Progress in SM predictions for rare $\boldsymbol{B}$ decays

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Based on
arXiv:2011.09813, 2206.03797, 23xx.xxxxx
in collaboration with
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Les Rencontres de Physique
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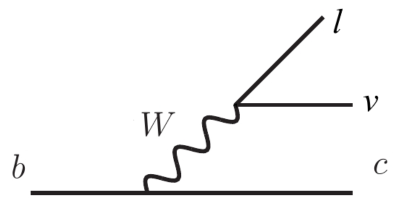
Particle Physics Phenomenology after the Higgs Discovery
Collaborative Research Center TRR 257

Universität Siegen
Center for Particle Physics Siegen

Introduction

Flavour changing currents

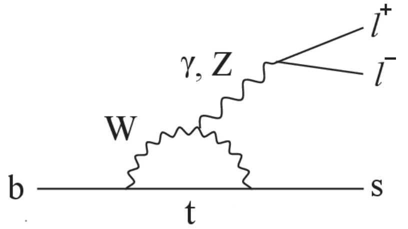
flavour changing charged currents (FCCC) occur at tree level (mediated by $W^{\pm}$) in the SM

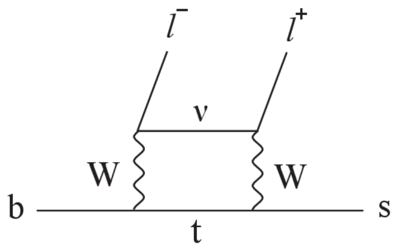


FCCC

flavour changing neutral currents (FCNC) absent at tree level in the SM
FCNC are loop, GIM and CKM **suppressed in the SM**

FCNC **sensitive to new physics** contributions
probe the SM through **indirect searches**





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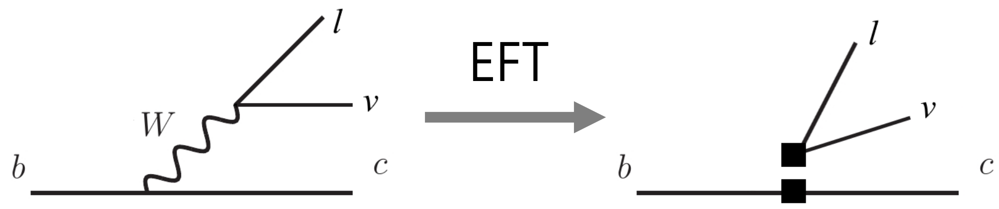
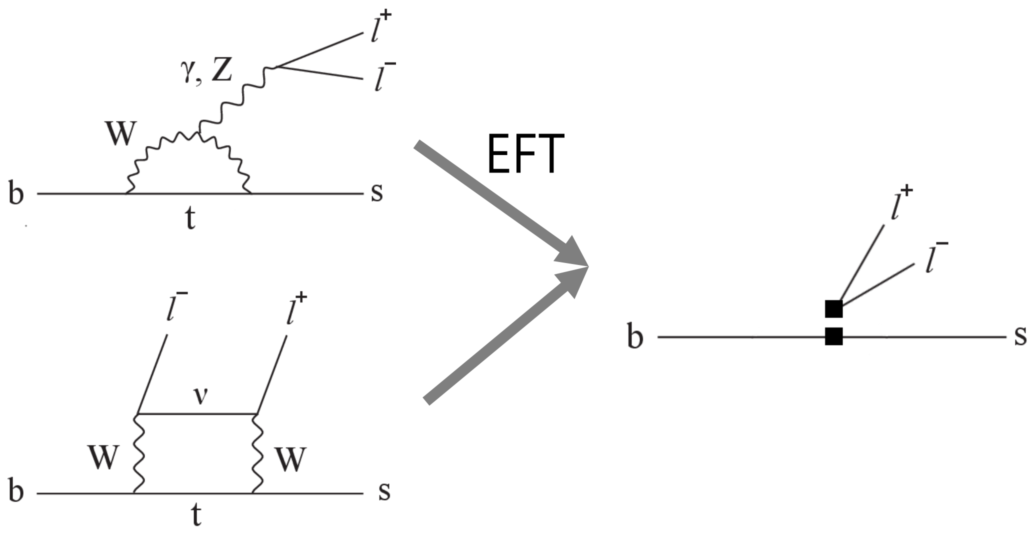
integrate out DOF heavier than the b

⇓

weak effective field theory



FCCC



FCNC

Hadronic matrix elements

study $\boldsymbol{b \to s\ell^+\ell^-}$ transitions using $\boldsymbol{B}$-meson, focus on to $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$
factorise decay amplitude as (neglecting QED corrections)

FCCC: $$\langle \bar{D}^{(*)}\ell\nu_\ell|\mathcal{O}_{eff}|B\rangle = \langle \ell\nu_\ell|\mathcal{O}_{lep}|0\rangle\langle D^{(*)}|\mathcal{O}_{had}|B\rangle$$

FCNC: $$\langle K^{(*)}\ell^+\ell^-|\mathcal{O}_{eff}|B\rangle = \langle \ell\ell|\mathcal{O}_{lep}|0\rangle\langle K^{(*)}|\mathcal{O}_{had}|B\rangle + \text{non−fact.}$$

leptonic matrix elements: perturbative objects, high accuracy

hadronic matrix elements: non-perturbative QCD effects, usually large uncertainties

Hadronic matrix elements

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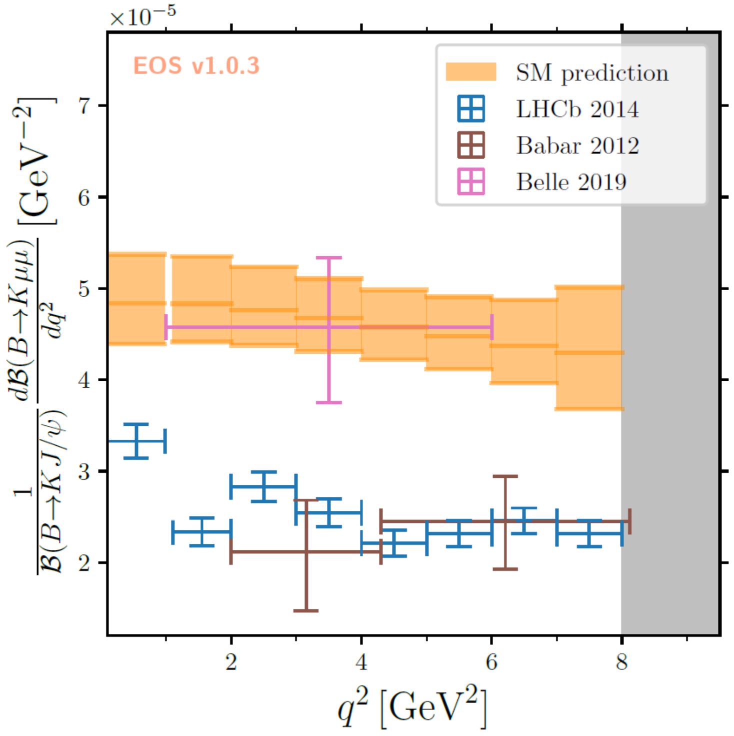
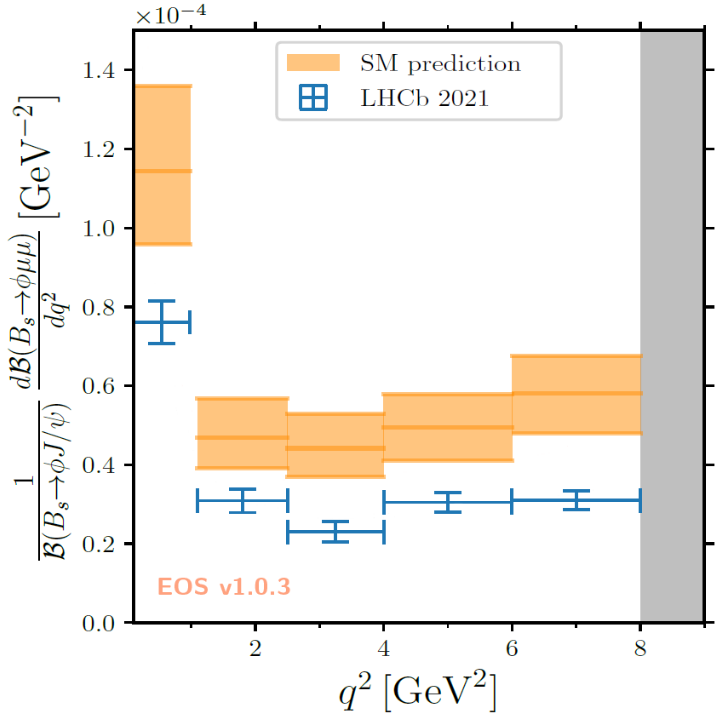
decay amplitudes depend on:

