The geometry of the SMEFT

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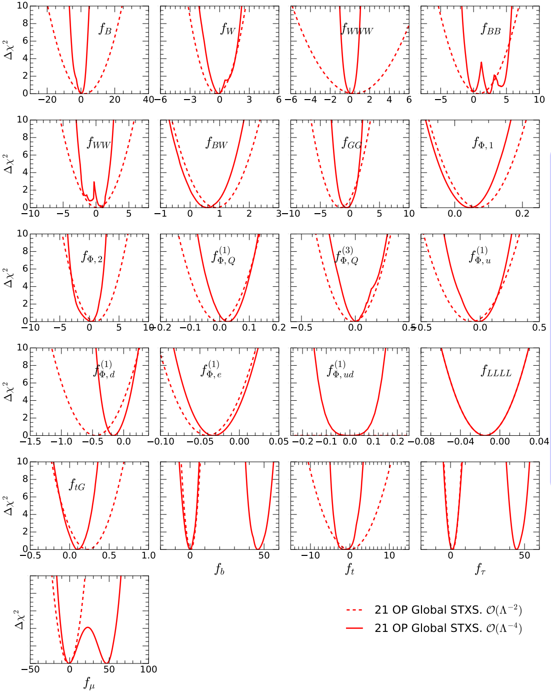

Almeida, Alves, Éboli, Gonzalez-Garcia
arXiv:2108.04828

Uses:

- EWPD
- EW diboson production
- Higgs data

dashed - $\mathcal{O}\left(\frac{1}{\Lambda^2}\right)$

solid - $\sigma\left(\mathcal{O}\frac{1}{\Lambda^4}\right) \times \text{BR}\left(\mathcal{O}\frac{1}{\Lambda^4}\right)$

Motivation for NLO in $1/\Lambda^2$ expansion

On $(D6)^2$ LHC EFT Working Group, arXiv:2201.04974

LHC EFT WG note
Truncation, validity, uncertainties

2. although they only constitute a partial set of $1/\Lambda^4$ corrections, the squares of amplitudes featuring a single dimension-six operator insertion provide a convenient proxy to estimate $1/\Lambda^4$ corrections, as they are well defined and unambiguous. They are indeed gauge invariant and can be translated exactly from one dimension-six operator basis to the other. See Appendix A for more detailed statements.

On the "Inverse problem," C. Zhang, arXiv:2112.11665

SMEFTs living on the edge: determining the UV theories from positivity and extremality

dimension-8 operators encode much more information about the UV than one would naively expect, which can be used to reverse engineer the UV physics from the SMEFT.

The Standard Model

The SM field content is:

$$\phi^I = \{\phi_1, \phi_2, \phi_3, \phi_4\} \quad \rightarrow \quad \text{Real 4 component scalar}$$

$$\left(H = \frac{1}{\sqrt{2}}\begin{bmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{bmatrix}\right)$$

$$W_\mu^A = \{W_\mu^1, W_\mu^2, W_\mu^3, B_\mu\} \quad \rightarrow \quad \text{Real 4 component vector}$$

$$G_\mu^C \quad \rightarrow \quad \text{gluons}$$

$$\psi \quad \rightarrow \quad \text{(chiral) fermions}$$

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The SM field content is:

$$\begin{aligned} \phi^I = \{\phi_1, \phi_2, \phi_3, \phi_4\} &\quad\rightarrow\quad \text{Real 4 component scalar} \\ &\quad\quad \left(H = \frac{1}{\sqrt{2}}\left[\begin{array}{c} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{array}\right]\right) \\ W^A_\mu = \{W^1_\mu, W^2_\mu, W^3_\mu, B_\mu\} &\quad\rightarrow\quad \text{Real 4 component vector} \\ G^C_\mu &\quad\rightarrow\quad \text{gluons} \\ \psi &\quad\rightarrow\quad \text{(chiral) fermions} \end{aligned}$$

