The muon $g-2$ within the SM

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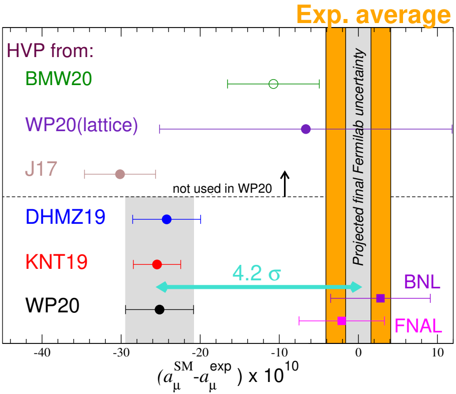
La Thuile 2023

Les Rencontres de Physique de la Vallée d'Aoste

Session on $(g-2)_\mu$



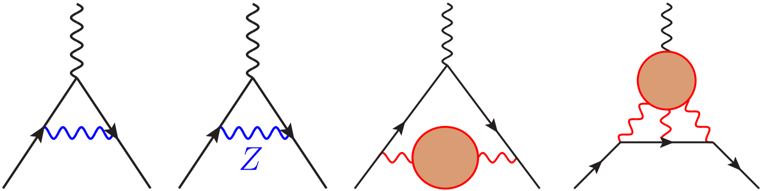
- **Muon anomalous magnetic moment**, $a_\mu = (g-2)_\mu/2$, in the SM

  $\hookrightarrow$ focus on data-driven methods this talk

- Lattice QCD talk by M. Marinković

- Experiment talk by P. Girotti

Anomalous magnetic moments of charged leptons



- **SM prediction for $(g-2)_\ell$**

$$a_\ell^{\text{SM}} = a_\ell^{\text{QED}} + a_\ell^{\text{EW}} + a_\ell^{\text{had}}$$

- For the electron: electroweak and hadronic contributions under control
- For a precision calculation need:
  - Independent input for α
  - Higher-order QED contributions
- For the muon: by far main uncertainty from the hadronic contributions
- For the tau: see backup SuperKEKB with electron polarization upgrade?

Anomalous magnetic moment of the electron: fine-structure constant

- Input from **atom interferometry**

$$\alpha^2 = \frac{4\pi R_\infty}{c} \times \frac{m_{\text{atom}}}{m_e} \times \frac{\hbar}{m_{\text{atom}}}$$

- With **Rb measurement** LKB 2011 (a_e^{exp} Harvard 2008)

$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.73(28) \times 10^{-12}$$

$$a_e^{\text{SM}} = 1{,}159{,}652{,}182.03(1)_{\text{5-loop}}(1)_{\text{had}}(72)_{\alpha(\text{Rb})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = -1.30(77) \times 10^{-12}[1.7\sigma]$$

$\hookrightarrow$ α limiting factor, but more than an order of magnitude to go in theory

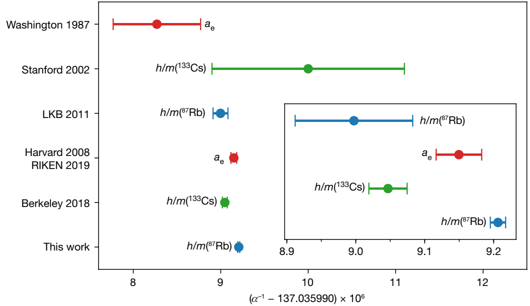
- With **Cs measurement** Berkeley 2018, Science 360 (2018) 191

$$a_e^{\text{SM}} = 1{,}159{,}652{,}181.61(1)_{\text{5-loop}}(1)_{\text{had}}(23)_{\alpha(\text{Cs})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = -0.88(36) \times 10^{-12}[2.5\sigma]$$

$\hookrightarrow$ for the first time a_e^{exp} limiting factor

Anomalous magnetic moment of the electron: fine-structure constant



LKB 2020

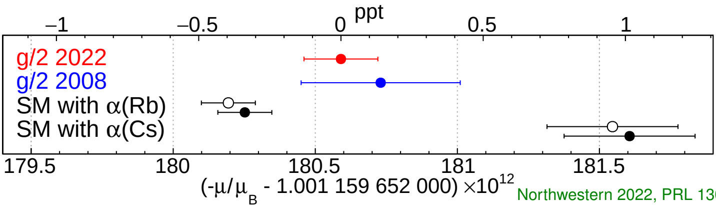
During the interferometer sequence, we apply a frequency ramp to compensate the Doppler shift induced by gravity. Nonlinearity in the delay of the optical phase-lock loop induces a residual phase shift that is measured and corrected for each spectrum. These systematic effects were not considered in our previous measurement[18] (see Fig. 1), which could explain the 2.4σ discrepancy between that measurement and the present one. Unfortunately, we do not have available data to evaluate retrospectively the contributions of the phase shift in the Raman phase-lock loop and of short-scale fluctuations in the laser intensity to the 2011 measurement. Thus, we cannot firmly state that these two effects are the cause of the 2.4σ discrepancy between our two measurements.

- Tensions
  - Berkeley 2018 vs. LKB 2020: 5.4σ
  - LKB 2011 vs. LKB 2020: 2.4σ

- With new **Rb measurement** LKB 2020, Nature 588 (2020) 61

$$a_e^{\text{SM}} = 1{,}159{,}652{,}180.25(1)_{\text{5-loop}}(1)_{\text{had}}(9)_{\alpha(\text{Rb})} \times 10^{-12}$$

$$a_e^{\text{exp}} - a_e^{\text{SM}} = 0.48(30) \times 10^{-12} [1.6\sigma]$$



- Latest development: new measurement of a_e^{exp}

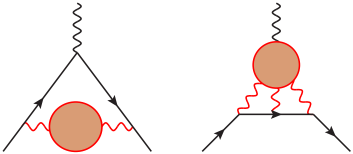
$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.59(13) \times 10^{-12}$$
$$a_e^{\text{exp}} - a_e^{\text{SM}}[\text{Cs}] = -1.02(26) \times 10^{-12}[3.9\sigma]$$
$$a_e^{\text{exp}} - a_e^{\text{SM}}[\text{Rb}] = 0.34(16) \times 10^{-12}[2.1\sigma]$$

- Another 4.8σ tension in 5-loop QED coefficient

  ↪ full circles Aoyama et al. 2019 vs. open circles Volkov 2019

- BSM sensitivity of a_e depends on resolution of this experimental 5σ discrepancy!