- local hadronic matrix elements
  (local form factors)
  $\langle K^{(*)}|\mathcal{O}(0)|B\rangle$
  $\langle D^{(*)}|\mathcal{O}(0)|B\rangle$
- nonlocal hadronic matrix elements
  (soft gluon contributions
  to the charm-loop)
  $\langle K^{(*)}|\mathcal{O}(0,x)|B\rangle$

SM predictions for BRs in rare decays

test the SM and constrain new physics by comparing theory predictions and exp. measurements of, e.g., branching ratios $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$





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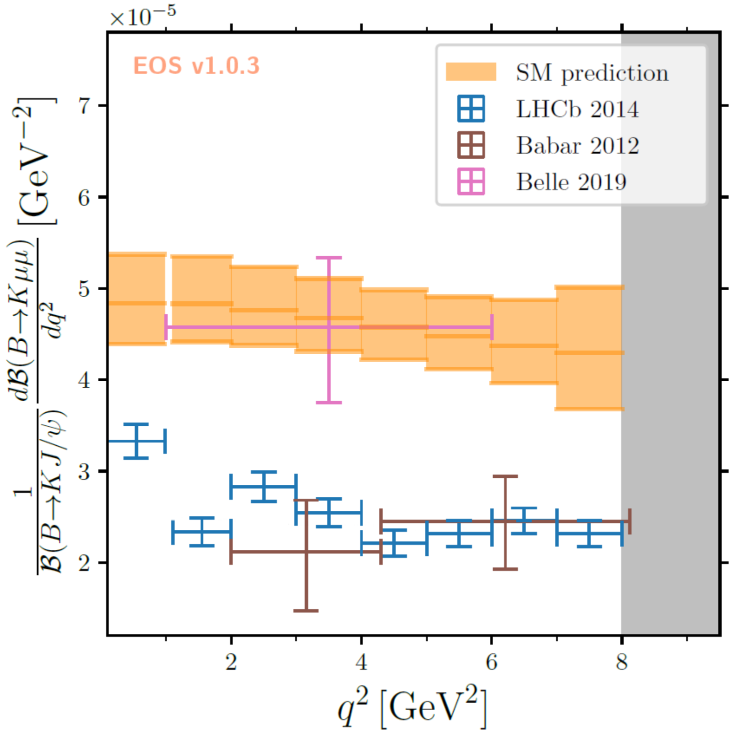
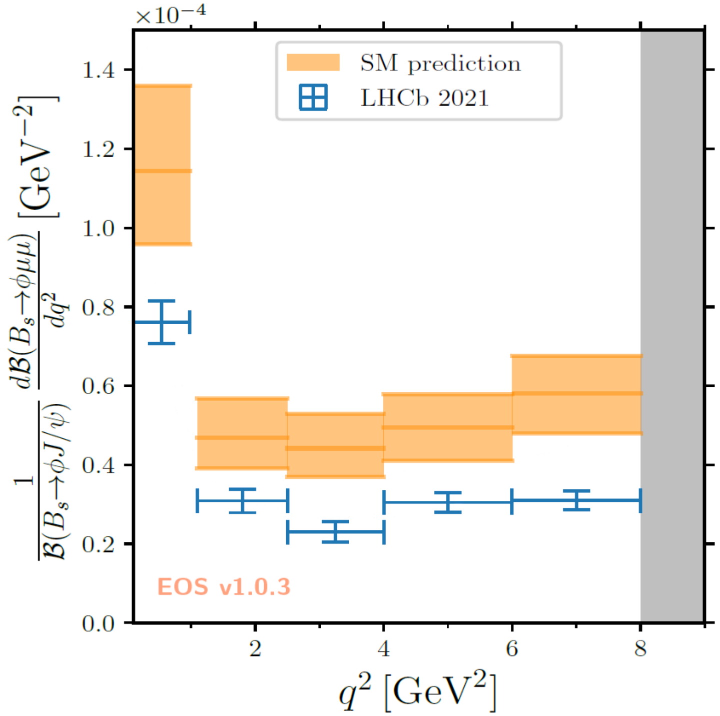




agreement between theory and experiment for LFU ratios R_K and R_{K^*},
but **tension remains for $b \to s\mu^+\mu^-$** observables $\Longrightarrow$ need to understand this tension

focus of this talk: how to obtain these SM predictions and what ingredients are needed

Theoretical framework

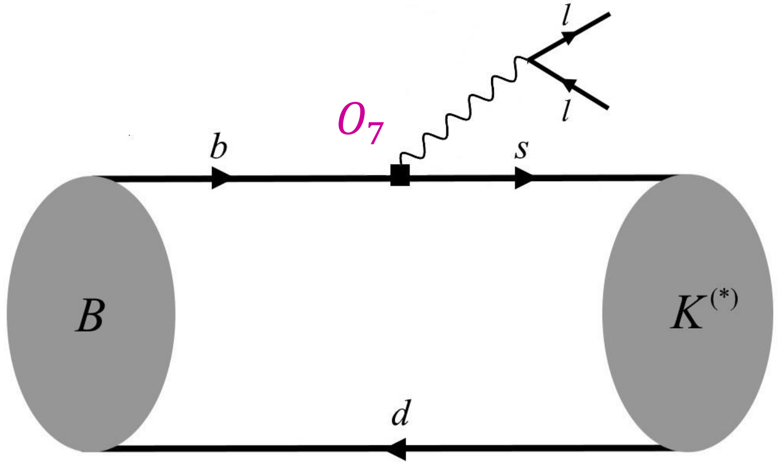
$b \to s\ell^+\ell^-$ effective Hamiltonian

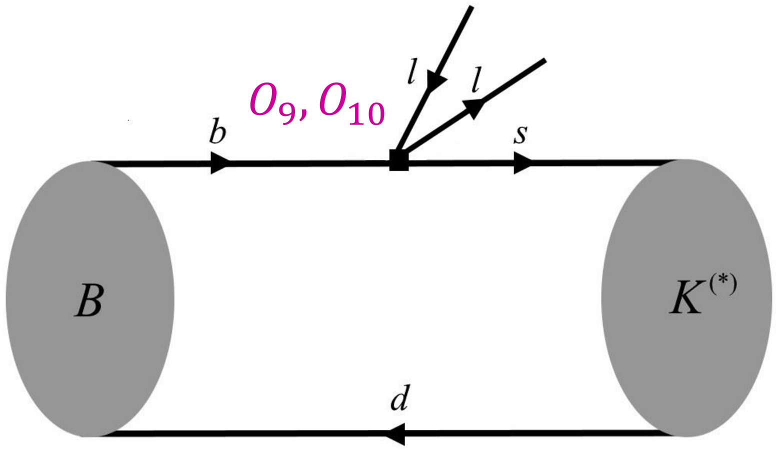
transitions described by the **effective Hamiltonian**

$$\mathcal{H}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \qquad \mu = m_b$$

main contributions to $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ in the SM given by local operators O_7, O_9, O_{10}

$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$ $\quad O_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \ell)$ $\quad O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \gamma_5 \ell)$

