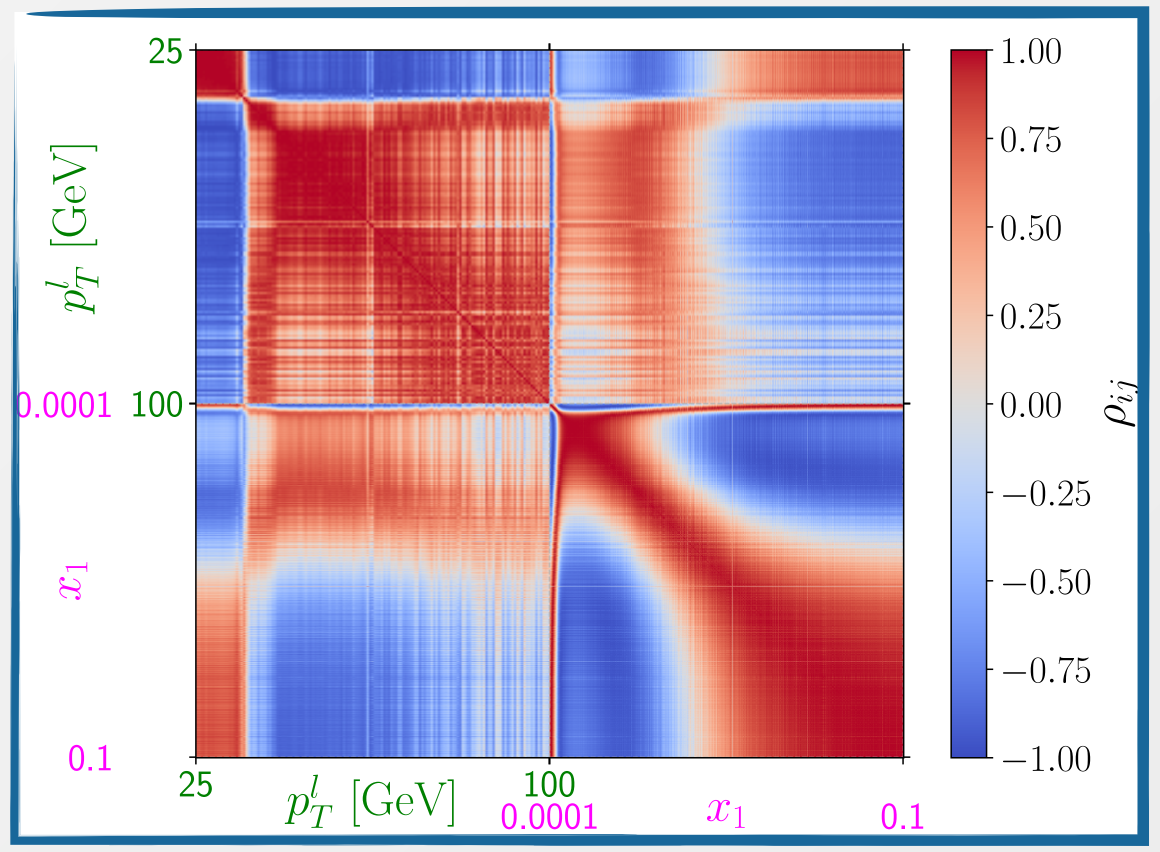


[Bagnaschi, Vicini 1910.04726]

Inserting the information about PDFs in the covariance matrix leads to a profiling action given by the data

PDF uncertainty can be reduced to the **few MeV level** thanks to the **strong anti correlated behaviour** of the two tails of $p_\perp^\ell$