The SM Lagrangian is then:

$$\begin{aligned} \mathcal{L} \quad = \quad & \delta_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J - \frac{1}{4}\delta_{AB}W^A_{\mu\nu}W^{B,\mu\nu} - \frac{1}{4}G^C_{\mu\nu}G^{C,\mu\nu} \\ & + i\bar{\Psi}\not{D}\Psi + i\bar{\psi}\not{D}\psi + Y_\psi \overset{(\sim)}{H}\bar{\Psi}\psi \\ & + \underbrace{\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}}_{\text{BFM gauge invar.}} \end{aligned}$$

(Denner et al. hep-ph/9406204)
(Helset et al. arXiv:1803.08001)

Ward Identities in the EW Standard Model

The master Ward Identity for the effective action (Γ):

$$
\begin{aligned}
\frac{\delta\Gamma}{\delta\alpha^B} = 0 \quad &= \quad \frac{\delta W^\mu_A}{\delta\alpha^B}\frac{\delta\Gamma}{\delta W^\mu_A} + \frac{\delta\phi^J}{\delta\alpha^B}\frac{\delta\Gamma}{\delta\phi^I} + \text{fermions} + \text{glu} \\
&= \quad \left(\partial^\mu\delta^A_B - \epsilon^A_{BC}g_2 W^{C,\mu}\right)\frac{\delta\Gamma}{\delta W^\mu_A} - \underbrace{\frac{\tilde{\gamma}^I_{B,J}}{2}}_{\text{generators}}\phi^J\frac{\delta\Gamma}{\delta\phi^I} + \cdots
\end{aligned}
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The Ward IDs we know and love $\rightarrow$ take variations w/r to fields and take the expectation:

$$\begin{aligned}\frac{\delta^2\Gamma}{\delta W^{A,\nu}\delta\alpha^B} \quad &\rightarrow \quad \partial^\mu\left(\frac{\delta^2\Gamma}{\delta W^{A,\nu}\delta W^{B,\mu}}\right)=\frac{\tilde{\gamma}^I_{B,J}}{2}\underbrace{\left\langle\phi^J\right\rangle}_{=v\delta^J_4}\frac{\delta^2\Gamma}{\delta W^{A,\nu}\delta\phi^I}\\ \frac{\delta^2\Gamma}{\delta\phi^K\delta\alpha^B} \quad &\rightarrow \quad \partial^\mu\left(\frac{\delta^2\Gamma}{\delta\phi^K\delta W^{B,\mu}}\right)=\frac{\tilde{\gamma}^I_{B,J}}{2}\left(v\delta^J_4\frac{\delta^2\Gamma}{\delta\phi^K\delta\phi^I}+\delta^I_K\frac{\delta\Gamma}{\delta\phi^I}\right)\end{aligned}$$

(TC, Helset, Trott, arXiv:1909.08470)

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Rotating to the mass eigenstates:

$$W^{A,\nu} \quad = \quad U^A_B\mathcal{A}^{B,\nu} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ i/\sqrt{2} & -i/\sqrt{2} & 0 & 0 \\ 0 & 0 & c_W & s_W \\ 0 & 0 & -s_W & c_W \end{bmatrix}\begin{bmatrix} W^{+,\nu} \\ W^{-,\nu} \\ Z^\nu \\ A^\nu \end{bmatrix}$$

$$\phi^J \quad = \quad V^J_K\Phi^K = \begin{bmatrix} -i/\sqrt{2} & i/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \Phi^+ \\ \Phi^- \\ \chi \\ h \end{bmatrix}$$

(TC, Helset, Trott, arXiv:1909.08470)