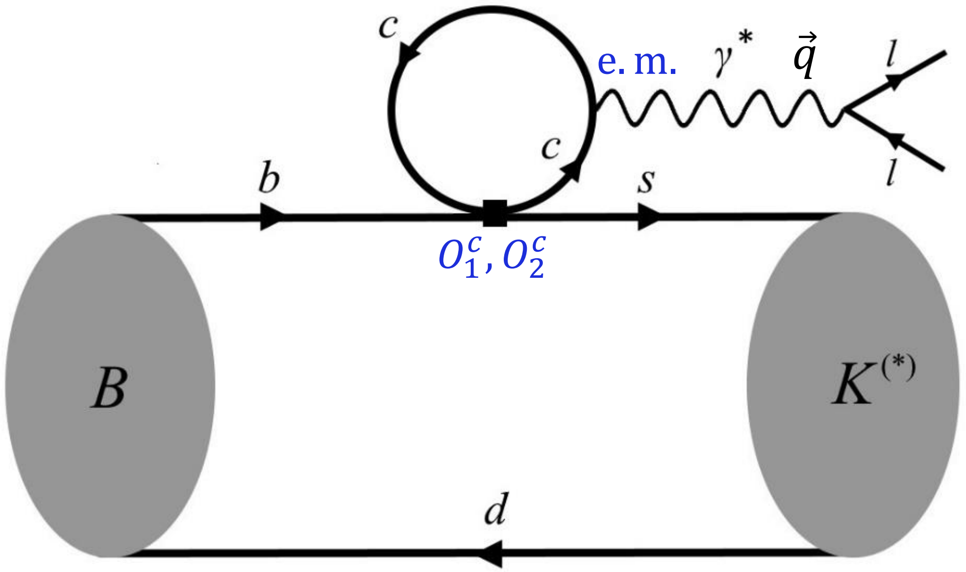




Charm loop in $B \to K^{(*)}\ell^+\ell^-$

additional non-local contributions come from O_1^c and O_2^c combined with the e.m. current (charm-loop contribution)

$$O_1^c = (\bar{s}_L\gamma^\mu c_L)(\bar{c}_L\gamma_\mu b_L) \qquad O_2^c = \left(\bar{s}_L^j\gamma^\mu c_L^i\right)\left(\bar{c}_L^i\gamma_\mu b_L^j\right)$$



Decay amplitude for $B \to K^{(*)}\ell^+\ell^-$ decays

calculate decay amplitudes precisely to probe the SM
B-anomalies: NP or underestimated systematic uncertainties?
(analogous formulas apply to $B_s \to \phi\ell^+\ell^-$ decays)

$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

local hadronic matrix elements

$$\mathcal{F}_\mu = \langle K^{(*)}(k)|O_{7,9,10}^{\text{had}}|B(k+q)\rangle$$

non-local hadronic matrix elements

$$\mathcal{H}_\mu = i\int d^4x\, e^{iq\cdot x}\langle K^{(*)}(k)|T\{j_\mu^{\text{em}}(x), (C_1 O_1^c + C_2 O_2^c)(0)\}|B(k+q)\rangle$$

Form factors definitions

form factors (FFs) **parametrize hadronic matrix elements**

FFs are functions of the momentum transfer squared $\boldsymbol{q^2}$

local FFs

$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\,\mathcal{F}_\lambda(q^2)$$

computed with lattice QCD and sum rules with good precision $\boldsymbol{\sim 10\%}$

non-local FFs

$$\mathcal{H}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\mathcal{H}_\lambda(q^2)$$

calculated using an **Operator Product Expansion (OPE)** or QCD factorization or …
(variety of approaches, most of them model-dependent)

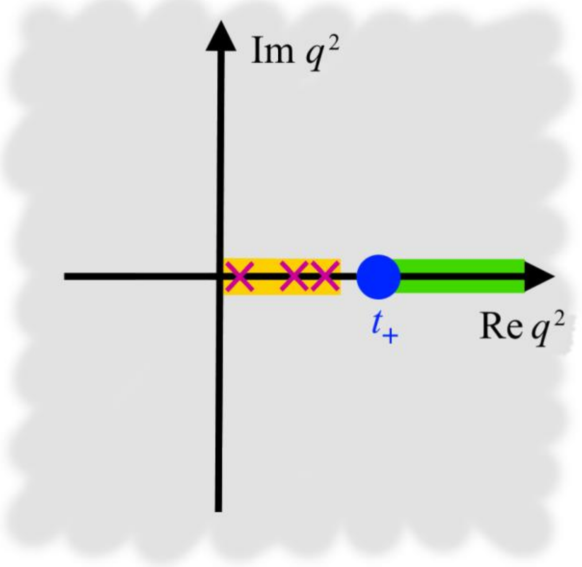
large uncertainties → reduce uncertainties for a better understanding of rare $\boldsymbol{B}$ decays

FF and SM predictions

Parametrization for $\mathcal{F}_\lambda$

obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by **combining**

- lattice QCD (**LQCD**) calculations at high q^2
- light-cone sum rule (**LCSR**) calculation at low q^2



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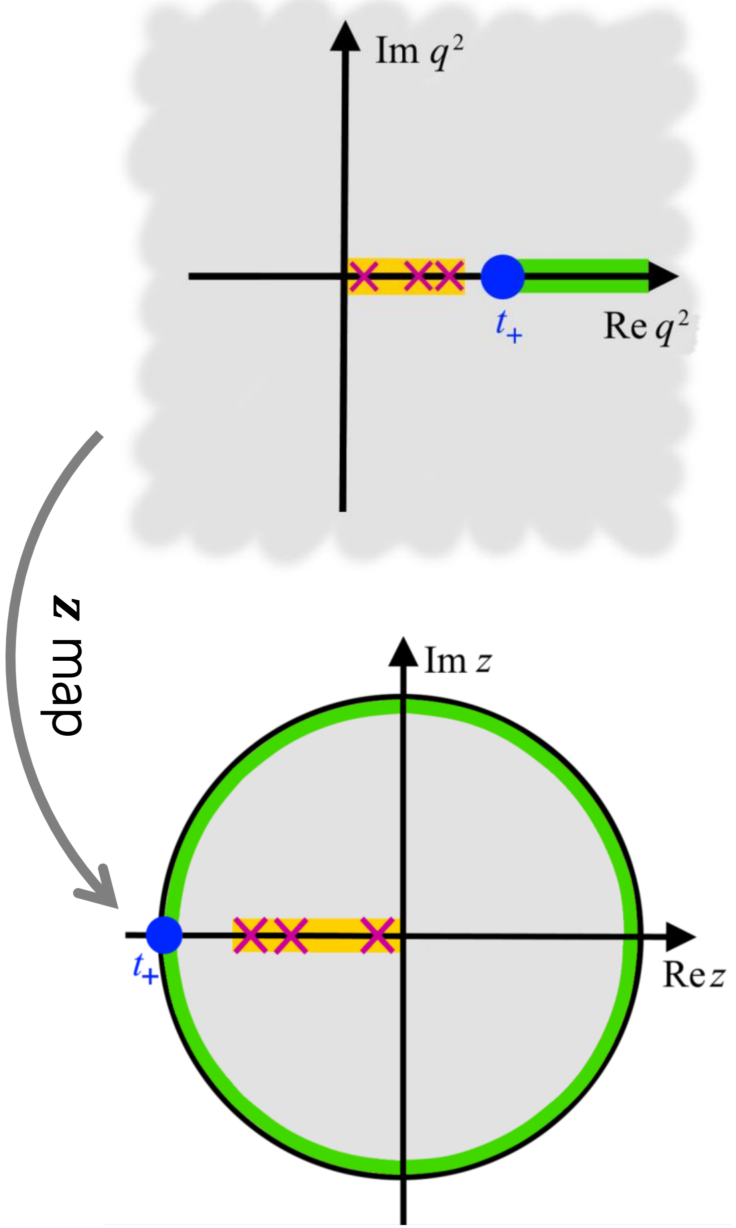
$\mathcal{F}_\lambda$ analytic functions of q^2 (branch cut for $q^2 > t_+ = \left(M_B + M_{K^{(*)}}\right)^2$)
define the map

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$

fit results to a $\boldsymbol{z}$ **parametrization** and use **dispersive bound** (standard approach) [Boyd/Grinstein/Lebed 1997]

$$\mathcal{F}_\lambda \propto \sum_{k=0}^{\infty} \alpha_k^{\mathcal{F}} z^k \qquad \sum_{k=0}^{\infty} \left|\alpha_k^{\mathcal{F}}\right|^2 < 1$$



Local form factors predictions

available theory inputs for local FFs $\mathcal{F}_\lambda$

$B \to K\,\ell^+\ell^-$:

- LQCD calculations at high q^2 and whole semilept. region
  [HPQCD 2013/2023] [FNAL/MILC 2015]
- LCSR at low q^2 [Khodjamirian/Rusov 2017]

$B \to K^*\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$:

