

# GW190521 as the dynamical capture of two (nonspinning) black holes

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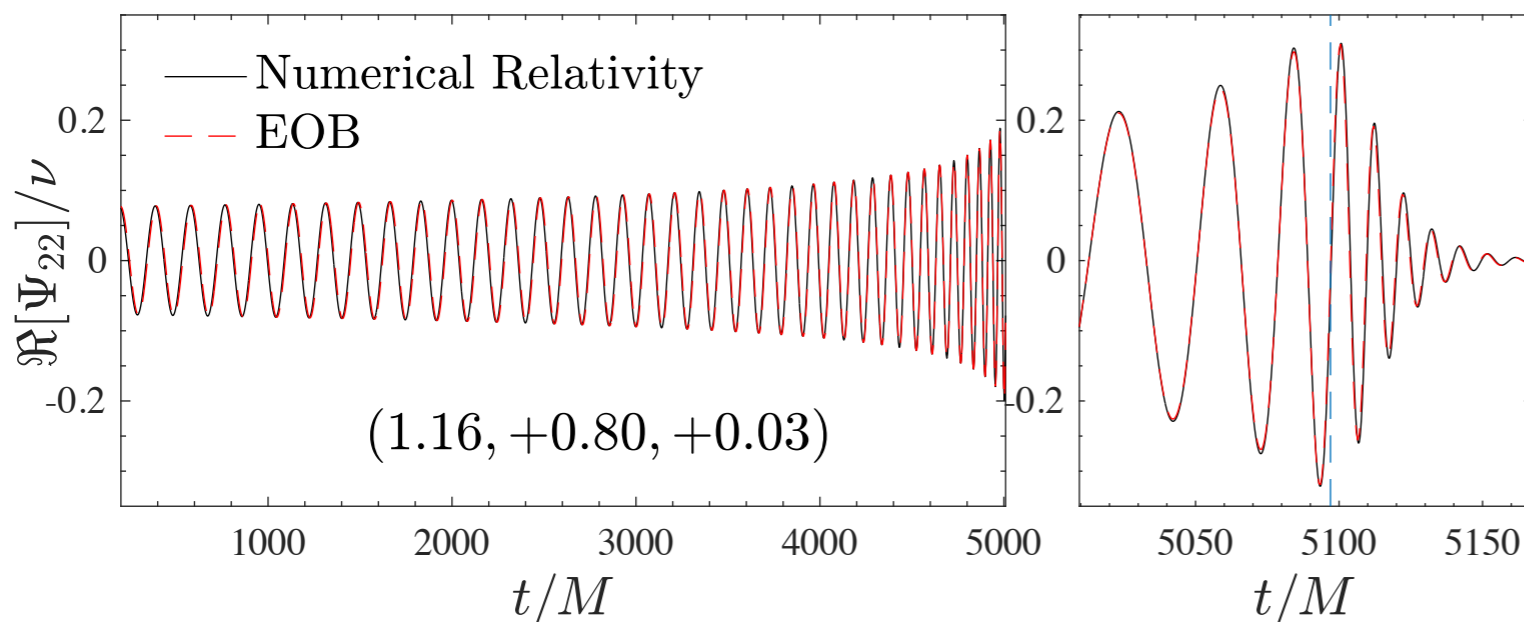
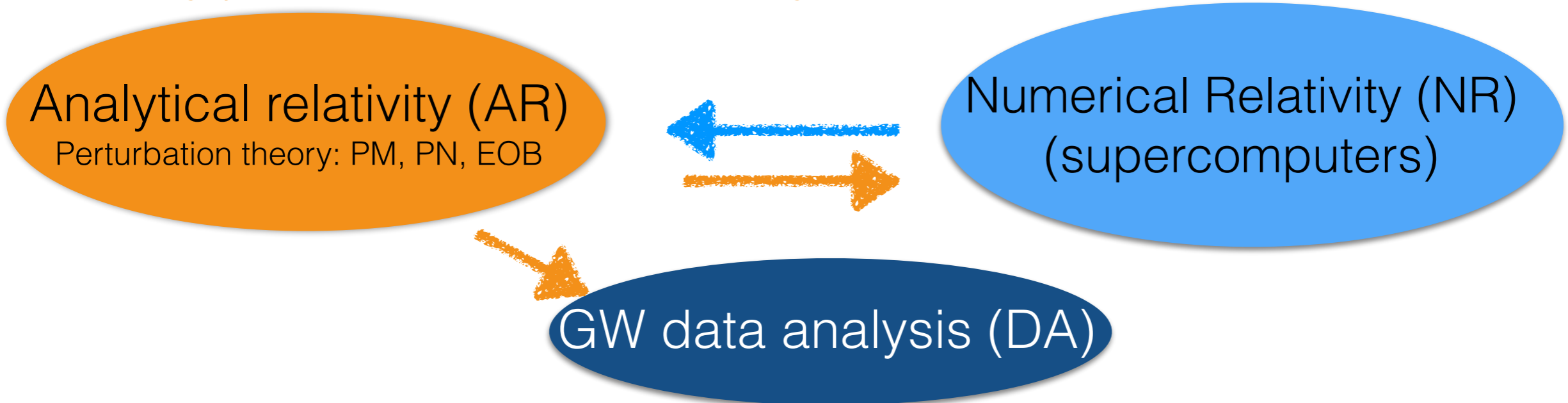
R. Gamba, M. Breschi, G. Carullo, S. Albanesi, P. Rettegno, S. Bernuzzi and A. Nagar,  
Nature Astronomy **7**, 11-17 (2023)

The logo for the VIRGO gravitational wave observatory, featuring a stylized green and blue wave pattern to the left of the word "VIRGO" in a bold, sans-serif font.The logo for INFN (Istituto Nazionale di Fisica Nucleare), consisting of a blue swoosh above the letters "INFN" and the full name "Istituto Nazionale di Fisica Nucleare" in a smaller font below.The logo for IHES (Institut des Hautes Études Scientifiques), featuring the letters "IHES" in blue above a stylized blue graphic of intersecting lines.

Jena University

# The Trinity of GW astronomy

- Compact Binary Coalescence (CBC, i.e. BBHs, BNS, BHNS)
- Analytical Relativity, Numerical Relativity and Data Analysis (AR/NR/DA)
- Waveform TEMPLATES (Theory)
- 2-body problem in General Relativity

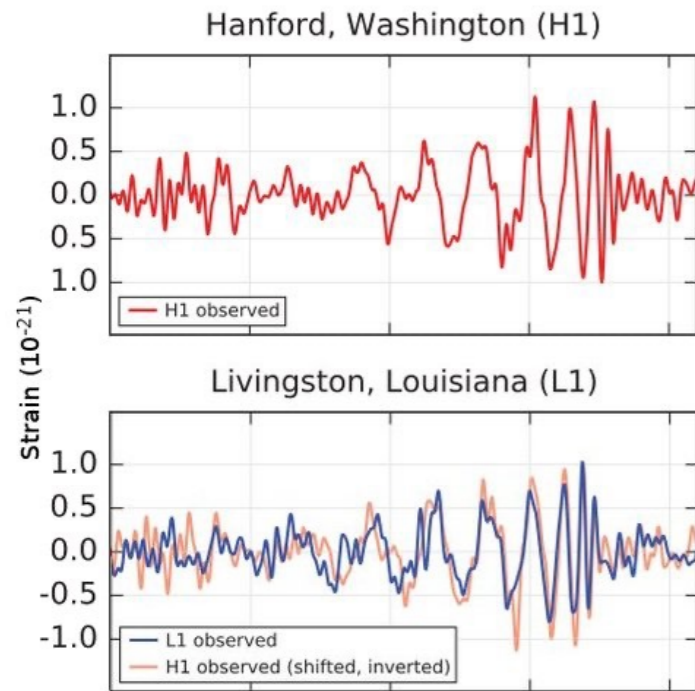


## Challenges:

- physical completeness (spin, tides, eccentricity, hyp)
- accuracy
- efficiency (AR vs NR)
- $10^7$  templates needed for a single event

The physics you can infer depends on your ability at modeling it!

# Why waveform templates?



$$\text{strain} = \frac{\delta L}{L}$$

Symmetric mass ratio

$$\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466$$

GW150914 parameters:

$$m_1 = 35.7 M_\odot$$

$$m_2 = 29.1 M_\odot$$

$$M_f = 61.8 M_\odot$$

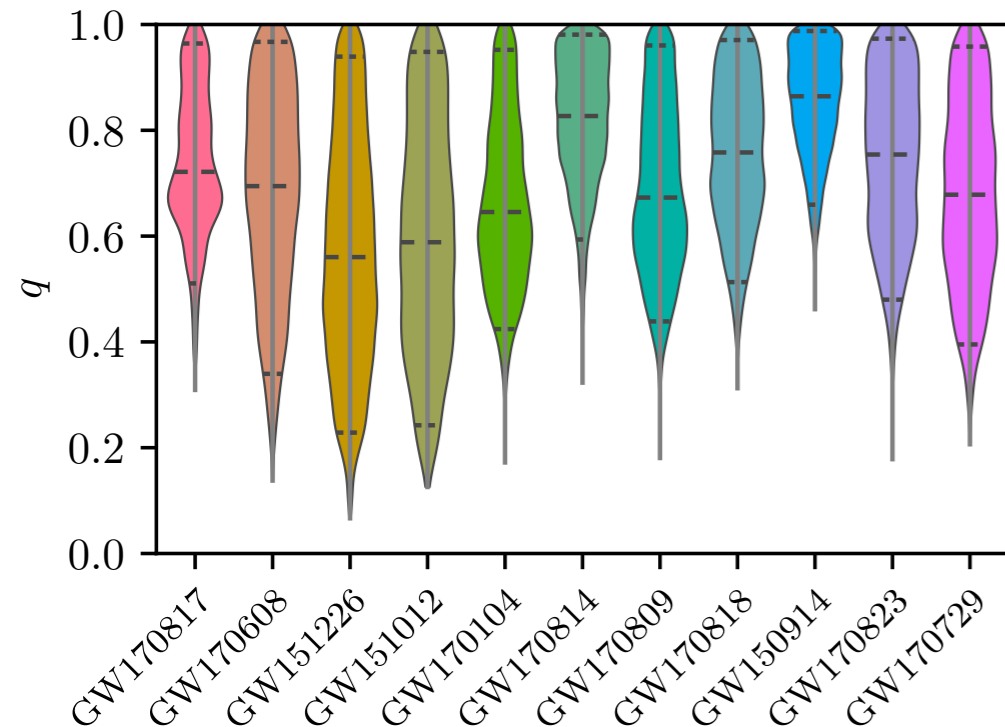
$$a_1 \equiv S_1 / (m_1^2) = 0.31^{+0.48}_{-0.28}$$

$$a_2 \equiv S_2 / (m_2^2) = 0.46^{+0.48}_{-0.42}$$

$$a_f \equiv \frac{J_f}{M_f^2} = 0.67$$

$$q \equiv \frac{m_1}{m_2} = 1.27$$

O2 events: GWTC-1



Matched filtering: detection and parameter estimation

$$\langle \text{output} | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h_{\text{template}}^*(f)$$