Hadronic effects in $(g-2)_\mu$



- **Hadronic vacuum polarization**: need hadronic two-point function

$$\Pi_{\mu\nu} = \langle 0|T\{j_\mu j_\nu\}|0\rangle$$

- **Hadronic light-by-light scattering**: need hadronic four-point function

$$\Pi_{\mu\nu\lambda\sigma} = \langle 0|T\{j_\mu j_\nu j_\lambda j_\sigma\}|0\rangle$$

- Here: focus on data-driven methods, for lattice QCD see next talk by M. Marinković

Muon $g-2$ Theory Initiative

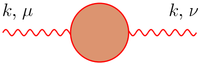
- Maximize the impact of the Fermilab and J-PARC experiments
  https://muon-gm2-theory.illinois.edu/

  $\hookrightarrow$ **quantify and reduce the theory uncertainties on the hadronic corrections**

- Summarize the theory status and assess reliability of uncertainty estimates
- Organize workshops to bring the different communities together:
  - First plenary workshop @ Fermilab: 3–6 June 2017
  - HVP workshop @ KEK: 12–14 Feb 2018
  - HLbL workshop @ UConn: 12–14 Mar 2018
  - Second plenary workshop @ Mainz: 18–22 June 2018
  - Third plenary workshop @ Seattle: 9–13 Sep 2019
  - Lattice HVP workshop (virtual): 16–20 Nov 2020
  - Fourth plenary workshop @ KEK (virtual): 28 June–2 July 2021
  - Fifth plenary workshop @ Edinburgh: 5–9 Sep 2022 https://indico.ph.ed.ac.uk/event/112/
  - Sixth plenary workshop @ Bern: 4–8 Sep 2023 https://indico.cern.ch/event/1258310/
- White paper (WP20) Phys. Rept. 887 (2020) 1: "The anomalous magnetic moment of the muon in the SM"

Hadronic vacuum polarization: how to connect to experiment

- General principles yield **direct connection with experiment**
  - **Gauge invariance**



$$= -i\left(k^2 g^{\mu\nu} - k^\mu k^\nu\right)\Pi(k^2)$$

  - **Analyticity**

$$\Pi_{\text{ren}} = \Pi(k^2) - \Pi(0) = \frac{k^2}{\pi}\int_{4M_\pi^2}^{\infty} ds \frac{\operatorname{Im}\Pi(s)}{s(s-k^2)}$$

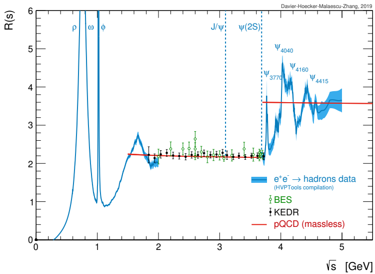
  - **Unitarity**

$$\operatorname{Im}\Pi(s) = -\frac{s}{4\pi\alpha}\sigma_{\text{tot}}\left(e^+e^- \to \text{hadrons}\right) = -\frac{\alpha}{3}R_{\text{had}}(s)$$

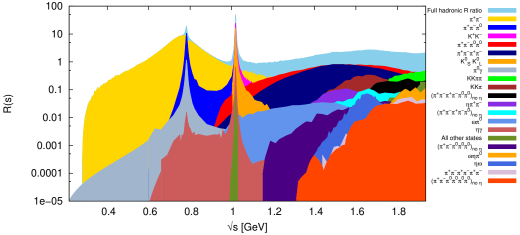
Master formula for HVP contribution to a_μ

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)$$

Hadronic vacuum polarization from e^+e^- data



Davier, Hoecker, Malaescu, Zhang 2019



Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
  - cross sections defined photon-inclusively
    $\hookrightarrow$ threshold $s_{\text{thr}} = M_{\pi^0}^2$ due to $\pi^0\gamma$ channel
  - up to about 2 GeV: sum of exclusive channels
  - above: inclusive data + narrow resonances + pQCD
- **Tensions in the data**: most notably between KLOE and BaBar 2π data

  $\hookrightarrow$ extensive discussion in WP of current status and consequences

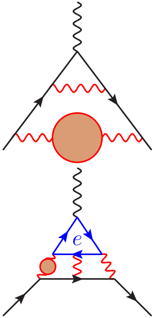
Data-driven determination of HVP: our recommendation from WP20

HVP from e^+e^- data

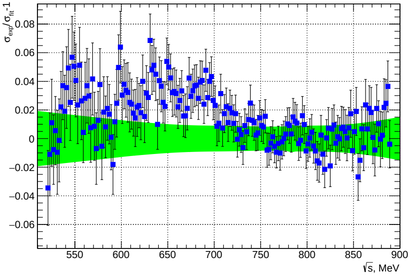
$$a_\mu^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11} = 6931(40) \times 10^{-11}$$

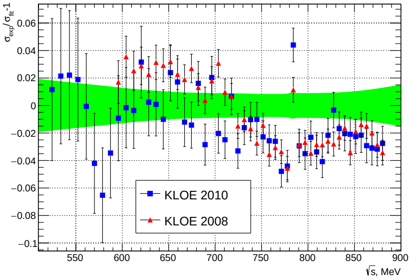
$$a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error
  $\hookrightarrow$ would get $4.2\sigma \rightarrow 4.8\sigma$ when ignoring additional systematics
- Systematic effect dominated by [fit w/o KLOE - fit w/o BaBar]/2
- a_μ^{HVP} includes NLO Calmet et al. 1976 and NNLO Kurz et al. 2014 iterations



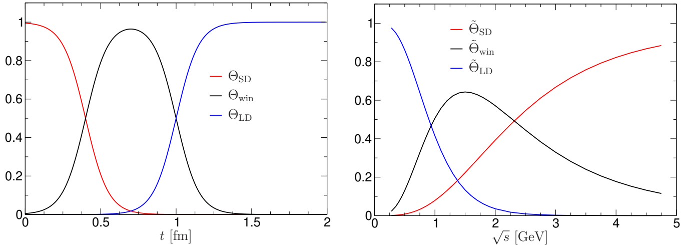
New data since WP20 (prior to CMD-3)





BaBar vs. SND 20 SND 2020 KLOE vs. SND 20

- New data from SND experiment not yet included in WP20 number
  $\hookrightarrow$ lie between BaBar and KLOE
- New data for 3π: BESIII, BaBar
- New data on inclusive region: BESIII (slight tension with pQCD)



- BMWc still **only complete calculation** at similar level of precision as e^+e^- data

$$a_\mu^{\text{HVP, LO}}[e^+e^-] = 6931(40) \times 10^{-11} \qquad a_\mu^{\text{HVP, LO}}[\text{BMWc}] = 7075(55) \times 10^{-11}$$

$\hookrightarrow$ globally 2.1σ

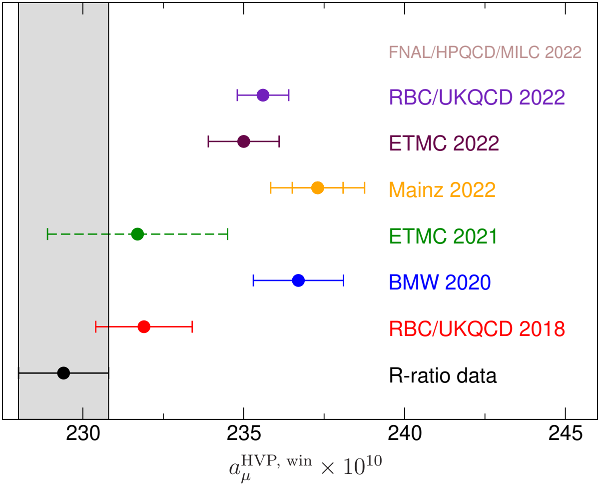
- Idea RBC/UKQCD 2018: define partial quantities

$$a_\mu^{\text{HVP, LO, win}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s)\tilde{\Theta}_{\text{win}}(s)$$