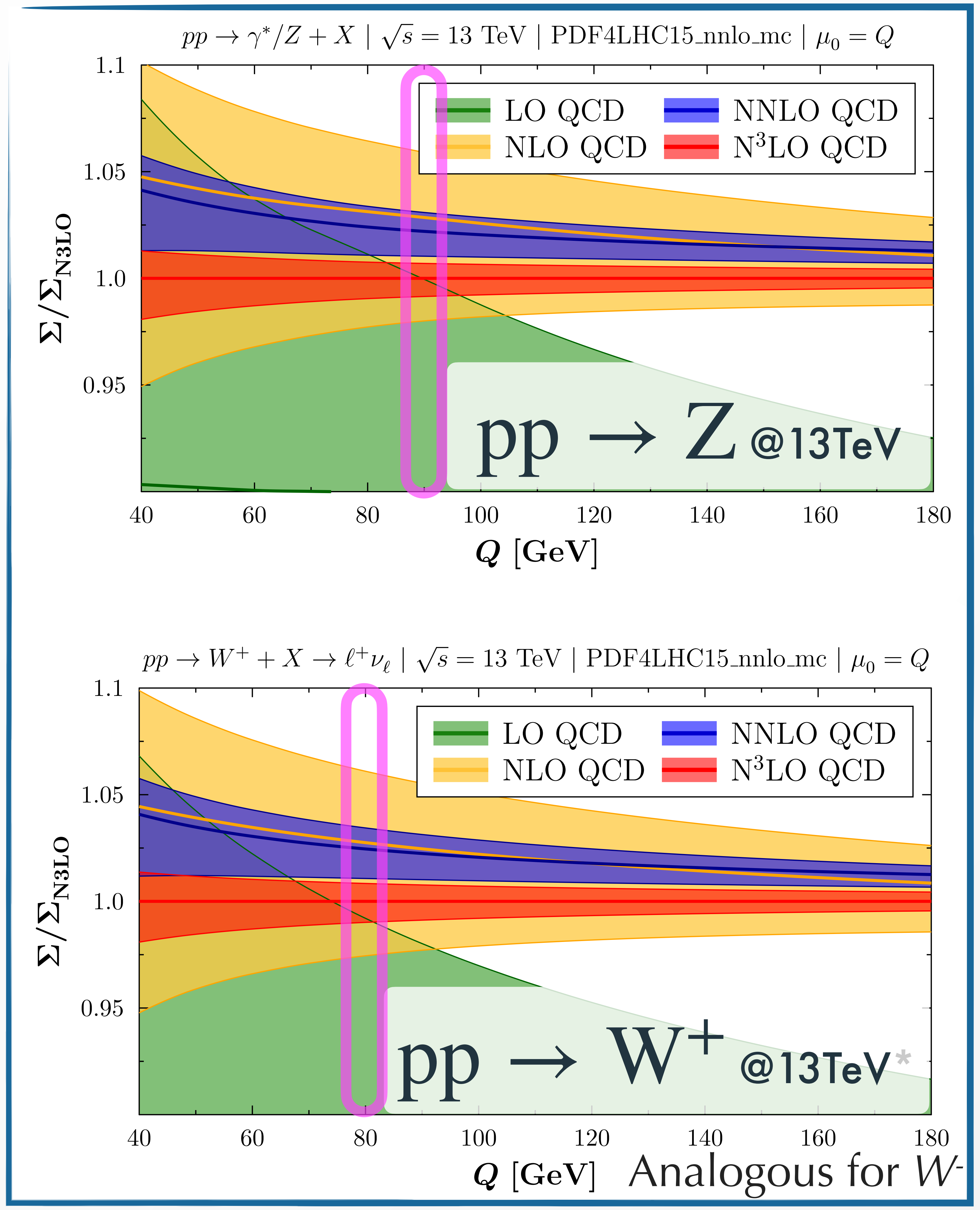
$$\rho_{ij} = \frac{\langle (\mathcal{O}_i - \langle \mathcal{O} \rangle)(\mathcal{O}_j - \langle \mathcal{O} \rangle) \rangle}{\sigma_i \sigma_j}$$



Drell-Yan production at N^3LO_{QCD}

Inclusive Drell-Yan cross section known analytically at N^3LO

[Duhr, Mistlberger '21]