Ward Identities in the EW Standard Model

Rotating to the mass eigenstates, after some simplification:

$$\frac{\delta^2\Gamma}{\delta W^{A,\nu}\delta\alpha^B} \quad \rightarrow \quad \partial^\mu\left(\frac{\delta^2\Gamma}{\delta\mathcal{A}^{X,\nu}\delta\mathcal{A}^{Y,\mu}}\right) = \frac{\delta^2\Gamma}{\delta\mathcal{A}^{Y,\nu}\delta\Phi^K} v(V^{-1})^K_I \frac{1}{2}\tilde{\gamma}^I_{W,4} U^W_X$$

$$\frac{\delta^2\Gamma}{\delta\phi^K\delta\alpha^B} \quad \rightarrow \quad \partial^\mu\left(\frac{\delta^2\Gamma}{\delta\mathcal{A}^{X,\mu}\delta\Phi^O}\right) = \frac{\delta^2\Gamma}{\delta\Phi^K\delta\Phi^O} v(V^{-1})^K_I \frac{1}{2}\tilde{\gamma}^I_{W,4} U^W_X + (V^{-1})^K_I \frac{1}{2}\tilde{\gamma}^I_{W,L} U^W_X V^L_O$$

These are ugly. But, choose X, Y/O and sum over other variables:

Ward Identities in the EW Standard Model

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These are ugly. But, choose X, Y/O and sum over other variables:

$$\partial^\mu\left(\frac{\delta^2\Gamma}{\delta A^\mu\delta A^\nu}\right) = 0 \qquad \text{(transversality of photon)}$$

$$\partial^\mu\left(\frac{\delta^2\Gamma}{\delta W^{+,\mu}\delta W^{-,\nu}}\right) = -im_W\frac{\delta^2\Gamma}{\delta\Phi^+\delta W^{-,\nu}} \qquad \left(m_W = \frac{g_2^2 v^2}{4}\right)$$

$$\partial^\mu\left(\frac{\delta^2\Gamma}{\delta W^{+\mu}\delta\Phi^-}\right) = -im_W\frac{\delta^2\Gamma}{\delta\Phi^+\delta\Phi^-} + \frac{ig_2}{2}\frac{\delta\Gamma}{\delta h}$$

(TC, Helset, Trott, arXiv:1909.08470)

Ward IDs in the SMEFT, motivating geoSMEFT

The SMEFT Lagrangian is then:

$$
\begin{aligned}
\mathcal{L} &= h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J - g_{AB}W^A_{\mu\nu}W^{B,\mu\nu} - \frac{1}{4}G^C_{\mu\nu}G^{C,\mu\nu} \\
&\quad + i\bar{\Psi}\not{D}\Psi + i\bar{\psi}\not{D}\psi + Y_\psi \overset{(\sim)}{H}\bar{\Psi}\psi \\
&\quad + \sum_{i,n}\frac{1}{\Lambda^n}Q_i + \underbrace{\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}}_{\text{BFM gauge invar.}} \\
h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J &= (D_\mu\phi)^I(D^\mu\phi)^I + c_{H\Box}(H^\dagger H)\Box(H^\dagger H) + \cdots \\
g_{AB}W^A_{\mu\nu}W^{B,\mu\nu} &= -\frac{1}{4}W^A_{\mu\nu}W^{A,\mu\nu} + c_{HB}(H^\dagger H)B^{\mu\nu}B_{\mu\nu} + \cdots
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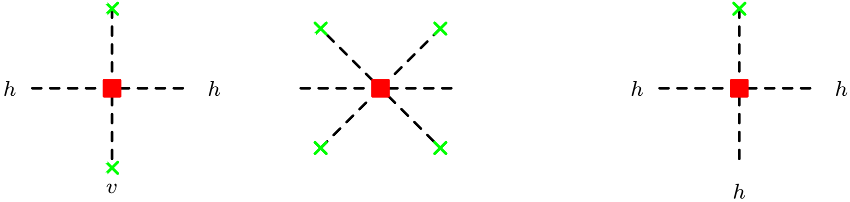
Ex: h_{IJ} generates 2+ point functions:



$\langle h_{IJ}\rangle$ $\langle h_{IJ}\rangle$ $\left\langle \frac{\delta h_{IJ}}{\delta h}\right\rangle$