$\hookrightarrow$ smaller systematic errors for same quantity in lattice QCD next talk

$\hookrightarrow$ tool for the comparison to e^+e^- data

A puzzle in the intermediate window: e^+e^- vs. lattice QCD



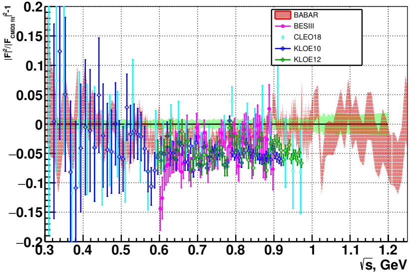
RBC/UKQCD 2022 supersedes RBC/UKQCD 2018

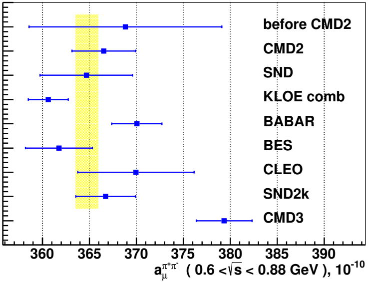
ETMC 2022 supersedes ETMC 2021

FNAL/HPQCD/MILC 2022 agrees for *ud* connected contribution, same for Aubin et al. 2022, χQCD 2022

R-ratio result from Colangelo et al. 2022

A new puzzle: $e^+e^- \to \pi^+\pi^-$ from CMD-3





CMD-3, 2302.08834

generally shows larger pion form factor in the whole energy range under discussion. The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ-meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. The source of this difference is unknown at the moment.

Where to go from here?

- **Need to understand the details of CMD-3 result**

  $\hookrightarrow$ seminar + discussion (online) organized by TI, likely in March
- New data on the 2π channel forthcoming:
  - New BaBar and KLOE analyses (a lot more data not analyzed so far)
  - Full statistics of SND
  - New data from BESIII and Belle II
- In addition:
  - **Improved lattice-QCD calculations for full HVP**, more windows
  - Further scrutiny of radiative corrections
  - Potentially τ data to be resurrected as a viable cross check if **progress on isospin breaking** allows (lattice QCD, dispersive)
  - Independent HVP determination from **MuonE**

Some first comments from analyticity and unitarity constraints

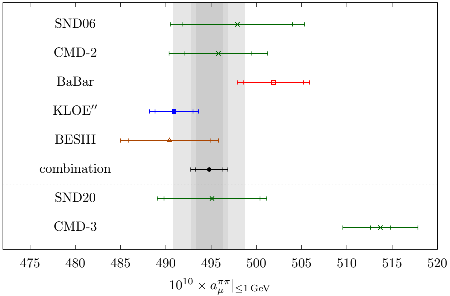
The pion form factor from dispersion relations

$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{elastic } \pi\pi \text{ scattering}} \times \underbrace{G_\omega(s)}_{\text{isospin-breaking } 3\pi \text{ cut}} \times \underbrace{G_{\text{in}}(s)}_{\text{inelastic effects: } 4\pi, \ldots}$$

- $e^+e^- \to \pi^+\pi^-$ cross section subject to strong constraints from **analyticity**, **unitarity**, **crossing symmetry**, leading to dispersive representation with few parameters Colangelo, MH, Stoffer, 2018, 2021, 2022, work in progress
  - **Elastic $\pi\pi$ scattering**: two values of phase shifts
  - **ρ–ω mixing**: ω pole parameters and residue
  - **Inelastic states**: conformal polynomial

$\hookrightarrow$ cross check on data, functional form for all $s \leq 1\,\text{GeV}^2$

Some first comments from analyticity and unitarity constraints

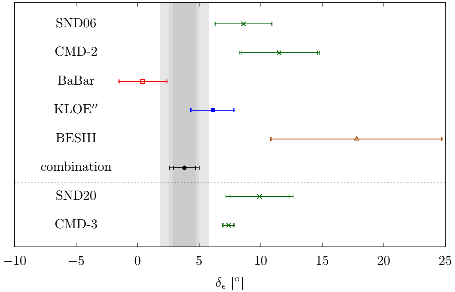


	$a_\mu^{\pi\pi}\|_{\leq 1\text{ GeV}}$	$a_\mu^{\pi\pi}\|_{[0.60,0.88]\text{ GeV}}$	$a_\mu^{\pi\pi}\|_{\text{win}}$
SND06	1.7σ	1.8σ	1.7σ
CMD-2	2.0σ	2.3σ	2.1σ
BaBar	2.9σ	3.3σ	3.1σ
KLOE″	4.8σ	5.6σ	5.4σ
BESIII	2.8σ	3.0σ	3.1σ
SND20	2.1σ	2.2σ	2.2σ
comb	3.7σ [5.0σ]	4.2σ [6.1σ]	3.8σ [5.7σ]

- Tensions in $a_\mu^{\pi\pi}|_{\leq 1\text{ GeV}}$ compared to CMD-3:
  - Inner/outer error: experiment/total (also shown: combination + BaBar/KLOE error)
  - Theory error dominated by order in conformal polynomial N
- No red flags for CMD-3 so far, but:
  - Large systematic error from N, correlated/anticorrelated for BaBar/other experiments
  - $\pi\pi$ phase shifts remain reasonable, main change in conformal polynomial

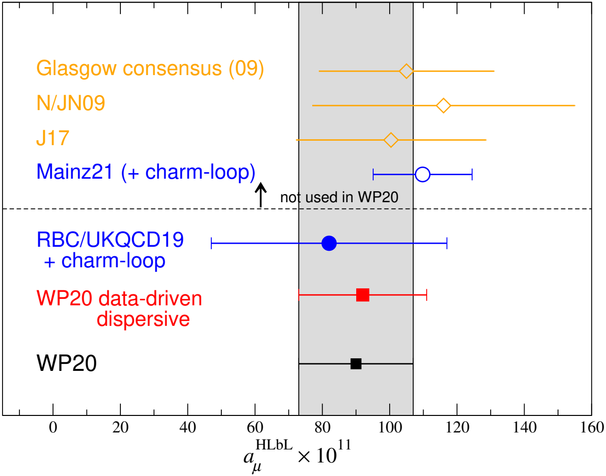
$\hookrightarrow$ suggests that inelastic effects could give a handle on the tension