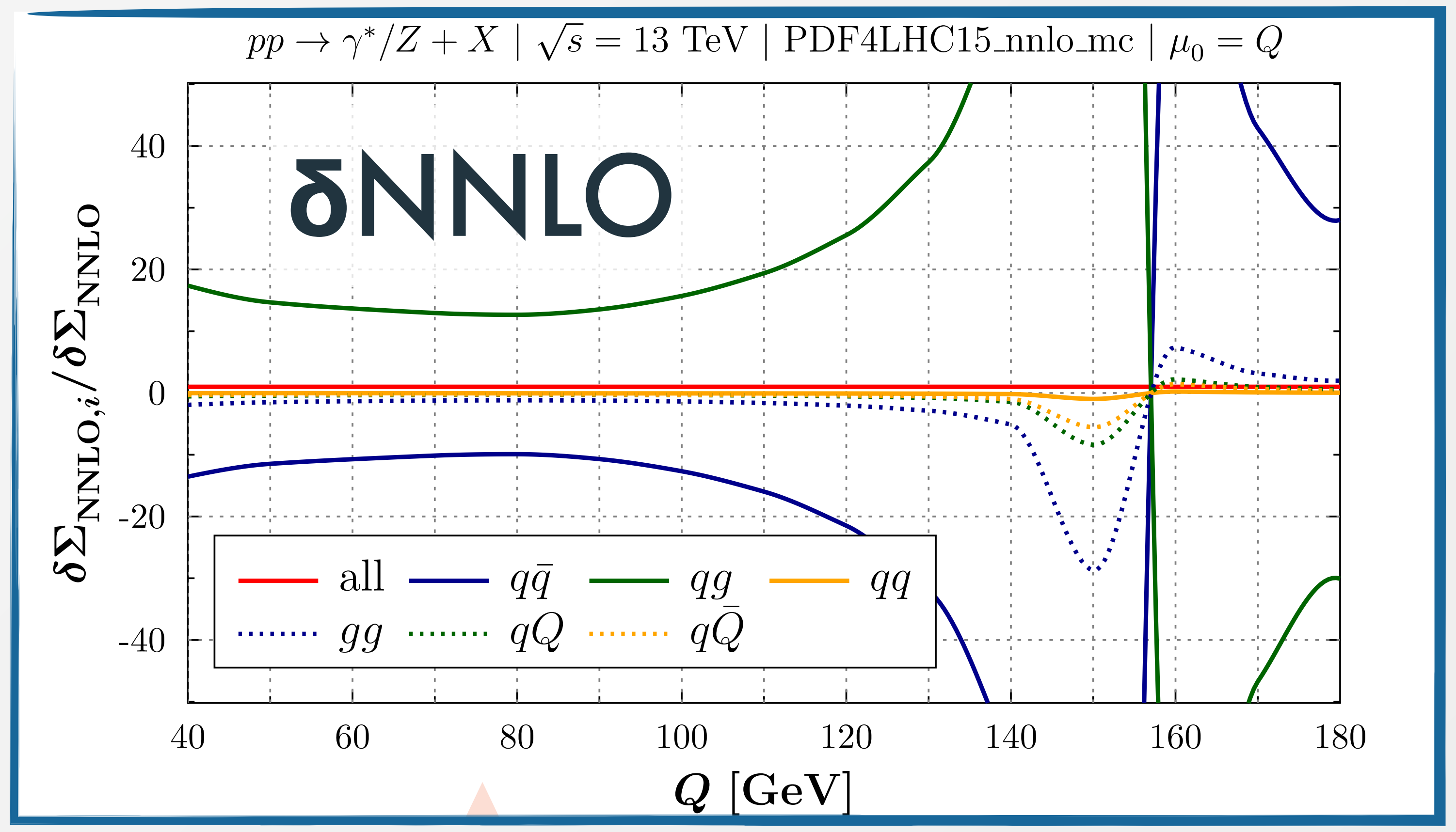


[Baglio, Duhr, Mistlberger, Szafron '22]

Resonance region: non-overlapping bands

Scale uncertainty does not reduce at N^3LO $\Delta^{scale}_{NNLO} \simeq \Delta^{scale}_{N^3LO}$

Large cancellations between different partonic channels



$\Delta(N^3LO\text{-}NNLO) \sim 2\%$ **Necessary for % level target**

Calculation at the fiducial level crucial step for the LHC precision programme

Understanding the Z and W correlations

Thanks to the availability of theoretical prediction at high accuracy, it is possible to assess reliably the behaviour of the perturbative series for crucial observables such as p_T^Z/p_T^W ratio