- LQCD calculations at high q^2
  [Horgan et al. 2015]
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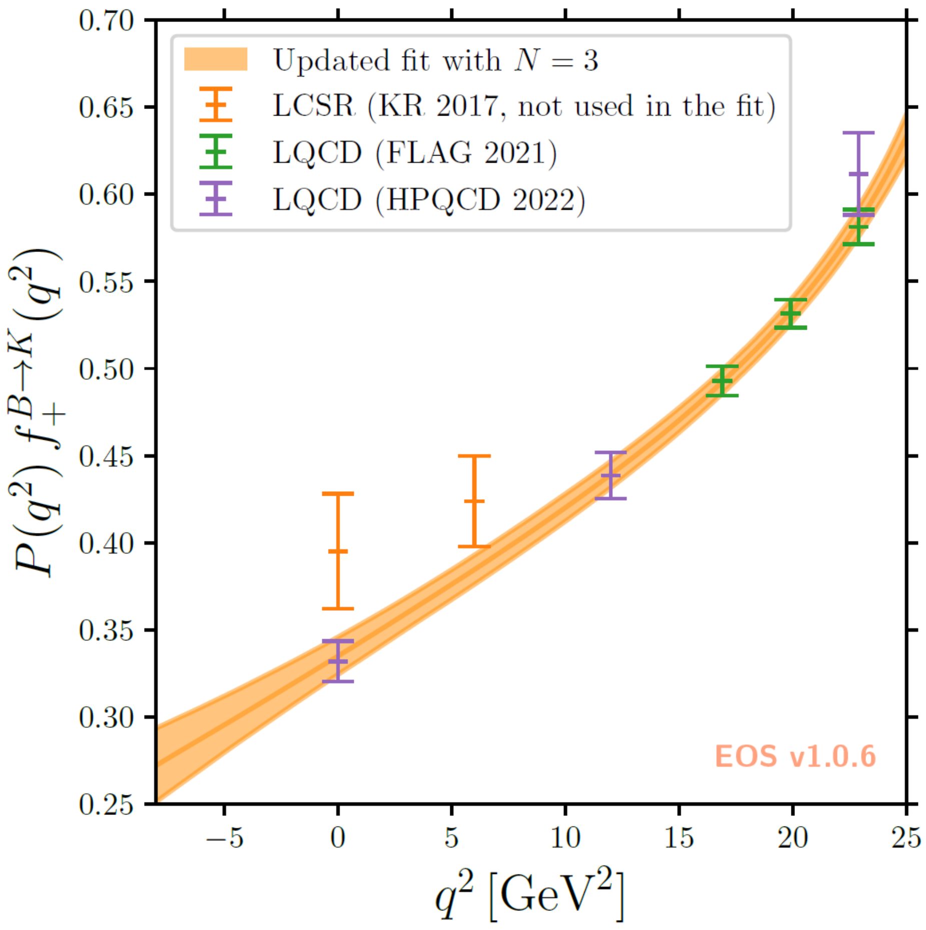
fit theory inputs to **improved BGL expansion**
(more stringent constraint, remove $B_s\pi$ branch cut)

more LQCD results needed for vector states



[Reboud/NG/van Dyk/Virto w.i.p.]

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

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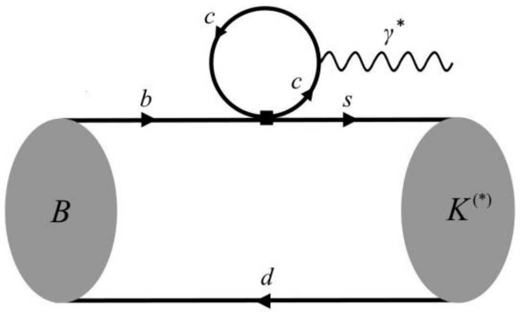
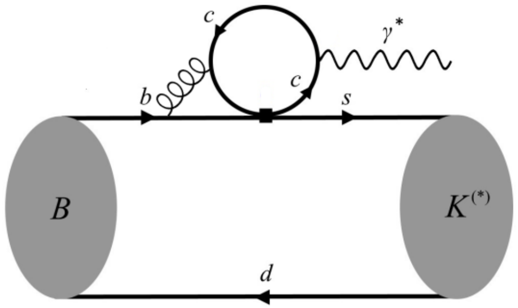
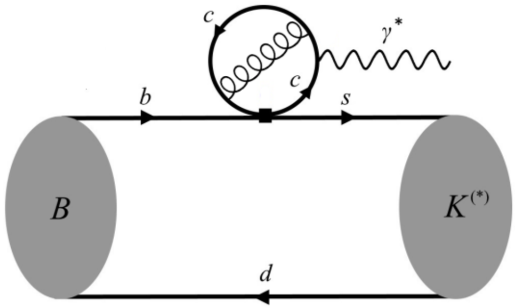
leading power (LO in α_s)



+ hard gluons (α_s) corrections





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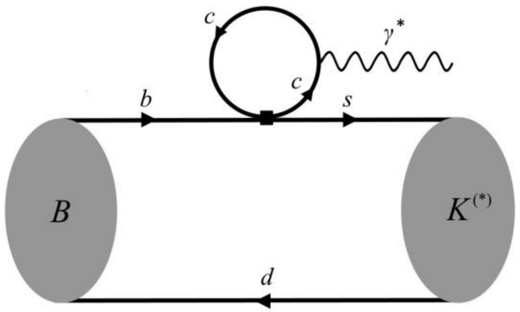

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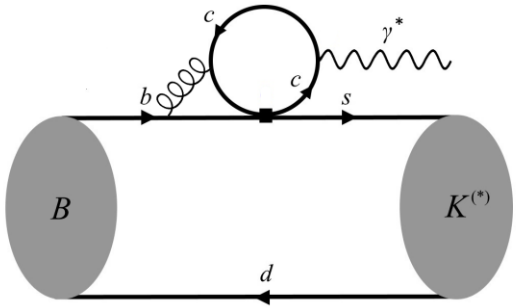
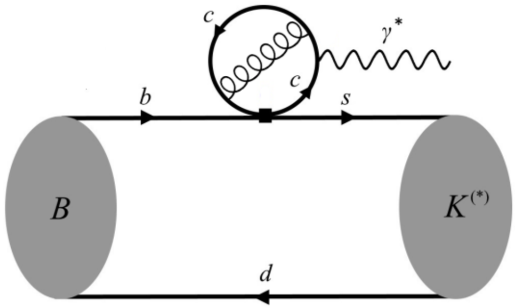
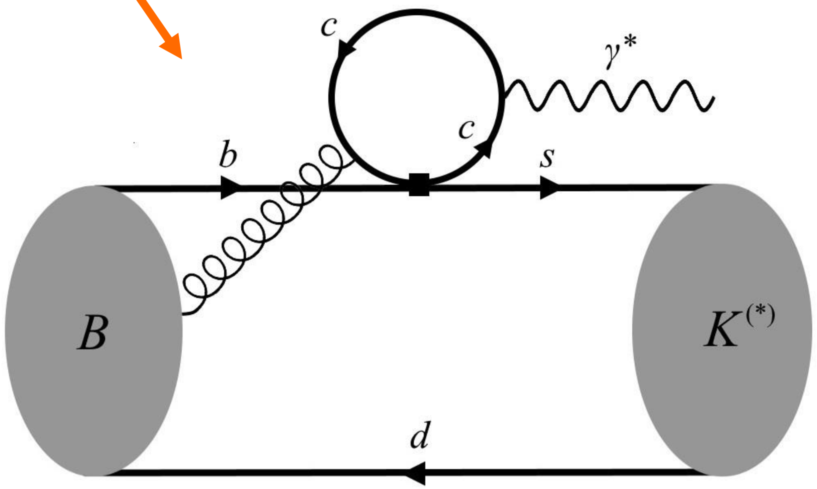




soft gluon correction
non-perturbative
$\Longrightarrow$ not α_s suppressed



[Khodjamirian et al. 2010]
[NG/van Dyk/Virto 2020]