Some first comments from analyticity and unitarity constraints



- Can also study consistency of hadronic parameters
  $\hookrightarrow$ **phase of the ρ–ω mixing parameter δ_ϵ**
  - δ_ϵ observable, since defined as a phase of a residue
  - δ_ϵ vanishes in isospin limit, but can be non-vanishing due to $\rho \to \pi^0\gamma, \eta\gamma, \pi\pi\gamma, \ldots \to \omega$
  - Combined-fit $\delta_\epsilon = 3.8(2.0)[1.2]^\circ$ agrees well with narrow-width expectation $\delta_\epsilon = 3.5(1.0)^\circ$, but **considerable spread among experiments**
  - Mass of the ω systematically too low compared to $e^+e^- \to 3\pi$

Hadronic light-by-light scattering: status



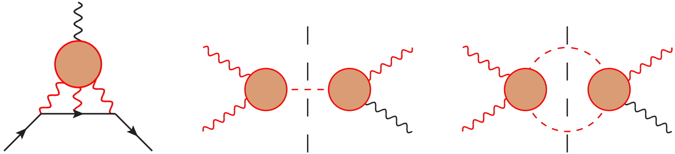
- Lattice QCD Mainz 2021, 2022:

$$a_\mu^{\text{HLbL}}[uds] = 107(15) \times 10^{-11}$$
$$a_\mu^{\text{HLbL}}[c] = 2.8(5) \times 10^{-11}$$

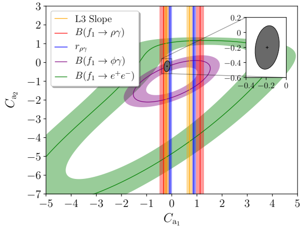
- Preliminary update from RBC/UKQCD 2022 also looks consistent

- Good agreement between lattice QCD and phenomenology at $\simeq 20 \times 10^{-11}$
- Need another factor of 2 for final Fermilab precision work in progress

Hadronic light-by-light scattering: data-driven, dispersive evaluations

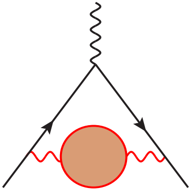


- Organized in terms of **hadronic intermediate states**, in close analogy to HVP Colangelo et al. 2014, ...
- Leading channels implemented with **data input for $\gamma^*\gamma^* \to$ hadrons**, e.g., $\pi^0 \to \gamma^*\gamma^*$
- Uncertainty dominated by subleading channels
  $\hookrightarrow$ **axial-vector mesons** $f_1(1285)$, $f_1(1420)$, $a_1(1260)$
- Transition form factors accessible in e^+e^- collisions
  $\hookrightarrow$ BESIII, Belle II (?)



Zanke, MH, Kubis 2021

- **Electron $g-2$**:
  - 5σ discrepancy in α currently the roadblock
- **Muon $g-2$**:
  - For HLbL agreement between lattice and phenomenology
    $\hookrightarrow$ another factor 2 looks feasible
  - HVP: puzzles in intermediate window and with CMD-3
  - New e^+e^- data and lattice calculations forthcoming
  - For prospects see also Snowmass contribution 2203.15810
  - WP update in preparation, with CMD-3 timeline unclear, but still aimed for 2023



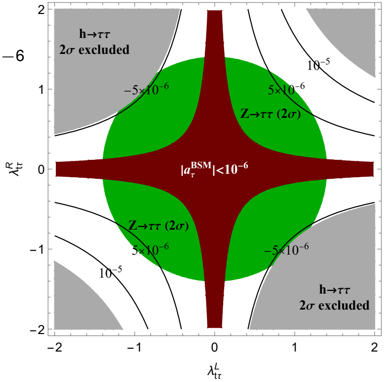
Sixth plenary TI workshop

What about $(g-2)_\tau$?

- Current status Abdallah et al. 2004, Keshavarzi et al. 2020

$$a_\tau^{\text{exp}} = -0.018(17) \quad vs. \quad a_\tau^{\text{SM}} = 1{,}177.171(39)\times 10^{-6}$$

- **Scaling arguments**:
  - Minimal flavor violation:
    $a_\tau^{\text{BSM}} \simeq a_\mu^{\text{BSM}} \left(\frac{m_\tau}{m_\mu}\right)^2 \simeq 0.7 \times 10^{-6}$
  - Electroweak contribution: $a_\tau^{\text{EW}} \simeq 0.5 \times 10^{-6}$
- **Concrete models**:
  - S_1 leptoquark model promising due to **chiral enhancement** with $\frac{m_t}{m_\tau}$
    $\hookrightarrow$ can get $a_\tau^{\text{BSM}} \simeq$ (few) $\times\, 10^{-6}$ without violating $h \to \tau\tau$ and $Z \to \tau\tau$



Crivellin, MH, Roney 2021

- Ultimate target has to be a measurement of a_τ at the level of 10^{-6}

$\hookrightarrow$ requires two-loop accuracy for theory throughout

Experimental prospects for $(g-2)_\tau$

- Many recent proposals, none of which seem to reach much beyond the Schwinger term
- Exception: $\boldsymbol{e^+e^- \to \tau^+\tau^-}$ **at Υ resonances** Bernabéu et al. 2007
  $\hookrightarrow$ quotes projections at 10^{-6} level
- Idea: study $e^+e^- \to \tau^+\tau^-$ cross section and asymmetries
  $\hookrightarrow$ could this be realized at Belle II Crivellin, MH, Roney 2021?
- Answer: yes, but requires **polarization upgrade of SuperKEK** to get access to transverse and longitudinal asymmetries
  $\hookrightarrow$ Hiroshima Workshop on Beam Polarization Feb 8+9, `https://indico.belle2.org/event/7500/`
- Idea: extract $F_2(s)$ at $s \simeq (10\,\text{GeV})^2$, but heavy new physics decouples
  $\hookrightarrow$ $a_\tau^{\text{BSM}} = F_2^{\text{exp}}(s) - F_2^{\text{SM}}(s)$ as long as $s \ll \Lambda^2_{\text{BSM}}$
- Bounds on light BSM become model dependent, but anyway better constrained in other processes

First attempt: total cross section

- **Differential cross section** for $e^+e^- \to \tau^+\tau^-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}\bigg[(2-\beta^2\sin^2\theta)\Big(|F_1|^2-\gamma^2|F_2|^2\Big) + 4\text{Re}\,\big(F_1F_2^*\big) + 2(1+\gamma^2)|F_2|^2\bigg]$$

  with scattering angle θ, $\beta=\sqrt{1-4m_\tau^2/s}$, $\gamma=\sqrt{s}/(2m_\tau)$

- Interference term $4\text{Re}\,\big(F_1F_2^*\big)$ sensitive to the sought two-loop effects

- Could be determined by fit to θ dependence

- But: need to measure total cross section at 10^{-6}

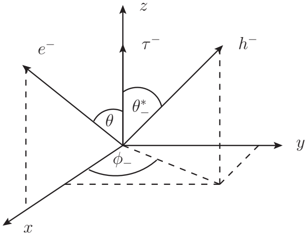
  $\hookrightarrow$ **can we use asymmetries instead**?