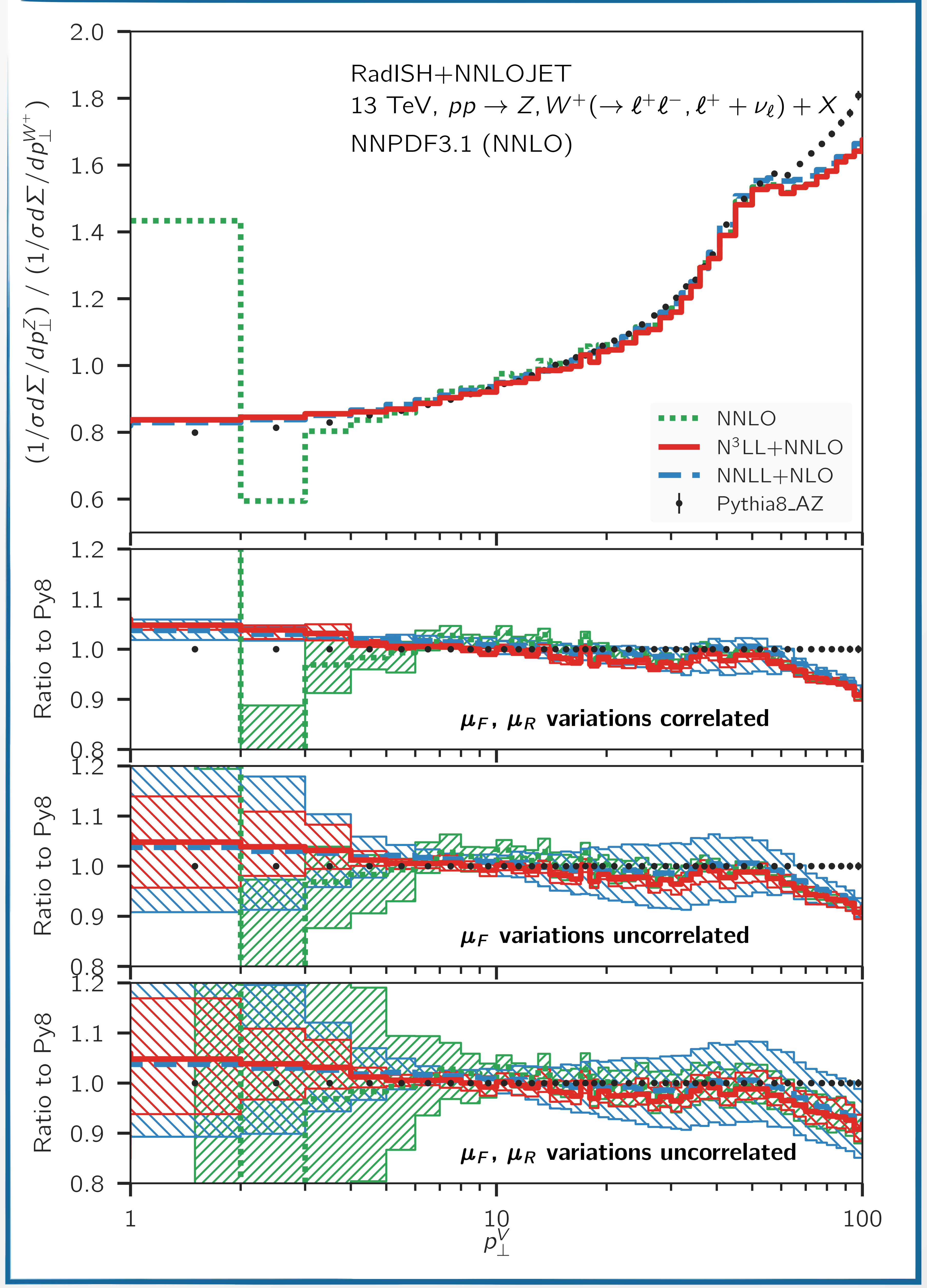
$$\frac{1}{\sigma^W}\frac{d\sigma^W}{p_\perp^W} \sim \frac{1}{\sigma^Z_{\rm data}}\frac{d\sigma^Z_{\rm data}}{p_\perp^Z}\,\frac{\frac{1}{\sigma^W_{\rm theory}}\frac{d\sigma^W_{\rm theory}}{p_\perp^W}}{\frac{1}{\sigma^Z_{\rm theory}}\frac{d\sigma^Z_{\rm theory}}{p_\perp^Z}}$$

Stability of the ratio indicates **high level of correlation** between the two spectra

Comparison with tuned event generator such as PYTHIA* however indicates that full correlation might be too strong an assumption