Ward IDs in the SMEFT, motivating geoSMEFT

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\end{aligned}$$

But the SMEFT has the same gauge symmetries of the SM, so:

$$\begin{aligned}
\partial^\mu\left(\frac{\delta^2\Gamma}{\delta A^\mu\delta A^\nu}\right) &= 0 \qquad \text{(transversality of photon)} \\
\partial^\mu\left(\frac{\delta^2\Gamma}{\delta W^{+,\mu}\delta W^{-,\nu}}\right) &= -i\bar{m}_W\frac{\delta^2\Gamma}{\delta\Phi^+\delta W^{-,\nu}} \qquad \left(\bar{m}_W = \frac{\bar{g}_2 v}{2}\langle h_{11}\rangle\right) \\
\partial^\mu\left(\frac{\delta^2\Gamma}{\delta W^{+\mu}\delta\Phi^-}\right) &= -i\bar{m}_W\frac{\delta^2\Gamma}{\delta\Phi^+\delta\Phi^-} + \frac{i\bar{g}_2}{2}\underbrace{\textit{(something?)}}_{=\sqrt{h}_{44}\sqrt{h}^{11}}\frac{\delta\Gamma}{\delta h} \qquad \left(\bar{g}_2 = g_2\sqrt{g}^{11}\right)
\end{aligned}$$

(TC, Helset, Trott, arXiv:1909.08470)
(Confirmed at 1-loop to D6, TC, Trott, arXiv:2010.02819)

The geoSMEFT, a first glance

So these h_{IJ} and g_{AB} are of major importance in the SMEFT.

Let's define them more formally,

$$
\begin{aligned}
\mathcal{L}_{\rm SMEFT} &= h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J + g_{AB}W^A_{\mu\nu}W^{B,\mu\nu} + \cdots \\
h_{IJ} &\equiv \frac{g^{\mu\nu}}{d}\frac{\delta^2\mathcal{L}_{\rm SMEFT}}{\delta(D_\mu\phi)^I\,\delta(D_\nu\phi^J)}\bigg|_{\{W,\psi,(D_\alpha\phi)^K\}\to 0} \qquad \Big(\text{i.e. } h = h(\phi)\Big) \\
g_{AB} &\equiv \frac{-2g^{\mu\nu}g^{\sigma\rho}}{d^2}\frac{\delta^2\mathcal{L}_{\rm SMEFT}}{\delta W^A_{\mu\sigma}\,\delta W^B_{\nu\rho}}\bigg|_{\{W,\psi,(D_\alpha\phi)^K\}\to 0} \qquad \Big(\text{i.e. } g = g(\phi)\Big)
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These are 4×4 matrices, defined to all orders in $1/\Lambda$

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\end{aligned}$$

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We can define their inverses, expectation values, and square roots:

$$\underbrace{g_{AB}\,g^{BC} = \delta^C_A \qquad h_{IJ}\,h^{JK} = \delta^K_I}_{\text{like metrics in GR}\to\text{curved field space}} \qquad \langle g\rangle_{AB} \quad \langle h\rangle_{IJ} \quad \sqrt{g}_{AB} = \sqrt{\langle g\rangle}_{AB} \quad \sqrt{h}_{IJ} = \sqrt{\langle h\rangle}_{IJ}$$

These "field-space connections" are tied to the geometry of the curved field-space.

⇒ the geometric SMEFT (geoSMEFT)

⇒ not necessarily new ideas (Alonso, Jenkins, Manohar, 1605.03602)

Some all-orders results in the SMEFT

These geometric quantities h_{IJ} and g_{AB} $\rightarrow$ derive all-orders in $1/\Lambda$ results:

1. Canonically normalized couplings to fermions:

$$
\begin{aligned}
D_\mu\psi &= \left[\partial_\mu + i\bar{g}_3 G^A_\mu + i\frac{\bar{g}_2}{\sqrt{2}}\left(W^+_\mu T^+ + W^- T^-\right) + i\bar{g}_Z\left(T_3 - \bar{s}^2_Z Q_\psi\right) Z_\mu + iQ_\psi \bar{e} A_\mu\right]\psi \\
\bar{g}_2 &= g_2\sqrt{g}^{11} \\
\bar{g}_Z &= \frac{g_2}{\bar{c}^2_Z}\left(\bar{c}\sqrt{g}^{33} - \bar{s}\sqrt{g}^{34}\right) \\
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$$\bar{g}_2 = g_2\sqrt{g}^{11}$$