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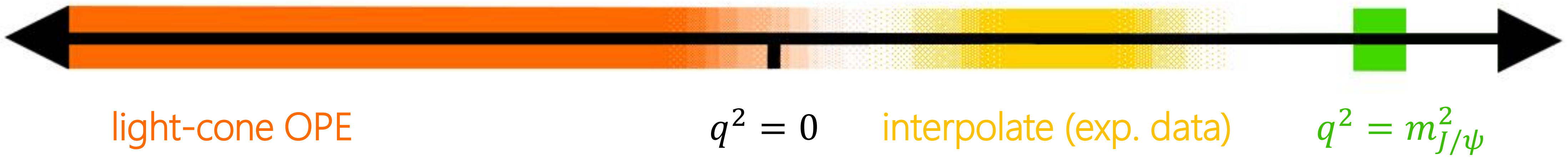
3. **new approach: interpolate** these two results to obtain theoretical predictions in the low q^2 $(0 < q^2 < 8\ \mathrm{GeV}^2)$ region $\Longrightarrow$ compare with experimental data



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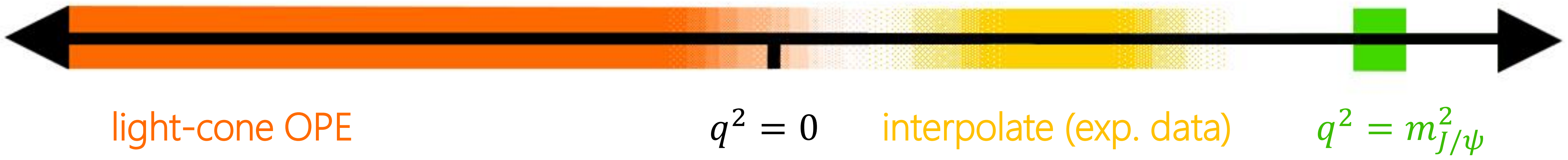
need a parametrization to interpolate $\mathcal{H}_\lambda$: which is the optimal parametrization?



Parametrizations for $\mathcal{H}_\lambda$

- q^2 parametrization [Jäger/Camalich 2012, Ciuchini et al. 2015]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\text{QCDF}}(q^2) + \mathcal{H}_\lambda^{\text{rest}}(0) + \frac{q^2}{M_B^2}\mathcal{H}_\lambda^{\text{rest},\prime}(0) + \frac{(q^2)^2}{M_B^4}\mathcal{H}_\lambda^{\text{rest},\prime\prime}(0) + \cdots$$

- dispersion relation [Khodjamirian et al. 2010]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda(0) + \sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi \mathcal{A}_\psi}{M_\psi^2\left(M_\psi^2 - q^2\right)} + \int_{4M_D^2}^{\infty} dt\, \frac{\rho(t)}{t(t-q^2)}$$

- z expansion [Bobeth/Chrzaszcz/van Dyk/Virto 2017]

$$\mathcal{H}_\lambda(z) \propto \sum_{n=0}^{\infty} c_n z^n$$

- we propose a new parametrization ($\hat{z}$ polynomials) [NG/van Dyk/Virto 2020]

$$\mathcal{H}_\lambda(\hat{z}) \propto \sum_{n=0}^{\infty} \beta_n p_n(\hat{z})$$

Dispersive bound for $\mathcal{H}_\lambda$

$\mathcal{H}_\lambda$ analytic functions of q^2 (branch cut for $q^2 > t_+ \neq \hat{t}_+ \equiv 4M_D^2$)
define the map:

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_+ - q^2} - \sqrt{\hat{t}_+}}{\sqrt{\hat{t}_+ - q^2} + \sqrt{\hat{t}_+}}$$

expand $\mathcal{H}_\lambda$ in orthogonal polynomials $p_n(\hat{z})$

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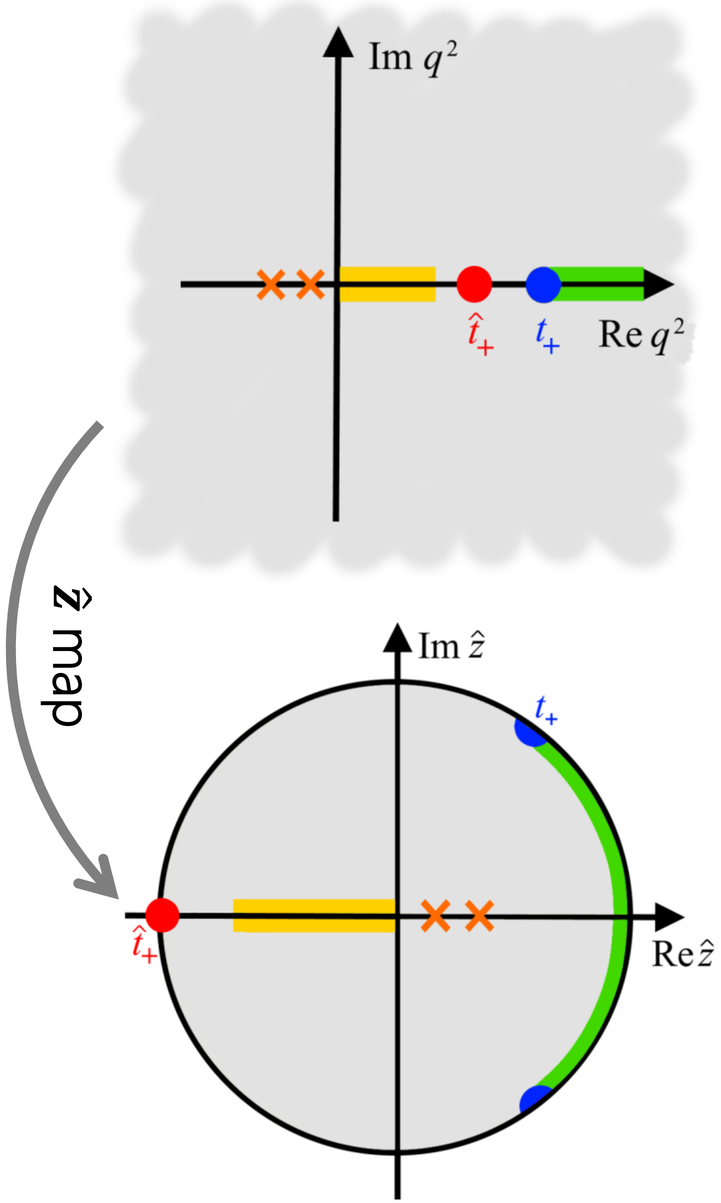

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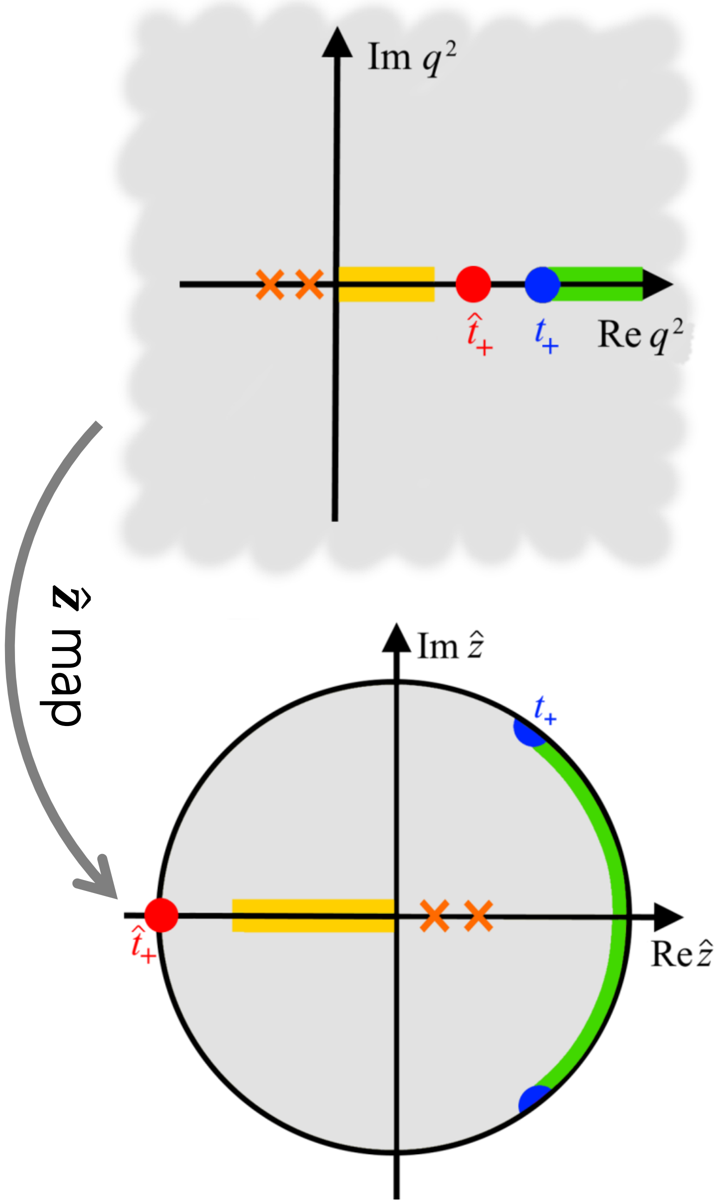
obtain **dispersive bound** using unitarity and duality

$$1 > \sum_{n=0}^{\infty} |\beta_n^{B\to K}|^2 + \sum_{\lambda} \left(2 \sum_{n=0}^{\infty} \left|\beta_{\lambda,n}^{B\to K^*}\right|^2 + \sum_{n=0}^{\infty} \left|\beta_{\lambda,n}^{B_s\to\phi}\right|^2 \right)$$

new model independent constraints [NG/van Dyk/Virto 2020]