* *"PYTHIA is not QCD"*

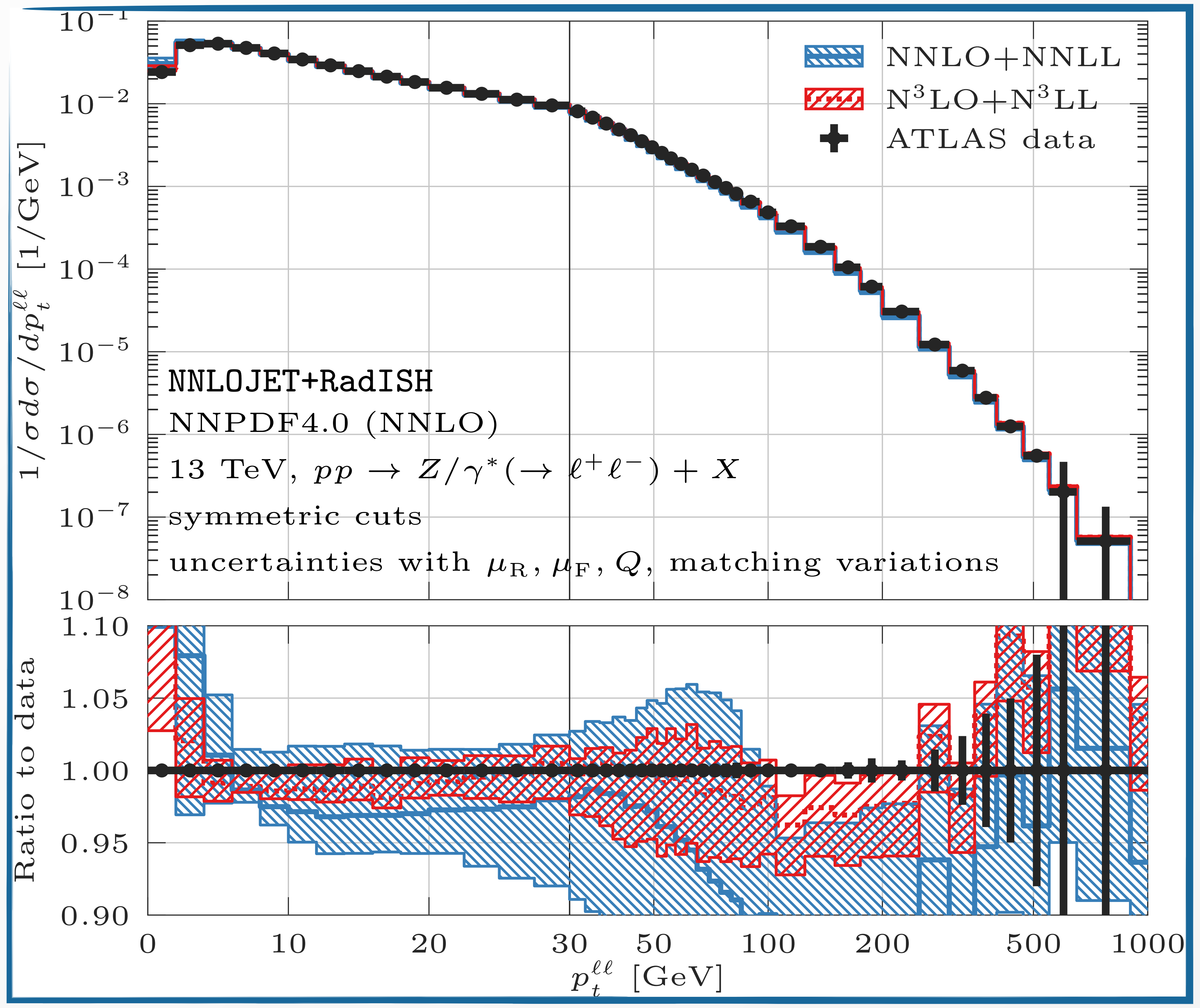
[Kirill Melnikov, QCD@LHC 2016]