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2. The geometric masses:

$$\bar{m}_W = \frac{\bar{g}_2}{2}\sqrt{h}_{11} v \qquad \bar{m}_Z = \frac{\bar{g}_Z}{2}\sqrt{h}_{33} \qquad \bar{m}_A = 0$$

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3. The geometric mixing angles (necessary for above):

$$\bar{s}^2 = \frac{\left(g_1\sqrt{g}^{44} - g_2\sqrt{g}^{34}\right)^2}{g_1^2\left[\left(\sqrt{g}^{34}\right)^2 + \left(\sqrt{g}^{44}\right)^2\right] + g_2^2\left[\left(\sqrt{g}^{33}\right)^2 + \left(\sqrt{g}^{34}\right)^2\right] - 2g_1 g_2\sqrt{g}^{34}\left(\sqrt{g}^{33} + \sqrt{g}^{44}\right)}$$

$$\bar{s}^2_Z = \frac{g_1\left(\sqrt{g}^{44}\bar{s} - \sqrt{g}^{34}\bar{c}\right)}{g_2\left(\sqrt{g}^{33}\bar{c} - \sqrt{g}^{34}\bar{s}\right) + g_1\left(\sqrt{g}^{44}\bar{s} - \sqrt{g}^{34}\bar{c}\right)}$$

The geoSMEFT

Let's go "full metric," or, how far can we take this defining objects like h_{IJ} and g_{AB}?

$$(D_\mu\phi)^I \qquad W^A_{\mu\nu} \qquad \Psi_L \qquad \psi_R \qquad D_\mu$$

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We'll take variations of $\mathcal{L}_{\text{SMEFT}}$ w/r to the fields above and see where we get:
(metrics/field-space connections must be Lorentz & $SU(3)$ invariant, ϕ is a colorless scalar)

$$h_{IJ} = \frac{g^{\mu\nu}}{d} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta(D_\mu \phi)^I \delta(D_\nu \phi)^J}\bigg|_{\text{fields}\to 0} \qquad \left(\text{scalar 2pts, } m_{W,Z}\right)$$

$$g_{AB} = \frac{-2g^{\mu\nu} g^{\sigma\rho}}{d^2} \frac{\delta^2 \mathcal{L}_{\text{SMEFT}}}{\delta W^A_{\mu\sigma} \delta W^B_{\nu\rho}}\bigg|_{\text{fields}\to 0} \qquad (\text{gauge 2pts, mixing})$$

$$\mathcal{Y} = \frac{\delta^2 \mathcal{L}}{\delta \Psi_L \delta \psi_r}\bigg|_{\text{fields}\to 0} \qquad \left(\text{Yukawas} \to \bar{m}_\psi\right)$$

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$$\mathcal{Y} = \frac{\delta^2\mathcal{L}}{\delta\Psi_L\delta\psi_r}\bigg|_{\rm fields\to 0} \qquad \text{(Yukawas} \to \bar{m}_\psi\text{)}$$

1. EOM $\to$ remove D_μ's $\to$ full set of SMEFT operators affecting 2pt functions! analogue to kinematics: $p_1 = -p_2 \to p_1^2 = m^2$
2. $\mathcal{Y}$ is the most intuitive metric:

$$\underbrace{\langle\mathcal{Y}\rangle \sim \bar{m}_\psi}_{\text{all}-\text{orders }\frac{1}{\Lambda}\text{ mass}} \qquad \underbrace{\left\langle\frac{\delta\mathcal{Y}}{\delta h}\right\rangle \sim h\bar{\Psi}_L\psi_R}_{\text{all}-\text{orders coupling}}$$

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$$h_{IJ} \qquad g_{AB} \qquad \mathcal{Y} \qquad \Rightarrow \qquad \text{all EW 2pt functions}$$

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$$h_{IJ} \quad g_{AB} \quad \mathcal{Y} \quad \Rightarrow \quad \text{all EW 2pt functions}$$