Non-local form factors predictions

theory inputs used to constrain non-local FFs $\mathcal{H}_\lambda$

- light-cone OPE calculation at negative q^2
- $B \to K^{(*)} J/\psi$ and $B_s \to \phi\, J/\psi$ measurements at $q^2 = m^2_{J/\psi}$
- dispersive bound

fit theory inputs to parametrization

$$\mathcal{H}_\lambda(\hat{z}) \propto \sum_{n=0}^{6} \beta_n p_n(\hat{z})$$

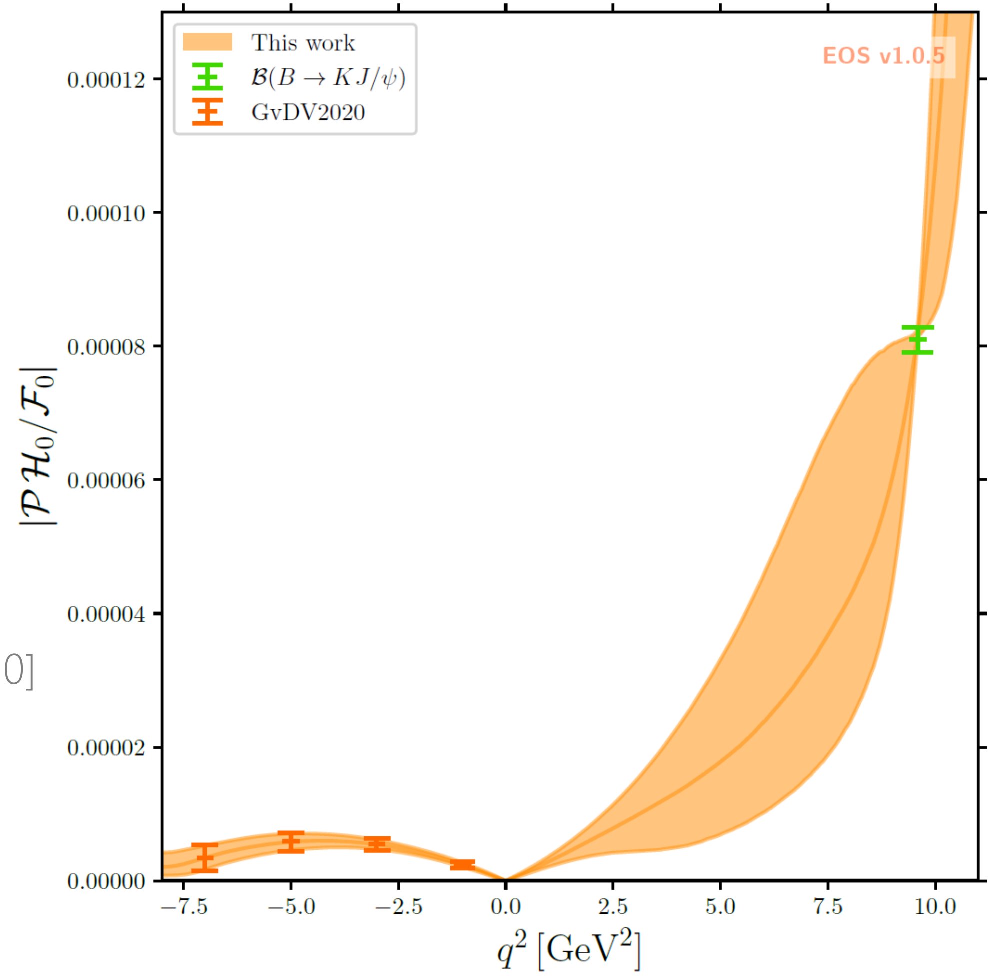
new approach to determine to obtain non-local FFs

[NG/van Dyk/Virto 2020]

systematically improvable approach
(more precise form factor results, saturate the bound,…)

one fit per decay channel (all p values $> 11\%$)

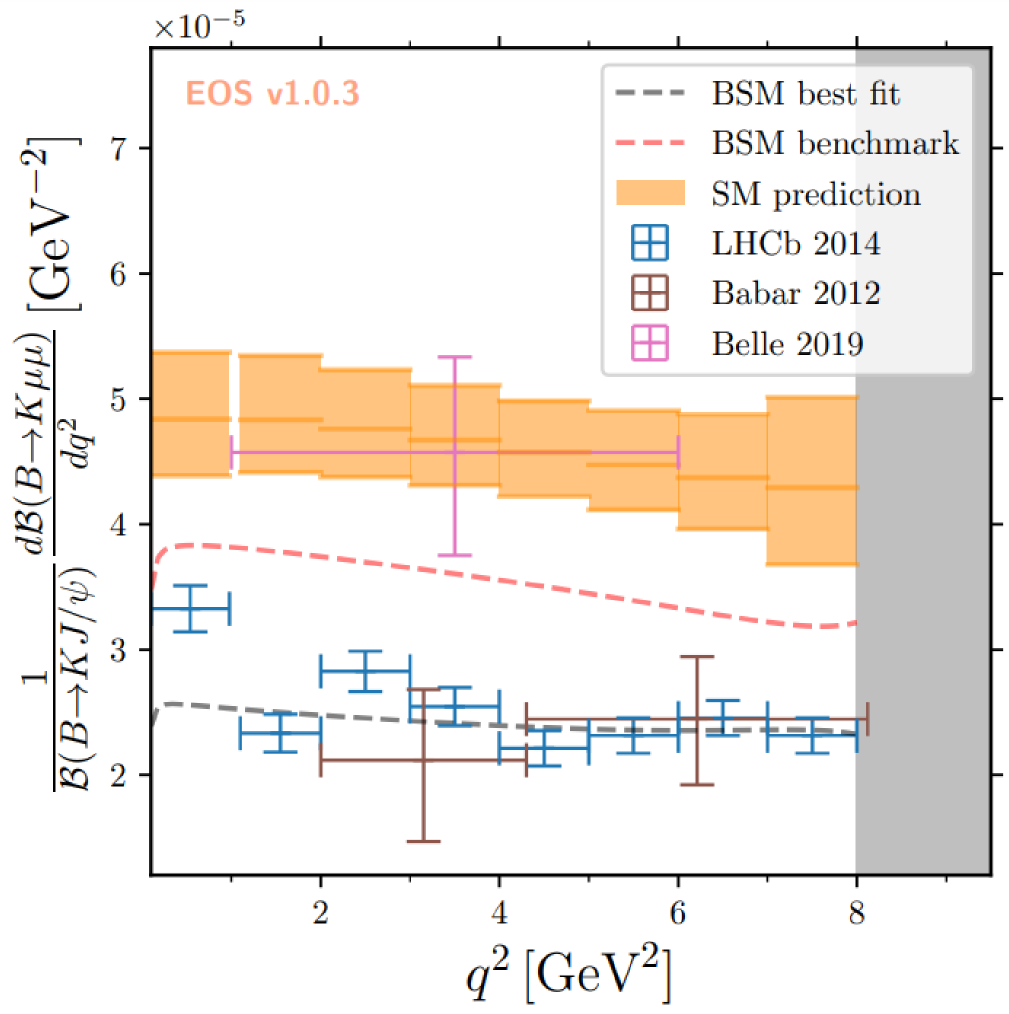
[NG/Reboud/van Dyk/Virto 2022]