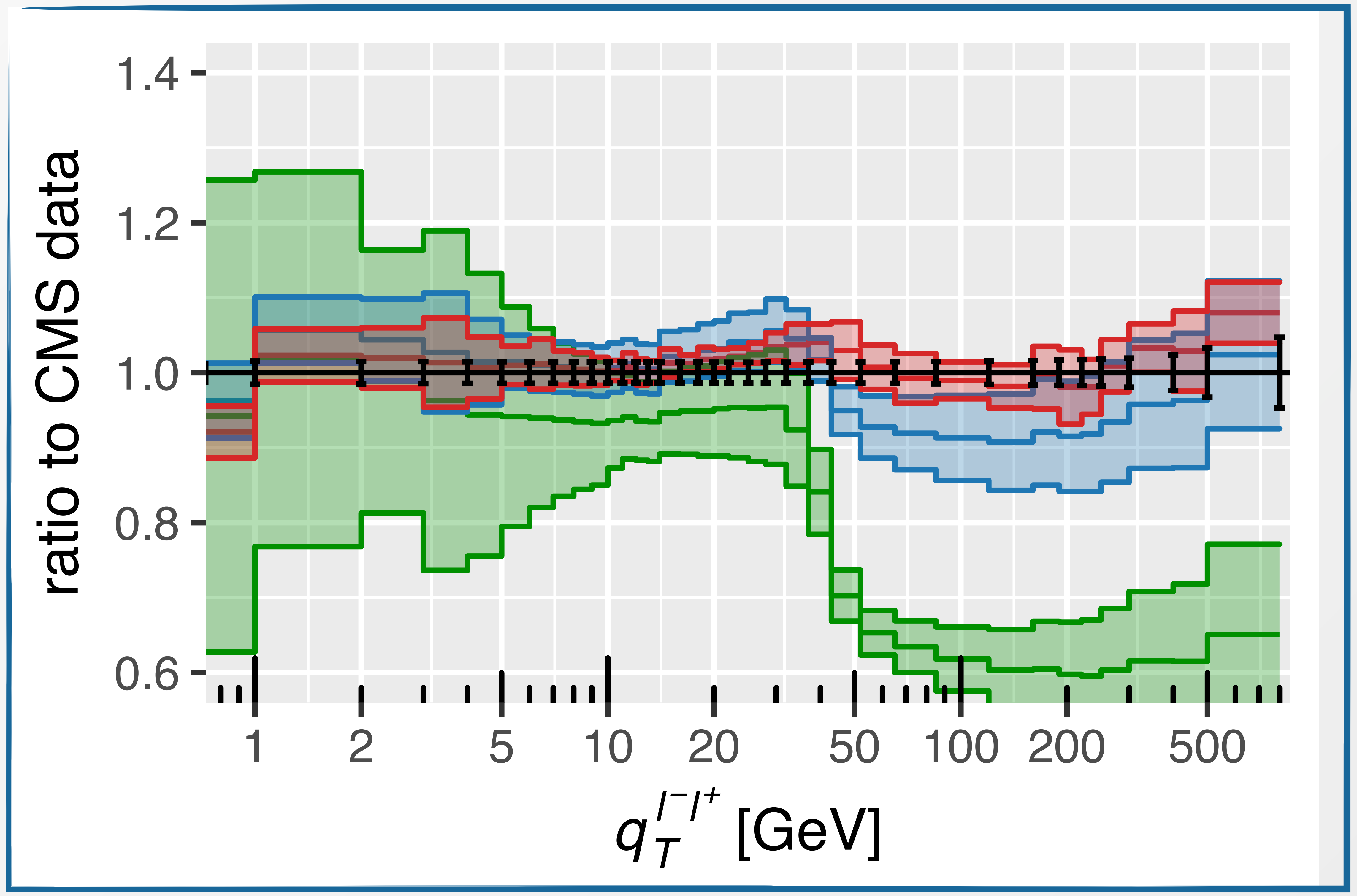
[Bizon, de Ridder, Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Walker, 1905.05171]

Description of experimental data at N^3LO_{QCD}+$N^3LL'_{QCD}$

The theoretical progress made in the the past 5 years has **significantly improved** the description of the experimental data, pinning down the theoretical uncertainties to the **few percent level** in the description of differential spectra



[Gehrmann, Glover, Huss, Chen, Monni, Re, LR, Torrielli, 2203.01565]



[Neumann, Campbell '22]

blue: $NNLL'_{QCD}$+$NNLO_{QCD}$

red: $N^3LL'_{QCD}$+N^3LO_{QCD}

Impact of QED and mixed QCD×QED corrections

Largest shifts induced by **QED FSR**

Subleading EW effects induce **few MeV** shifts

$pp \to W^+$, $\sqrt{s} = 14$ TeV		M_W shifts (MeV)			
Templates accuracy: LO		$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$	
Pseudo–data accuracy		M_T	p_T^ℓ	M_T	p_T^ℓ
1	HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
2	HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
3	HORACE NLO-EW with QED shower	-90±1	-94±1	-177±1	-190±2
4	HORACE FSR-LL + Pairs	-94±1	-102±1	-182±2	-199±1
5	PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2

$p\bar{p} \to W^+$, $\sqrt{s} = 1.96$ TeV			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD$_{PS}$			$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
Pseudodata accuracy		QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
1	NLO-QCD+(QCD+QED)$_{PS}$	PYTHIA	-91±1	-308±4	-37±1	-116±4
2	NLO-QCD+(QCD+QED)$_{PS}$	PHOTOS	-83±1	-282±4	-36±1	-114±3
3	NLO-(QCD+EW)-`two-rad`+(QCD+QED)$_{PS}$	PYTHIA	-86±1	-291±3	-38±1	-115±3
4	NLO-(QCD+EW)-`two-rad`+(QCD+QED)$_{PS}$	PHOTOS	-85±1	-290±4	-37±2	-113±3