We'll take variations of $\mathcal{L}_{\text{SMEFT}}$ w/r to the fields above and see where we get:

$$\kappa_{IJ}^{A} = \frac{g^{\mu\rho}g^{\nu\sigma}}{2d^2}\frac{\delta^3\mathcal{L}_{\text{SMEFT}}}{\delta(D_\mu\phi)^I\delta(D_\nu\phi)^J\delta W_{\rho\sigma}^{A}}\bigg|_{\text{fields}\to 0}$$

$$f_{ABC} = \frac{g^{\nu\rho}g^{\sigma\alpha}g^{\beta\mu}}{3!d^3}\frac{\delta^3\mathcal{L}_{\text{SMEFT}}}{\delta W_{\mu\nu}^{A}\delta W_{\rho\sigma}^{B}\delta W_{\alpha\beta}^{C}}\bigg|_{\text{fields}\to 0}$$

$$L_{J,A} = \frac{\delta^2\mathcal{L}_{\text{SMEFT}}}{\delta(D^\mu\phi)^J\delta(\bar\psi\gamma_\mu\sigma_A\psi)}\bigg|_{\text{fields}\to 0}$$

$$d_A = \frac{\delta^2\mathcal{L}_{\text{SMEFT}}}{\delta(\bar\Psi_L\sigma_{\mu\nu}\psi_R)\delta W_{\mu\nu}^{A}}\bigg|_{\text{fields}\to 0}$$

1. EOM $\to$ remove D_μ's $\to$ full set of SMEFT operators affecting 3pt functions! analogue to kinematics: $p_i = -p_j - p_k \to p_i \cdot p_j = \frac{1}{2}\left(m_k^2 - m_i^2 - m_j^2\right)$
2. These plus four-fermion operators fully span the Warsaw basis of D6 operators.

The geoSMEFT

Let's go "full metric," or, how far can we take this defining objects like h_{IJ} and g_{AB}?

	h_{IJ}	g_{AB}	$\mathcal{Y}$	$\Rightarrow$	all EW 2pt functions
κ_{IJ}^{A}	f_{ABC}	$L_{J,A}$	d_A	$\Rightarrow$	all EW 3pt functions

Unfortunately, we can't do the same for 4pt functions,

The geoSMEFT

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Unfortunately, we can't do the same for 4pt functions,
(kinematic ex. is illustrative, but
real check Hilbert series: Lehman & Martin 1503.07537+ & Henning et al. 1512.03433+)

$$p_i = -p_j - p_k - p_l \qquad \rightarrow \qquad s + t + u = \sum_i m_i$$

Summary:

1. All two- and three-point functions to all orders in $\frac{1}{\Lambda}$ can be defined in the geoSMEFT
2. This includes a partial set of 4+ point functions
   e.g. $(v^2 + 2vh + h^2)F^2$ & $(v^2 + 2vh + h^2)F^3$
3. Essentially resumming the v expansion
   as well as identifying all kinematic forms of 2- and 3-pt functions
   $\Rightarrow$ a return to effective vertices *including* correlations required by gauge invar.

Parameter counting on the Z-pole

operator form	shifts:
$h_{IJ}(D\phi)^I(D\phi)^J$	SM 3–point functions + Masses
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	SM triple gauge couplings + $h(\partial V)^2$ + mixing angles
$L^{\psi}_J(D^\mu\phi)^J(\bar{\psi}\Gamma_\mu\psi)$	SM gauge-fermion couplings

$$\psi = \{Q, L, u_R, d_R, e_R\}$$

$$g^{Z,\psi}_{\rm eff} = \frac{\bar{g}_Z}{2}\left[(2\, s^2_{\theta_Z} Q_\psi - \sigma_3) + \bar{v}_T \langle L^{\psi}_{3,4}\rangle + \sigma_3 \bar{v}_T \langle L^{\psi}_{3,3}\rangle\right]$$