Analytical formalism:  
theoretical understanding  
of the coalescence process

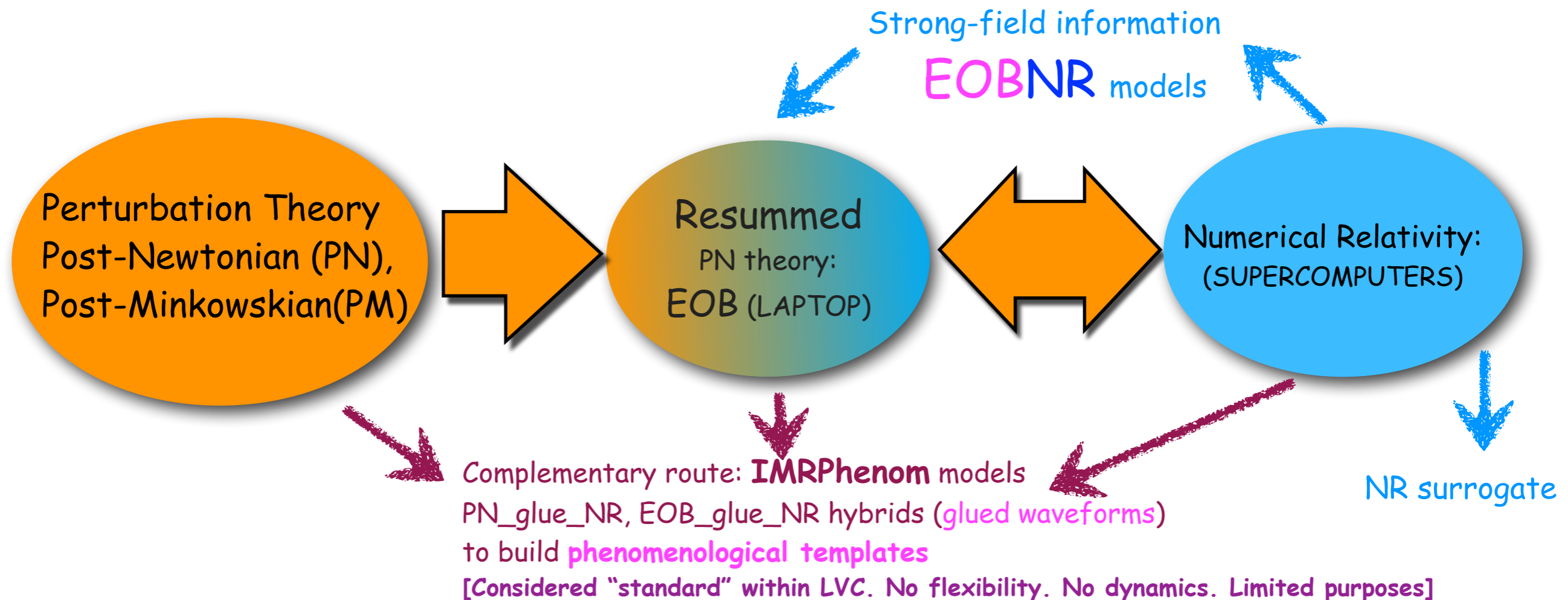
# 2-body problem in GR

**Hamiltonian:** conservative part of the dynamics

**Radiation reaction:** mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) **orbit CIRCULARIZES** and **SHRINKS** with time

## Waveform



# Analytical Effective-One-Body approach

Provides a complete description of dynamics and radiation from relativistic binaries  
(Recent development and connection with scattering amplitudes)

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

- (1) Replace two-body dynamics  $(m_1, m_2)$  by dynamics of a particle  $(\mu \equiv m_1 m_2 / (m_1 + m_2))$  in an effective metric  $g_{\mu\nu}^{\text{eff}}(u)$ , with

$$u \equiv GM/c^2 R, \quad M \equiv m_1 + m_2$$

- (2) Systematically use **RESUMMATION** of PN expressions (both  $g_{\mu\nu}^{\text{eff}}$  and  $\mathcal{F}_{RR}$ ) based on various physical requirements

- (3) Require **continuous deformation w.r.t.**

$$\nu \equiv \mu/M \equiv m_1 m_2 / (m_1 + m_2)^2 \text{ in the interval } 0 \leq \nu \leq \frac{1}{4}$$

$$A_{5\text{PN}}^{\text{Taylor}} = \underbrace{1}_{1\text{PN}} - \underbrace{2u}_{2\text{PN}} + \underbrace{2\nu u^3}_{3\text{PN}} + \underbrace{\left(\frac{94}{3} - \frac{41}{32}\pi^2\right)}_{3\text{PN}} \nu u^4 + \underbrace{\nu[a_5^c(\nu) + a_5^{\text{ln}} \ln u]}_{4\text{PN}} u^5 + \underbrace{\nu[a_6^c(\nu) + a_6^{\text{ln}} \ln u]}_{5\text{PN}} u^6$$