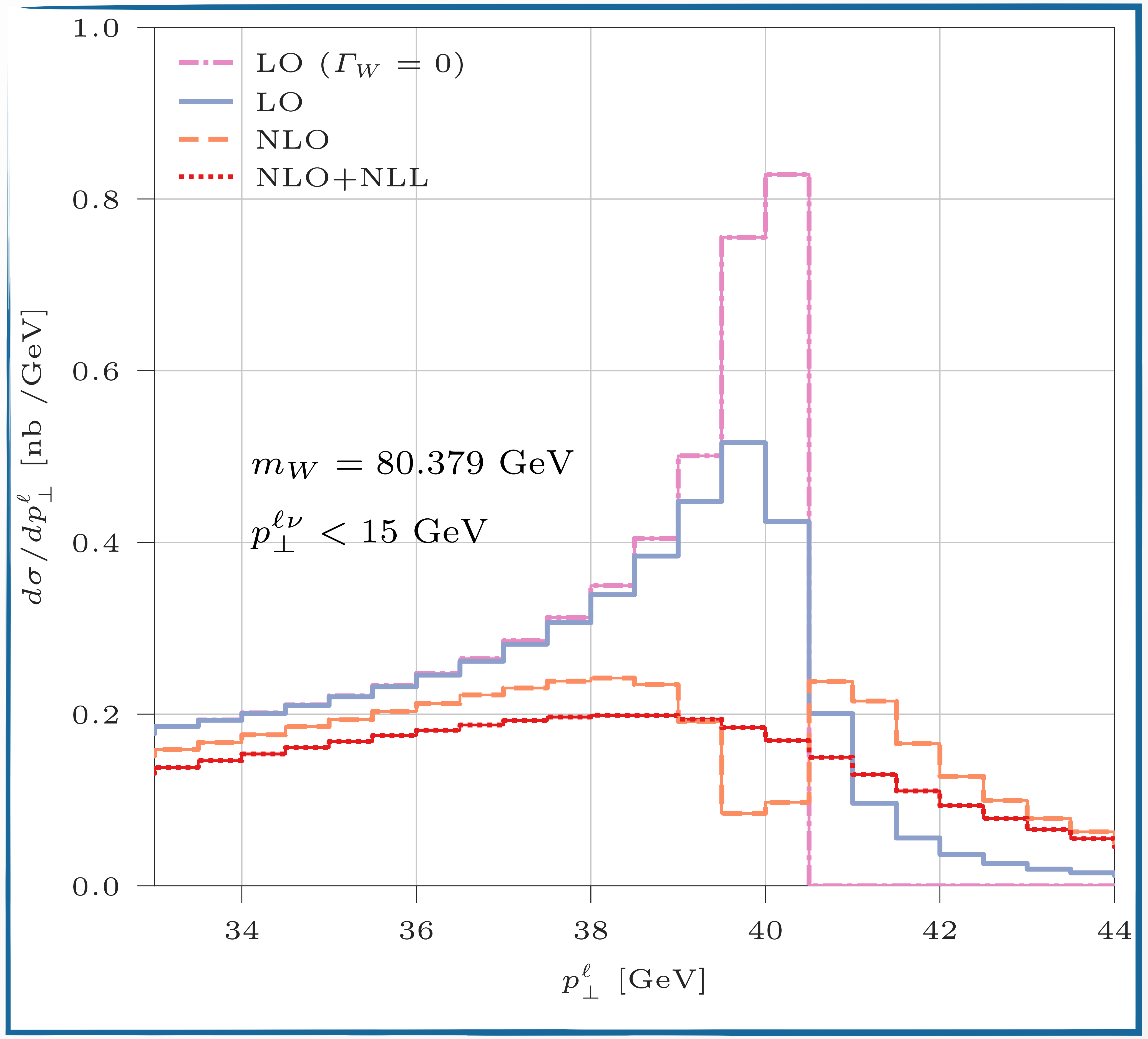
[Carloni Calame, Chiesa, Martinez, Montagna, Nicrosini, Piccinini, Vicini 1612.02841]

Analyses do include the bulk of the QCD×QED corrections

The impact on the m_W shifts of the mixed QCD×QED corrections strongly depends on the underlying QCD model

Note: in this approach non-factorizable contributions are neglected

The lepton transverse momentum distribution in CC DY



The lepton transverse momentum distribution features a **Jacobian peak** at $p_T^\ell \sim m_W/2$

At **LO**, in the narrow width approx., the distribution features a kinematical endpoint at $m_W/2$

Width effects broaden the distribution above $m_W/2$

Beyond LO, sensitivity to **soft radiation** creates unphysical instabilities around $m_W/2$ in fixed-order computations [Catani, Webber '97]