- Usual forward–backward asymmetry ($z=\cos\theta$)

$$\sigma_{\text{FB}} = 2\pi\bigg[\int_0^1 dz\frac{d\sigma}{d\Omega} - \int_{-1}^0 dz\frac{d\sigma}{d\Omega}\bigg]$$

  alone does not help

Second attempt: normal asymmetry



- Idea: use **polarization information of the $\tau^\pm$**

  $\hookrightarrow$ semileptonic decays $\tau^\pm \to h^\pm \overset{(-)}{\nu}_\tau$, $h = \pi, \rho, \ldots$

  Bernabéu et al. 2007

- Polarization characterized by

$$\mathbf{n}^*_\pm = \mp\alpha_\pm \begin{pmatrix} \sin\theta^*_\pm \cos\phi_\pm \\ \sin\theta^*_\pm \sin\phi_\pm \\ \cos\theta^*_\pm \end{pmatrix} \qquad \alpha_\pm \equiv \frac{m_\tau^2 - 2m_{h^\pm}^2}{m_\tau^2 + 2m_{h^\pm}^2} = \begin{cases} 0.97 & h^\pm = \pi^\pm \\ 0.46 & h^\pm = \rho^\pm \end{cases}$$

  $\hookrightarrow$ angles in $\tau^\pm$ rest frame

- **Normal asymmetry**

$$A_N^\pm = \frac{\sigma_L^\pm - \sigma_R^\pm}{\sigma} \propto \text{Im}\, F_2(s) \qquad \sigma_L^\pm = \int_\pi^{2\pi} d\phi_\pm \frac{d\sigma_{\text{FB}}}{d\phi_\pm} \quad \sigma_R^\pm = \int_0^\pi d\phi_\pm \frac{d\sigma_{\text{FB}}}{d\phi_\pm}$$

  $\hookrightarrow$ only get the imaginary part, **need electron polarization**

Third attempt: electron polarization

- **Transverse and longitudinal asymmetries** Bernabéu et al. 2007

$$A_T^\pm = \frac{\sigma_R^\pm - \sigma_L^\pm}{\sigma} \qquad A_L^\pm = \frac{\sigma_{\text{FB},\,R}^\pm - \sigma_{\text{FB},\,L}^\pm}{\sigma}$$

- Constructed based on helicity difference

$$d\sigma_{\text{pol}}^S = \frac{1}{2}\Big(d\sigma^{S\lambda}\big|_{\lambda=1} - d\sigma^{S\lambda}\big|_{\lambda=-1}\Big)$$

and then integrating over angles

$$\sigma_R^\pm = \int_{-\pi/2}^{\pi/2} d\phi_\pm \frac{d\sigma_{\text{pol}}^S}{d\phi_\pm} \qquad \sigma_L^\pm = \int_{\pi/2}^{3\pi/2} d\phi_\pm \frac{d\sigma_{\text{pol}}^S}{d\phi_\pm} \qquad \sigma_{\text{FB},\,R}^\pm = \int_0^1 dz_\pm^* \frac{d\sigma_{\text{FB, pol}}^S}{dz_\pm^*} \qquad \sigma_{\text{FB},\,L}^\pm = \int_{-1}^0 dz_\pm^* \frac{d\sigma_{\text{FB, pol}}^S}{dz_\pm^*}$$

- Linear combination

$$A_T^\pm - \frac{\pi}{2\gamma} A_L^\pm = \mp\alpha_\pm \frac{\pi^2\alpha^2\beta^3\gamma}{4s\sigma}\Big[\text{Re}\,\big(F_2 F_1^*\big) + |F_2|^2\Big]$$

isolates the interesting interference effect

How to make use of this?

Contributions to Re $F_2^{\text{eff}}(s)$	$s=0$	$s=(10\,\text{GeV})^2$
1-loop QED	1161.41	−265.90
e loop	10.92	−2.43
μ loop	1.95	−0.34
2-loop QED (mass independent)	−0.42	−0.24
HVP	3.33	−0.33
EW	0.47	0.47
total	1177.66	−268.77

$$\text{Re}\,F_2^{\text{eff}}((10\,\text{GeV})^2) \simeq \mp\frac{0.73}{\alpha_\pm}\Big(A_T^\pm - 0.56A_L^\pm\Big)$$

- **Strategy**:
  - Measure effective $F_2(s)$

$$\text{Re}\,F_2^{\text{eff}} = \mp\frac{8(3-\beta^2)}{3\pi\gamma\beta^2\alpha_\pm}\Big(A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm\Big)$$

  - Compare measurement to SM prediction for Re F_2^{eff}
  - Difference gives constraint on a_τ^{BSM}
  - A measurement of $A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm$ at $\lesssim 1\%$ would already be competitive with current limits

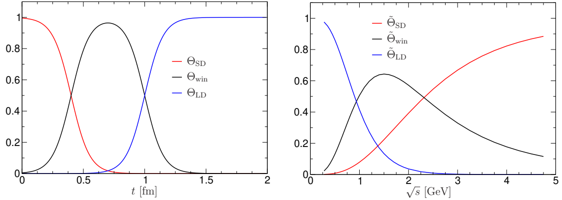
How to make use of this?

- **Challenges**:
  - Cancellation in $A_T^{\pm} - \frac{\pi}{2\gamma} A_L^{\pm}$: $A_{T,L}^{\pm} = \mathcal{O}(1)$, difference $\mathcal{O}(\alpha)$
  - Two-loop calculation in SM see 2111.10378 for form factor and radiative corrections
  - Form factor only dominates for resonant $\tau^+\tau^-$ pairs

$$|H(M_\Upsilon)|^2 = \left(\frac{3}{\alpha}\text{Br}(\Upsilon \to e^+e^-)\right)^2 \simeq 100$$

  - However: continuum pairs dominate even at $\Upsilon(nS)$, $n = 1, 2, 3$, due to energy spread
  - Should consider $A_T^{\pm}$, $A_L^{\pm}$ also for nonresonant $\tau^+\tau^-$, but requires substantial investment in theory for SM prediction (box diagrams, ...)

What can we conclude about the difference at the moment?