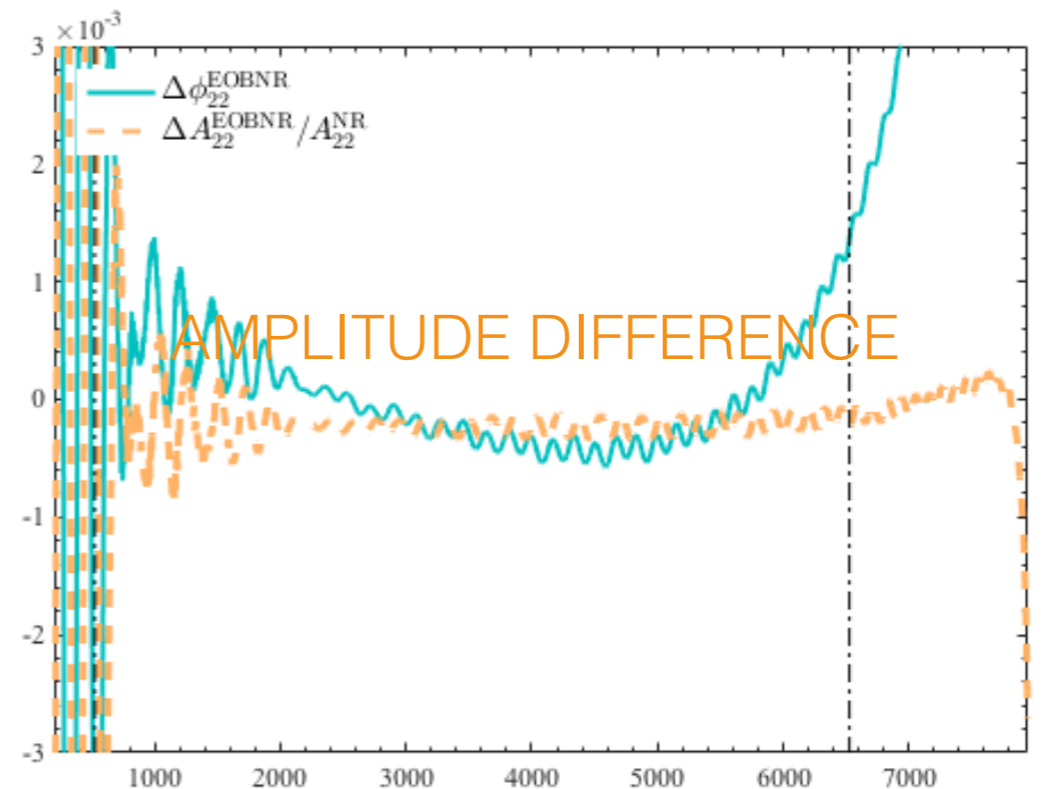
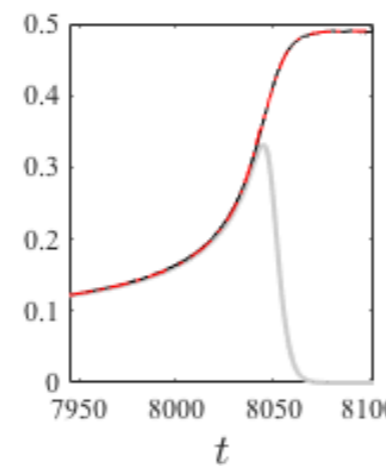
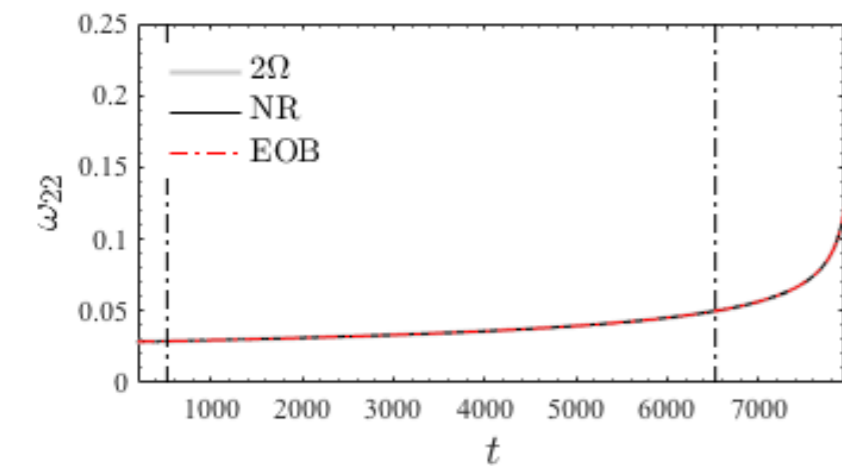
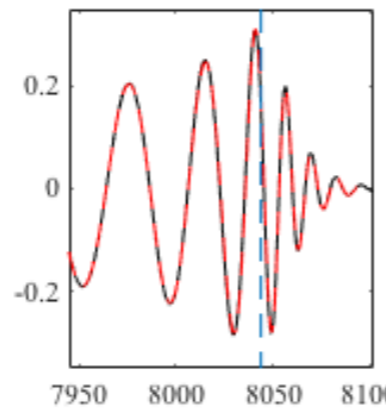
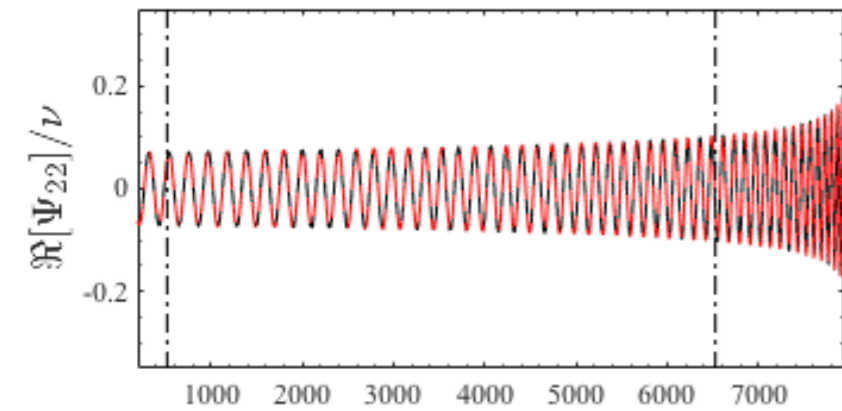
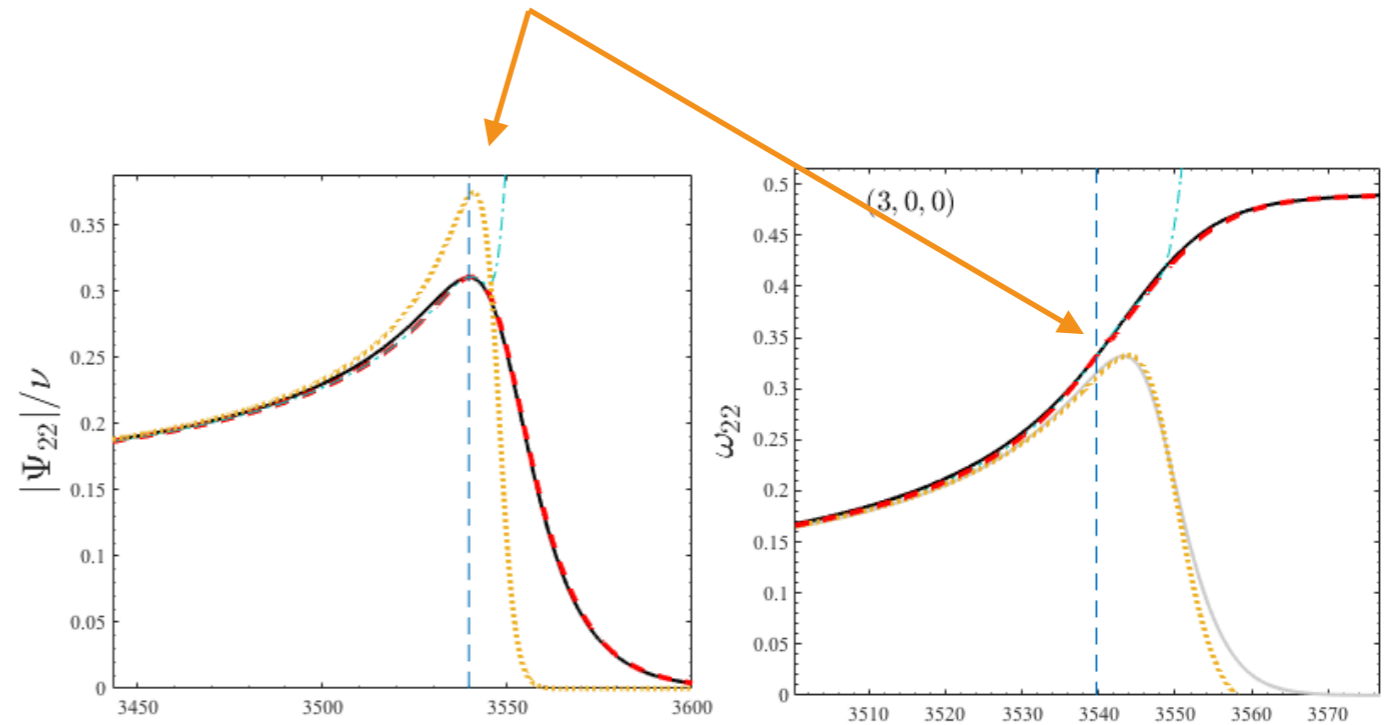
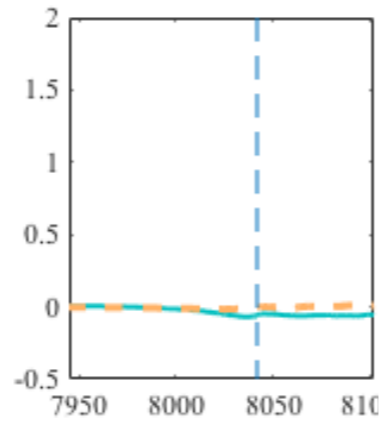
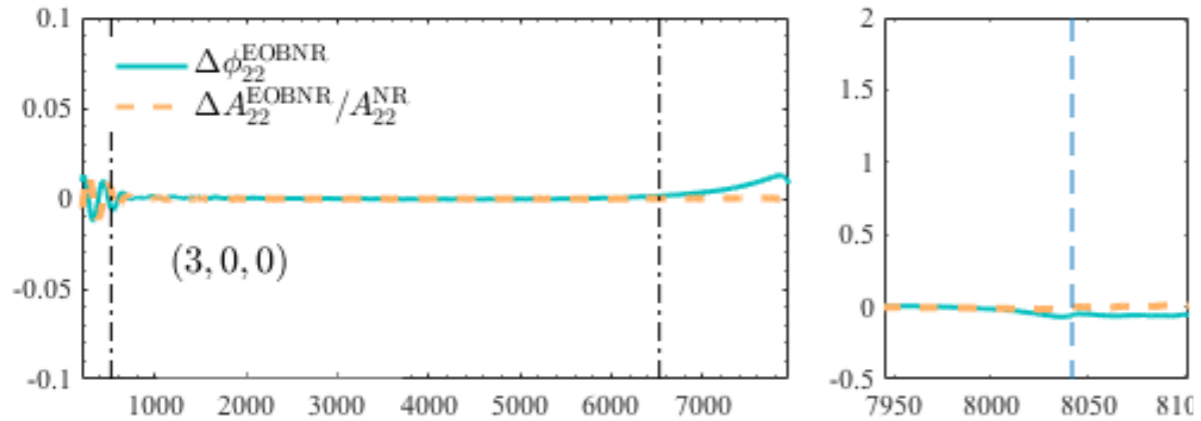


# For example (TEOBResumS vs NR: SXS:1221)

$q=m_1/m_2=3$

MERGER

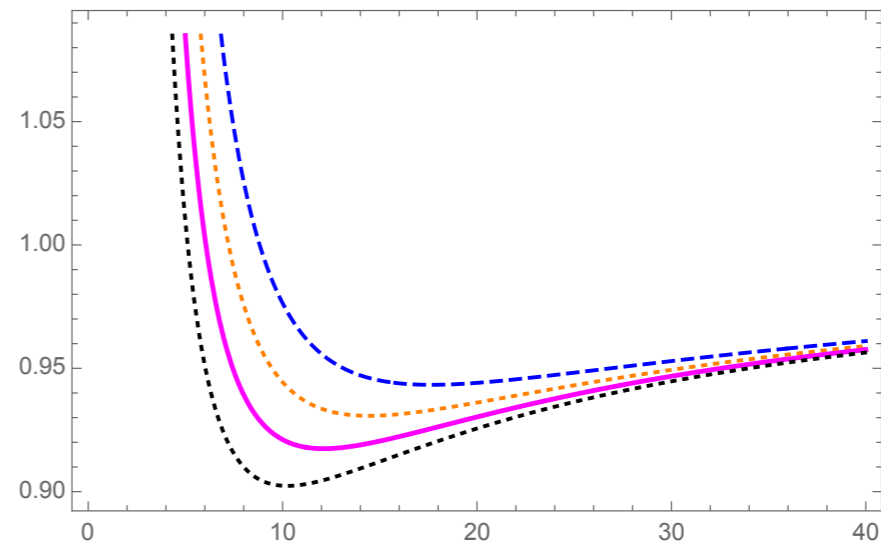
Analytical resummed waveform  
(+ a6c-NR-informed)