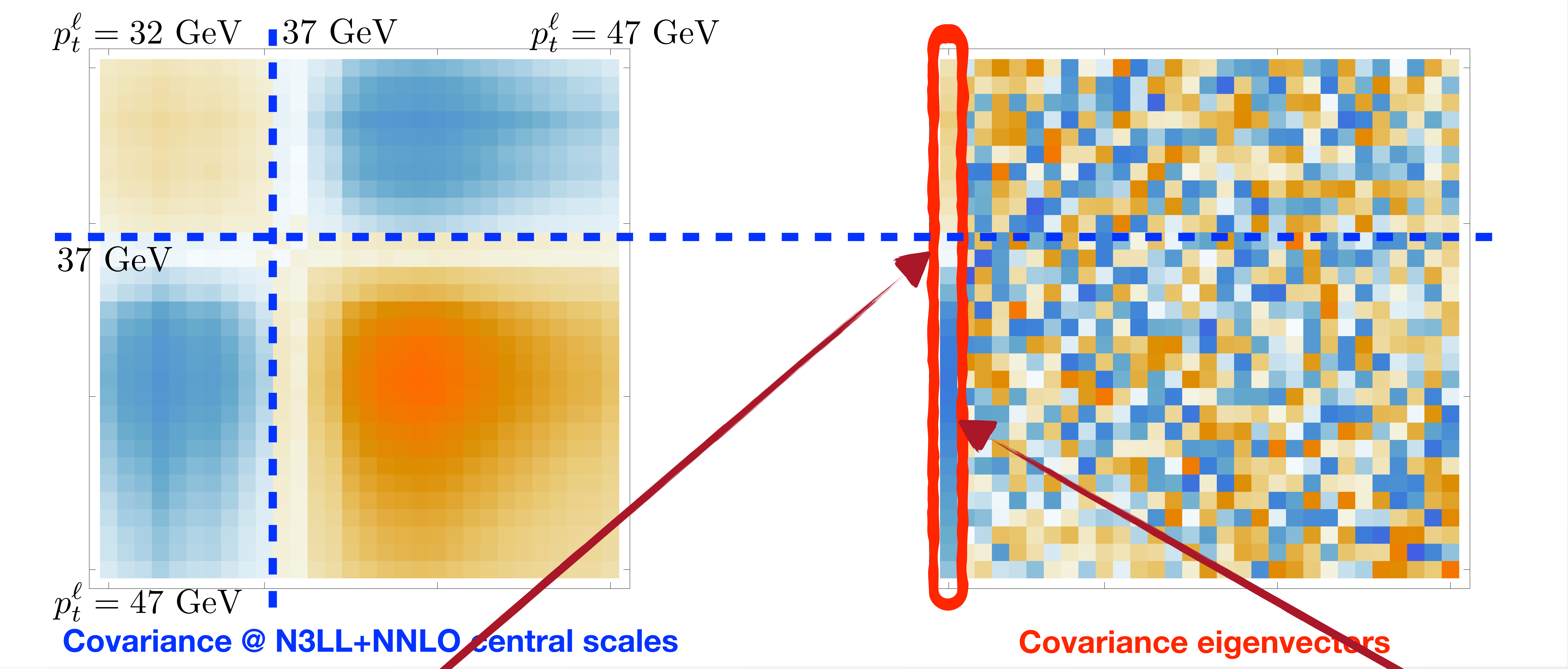
All-order resummation effects cure such instabilities ad provide **physical prediction**

Presence of the endpoint makes the distribution particularly sensitive to m_W

$p_\perp^\ell$ covariance matrix

$$C_{ij}^{(m_W)} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle \qquad \langle x \rangle = \frac{1}{N} \sum_{k=1}^{N} x_k$$

Eigenvalues represent the sensitivity of each eigenvectors (bin combination) to m_W variations.



Coefficients of the dominant eigenvector change sign around $p_\perp^\ell \sim 37$ GeV.

First eigenvalue dominates: bulk of m_W sensitivity captured by a single bin combination.

Progress in mixed QCD×EW corrections

Complete set of corrections to neutral and charged current Drell-Yan production recently obtained by two groups

NNLO QCD-EW corrections to charged-current DY (2-loop contributions in pole approximation).
[Buonocore, Grazzini, Kallweit, Savoini, Tramontano 2102.12539]

exact NNLO QCD-EW corrections to neutral-current DY
[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini, 2106.11953]
[Armadillo, Bonciani, Devoto, Rana, Vicini 2201.01754]
[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, Signorile-Signorile 2203.11237]

Impact of mixed $\mathcal{O}(\alpha_s\alpha)$ corrections estimated to be potentially relevant for $\mathcal{O}(10\,\mathrm{MeV})$ extraction at the LHC
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch 2103.02671]

Matching of such corrections to **QCD and QED all-order resummation** of high relevance for accurate and precise analysis of the p_T^ℓ distribution

Combination of QCD+QED resummation so far available only for Z/W production without decays
[Autieri, Cieri, Ferrera, Sborlini ’18, ’23]