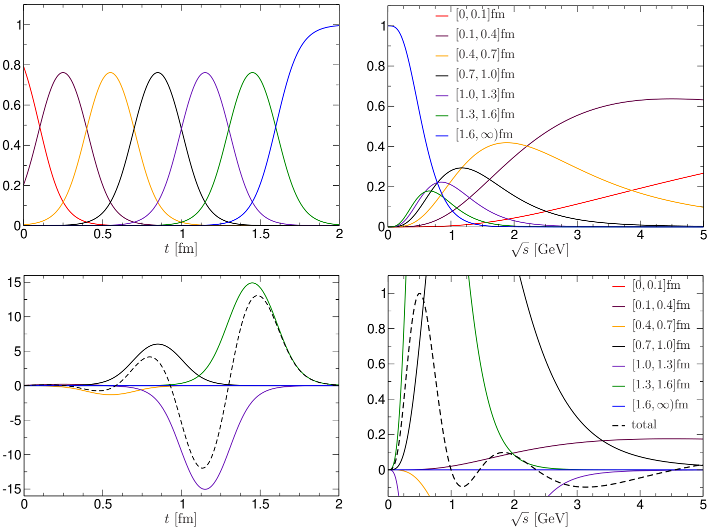


- Difference in full HVP between BMWc and e^+e^- about $14.4(6.8) \times 10^{-10}$, thereof $7.3(2.0) \times 10^{-10}$ from intermediate window
- Can one modify the 2π cross section to accommodate change? Colangelo et al. 2022

  $\hookrightarrow$ yes, but **not simultaneously for full HVP and window**
- Assuming
  - uniform shifts in low-energy $\pi\pi$ region
  - no significant negative shifts

  $\hookrightarrow$ at least $\simeq 40\%$ from **above 1 GeV**
- Changes above $\simeq 2\,\text{GeV}$ constrained by hadronic running of α BMWc, Mainz

Window quantities: the inverse Laplace problem



Colangelo et al. 2022

↪ localization in energy entails **strong cancellation in Euclidean time**

2π channel: isospin breaking and ω mass

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \text{Re}\,\epsilon_\omega$	δ_ϵ [°]	$10^{10} \times a_\mu^{\pi\pi}\|_{\leq 1\,\text{GeV}}$
SND06	1.40	5.3%	781.49(32)(2)	2.03(5)(2)		499.7(6.9)(4.1)
	1.08	35%	782.11(32)(2)	1.98(4)(2)	8.5(2.3)(0.3)	497.8(6.1)(4.9)
CMD-2	1.18	14%	781.98(29)(1)	1.88(6)(2)		496.9(4.0)(2.3)
	1.01	45%	782.64(33)(4)	1.85(6)(4)	11.4(3.1)(1.0)	495.8(3.7)(4.2)
BaBar	1.14	5.7%	781.86(14)(1)	2.04(3)(2)		501.9(3.3)(2.0)
	1.14	5.5%	781.93(18)(4)	2.03(4)(1)	1.3(1.9)(0.7)	501.9(3.3)(1.8)
KLOE''	1.20	3.1%	781.81(16)(3)	1.98(4)(1)		491.8(2.1)(1.8)
	1.13	10%	782.42(23)(5)	1.95(4)(2)	6.1(1.7)(0.6)	490.8(2.0)(1.7)
BESIII	1.12	25%	782.18(51)(7)	2.01(19)(9)		490.8(4.8)(3.9)
	1.02	44%	783.05(60)(2)	1.99(19)(7)	17.6(6.9)(1.2)	490.3(4.5)(3.1)
SND20	2.93	3.3×10^{-7}	781.79(30)(6)	2.04(6)(3)		494.2(6.7)(9.0)
	1.87	4.1×10^{-3}	782.37(28)(6)	2.02(5)(2)	10.1(2.4)(1.4)	494.9(5.3)(3.1)

Colangelo et al. 2022

- Mysteries in the fit:
  - Phase of the ρ–ω mixing parameter varies widely among experiments
  - Resulting value of M_ω at odds with 3π, $\pi^0\gamma$ channel

$\hookrightarrow$ hopefully forthcoming data will shed some light

Relation to global electroweak fit

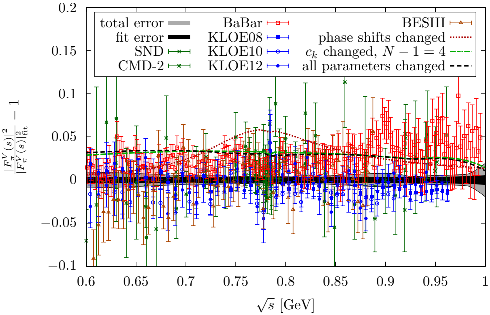
Hadronic running of α

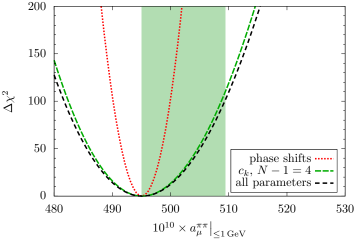
$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ enters as input in **global electroweak fit**

  $\hookrightarrow$ integral weighted more strongly towards high energy Passera, Marciano, Sirlin 2008
- Changes in $R_{\text{had}}(s)$ have to occur at low energies, $\lesssim 2\,\text{GeV}$ Crivellin et al. 2020, Keshavarzi et al. 2020, Malaescu et al. 2020
- This seems to happen for BMWc calculation (translated from the space-like), with only moderate increase of tensions in the electroweak fit ($\sim 1.8\sigma \to 2.4\sigma$)

  $\hookrightarrow$ need **large changes in low-energy cross section**
- Similar conclusion from Mainz 2022 calculation of hadronic running



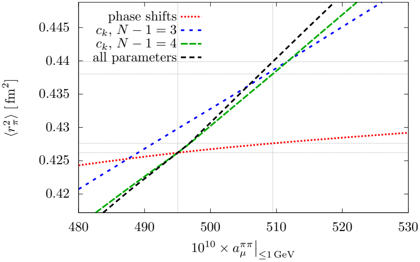


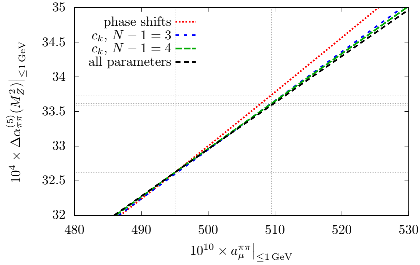
Colangelo, MH, Stoffer 2020

- Changes in 2π cross section **cannot be arbitrary** due to analyticity/unitarity constraints, but increase is actually possible
- Three scenarios:
  1. "Low-energy" scenario: **$\pi\pi$ phase shifts**
  2. "High-energy" scenario: **conformal polynomial**
  3. Combined scenario

$\hookrightarrow$ 2. and 3. lead to uniform shift, 1. concentrated in ρ region

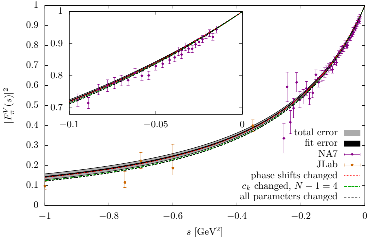
Correlations



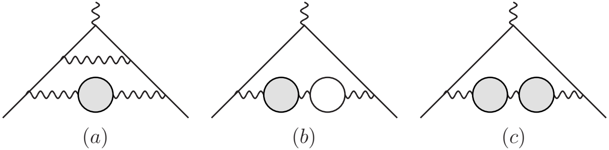


Correlations with other observables:

- **Pion charge radius $\langle r_\pi^2 \rangle$**
  - $\hookrightarrow$ significant change in scenarios 2. and 3.
  - $\hookrightarrow$ can be tested in lattice QCD
- **Hadronic running of α**
- **Space-like pion form factor**



FAQ 1: do e^+e^- data and lattice really measure the same thing?