# Effective potentials

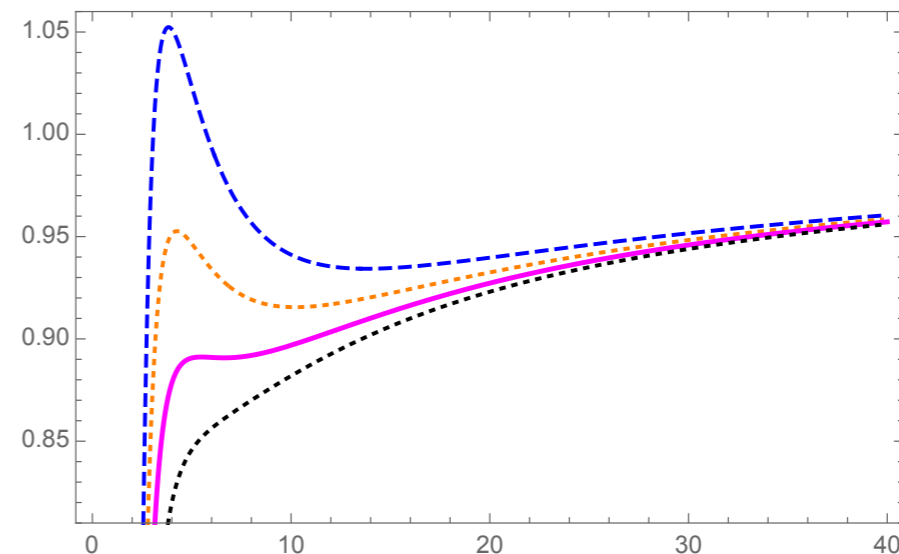
Newtonian gravity (any mass ratio):  
circular orbits are always stable. No plunge.

$$W_{\text{Newt}}^{\text{eff}} = 1 - \frac{2}{r} + \frac{p_{\varphi}^2}{r^2}$$



Test-body on Schwarzschild black hole:  
last stable orbit (LSO) at  $r=6M$ ; plunge

$$W_{\text{Schwarzschild}}^{\text{eff}} = \left(1 - \frac{2}{r}\right) \left(1 + \frac{p_{\varphi}^2}{r^2}\right)$$



**EOB**, Black-hole binary, any mass ratio:  
last stable orbit (LSO) at  $r < 6M$  plunge

$$W_{\text{EOB}}^{\text{eff}} = A(r; \nu) \left(1 + \frac{p_{\varphi}^2}{r^2}\right)$$

$\mathcal{V}$ -deformation of the Schwarzschild case!

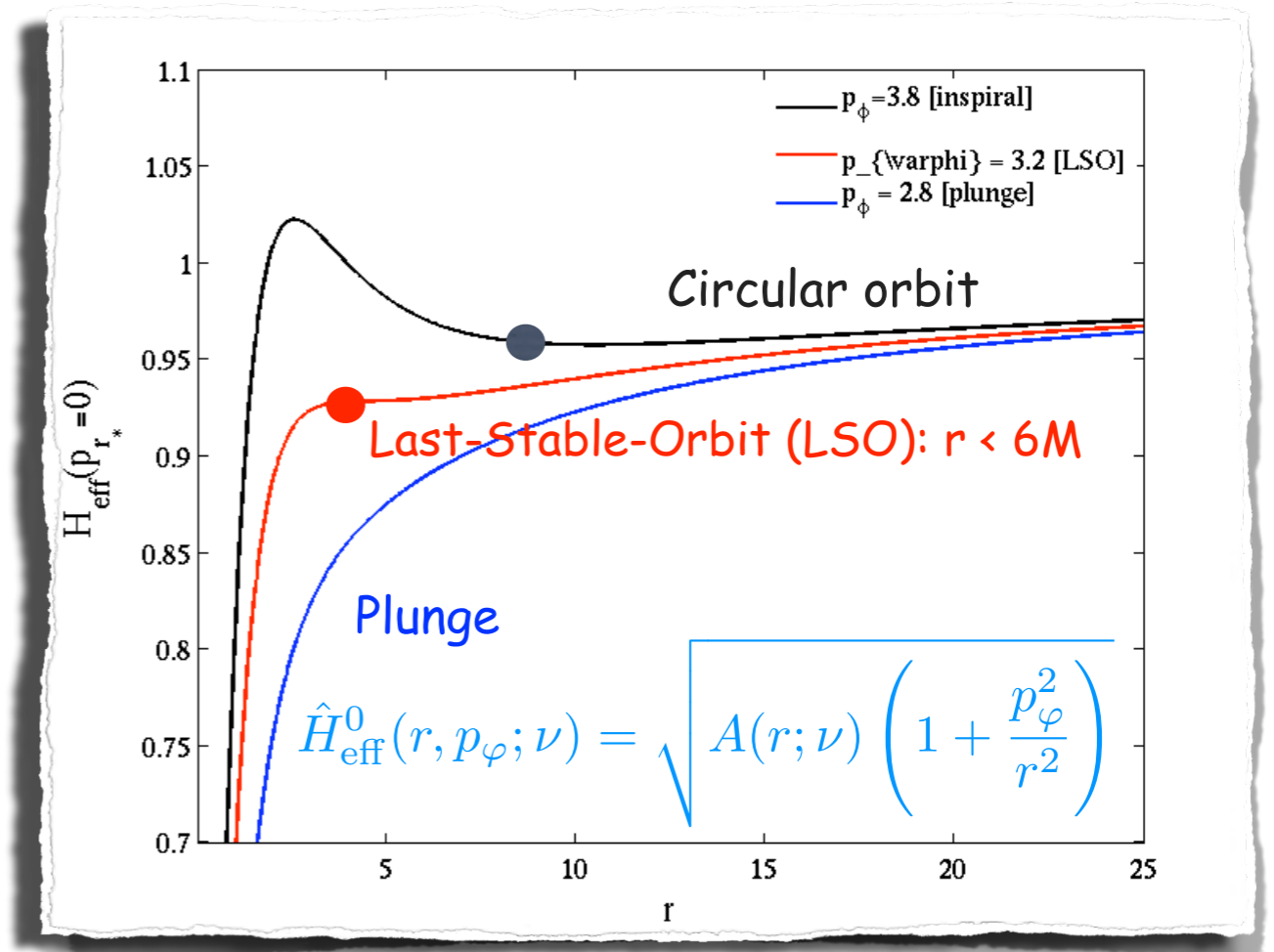
# Hamilton's equations & radiation reaction

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_{\varphi}} \equiv \Omega$$

$$\dot{p}_{r_*} = - \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_{\varphi} = \hat{\mathcal{F}}_{\varphi}$$



- ▶ The system must radiate angular momentum
- ▶ How? Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- ▶ Need flux resummation

$$\hat{\mathcal{F}}_{\varphi}^{\text{Taylor}} = -\frac{32}{5} \nu \Omega^5 r_{\Omega}^4 \hat{F}^{\text{Taylor}}(v_{\varphi}) \rightarrow$$

Plus horizon contribution [AN&Akcaay2012]

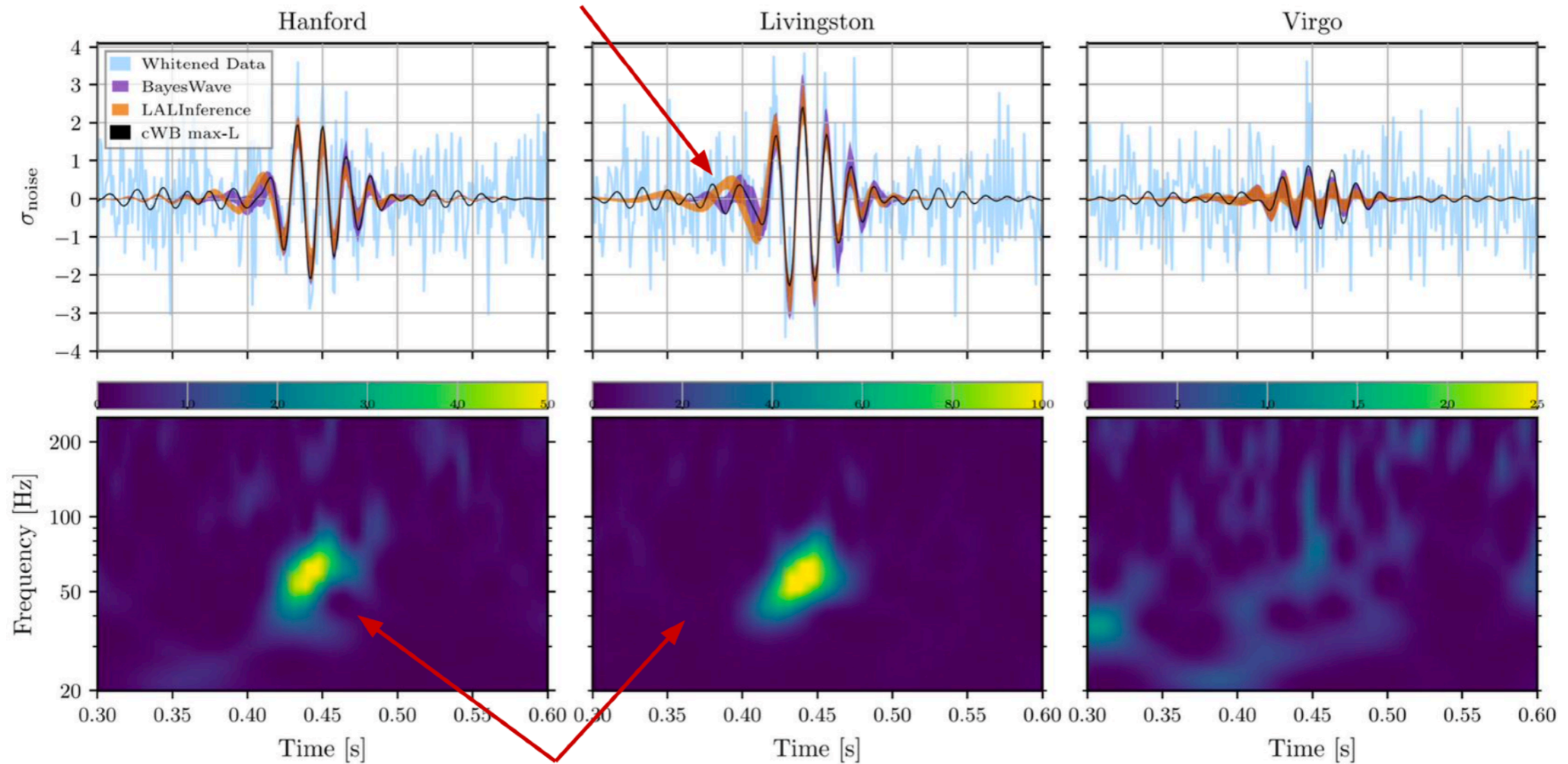
Resummation multipole by multipole  
(Damour&Nagar 2007,  
Damour, Iyer & Nagar 2008,  
Damour & Nagar, 2009)