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$$\bar{m}^2_Z = \frac{\bar{g}_Z}{4}^2 \langle h_{33}\rangle \bar{v}^2_T \qquad s^2_{\theta_Z} = f(\langle g_{AB}\rangle, g_1, g_2)$$

$$\Gamma_{Z\to\bar{\psi}\psi} = \frac{N^\psi_c}{24\pi} \bar{m}_Z |g^{Z,\psi}_{\rm eff}|^2 \left(1 - \frac{4\bar{m}^2_\psi}{\bar{m}^2_Z}\right)^{3/2}$$

(TC, Helset, Martin, Trott, arXiv:2102.02819)

Parameter counting on the Z-pole

$$g_{\text{eff}}^{Z,\psi} = \frac{\bar{g}_Z}{2}\left[(2\, s_{\theta_Z}^2 Q_\psi - \sigma_3) + \bar{v}_T \langle L_{3,4}^{\psi}\rangle + \sigma_3 \bar{v}_T \langle L_{3,3}^{\psi}\rangle\right]$$

$$\bar{m}_Z^2 = \frac{\bar{g}_Z}{4}^2 \langle h_{33}\rangle \bar{v}_T^2 \qquad s_{\theta_Z}^2 = f(\langle g_{AB}\rangle, g_1, g_2)$$

Number of parameters at each order (for LH fermion):

	geoSMEFT	D6	D(6+n)
$\bar{g}_Z$	1	3 $\{c_{HW}, c_{HB}, c_{HWB}\}$	4 $\{c_{HW}^{(6+n)}, c_{HW,2}^{(6+n)}, c_{HB}^{(6+n)}, c_{HWB}^{(6+n)}\}$
s_{θ_Z}	1	3 $\{c_{HW}, c_{HB}, c_{HWB}\}$	4 $\{c_{HW}^{(6+n)}, c_{HW,2}^{(6+n)}, c_{HB}^{(6+n)}, c_{HWB}^{(6+n)}\}$
$\langle h_{33}\rangle$	1	1 $\{c_{HD}\}$	2 $\{c_{HD}^{(6+n)}, c_{HD,2}^{(6+n)}\}$
$\langle L_{3,4}^{\psi}\rangle$	1	1 $\{c_{Hl}^{(3)}\}$	2 $\{c_{Hl}^{(6+n,2)}, c_{Hl}^{(6+n,3)}\}$
$\langle L_{3,3}^{\psi}\rangle$	1	1 $\{c_{Hl}^{(1)}\}$	1 $\{c_{Hl}^{(6+n,1)}\}$
sum:	3+2	4+2	4+5

Pheno in the geoSMEFT

The following calculations have been completed using the geoSMEFT:

1. Z-pole, TC, Helset, Martin, Trott, arXiv:2102.02819
   tree level to $1/\Lambda^4$
2. $ggh + h\gamma\gamma$, TC, Martin, Trott, arXiv:2107:07470
   tree level to $1/\Lambda^4$, one-loop to $1/\Lambda^2$
3. Various simple examples: $hZ\gamma$, hZZ^*, Hay, Helset, Martin, Trott, arXiv:2007.00565
   translation to $\{\hat{\alpha}, \hat{G}_F, \hat{m}_Z\}$ & $\{\hat{m}_W, \hat{G}_F, \hat{m}_Z\}$ input parameter schemes
   all orders, and $1/\Lambda^4$
4. $h \to \gamma\bar{\psi}\psi$, TC, Rasmussen arXiv:2110.03694
   tree level $1/\Lambda^4$ + SM-loop interference w $1/\Lambda^4$
5. The one-loop all orders tadpole, TC arXiv:2106.10284

Forthcoming (2023):

1. EWPD + TGVs fit to $1/\Lambda^4$, TC, Gonzalez-Garcia, Éboli, Reimitz, Martines
2. $pp \to h\bar{\psi}\psi$ to $1/\Lambda^4$, TC, Martin
3. The 2pt Ward Identities at one-loop in the geoSMEFT, TC

The future

geoSMEFT goals for the future:

1. Formulate field-space connections corresponding to all operators in D8 bases
   $\Rightarrow$ can calculate to $1/\Lambda^4$ and sum the v expansion
   (Murphy, arXiv:2005.00059, Li et al, arXiv:2005.00008)