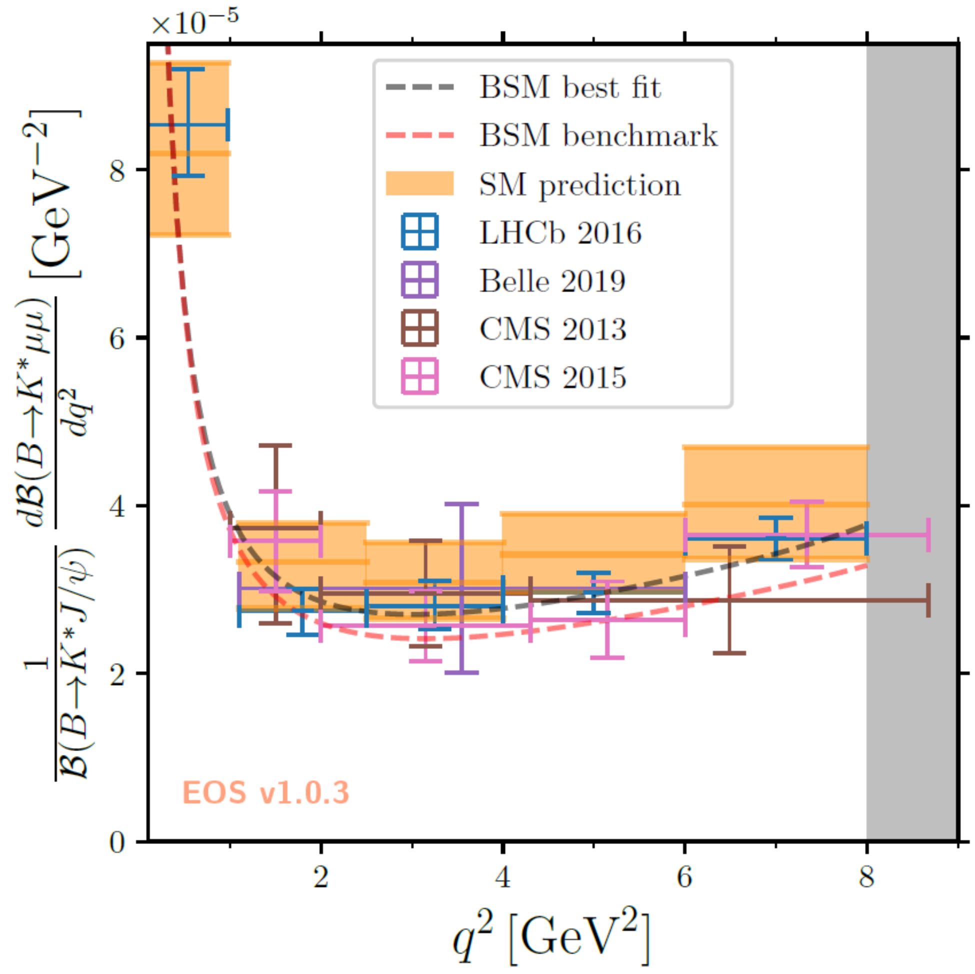
Comparison with measurements for $B \to K\mu^+\mu^-$

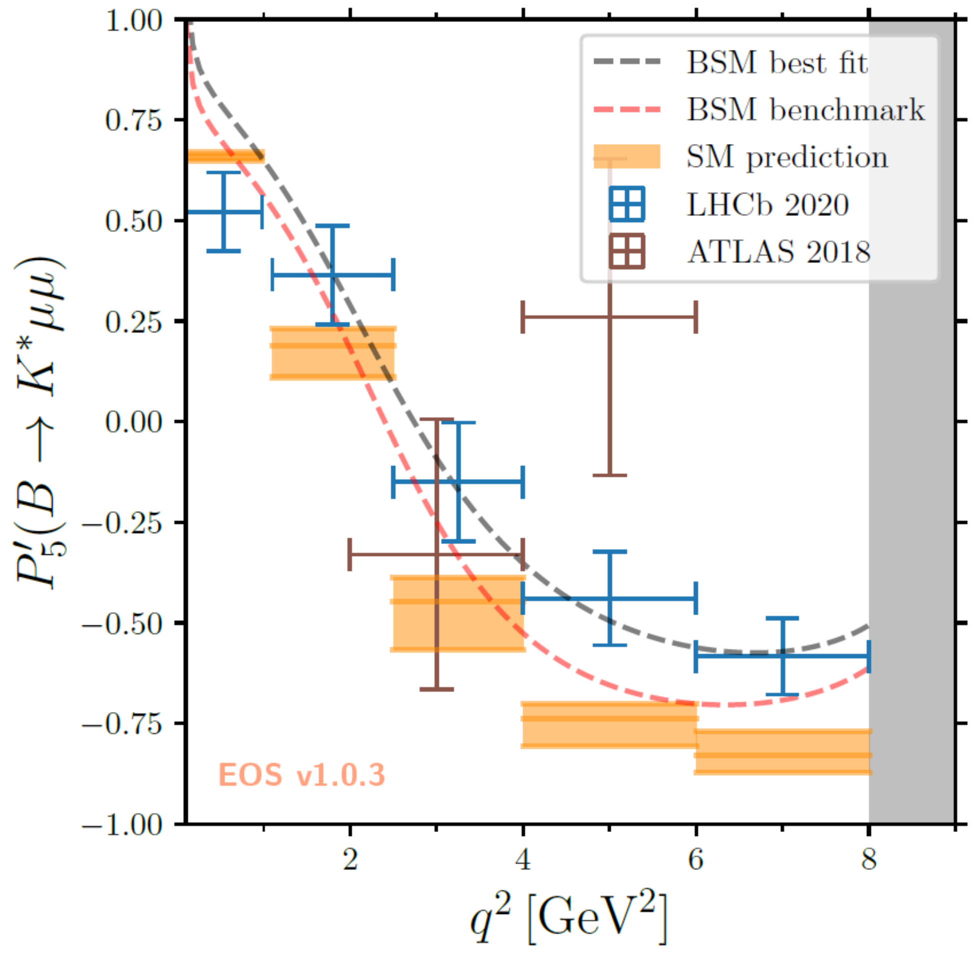
using our local and non-local FFs values
we predict branching ratios and angular observables in $B \to K\mu^+\mu^-$, $B \to K^*\mu^+\mu^-$, and $B_s \to \phi\mu^+\mu^-$ in the SM

- we do not use QCD factorization
- **sizable tension** between SM predictions and experimental results
- "BSM best fit" $\longrightarrow$ best fit point of our BSM fit ($C_{9,10} = C_{9,10}^{\mathrm{SM}} + C_{9,10}^{\mathrm{NP}\mu}$)
- "BSM benchmark" $\longrightarrow$ set $C_9^{\mathrm{NP}\mu} = -C_{10}^{\mathrm{NP}\mu} = -0.5$



Comparison with measurements for $B \to K^*\mu^+\mu^-$





tension **smaller** than in other works in the literature ⟶ inputs for the local FFs $\mathcal{F}_\lambda$