# GW190521: A Binary Black Hole Merger with a Total Mass of $150 M_{\odot}$

## GW190521

just 3-4 cycles total,  
~1+ringdown?



No real “chirp”, frequency  $\sim 50$ -100 Hz

estimated false-alarm rate of 1 in 4900 yr using a search sensitive to generic transients. If GW190521 is from a quasicircular binary inspiral, then the detected signal is consistent with the merger of two black holes with masses of  $85^{+21}_{-14} M_{\odot}$  and  $66^{+17}_{-18} M_{\odot}$  (90% credible intervals). We infer that the primary black

# GW190521

**GW190521: A Binary Black Hole Merger with a Total Mass of  $150 M_{\odot}$**

R. Abbott *et al.*<sup>\*</sup>  
(LIGO Scientific Collaboration and Virgo Collaboration)

Quasi-circular BBH, large in-plane spins

Highly eccentric BBH, large in-plane spins

**GW190521 as a Highly Eccentric Black Hole Merger**

V. Gayathri,<sup>1</sup> J. Healy,<sup>2</sup> J. Lange,<sup>2</sup> B. O'Brien,<sup>1</sup> M. Szczepańczyk,<sup>1</sup> I. Bartos,<sup>1,\*</sup> M. Campanelli,<sup>2</sup> S. Klimentko,<sup>1</sup> C. Lousto,<sup>2</sup> and R. O'Shaughnessy<sup>2,†</sup>

**GW190521 as a merger of Proca stars:  
a potential new vector boson of  $8.7 \times 10^{-13}$  eV**

Juan Calderón Bustillo,<sup>1,2,3,4,\*</sup> Nicolas Sanchis-Gual,<sup>5,6,†</sup> Alejandro Torres-Forné,<sup>7,8,9</sup> José A. Font,<sup>8,9</sup> Avi Vajpeyi,<sup>3,4</sup> Rory Smith,<sup>3,4</sup> Carlos Herdeiro,<sup>6</sup> Eugen Radu,<sup>6</sup> and Samson H. W. Leong<sup>2</sup>

Proca stars

High mass ratio BBH ( $q \sim 5-10$ ), plus large in-plane spins

**GW190521 may be an intermediate mass ratio inspiral**

ALEXANDER H. NITZ<sup>1,2</sup> AND COLLIN D. CAPANO<sup>1,2</sup>

...and more!

# Effective-one-body waveforms from dynamical captures in black hole binaries

Alessandro Nagar<sup>1,2</sup>, Piero Rettegno<sup>1,3</sup>, Rossella Gamba<sup>4</sup>, and Sebastiano Bernuzzi<sup>4</sup>

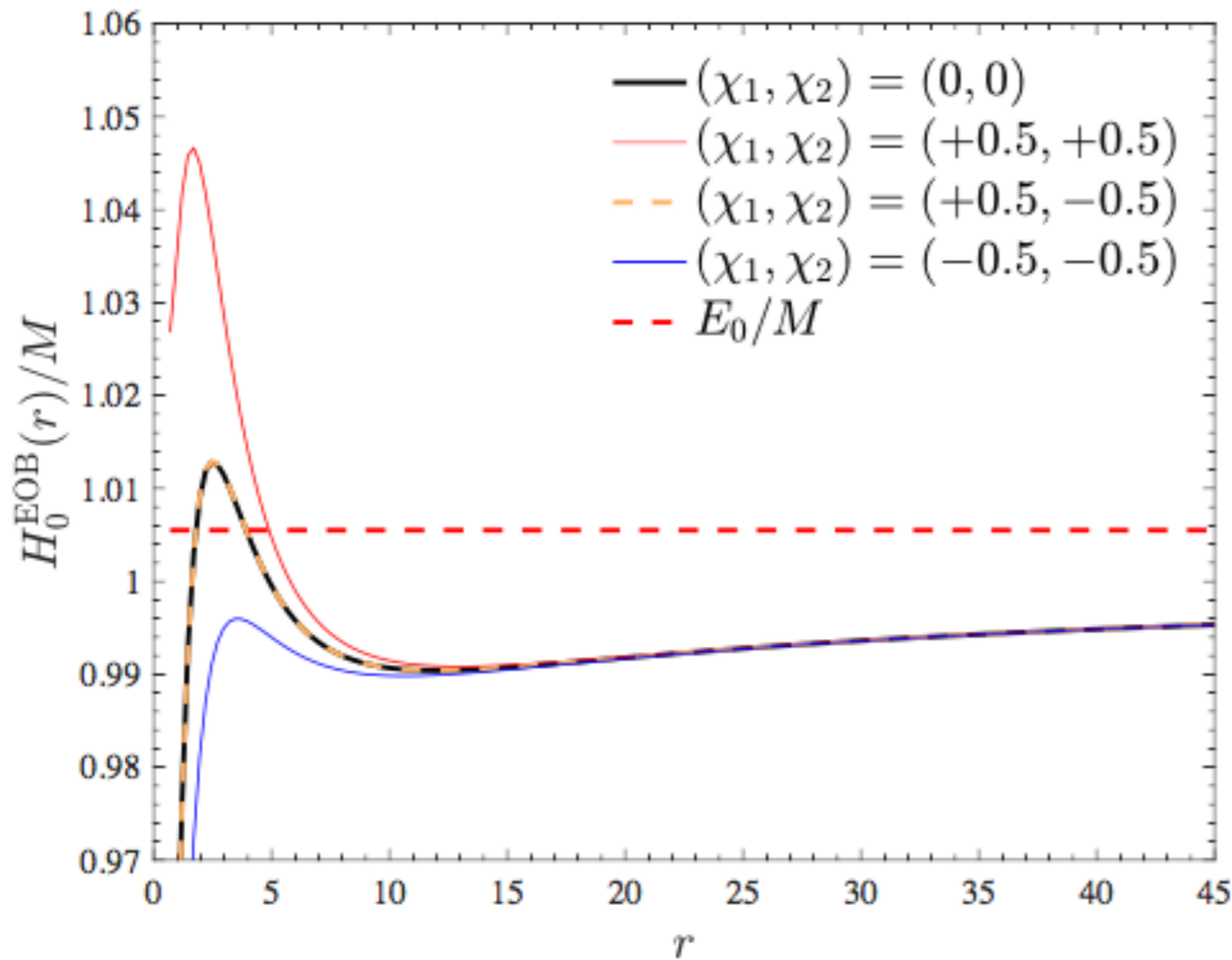
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<sup>4</sup> *Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität Jena, 07743, Jena, Germany*