2. Expand into full one-loop calculations in the geoSMEFT

   $\Rightarrow$ all 2pt functions,
   $\Rightarrow$ important one-loop processes like $Z\bar{\psi}\psi$, $h\gamma\gamma$, ggh, $h\bar{\psi}\psi$, top physics, etc

3. Matching to the geoSMEFT (can it be done?)

4. RGEs $\rightarrow$ nonlinear σ by Howe, Papadopoulos, Stelle, 1988
   (Background field method was critical here)

5. geoSMEFT as a QFT:

$$\mathcal{M} \sim \langle T\{\phi_1 \cdots \phi_n\}\rangle \quad \sim \quad \frac{\delta^n}{\delta\phi_1 \cdots \delta\phi_n} \int \mathcal{D}\phi \exp\left(iS_{\text{SMEFT}}\right)$$

$$\Leftrightarrow \quad \text{Metrics} \sim \frac{\delta^n}{\delta\phi_1 \cdots \delta\phi_n} \mathcal{L}_{\text{SMEFT}}$$

Conclusions

1. fits to D8 $\rightarrow$ resolving "inverse problem"
2. the SMEFT is the SM on a curved field space $\rightarrow$ geometry
3. the geoSMEFT takes this into account:
   - certain quantities can be defined to all orders in $1/\Lambda$, m_V, m_ψ, mixing, $\cdots$
   - all two- and three-point kinematic structures can be defined to all orders

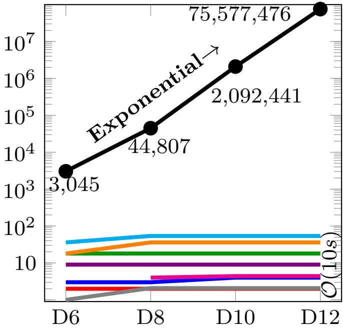
     $\rightarrow$ effective vertices (that respect symmetries of SMEFT)
     $\rightarrow$ think resonance physics, e.g. EWPD, can be defined to all orders

     $\rightarrow$ ¿ if kinematic forms saturated, can safely include partial higher orders?
4. How does the geoSMEFT (or simpler EFTs) look radiatively?
   - Helset, Jenkins, Manohar, arXiv:2212.03253
   - Helset, Jenkins, Manohar, arXiv:2210.08000
   - Cheung, Helset, Parra-Martinez, arXiv:2202.06972
   - Howe, Papadopoulos, Stelle, 1988

Number of parameters in the (geo)SMEFT

At D6 we have 59 op forms → 2499 free parameters
At D8 we have 895 op forms → way more...

Operator form:	Mass Dimension 6	8	10
$h_{IJ}(D_\mu\phi)^I(D^\mu\phi)^J$	2	2	2
$g_{AB}W^A_{\mu\nu}W^{B,\mu\nu}$	3	4	4
$k_{IJA}(D^\mu\phi)^I(D^\nu\phi)^J W^A_{\mu\nu}$	0	3	4
$f_{ABC}W^A_{\mu\nu}W^{B,\nu\rho}W^{C,\mu}_\rho$	1	2	2
$Y^\psi_{pr}\bar{\Psi}_L\psi_R + h.c.$	$2N_f^2$	$2N_f^2$	$2N_f^2$
$d^{\psi,pr}_A\bar{\Psi}_L\sigma_{\mu\nu}\psi_R W^{\mu\nu}_A + h.c.$	$4N_f^2$	$6N_f^2$	$6N_f^2$
$L^{\psi_R}_{pr,J,A}(D^\mu\phi)^J(\bar{\psi}_{p,R}\gamma_\mu\sigma_A\psi_{r,R})$	N_f^2	N_f^2	N_f^2
$L^{\Psi_L}_{pr,J,A}(D^\mu\phi)^J(\bar{\Psi}_{p,L}\gamma_\mu\sigma_A\Psi_{r,L})$	$2N_f^2$	$4N_f^2$	$4N_f^2$



This counting is still in Wilson coefficients...
Can fits be done in terms of geometric quantities instead?