- Conventions for **bare cross section**
  - Includes radiative intermediate states and final-state radiation: $\pi^0\gamma, \eta\gamma, \pi\pi\gamma, \ldots$
  - Initial-state radiation and VP subtracted to avoid double counting
- NLO HVP insertions

$$a_\mu^{\text{HVP, NLO}} \simeq [\underbrace{-20.7}_{(a)} + \underbrace{10.6}_{(b)} + \underbrace{0.3}_{(c)}] \times 10^{-10} = -9.8 \times 10^{-10}$$

$\hookrightarrow$ dominant VP effect from leptons, HVP iteration very small

- Important point: **no need to specify hadronic resonances**

  $\hookrightarrow$ calculation set up in terms of decay channels

FAQ 1: do e^+e^- data and lattice really measure the same thing?

- HVP in subtraction determined iteratively (converges with α) and self-consistently

$$\alpha(q^2)=\frac{\alpha(0)}{1-\Delta\alpha_{\text{lep}}(q^2)-\Delta\alpha_{\text{had}}(q^2)} \qquad \Delta\alpha_{\text{had}}(q^2)=-\frac{\alpha q^2}{3\pi}P\int_{s_{\text{thr}}}^{\infty}\text{d}s\frac{R_{\text{had}}(s)}{s(s-q^2)}$$

- Subtlety for very narrow $c\bar{c}$ and $b\bar{b}$ resonances (ω and ϕ perfectly fine)

  $\hookrightarrow$ Dyson series does not converge Jegerlehner

- Solution: take out resonance that is being corrected in R_{had} in VP undressing

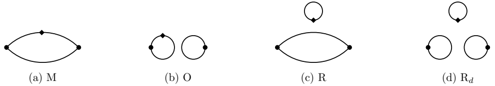
- How to match all of this on the lattice?

- Need to calculate all sorts of **isospin-breaking (IB) corrections**

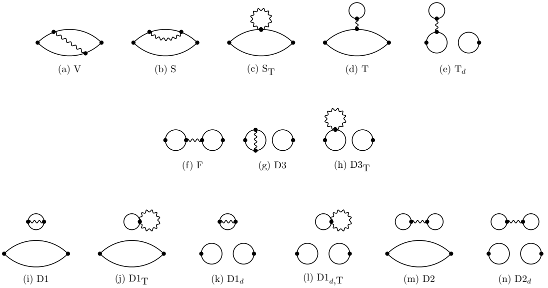
  $\hookrightarrow$ e^2 (QED) and $\delta=m_u-m_d$ (strong IB) corrections

FAQ 1: do e^+e^- data and lattice really measure the same thing?

- **Strong isospin breaking** $\propto m_u - m_d$



- **QED effects** $\propto \alpha$



plots from Gülpers et al. 2018

- Diagram (f) F critical for consistent VP subtraction

  $\hookrightarrow$ same diagram without additional gluons is subtracted RBC/UKQCD 2018

FAQ 1: do e^+e^- data and lattice really measure the same thing?

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	−0.09(7)	–	−7.62(14)	–	−7.67(22)	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	−0.29(1)	0.44(2)	−1.71(9)	2.63(14)	−1.24(6)	1.91(10)	−3.24(17)	4.98(26)
kaon mass ($\bar{K}^0K^0$)	0.00(0)	−0.41(2)	−0.01(0)	−2.44(12)	−0.01(0)	−1.78(9)	−0.02(0)	−4.62(23)
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	−2.44(16)	2.92(17)	−0.68(29)	4.04(39)
BMWc 2020	–	–	−0.09(6)	0.52(4)	–	–	−1.5(6)	1.9(1.2)
RBC/UKQCD 2018	–	–	0.0(2)	0.1(3)	–	–	−1.0(6.6)	10.6(8.0)
JLM 2021	–	–	–	–	–	–	–	3.32(89)

- Note: error estimates only refer to the effects included

  ↪ **additional channels missing** (most relevant for SD and int window)

- Reasonable agreement with BMWc 2020, RBC/UKQCD 2018, and James, Lewis, Maltman 2021

  ↪ if anything, the result would become even larger with pheno estimates

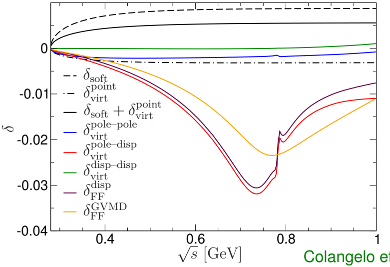
FAQ 2: can we trust radiative corrections/MC generators?

- Typical objection: can we really trust scalar QED in the MC generator?
- Report by Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

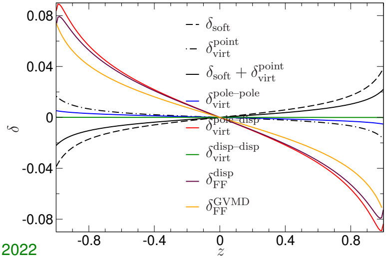
  $\hookrightarrow$ Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data (0912.0749)
- Never just use scalar QED, include pion form factor wherever possible

  $\hookrightarrow$ **FsQED**
- From the point of view of dispersion relations, this captures the **leading infrared enhanced effects**
- Existing NLO calculations do not point to (significant) center-of-mass-energy dependent effects Campanario et al. 2019
- Could there be subtleties in how the form factor is implemented or from pion rescattering?

FAQ 2: can we trust radiative corrections/MC generators?



Colangelo et al. 2022



- Test case: **forward–backward asymmetry** (*C*-odd)
- Large corrections found in GVMD model Ignatov, Lee 2022
- Can be reproduced using dispersion relations

  ↪ effect still comes from **infrared enhanced contributions**
- Relevant effects for the *C*-even contribution?

FAQ 3: what about the τ data?

- Why did people stop using $\tau \to \pi\pi\nu_\tau$ data?
  - Better precision from e^+e^-
  - **IB corrections not under sufficient control**
- If this issue could be solved, would yield very useful cross check

  $\hookrightarrow$ new data at least on spectrum from Belle II
- New developments from the lattice talk by M. Bruno at Edinburgh

  $\hookrightarrow$ re-using HLbL lattice data
- Long-distance QED (G_{EM}) still taken from phenomenology for the time being

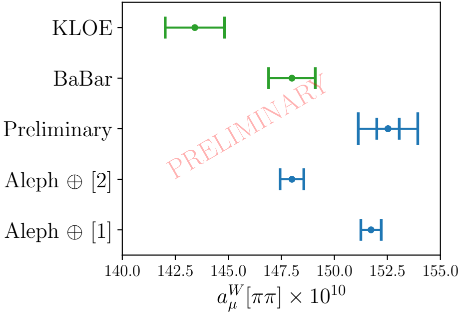
  $\hookrightarrow$ dispersive methods?

talk by M. Bruno at Edinburgh

Window fever - τ

my PRELIMINARY analysis of exp. + latt. data

- only exp. errs, no attempt at estimating sys. errs for [1] and [2]
- LQCD syst. errs require further investigation/improvements



Isospin-breaking:

[1]: w/o $\rho\gamma$ mixing

[2]: w/ $\rho\gamma$ mixing

What is $\rho\gamma$? too much to say, too little time to explain everything...