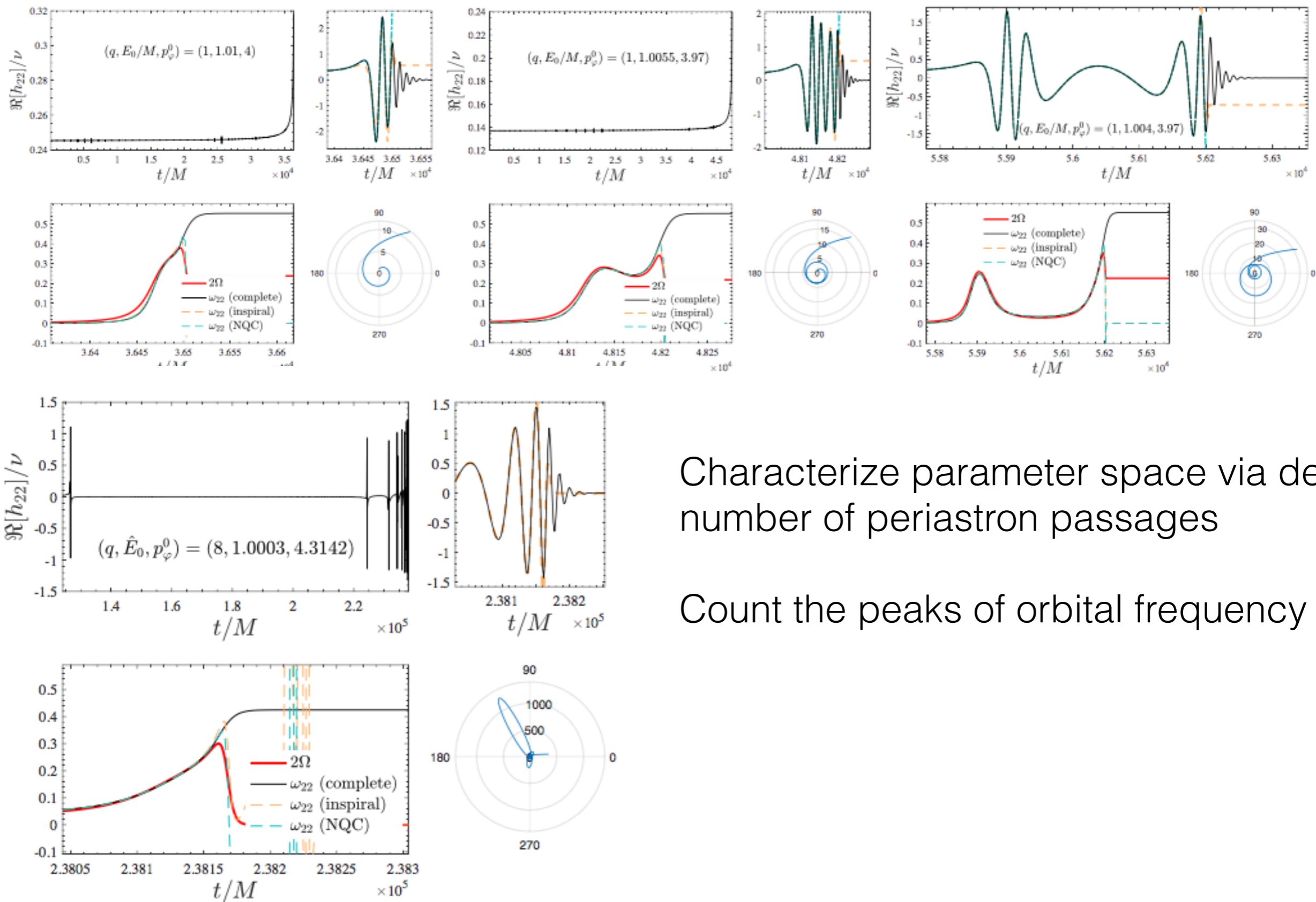
(Dated: June 17, 2022)



Chiamarello, D. & Nagar, A. Faithful analytical effective-one-body waveform model for spin-aligned, moderately eccentric, coalescing black hole binaries. *Phys. Rev. D.* **101**, 101501 (2020).  
Nagar, A., Rettegno, P., Gamba, R. & Bernuzzi, S. Effective-one-body waveforms from dynamical captures in black hole binaries. *Phys. Rev. D.* **103**, 064013 (2021).



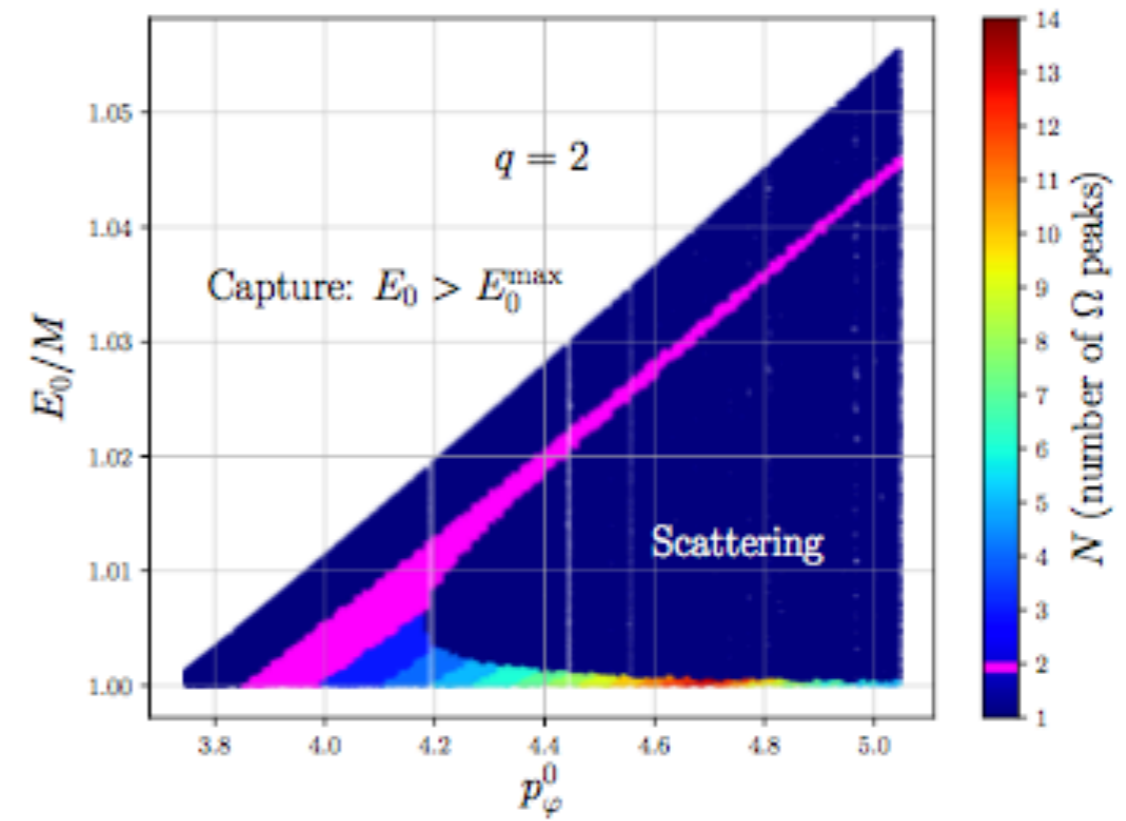
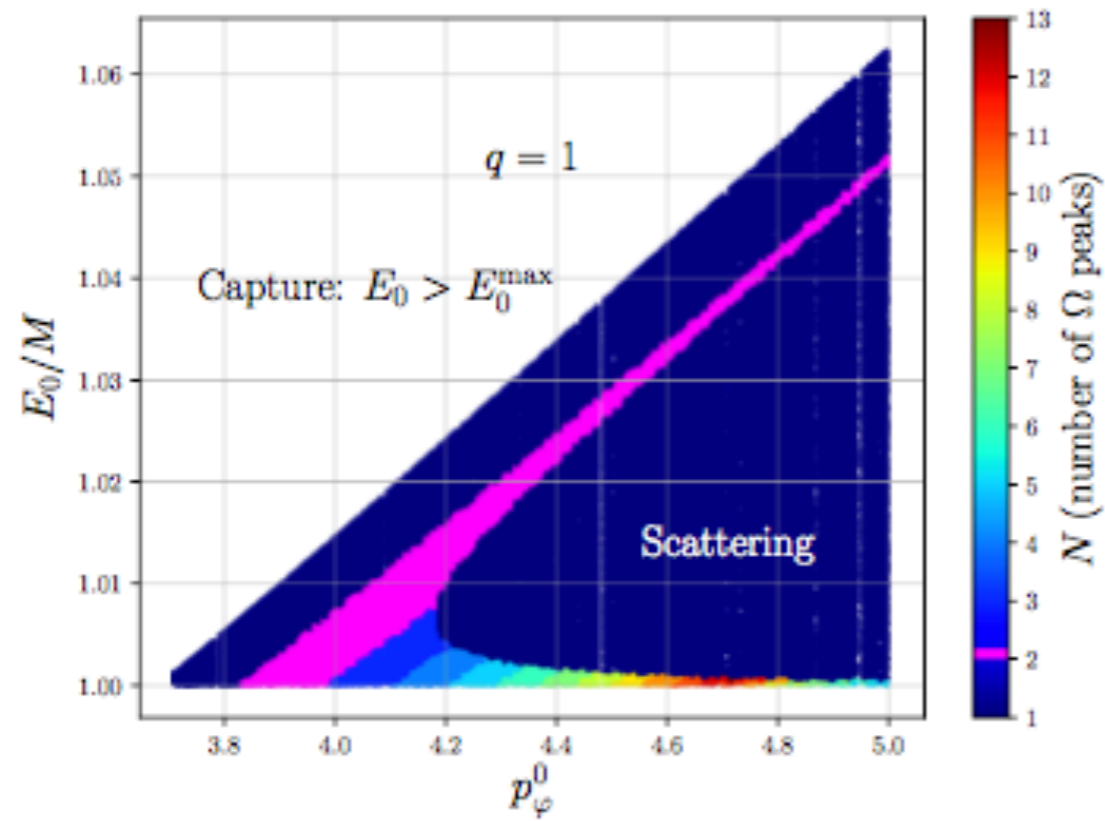
# Analytic predictions



Characterize parameter space via de number of periastron passages

Count the peaks of orbital frequency

# Parameter space



Parameter space characterized by energy and angular momentum







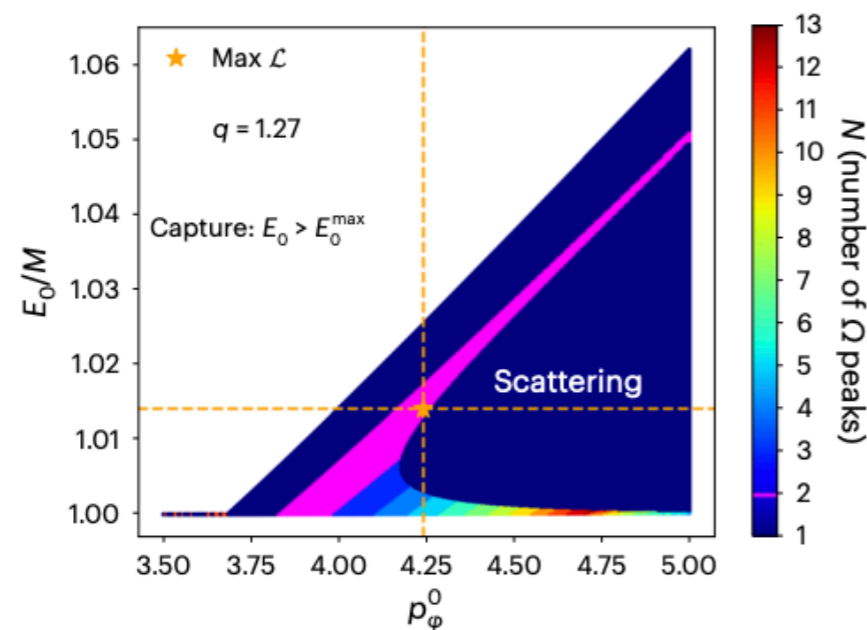
# GW190521 as a dynamical capture of two nonspinning black holes