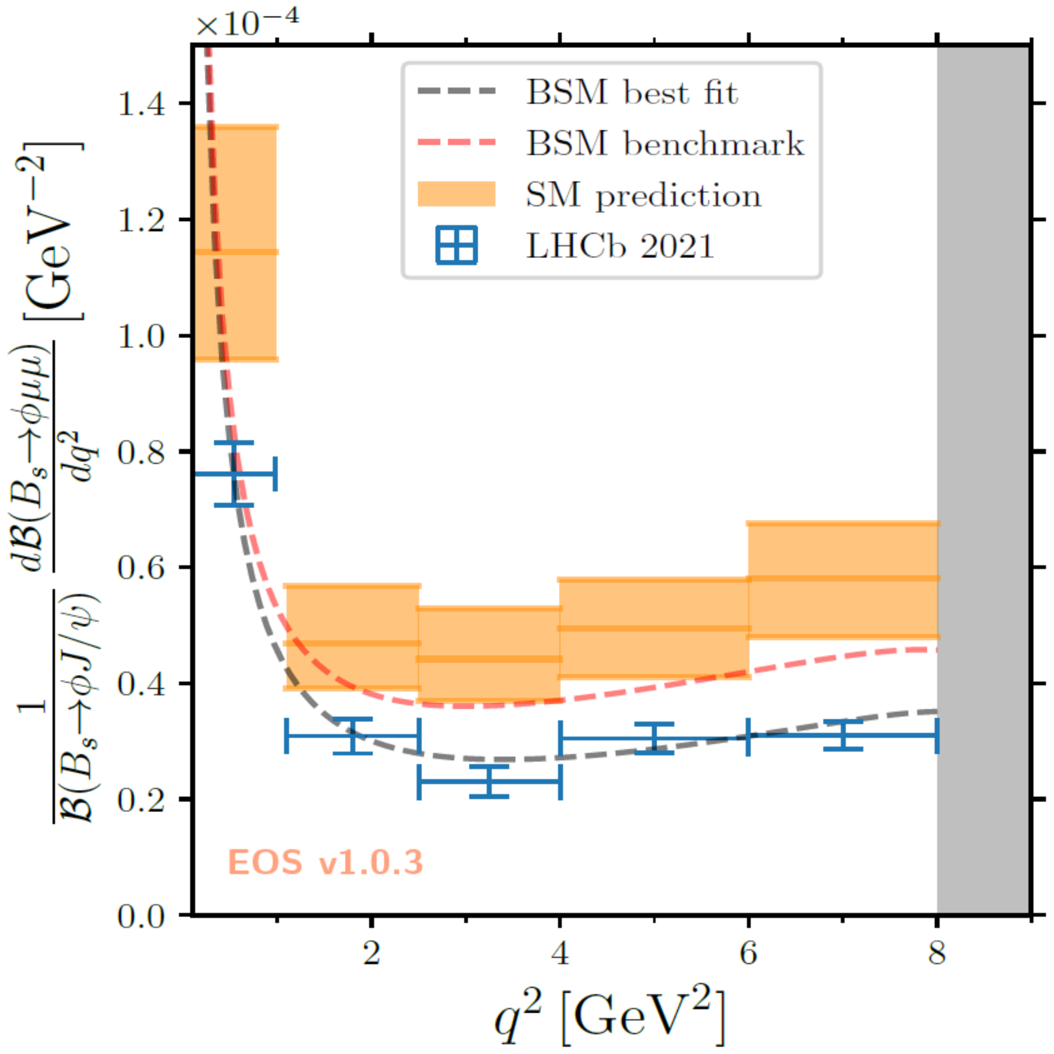
Comparison with measurements for $B_s \to \phi\mu^+\mu^-$

- **consistent picture** with the deviations in $B \to K\mu^+\mu^-$, $B \to K^*\mu^+\mu^-$

- **main source of unc. local FFs** $\mathcal{F}_\lambda$

- **choice of theory inputs** (local FFs $\mathcal{F}_\lambda$) is decisive
  $\longrightarrow$ usage of light-meson LCSRs rather than B-meson LCSRs yields much larger tension w.r.t. data

- **precise LQCD** calculations at low q^2 **essential** to have more reliable theoretical predictions (already available for $B \to K\ell^+\ell^-$)



Global fit to $b \to s\mu^+\mu^-$ (setup)

use our predictions for the local and non-local FFs as priors

fit the Wilson coefficients $C_9^{\mathrm{NP}\mu}$ and $C_{10}^{\mathrm{NP}\mu}$ to the available experimental measurements in $b \to s\mu^+\mu^-$ transitions
$(C_{9,10} = C_{9,10}^{\mathrm{SM}} + C_{9,10}^{\mathrm{NP}\mu})$

we perform **three fits**, one for each set of the following set of experimental measurements:
(BRs, angular observables, binned and not binned)

- $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- $B \to K^*\mu^+\mu^-$
- $B_s \to \phi\mu^+\mu^-$

combined fit would be very challenging ⟶ **130 nuisance parameter**

Global fit to $b \to s\mu^+\mu^-$ (results)

we **obtain good fits**, agreement between the three fits

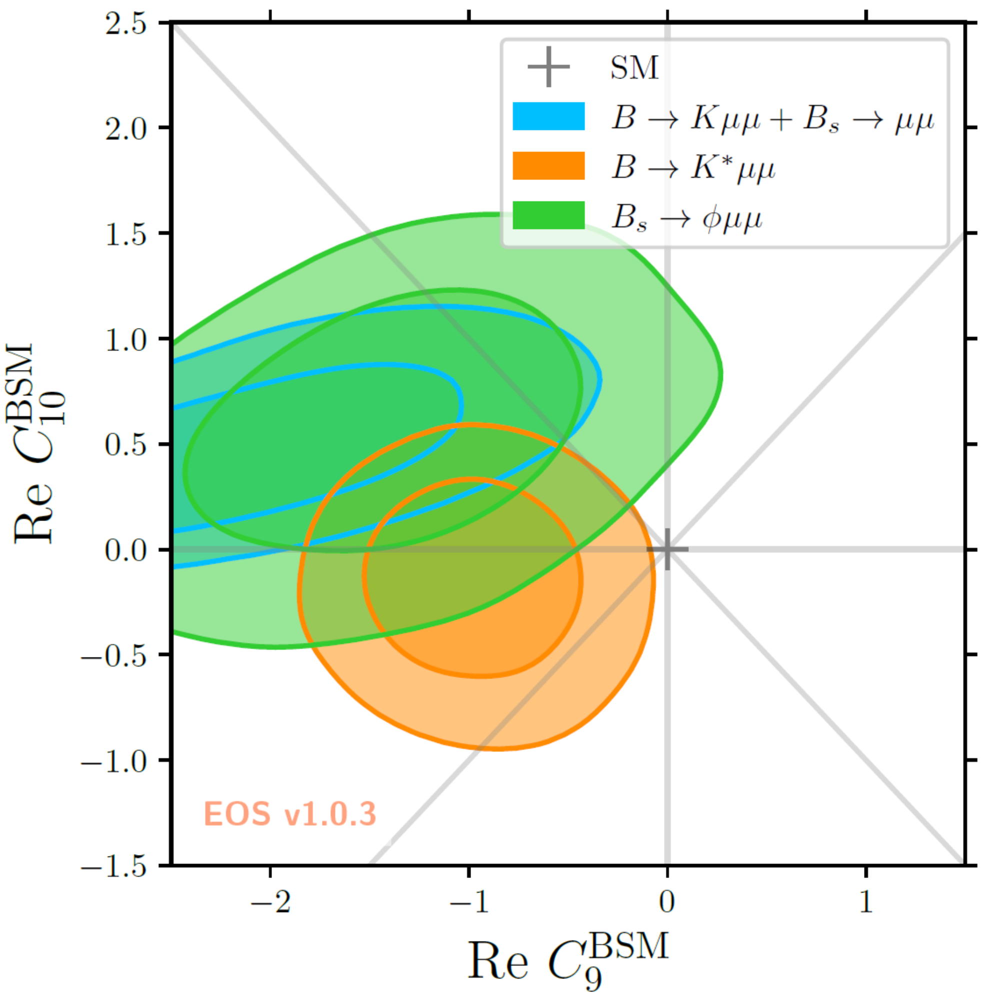
substantial tension w.r.t. SM (in agreement with the literature)

pulls (p value of the SM hypothesis):

- 5.7σ for $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- 2.7σ for $B \to K^*\mu^+\mu^-$
- 2.6σ for $B_s \to \phi\mu^+\mu^-$

local FFs $\mathcal{F}_\lambda$ main uncertainties

non-local FFs $\mathcal{H}_\lambda$ cannot explain this tension



Summary and conclusion

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1. reassess BGL parametrization for local FFs $\mathcal{F}_\lambda$ to consider below threshold branch cut and obtain more constraining dispersive bound

   combine theory inputs in new dispersive analysis of the local FFs $\mathcal{F}_\lambda$

   [Reboud/NG/van Dyk/Virto w.i.p.]

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2. new approach for non-local FFs $\mathcal{H}_\lambda$ that combines our OPE calculation at $q^2 < 0$, experimental data for $B \to K^{(*)} J/\psi$, and a dispersive bound

   dispersive bound allows to control truncation error [NG/van Dyk/Virto 2020]

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3. obtain improved SM predictions using local and non-local FFs results

   tension between theory and experiment $\Longrightarrow$ understand the origin of this tension

   [NG/Reboud/van Dyk/Virto 2022]

Thank you!