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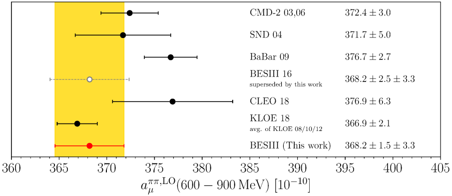
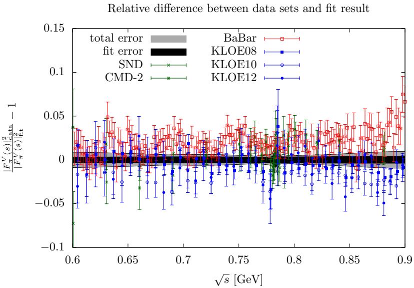
Cross checks from analyticity and unitarity



BESIII 2009.05011



Colangelo, MH, Stoffer 2018

- For "simple" channels $e^+e^- \to 2\pi, 3\pi$ can derive form of the cross section from **general principles of QCD** (analyticity, unitarity, crossing symmetry)
  $\hookrightarrow$ strong cross check on the data sets (covering about 80% of HVP)
- Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests

Merging procedure

- How to deal with tensions?

  $\hookrightarrow$ extensive discussion at TI workshops

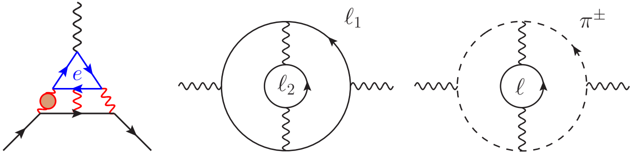
- Errors **systematics dominated**

  $\hookrightarrow$ scale factor not adequate/sufficient

- There was broad consensus to adopt **conservative error estimates**
- **Merging procedure**
  - Take average of central values from different analyses channel by channel (including analyticity/unitarity constraints)
  - In each channel: take biggest uncertainty from DHMZ/KNT, add half their difference as additional systematic effect
  - Exception: in 2π channel this additional systematic uncertainty taken as [fit w/o KLOE - fit w/o BaBar]/2
  - Take interchannel correlations from DHMZ analysis

  $\hookrightarrow$ **covers tensions in the data and accounts for different methodologies for the combination of data sets**

A note on higher-order hadronic effects



- Generic scaling of $\mathcal{O}(\alpha^4)$ effects: $\left(\frac{\alpha}{\pi}\right)^4 \simeq 3 \times 10^{-11}$
- Enhancements (numerical or $\log \frac{m_e}{m_\mu}$) can make such effects relevant Kurz et al. 2014
- NLO HLbL small Colangelo et al. 2014
- Mixed hadronic and leptonic contributions with inner electron potentially dangerous
  $\hookrightarrow$ could affect LO HVP via radiation of e^+e^- pairs, but $\lesssim 1 \times 10^{-11}$ MH, Teubner 2022

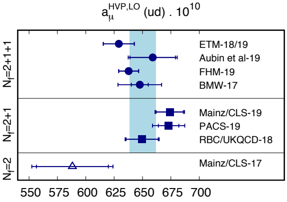
Lattice QCD calculations of HVP

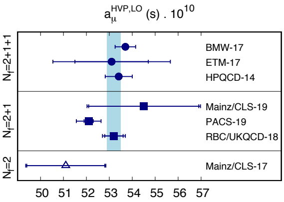
HVP from lattice QCD

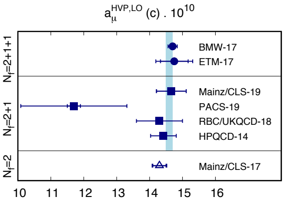
$$a_\mu^{\text{HVP, LO}} = a_{\mu,\,\text{conn}}^{\text{HVP, LO}}(ud) + \sum_{q=s,c,b} a_{\mu,\,\text{conn}}^{\text{HVP, LO}}(q) + a_{\mu,\,\text{disc}}^{\text{HVP, LO}} + a_{\mu,\,\text{IB}}^{\text{HVP, LO}}$$
$$= 7116(184) \times 10^{-11}$$

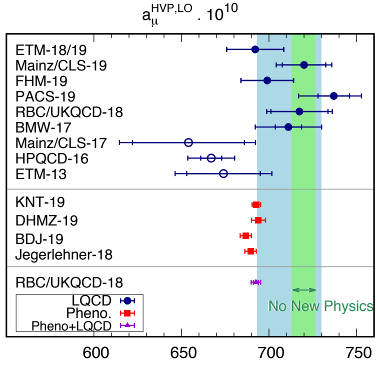
- Basic differences to data-driven approach:
  - Calculation in **space-like**, not time-like kinematics
  - **Decomposition by flavor**, not hadronic channel
  - Disconnected diagrams and isospin breaking calculated as corrections
- WP discussion includes:
  - Detailed discussion of computational strategy (e.g., schemes for isospin breaking)
  - Comparisons of calculations available as of the deadline 31 March, 2020
  - Averages of subquantities and total HVP

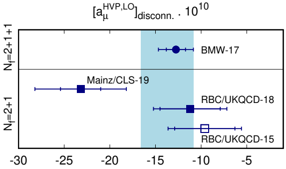
HVP from lattice QCD: averages











\+ isospin-breaking corrections

↪ many different calculations required for full HVP

Hadronic running of α and global EW fit

	e^+e^- KNT, DHMZ	EW fit HEPFit	EW fit GFitter	guess based on BMWc
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	270.2(3.0)	271.6(3.9)	277.8(1.3)
difference to e^+e^-		-1.8σ	-1.1σ	$+1.0\sigma$

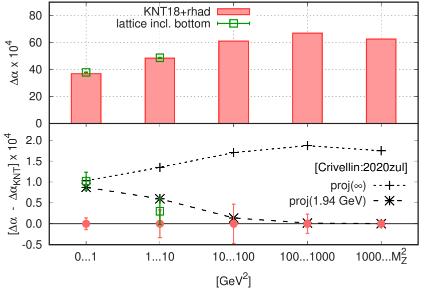
- **Time-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha M_Z^2}{3\pi} P \int_{s_{\text{thr}}}^{\infty} ds \frac{R_{\text{had}}(s)}{s(M_Z^2 - s)}$$

- **Space-like formulation**:

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \frac{\alpha}{\pi}\hat{\Pi}(-M_Z^2) + \frac{\alpha}{\pi}\left(\hat{\Pi}(M_Z^2) - \hat{\Pi}(-M_Z^2)\right)$$

- Global EW fit
  - Difference between HEPFit and GFitter implementation mainly treatment of M_W
  - Pull goes into **opposite direction**



BMWc 2020