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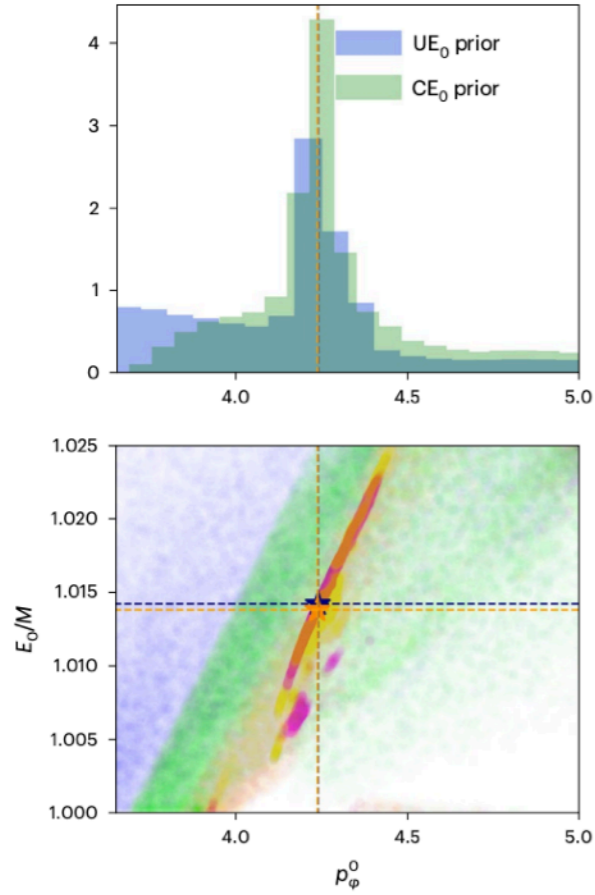
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 R. Gamba <sup>1</sup>, M. Breschi <sup>1</sup>, G. Carullo <sup>1,2,3</sup>, S. Albanesi <sup>4,5</sup>, P. Rettegno <sup>4,5</sup>,  
 S. Bernuzzi<sup>1</sup> & A. Nagar <sup>5,6</sup> ✉


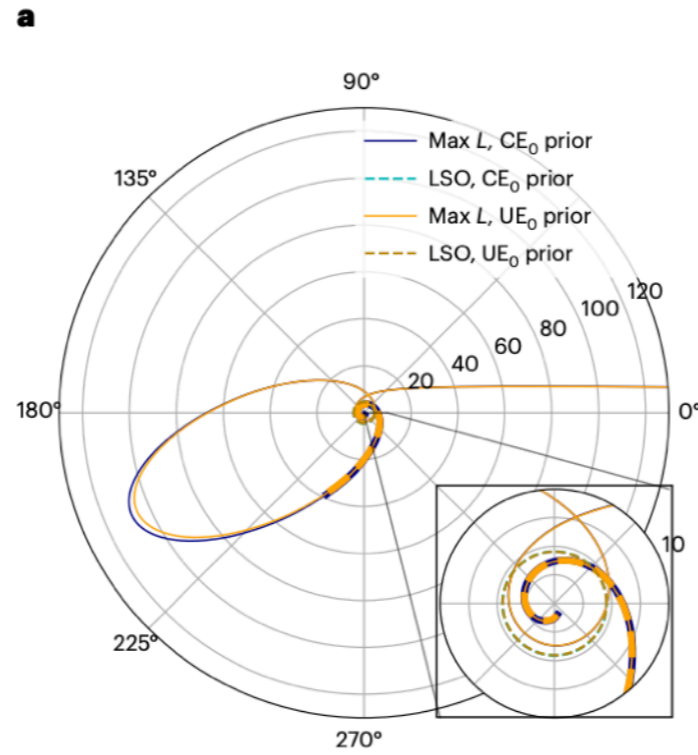
**Fig. 1 | Number of encounters as a function of the initial energy and angular momentum.** Parameter space for nonspinning hyperbolic encounters predicted using the TEOBResumS EOB model and fixing  $q \equiv m_1/m_2 = 1.27$ . Here  $(p_\phi^0, E_0)$  are the EOB initial angular momentum and energy, respectively, while  $E_0^{\max}$  is the value corresponding to unstable circular orbit. For  $E_0 < E_0^{\max}$ , the colour scale labels the number of peaks (that is, of periastron passages)  $N$  of the EOB orbital frequency  $\Omega$ . The orange star and dashed lines label the maximum likelihood values  $(\bar{p}_\phi^0, \bar{E}_0)$  corresponding to the constrained analysis (Table 1).

Gravitational waves from ~90 black hole binary systems have been detected and their progenitors' properties inferred<sup>1</sup> so far by the Laser Interferometer Gravitational-Wave Observatory<sup>2</sup> and Virgo<sup>3</sup> experiments. This has allowed the scientific community to draw conclusions on the formation channels of black holes in binaries, informing population models and at times defying our understanding of black hole astrophysics. The most challenging event detected so far is the short-duration gravitational-wave transient GW190521 (refs. 4,5). We analyse this signal under the hypothesis that it was generated by the merger of two nonspinning black holes on hyperbolic orbits. The configuration best matching the data corresponds to two black holes of source-frame masses of  $81_{-25}^{+62} M_\odot$  and  $52_{-32}^{+32} M_\odot$  undergoing two encounters and then merging into an intermediate-mass black hole. We find that the hyperbolic merger hypothesis is favoured with respect to a quasi-circular merger with precessing spins with Bayes' factors larger than 4,300 to 1, although this number will be reduced by the currently uncertain prior odds. Our results suggest that GW190521 might be the first gravitational wave detection from the dynamical capture of two stellar-mass nonspinning black holes.

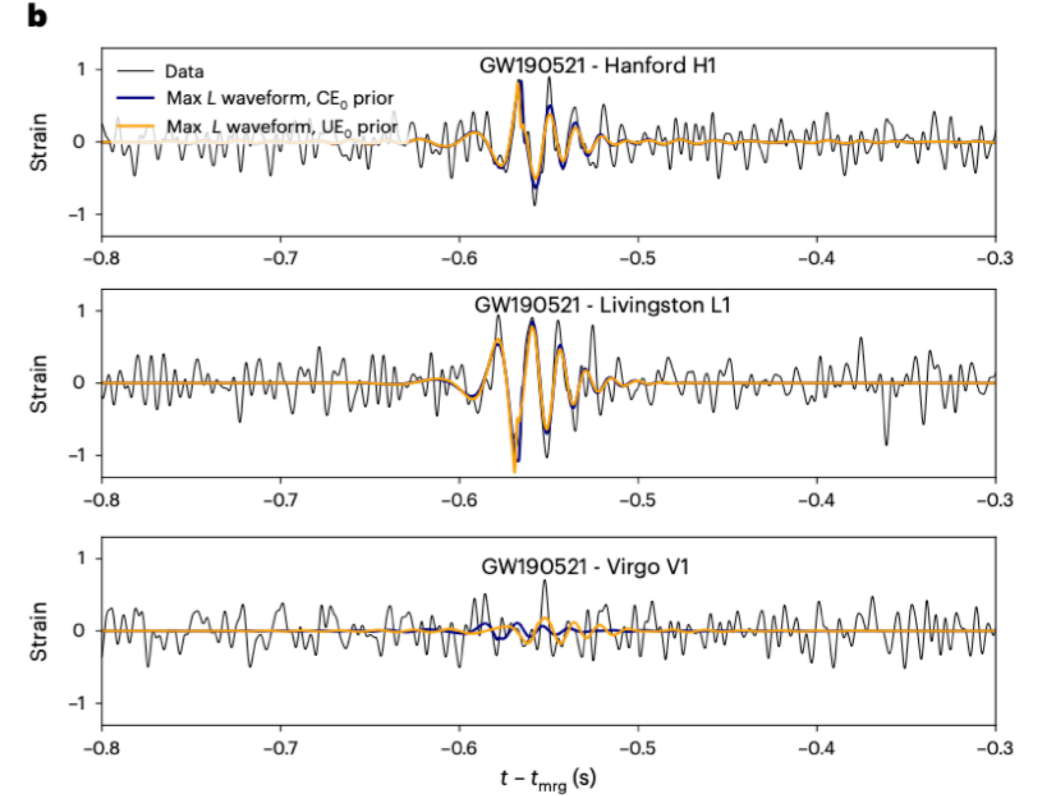




**Fig. 2 | Energy–angular momentum marginalized two-dimensional posterior.** Marginalized two-dimensional posterior distributions of the initial energy  $E_0$  and initial angular momentum  $p_\phi^0$  for the constrained ( $CE_0$ ) and unconstrained ( $UE_0$ ) energy prior choices. The colours highlight the different waveform



**Fig. 3 | Maximum likelihood (Max  $L$ ) configurations with the two different energy priors.** Unconstrained ( $UE_0$ ) and constrained ( $CE_0$ ) energy priors are shown in shades of orange and blue, respectively. **a**, The  $(r, \varphi)$  EOB relative orbit. **b**, The waveform templates projected onto the three detectors compared with the whitened LIGO-Virgo data (thin black line) around the time of GW190521. The most probable last stable orbits are highlighted with gold ( $UE_0$  prior) and cyan ( $CE_0$  prior) dashed lines and are located, respectively, at  $\bar{r}_{\text{LSO}} = 4.54$  and  $\bar{r}_{\text{LSO}} = 4.52$ . Corresponding mass ratios are  $\bar{q} = 1.04$  and  $\bar{q} = 1.27$ . The inset



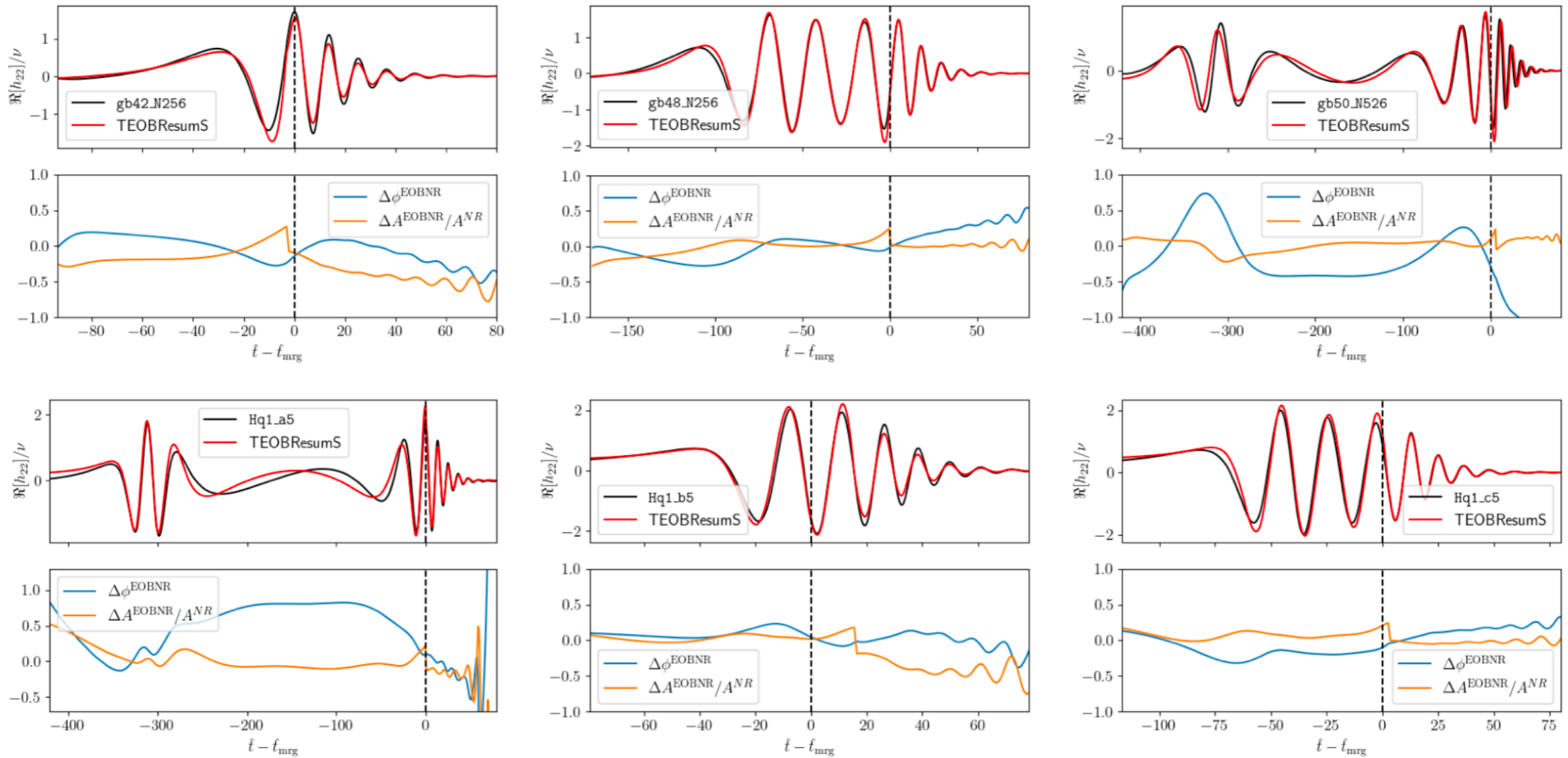
highlights the first close encounter, which is then followed by a highly eccentric orbit that eventually ends up with a plunge and merger phase. The part of the trajectory from  $-(t_{\text{GPS}} - 0.8)$  s to merger time  $t_{\text{mrg}}$ , which contributes to the second GW burst, is highlighted with thicker lines in the plot. Note that the GW bursts corresponding to the first encounter occur  $\sim 4$  s before the GW190521 time; their magnitude is comparable to the detector noise, and they are outside the segment of data analysed.

**Table 1 | Source parameters of GW190521 with median values and 90% credible intervals quoted and natural logarithms reported**

Reference <sup>a</sup>	This paper					LVK <sup>4</sup>	Gayathri et al. <sup>15</sup>	Romero-Shaw et al. <sup>16</sup>
Waveform	TEOBResumS <sup>30,31</sup>	TEOBResumS <sup>30,31</sup>	TEOBResumSP <sup>44b</sup>	NRSur7dq4 (ref. <sup>42</sup> )	NRSur7dq4 (ref. <sup>42</sup> )	NRSur7dq4 (ref. <sup>42</sup> )	NR <sup>47</sup>	SEOBNRE <sup>52</sup>
$E_0$ prior	Unconstrained (UE <sub>0</sub> )		Constrained (CE <sub>0</sub> )	–	–	–	–	–
Multipoles	$(\ell,  m )=(2, 2)$		$(\ell,  m )=(2, 2)$	$(\ell,  m )=(2, 2)$	$(\ell,  m )=(2, 2)$	$\ell \leq 4$	$\ell \leq 4$	–
$m_1 (M_\odot)$	$85^{+88}_{-22}$	$81^{+62}_{-25}$	$90^{+19}_{-14}$	$102^{+35}_{-23}$	$84^{+17}_{-12}$	$85^{+21}_{-14}$	$102^{+7}_{-11}$	$92^{+26}_{-16}$
$m_2 (M_\odot)$	$59^{+18}_{-37}$	$52^{+32}_{-32}$	$66^{+10}_{-8}$	$64^{+19}_{-25}$	$71^{+16}_{-18}$	$66^{+17}_{-18}$	$102^{+7}_{-11}$	$69^{+18}_{-19}$
$M_{\text{source}} (M_\odot)^c$	$151^{+73}_{-51}$	$130^{+75}_{-43}$	$156^{+25}_{-15}$	$164^{+40}_{-23}$	$153^{+29}_{-19}$	$150^{+29}_{-17}$	–	–
$m_2/m_1 \leq 1$	$0.69^{+0.27}_{-0.52}$	$0.63^{+0.31}_{-0.43}$	$0.73^{+0.21}_{-0.15}$	$0.62^{+0.32}_{-0.30}$	$0.86^{+0.12}_{-0.30}$	$0.79^{+0.19}_{-0.29}$	–	–
$\chi_{\text{eff}}^d$	–	–	$-0.05^{+0.09}_{-0.12}$	$0.01^{+0.24}_{-0.26}$	$-0.03^{+0.25}_{-0.26}$	$0.08^{+0.27}_{-0.36}$	0	$0.0^{+0.2}_{-0.2}$
$\chi_p^e$	–	–	$0.72^{+0.16}_{-0.22}$	$0.71^{+0.22}_{-0.36}$	$0.79^{+0.16}_{-0.40}$	$0.68^{+0.25}_{-0.37}$	0.7	–
Eccentricity	–	–	–	–	–	–	0.67	0.11 <sup>f</sup>
$E_0/M$	$1.014^{+0.009}_{-0.012}$	$1.014^{+0.010}_{-0.012}$	–	–	–	–	–	–
$p_\phi^0$	$4.18^{+0.50}_{-0.62}$	$4.24^{+0.57}_{-0.37}$	–	–	–	–	–	–
Luminosity distance $D_L$ (Gpc)	$4.7^{+4.8}_{-2.7}$	$6.1^{+3.3}_{-3.7}$	$4.5^{+1.2}_{-1.2}$	$3.9^{+2.3}_{-1.9}$	$4.8^{+2.3}_{-2.2}$	$5.3^{+2.4}_{-2.6}$	$1.84^{+1.07}_{-0.054}$	$4.1^{+1.8}_{-1.8}$
$\text{SNR}_{\text{max}}$	15.2	15.4	14.7	14.7	14.6	15.4	–	–
$\log(L)_{\text{max}}$	123.2	123.0	106.0	107.0	105.6	–	–	–
$\log B_{\text{noise}}^{\text{signal}}$	$84.00 \pm 0.18$	$83.30 \pm 0.18$	$72.95 \pm 0.08$	$74.76 \pm 0.11$	$74.86 \pm 0.11$	–	–	–

<sup>a</sup>Results of other analyses<sup>4,15,16</sup> are included for reference <sup>b</sup>Spin results obtained at a reference frequency of 5 Hz <sup>c</sup>Total mass in the frame of the source <sup>d</sup>Effective spin along the orbital angular momentum <sup>e</sup>Effective precessing spin <sup>f</sup>Lower limit at 10 Hz

# Model validation: EOB/NR



Consistency with (limited) NR simulations

More comparisons/model improvement is in progress

# Conclusions

The hypothesis of GW190521 as the result of a dynamical capture seems favored with respect to the mainstream hypothesis of a quasi-circular binary with precessing spins

More work on the modeling side is needed, though very challenging.

The interesting part of parameter space is narrow and very sensitive to initial conditions. Corners are difficult to cover (especially less-circular captures)

NR-informed merger-ringdown part is necessary.

Analyses including spins are (hopefully!) in progress